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ON THE SPIN OF K MESONS FROM THE ANALYSIS OF ANTIPROTON ANNIHILATIONS IN NUCLEAR EMULSIONS

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Jack Sandweiss

(Thesis)

October 31, 1956

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ABSTRACT

The antiproton-nucleon annihilation process has been shown to proceed primarily through pion production with occasional emission of K mesons. The fraction of stars yielding K - $\overline{K}$ pairs (conservation of "strangeness" is assumed) depends upon the spin, I, of the K mesons. A statistical-theory method has been developed for analyzing the annihilation data for information regarding the K-mesons' spin. The theory employed is essentially that of Fermi; however, conservation of angular momentum has been included and the interaction volume is treated as an adjustable parameter. We find I = 0 most likely for the K mesons but, with present data, cannot exclude higher values. Under favorable circumstances this analysis may also yield information about the number of distinct types of K mesons. Various experimental tests have been devised to investigate the validity of the assumptions of the statistical-theory analysis. The existing data are consistent with the validity of these assumptions.

I. INTRODUCTION

A program for the search for and the study of antiprotons in nuclear emulsions was initiated[1,2] concurrently with the counter experiment at the Berkeley Bevatron that demonstrated the existence of antiprotons.[3] The first aim of the emulsion program was to provide the proof for the annihilation process. Once this was accomplished,[4] the emphasis was shifted to a study of the annihilation process and the antiproton interactions in nuclear emulsion. This report is an outgrowth of recent investigations of these subjects.[5]

The antiproton-nucleon annihilation process has been shown to proceed primarily through pion production with occasional emission of K mesons.[5] The characteristics of the annihilations involving K mesons--in particular, the probability for their emission--in general depend upon their spin. This suggests that the study of antiproton-nucleon annihilations may yield some evidence concerning the spin of the K mesons. We have analyzed the annihilation process according to the statistical

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[1] Chamberlain, Chupp, Goldhaber, Segrè, Wiegand, Amaldi, Baroni, Castagnoli, Franzinetti, and Manfredini, Phys. Rev. 101, 909 (1956); and Nuovo Cimento 3, 447 (1956).

[2] Stork, Birge, Haddock, Kerth, Sandweiss, and Whitehead, Search for Antiprotons, UCRL-3505, Sept. 1956; also Phys. Rev. (to be published).

[3] Chamberlain, Segrè, Wiegand, and Ypsilantis, Phys. Rev. 100, 947 (1955).

[4] Chamberlain, Chupp, Ekspong, Goldhaber, Goldhaber, Lofgren, Segrè, Wiegand, Amaldi, Baroni, Castagnoli, Franzinetti, and Manfredini, Phys. Rev. 102, 921 (1956).

[5] Barkas, Birge, Chupp, Ekspong, Goldhaber, Goldhaber, Heckman, Perkins, Sandweiss, Segrè, Smith, Stork, Van Rossum, Amaldi, Baroni, Castagnoli, Franzinetti, and Manfredini, to be published in Physical Review.

theory. The theory employed is essentially that proposed by Fermi,[6] but with two important modifications. Conservation of angular momentum has been included and the interaction volume Ω, which Fermi takes as $\sim \frac{4}{3}\pi \left(\frac{\hbar}{M_\pi c}\right)^3$, has been treated as a completely adjustable parameter.

This method for studying the spin of K mesons is in a sense complementary to the Dalitz analysis for the spin of the Tau meson.[7] The Dalitz analysis has no way of excluding high spins, whereas the statistical-theory analysis of annihilation stars is, in principle, capable of measuring the spin regardless of its magnitude. In actuality we shall show that the method can determine the spin only if it happens to be one or zero. For higher spins it can only show that the spin is greater than one. Also, of course, this method is applicable not only to τ mesons but also to all K mesons emitted in antiproton-nucleon annihilations.

Finally, we note that the probability for producing a K meson depends upon the number of different types of K mesons. Thus, this analysis should, in principle, be able to give some evidence on this point. We shall see that under favorable circumstances this is indeed so.

The crucial question of the validity of the statistical-theory analysis can be answered only by experimental test. We have devised a rather complete series of such tests, and they are presented in Section IV.

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[6] E. Fermi, Progr. Theoret. Phys. (Japan) S, 570 (1950).

Application to the annihilation process: R. Gatto, Nuovo Cimento 3, 468 (1956); G. Sudarshan, Phys. Rev. 103, 777 (1956) [We found that in the Sudarshan paper the factor $(0.945\frac{\Omega}{\Omega_0})^{N-1}$ occurring in Formula (4) is in error and should read $(5.2\frac{\Omega}{\Omega_0})^{N-1}$, and consequently the calculations presented were actually made for an interaction volume of (0.19) $4/3\,\pi\,(\frac{\hbar}{M_\pi c})^3$];

S. Belenky, V. Maximenko, A. Nikishov, and I. Rosental, paper presented at the Moscow Conference on High-Energy Physics, May 1956.

[7] R. H. Dalitz: Proc. Phys. Soc. 64, 710 (1953);

R. H. Dalitz: Phil. Mag. 44, 1068 (1953);

R. H. Dalitz: Phys. Rev. 94, 1046 (1954).

II. STATISTICAL-THEORY ANALYSIS OF ANNIHILATION STARS

For convenience, we list the assumptions that have been made in this analysis.

1. Strangeness[8,9] is conserved. Because there is not enough energy in annihilations at rest to form four K mesons, the number of K mesons emitted in annihilations at rest must be either zero or two. So far as the data of Reference 5 are concerned, even the annihilations in flight have barely enough energy to produce four K particles.

2. Total isotopic spin is conserved, i.e., charge independence holds.

3. The square of the matrix element for the annihilation into N_π pions and N_K K mesons can be represented by const. x $\Omega^{N_\pi+N_K-1}$. This is the basic assumption of Fermi's statistical theory, in which Ω is the interaction volume and is taken to be $\sim \frac{4}{3}\pi\left(\frac{\hbar}{M_\pi c}\right)^3$. We adopt a more phenomenological approach and treat Ω as a completely adjustable parameter.

4. The K mesons have isotopic spin T = 1/2.[8]

5. The K mesons all have the same spin, I.

The last assumption is not essential to the analysis; however, it is the simplest and is suggested by the near equality of the masses of the various types of K mesons.[10] In Appendix II it is shown how the analysis can be readily extended to cover K mesons of different spins. Finally, we have considered both the case in which there is only one type of K meson (of course, with different decay modes), and the case in which there are two distinct types of K mesons, i.e., two T = 1/2 multiplets of "strange" bosons. That is, we have considered the two

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[8] M. Gell-Mann and A. Pais, Proceedings of the Fifth Annual Rochester Conference on High-Energy Physics, 1955 (Interscience, New York, 1955).

[9] T. Nakano and K. Nishijima, Progr. Theoret. Phys. (Japan) 10, 581 (1953).

[10] A. M. Shapiro, Revs. Modern Phys. 28, 164 (1956).

possible outcomes of the τ-θ dilemma,[11] i.e., τ mesons and θ mesons are the same or τ mesons and θ mesons are different.

