Title

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Publication Date

1965-10-01

UCRL-16440

University of California

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Berkeley, California

Submitted to the Proceedings of the V International Conference on High Energy Accelerators, Frascati, 1965

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AEC Contract No. W-7405-eng-48

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The purpose of this paper is twofold: to introduce the reader to the subject of instabilities exhibited by relativistic particle beams, and to summarize the present state of our knowledge concerning these phenomena.

Most of the material in the first part of the paper is not new. It has been known to some specialists for a good many years; what is new is that the problems that can be solved are now of much more interest to the general community of accelerator physicists. Consequently, many accelerator physicists who have not paid much attention to these matters may now want to become informed; it is my hope that this paper will provide an introduction to the field.

The second part of the article consists of two sections. The first summarizes the experimental information presently available, with emphasis upon the degree to which it confirms or disagrees with theory. Our current level of understanding is delineated: considering the generality and reliability of the theoretical analysis as well as

* This work was done under the auspices of the U. S. Atomic Energy Commission.

the degree of experimental confirmation, the author expresses his opinion as to what can be considered relatively well established. The final section contains a discussion of subjects needing further investigation and, consequently, supplements the discussion of areas of understanding by describing the peripheral areas of uncertainty.

I. INTRODUCTION TO THE PHYSICS OF BEAM INSTABILITY

Section I.1 below consists of a categorization of the diverse phenomena associated with self-fields of relativistic particle beams. An important part of this section is an extended bibliography of the many theoretical papers on instabilities of beams in particle accelerators. Section I.2 discusses the two different mathematical methods that have been employed to analyze instabilities; Section I.3 consists of three examples that have been selected to demonstrate both the variety of physical phenomena and the methods employed for their analysis.

1. Categorization of Self-Field Phenomena

The physical phenomena associated with self-fields may be either those in which the self-field is static, or those in which the self-field takes on a dynamical behavior. In Table I, these two categories are listed with a number of different subcategories. Numerous references have been indicated in the table, including the majority of theoretical papers on the subject prior to this conference. Experimental papers have not been included; comparison with experiment will be made in Section II.1, and appropriate references given there. Similarly, contributions to this conference are not referenced, but are discussed in Section II.1. Although an effort has been made to make the bibliography relatively complete, surely many papers--especially in the non-English literature--have escaped this reviewer's attention; the bibliography should, nevertheless, serve as a useful guide to the literature.

The phenomena in which the self-field is static are basically simpler, as is evidenced by the historical priority of their investigation. Most of these effects are not instabilities and are included only for completeness and orientation of the reader. Phenomena of the class in which the self-field is dynamic are more difficult to envision and, in general, are associated with instabilities, or potential instabilities. Sometimes, as in the negative-mass instability, the self-field motion is rather simply described. (Here, for an initially uniform beam and in the frame of reference in which the unperturbed particles are at rest, the instability corresponds to an exponential growth of a small density fluctuation.) In other cases, such as the transverse coherent resistive instability, the self-field motion is most easily described in a frame of reference that is neither the laboratory frame nor the frame in which the particles are at rest. A mathematical approach (and associated physical reasoning) that concentrates on the particles, and does not ascribe dynamical variables to the self-field, is clearly not particularly convenient for the analysis of such cases.

2. Mathematical Methods

Two different methods have been employed to study self-field phenomena. The first is the Single Particle Motion approach, summarized in Fig. 1. In this method one assumes a current and charge distribution from which one computes self-fields and then determines single-particle motion. This method is particularly effective when the charge and current distributions are known, as, for example, in the instability studies of a single particle interacting with an intense beam of a storage ring. The method can be employed even when the charge and current distributions are not known, by making the calculation self-consistent. An example of this is given in Section I.3c, where the charge distribution is characterized by a few simple parameters which are easily determined self-consistently. This method is often difficult to apply in problems where the self-field has dynamical properties, and does not--in an obvious way--yield the phenomenon of Landau damping.

The second approach is the Collisionless Boltzmann Equation (or Vlasov Equation, or--in the USSR--the Kinetic Equation) method. This is a very powerful theoretical technique that has proved essential in the study of plasmas; it is equally effective when employed to study the instabilities of relativistic particle beams. More than that, it is a straightforward approach (it is "easy to use"), and the resulting suggestions for controlling instabilities are often simple in concept yet both unobvious and strikingly effective.

The essentials of the approach are indicated in Fig. 2. One can readily see that the method involves characterizing the properties of the system with a Hamiltonian that is a functional of the (unknown) distribution function. As in the Single Particle Motion approach, one must employ Maxwell's equations and Hamilton's equations. The new feature, in this approach, is the solution of the equation $d\psi/dt = 0$. The basis of this equation is well known, and amply discussed in the literature; it is just Liouville's theorem with the subtlety that the Hamiltonian is a functional of the distribution function itself. For long-range forces, in which case direct particle-particle collisions are unimportant compared with particle-particle interactions mediated by the self-field, this equation is a very good approximation.

For static self-field phenomena $\psi(q,p,t)$ is independent of time and the Boltzmann equation becomes a time-independent but nonlinear equation. This equation was first employed, in accelerator physics, to study longitudinal space-charge effects.[5] In this initial work attention was limited to distribution functions corresponding to a uniform density in a restricted region of synchrotron phase space; the analysis yielded self-consistent "bucket" shapes. Our interest is in instabilities, so no further attention will be devoted, here, to the problem of determining stationary distribution functions.

Almost all investigations of dynamical self-field effects have proceeded from the linearized Boltzmann equation. This is not really a compromising approximation, as our concern is normally not with the mode of development of an instability, but only with the criterion for its onset--which is given exactly even in linear approximation. Thus the linear theory is excellent for obtaining thresholds and for suggesting ways of avoiding instabilities; the growth rates, however, are valid only for small growth. Some nonlinear work is described in Ref. 13.

