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Laser Acceleration in Vacuum with an Open Iris-Loaded Structure

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Abstract

An open iris-loaded waveguide structure is considered for laser acceleration of highly relativistic particle in vacuum. Complete characterization of eigenmodes are given in analytical form for the structure. In particular acceleration performance of the dominant TM mode is evaluated in detail. Transparent scaling laws are derived, and through which significant advantages over other vacuum laser acceleration schemes are demonstrated. The entire parameter space is searched and it is found that below the laser damage threshold of the structure an acceleration gradient around 1 GV/m can be obtained over a phase slippage length of 10s of cm with TWs laser in the wavelength range from 1 to 10 μm.

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The Open Iris-Loaded waveguide Structure (OILS) considered in this paper is made of a series of equally spaced thin screens each having a circular aperture. It differs from the usual iris-loaded linac structure in the following aspects: first of all it does not have side walls thus called an open structure; secondly all its characteristic dimensions, i.e., radius of the aperture (a) and separation between the adjacent apertures (L), are much larger than laser wavelength (λ). As a result, the eigenmodes are determined dominantly by diffraction. Recently, Pantell [1] presented a calculation of the longitudinal field component in the structure using the numerical method of Fox-Li [2,3] and discussed the possibility of laser acceleration. But, soon after that Pantell [4] claimed that an additional fast axial phase oscillation absent from the Fox-Li's original calculations [2,3] was found and thus concluded that the structure is not suitable for laser acceleration.

However Pantell's conclusion is incorrect, as will be shown later through rigorous analysis. The numerical method of Fox-Li is not an appropriate approach for the acceleration problem under study. To demonstrate net energy gain for a test particle, the longitudinal electric field everywhere along the particle trajectory needs to be calculated. The kernel of the Fresnel integral used in Fox-Li's method becomes increasingly fast oscillatory for the field at locations close to the diffracting aperture, making it extremely cumbersome to calculate the integral and the method prone to various sources of numerical and systematic errors. To avoid the numerical difficulties fully analytical approach is taken in this paper.

It is found that OILS offers significant advantages over other vacuum laser acceleration schemes [5–9]. First of all, it provides effective guiding for the acceleration field, leading to longer interaction length and higher energy gain per stage. Here the interaction length is limited by either diffraction length or phase velocity slippage length or group velocity slippage length, whichever shorter. Secondly, it supports eigenmode with a higher ratio of acceleration to surface fields, thus it is desirable for ameliorating power damage to the structure. And finally, it has a larger transverse dimension along the beam passage, thus it is favorable for reducing such deteriorating effects as radiative energy loss and wakefields.

Starting from source free Maxwell Equations in vacuum, we seek TM mode with three nonvanishing components: E_z, E_r, H_φ. Other types of modes can be obtained following essentially the same procedures. For open waveguide without side walls to match boundary condition it is convenient to solve first for the radial component E_r and then express E_z and H_φ in terms of E_r. Taking the time dependence as $\exp(-i\omega t)$ and assuming all field components are independent of φ, it is shown that $E_r(r, z)$ satisfies the following wave equation

$$\frac{\partial^2 E_r}{\partial r^2} + \frac{1}{r}\frac{\partial E_r}{\partial r} - \frac{1}{r^2}E_r + \frac{\partial^2 E_r}{\partial z^2} + k^2 E_r = 0 , \tag{1}$$

where $k = \omega/c$. Under the conditions $L \gg \lambda$ and $a \gg \lambda$, the wave equation can be further reduced to a parabolic equation

$$\frac{\partial^2 \Psi}{\partial r^2} + \frac{1}{r}\frac{\partial \Psi}{\partial r} - \frac{1}{r^2}\Psi + 2ik\frac{\partial \Psi}{\partial z} = 0 , \tag{2}$$

where $\Psi(r, z)$ is an envelope slowly varying in z, defined by $E_r(r, z) = \Psi(r, z)\exp(ikz)$. Eq.(2) is identified with that of azimuthal mode number $m = 1$, and its solution may be labeled as $\Psi_{1n}(r, z)$ or TM_{1n}, where n is the radial mode index. Notice that the terminology for the mode index adopted here follows from that of Vainshtein [10], which is defined according to the solution for the transverse component of the mode and therefore different from the usual definitions for closed cylindrical waveguides.