We can write the probability, $P(N_\pi, N_K)$, for annihilation into N_π pions and N_K K mesons as

$$P(N_\pi, N_K) = \text{const.} \times \left(\frac{\Omega}{8\pi^3\hbar^3}\right)^{N_\pi+N_K-1} G_I G_T S \frac{dQ}{dW}. \qquad \text{(II-1)}$$

Equation (II-1) follows from the usual transition-rate formula and Assumption 3 above. The factor $\frac{dQ}{dW}$ is the volume in momentum space available to the system, subject to the conservation of energy and momentum:

$$\frac{dQ}{dW} = \int \vec{\delta}\,(\sum_i \vec{P}_i)\,\delta(W - \sum_i E_i) \prod_i d^3\,P_i\,. \qquad \text{(II-1a)}$$

In equation (II-1a) the index i runs over the $N_\pi + N_K$ particles in the system, P_i and E_i are the momentum and total energy, respectively, of the i^{th} particle, W is the total energy of the system (for annihilation at rest $W = 2Mc^2$, M = nucleon mass). All the momentum-space integrals have been calculated by means of the saddle-point approximation method of Fialho.[12]

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[11] The τ (which decays into three pions) and the θ (which decays into two pions) have been shown to have the same mass within experimental error.[10] Furthermore, the lifetimes of the τ^+ and θ^+ are the same within experimental error (V. Fitch and R. Motley, Phys. Rev. 101, 496 (1956), and Phys. Rev. (to be published); and Alvarez, Crawford, Good, and Stevenson, Phys. Rev. 101, 503 (1956), and Phys. Rev. (to be published)). The simplest explanation of the above similarities between τ and θ is that they are the same particle. However, the Dalitz analysis for the spin parity of the τ indicates that if the spin of the τ is less than five the τ and θ are different particles.[7] This is known as the "τ-θ or Dalitz Dilemma". For a summary of the present situation see the talks by Gottstein, Yang, and Dalitz in the Proceedings of the Sixth Annual Rochester Conference on High-Energy Physics (Interscience, New York, 1956).

[12] Gabriel E. A. Fialho, Phase-Space Calculations (Thesis), Columbia University, Nevis Report 22, Feb. 1956.

The factor S takes into account the indistinguishability of the pions,

$$S = \frac{1}{N_\pi!} . \tag{II-1b}$$

We note that if two K mesons are emitted they must have opposite strangeness and hence are distinguishable particles. The factor G_T is an isotopic-spin weight factor taking into account the different charge states available to the system. G_T is the number of ways a system of N_K particles, each with $T = 1/2$, and N_π particles, each with $T = 1$, can form a state of given total isotopic spin. If the antiproton annihilates with a proton the system is in a mixed state, total isotopic spins 1 and 0 occurring with equal probability. If the antiproton annihilates with a neutron the system is in a state of total isotopic spin 1. Tables I and II give G_T as a function of (N_π, N_K) for total isotopic spins of 0 and 1, respectively.

Table I

The isotopic-spin weight factor, G_T, as a function of (N_π, N_K) for a total isotopic spin of zero.

N_K	N_π							
	0	1	2	3	4	5	6	7
0	-	-	1	1	3	6	15	36
2	1	1	2	4	9	21	-	-

Table II

The isotopic-spin weight factor, G_T, as a function of (N_π, N_K) for a total isotopic spin of one.

N_K	N_π							
	0	1	2	3	4	5	6	7
0	-	-	1	3	6	15	36	91
2	1	2	4	9	21	51	-	-

For annihilation in nuclear emulsion we compute G_T according to the relative numbers of neutrons and protons present in emulsion. Since the neutron-to-proton ratio in emulsion is 1.2/1.0, we assume G_T is given by

$$G_T = \frac{17}{22} G_T (T = 1) + \frac{5}{22} G_T (T = 0) . \qquad \text{(II-1c)}$$

The factor G_I is the statistical weight factor for the K mesons' spin. If conservation of angular momentum is disregarded, G_I is given by

$$G_I = (2I + 1)^2 . \qquad \text{(II-1d)}$$

It is clear, however, that conservation of angular momentum may impose a serious restriction upon the allowed configurations of the spin system. Qualitatively, we expect the restriction to be most severe for high spins and (or) low values of the total angular momentum of the system. In Appendix I, we have made an estimate of G_I including conservation of angular momentum. The approximations are such that this estimate probably gives values of G_I somewhat lower than the true ones. Table III lists G_I as a function of the K mesons' spin, I.

Table III

Estimate of the spin weight factor G_I for different values of the K mesons' spin, I, when conservation of angular momentum has been included. For comparison, G_I without conservation of angular momentum is also shown.

I	0	1	2	3	4
G_I	1	6.9	13.8	14.3	14.6
$(2I + 1)^2$	1	9	25	49	81

We see from Table III that spins of 2 and higher are associated with essentially the same statistical weights. Consequently, this analysis can be used to determine the spin only if the spin is 0 or 1. If the spin is 2 or higher, this analysis shows only that the spin is greater than 1.

In all of this analysis we understand N_K to be the number of K mesons, regardless of their type. Thus, if there are two distinct types of K meson, i.e., two T = 1/2 multiplets of strange bosons, then $P(N_\pi, N_K = 2)$ must be multiplied by a factor of 2:

$$P(N_\pi, N_K) = \text{const.} \times (1 + \delta_{N_K, 2}) \left(\frac{\Omega}{8\pi^3 \hbar^3}\right)^{N_\pi + N_K - 1} G_I G_T S \frac{dQ}{dW} . \quad \text{(II-2)}$$

Formula (II-2) replaces (II-1) when we consider two families of K mesons. The factors G_I, G_T, and S are the same in both (II-1) and (II-2). The normalizing constant must be determined separately in each case from the condition

$$\sum_{N_\pi, N_K} P(N_\pi, N_K) = 1 . \quad \text{(II-3)}$$

The $P(N_\pi, N_K)$ have been calculated from Eqs. (II-1) and (II-2) with a range of values of the constant Ω, varying from $\Omega = \Omega_o$ to $\Omega = 100\ \Omega_o$, $\Omega_o = \frac{4}{3} \pi \left(\frac{\hbar}{M_\pi c}\right)^3$. The calculations have been made for G_I as given by (II-1d) and by Table III. For clarity we systematically enumerate the cases for which $P(N_\pi, N_K)$ has been computed:

1. One family of K mesons, G_I given by $(2I + 1)^2$ and a range of Ω; Ω_o to $100\ \Omega_o$;

2. One family of K mesons, G_I given by Table III, and a range of Ω; Ω_o to $100\ \Omega_o$;

3. Two famililies of K mesons, G_I given by Table III, and a range of Ω; Ω_o to $100\ \Omega_o$.

Although Case 1, above, is certainly not correct (it does not conserve angular momentum), it has been computed because it forms a very convenient basis for calculating other cases. The results for Case 1 are given in Appendix II.

A convenient summary of the $P(N_\pi, N_K)$ calculations is a diagram in which the average number of pions $\overline{N}_\pi$ is plotted against the probability P_K of emission of a pair of K mesons (regardless of type). For clarity we give the explicit definitions of $\overline{N}_\pi$ and P_K:

$$\overline{N}_\pi = \sum_{N_\pi, N_K} N_\pi \, P(N_\pi, N_K) ,$$

$$P_K = \sum_{N_\pi} P(N_\pi, N_K = 2) . \qquad \text{(II-4)}$$

On an $(\overline{N}_\pi, P_K)$ diagram the points corresponding to one choice for the K mesons' spin, I, and various choices for Ω form a line which we will call the iso-spin line for spin I. Conversely, we can consider the points corresponding to one value of Ω and various choices of the K mesons' spin, I. These points form a line which we call an iso-Ω line. Figure 1 shows the $(\overline{N}_\pi, P_K)$ diagram for Case 1, above. Both the iso-spin lines and the iso-Ω lines are indicated, and the experimental point (see Section III), with its errors, is shown for reference. We note that the experimental point corresponds to a value of Ω of $\sim 20\ \Omega_o$. The iso-Ω lines are easily seen to be straight lines.

$$\overline{N}_\pi = P_K \overline{N}_{\pi, K} + (1 - P_K) \overline{N}_{\pi, \pi} \qquad \text{(II-5)}$$

In Eq. (II-5), $\overline{N}_{\pi, K}$ denotes the average number of pions emitted in association with K-meson pairs, and $\overline{N}_{\pi, \pi}$ denotes the average number of pions emitted when no K mesons are produced. Because the factor G_I in Eq. II-1 and the factor $(1 + \delta_{N_K, 2}) G_I$ in Eq. II-2 are both unity for $N_K = 0$, the quantity $\overline{N}_{\pi, \pi}$ depends only upon Ω. Similarly, because G_I or $(1 + \delta_{N_K, 2}) G_I$ occurs in all the $P(N_\pi, N_K = 2)$, the quantity $\overline{N}_{\pi, K}$ depends only upon Ω. From Eq. II-5 we thus see that the iso-Ω lines are indeed straight lines and that their locations are the same for the three cases considered.