Dynamical studies require, first, a static solution $\psi_{eq}(q,p)$. Linearizing the Boltzmann equation by letting

$$\psi(q,p,t) = \psi_{eq}(q,p) + \psi_1(q,p,t) , \qquad (1)$$

one obtains

$$\frac{\partial\psi_1}{\partial t} + \frac{\partial\psi_{eq}}{\partial q}\left(\frac{dq}{dt}\right)_1 + \frac{\partial\psi_1}{\partial q}\left(\frac{dq}{dt}\right)_{eq} + \frac{\partial\psi_{eq}}{\partial p}\left(\frac{dp}{dt}\right)_1 + \frac{\partial\psi_1}{\partial p}\left(\frac{dp}{dt}\right)_{eq} = 0 , \tag{2}$$

where

$$\left(\frac{dq}{dt}\right)_{eq} = \frac{\partial H(q,p,\psi_{eq})}{\partial p}$$

$$\left(\frac{dp}{dt}\right)_{eq} = -\frac{\partial H(q,p,\psi_{eq})}{\partial q}$$

$$\left(\frac{dq}{dt}\right)_1 = \frac{\partial}{\partial p}\left[\frac{\partial H(q,p,\psi_{eq})}{\partial\psi_{eq}}\ \psi_1\right]$$

$$\left(\frac{dp}{dt}\right)_1 = -\frac{\partial}{\partial q}\left[\frac{\partial H(q,p,\psi_{eq})}{\partial\psi_{eq}}\ \psi_1\right] , \tag{3}$$

and the partial derivatives include differentiation of the q and p within ψ_{eq} and ψ_1. The equation is still (usually) a partial differential integral equation--and time dependent--but it is linear. This approach was first used to study the negative-mass instability,[10,11] an application discussed in detail below.

The reader can find some general comments concerning mathematical approaches in Ref. 29. Finally, it should be emphasized that Landau damping[30] is automatically contained in the linearized Boltzmann equation approach, as will be seen in the example to follow. Some of the mathematical complexities associated with Landau damping are discussed in Ref. 31, while Ref. 32 gives a particularly lucid--and expansive--discussion of the phenomenon. The reader first approaching this subject should find the articles by Landau[30] and Hereward[32] most illuminating.

3. Examples

In this section we discuss first a static self-field effect, employing the Single Particle Motion method. Then we study a dynamic self-field instability by use of the Collisionless Boltzmann Equation approach. Finally, in large part because of the interesting new physical results obtained, we study the transverse coherent resistive instability of a tightly bunched beam.

(a) Incoherent Transverse Space-Charge Limit by Single Particle Motion Method

Proceeding according to the general outline of Fig. 1, we first assume a charge and current distribution, which in this calculation is taken to be a uniform beam of circular cross section, with minor radius a and major radius R. The azimuthal direction is $\hat{\underset{\sim}{j}}$ and the vertical direction $\hat{\underset{\sim}{k}}$, so that

$$\rho = \begin{cases} \dfrac{Ne}{(2\pi R)(\pi a^2)} & \text{for} \quad r \leq a , \\[2ex] 0 & \text{for} \quad r > a , \end{cases}$$

$$\underset{\sim}{j} = \rho \beta c \, \hat{\underset{\sim}{j}} , \tag{4}$$

where the beam has been taken to have N particles of velocity βc. If we ignore (the negligible) effects of curvature associated with the major radius R, then Maxwell's equations imply

$$\underset{\sim}{\mathcal{E}}_{\text{self}} = 2\pi\rho(z\,\underset{\sim}{\hat{k}} + x\,\underset{\sim}{\hat{i}})$$

$$\underset{\sim}{\mathcal{H}}_{\text{self}} = 2\pi\rho\beta(z\,\underset{\sim}{\hat{i}} - x\,\underset{\sim}{\hat{k}}) . \qquad (5)$$

The Lorentz force equation, plus Hamilton's equations (in this simple case just $\underset{\sim}{F} = \gamma\, m_0\, \underset{\sim}{a}$), imply, for motion in the $\hat{k}$ direction,

$$\gamma\, m_0 \frac{d^2z}{dt^2} = e(\mathcal{E}_z - \beta\,\mathcal{H}_x)$$

$$\gamma\, m_0 \frac{d^2z}{dt^2} = 2\pi\rho e(1 - \beta^2)z - e\beta \left.\frac{\partial\mathcal{H}_0}{\partial z}\right|_{z=0} z . \qquad (6)$$

We have included the external field, of course; letting

$$Q_0^{\,2} = n \equiv \frac{R}{\mathcal{H}_0} \left.\frac{\partial\mathcal{H}_0}{\partial z}\right|_{z=0} , \qquad (7)$$

and changing to θ as the independent variable, we obtain

$$\frac{d^2z}{d\theta^2} + Q_0^{\,2}\, z = \frac{2\pi R^2 e\rho}{\gamma^3 m_0 \beta^2 c^2}\, z . \qquad (8)$$

Noting that the solution to Eq. (8) has a θ dependence of the form $\exp(i\,Q\,\theta)$, and introducing the classical particle radius $r_0 = e^2/m_0c^2$, we obtain

$$Q^2 - Q_0^2 = -\frac{R\, r_0\, N}{\pi\, a^2\, \gamma^3\, \beta^2}\,, \tag{9}$$

where $\gamma = (1 - \beta^2)^{-1/2} = E/m_0c^2$.

Finally, effects not explicitly in our Hamiltonian--namely, machine imperfections--limit the Q value to nonintegral and non-half-integral values, so that $Q^2 - Q_0^2$ is restricted. Letting $\Delta Q^2 \equiv Q^2 - Q_0^2$, we have the result, equivalent to that first obtained by Kerst:[1]

$$N = -\frac{\pi\, a^2\, \gamma^3\, \beta^2}{R\, r_0}\, \Delta Q^2\,, \tag{10}$$

with ΔQ^2 typically of the order of 0.25 (for non-AG devices).

It now remains to be demonstrated that the single-particle motion is consistent with the assumed charge and current distribution; in this case--if the effects of nonlinearities in the external field and the effect of the machine imperfections are ignored--it is true. This point is discussed more carefully in Refs. 2 and 3, where the Boltzmann equation method is employed.