Our goal is to solve Eq.(2) for the eigenmodes of an iris-loaded waveguide. This problem is shown to be equivalent to finding the eigenmodes of a Fabry-Perot optical resonator with two circular plane mirrors of radius a, separated by a distance L [2]. Under appropriate boundary condition on the mirrors and with the special technique of Wiener-Hopf, Vainshtein [10] obtained analytical solution of the eigenmodes to all orders for such a resonator. Once the eigenmode is obtained for the resonator in the form of a standing wave, it is straightforward to convert it to a traveling wave eigenmode for the equivalent waveguide. With all spatio-temporal dependence explicitly included, the solution for TM_{1n} mode reads

$$E_z = E_a J_0(k_r r)\exp[i(k_z z - \omega t)] , \tag{3}$$

$$E_r = -i(k_z/k_r)E_a J_1(k_r r)\exp[i(k_z z - \omega t)] , \tag{4}$$

$$H_\varphi = E_r/Z_{TM} \ , \tag{5}$$

$$k_r = \frac{\nu_{1n}}{a\left[1+\eta(1 \ + \ i)/M\right]} \ , \qquad k_z = k - k_r^2/2k \ , \tag{6}$$

where $M = \sqrt{8\pi N}$, N is the Fresnel number defined by $N = a^2/\lambda L$, $\eta = -\zeta(1/2)/\sqrt{\pi} \approx 0.824$, $\zeta(z)$ is Riemann's Zeta function, $Z_{TM} = (k_z/k)Z_0$, Z_{TM} is the impedance of TM wave, Z_0 is the vacuum impedance, ν_{1n} is the nth zero of Bessel function satisfying $J_1(\nu_{1n}) = 0$, and E_a is peak acceleration gradient. As a result of slowly varying envelope approximation, the modes are characterized by $|k_z| \sim k$ and $|k_r| \ll k$. Vainshtein's solution is asymptotically more accurate for $N > 1$ [3,10], which happens to be the regime favorable for laser acceleration.

To evaluate the performance of OILS for acceleration, let's introduce a complex quantity: $\alpha \equiv \alpha_r + i\alpha_i = ik_r^2/2k$, and from Eq.(6) we have

$$\alpha_r = \frac{4\nu_{1n}^2\eta(M+\eta)}{L\left[(M+\eta)^2+\eta^2\right]^2} \ , \tag{7}$$

$$\alpha_i = \frac{2\nu_{1n}^2 M(M+2\eta)}{L\left[(M+\eta)^2+\eta^2\right]^2} \ . \tag{8}$$

The $\{z, t\}$ dependence for all field components becomes

$$\{E_z, E_r, H_\varphi\} \ \sim \ e^{i[(k-\alpha_i)z-\omega t]-\alpha_r z} \ , \tag{9}$$

and power flow through the waveguide is simply

$$P(z) \ = \ P_0 e^{-2\alpha_r z} \ , \tag{10}$$

$$P_0 = \frac{\pi a^2 |k_z/k_r|^2 E_a^2}{Re(Z_{TM})} \int_0^1 |J_1(k_r a\rho)|^2 \rho d\rho \ . \tag{11}$$

As seen from Eqs.(9,10), the mode is characterized by a phase velocity, $v_p = \omega/(k - \alpha_i)$, larger than c, and a power attenuation due to diffraction loss at the apertures. However, for sufficiently large N both phase slippage per cell, $\phi_c = \alpha_i L$, and fractional power loss per cell, $\alpha_c = 2\alpha_r L$, can be made as small as desired, as seen from Eqs.(7,8). Let's define a phase slippage length, $\mathrm{L_s} = \pi/\alpha_\mathrm{i}$, over which an acceleration phase slippage of π is experienced by a highly relativistic particle ($\mathrm{v} \approx \mathrm{c}$) moving along the z-axis. The energy gain of the test particle traversing a slippage length can be calculated as

$$\Delta \mathrm{W_s} = \mathrm{eE_a} \int_0^{\mathrm{L_s}} \sin(\alpha_\mathrm{i} \mathrm{z}) \mathrm{e}^{-\alpha_\mathrm{r} \mathrm{z}} \mathrm{dz} = \mathrm{eE_a L_s T_s} \ , \tag{12}$$

where $\mathrm{T_s}$ is a reduction factor due to a full π phase slippage and the attenuation of the acceleration field over a slippage length, given by

$$\mathrm{T_s} = \frac{1 + \mathrm{e}^{-(\alpha_\mathrm{r}/\alpha_\mathrm{i})\pi}}{\pi[1 + (\alpha_\mathrm{r}/\alpha_\mathrm{i})^2]} \ . \tag{13}$$