The results for Cases 2 and 3 are given in the next section along with a discussion of the status of current experimental work.

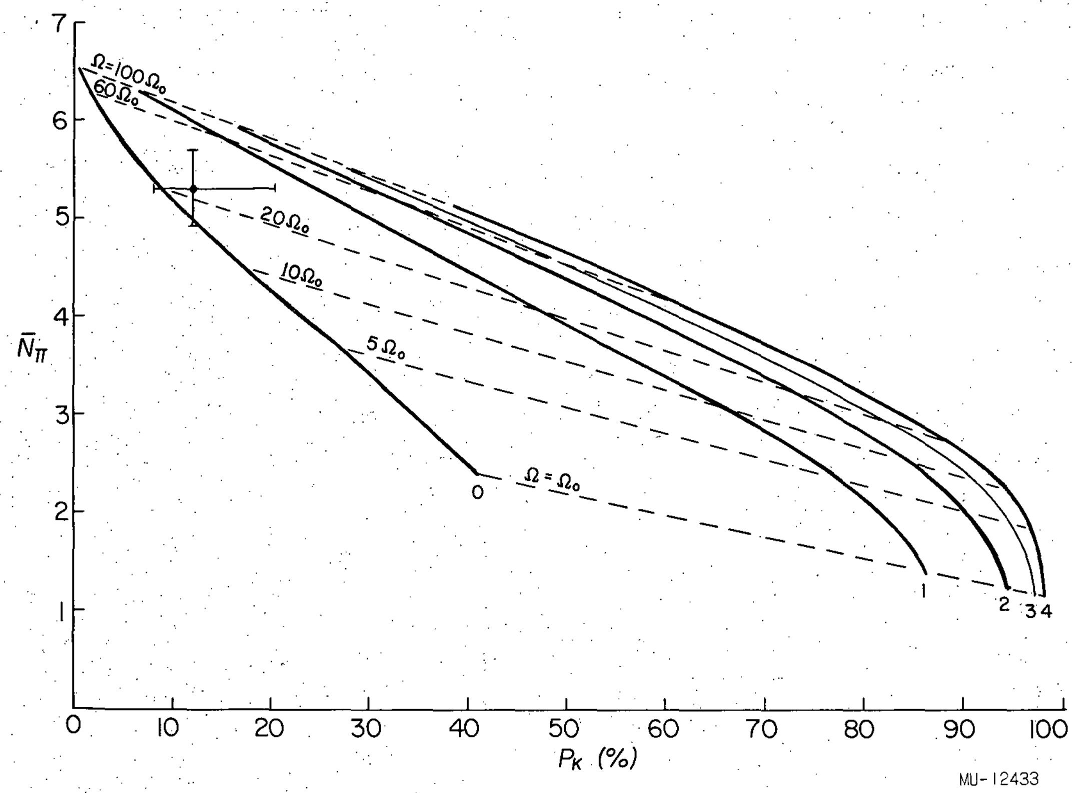

Fig. 1. The iso-spin lines (solid) and iso-Ω lines (dashed) computed from Eq. (II-1) with G_I taken as $(2I + 1)^2$ (Case 1). $\bar{N}_\pi$ is the average number of pions emitted and P_K is the probability for emitting a K-meson pair. The experimental point,[5] with its errors, is indicated.

III. COMPARISON WITH EXPERIMENT

The main portion of the data referred to in this analysis is drawn from the "Antiproton Collaboration Experiment" (hereinafter denoted by ACE).[5] In all, 36 examples of antiproton annihilation in nuclear emulsion were observed. The average number of pions emitted per star, $\overline{N}_\pi$, was found to be 5.3 ± 0.4, and in three stars there was evidence of charged K-meson emission. It was also shown in ACE that on the average one pion was absorbed by the nucleus on which the annihilation took place. In estimating P_K (the probability for emitting a K-meson pair) we have assumed that half the K mesons (those with strangeness of -1) are absorbed at the same rate as pions, while the remaining half (those with strangeness +1) are not absorbed at all.[13] Because the emulsion technique does not detect neutral particles it is necessary to make a correction for the cases in which both K mesons are neutral. To do this we need to estimate the fraction, F, of the total number of annihilations yielding K mesons, in which both K mesons are neutral. From the conservation of total isotopic spin this fraction can be computed for each of the annihilation modes $N_K = 2$, $N_\pi = 0, 1, \cdots 4$. Table IV gives the results of this computation for antiproton annihilations in nuclear emulsion. For the present work we have taken $F = 0.2$. With the above assumptions, we find P_K to be $11.1^{+8.2}_{-4.1}\%$. The errors quoted are chosen so that the true P_K has a 70% probability of lying within these limits.[14]

Figures 2 and 3 show the experimental "point" and the iso-spin lines (with conservation of angular momentum) for Cases 2 and 3 of Section II. That is, Fig. 2 corresponds to the existence of only one $T = 1/2$ multiplet of K particles and Fig. 3 corresponds to the existence of two $T = 1/2$ multiplets of K particles. We see that although the

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[13] Lannutti, Chupp, Goldhaber, Goldhaber, Helmy, Iloff, Pevsner, and Ritson, Phys. Rev. 101, 1617 (1956). See also the talks by S. Goldhaber and N. Dallaporta, Sixth Annual Rochester Conference on High-Energy Physics (Interscience, New York, 1956).

[14] E. C. Molina, Poissons Exponential Binomial Limit, First Edition (Van Nostrand, New York, 1942).