The electrodynamics in the above calculation is rather poorly done; no effect of the surrounding media has been included. Simply by improving this aspect of the analysis one can arrive at the formula[4]

$$N = \frac{\pi\, h^2\, \gamma\, \Delta Q^2}{2Rr_0}\, Z^{-1}\,,$$

with

$$Z = \epsilon_1 \left[1 + \frac{(1 - \eta B\gamma^2)}{B(\gamma^2 - 1)} \right] + \epsilon_2 \frac{h^2}{g^2} + \frac{(1 - \eta B\gamma^2)\, h^2}{B(\gamma^2 - 1)\, b(a + b)} \quad , \tag{11}$$

appropriate to an elliptical beam of major radius a and minor radius b, between conducting walls (vacuum tank) with separation $2h$, and iron (magnet) surfaces of separation $2g$. The coefficients are, for parallel plane iron and conducting surfaces, $\epsilon_1 = \pi^2/48$ and $\epsilon_2 = \pi^2/24$. The coefficient η is the fraction of the beam neutralized, and B is the percent of the circumference occupied by beam. Details of the derivation, and coefficients for more complicated geometry, are given by Laslett;[4] it is sufficient to notice that the presence of the surrounding media can have significant consequences. For example, Eq. (11) implies that at high energy N increases only lineally with γ --while Eq. (10) (incorrectly) predicts a γ^3 dependence.

(b) Negative-Mass Instability by Collisionless Boltzmann Equation Method

We turn now to one of the most straightforward applications of the Boltzmann Equation method; namely, the study of small density fluctuations in an otherwise azimuthally uniform beam of particles. We consider, here, only the longitudinal degree of freedom and (guided by deeper insight) employ the azimuthal angle ϕ and its time derivative $\dot{\phi}$ as independent variables, even though they are not a set of canonically conjugate coordinates and momenta.

For this case $\psi_{eq}(\phi, \dot{\phi}) = \psi_{eq}(\dot{\phi})$, since the unperturbed beam is assumed to be azimuthally uniform but having a possible spread in particle energies. If we let

$$\psi(\phi, \dot{\phi}, t) = \psi_{eq}(\dot{\phi}) + \psi_1(\phi, \dot{\phi}, t) , \tag{12}$$

the linearized Boltzmann equation becomes

$$\frac{\partial \psi_1}{\partial t} + \dot{\phi} \frac{\partial \psi_1}{\partial \phi} + \left(\frac{d\dot{\phi}}{dt}\right)_1 \frac{\partial \psi_{eq}}{\partial \dot{\phi}} = 0 . \tag{13}$$

Hamilton's equations imply

$$\left(\frac{d\dot{\phi}}{dt}\right)_1 = 2\pi \frac{df}{dt} = 2\pi \left(\frac{df}{dE}\right)_{eq} \frac{dE}{dt} ,$$

$$\frac{dE}{dt} = 2\pi f/R \; e \; \mathcal{E} , \tag{14}$$

where f is the particle frequency, E is the particle energy, and $\mathcal{E}$ is the longitudinal electric field. In this problem $(d\dot{\phi}/dt)_{eq}$ is zero; that is, only the perturbed distribution has any associated field $\mathcal{E}$. Solving Maxwell's equations--details are given in Ref. 10--we find

$$\mathcal{E} \approx - \frac{eg}{\gamma^2 R^2} \frac{\partial}{\partial \phi} \int \psi_1(\phi, \dot{\phi}, t) d\dot{\phi} , \tag{15}$$

where g is a geometrical factor, which for a circular beam of minor radius a between conducting planes separated by a distance G is

$$g = 1 + 2 \ln\left(\frac{2G}{\pi a}\right) . \tag{16}$$

The formula for $\mathcal{E}$ should be rather evident: The integral is simply the charge at azimuth ϕ, and the field is proportional to the charge gradient; the factor R^{-2} is required on dimensional grounds, and the γ^{-2} takes account of the Lorentz contraction in the azimuthal direction. Combining these equations, we have

$$\frac{\partial\psi_1}{\partial t} + \dot{\phi}\,\frac{\partial\psi_1}{\partial\phi} - \frac{4\pi^2\left(f\,\frac{df}{dE}\right)_{eq} e^2 g\,\frac{d\psi_{eq}}{d\dot{\phi}}}{\gamma^2 R}\,\frac{\partial}{\partial\dot{\phi}}\int\psi_1\,d\dot{\phi} = 0\,, \tag{17}$$

which is a linear partial differential integral equation with three independent variables. But it can easily be solved! Assume ψ_1 is of the form

$$\psi_1(\phi,\dot{\phi},t) = \overline{\Psi}_1(\dot{\phi})\,e^{i(n\phi-\omega t)}\,, \tag{18}$$

where n is an integer (because of the boundary condition on ϕ), and ω is to be obtained from the equation. We find

$$i(n\dot{\phi}-\omega)\overline{\Psi}_1(\dot{\phi}) = -\frac{4\pi^2\left(f\,\frac{df}{dE}\right)_{eq} e^2 g\,i\,n}{\gamma^2 R}\,\frac{d\psi_{eq}(\dot{\phi})}{d\dot{\phi}}\int\overline{\Psi}_1(\dot{\phi})d\dot{\phi} = 0\,, \tag{19}$$

from which it is clear that

$$\overline{\Psi}_1(\dot{\phi}) = \frac{[\text{Constant}]}{n\dot{\phi}-\omega}\,\frac{d\psi_{eq}(\dot{\phi})}{d\dot{\phi}}\,. \tag{20}$$

Inserting this, we obtain--after canceling the constant--

$$1 = \frac{4\pi^2 e^2 gn\left(f\,\frac{df}{dE}\right)_{eq}}{\gamma^2 R}\int\frac{\frac{d\psi_{eq}(\dot{\phi})}{d\dot{\phi}}}{n\dot{\phi}-\omega}\,d\dot{\phi}\,, \tag{21}$$

which is a dispersion relation, i.e., an equation for ω as a function of n. We can see the implications of Eq. (21) by taking a simple example for ψ_{eq}, namely a beam with a uniform spread of particle energies within a band of full width ΔE. Thus, take

$$\psi_{eq} = \begin{cases} \dfrac{N}{(2\pi(2\Delta)} & \text{for} \quad \dot{\phi}_{eq} - \Delta < \dot{\phi} < \dot{\phi}_{eq} + \Delta , \\ 0 & \text{otherwise,} \end{cases} \tag{22}$$

where

$$2\Delta = 2\pi \left(\frac{df}{dE}\right)_{eq} (\Delta E) . \tag{23}$$

Clearly $d\psi_{eq}/d\dot{\phi}$ contains two δ functions; the integral in Eq. (21) is trivial, and one readily obtains

$$\frac{\omega}{n} = \dot{\phi}_{eq} \pm \left[\frac{2\pi e^2 gN}{\gamma^2 R} \left(f \frac{df}{dE} \right)_{eq} + \pi^2 \left(\frac{df}{dE} \right)^2_{eq} (\Delta E)^2 \right]^{1/2} . \tag{24}$$

One can see that if (ΔE) is very small and df/dE is negative, then ω will have a complex term and the perturbation will grow exponentially. On the other hand, there exists an energy spread (ΔE) that will stabilize the beam for any given intensity N, which is just Landau damping. The physics is described in detail in Refs. 10 and 11, along with an expansive discussion, in Ref. 10, of the Landau damping--namely, the proper definition of the singularity in Eq. (21) as well as the dependence of the result upon the choice of $\psi_{eq}(\dot{\phi})$.