Two other figures of merit, the shunt impedance per unit length $\mathrm{Z_L}$ and the Q factor, are given here for comparison with traditional microwave acceleration structures

$$\mathrm{Z_L} = \frac{|\mathrm{E_z}(\mathrm{r}=0,\mathrm{z})|^2}{-\mathrm{dP/dz}} = \frac{\mathrm{E_a^2}}{2\alpha_\mathrm{r} \mathrm{P_0}} \ , \tag{14}$$

$$\mathrm{Q} = \frac{\omega \mathrm{U}}{-\mathrm{dP/dz}} = \frac{\pi}{\lambda \alpha_\mathrm{r} (\mathrm{v_g/c})} \approx \frac{\pi}{\lambda \alpha_\mathrm{r}} \ , \tag{15}$$

where U is the field energy per unit length given by $\mathrm{U} = \mathrm{P/v_g}$, and $\mathrm{v_g}$ is the group velocity very close to c. Another important concern for high gradient acceleration by a waveguide is the surface field on the structure which is limited by laser damage threshold. For OILS, we define an edge field by $\mathrm{E_e} = |\mathrm{E_r}(\mathrm{r}=\mathrm{a},\mathrm{z}=0)|$.

Before going to the exploration of OILS performance using the general formulas just derived, it is instructive to look at more transparent weak diffraction limit and scaling laws for several important performance parameters. We will consider the dominant radial mode with $\mathrm{n} = 1$ and $\nu_{11} = 3.832$. Keeping only the leading term while taking the limit $\mathrm{N} \gg 1$, we have

$$\frac{\mathrm{E_a}}{\mathrm{E_e}} \ \longrightarrow \ \frac{\sqrt{\mathrm{N}}(\lambda/\mathrm{a})}{\sqrt{\pi}\eta\, |\mathrm{J_0}(\nu_{11})|} \ \approx \ 1.7\sqrt{\mathrm{N}}(\lambda/\mathrm{a}) \ , \tag{16}$$

$$\frac{\mathrm{I_e}}{\mathrm{I_{av}}} \ \longrightarrow \ \frac{(\nu_{11}\eta)^2}{4\pi \mathrm{N}} \ \approx \ 0.79/\mathrm{N} \ , \tag{17}$$

$$\alpha_{\mathrm{s}} \longrightarrow 1 - \mathrm{e}^{-\eta\sqrt{2\pi/\mathrm{N}}} \approx 1 - \mathrm{e}^{-2.1/\sqrt{\mathrm{N}}} \ , \tag{18}$$

$$\mathrm{L_s} \ \longrightarrow \ \frac{4\pi^2 \mathrm{a}^2}{\nu_{11}^2 \lambda} \ \approx \ 2.7\ \mathrm{a}^2/\lambda \ = \ 2.7\ \mathrm{N\ L} \ , \tag{19}$$

$$\Delta \mathrm{W_s[MeV]} \ \longrightarrow \ 2.9\sqrt{\mathrm{N}}\ \mathrm{a[mm]}\ \mathrm{E_e[GV/m]} \ , \tag{20}$$

$$\mathrm{I_{av}[PW/cm^2]} \ \longrightarrow \ 1.7 \times 10^{-4} \mathrm{N E_e^2[GV/m]} \ , \tag{21}$$

where $\mathrm{I_e}$ is the laser intensity at the aperture edge, $\mathrm{I_{av}}$ is the laser intensity averaged over the waveguide cross section at $\mathrm{z} = 0$, thus the required laser power is determined by $\mathrm{P_0} = \pi \mathrm{a}^2 \mathrm{I_{av}}$, and α_s is the fractional power loss per slippage length. Also we have $\mathrm{T_s} \rightarrow 2/\pi$, and with definition $\Gamma = \mathrm{k} - \alpha_{\mathrm{i}}$, the phase and group velocity of the TM_{11} mode

$$\mathrm{v_p} = \frac{\omega}{\Gamma} \approx \frac{\mathrm{c}}{1 - \frac{1}{2}\left(\frac{\nu_{11}\lambda}{2\pi \mathrm{a}}\right)^2} \ , \tag{22}$$

$$\mathrm{v_g} = \frac{\mathrm{d}\omega}{\mathrm{d}\Gamma} \approx \frac{\mathrm{c}}{1 + \frac{1}{2}\left(\frac{\nu_{11}\lambda}{2\pi \mathrm{a}}\right)^2} \ . \tag{23}$$