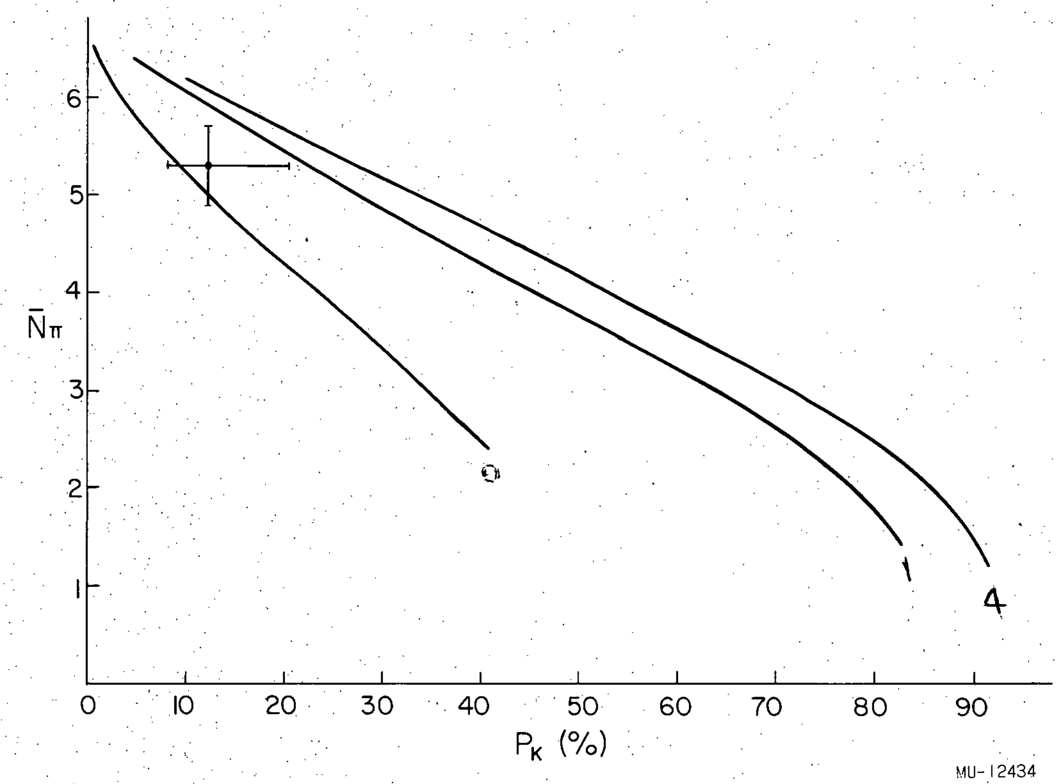

Fig. 2. The iso-spin lines, for spins of 0, 1, and 4, as computed from Eq. (II-1)--i.e., one family of K mesons--with conservation of angular momentum included in the estimates of the G_I. The experimental point, with its errors, is indicated.

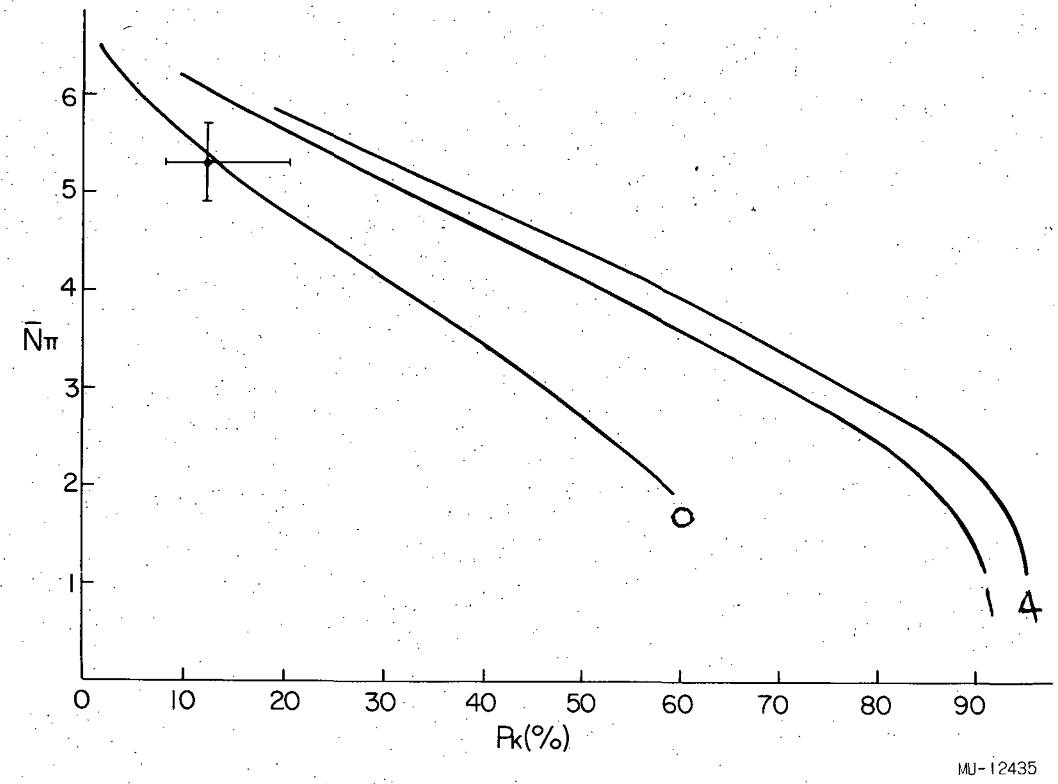

Fig. 3. The iso-spin lines (for spins of 0, 1, and 4) as computed from Eq. (II-2)--i.e., two "families" of K mesons--with conservation of angular momentum included in the estimates of the G_I. The experimental point, with its errors, is indicated.

present data favor spin 0 for the K mesons the data are not significant enough to exclude higher spins. The spacing between the spin 0 line and the spin 1 line is large enough so that if the spin is 0, this method is capable of demonstrating it with data that should be available in the not too distant future. We note that if the spin is 0 or 1, we may be able, with better data, to have evidence on the question of the number of distinct families of K mesons.

Table IV

The fraction F of the total number of annihilations yielding K mesons, in which both K mesons are neutral. The fraction F is given as a function of the number of pions N_π emitted in association with the K mesons. This table is computed for antiproton annihilations in nuclear emulsion.

N_π	0	1	2	3	4
F	0.114	0.193	0.154	0.157	0.204

IV. TESTS FOR THE VALIDITY OF THE STATISTICAL-THEORY ANALYSIS

In this section we examine the possible ways in which the foregoing analysis can be experimentally verified. In the preceding section we have studied the situation with regard to the quantities $\overline{N}_\pi$ and P_K. We have seen that fixing the values of $\overline{N}_\pi$ and P_K determines the parameter Ω, and that existing data imply $\Omega \sim 20\ \Omega_o$. There are a number of quantities which, in the statistical theory, are determined solely by the value of Ω and are independent, or nearly independent, of the assumptions about the spin of the K mesons or the number of distinct families of K mesons. By comparing the theoretical predictions for these quantities with experiment, we should have a sensitive test for the validity of the basic assumptions.

The first group of quantities we shall consider are the momentum and (or) energy spectra of the emitted pions and K mesons. The theoretical spectra are computed as follows: For annihilations into a definite set of $(N_\pi,\ N_K)$ the spectra of both the pions and the K mesons are determined by the relative volumes of momentum space available to the system when the momentum of the particle whose spectrum we are computing takes on different values. For example, the pion-momentum spectrum for the definite mode $(N_\pi,\ N_K)$ is given by

$$\frac{dN}{dP_\pi}(N_\pi,\ N_K) = \text{const.} \times P_\pi^{\ 2}\ \frac{dQ}{dW'}(\vec{P}',\ W')\ , \qquad \text{(IV-1)}$$

where

$$\vec{P}' = -\vec{P}_\pi$$

$$W' = W - \sqrt{P_\pi^{\ 2} C^2 + M_\pi^{\ 2} C^4}\ .$$

Here $\frac{dQ}{dW'}(\vec{P}',\ W')$ is the volume in momentum space available to the system $(N_\pi - 1,\ N_K)$ subject to the conditions that the total momentum be $\vec{P}'$ and the total energy W':

$$\frac{dQ}{dW'}(\vec{P}',\ W') = \int \vec{\delta}\,(\vec{P}' - \sum_i \vec{P}_i)\,\delta(W' - \sum_i E_i) \prod_i d^3P_i\ . \qquad \text{(IV-1a)}$$

The index i in Eq. (IV-1a) runs over the N_π - 1 pions and the N_K K mesons. The constant in Eq. (IV-1) has been chosen so that all the spectra have the same area. The pion-momentum spectrum for all annihilation modes is then determined by mixing the $\frac{dN}{dP_\pi}(N_\pi, N_K)$ according to their relative probabilities:

$$\frac{dN}{dP_\pi} = \text{const.} \times \sum_{N_\pi, N_K} P(N_\pi, N_K) \frac{dN}{dP_\pi}(N_\pi, N_K) . \qquad \text{(IV-2)}$$

The constant in Eq. (IV-2) is adjusted so that the area of the theoretical spectrum is the same as the area of the experimental spectrum. The momentum spectrum of the K mesons is, of course, computed in exactly the same fashion. The momentum-space integrals $\frac{dQ}{dW'}(\vec{P}', W')$ have been computed by means of the saddle-point approximation method of Fialho.[12] Finally, the energy spectra, if desired, can be obtained by transforming the momentum spectra.

