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Numerical Modeling of Dissolved Oxygen Dynamics Using a Finite-Difference Solution to the Streeter-Phelps Equation: Applications to a Small Stream and an Engineered River System

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Numerical Modeling of Dissolved Oxygen Dynamics Using a Finite-Difference Solution to the Streeter-Phelps Equation: Applications to a Small Stream and an Engineered River System

A thesis submitted in partial satisfaction

of the requirements for the degree Master of Science

in Civil Engineering

By

Hanifah Amata Abatcha

2026

ABSTRACT OF THE THESIS

Numerical Modeling of Dissolved Oxygen Dynamics Using a Finite-Difference Solution to the Streeter-Phelps Equation: Applications to a Small Stream and an Engineered River System

by

Hanifah Amata Abatcha

Master of Science in Civil Engineering

University of California, Los Angeles, 2026

Professor Regan Patterson, Chair

Dissolved Oxygen (DO) is an important indicator of stream health, and modeling dissolved oxygen (DO) dynamics can provide insight into the processes governing oxygen availability in urban waterways. The Streeter-Phelps model (SPM) provides a foundational framework for representing DO dynamics, but its simplified formulation may not fully represent heterogeneous urban river systems. This study evaluates a finite-difference implementation of the SPM using field measurements from the UCLA Botanical Garden stream and characterizes preliminary hydraulic and water quality conditions in the Los Angeles River to inform future model application. Sensitivity analysis of the Botanical Garden model revealed that DO predictions were most sensitive to reaeration, highlighting the importance of hydraulic conditions in governing modeled DO dynamics. The final model reproduced observed DO values with an

RMSE of 0.3364, however, achieving this fit required a high initial BOD value of 50 mg/L, which is not representative of the study system. The baseline model, using an initial BOD of 2.26 mg/L, did not reproduce the observed variability in the field data. Increasing the initial BOD improved the modeled DO profile, but could not fully account for the observed DO dynamics without reaching physically unrealistic concentrations. These findings suggest that additional oxygen sink terms, such as sediment oxygen demand and photosynthesis and respiration, are needed to better represent the distributed oxygen demand within the stream. Preliminary measurements from the LA River highlight the need for spatially resolved monitoring and site-specific model parameterization in future applications. Future modeling of the LA River should also consider more rigorous BOD measurements and additional oxygen sink terms. Overall, the findings demonstrate both the utility and limitations of applying a simplified mechanistic framework to evaluate DO dynamics in heterogeneous urban stream systems.

The thesis of Hanifah Amata Abatcha is approved.

Michael K. Stenstrom

Mekonnen Gebremichael

Regan Patterson, Committee Chair

University of California, Los Angeles

2026

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Acknowledgements

I would first like to thank Dr. Regan F. Patterson for her continued support and guidance in navigating this thesis. Her mentorship proved vital for navigating communication among the various partnerships and remaining organized and focused on the research path.

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1 Introduction

1.1 Background

Despite variability in hydraulics and boundary conditions, dissolved oxygen (DO) remains an effective indicator of a stream's ability to support aquatic life and maintain aesthetic quality. DO is controlled by the balance between oxygen-consuming processes, such as microbial oxidation of organic matter, and atmospheric reaeration. The Streeter-Phelps mass balance provides a framework for modeling these dynamics. It assumes steady-state, one-dimensional flow and represents oxygen sinks and reaeration through simplified terms and empirical formulations. However, its conceptual simplicity makes it an effective foundation for extending reaeration and deoxygenation parameters.

The UCLA Botanical Garden is home to a stream that is the center of the garden. It consists of a series of reaches and pools of varying sizes with water pumped as inflow, and provides a relatively controlled urban stream system in which to evaluate the behavior and limitations for the Streeter-Phelps framework. In contrast, the Sepulveda Basin reach of the Los Angeles River represents a larger riparian urban system influenced by variable hydraulic conditions, tributary inputs, stormwater runoff, and treated effluent. Both systems are home to naturalized vegetation, wildlife, and sediment lined river beds.

1.2 Research Motivation

This study aims to develop and evaluate the Streeter–Phelps model in two contrasting urban stream environments: the UCLA Botanical Garden, which provides relatively controlled conditions for examining model behavior and parameter sensitivity, and the Sepulveda Basin reach of the Los Angeles River, which represents a larger and more complex urban waterway. Specifically the objectives are to: 1) formulate and evaluate a finite-difference implementation of the Streeter-Phelps model using the UCLA Botanical Garden stream; (2) assess model performance using in-situ DO measurements and identify structural limitations in representing observed DO dynamics; and, (3) apply the modeling framework to the more hydraulically and environmentally complex Los Angeles River to assess its applicability under less controlled conditions and identify modifications needed for future application.

2 Literature Review

2.1 Dissolved Oxygen Modeling in Streams

2.1.1 Streeter - Phelps Foundations

Dissolved Oxygen (DO) is often used as an indicator of water quality in surface waters because oxygen supports biological processes, maintains ecological balance, and indicates a surface water source's ability to self-purify after being contaminated with pollutants (Wu and Yu, 2021). Low levels are often associated with unbalanced ecosystems, fish mortality, aesthetic annoyances, and more (Thomann, 1987). This makes DO a tangential indicator of general health in aquatic systems. In an age where anthropogenic discharge tends to accumulate in water systems due to developing waste management practices, understanding the history and trajectory of water quality assessment is paramount to future decision making.

As a result, water quality monitoring and, by extension, modeling are useful tools for investigating water quality trends. This paper explores a classical approach to water quality assessment, the Streeter-Phelps (SP) model (SPM). Streeter and Phelps compiled a mass balance that describes the change from the current dissolved oxygen level in the stream relative to the saturated DO concentration (the deficit) as a function of the rate at which oxygen is replenished in the water body and the amount of dissolved oxygen that microorganisms would consume to oxidize biodegradable organic matter – the biochemical oxygen demand (BOD). An increase in the decay of organic matter increases the DO deficit while an increase in reaeration decreases the deficit. The model outputs the DO sag curve, a representation of the change in DO from a point source of pollution. From the conceptual model of the sag curve in Figure 1, DO starts at a saturation value and has the characteristic dip once a point source of pollution is introduced as microorganisms consume dissolved oxygen to oxidize organic matter; the decay is far faster than the reaeration of the system so it continues to dip until much of the organic matter has degraded, and reaeration has sufficiently replenished the system. At this point, the DO sag curve rises to reflect the increase in DO.

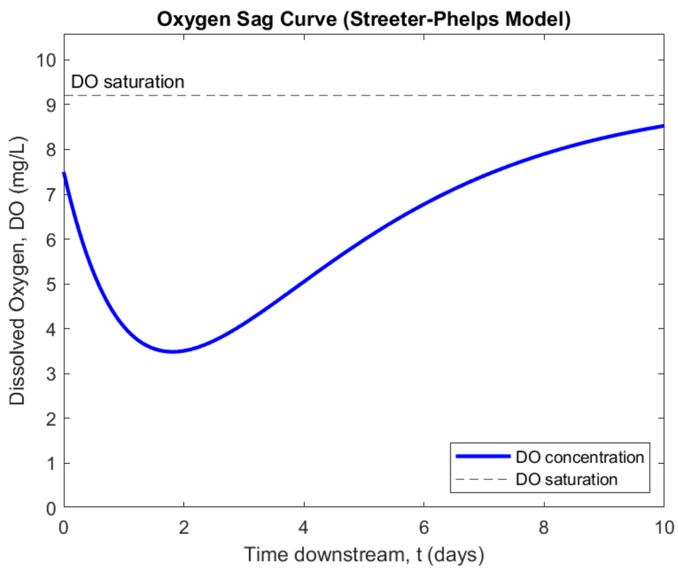

Figure 1. *Oxygen sag curve generated using the Streeter-Phelps model, showing dissolved oxygen (DO) concentration (blue) declining after pollutant discharge and recovering as reaeration dominates. The dashed line indicates the DO saturation level.*

2.1.2 Assumptions and Limitations

Although models are not a comprehensive view of a system, information about system processes can be estimated through appropriate assumptions and considerations; models represent the underlying physics, chemistry, and biology of a system. This model assumes steady-state, one-dimensional flow which neglects lateral and vertical variations in DO concentration that can be significant in more hydraulically complex water bodies (Thomann, 1987). This assumption excludes time-varying inputs such as diurnal photosynthesis and respiration by aquatic plants, non-point source pollution loadings, and seasonal fluctuations in temperature and streamflow, all of which meaningfully alter the shape and magnitude of the DO sag curve. The model further assumes that BOD decay and reaeration are the only significant processes governing the oxygen deficit (Thomann, 1987). Additionally, the deoxygenation and reaeration coefficients are treated as constant over time and uniform over space despite being sensitive to spatially specific hydraulic conditions.

As a result, this literature review not only examines determination, calibration, and validation of the parameters, DO sinks, and sources, but also describes the science that motivates the resulting decisions. Research regarding the modification and application of the SPM can be divided into two categories: the addition of hydraulic modeling to devise more representative reaeration constants and the addition of oxygen sinks. Researchers have also been known to conduct parameter calibration on the original SP mass balance system (Gotovtsev, 2010).

2.1.3 Historical Development

The SP equation was first used by Streeter and Phelps in 1925 to manage water quality in the Ohio River (Streeter and Phelps, 1925). Researchers continued to advance the equation's applicability, but it wasn't until the 1970s that it saw significant practical use (Lung, 2023). Initially, water pollution control efforts were based on achieving ambient water quality standards, which was economically efficient but difficult to administer in practice, since translating ambient standards into discharge limits for individual dischargers required field data that was often unavailable (Lung, 2023). In 1972, Congress passed the Clean Water Act, which implemented a minimum level of pollution control based on available wastewater treatment technology, applied uniformly regardless of a receiving water's specific characteristics (Lung, 2023). The technology-based approach did not account for existing pollution levels in the receiving stream; instead, it focused solely on a treatment plant's capacity to treat wastewater to a fixed standard. When technology-based controls alone proved insufficient to meet water quality standards, the focus shifted to a stream's assimilative capacity; how much additional pollution it could absorb while still meeting water quality standards. This shift led to more advanced water-quality-based efforts, such as Total Maximum Daily Loads (TMDLs) (Lung, 2023), where the SP formulation was used to quantify pollutant impacts on specific receiving waters.

2.2 Estimating Model Parameters in Natural and Urban Streams

Reaeration of the water system is determined by the oxygen transfer from the atmosphere and photosynthesis of aquatic life. There are three groups in literature that describe the gas transfer from the atmosphere to the water body: conceptual models such as the two film theories (oxygen transfer through air-water interfaces), eddy diffusivity models, and hydrodynamic models (N. T. Nguyen, 2014). All three types of models rely on a region that is dominated by molecular diffusion which controls the gas transfer process. As for photosynthesis of aquatic life, it relies on the balance between photosynthetic abilities during the day, the loss of solar energy in the evening, and the additional nutrients needed for the completion of the process (Thomann, 1987). Quantifying reaeration comes in two parts: empirical formulations that relate stream hydraulics and general channel characteristics to reaeration rates or developing hydraulic models that directly couple flow dynamics to reaeration process.

2.2.1 Reaeration Processes

The two film theory is the most discussed and applied gas transfer process in terms of reaeration of streams, although many more are fundamental to the topic such as surface renewal or penetration theory. This study will explore the two film theory as a consequence of its relevance in historical adaptations to SPM modifications.