The reader should appreciate--from the block diagram of Fig. 2, and this example--the general features of an instability calculation using the Boltzmann Equation Method. Often, in the literature, the basic simplicity is obscured by very involved mathematical details. Take, for example, the rather impenetrable paper (LNS)[16] on the transverse resistive wall instability for a uniform beam. The analysis is fundamentally no more complicated than in the above example: One assumes a circular beam of minor radius a , inside a circular tank of minor radius b , having walls of conductivity σ . The equilibrium distribution function is chosen to be of the form

$$\psi_{eq} = \frac{N}{(2\pi)^2 R} f(x, E) , \qquad (25)$$

where $f(x,E)$ describes the distribution of betatron amplitudes and energy in the beam, and is normalized

$$\int f(x,E)x \, dx \, dE = 1 . \qquad (26)$$

Assumming a wave of the form $\exp[i(n\theta - \omega t)]$, one finds a dispersion relation

$$1 = Q_0 \omega_0 [U + (1 + i)V]I , \qquad (27)$$

where ω_0 is the average revolution frequency; U and V come from Maxwell's equations,

$$U = -\frac{e^2 N}{2\pi Q_0 \omega_0 \gamma^3 m_0 R a^2}\left(1 - \frac{a^2}{b^2}\right)$$

and

$$V = \frac{e^2 N \beta}{\pi Q_0 \gamma m_0 b^3 \omega}\left(\frac{|\omega|}{8\pi\sigma}\right)^{1/2}, \tag{28}$$

and all other symbols have been defined previously. The dispersion integral I is

$$I = \int \frac{\frac{\partial f(x,E)}{\partial x} x^2 \, dx \, dE}{[\omega - n\Omega(x,E)]^2 - [Q(x,E)\Omega(x,E)]^2}, \tag{29}$$

where $\Omega(x,E)$ is the circulation frequency and $Q(x,E)$ the Q value for a particle of betatron amplitude x and energy E $(\Omega \approx \omega_0)$. The theory is evidently similar--in structure--to the simpler problem, but the increased difficulty associated with solving both Maxwell's equations and the linearized Boltzmann equation should be emphasized.

It is not difficult to obtain from Eqs. (27), (28), and (29) the main results of LNS: If we take $f(x,E) = \delta(E)\delta(x)/x$, then

$$\omega = (n \pm Q_0)\omega_0 \mp (U + V + iV)\ ; \tag{30}$$

for $n > Q$ (V changes sign as ω does) and the lower choice of sign there is an instability with growth time

$$\tau_0 \equiv V^{-1} = \frac{\pi Q_0 \gamma b^3}{r_0 N c}\left[\frac{8\pi\sigma(n - Q_0)}{\beta c R}\right]^{1/2}. \tag{31}$$

As in the negative-mass problem, Landau damping can prevent an instability, and one can show[16] that the spread in the quantity $S \equiv [n - Q(x,E)]\Omega(x,E)$ required to prevent growth is $\Delta S \gtrsim |U| + V$.

(c) Transverse Resistive Instability for a Bunched Beam by Single Particle Motion Method

We consider, for simplicity, the case of a single bunch. Since the bunch is assumed rigid, the only dynamical variable is its transverse coordinate. The dynamics is very simple--much as in the first example above--but the electrodynamics is more complicated. Consequently we concentrate first on the solution of Maxwell's equations, following the analysis of Robinson.[21]

The important point--in fact the physical basis of the instability --is that in a resistive vacuum tank, fields due to a particle decay, after the particle has left, only very slowly in time. The decay is so slow that a bunch traveling about a circular accelerator returns soon enough to be subject to its own wake field. Clearly, depending upon its phase--relative to the wake field--the motion can be damped or undamped. We shall see this in detail, but first we must compute the wake field of an oscillating charge.

Consider a conducting medium, of conductivity σ , located above the $y = 0$ plane, and subject to the fields of a particle moving with velocity βc in the $\hat{k}$ direction. Within the conductor,

$$\frac{\partial \mathcal{B}_x}{\partial y} = \frac{4\pi}{c} j_z ,$$

$$\frac{\partial \mathcal{E}_z}{\partial y} = -\frac{1}{c}\frac{\partial \mathcal{B}_x}{\partial t}$$

$$j_z = \sigma \mathcal{E}_z , \qquad (32)$$

which clearly yields[33]

$$\frac{\partial^2 j_z}{\partial y^2} = -\frac{4\pi\sigma}{c^2}\frac{\partial j_z}{\partial t}, \tag{33}$$

with a solution

$$j_z = \frac{[\text{Constant}]}{t^{1/2}} \exp\left[-\frac{R}{4\beta c}\frac{y^2}{t}\right], \tag{34}$$

where $R = 4\pi\beta\sigma/c$. We may evaluate the constant in Eq. (34) by equating the integral of j_z (over y) to the product of $c/4\pi$ times the change in $\mathcal{B}_x$, obtaining

$$j_z = \frac{\mathcal{B}_{x0}}{4\pi^{3/2}}\left(\frac{Rc}{\beta}\right)^{1/2}\frac{1}{t^{1/2}} \exp\left[-\frac{R}{4\beta c}\frac{y^2}{t}\right], \tag{35}$$

where the field $\mathcal{B}_x$ has been taken as a step function that is zero for $t < 0$, and equal to $\mathcal{B}_{x0}$ for $t \geqslant 0$. Consequently the electric field at the metal surface due to a general time-dependent magnetic field is

$$\mathcal{E}_z(y=0,t) = \frac{1}{4\pi^{3/2}\sigma}\left(\frac{Rc}{\beta}\right)^{1/2}\int_{-\infty}^{t}\frac{\partial \mathcal{B}_x(t')}{\partial t'}\frac{dt'}{(t-t')^{1/2}}. \tag{36}$$