The energy spectrum of all pions emitted in annihilation stars is reproduced from ACE in Fig. 4. The solid curve shows the predicted spectrum for $\Omega = 20\ \Omega_o$. The theoretical curve has been computed without taking into account the pions emitted in association with K mesons. Since such pions are a small fraction of the total number of pions and, in addition, have a spectrum quite similar to the over-all spectrum, the error caused by neglecting them in the theoretical spectrum is negligible. The agreement between the experimental and theoretical distributions is seen to be quite good.

We next examine the momentum spectrum of the K mesons. For $\Omega = 20\ \Omega_o$, the only annihilation modes involving K mesons and occurring with a nonnegligible probability are those of N_π equal to 1, 2, or 3. Figure 5 shows the K meson momentum spectra for these three modes. Figure 6 shows the experimental distribution and the theoretical distribution for $\Omega = 20\ \Omega_o$. The two spectra are consistent but the data are still too meager to allow any definite conclusions. Four of the K mesons in Fig. 6 were taken from ACE and two were found in a new exposure. One was found by Dr. Gerson Goldhaber[15] and one by

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[15] G. Goldhaber and L. R. Janneau, Example of a Positive $K_{\mu 2}$ Meson from an Antiproton Annihilation Star, UCRL-3565, Oct. 1956; and Bull. Am. Phys. Soc. (to be published).

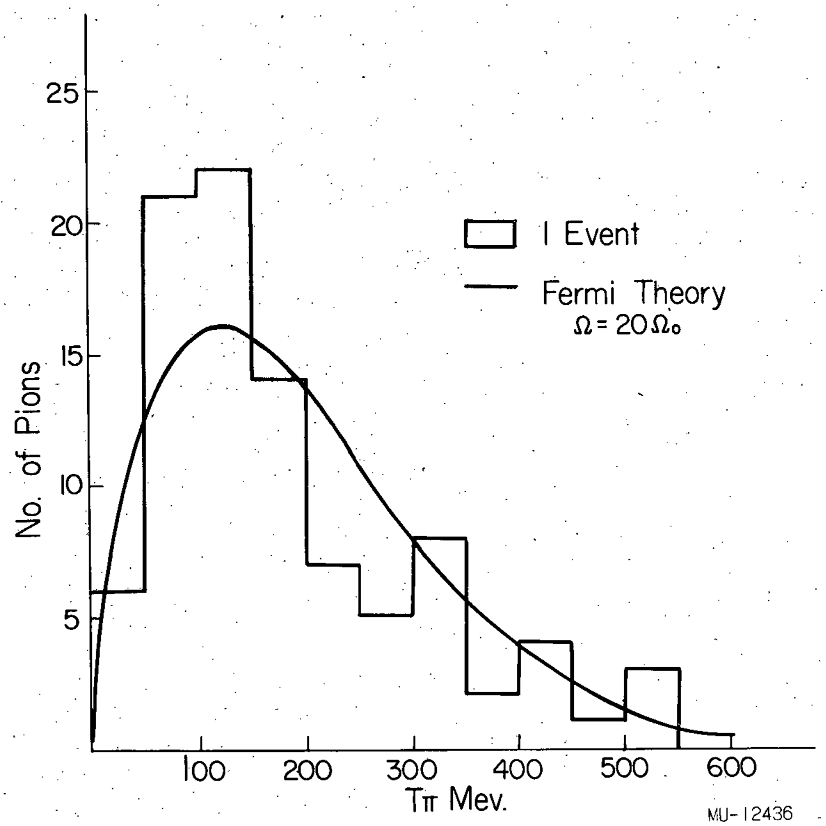

Fig. 4. The energy spectrum of all pions emitted in annihilation stars. (Experimental data are from Reference 5.)

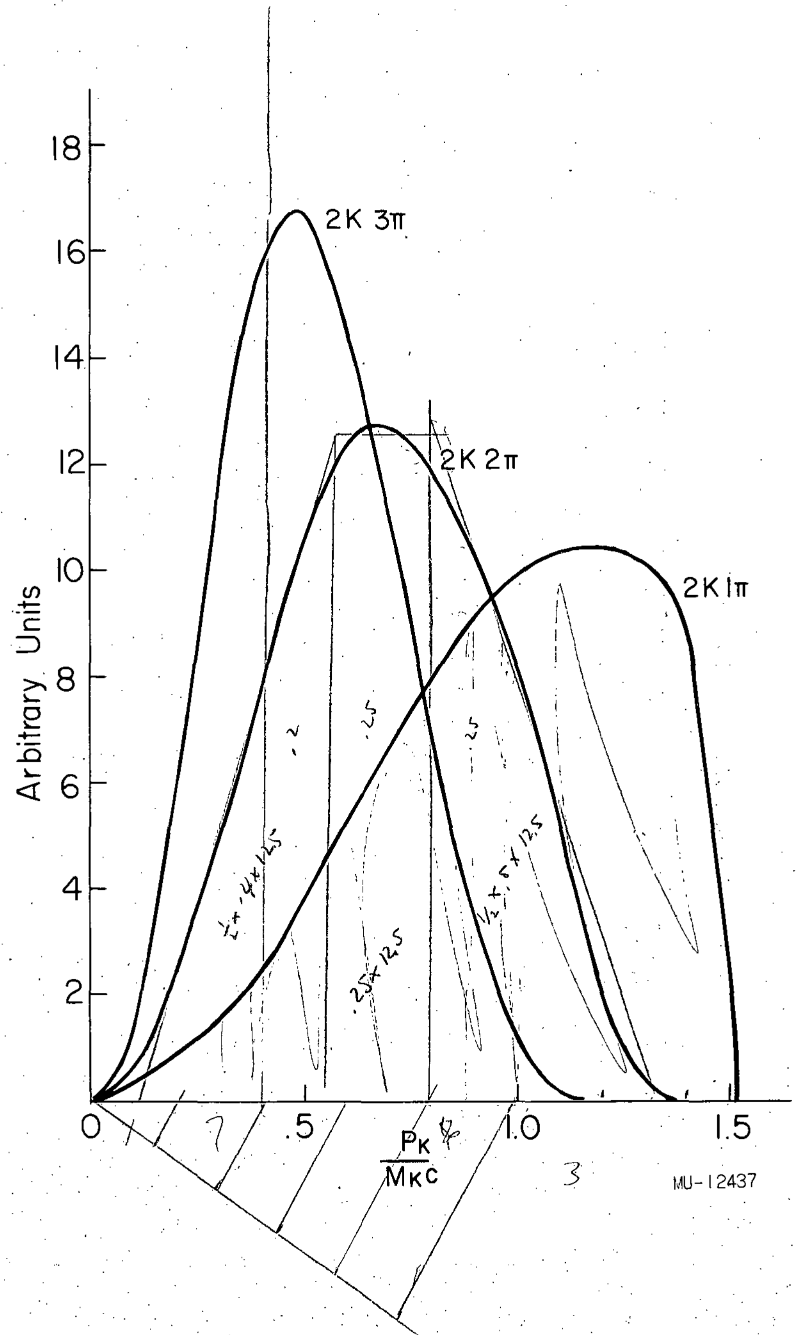

Fig. 5. The momentum spectra of K mesons emitted in the annihilation modes ($N_K = 2$, $N_\pi = 1, 2, 3$) as computed from the statistical theory. All curves are normalized to the same area.