Given Eq.(22) a critical energy can be defined by $\gamma_{\mathrm{c}} = 2\pi \mathrm{a}/\nu_{11}\lambda$. A particle is considered highly relativistic if its Lorentz factor satisfies $\gamma \gg \gamma_{\mathrm{c}}$, and under this condition Eqs.(19,20) are expected to hold in good accuracy.

The most important characteristics of OILS as a potentially attractive structure for laser acceleration is revealed by Eq.(16). It is shown that the ratio of acceleration to edge (surface) field is enhanced from the usual scaling for all near field accelerations [9,11], λ/a, by a large factor, $\sqrt{\mathrm{N}}$. As a result, substantial acceleration gradient can be obtained on the axis, even though the boundary is hundreds of wavelengths away. To understand this attractive and unique scaling it is instructive to look at the transverse profiles of the eigenmode.

The transverse profiles for $\mathrm{E_z}$ and $\mathrm{E_r}$, given by Eqs.(3,4), are determined by $\mathrm{J_0(k_r r)}$ and $\mathrm{J_1(k_r r)}$ respectively and dependent only on the Fresnel number N. The relative amplitude of the profiles for $\mathrm{N} = 250$ case are shown in Fig.1 as functions of r/a. The larger the N, the smaller the edge field, the smaller the portion of the field to be clipped off by the apertures, the smaller the diffraction loss and phase slippage. Notice that according to Eq.(16) the condition $|\mathrm{E_r}(\mathrm{r}=\mathrm{a})| > |\mathrm{E_z}(\mathrm{r}=0)|$ always holds, indicating the field is dominantly transverse. It is seen clearly from Fig.1 and also revealed by Eq.(17) that the field energy is concentrated in the vacuum region away from the boundary, quite different from the schemes [9,11] where the dominant transverse field component is peaked at the boundary.

Due to the difference between the wave impedance $\mathrm{Z_{TM}}$ and the vacuum impedance $\mathrm{Z_0}$, the transverse electric and magnetic force do not cancel to the order of $1/\gamma^2$ for highly relativistic particle, instead a small but finite ratio of momentum transfer rate remains as $\Delta\mathrm{P}_\perp/\Delta\mathrm{P}_\parallel \approx (\nu_{11}^2/8\pi)\,(\lambda/\mathrm{a})\,(\mathrm{r}/\mathrm{a})$ for a particle near the axis.

With the parameter definitions and general formulas given, we are now ready to evaluate OILS performance. It is noted that there are only four independent parameters and they are chosen for convenience to be $\{\lambda,\ \mathrm{E_e},\ \mathrm{a}/\lambda,\ \mathrm{N}\}$. We are interested in only a few discrete wavelengths where bright sources are available, in particular at 1 and 10 μm. The maximum tolerable edge field $\mathrm{E_e}$ is determined by laser damage threshold, which can often be set at a constant value given laser wavelength and material of the structure. It can be inferred from the experimental data [12,13] that 10 GV/m seems to be a reasonable upper limit for $\mathrm{E_e}$ at $\lambda = 1\mu$m. Thus we are left with only two independent parameters $\{\mathrm{a}/\lambda,\ \mathrm{N}\}$ to vary, all dependent parameters can therefore be conveniently visualized in contour plots.

Shown in Fig.2-4 are the peak acceleration gradient, energy gain per slippage length, and the required laser power, respectively, for $\{\lambda = 1\mu\mathrm{m}, \mathrm{E_e} = 10\mathrm{GV/m}\}$. Four examples including $\{\lambda = 10\mu\mathrm{m}, \mathrm{E_e} = 5\mathrm{GV/m}\}$ cases with more complete listing of performance parameters are given in Table I. All these results can be readily scaled to other parameter regime of interest using the scaling laws derived.