For a pulse of charge moving parallel to the surface, $\mathcal{B}_x = \beta\,\mathcal{E}_y$; while

$$\int_{-\infty}^{\infty}\frac{\partial \mathcal{E}_y}{\partial t'}\,dt' = 0 \tag{37}$$

and

$$\int_{-\infty}^{\infty} \mathcal{E}_y \, dt' = \frac{2Ne}{b\beta c} . \tag{38}$$

The last is valid for a pulse of length L inside a tube of radius b , when L >> b . Expanding the denominator in Eq. (36) for t large, and using Eqs. (37) and (38), yields

$$\mathcal{E}_z(y=0,t) = \frac{Ne}{bc} \left(\frac{\beta}{\pi Rc}\right)^{1/2} \frac{1}{t^{3/2}} . \tag{39}$$

This result, of Robinson, is valid for long times $(t \gg L/c)$, but because the conductor was assumed planar it becomes incorrect for time $t > Rb^2/\beta c$, which is a very long time;[19] thus Eq. (39) suffices for our purposes. Notice that $\mathcal{E}_z$ is falling off only algebraically in time.

An analogous calculation[21] for an oscillating charge having an amplitude $\xi \exp(i\omega t)$ in the $\hat{j}$ direction, moving in the $\hat{k}$ direction with speed βc, and passing the point of observation at time t' , yields

$$\mathcal{B}_x(t) = \frac{Ne \, \xi \, e^{i\omega t'}}{\pi \, b^3 \, \sigma^{1/2} (t - t')^{1/2}} \tag{40}$$

for the wake field. In this case $\mathcal{B}_x$ is larger than $\mathcal{E}_y$ in the asymptotic régime. Notice that $\mathcal{B}_x$ has the same phase as that of the particle at the moment when it passes the point of observation; subsequently $\mathcal{B}_x$ simply decreases slowly $(t^{-1/2})$ in time (not oscillating, for example). The range of validity of Eq. (40) is the same as that of Eq. (39).

We are now in a position to follow the Single Particle Motion approach, and quickly obtain results concerning the resistive instability. Assume the transverse motion is of the form

$$\xi \exp(i\, Q\, \omega_0\, t) \, , \tag{41}$$

where the value Q (presumably near to Q_0) is to be determined in the analysis. From Eq. (40), ignoring the major radius curvature of the vacuum tank (a very good approximation), the asymptotic wake field from the previous turn is proportional to

$$\frac{\xi \exp\left[i\, Q\, \omega_0 (t - 2\pi/\omega_0)\right]}{(2\pi/\omega_0)^{1/2}} \, . \tag{42}$$

Consequently, as in Eq. (6), but summing over all previous turns, we have

$$\left[\frac{d^2}{dt^2} + Q_0^{\,2}\, \omega_0^{\,2}\right] \xi \exp(i\, Q\, \omega_0\, t)$$
$$= k_1\, \xi \exp(i\, Q\, \omega_0\, t) \left[\frac{e^{-i2\pi Q}}{(2\pi/\omega_0)^{1/2}} + \frac{e^{-i4\pi Q}}{(4\pi/\omega_0)^{1/2}} + \cdots\right] , \tag{43}$$

where k_1 is a positive constant. We have neglected in Eq. (43) the "local" fields which have, in fact, a negligible effect on this particular calculation. The local fields are, however, important for the proper computation of thesholds. Ignoring the slow variation of amplitude (the general result is the same when amplitude variation is included[7,18,22]), we have

$$Q_0^2 - Q^2 = \frac{k\, e^{-i2\pi Q}}{1 - e^{-i2\pi Q}}, \tag{44}$$

with k a positive constant. Thus

$$Q \approx Q_0 \left[1 - \frac{k\, e^{-i2\pi Q}}{2\, Q_0^2 (1 - e^{-i2\pi Q})} \right], \tag{45}$$

where the positive sign must be taken to be consistent with the initial assumption of Eq. (41). Now,

$$\mathrm{Im}\, Q = \frac{k \sin 2\pi Q}{4\, Q_0 (1 - \cos 2\pi Q)}, \tag{46}$$

which, since instability occurs (Eq. 41) for $\mathrm{Im}\, Q < 0$, implies instability when $I + \frac{1}{2} < Q < I$, where I is an integer. Correspondingly when $I < Q < I + \frac{1}{2}$, the motion is stable.

It should be noted that the instability can be prevented by Landau damping; the criterion for stability can be obtained from the Boltzmann equation approach.[7,18,22] Conversely, the stable zones remain stable in the more complete analysis; this result has yet to be confirmed by experiment.

Extension of the theory to many bunches is straightforward,[22] as is the extension to two beams in an electron-positron storage ring.[23]

II. PRESENT STATE OF KNOWLEDGE OF INSTABILITIES

If I were to summarize in a paragraph the main content of this section--and such a summary must perforce be inexact--I would observe that the need to conquer a diversity of practical problems associated with the instabilities of relativistic particle beams has precipitated considerable activity during the past few years. This activity--by both experimentalists and theorists--has resulted in a tremendous increase in our understanding of the diversity of profound and subtle aspects of cooperative behavior exhibited by these many-particle systems. Concomitant with our increased knowledge there has come the ability to design and construct particle-handling devices in which we expect to be able to control, avoid, or operate successfully despite all presently known beam instabilities. There are, of course, new subjects to be investigated theoretically and many predictions to be confirmed experimentally, but the present spirit is one of confidence--brought forth, we trust, from understanding rather than ignorance.

1. Theory and Experiment

It is convenient, in reviewing our present situation, to follow the categorization of effects as outlined in Table I.