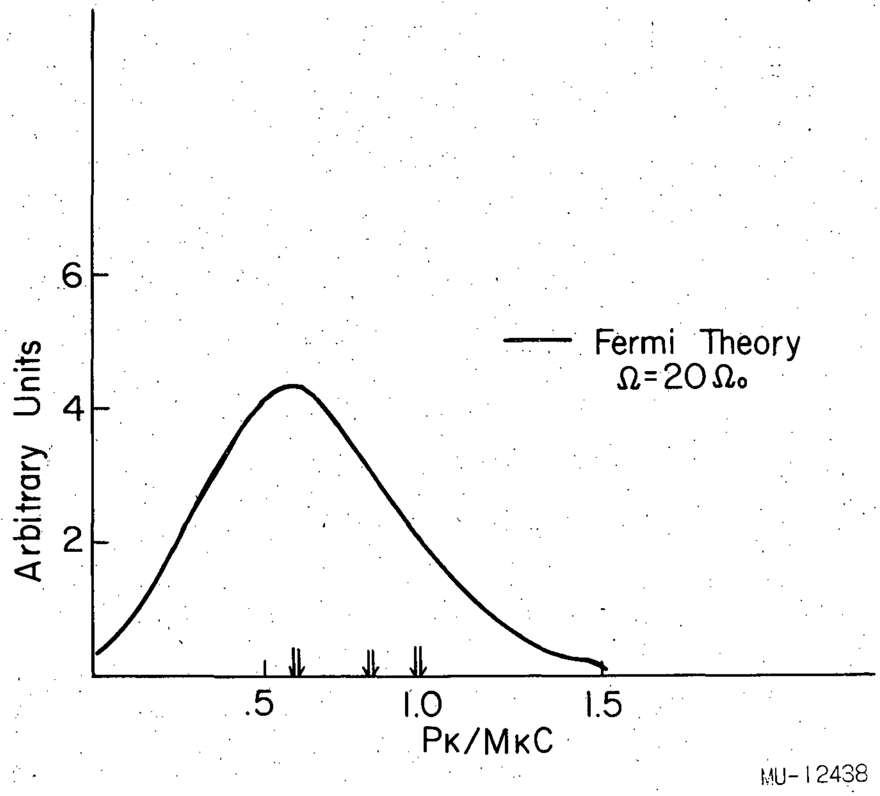

Fig. 6. The momentum spectrum of K mesons emitted in annihilation stars. (The data are from References 5, 15, 16.)

Drs. Sulamith Goldhaber and Warren Chupp.[16] The author is indeed grateful for permission to use their data in this work.

The last spectrum that we consider is the momentum spectrum of pions emitted in association with K mesons. Figure 7 shows the momentum spectra for the three modes $N_K = 2$, $N_\pi = 1, 2, 3$. Figure 8 shows the experimental distribution and the theoretical distribution for $\Omega = 20\ \Omega_o$. The shaded square represents a pion that may have scattered inelastically. Although the data are again somewhat meager, the agreement appears to be fairly good. The data are from ACE and from Reference 16.

Agreement between the theoretical and experimental momentum spectra is significant in that it implies that there are no strong resonances affecting the annihilation process. Such resonances, if present, would very likely invalidate the statistical-theory analysis. The present status can be summarized by stating that the pion spectra appear to be in good agreement and that the K-meson spectrum is inconclusive because of insufficient data.

The other quantities which in the statistical theory depend only upon the parameter Ω are the relative probabilities of the annihilation modes $N_K = 2$, $N_\pi = 0, \cdots 5$.

From the results presented in Appendix II, these probabilities can be computed for various values of Ω. Table V gives the distribution, in percent, for the different annihilation modes involving K mesons for $\Omega = 20\ \Omega_o$.

The comparison of these relative frequencies with experiment will constitute a test which is in a sense complementary to the test afforded by the momentum spectra. Comparison of the experimental and theoretical momentum spectra tests for strong interactions in states of definite energy. Comparison of the experimental and theoretical values for the relative frequencies of the various modes involving K mesons tests for strong interactions occurring for particular sets of emitted particles. Unfortunately, there are no data available at present for comparison with the above probabilities.

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[16] S. Goldhaber and W. W. Chupp, private communication.

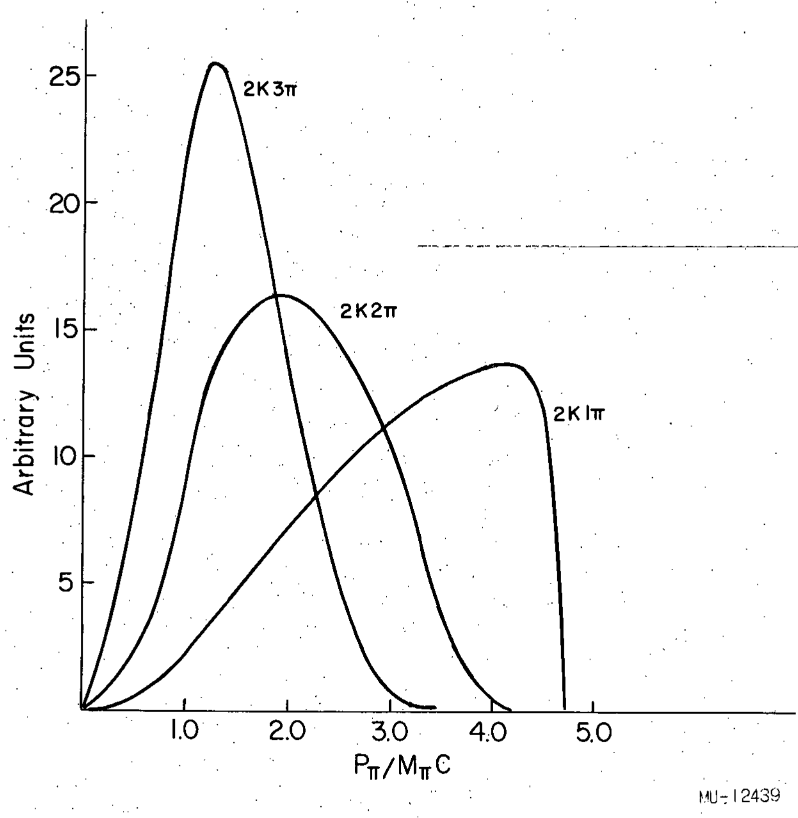

Fig. 7. The momentum spectrum computed from the statistical theory of pions emitted in the annihilation modes (N_K = 2, N_π = 1, 2, 3).