Up to now, we have not specified the properties of the screens required for making the iris-loaded structure, the only assumption on the boundary condition that matters for the diffraction calculation requires the part of the wave intercepted by the screen be considered lost. In this regard, the screen could be either totally absorbing or totally reflecting as long as the reflected portion of the wave is directed out of the waveguide.

The wave-particle interaction has to be terminated before the particle slips out of the accelerating phase. The simplest way to do this would be to deflect the particle out of the field region. For phase slippage length 10s of cm long this is certainly doable if the energy of the particle is not too high to cause severe radiative loss of energy. For extremely relativistic particle where trajectory deflection is no longer acceptable, other effective methods of termination have to be considered or invented.

In conclusion, I have presented a systematic evaluation of laser acceleration with OILS assuming well established eigenmode. Analysis of planar and rectangular structures can be carried out parallel to what's done here. The scaling laws derived from the powerful analytical approach of Vainshtein have both revealed the simplicity of the acceleration mechanism and uncovered somewhat surprisingly favorable characteristics of such a diffraction dominated structure. However I have not touched upon such critical issues as wakefield, beam loading, and ways to couple laser power in and out of the structure without significantly degrading acceleration performance. I do want to point out in passing that the axicon scheme used to generate radially polarized laser beam for the inverse Cherenkov laser acceleration experiment [14] appears to be an interesting candidate for OILS mode injection. Further investigation on this and other critical issues will be presented in a forthcoming paper. This work was supported by DOE under contract No. DE-AC03-76SF00098.

REFERENCES

[1] R. Pantell, talk presented at FEL conference, Aug. 1996.

[2] A. G. Fox and T. Li, Bell Sys. Tech. J. **40**, 453 (1961).

[3] T. Li, Bell Sys. Tech. J. **44**, 917 (1965).

[4] R. Pantell, private comm. at AAC workshop, Oct. 1996.

[5] L. C. Steinhauer et al., J. Appl. Phys., **72**, 3237 (1992).

[6] E. Esarey et al., Phys. Rev. E, **52**, 5443 (1995).

[7] Y. C. Huang et al., Appl. Phys. Lett. **68**, 753 (1996).

[8] B. Hafizi et al., Phys. Rev. E, **55**, 3539 (1997).

[9] P. Sprangle et al., Phys. Rev. E, **55**, 5964 (1997).

[10] L. A. Vainshtein, Soviet Physics JETP **17**, 709 (1963).

[11] R. B. Palmer, AIP Conf. Proc. **335**, 90 (1995).

[12] D. Du et al., Appl. Phys. Lett. **64**, 3071 (1994).

[13] B. C. Stuart et al., Phys. Rev. Lett. **74**, 2248 (1995).

[14] W. D. Kimura et al., Phys. Rev. Lett. **74**, 546 (1995).

FIGURES

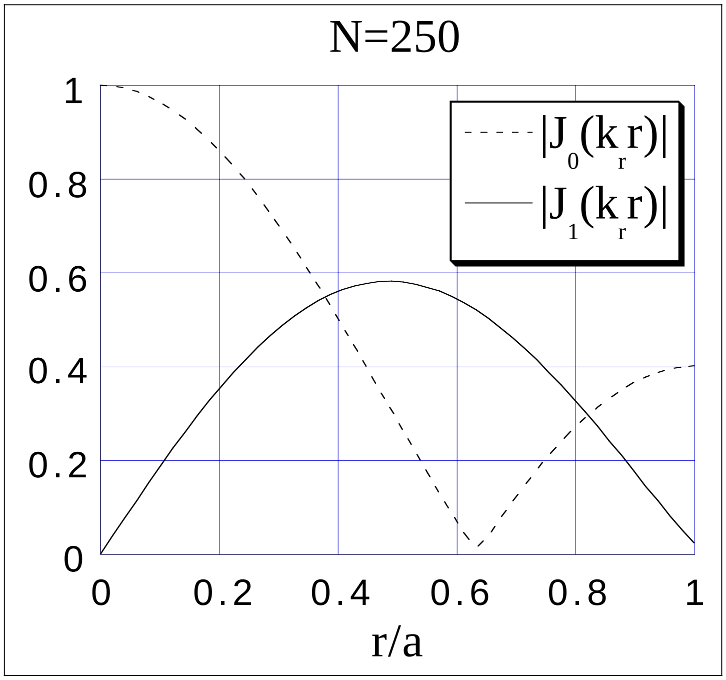

FIG. 1. $|J_0(k_r r)|$ and $|J_1(k_r r)|$ vs. r/a at N = 250.