The two film theory is based on Fick's law that describes the flux of oxygen from the atmosphere into the water as proportional to the dissolved oxygen gradient in water, the transfer is signed positive as oxygen enters the water from the atmosphere. The air is one film and the water is another. Associated with each film is resistance that inhibits the transfer of oxygen. The interface between the two is the root of variation between progressing theories; researchers tend to change the thickness of the interface layer based on careful measurement or theoretical models.

One of the first descriptions of the model is Whitman's: the stagnant boundary layer is a thin layer between the bulk of the fluid that is perfectly mixed in terms of dissolved oxygen concentration, and the air (Lewis and Whitman, 1924). Transport in this stagnant layer is governed by molecular diffusion instead of turbulence. This implication is crucial to understanding future progressions in the interface layer thickness. The next theory proposed a thinner version of the stagnant layer. The concentration boundary layer describes the region where diffusion and advection may occur (Herlina, 2004). However, using photokinetic mechanisms such as non-linear spectroscopy, researchers have found the measured concentration boundary layer is thinner than the theoretical layer because in actuality, turbulence contributes to transport more meaningfully than the theoretical model suggests.

Special attention is given to the symmetry of these progressively smaller interface layers. The diffusion controlled layer gets thinner as turbulence increases because turbulence enhances mixing of the bulk fluid toward the diffusion dominated boundary (interface layer). The transfer velocity increases as turbulence increases so the bulk solution is able to reach the surface quicker allowing the solution at the interface to be replaced with the bulk fluid which maintains the higher concentration gradient which enhances the transfer of oxygen from the atmosphere to the water. As a result, models that account for the transport dynamics such as waves or stirred grids of the bulk fluid, tend to have a thinner interface layer.

Eddy diffusivity is a more fundamental way of looking at the gas transfer process. Instead of assuming a stagnant thin film where oxygen transport is described only from the perspective of molecular diffusion, eddy diffusivity incorporates the shift from a turbulence dominated reaeration in the bulk of the fluid to a diffusion dominated reaeration near the surface.

Near a wall, velocity fluctuations go to zero, the eddy diffusivity coefficient goes to zero, and molecular diffusion becomes the governing characteristic for the mass (oxygen) flux (Son and Hanratty, 1967). This visualization is superimposed onto fluid systems with an air-water interface; near the interface has the same attributes as near the wall. Turbulence in the bulk of the fluid is transformed from the main contributor to reaeration to molecular diffusion as the system approaches the surfaces of the fluid (Son and Hanratty, 1967).

Both the two-film and eddy diffusivity theory provide a visualization for the thinness of the molecular diffusion dominated layer of a fluid but it is the hydrodynamic models that explain why the layer is thin (Brumley and Jirka, 1988; Fortescue and Pearson, 1967). There exist many hydrodynamic models, this paper will discuss only two, the penetration and surface renewal theories.

Penetration theory describes that oxygen is transferred from the atmosphere into the diffuse layer, or diffuse dominated region, by turbulent eddies rising from the bulk fluid to the surface, remaining in the diffuse layer for a fixed amount of time while absorbing oxygen, then returning to the bulk fluid (Harriott, 1962).

Higbie's penetration theory rests on the assumption that all eddies have the same exposure time; however, this is unrealistic (Harriott, 1962). Danckwerts' surface renewal theory shows how surface renewal is a more unpredictable, erratic process (O'Connor and Dobbins, 1958). The interface is made up of constituents exposed to the atmosphere for varying lengths of time; they are constantly exchanging mass with the bulk phase of the system.

2.2.2 Photosynthesis of Algae and Aquatic plants

Reaeration can occur by the physical transfer of gas or by the photosynthetic ability of aquatic plants. Chlorophyll containing plants can utilize solar energy and convert water and carbon dioxide into glucose with oxygen as a byproduct (Thomann, 1987). As photosynthesis only occurs in daylight hours, measured DO values fluctuate based on the diurnal cycle. Minimum values occur in the early mornings while maximum values occur in the mid-afternoon (Thomann, 1987). However, since plant life proportionally respires, there is an inherent balance between the oxygen production by photosynthesis and the consumption by respiration.

Studies analyze the effect of photosynthesis and respiration by studying the diurnal curve which describes how dissolved oxygen changes over space and time. J. B. Erdmann assessed the diurnal cycle of the Charles River (Erdmann, 1979). Using the SP mass balance as a starting term, he added on the average gross photosynthetic production of DO (p) and the average respiration rate (R). The modified mass balance describing the time rate of change of DO concentration is below.

$$\frac{dDO}{dt} = k_2 D + p - R$$

Note, this equation was modified to match the variables consistent in this paper. Erdmann used numerical methods to discretize the time dimension as opposed to traditional spatial perspective. He found that the upstream photosynthesis-to-respiration ratio was 0.32 and downstream was 0.96, implying that photosynthesis produces about 32% of the total oxygen needed for life to respire upstream and 96% downstream. Upstream, reaeration from the atmosphere supplements the oxygen demand while downstream, the photosynthetic ability can support much more of the respiration demand.

Consistent with the broader oxygen demand literature, plant and animal respiration is generally treated as a secondary, diurnally-modulating factor rather than a primary driver of sustained DO depletion, which is instead attributed to microbial decomposition of organic matter (Thomann, 1987; U.S. Environmental Protection Agency, n.d.). However, the diurnal cycle of photosynthesis and respiration in aquatic plants is worth acknowledging: plants produce DO during the day and consume it at night, which can drive short-term fluctuations in the DO curve and, in some cases, lead to supersaturated DO conditions during peak photosynthetic activity. Additionally, diurnal photosynthesis/respiration is a dynamic, time-varying process that conflicts with the SP model's steady-state framing.

2.2.3 Empirical Formulations for the Reaeration Constant

Determining reaeration is accomplished by using empirical formulations derived from the physical and hydraulic characteristics of the stream bed, or by coupling water quality models with hydraulic models. Empirical models are beneficial because their equations are backed by data, allowing researchers to build on specific case studies and generalize them further. However, models lose resolution when researchers develop parameters based on previous experiments conducted under different conditions. The limitations of hydraulic models mirror the limitations of modeling in general: the abstraction is constrained by its underlying assumptions.

This paper will explore five commonly used reaeration empirical formulas, as well as common uses of coupled SP and hydraulic models.

The basis of O'Connor and Dobbin's work rests on Higbie's surface penetration theory previously discussed. The exposure time which was left ambiguous to the context in Higbie's original theory is defined and solved in this formulation by relating the ratio of the depth of the channel and velocity of the stream to the exposure time of the eddy. Unlike deriving a formulation from first principles (Higbie's penetration theory), Churchill et. al used field measurements of actual reaeration rates in rivers and applied dimensional analysis and regression techniques to relate the observed reaeration directly with hydraulic properties of the stream channel.

The Tennessee Valley Authority river reaches were largely regulated, relatively deep and fast-moving, and were virtually free of organic pollution, making oxygen sink terms negligible. Influent water sources are already oxygen depleted. They measured the actual DO along the reach and calculated the deficit. Since the oxygen sink terms were negligible and the system was steadily increasing in DO levels, the change is attributed to the reaeration rate (Bennet and Rathbun, 1972). He used dimensional analysis to identify which hydraulic variables govern reaeration and multiple regression algorithms to fit the exponents and coefficients to the measured field data. The depth component is inversely proportional to reaeration to a power of 1.673. Compared to the O'Connor-Dobbins formulation that represents shallow streams, in deeper streams depth suppresses surface renewal stronger than penetration theory implies; eddies are not replenished as readily in deeper channels than in shallower ones.

Compared to the TVA site that is relatively unpolluted and deep, Owens et al. were working in shallow, fast-moving waters of British streams. Researchers in this study saw a gap in results when they applied the Churchill formulation to shallow, turbulent streams.

They used a similar methodology to the Churchill team, measuring the deficit, and applying dimensional analysis and regression techniques. The key difference is the chosen stretches were shallow. The published reaeration coefficient is described as:

$$k_2 = \frac{5.34 \cdot U^{0.67}}{H^{1.85}}$$

where k_2 is in units of /day and U and H are in SI units (Owens et al., 1964).

The velocity term is now to the power of 0.67, noticeably less than the Churchill. In shallow streams, velocity does not contribute as meaningfully to reaeration since the shallow stream nature provides enough vigorous mixing to reaerate, changes or magnitudes of velocity are less of a factor. However, the high depth exponent shows that even small changes in depth, result in significant changes in surface renewal.

For both the Owens and Churchill formulation, they are valid only for a specific range of depths. Owens tends to produce abnormally high k2 values for systems with depths greater than 0.7 m (Gualtieri et al., 2002).

Similarly to O'Connor and Dobbins, Thackston and Krenkel rooted their formulation in the surface renewal theory and extended it by adding a shear velocity component to the coefficient. The reaeration coefficient is proportional to the vertical mass transfer coefficient at the surface and inversely proportional to the mean depth (Thackston and Krenkel, 1969).

Tsivoglou and Wallace's work was unlike the literature previously described. They measured the gas transfer capacity of a stream using a gaseous tracer technique.

The tracer approach produces far more reliable results than previous DO balance approaches since it does not rely on categorizing oxygen sources and sinks (Bennett and Rathbun, 1972). Subsequent literature such as the USGS database work use tracer based k_2 measurements for validation and development of predictive equations because of its notable accuracy (Rathbun, 1977; Kilpatrick et al., 1989; Melching et al., 1999).

2.2.4 Challenges Under Varying Hydraulic Conditions

The shallow water equations (SWE) are a 2D simplification of the Navier - Stokes equations by averaging depth. It is valid under the assumption that the horizontal scale is much larger than the stream depth. The core objective of using the SWE is to resolve depth, velocity, and water surface elevation in space and time. In environments where streams are subject to expansion and constriction of river width, recognizing how velocity gradients change by changes in cross-sectional geometry; it generates recirculation patterns that govern how contaminants are transported and where oxygen deficits accumulate.

The output from the SWE equation, depth, depth average velocity, and water surface elevation are direct inputs to the O'Connor-Dobbins, Churchill, and Owens-Gibbs formulation and velocity and slope combined for the Tsivoglou-Wallace formulation. Instead of applying these equations with a single reach averaged estimate of depth averaged velocity and depth leaving the system subject to researcher measurement error, the SWE resolves these parameters spatially accounting for geometry and continuously modifies them along the reach. The mass transport representation shifts from a 3D advection-diffusion equation from Fick's law to a 2D form where the molecular diffusion term is replaced by turbulent diffusion to account for the turbulent nature of natural streams (Chaudhry M.H, 2008).

Wu and Yu (2021) demonstrated that the sudden constriction of a channel cross section increases flow velocity, reducing reaeration travel time and producing a more uniform DO distribution less prone to low aeration zones. A degree of expansion in the river is conducive to reaeration and can also accelerate the consumption of pollutants through the water body's self-purification. Channel expansion induces flow separation and recirculation patterns that elevate the reaeration coefficient downstream, while creating dead zones where asymmetrical concentration patterns develop due to the recirculation effects, areas of stagnation caused by geometrical irregularities in the flow boundary. The effects of expansion and constriction are specifically revealed through spatially distributed velocity and depth fields that the SWE provides; reach averages hydraulics tend to miss this representation. They further show the self-purification capacity in the design stage of hydraulic structures suggesting that the SWE framework is not only physically accurate for predicting reaeration but also a practical instrument for engineering applications where DO management is a design consideration. Another form of hydraulic modeling is the HEC-RAS tool. The Saint-Vanant equations form its governing equations which are a 1D form of this same SWE framework.