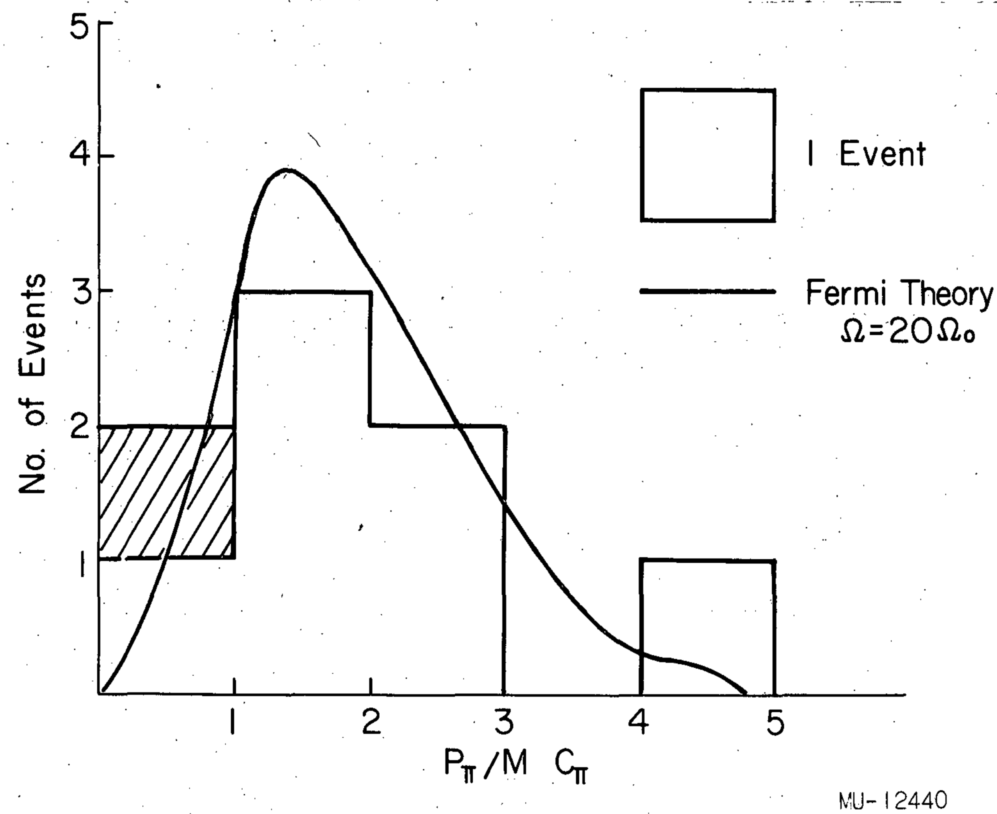

Fig. 8. The momentum spectrum of pions emitted in association with K mesons. The shaded square represents a pion that may have suffered an inelastic scattering in the residual nucleus. (The data are from References 5, 15, 16.)

Table V

Distribution in percent for the different annihilation modes involving K mesons for $\Omega = 20\ \Omega_o$.

N_K	N_π	Probability (%)
3	0	0.0
2	1	7.8
2	2	51.1
2	3	41.1
2	4	0.0
2	5	0.0

V. SUMMARY

We have proposed a method of measuring the spin of the K mesons by applying a statistical-theory analysis to antiproton-nucleon annihilation stars. We have shown that this method is most sensitive if the spin of the K mesons happens to be 0, and that it is still applicable, although more difficult to use (i.e., requires more accurate data), if the spin of the K mesons is 1. Because of the restriction imposed on the orientations of the spin system by the conservation of angular momentum this method cannot resolve a spin of 2 from higher spins. It has also been shown that if the spin is 0, this analysis may give some evidence as to the number of families of K mesons, i.e., the number of distinct T = 1/2 multiplets of "strange" bosons. It is interesting to note that the present status of the Dalitz analysis for the spin-parity of the τ meson[11, 17] indicates that if the spin of the τ is less than 5 the τ and θ are not the same particle. However, the data are not yet good enough for the Dalitz analysis to completely determine the spin of the τ meson.

The existing data, as analyzed by this method, favor spin 0 for the K mesons but are still too meager to allow a definite conclusion.

We have also proposed a number of experimental tests of the statistical-theory analysis. These tests are particularly sensitive to strong resonances in the annihilation process, such as can seriously distort the characteristics of the process from those predicted by the statistical theory. The existing data show no signs of such resonances and are consistent with the validity of the statistical-theory analysis. Considerably more data are necessary, however, before the question of the validity of this analysis is settled. Of special interest are the momentum spectrum of the K mesons emitted and the relative frequencies of occurrence of the different annihilation modes involving emission of K mesons.

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[17] Roy P. Haddock, Analysis of Bevatron Tau Mesons (Thesis), UCRL-3580, Nov. 1956.

ACKNOWLEDGMENTS

I am indebted to Dr. Maurice Goldhaber for suggesting that the antiproton stars could be used to study the spin of the K mesons, and for several very stimulating discussions on the subject. Another important incentive to beginning this problem was the work of Dr. Donald H. Stork in connection with the statistical-theory analysis presented in the Antiproton Collaboration Experiment.[5] I am also grateful to Dr. Joseph V. Lepore, Dr. Steven Gasiorowicz, and Mr. Roy P. Haddock for several illuminating discussions concerning the estimation of the effect of including conservation of angular momentum in the statistical theory. I wish to thank Drs. Gerson Goldhaber, Sulamith Goldhaber, and Warren Chupp for permission to use their data prior to publication.

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APPENDIX I. THE SPIN WEIGHT FACTOR WITH CONSERVATION OF ANGULAR MOMENTUM

In the statistical theory all states available to the system (i.e., satisfying the various conservation laws) are considered to be equally probable. Thus the spin weight factor G_I that occurs in Eqs. (II-1) (II-2) represents the ratio of the number of states available to the system ($N_K = 2$, N_π) for K mesons of spin I to the number of states available to the system ($N_K = 2$, N_π) for K mesons of spin 0. To determine the G_I we thus need to count the number of states available to the system ($N_K = 2$, N_π) for various values of I. The number of available states depends not only upon I but also upon the total number of particles $N_\pi + N_K$, upon the oribital angular momenta of the individual particles, and upon the total angular momentum J of the system. We have considered only the system ($N_K = 2$, $N_\pi = 2$). This is justified because ($N_K = 2$, $N_\pi = 2$) is considerably more probable (Table V) than ($N_K = 2$, $N_\pi = 1$, 0) and because neglecting ($N_K = 2$, $N_\pi = 3$) can only cause us to underestimate the G_I. We note that underestimating the G_I has the effect of "moving" the iso-spin lines closer to the spin 0 line (which is unchanged, of course), so that the value of the K mesons' spin as determined by this analysis is an upper limit to the true value.

Next we have assumed that the maximum orbital angular momentum L that a $\left\{\begin{matrix}\text{pion}\\ \text{K meson}\end{matrix}\right\}$ may have is the angular momentum of the $\left\{\begin{matrix}\text{pion}\\ \text{K meson}\end{matrix}\right\}$ of average momentum, as determined by the momentum spectra for ($N_K = 2$, $N_\pi = 2$), at a distance $\left(\frac{\hbar}{M_\pi c}\right)$ from the center of mass of the system. Using the momentum spectra of Figs. 5 and 7, we find that the above assumption implies that the maximum orbital angular momentum for both pions and K mesons is 2. There is a further complication which arises because the two pions, if they happen to be of the same charge, cannot be in states of odd relative angular momentum. For this reason we have computed the G_I separately for the two cases (like pions or unlike pions). Via the enumeration of 48,661 different angular momentum states we have computed Table VI.

It can readily be shown from conservation of total isotopic spin (assuming T = 1 for pions and T = 1/2 for K mesons) that for the annihilation of antiprotons in nuclear emulsion the two pions will be identical 0.4 of the time. Using this and Table VI, we have computed Table VII.