The major instability associated with a static self-field is that of a single particle in one beam of a storage ring interacting with the intense nonlinear field of the other stored beam. The theory has been discussed by Courant[7] and in a contribution to this meeting by Beck and Gendreau; the comparison of theory with phenomena observed on the Princeton-Stanford electron storage ring is given in a contribution by Barber et al. Experimental observatons at Novosibirsk are reported in a contribution by Auslander et al. The agreement is good. For proton storage rings the effect is of much more concern because of the absence of any radiation damping. There are profound questions concerning the long-time stability of single-particle motion in nonlinear fields, and, in particular, in the necessarily somewhat stochastic fields associated with an intense beam. The CERN group has attacked these questions theoretically, and also computationally;[34] the numerical work (which is still in progress) indicates that if resonances are avoided in accordance with the work of Courant,[7] then there is no observable long-term growth --at least in the first (one-dimensional) model--but the theoretical studies by Schoch indicate that more complicated models may exhibit observable growths. Experimental studies on long-time beam stability employing the CERN PS and also the CERN electron model are reported in contributions by Baconnier, de Raad and Steinbach, and Pentz; again, with no beam growth in the (necessarily short) times available for observation. This subject is of immediate concern only to the CERN group, and it is being very actively investigated by them; judging from the reports on the CERN IRS, presented to this meeting, there already exist optimistic opinions on the outcome of these studies.

Another beam instability, or at least an effect which has the consequence of leading to beam enlargement, is the Touschek or AdA Effect.[35] This is not a self-field effect, but rather an incoherent particle-particle interaction within a single intense beam. It seems to be well understood theoretically, and the theory is very well confirmed by observations on a number of different storage rings. One aspect of this phenomenon is analyzed in the contribution by Bruck and LeDuff.

Moving on to the dynamic self-field phenomena, we consider, first, the negative mass instability. The linear theory[10,11] has been checked most completely in experiments on the Bevatron in which the predicted functional dependence of threshold upon energy spread was confirmed.[36] Similar experiments on the Cosmotron[37] were, for diverse technical reasons, not definitive, although--like the Bevatron--in rather good quantitative agreement with the theory. The instability has also been observed and studied at a number of other accelerators.[38,39] Nonlinear effects, self-stabilized bunches, and even the interaction of one self-stabilized bunch with another have been extensively explored in a series of beautiful experiments by Barton and Nielsen;[40] similar observations are reported in the contribution of Samoilov and Sokolov. An initial attempt at a theoretical analysis is contained in the contribution of Perelstein; Ref. 40 also has some contributions to the theory of the negative-mass instability in the nonlinear regime. I think, in summary, we can feel confident about the basic correctness of the linear theory, and put some reliance upon the quantitative predictions of threshold criteria.

The longitudinal resistive instability of a uniform beam has been observed, if at all, only at MURA. The confrontation of theory with definitive experiment is difficult; the MURA group has attempted to differentiate the resistive-wall instability from an alternative hypothesis of a two-stream instability by measuring the sign of the (small) frequency shift associated with the coherent motion. The experiment[41] yields a sign in agreement with the resistive-wall theory, and the measured threshold and growth rate are also in quantitative agreement with this theory. Naturally, confidence in this theory must be somewhat restrained until the effect has been observed and studied in more detail at a number of laboratories; one of the strong arguments in favor of the theory is that it very closely parallels the theories of the negative-mass and transverse resistive-wall instabilities which are, themselves, so well confirmed experimentally.

The transverse coherent instability of a uniform beam has been studied by the MURA group and reported[41]--in greater detail in a series of internal reports.[42] The qualitative agreement with theory (the instability is observed for $n \geqslant 3$ and $Q_0 = 2.7$) is good, but quantitative comparison with the linear theory indicated observed growth rates up to 100 times the theoretical values and thresholds at significantly smaller currents than the theoretical values (less than 1/50). Recently, however, the MURA group has changed the termination of their clearing electrodes, with the dramatic effect of converting the $n = 5$ mode from growing at 100 times the theoretical growth rate to damping at approximately the same rate![43] Theoretical analysis--still in a preliminary state--by Laslett[44] indicates that clearing electrodes can have a significant effect on the phenomenon. In particular, for the MURA 50-MeV electron machine resonances are likely for $n \gtrsim 4$, and the clearing electrode can easily become the dominant element. The present situation is, then, one of uncertainty: Further theoretical or experimental work needs to be done before one will be able to compare the MURA experiments with the LNS theory;[16] either the experiment must be modified to approximate the simple geometry of the theory, or the theory must be extended so as to include the actual geometry of the MURA experiment.

Transverse coherent instabilities of bunched beams have been observed at a number of accelerators: the Cosmotron,[45] the Princeton-Stanford storage rings (see the report by Barber et al.), the Argonne ZGS (see the report by Martin et al.), Nimrod (see the contribution by Gray), the AGS (see the contribution by van Steenbergen), and the CERN PS.[25] The comparison of theory and experiment is, in general, surprisingly good; more detailed comparisons can be made following numerical evaluation of the recent theory of bunched beams.[22] I think that the resistive theories can be considered basically confirmed by experiment, but this statement is correct only when the theories are extended to include more general situations than the idealized geometry employed in the work of LNS. In particular, the influence of ions,[25] and various media and diverse walls[26-28] (see also the contribution of Balbekov and Kolomenskij), must be included in the analysis. Perhaps the most important result of the various experiments is the clean demonstration of the control of the instability, either by feedback or by artificially increasing the Landau damping with nonlinear lenses.

There is a phenomenon in linacs that is closely related to the effect just discussed: namely, the interaction of the bunched beam with the transverse modes of the linac rf cavities. Analysis of this instability (see the contribution by Gluckstern and Butler) appears to agree well with observation, thus constituting another area of confidence.

Two-beam transverse coherent instabilities have been observed on the Princeton-Stanford electron storage rings (contribution of Barber et al.). Unfortunately--from the point of view of learning about instabilities--the instability threshold was greatly increased by separating the Q values of the two beams (with quadrupoles) and increasing the vertical thickness of the beams (with skewed quadrupoles); thus an experiment on quantum electrodynamics became possible and the rings have since been devoted exclusively to the experiment. The sole comparison of theory and experiment consists of noting that the theory suggested the modifications that did, in fact, prove successful. Quantitative comparisons with the many detailed predictions of the theory[23] will have to await observations at Novosibirsk, Orsay, or Frascati, or subsequent work at Stanford. Thus although the comparison of theory with experiment is scant, the theory is being taken seriously and is forming the basis for the design and construction of a number of facilities. A variety of ways to avoid coherent instabilities, such as a proper choice of Q values, feedback, use of octupoles, or loading of the vacuum tank with dielectric, are discussed in Refs. 46 and 47.