HEC-RAS is a 1D steady flow hydraulic model designed for channel analysis, developed by the US Army Corps of Engineers. Rather than resolving the full 2D velocity field, HEC-RAS reduces the problem to cross section averaged hydraulic characteristics, flow velocity and water depth, using the longitudinal axis of the channel. HEC-RAS provides a computationally efficient alternative to the full 2D resolution.

Fan et al (2012) combined HEC-RAS with a modified SP equation to assess water quality of the Tan-Sui River and its tributaries in Taiwan, employing HEC-RAS specifically to calculate the reaeration coefficients and assess the impact of tidal dynamics on water quality. The coupling strategy mirrors the framework discussed in Wu and Yu (2021), hydraulics and biogeochemistry are treated as separate modules with HEC-RAS resolving velocity and depth, and the modified SP equation handling BOD degradation, nitrogen oxygen demand, and reaeration. Critically, the hydraulic outputs from HEC-RAS feed directly into the empirical equations from section 2.3 which replaces the reach-averaged or literature derived hydraulic constraint that would otherwise introduce significant errors.

Water quality modeling has been shown to be a useful tool for water resources management. However, the use of a basic model for water quality simulation becomes unavoidable when available data on hydraulic characteristics and water quality monitoring are limited. This tradeoff is central to evaluating when a hydraulic model like HEC-RAS is warranted versus when carefully collected field measurements of velocity and depth can serve as adequate substitutes. In reaches that are small and accessible enough to support detailed field sampling events, direct measurement may bypass the need for a hydraulic model entirely while achieving comparable accuracy in the hydraulic inputs to reaeration equations.

In river sections without tidal influences, water quality was sensitive to the BOD and NH_3-N degradation constants, while tidal dynamics introduced additional complexity in the lower sections of the river where the SPM's assumption of unidirectional steady flow breaks down. This reveals an important limitation, the 1D framework in both the SPM and HEC-RAS become less accurate as the flow regime departs from the assumptions on which the equations are founded on, particularly in tidally influenced or geometrically complex reaches.

Similarly, Numerical solutions can address the challenge of categorizing stream hydraulics by resolving flow conditions at discrete points along the reach, allowing spatial variations in depth and velocity to be incorporated into the model rather than relying on a single reach-averaged estimate.

The SP mass balance equation in its original closed form derivation yields an analytical solution for the DO deficit as a function of travel time downstream, given constant values for deoxygenation and reaeration coefficients. This simple SPM is based on assumptions that a single BOD input is distributed evenly at the cross section of a stream or river and that it moves as a plug flow with no mixing. Under these governing differential equations can be solved exactly, producing an explicit expression for the oxygen sag curve that requires no iterative computation. This simplicity is both the primary strength and limitation of the closed form approach. It is computationally inexpensive but it is valid only under restricted assumptions from which it was derived.

In practice, real river systems violate these assumptions. Flow velocities vary spatially, channel geometries are not constant, and multiple distributed sources of BOD and other oxygen sinks are active simultaneously along the reach. Numerical solutions to the SP mass balance are essential for handling those non-ideal conditions (Erdmann, 1979). By discretizing the spatial domain into grid cells and the time domain into steps, numerical methods approximate the governing differential equations at each node, allowing hydraulic parameters and sink terms to vary along the reach rather than being held constant (Wu and Yu, 2021).

When the model is extended to include additional physical processes, there exists a lack of feedback between the DO concentration and the rate of organic matter oxidation, which can result in physically incorrect solutions where DO concentration becomes negative (Gotovtsev, 2010). When a feedback term, coupled DO concentrations to the oxidation rate, the system of equations is no longer analytical or closed form and can be solved numerically (Gotovtsev, 2010).

However, the closed form approach has remained widely used because of its practicality since the data requirement often can't be met. Closed form solutions are also used as validation for numerical implementations even if the numerical version is ultimately adopted (Wu and Yu, 2021). Considering the historical significance of the closed form solution, it often serves as the ground truth and assesses whether the numerical version behaves correctly before applying it to situations where analytical methods are no longer valid.

2.2.5 Oxygen Sink terms

For the purposes of this discussion, microbial communities all use oxygen as the electron acceptor to oxidize organic compounds.

Carbonaceous oxygen demand is the consumption of complex carbonaceous material by microbes in the water column (Thomann, 1987 pg. 293). The decay rate of carbonaceous material is first order, as the concentration of the material decreases, the rate of oxygen consumption to degrade, decreases. In the SPM oxygen depletion is represented by the sag curve, the rate represents only CBOD dynamics since carbonaceous material is carried downstream and gets consumed as it travels. At the pollution source point, CBOD is highest and oxygen consumption is fastest so the oxygen deficit of the system is high. As concentration travels downstream and decreases, the deficit decreases as reaeration competes and the deficit decreases. This characteristic sag is a representation of transport, depletion, and recovery of a system and its pollutants, CBOD embodies this phenomenon and therefore represents the deoxygenation process in the SPM.

Nitrogenous oxygen demand is the oxygen consumed by microorganisms as they oxidize nitrogen-containing compounds such as ammonia through nitrification. Sources include: sewage effluent which is a primary source rich in ammonia from urea and protein breakdown, agricultural runoff such as fertilizers and animal waste, industrial discharge, and internal recycling as sediments release ammonia back into the water column (Chapra S.C. 1997). Since nitrifiers are slower growers, their consumption lags behind carbonaceous oxygen demand. As heterotrophic activity dominates, nitrifiers tend to slow activity. As a result, CBOD tends to take place first and produce the sag while NBOD dynamics take place after.

Yustiani's study regarding long-term BOD rates, reveals the definitional nuance in BOD. BOD = CBOD + NBOD, since NBOD is only apparent when heterotrophic activity is less competitive or when a long enough time has passed, long-term deoxygenation studies reveal both heterotrophic and nitrifying activity.

COD = Biodegradable OD + non-Biodegradable OD + chemical inorganic demand (Chapra S.C. 1997). The term BOD previously discussed addresses the biodegradable OD fraction of the COD. COD is useful to build perspective about the oxygen sinks in a system, however, even though it may appear more representative of demand activity, it measures oxidation potential rather than actual biological oxidation rate (Chapra S.C. 1997). Non-biodegradable sources do not affect the sag curve for the SPM because microbes are unable to metabolize them at a river timescale (Chapra S.C. 1997).

BOD accounts for microbial activity in the water column only. Sediment oxygen demand represents all oxygen consuming activity that takes place at and below the sediment-water interface (Chapra S.C. 1997). It can be divided between biological and chemical oxidation. Biological SOD concerns aerobic heterotrophs that consume oxygen, nitrifying bacteria oxidizing ammonia, microbial respiration in the entire benthic zone (water column and sediment layer), bioturbation by benthic invertebrates that physically mix the sediment renewing organic matter to sediments (Chapra S.C. 1997). Chemical SOD concerns refers to processes such as oxidation or reduced iron or manganese, and the oxidation of sulfides and methane from methanogenic bacteria (Chapra S.C. 1997).

Both NOD and SOD are commonly added sink terms to the original SPM to capture activity that may heavily affect oxygen dynamics in a stream but are otherwise left out of the original model. Codified versions of DO based water quality models that build off SP with added oxygen sink terms include the QUAL (1970) and WASP in 1983 models by the USEPA. QUAL2k simulates dozens of factors that influence oxygen dynamics including nitrogen and phosphorus (demand by nutrients), algae (photosynthesis and respiration dynamics), pH, and temperature (Wang et al., 2013). The WASP, Quasar (1997), MIKE, BASINS, and EFDC models extend the subject into two- or three- dimensional spaces and can represent non-point source pollutants (Wang et al., 2013).

2.2.6 Determining Deoxygenation Constant

Determining the deoxygenation coefficient follows a similar breakdown as reaeration as in there are empirical as well as theoretical based formulations. However, in terms of the deoxygenation coefficient for the SPM, the formulation is concerned only with carbonaceous oxygen demand, oxygen demand exclusively from the water column. Researchers add nuance to the original SP mass balance by adding additional oxygen sink terms that better represent the complex removal rates.

consider

Hydroscience Inc. expresses BOD decay as a first-order process:

$$L = L_0 e^{-k_1 x/U}$$

where L is the BOD decay concentration at a downstream distance x, L_0 is the initial BOD concentration, U is stream velocity and, k_d is the deoxygenation coefficient. Hydroscience found that k_l can be estimated from field measurements using the slope of a semi logarithmic relationship between observed BOD concentration and downstream distance. Their empirical relationship between the deoxygenation rate and stream depth is described as:

$$k_1 = 0.3 * \left(\frac{H}{8}\right)^{-0.434}$$

where H is channel depth in feet (Hydroscience, 1971/1972). The formulation implies that shallower depths experience greater deoxygenation rates because of increased interaction between organic matter and biologically active surfaces such as streambeds. Deeper channels reduce this interaction and may decrease oxidative removal rates. Similar to the reaeration formulations earlier discussed, this formulation is only as good as the context in which it was developed. It was based primarily on conventional wastewater-impacted streams therefore, application to urban streams with geometrical complex channels and non-point source inputs. Other contexts may require additional calibration. The corporation analyzed observed stream behavior and related it to measured deoxygenation rates.

2.3 Advancing in DO Modeling for Complex Stream Conditions

An alternative approach to the hydroscience formulation is the use of long-term BOD analysis to directly estimate deoxygenation rates from observed oxygen consumption. Degradation follows the first order decay kinetics. Conventional incubation periods are about 5 days (G. C. Dezler and S.W. McKenzie, 2003). By extending incubation periods beyond the conventional BOD test length, long-term analysis captures the complete oxygen demand associated with organic matter degradation and provides site specific estimates of deoxygenation rates (Yustiani et al., 2023). This approach is especially productive for urban river systems, where organic matter inputs may be highly variable and continuous (Yustiani et al., 2023). Sources include: stormwater runoff, wastewater influences, and biological processes such as algal growth and decay. These continuous inputs can alter observed oxygen demand patterns and may not be adequately represented by the general empirical relationships.

Applying the long-term BOD analysis to a highly polluted river system in Indonesia, they found the estimated deoxygenation coefficients range from 0.23 - 0.29 day^{-1} (Yustiani et al., 2023). When compared with the Hydroscience formulation which produced 040 - 0.81 day^{-1}, the Hydroscience formulation yielded much higher results (Yustiani et al., 2023).

Long-term BOD analysis may provide a more representative estimate of deoxygenation rates in urban rivers because it captures the cumulative oxygen demand associated with complex organic matter sources and prolonged degradation processes. Shorter incubation periods may capture only a fraction of the total demand and potentially much less if waters are heavily polluted rivers that have compounds that slowly biodegrade. However, for long incubation periods, other oxygen sink terms such as nitrogen oxygen demand may overlap with the carbonaceous demand skewing measurements. Concerns with BOD tests in general still apply to this situation, lab conditions and bottle effects don’t accurately reflect the river system.

Long-term BOD tests are competitive alternatives to empirical equations as they represent site-specific characteristics that are otherwise missed in empirical assumptions.

2.3.1 Spatially Distributed Modeling Approach

The classic SP formulation treats the deoxygenation and reaeration processes as independent. The deoxygenation rate constant k_1 is applied to BOD concentration as a first order reaction regardless of existing DO level, while k_2 drives reaeration in proportion to the DO deficit. The decoupling is convenient mathematically and produces a manageable, closed-form solution but it obscures the physical reality: the rate at which organic matter is oxidized is not independent of the available DO.