Table VI

The statistical weight G_I for the system ($N_K = 2$, $N_\pi = 2$), as a function of the K mesons' spin I and the total angular momentum J, as determined from consideration of all states with orbital angular momentum up to and including 2.

	Like pions					Unlike pions				
J	S_K					S_K				
	0	1	2	3	4	0	1	2	3	4
0	1	5.96	9.81	11.38	11.54	1	6.46	10.39	11.96	12.09
1	1	7.29	12.89	15.05	15.40	1	7.27	12.41	14.45	14.72
2	1	7.07	13.37	16.37	16.61	1	7.33	13.69	16.54	17.23
3	1	8.21	17.41	23.65	25.48	1	8.00	17.09	21.83	23.56
4	1	8.80	21.82	33.07	39.05	1	8.95	21.62	33.91	39.01

Table VII

The statistical weight G_I for the system ($N_K = 2$, $N_\pi = 2$), as a function of the K mesons' spin I, and the total angular momentum J.

J	S_K				
	0	1	2	3	4
0	1	6.27	10.16	11.73	11.88
1	1	7.27	12.60	14.68	14.99
2	1	7.23	13.57	16.47	16.99
3	1	8.08	17.21	22.54	24.31
4	1	8.89	21.70	33.58	39.03

Bethe and Hamilton[18] have estimated that for antiproton annihilation on hydrogen, the annihilation most likely takes place from an S or P state. In complex nuclei the annihilation undoubtedly takes place from states of higher orbital angular momentum. However, the angular momentum, which must be considered for the above analysis, is the angular momentum of the emitted particles in their own center-of-mass frame. Because there are, as yet, no reliable estimates of this quantity we have assumed that the total angular momentum is the same as for annihilations with hydrogen. Table III of the text has been computed from Table VII with the assumption that total angular momenta of 0, 1, and 2 are all equally probable.

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[18] H. A. Bethe and G. Hamilton, Nuovo Cimento 4, 1 (1956).

APPENDIX II. PROBABILITY DISTRIBUTIONS FOR VARIOUS ANNIHILATION MODES

Tables VIII through XII give the probability $P(N_K, N_\pi)$ for annihilation into N_K K mesons and N_π pions, according to Eq. (II-1), with the spin weight factor G_I taken as $(2I + 1)^2$.

Table VIII

The probability $P(N_K, N_\pi)$ in percent for $G_I = 1$.

N_K	N_π	$\Omega = 1$	5	10	20	60	100
0	2	3.8	0.3	0.0	0.0	0.0	0.0
	3	37.4	13.2	4.6	1.0	0.0	0.0
	4	14.5	25.5	17.9	8.0	1.0	0.3
	5	2.9	25.8	36.1	32.3	11.6	6.2
	6	0.2	7.0	19.5	34.8	37.5	29.7
	7	0.0	0.7	4.2	14.8	48.0	63.2
2	0	5.9	0.4	0.0	0.0	0.0	0.0
	1	26.7	9.4	3.3	0.7	0.0	0.0
	2	8.3	14.6	10.2	4.6	0.5	0.0
	3	0.3	3.0	4.1	3.7	1.3	0.6
	4	0.0	0.0	0.0	0.0	0.0	0.0
	5	0.0	0.0	0.0	0.0	0.0	0.0

Table IX

The probability $P(N_K, N_\pi)$ in percent for $G_I = 9$.

N_K	N_π	$\Omega = 1$	5	10	20	60	100
0	2	0.9	0.1	0.0	0.0	0.0	0.0
	3	8.7	4.1	1.9	0.6	0.0	0.0
	4	3.4	8.0	7.4	4.6	0.8	0.3
	5	0.8	8.1	14.9	18.8	10.1	5.8
	6	0.0	2.2	8.1	20.2	32.7	29.1
	7	0.0	0.2	1.7	8.6	41.8	58.3
2	0	12.4	1.2	0.3	0.0	0.0	0.0
	1	55.8	26.6	12.3	3.9	0.2	0.0
	2	17.3	41.2	38.1	23.9	4.3	1.3
	3	0.7	8.3	15.4	19.3	10.4	5.2
	4	0.0	0.0	0.0	0.0	0.0	0.0
	5	0.0	0.0	0.0	0.0	0.0	0.0

Table X

The probability $P(N_K, N_\pi)$ in percent for $G_I = 25$.

N_K	N_π	$\Omega = 1$	5	10	20	60	100
0	2	0.3	0.0	0.0	0.0	0.0	0.0
	3	3.4	1.7	0.9	0.3	0.0	0.0
	4	1.3	3.4	3.4	2.5	0.7	0.2
	5	0.3	3.4	6.9	10.2	8.0	5.2
	6	0.0	0.9	3.7	11.0	25.9	26.1
	7	0.0	0.0	0.8	4.7	33.1	52.2
2	0	13.6	1.4	0.3	0.1	0.0	0.0
	1	61.2	31.1	15.7	5.8	0.5	0.1
	2	19.0	48.3	48.7	36.1	9.4	3.2
	3	0.8	9.7	19.6	29.2	22.8	13.0
	4	0.0	0.0	0.0	0.0	0.0	0.0
	5	0.0	0.0	0.0	0.0	0.0	0.0

Table XI

The probability $P(N_K, N_\pi)$ in percent for $G_I = 49$.

N_K	N_π	$\Omega = 1$	5	10	20	60	100
0	2	0.2	0.0	0.0	0.0	0.0	0.0
	3	1.9	0.9	0.5	0.2	0.0	0.0
	4	0.7	1.8	1.9	1.5	0.5	0.2
	5	0.2	1.8	3.8	6.1	6.1	4.5
	6	0.0	0.5	2.0	6.5	19.6	22.5
	7	0.0	0.0	0.4	2.8	25.1	45.1
2	0	14.5	1.5	0.4	0.1	0.0	0.0
	1	62.4	32.6	17.0	6.8	0.8	0.2
	2	19.4	50.6	52.7	42.1	14.0	5.4
	3	0.8	10.2	21.3	33.9	33.9	22.0
	4	0.0	0.0	0.0	0.0	0.0	0.1
	5	0.0	0.0	0.0	0.0	0.0	0.0

Table XII

The probability $P(N_K, N_\pi)$ in percent for $G_I = 81$.

N_K	N_π	$\Omega = 1$	5	10	20	60	100
0	2	0.1	0.0	0.0	0.0	0.0	0.0
	3	1.1	0.6	0.3	0.1	0.0	0.0
	4	0.4	1.1	1.2	1.0	0.4	0.2
	5	0.1	1.1	2.4	3.9	4.6	3.8
	6	0.0	0.3	1.3	4.2	14.9	19.1
	7	0.0	0.0	0.3	1.8	19.0	38.2
2	0	14.2	1.5	0.4	0.1	0.0	0.0
	1	63.6	33.3	17.6	7.3	0.9	0.2
	2	19.7	51.7	54.6	45.1	17.6	7.6
	3	0.8	10.4	22.0	36.4	42.5	30.8
	4	0.0	0.0	0.0	0.0	0.1	0.1
	5	0.0	0.0	0.0	0.0	0.0	0.0

The computation of $P(N_\pi, N_K)$ for the case in which the K mesons have different spins is simply the computation of Eq. (II-1) with a different set of G_I. If the new G_I are known, Tables VIII through XII can be used to facilitate the calculation. The method is to select that old G_I which is closest to the new one; then multiplication of the $P(N_\pi, N_K = 2)$ for G_I^{old} by the ratio G_I^{new}/G_I^{old} converts them into nonnormalized probabilities for the G_I^{new}. Renormalizing then gives the desired result.