The final instability to be discussed is the interaction of intense beams with rf cavities. A contribution to this meeting by Auslander et al. describes both experimental and theoretical work on this subject. The paper of Lebedev and Zhilkov presents a sophisticated theory. The subject appears to be well understood, and one can evidently feel confident concerning our mastery of it.

2. Areas Requiring Further Investigation

The preceding section was primarily devoted to the comparison of theory and experiment. It was seen that there are only a few experimental observations (primarily associated with nonlinear phenomena) that are not understood; or, at least, for which an explanation has not been put forth. On the other hand, there is a wealth of theoretical work that awaits experimental confirmation. The further areas of investigation for experimentalists is thus relatively clear: We are primarily interested in avoiding instabilities, consequently emphasis should center on the small-amplitude regime, and, in particular, on confronting the theoretical threshold formulas (with their multitudinous dependence upon machine and beam parameters) with experimental checks.

In consideration or those areas requiring further theoretical investigation the comments are, necessarily, of a more technical nature than in the rest of this paper; they are primarily addressed to those working on beam instability problems, but should prove of general interest by indicating the directions that further theoretical work may be expected to take. It is convenient, once again, to consider the instabilities one by one, following the order of Table I.

We consider, first, the negative-mass instability. The most interesting question is: How serious is it? Experimental evidence appears conflictive, or at best unclear; the instability seems to result in beam loss in some accelerators like the Cosmotron,[40] but in the Bevatron, beam loss (which is unexplained) doesn't seem to correlate with the presence or absence of longitudinal structure in the beam (Chupp, Elioff, and Wenzel contribution to this conference). Theoretical arguments have been presented for longitudinal bunching's leading to beam loss: (i) by leading to local increases in charge density, as bunches pass each other in an rf bucket during synchrotron motion, and hence to loss by exceeding the transverse space-charge limit, and (ii) by self-stabilized bunches' affecting each other in such a way as to eject a bunch from a stable rf bucket. Neither of these mechanisms has yet been described quantitatively, although the second has been likened to Brownian motion in a potential well (the noise being an approximation to the fields of the many bunches).

A second question, of some interest, is: What is the effect of rf longitudinal bunching on the negative-mass instability? The present theories are for uniform beams; they seem, however, to fit experiments on bunched beams, which fact should be understood, if possible. Perhaps related to this, are the very curious, and unexplained, phenomena reported by Maloy. He observes that at one (intensity-dependent) point in the acceleration cycle at Cal Tech particles move freely from one rf bucket to another, with most of the particles concentrating in two of the four buckets for a short time and then subsequently redistributing themselves approximately equally!

Perhaps the most exciting subject, apropos the negative-mass instability, is the recent suggestion by Briggs and Neil[48] that it can be prevented by appropriate choice of vacuum chamber wall material!

Further theoretical work would be interesting--in suggesting a variety of materials and design--but experimental work is the most pressing.

In regard to the transverse resistive instability in a uniform beam, we have already commented upon the special experimental and theoretical work needed in association with the MURA accelerator. More generally, and closely related to the work of Briggs and Neil, further theoretical work must be done on the effect of various wall materials as well as that of ions and associated low-energy electrons. Also of importance is removing some of the approximations in the present theory (none believed to be severe, but presumably of some quantitative significance) such as (i) including resistance in all the vacuum tank walls, and (ii) including longitudinal forces in the solution of the Boltzmann equation.

The theoretical work on bunched beams is very recent; some extensions of it are obvious, and will be worked out as time permits. This includes, for example, (i) more careful evaluation of the fields associated with a bunch to include the case in which bunches are sufficiently close that near fields (in contrast to wake fields) become important, and (ii) numerical studies of the many-bunch problem to bridge the gap between the soluble problem of all bunches of equal intensity and the soluble case of very different bunches (see Ref. 22).

A more complicated problem is to include--as must be done for the uniform beam also--the effect of ions and low-energy electrons. Only then can comparison be made, in detail, with the observed pressure-dependent instabilities on the CERN PS (in mode $n = 6$, with $Q = 6.3$)

and on the AGS (in modes $n = 8, 9$, with $Q = 8.7$). Perhaps an extension of Hereward's work[25] to bunched beams will suffice, but an incorporation of his ion-production mechanism and the dielectric properties associated with neutralization[26,28] into one theoretical structure is clearly desirable.

Of particular importance is further study of the nonlinear "window shade" phenomena observed by the MURA group;[42] not because large-amplitude nonlinear effects are themselves important, but because the proposed theory would appear to suggest a mechanism by which instabilities can develop in the regime that according to linear theory is stable.

A further important topic is the question of possible coherent motion within the bunches (which have been assumed rigid in the analysis to date). One expects these high-order modes normally to be strongly damped by rf mixing, but quantitative results are needed to ascertain the intensity at which this is no longer true.

In regard to two-beam coherent motion, topics requiring further study have already been discussed to a limited extent; we will refrain from further comments primarily because the theory is in a state of very rapid development--stimulated, as it is, by the considerable interest in its predictions--so that problems recorded here would most likely be solved before this article appeared in print.

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Table I. Physical phenomena associated with self-fields.

Static self-field effects	References
1. Incoherent transverse space charge limit	
Linear approximation	1,2
Nonlinear approximation	3
2. Influence of surrounding media on transverse space-charge limit	4
3. Longitudinal space-charge limits	5
4. Single particle--intense beam interactions	
Linear approximation	6
Nonlinear approximation	7
5. Beam-rf cavity interaction	8,9
Dynamic self-field effects	
1. Negative mass instability	
Linear approximation	10-12
Nonlinear approximation	13
2. Longitudinal resistive instability of a uniform beam	14
3. Longitudinal resistive instability of a bunched beam	15
4. Transverse coherent resistive wall instability of a uniform beam	16,17
5. Transverse coherent resistive wall instability of a bunched beam	7,18-22
6. Two-beam transverse coherent instability	23
7. Beam-rf cavity instabilities	24
8. Transverse coherent instability with general resistive media	25-28

FIGURE CAPTIONS

Fig. 1. A block diagram of the Single Particle Motion approach to self-field phenomena.

Fig. 2. A block diagram of the Collisionless Boltzmann Equation approach to a self-field phenomena. The symbols q and p represent the set of generalized coordinates and momenta describing the dynamical behavior of a particle. The partial derivatives include differentiation with respect to the q and p dependence introduced through the arguments of the distribution function.