In heavily polluted systems where the oxygen sag is deep, the classic formulation can drive DO below zero which is physically impossible. This signals that the model has left the domain of validity of its assumptions. Researchers have addressed this limitation in three stages as outlined in Gotovtsev's 2010 paper. Stage one is running the SPM and manually calibrating the system until agreement was achieved between model values and field data. Stage two modified the governing equations by introducing new terms, NOD, and paired rate constants, the associated degradation rate. Stage three recognized that the degradation rates are not constant, they are functions of the flow and concentration conditions in the stream, particularly, the deoxygenation rate depends on the available DO.

The Michaelis-Menten formulation, a methodology developed for enzyme kinetics, describes reaction rates as a function of substrate concentration which captures the transition between kinetic regimes that first-order decay cannot. At low BOD concentrations, the Michaelis-menten equation tends toward first order decay recovering the classical SP result s a limiting case but at high BOD concentrations, it follows a zero-order reaction. Degradation rate becomes independent of BOD concentration because the process is limited by the available DO rather than the substrate itself (Gotovtsev, 2010). An increase in BOD concentration doesn't necessarily produce a proportional increase in degradation rate when DO availability becomes the limiting constraint (Fan and Wang, 2008; Gotovtsev, 2010).

The inclusion of the Michaelis-Menten equation does not eliminate the possibility for physically incorrect solutions. Hence, Gotovtsev developed a closed form modification to address the physically incorrect edge cases and relates the concentration of dissolved oxygen and the oxidation rate of organic matter. k_1 becomes a function of the DO deficit rather than treating it as a constant, coupling the deoxygenation and reaeration terms in a way that the original formulation does not. Gotovtsev proposes a workflow that will streamline the use of his proposed closed form solution, if the initial BOD concentration is above a specified value, the closed form solution becomes pertinent. The general form of the solution also accounts for the situation when oxidation stops at a nonzero DO concentration, which occurs when microorganisms responsible for organic matter degrade and are inhibited by toxic substances in the water, this is analogous to anoxic conditions in environments associated with high SOD discussed in section 4.

Gotovtsev's closed formulation builds on the foundational work of Shishkin (1976), who first proposed a coupling between organic matter degradation and DO concentration. While Shishkin's contribution remained analytically underdeveloped, it laid the groundwork for the closed system and has since made significant contributions to computational water quality modeling by extending the SP framework to higher-pollution contexts where the classical formulation breaks down.

2.3.2 Novel Urban Stream Applications

The SPM is traditionally applied as a forward problem. Given known initial conditions, rate constants, and pollutant loads, the model predicts the DO distribution and oxygen sag curve downstream. However, an inverse approach may be used to determine an upstream pollution source if downstream water quality is known by field measurements (Long, 2019). Long represents the first application of the inverse problem formulation to the Streeter-Phelps framework specifically, extending a technique previously applied to advection-dispersion models to the DO-BOD system (Long, 2019).

A further departure from deterministic application of the SPM is embedding the classical framework within a Bayesian statistical approach to predict hypoxic volume in the Chesapeake Bay (Liu and Arhonditsis, 2011). Rather than treating the model's parameters and initial conditions as fixed, known quantities, this approach explicitly characterizes the uncertainty surrounding them. Monte Carlo simulation is used to propagate the uncertainty through to the final prediction. This is a meaningful difference from the deterministic SP applications discussed in the review; instead of producing a single sag curve from a single set of input values, the Bayesian approach produces a distribution of possible outcomes, giving a quantitative sense of confidence around the predicted hypoxic volume (Liu and Arhonditsis, 2011). This uncertainty-based framing is relevant given the coupling between the organic matter decay and DO availability discussed in section 5. Gotovtsev (2010) demonstrated that the classical SP model can produce physically unrealistic results at high initial BOD concentrations, and the sensitivity model output to initial conditions is not uniform across the full range of possible inputs. By applying Bayesian uncertainty analysis, Liu et al. directly addresses this limitation. Instead of assuming a single point estimate of the initial deficit is reliable, the Bayesian framework characterizes the range of plausible parameter values and assesses how much room exists before model prediction becomes reliable. This approach is well suited for large-scale ecologically consequential predictions like hypoxic volume in an estuary, where the cost of an overconfident point estimate could lead to flawed management (Liu and Arhonditsis, 2011).

Non-traditional applications of SP are not limited to methodology but may also be innovative depending on the scale of production. Athirah et al. (2020) apply the SP framework to assess the water quality impact of household greywater discharge from village houses into the receiving stream. This shifts the perspective of application from a single, large, point-source effluent to decentralized, low-volume, domestic discharge. They use two independent methods to estimate the deoxygenation coefficient, the hydroscience empirical equation and the Thomas slope method, a graphical and mathematical technique used to determine the ultimate BOD and/or reaeration technique (Athirah et al., 2020). They found that k_1 varies measurably along the length of the stream which reinforces the idea established in earlier sections; treating deoxygenation and reaeration as spatially uniform is a simplification that often does not hold over relatively short stream reaches and that empirically derived values that are site specific tend to provide a more accurate representation of the true kinetic conditions of a system.

2.4 Gaps in the Literature

2.4.1 Limited Validation in Urbanized Systems

The literature reviewed here traces the SPM's century long evolution from a simple closed-form oxygen sag equation into a flexible framework capable of incorporating hydraulic heterogeneity, nonlinear feedback between deoxygenation and reaeration probabilistic uncertainty, and even inverse, pollution-source attribution. What emerges from sections 2.1 - 2.4 is a consistent tension: the classical SPM's predictive accuracy depends on how well its governing parameters reaeration and deoxygenation constants represent the specific hydraulic and biochemical conditions of the stream being modeled.

2.4.2 Challenges Transferring Parameters Across sites

As described by the Owens-Gibbs, O'Connor - Dobbins, Hydroscience, and Churchill formulations, empirical equations are developed around specific hydraulic conditions making the transfer of parameters across sites difficult. Athirah et al. (2020), for example, estimated k_1 using both the Hydroscience equations and the Thomas Slope method and found that the deoxygenation varied along the stream, highlighting the limitations of assuming a single transferable value. Although empirical formulations provide a practical way of estimating parameters, their dependence on hydraulic and site characteristics limits their transferability across streams.

2.4.3 Need for Site Specific Modeling Assessment

However, the application of generalized DO modeling frameworks to individual streams remains limited by site-specific hydraulic and environmental conditions that may not be captured by standardized parameter formulations. This thesis addresses this gap by developing and evaluating a finite-difference Streeter–Phelps modeling framework. First, the framework is formulated and evaluated using the UCLA Botanical Garden stream to examine parameter sensitivity and model behavior under relatively controlled conditions. Second, in-situ DO measurements are used to assess model performance and identify structural limitations in representing observed DO dynamics. Finally, the framework is applied to the more hydraulically and environmentally complex Los Angeles River to assess its applicability under less controlled conditions and identify limitations associated with applying the Streeter–Phelps framework to a larger engineered urban system.

3 Methods

3.1 Site Selection

3.1.1 Botanical Garden

The UCLA Botanical Garden was selected as the primary study site because it provides an accessible urban stream system in which dissolved oxygen (DO) dynamics can be monitored repeatedly to develop, calibrate, and evaluate the Streeter-Phelps model. In addition, garden caretakers have expressed concerns regarding water quality and the health of the koi and carp inhabiting the stream, making DO a relevant indicator of overall stream health.

The study site consists of a recirculating stream approximately 130 meters in length located at the lowest elevation of the Botanical Garden (Figure 2). It contains approximately 50,000 gallons of water sourced from the municipal water supply, which is recirculated through the stream using a series of pumps located at the upstream and downstream ends of the stream. Approximately 10,000 gallons of water are siphoned from the stream weekly to irrigate the east side of the garden, and the lost volume is replenished with municipal water. Although the system is recirculated by pumps, water movement throughout most of the stream is driven by gravity.

The stream includes several distinct hydraulic features that influence oxygen transfer. On the left-hand side of the stream (see Figure 2), there is a waterfall with three pools. Following this is a narrow stretch of shallow stream bed that drains into the first pond, the koi pond. The width of the stream then narrows again into another shallow stretch that eventually flows into the lily pond. Some of the smaller fish from the koi pond have traveled downstream and now reside in the lily pond. The lily pond directly drains into the bog pond. At the end of the bog pond, another waterfall collects the downstream flow and pumps it back to the first waterfall, completing the recirculating system. The narrow stretches of the stream are lined with gravel and sand, while the ponds are lined with concrete. A thick layer of sediment has accumulated at the bottom of each pond.

Several characteristics of the system are expected to influence dissolved oxygen dynamics. Since the stream sits at the lowest section of the garden and extends throughout the space, heavy storms and irrigation runoff introduce nutrients and sediment that settle into the system. Coupled with the continuous recirculation and pumping, resuspended sediment creates turbid conditions. The accumulation of nutrients, combined with favorable sunlight conditions, promotes the growth of single-celled algae. These characteristics make the Botanical Garden an appropriate site for evaluating Streeter-Phelps model performance under controlled but hydraulically heterogeneous urban stream conditions.

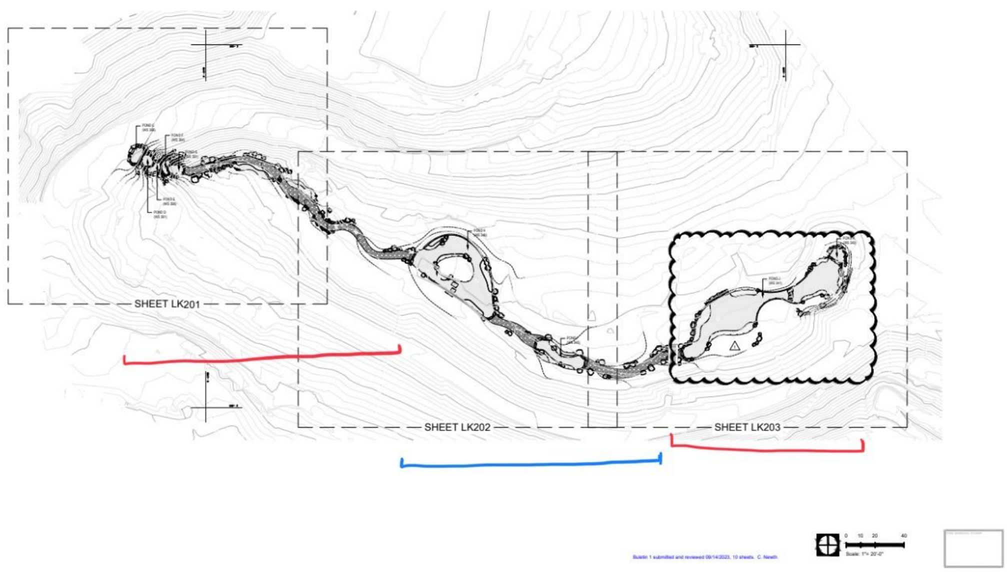

Figure 2. *Map of the UCLA Botanical Garden study site showing the stream configuration, major ponds, waterfalls, and areas exposed to direct sunlight (red) and tree canopy cover (blue).*

3.1.2 Los Angeles River – Sepulveda Basin

The Los Angeles River was selected as the second study site to evaluate the applicability of the Streeter-Phelps model within a larger urban river system. Whereas the Botanical Garden provides a controlled recirculating stream for model development and calibration, the Los Angeles River offers an opportunity to examine dissolved oxygen dynamics under more complex hydraulic and environmental conditions representative of urban waterways. The study also supports ongoing efforts to better understand water quality as part of the Los Angeles River Master Plan, which seeks to restore and rehabilitate the river (Los Angeles River Master Plan, 2020).

The Los Angeles River extends approximately 51 miles from Canoga Park, Los Angeles to Long Beach, California. The U.S. Army Corps of Engineers (the Corps) partitioned the river into 5-mile reaches to identify and mitigate flood-prone areas by constructing concrete-lined channels, with the goal of reducing flood events. Under the Los Angeles River Ecosystem Restoration Project, also led by the Corps, many of these concrete structures are being removed to restore the river to a more natural habitat with native vegetation. This study focuses on the Sepulveda Basin, a 5-mile reach that has retained riparian characteristics and represents one of the river's most naturalized sections. Unlike the Botanical Garden, most of the inflow to the river consists of wastewater effluent and storm runoff, resulting in more variable hydraulic and water quality conditions. Discussions with Friends of the Los Angeles River (FOLAR), a community-based organization that advocates for the river and engages local residents through educational programs, indicated that water quality varies over time. Following storm events, runoff introduces pollutants into the river. Outside of storm events, water quality is generally considered to be relatively good. Although the SPM was originally developed for simpler river systems, these characteristics provide a suitable setting for evaluating its applicability under more complex field conditions.

3.2 Sampling Framework

To characterize longitudinal changes in dissolved oxygen concentrations, the sampling framework for the Botanical Garden was informed by a previous study of dissolved oxygen dynamics in the Wen-Rui Tang River in Wenzhou China (Li et al. 2013). Li et al. (2013) recommended selecting sampling intervals three to five times smaller than the smallest expected spatial variation in dissolved oxygen concentrations. Following this guideline, the Botanical Garden stream was sampled every 2–3 meters, with more frequent sampling in the narrow stream reaches and less frequent sampling within the larger ponds. Sampling within the Botanical Garden was conducted with permission from the garden managers.

Proposed sampling locations for the Sepulveda Basin were selected using a more diagnostic approach to assess dissolved oxygen conditions throughout the river. Locations were selected at points where changes in hydraulic conditions, inflows, or other features could potentially influence DO levels. Figure 3 shows the proposed sampling locations. Locations 1 and 2 were located upstream and downstream of the busway bridge, respectively. Location 3 was located at the confluence of Bull Creek and the Los Angeles River. Location 4 was located adjacent to the Woodley Lakes Golf Course. Location 5 was selected to capture the river downstream of the Donald C. Tillman Water Reclamation Plant effluent discharge. Location 6 was located at the confluence of Encino Creek and the Los Angeles River. Locations 7 and 8 were located upstream and downstream of the Sepulveda Dam, respectively. Location 7 also corresponded to the confluence of the Los Angeles River and a tributary.

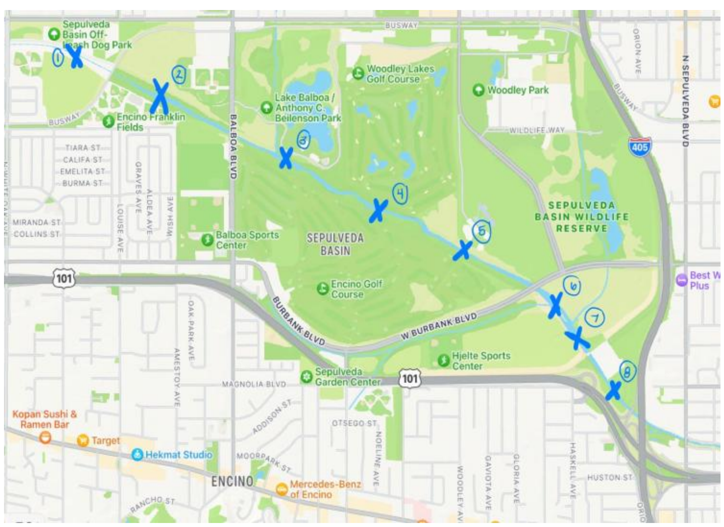

Figure 3. *Proposed sampling locations within the Sepulveda Basin reach of the Los Angeles River. Sampling locations were selected to characterize spatial variation in dissolved oxygen conditions and capture potential influences from tributary inputs, effluent discharge, and hydraulic transitions.*

Several of the proposed sampling locations were ultimately inaccessible because of logistical constraints. Figure 4 shows the locations that were successfully sampled. Sampling along the Los Angeles River was conducted in collaboration with the Corps.

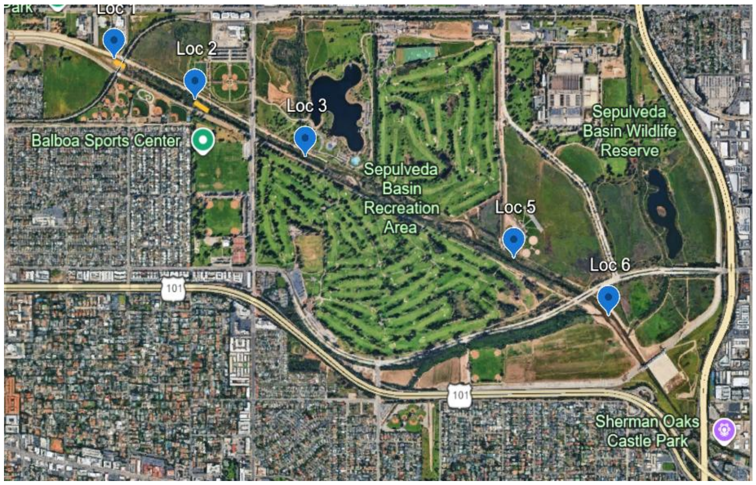

Figure 4. *Actual sampling locations within the Sepulveda Basin study reach. Locations 1, 2, and 3 were sampled as originally proposed. Two samples were collected at Location 2 to capture changes in stream conditions observed upstream and downstream of the levee.*

3.3 Instrumentation

Dissolved oxygen (DO) concentration and temperature were measured using a YSI DO probe, and conductivity was measured using a conductivity meter. Stream velocity was measured using a Marsh-McBirney flow meter. Instrument calibration and measurement procedures were conducted according to manufacturer specifications.

3.4 Numerical Implementation

3.4.1 Governing Equations and Numerical Solution

The numerical implementation was based on the one-dimensional diffusion-advection reaction form of the Streeter-Phelps model. The governing transport equation was first simplified by evaluating the relative importance of diffusion and advection, followed by numerical discretization using a finite-difference approach:

$$\frac{\delta C}{\delta t} = K\frac{\delta^2 C}{\delta x^2} - V\frac{\delta C}{\delta x} - kC$$

where C is the concentration of the constituent, K is the diffusion coefficient, V is the stream velocity, k is the reaction rate constant, x is distance, and t is time. The governing transport equation can be adapted to describe both BOD decay and dissolved oxygen deficit. Because advection was assumed to dominate transport, the diffusion term was neglected, reducing the governing equation to a one-dimensional advection-reaction transport equation.

The BOD transport equation becomes:

$$\frac{\delta L}{\delta x} = -k_1 L$$

where L is the biochemical oxygen demand concentration and k_1 is the deoxygenation rate. Applying a first-order forward finite-difference approximation to the spatial derivative yields:

$$L_{i+1} = -dtk_d L_i + L_i$$

where $dt = \frac{dx}{V}$ represents the travel time required for water to move one computational grid spacing dx. This study used a computational grid spacing of 0.001 m.

The dissolved oxygen deficit transport equation is:

$$\frac{\delta D}{\delta x} = k_1 L - k_2 D$$

where D is the dissolved oxygen deficit and k_2 is the reaeration rate.

Applying the same forward finite-difference scheme gives

$$D_{i+1} = dt\,(k_1 L_i - k_2 D_i) + D_i$$

Although centered finite-difference schemes generally reduce truncation error, they may become unstable for purely advective transport in the absence of diffusion. Therefore, a first-order forward finite-difference scheme was selected to maintain numerical stability under the assumed transport conditions.

3.4.2 Model Parameterization

Although numerous empirical formulations for estimating deoxygenation and reaeration coefficients have been proposed in the literature, a single formulation for each coefficient was selected for implementation in the baseline SPM. The Hydroscience formulation was chosen to estimate the deoxygenation coefficient because of its widespread application in stream water quality modeling, while the Owens-Gibbs formulation was selected to estimate the reaeration coefficient because it is appropriate for relatively shallow streams. Representative deoxygenation and reaeration coefficients reported in the literature were used during the initial model implementation to verify that the numerical solution behaved as expected before applying the model to the study sites.

3.4.3 Additional Oxygen Sink Terms

To evaluate potential limitations of the original SPM formulation, this study also evaluates additional oxygen sink terms that may better represent the observed dissolved oxygen dynamics within the study systems.

3.4.4 Model Inputs and Boundary Conditions

The initial BOD concentration used in the model was estimated from a 5-day BOD test and established as 2.56 mg/L. Measured velocity and depth values were used to estimate the reaeration coefficient using the selected empirical formulation. Dissolved oxygen saturation concentrations were estimated using the USGS DO Table software based on measured temperature and conductivity values.

4 Results

4.1 Observed Hydraulic and Water Quality Conditions

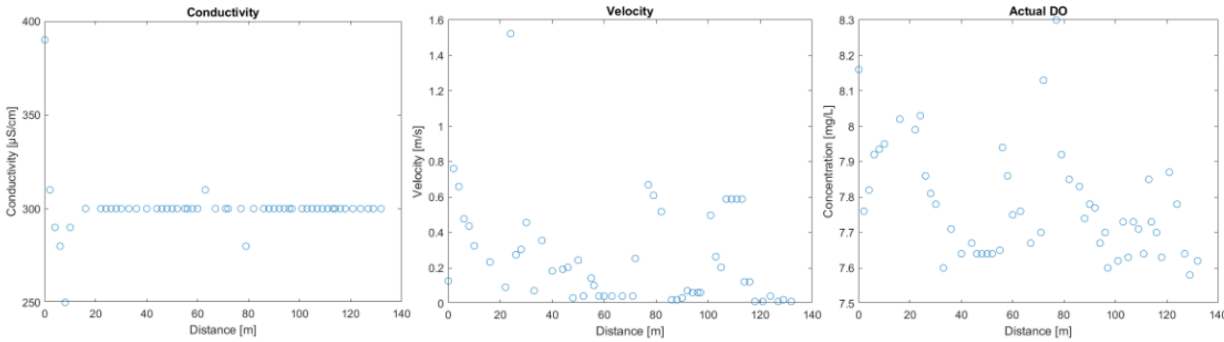

Figure 5*. Preliminary water quality measurements collected at the UCLA Botanical Garden stream. Spatial variation in dissolved oxygen (DO), electrical conductivity, and stream velocity measured along the study reach prior to model development.*

Preliminary field measurements indicated generally stable hydraulic and water quality conditions throughout the Botanical Garden study reach (Figure 5). Conductivity remained relatively constant at approximately 300 µS/cm, comparable to typical municipal water conductivity (approximately 200 µS/cm). Stream velocities varied substantially, ranging from stagnant conditions in pooled sections to a maximum of approximately 1.5 m/s in faster-flowing reaches. Higher velocities were consistently observed in areas of turbulence, whereas little to no flow occurred within stagnant pools. DO concentrations remained consistently high throughout the study period, ranging from 7.5 to 8.3 mg/L, with no evidence of significant oxygen depletion. Similar to stream velocity, DO concentrations tended to be highest in turbulent sections of the stream and slightly lower in stagnant areas, although all measured values remained within a relatively narrow range.