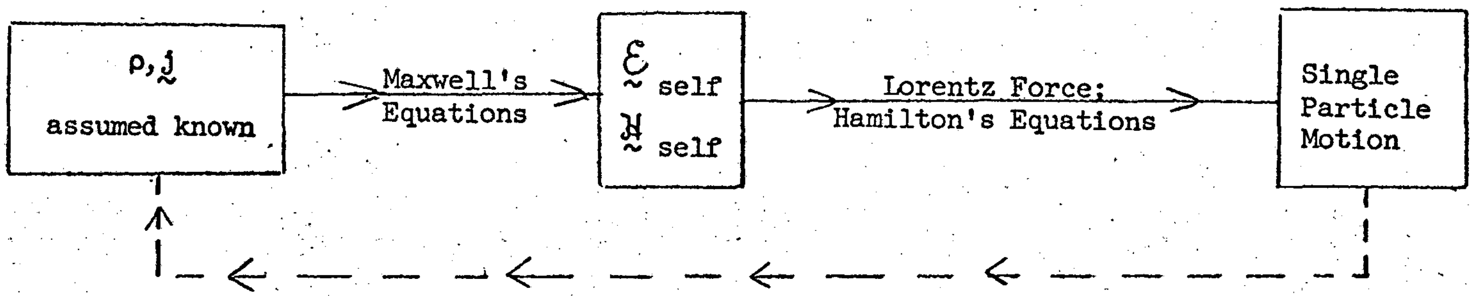
ρ, j
assumed known
Maxwell's Equations
ℰ self
ℋ self
Lorentz Force; Hamilton's Equations
Single Particle Motion

Fig. 1

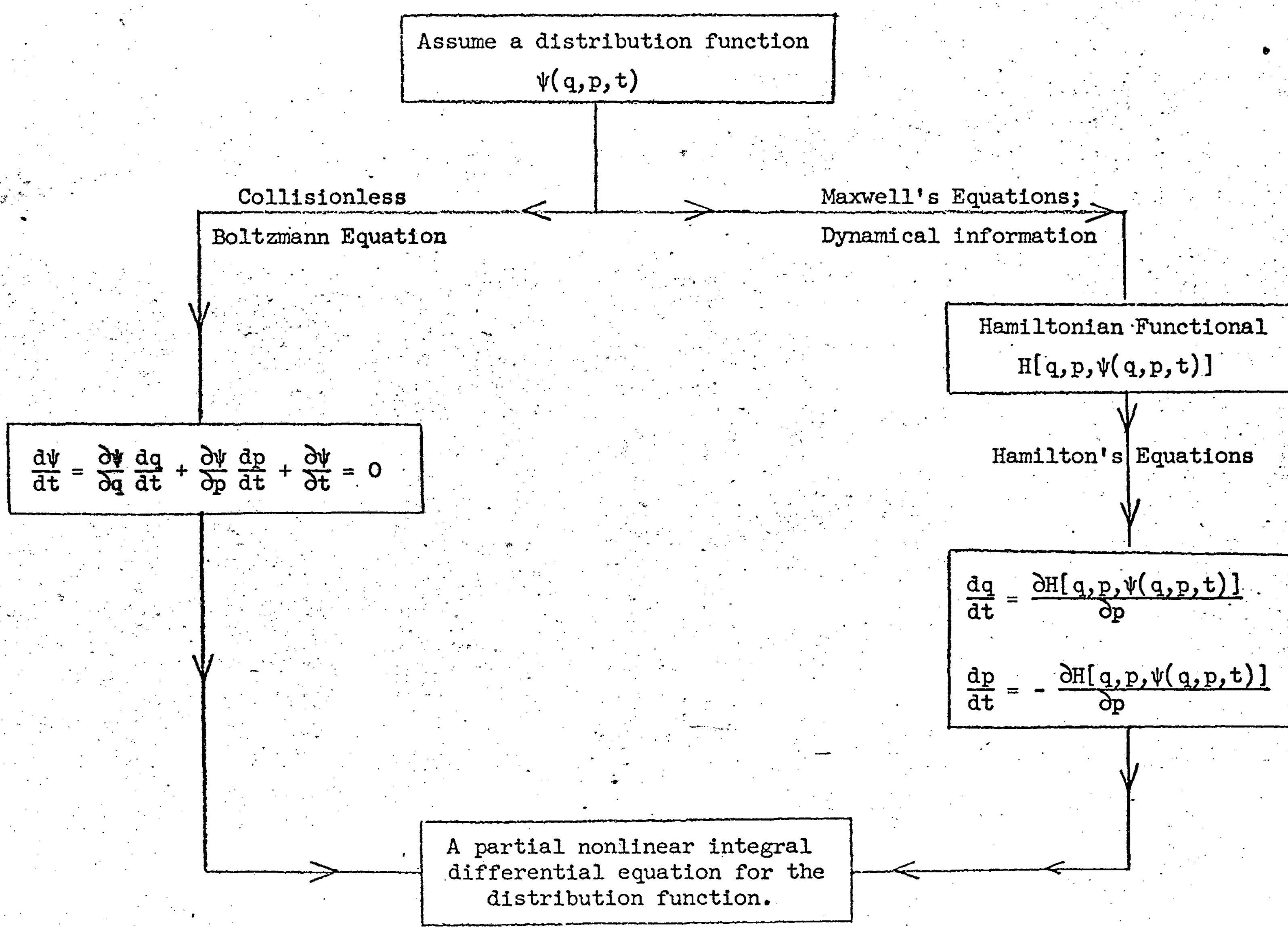
Assume a distribution function $\psi(q,p,t)$
Collisionless Boltzmann Equation
Maxwell's Equations; Dynamical information
Hamiltonian Functional $H[q,p,\psi(q,p,t)]$
Hamilton's Equations
$\frac{d\psi}{dt} = \frac{\partial\psi}{\partial q}\frac{dq}{dt} + \frac{\partial\psi}{\partial p}\frac{dp}{dt} + \frac{\partial\psi}{\partial t} = 0$
$\frac{dq}{dt} = \frac{\partial H[q,p,\psi(q,p,t)]}{\partial p}$
$\frac{dp}{dt} = - \frac{\partial H[q,p,\psi(q,p,t)]}{\partial p}$
A partial nonlinear integral differential equation for the distribution function.

Fig. 2