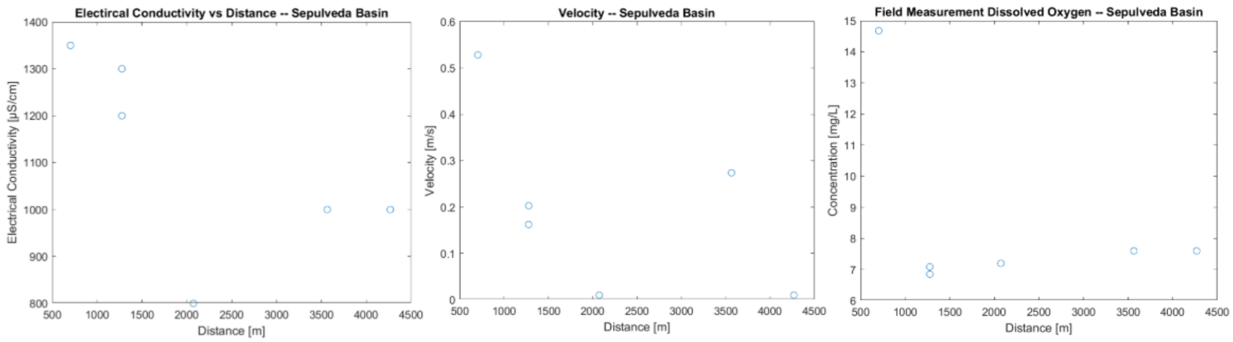

Figure 6. *Preliminary water quality measurements collected within the Sepulveda Basin study reach of the Los Angeles River. Spatial variation in dissolved oxygen (DO), electrical conductivity, and stream velocity measured during preliminary field sampling.*

Preliminary field measurements at the Sepulveda Basin study reach indicated greater variability in water quality than was observed at the UCLA Botanical Garden stream (Figure 6). Electrical conductivity ranged from approximately 1,000 to 1,350 µS/cm. Stream velocities were generally low, varying from no measurable flow in stagnant pools to approximately 0.6 m/s in flowing sections. Dissolved oxygen concentrations were typically between 7 and 8 mg/L; however, one sampling location recorded a substantially higher value of 14.68 mg/L, representing an isolated outlier within the dataset. The outlier was not removed from this analysis because, although it represents an isolated observation, it provides perspective on the range of DO conditions that may occur within the study reach and can inform future modeling decisions.

The preliminary results highlight differences between the Botanical Garden and the Sepulveda Basin. Relatively consistent conductivity in the Botanical Garden, combined with greater variability in stream velocity, reflects the controlled nature of its recirculating stream system. In contrast, the Sepulveda Basin exhibited greater variability in conductivity and generally lower and more spatially variable velocities, reflecting the more complex hydraulic and water quality conditions of the LA River. The greater variability in DO observed in the Sepulveda Basin, including the isolated high DO measurement, further suggests that localized conditions may influence oxygen dynamics throughout the study reach. These differences highlight the need for greater spatial resolution and potentially site-specific parameterization when applying the modeling framework developed for the Botanical Garden to a larger urban river system.

4.2 Baseline SPM Performance

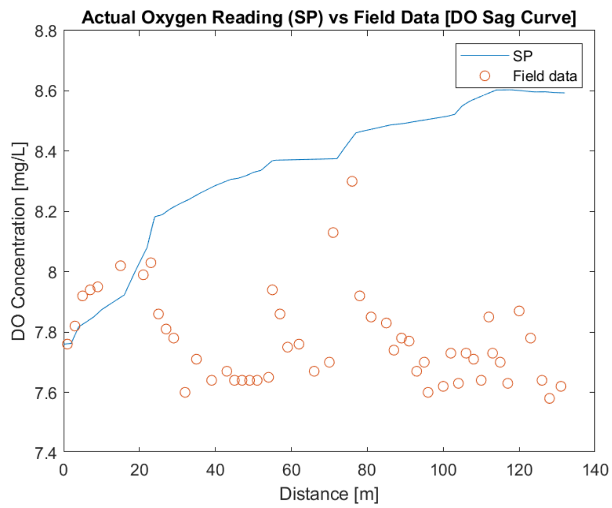

***Figure** 7. Baseline SPM performance.*

Figure 7 describes the baseline performance of the SPM. Reaeration was formulated using the shallow-water Owens-Gibbs formulation, deoxygenation was estimated using the Hydroscience equation, and an initial BOD value of 2.26 mg/L was derived from the 7-day BOD test. Initial model results tended not to follow the trends of the observed DO field data. Once the system recovered from the initial oxygen deficit at approximately 20 m, the model did not dip again to produce the up-and-down behavior observed in the field data. This suggests that a higher initial BOD value may be needed to better mimic the observed behavior and motivated the subsequent calibration study. Although adding additional oxygen sink terms may better represent the distributed oxygen demand that appears to occur throughout the system, this study focuses on increasing the initial load through calibration, while additional oxygen sink terms are considered conceptually.

4.3 Model Calibration

Manual calibration was performed by increasing the initial BOD value to pull the sag curve lower and better represent the observed DO trends. The initial BOD value of 2.26 mg/L was progressively increased, while the other model parameters were held constant. Increasing the initial BOD resulted in a lower modeled DO profile and allowed the model to better reproduce the observed decreases in DO throughout the stream. A final initial BOD value of 50 mg/L was selected based on the resulting model fit to the observed DO data. Although increasing the initial BOD value to 100 mg/L yielded a lower RMSE, this value was considered unrealistic given the stream characteristics, as the stream does not exhibit characteristics of a heavily polluted system.

4.4 Final SPM Performance

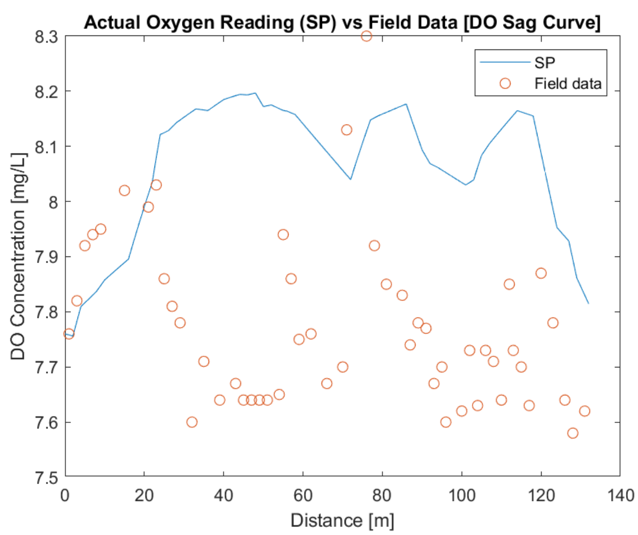

Figure 8. *Comparison of modeled and measured dissolved oxygen concentrations along the UCLA Botanical Garden study reach. The numerical Streeter–Phelps model reproduces the observed spatial variation in dissolved oxygen, with an overall RMSE of 0.3364 mg/L.*

Figure 8 compares the dissolved oxygen (DO) concentrations predicted by the SPM adjusting for higher initial BOD value with field measurements collected along the study reach.

Overall, the modeled DO concentrations closely matched the observed values, with both datasets exhibiting similar spatial patterns throughout the stream. The model reproduced the overall spatial pattern of DO concentrations along the stream reach, with relatively small differences between modeled and observed values. Quantitative evaluation yielded a root mean square error (RMSE) of 0.3364 mg/L, indicating good agreement between predicted and measured DO concentrations. While minor discrepancies were observed at several sampling locations, the model captured the overall distribution and magnitude of DO concentrations along the stream reach.

Overall, the model reproduced the general rises and falls observed in the measured dissolved oxygen profile, particularly within the first 20 m of the stream and again over approximately the last 100 m of the study reach. The greatest discrepancy between modeled and measured dissolved oxygen occurred between approximately 60 and 80 m, corresponding to the lily pond. This section of the stream is characterized by greater hydraulic complexity as flow transitions from the koi pond into narrower channel reaches with rapidly changing velocities and depths. These localized hydraulic conditions are difficult to fully represent using the one-dimensional Streeter–Phelps framework and likely contribute to the observed differences between modeled and measured dissolved oxygen.

Once the modeled system recovered from the initially elevated oxygen demand, dissolved oxygen concentrations generally remained higher than those measured in the field and did not exhibit the same degree of localized depletion. The modest decreases in modeled dissolved oxygen primarily reflected reductions in stream velocity and the associated decrease in atmospheric reaeration. This observation is supported by the sensitivity analysis, which demonstrated that the model is substantially more sensitive to changes in the reaeration coefficient than to changes in either the deoxygenation coefficient or the initial biochemical oxygen demand. Under the hydraulic conditions present within the Botanical Garden stream, velocity—and therefore reaeration—appears to be the dominant control on modeled dissolved oxygen dynamics.

The inability of the model to reproduce the magnitude of the observed dissolved oxygen depressions suggests that additional oxygen sink terms are present within the stream but are not represented by the current model formulation. Calibration studies explored increasing the initial BOD to reproduce these localized decreases. Although the calibrated model more closely reproduced some of the observed dissolved oxygen patterns, it did not fully capture the repeated increases and decreases observed throughout the stream. This suggests that increasing the initial BOD alone cannot account for the spatial variability in dissolved oxygen and that additional oxygen source and sink processes may contribute to the observed behavior.

4.5 Sensitivity Analysis

To isolate the influence of each model parameter, the sensitivity analysis was performed using fixed baseline values representative of those reported in the literature ($k_1 = 0.05\ day^{-1}$, $k_2 = 2\ day^{-1}$, and an initial BOD concentration of 2.26 mg/L). These values were held constant while individual parameters were varied independently. For subsequent model simulations, site-specific deoxygenation and reaeration coefficients were estimated using the Hydroscience and Owens empirical formulations to better represent the hydraulic characteristics of the study stream.

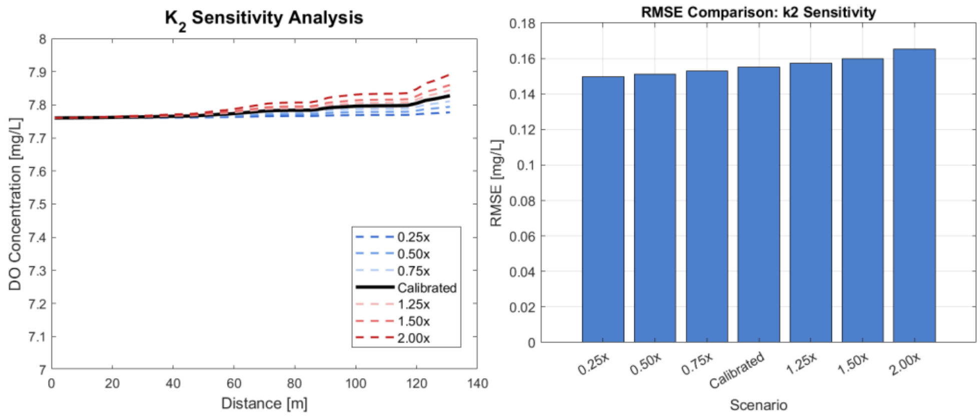

Figure 9. *Sensitivity of modeled dissolved oxygen concentrations to variations in the reaeration coefficient (k_2).*

The model was more sensitive to changes in the reaeration coefficient than to changes in the deoxygenation coefficient. Decreasing k_2 by successive multipliers consistently reduced the RMSE, whereas increasing k_2 resulted in progressively larger RMSE values. This trend suggests that the model predictions were more responsive to variations in the reaeration coefficient than to changes in the deoxygenation coefficient. As a result, uncertainty in representing site-specific system complexity through empirical formulations may contribute to the greater sensitivity of the model to the reaeration coefficient.

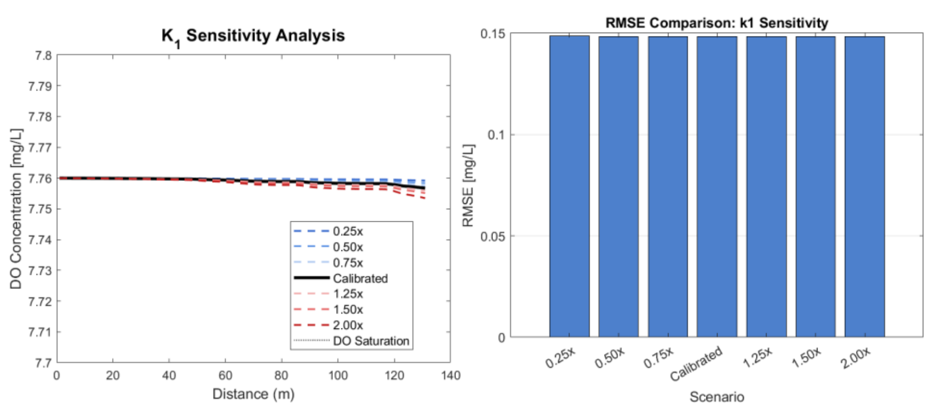

Figure 10. *Sensitivity of modeled dissolved oxygen concentrations to variations in the deoxygenation coefficient (k_1).*

The model exhibited relatively low sensitivity to changes in the deoxygenation coefficient. Decreasing k_1 produced a slight increase in RMSE, while increasing k_1 resulted in a small decrease in RMSE. However, these changes were minimal across the range of values evaluated. For example, reducing k_1 to 25% of its baseline value increased the RMSE to 0.1485, whereas doubling k_1 decreased the RMSE to 0.14819. These results indicate that model performance was only weakly influenced by variations in the deoxygenation coefficient over the tested range.

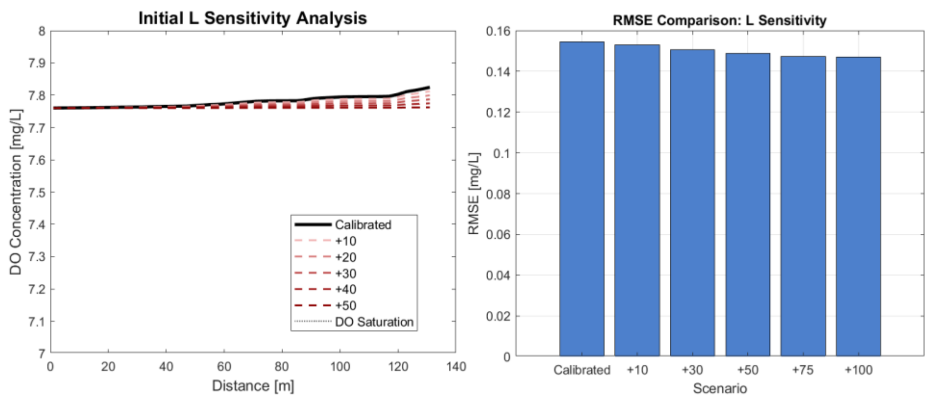

Figure 11. *Sensitivity of modeled dissolved oxygen concentrations to variations in the initial BOD (L_0).*

The sensitivity of the model to the initial BOD was evaluated by increasing the baseline concentration of 2.26 mg/L by 10, 30, 50, 75, and 100 mg/L. Increasing the initial BOD produced only modest changes in model performance. The RMSE decreased from 0.15457 for the baseline simulation to 0.14712 when the initial BOD was increased by 100 mg/L. Overall, variations in the initial BOD concentration produced only minor changes in model fit over the range examined.

5 Discussion

5.1 Model Performance

Although the model demonstrated reasonable agreement with measured dissolved oxygen concentrations within the Botanical Garden stream, the results highlight limitations in applying a traditional SPM to heterogeneous urban stream systems.

These findings point to a structural mismatch between the assumptions underlying the SPM and the processes governing dissolved oxygen in highly heterogeneous urban streams. The SPM was originally developed to describe oxygen depletion resulting from point-source organic pollution, whereas the Botanical Garden stream is more likely influenced by spatially distributed oxygen sources and sinks that vary throughout the reach. Processes such as photosynthesis and respiration by aquatic vegetation and algae, as well as sediment oxygen demand associated with benthic microbial communities, are not explicitly represented in the present model but may contribute to the observed dissolved oxygen dynamics. Incorporating these processes may improve model representation while maintaining physically realistic parameter values (Chapra and Di Toro, 1991).

5.2 UCLA Botanical Garden Implications

The UCLA Botanical Garden application demonstrates the value of combining field measurements with a mechanistic DO model to characterize oxygen dynamics within a small, heterogeneous urban stream system. The model was informed by antecedent velocity, conductivity, and depth measurements and provided a mechanistic understanding of how changing stream conditions inform DO dynamics. Specifically, model results and sensitivity analysis demonstrated that changes in velocity and reaeration strongly influenced modeled DO dynamics, highlighting the importance of accurately representing hydraulic conditions when modeling DO. However, calibration also demonstrated that increasing the initial BOD alone was not sufficient to reproduce the observed spatial variability in DO, as an initial BOD value of approximately 50 mg/L was required to produce a qualitatively similar DO profile despite being unrealistic for the Botanical Garden stream. This suggests that additional oxygen source and sink processes may contribute to the observed DO dynamics. While an elevated initial BOD is not representative of the current stream characteristics, a forecasted initial BOD value could be used within the framework as a predictive scenario to evaluate how DO may respond to increased organic loading or altered flow conditions. Based on the working model analysis, (see Appendix), the stream currently maintains relatively high DO levels throughout the study reach. These results suggest that the stream may have sufficient natural reaeration capacity to buffer moderate increases in organic inputs over 130m stream length; however, additional modeling would be needed to evaluate this potential response directly.

However, differences between modeled and measured DO concentrations suggest that processes beyond the current Streeter-Phelps formulation likely influence oxygen dynamics within the stream. The SPM primarily represents oxygen demand associated with organic matter decay and atmospheric aeration, whereas the Botanical Garden stream is influenced by spatially distributed oxygen sources and sinks. These processes, most plausibly photosynthesis, respiration, and sediment oxygen demand may contribute to localized DO variability and should be considered in future monitoring and modeling efforts, particularly during low-flow conditions when residence times increase and reaeration capacity is reduced.

5.3 Implications for LA River Modeling

The modeling framework developed for the UCLA Botanical Garden provides a foundation for future application to larger and more complex urban waterways, including the

Sepulveda Basin reach of the Los Angeles River. Although the Botanical Garden stream provided a simplified system for model development and validation, the sampling approach, parameter estimation framework, and model structure can inform future DO assessments in more heterogeneous river systems.

The hydraulic and water quality heterogeneity of the LA river requires modifications to the current framework developed for the UCLA Botanical Garden. The Botanical Garden results demonstrated changes in hydraulic conditions, particularly velocity, can strongly influence modeled DO dynamics, while the inability of the model to reproduce some of the observed DO variability suggests that additional site-specific processes may be important. These considerations become increasingly relevant in the LA river, where hydraulic and water quality conditions vary substantially across the system. Since specific stretches of the river may exhibit distinct characteristics, developing individual models for the characteristically different reaches may provide a more representative approach than applying a single model across the entire study reach.

The preliminary LA River data also demonstrated greater variability in DO than was observed at the Botanical Garden, including an isolated DO value of 14.68 mg/L. Although this value may be influenced by instrumentation error, it also highlights the possibility that site-specific characteristics contribute to differences in DO throughout the river. More spatially resolved field measurements could therefore help distinguish localized DO variability from measurement uncertainty and provide more appropriate inputs for future model development.

In addition to carbonaceous biochemical oxygen demand, future modeling efforts could incorporate additional oxygen sink terms relevant to urban river systems. These may include nitrogenous oxygen demand associated with treated effluent discharged from the Donald C.

Tillman Water Reclamation Plant, as well as photosynthesis and respiration associated with the algal communities observed throughout the river. Including these processes may better represent the controls on dissolved oxygen dynamics within the Sepulveda Basin and improve the predictive capability of future modeling efforts.

5.4 Limitations

Several limitations should be considered when interpreting the results of this study. First, the Streeter-Phelps model is simplified to a 1D system, which neglects potential diffusion and dispersion dynamics. Model performance was evaluated by comparing instrumentation-measured DO to modeled DO at multiple locations along the stream. Because stream velocity is an important input to the model, uncertainty in measured velocity and its spatial variability may influence model predictions.

A second limitation is the representation of oxygen sinks in the current model. This study identified potential oxygen sink terms that may provide better correlation between modeled and observed results; however, further laboratory and field study would be needed to quantify and incorporate additional sink terms such as sediment oxygen demand (SOD) and photosynthesis and respiration dynamics. The initial BOD concentration also represents an important source of uncertainty. The study used a preliminary BOD test to estimate the initial oxygen demand. Implementing a more robust, laboratory-based BOD test may provide a more accurate estimate of the initial BOD and improve model performance.

In addition, the empirical formulations used to estimate model parameters provide approximations of complex physical processes. These formulations may not fully represent site-specific conditions. These parameter uncertainties may become increasingly important when applying the framework to more complex urban systems, where hydraulic conditions and oxygen sources and sinks can vary substantially.

6 Conclusion

This study evaluated the applicability of the Streeter–Phelps model (SPM) for representing dissolved oxygen (DO) dynamics in an urban stream and examined how the framework could inform future modeling of more complex urban waterways. The UCLA Botanical Garden provided a controlled setting for evaluating the model, while preliminary measurement data from the Los Angeles River provided insight into the additional challenges associated with applying the framework to a more complex urban river system. In the Botanical Garden, widely varying velocities demonstrated the influence of the pump networks that recirculate water throughout the stream. In the Sepulveda Basin of the Los Angeles River, variable conductivity and velocity reflected a more hydraulically complex system with less controlled conditions. The sensitivity analysis revealed that model performance was most sensitive to the reaeration coefficient, emphasizing the importance of accurately representing hydraulic conditions when modeling DO dynamics. Calibration showed that perturbing the initial BOD produced a closer resemblance to observed DO patterns. Since the system is heavily influenced by reaeration, a sufficiently large initial oxygen demand was needed to reproduce the observed variability in modeled DO. The resulting model captured the general highs and lows observed in the field-measured DO, but required a substantially higher initial BOD concentration than would be expected for the study system. This suggests that the current SPM formulation does not fully represent the distributed oxygen demands present within the stream.

The application to the Los Angeles River highlights the need for site-specific monitoring, parameterization, and model development when extending DO modeling to larger urban waterways. The variable hydraulics and water quality conditions observed in the Sepulveda Basin suggest that the river may require models developed for distinct sections of the system rather than treating the entire reach as a single uniform system. Additional oxygen sink terms, such as nitrogen oxygen demand and biological processes associated with algal production and respiration, could better represent the distributed pollution and oxygen dynamics of the LA River.

Further work should apply the SPM to hydraulically distinct sections of the Los Angeles River and evaluate whether additional processes improve model performance under the more complex conditions of the river. Future work may also involve greater collaboration with an established laboratory to perform the necessary testing to quantify and incorporate additional oxygen sink terms, such as sediment oxygen demand and photosynthesis and respiration dynamics. These additions may help improve the model's ability to represent the more complex oxygen dynamics expected within the LA River and other complex urban waterways.

Overall, this study demonstrates both the utility and limitations of the Streeter–Phelps framework for representing DO dynamics in urban streams. The framework provides a useful baseline for evaluating the relative influence of key parameters and for simulating stream response to an organic load of a similar magnitude, but its assumptions may limit its ability to represent spatially distributed oxygen sources and sinks in heterogeneous urban waterways. Incorporating additional oxygen-demanding processes will be important for improving model representation and applying the model to more complex systems such as the Los Angeles River.

Appendix

Working Model

Prior to applying the numerical Streeter–Phelps model to the UCLA Botanical Garden stream, a series of verification tests were conducted to confirm that the model reproduced the expected physical behavior under simplified conditions. These tests isolate the effects of individual model parameters and demonstrate that the numerical implementation is consistent with the governing dissolved oxygen mass balance.

Constant Dissolved Oxygen with No Reaction Process

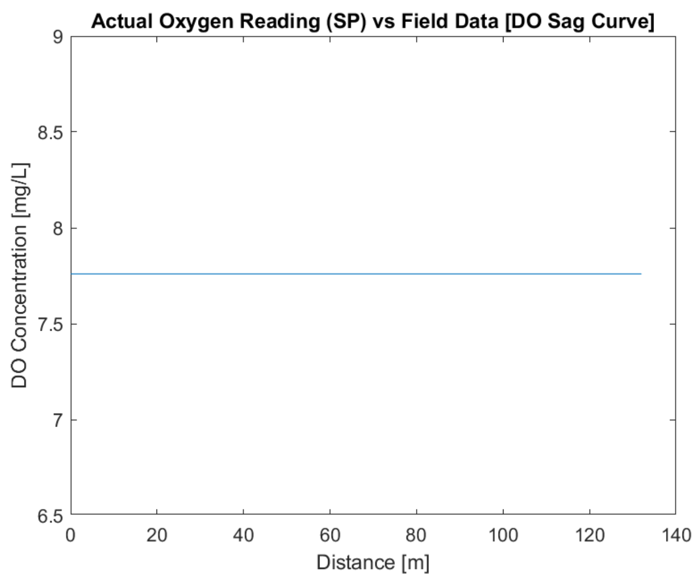

Figure 12. *Verification of dissolved oxygen conservation with deoxygenation and reaeration coefficients set to zero.*

To verify that the numerical implementation conserves dissolved oxygen in the absence of reaction processes, both the deoxygenation coefficient (k_1) and reaeration coefficient (k_2) were set to zero. Under these conditions, no oxygen is consumed through biochemical oxygen demand and no oxygen is transferred from the atmosphere to the stream. Consequently, the dissolved oxygen concentration should remain constant throughout the modeled reach.

As shown in Figure A.1, the dissolved oxygen concentration remained at its initial value of 7.76 mg/L over the entire stream length, confirming that the numerical model conserves dissolved oxygen when no reaction terms are present.

Verification of Atmospheric Reaeration

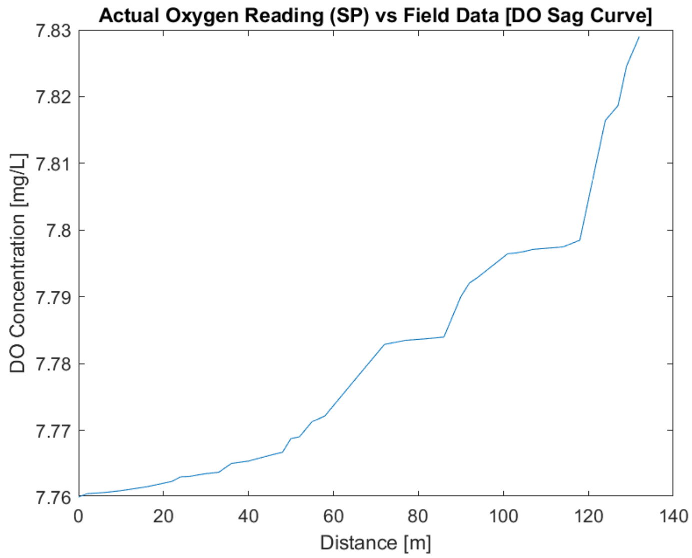

Figure 13. *Verification of atmospheric reaeration using a reaeration coefficient of 2 day^{-1} and segmented stream velocity.*

To verify the implementation of the reaeration term, the deoxygenation coefficient was set to zero ($k_1 = 0$ day^{-1}) while the reaeration coefficient was assigned a representative literature value of 2 day^{-1}. Under these conditions, dissolved oxygen is replenished from the atmosphere without any oxygen consumption.

Because oxygen demand is absent, dissolved oxygen is expected to increase toward saturation without exhibiting the characteristic SP oxygen sag. The selected reaeration coefficient represents a stream capable of replenishing approximately twice its existing oxygen deficit per day. Given the initial DO concentration of 7.76 mg/L and a saturation concentration of approximately 9.8 mg/L, the initial oxygen deficit is approximately 2 mg/L, making the selected coefficient appropriate for demonstrating the reaeration process.

Figure A.2 shows dissolved oxygen increasing monotonically toward saturation, confirming that the atmospheric reaeration term is implemented correctly.

Verification of BOD Decay

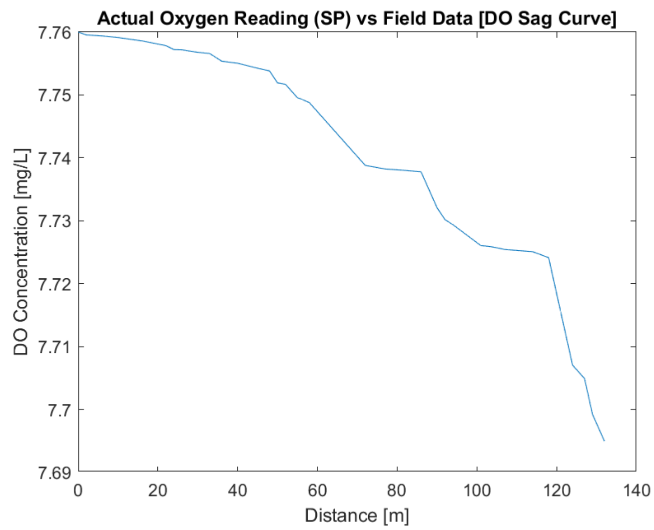

Figure 14. *Verification of first-order BOD decay using a deoxygenation coefficient of 0.05 day^{-1} and segmented stream velocity.*

The deoxygenation process was verified by assigning a representative literature value of 0.05 day^{-1} to the deoxygenation coefficient while setting the reaeration coefficient to zero (k_2 = 0 day^{-1}). Under these conditions, biochemical oxygen demand is consumed through first-order decay without atmospheric oxygen replenishment.

As expected, the biochemical oxygen demand decreases progressively along the stream as organic matter is oxidized. This behavior is illustrated in Figure A.3, confirming that the first-order BOD decay term is functioning as intended.

Development of DO Sag

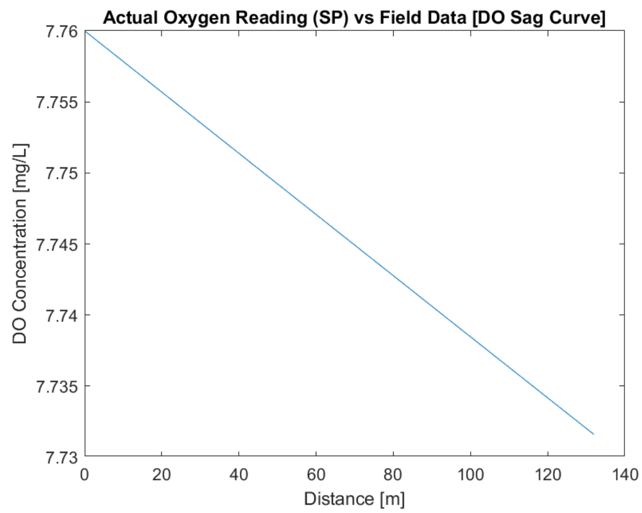

Figure 15. *Evaluation of dissolved oxygen sag development under representative hydraulic and reaction conditions.*

After verifying the individual reaction processes, simulations were performed to determine whether the numerical model could reproduce the characteristic Streeter–Phelps dissolved oxygen sag.

Representative parameter values were selected to promote oxygen depletion: Initial BOD = 10 mg/L, k_1 = 0.1 day^{-1}, k_2 = 0.2 day^{-1}, Velocity = 0.01 m/s. These conditions increase oxygen demand while reducing atmospheric reaeration and stream velocity, thereby maximizing the opportunity for dissolved oxygen depletion.

Despite these conditions, Figure A.4 shows that no visible oxygen sag developed within the 20-m study reach. This behavior is consistent with the physical characteristics of the Botanical Garden stream rather than a limitation of the numerical model. Even at the relatively slow observed velocities, travel time through the modeled reach is only on the order of 0.002 - 0.02 days, which is insufficient for biochemical oxygen demand decay and atmospheric reaeration to compete long enough to produce the classical SP sag.

Effect of Stream Length on Oxygen Sag Development

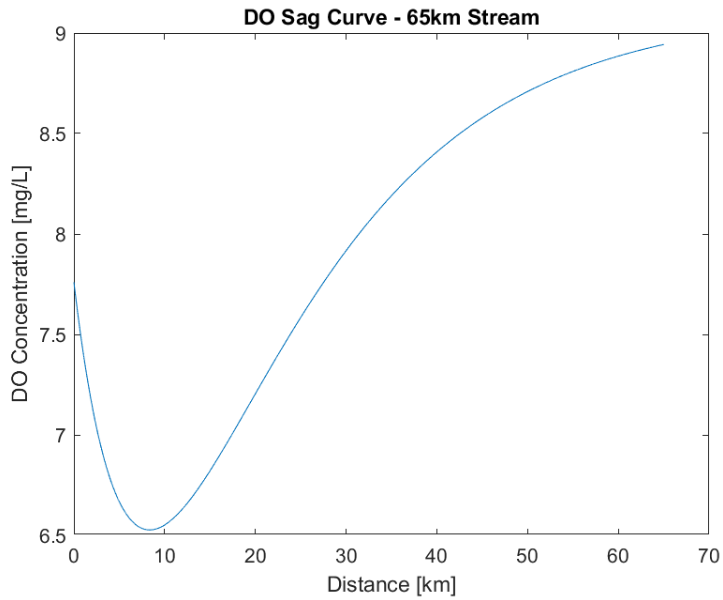

Figure 16. *DO sag development following extension of the modeled stream length.*

To further demonstrate that the absence of an oxygen sag is controlled by stream length rather than model formulation, the stream length was extended while maintaining representative hydraulic and reaction parameters.

As shown in Figure A.5, extending the stream allows sufficient travel time for oxygen consumption and atmospheric reaeration to compete, resulting in the development of the characteristic dissolved oxygen sag curve. Under the hydraulic conditions representative of the Botanical Garden stream, appreciable oxygen depletion does not occur until approximately 10 km downstream. Because the actual stream is only approximately 140 m long, the study reach captures only the initial, nearly linear portion of the Streeter–Phelps curve before significant oxygen depletion can occur.

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