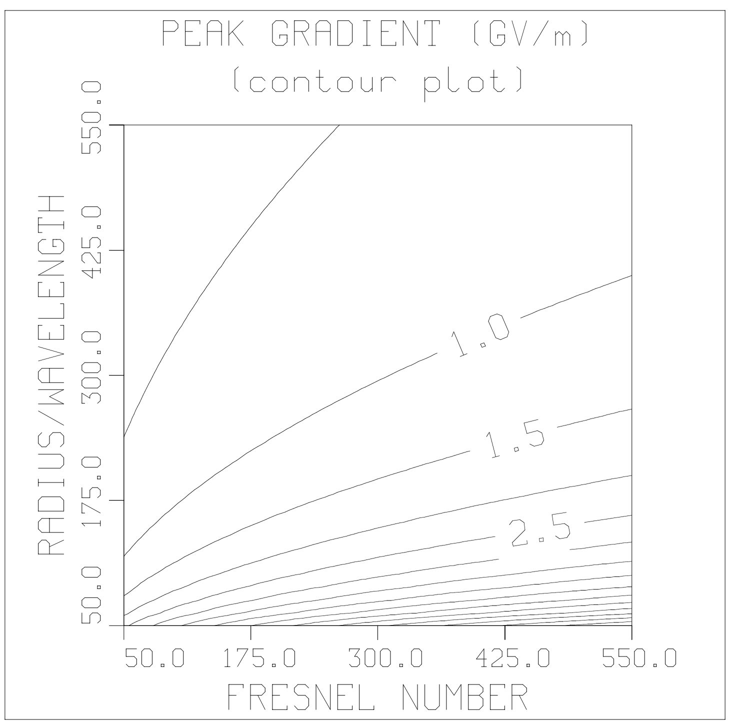

FIG. 2. E_a vs. $\{N, a/\lambda\}$ at $\{\lambda = 1\mu m, E_e = 10 GV/m\}$.

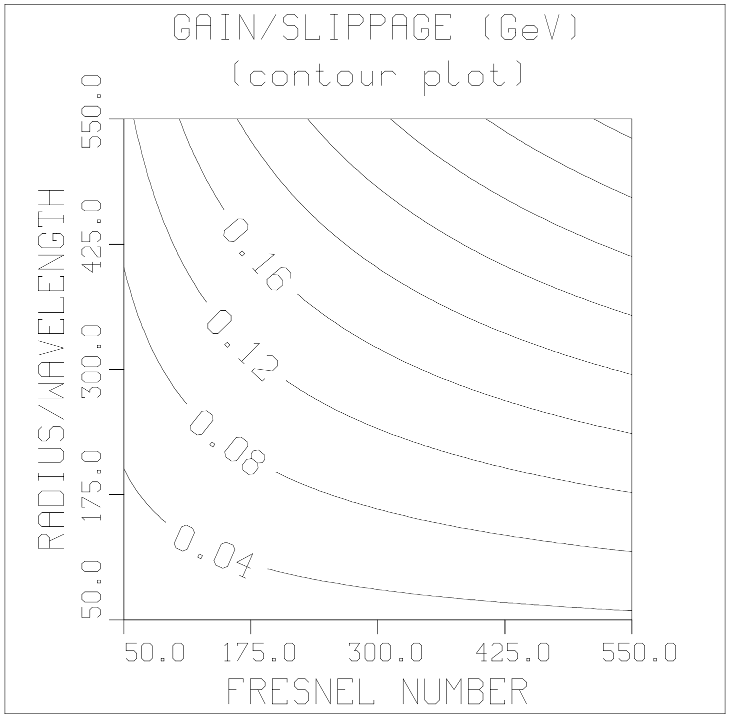

FIG. 3. ΔW_s vs. $\{N, a/\lambda\}$ at $\{\lambda = 1\mu m, E_e = 10GV/m\}$.

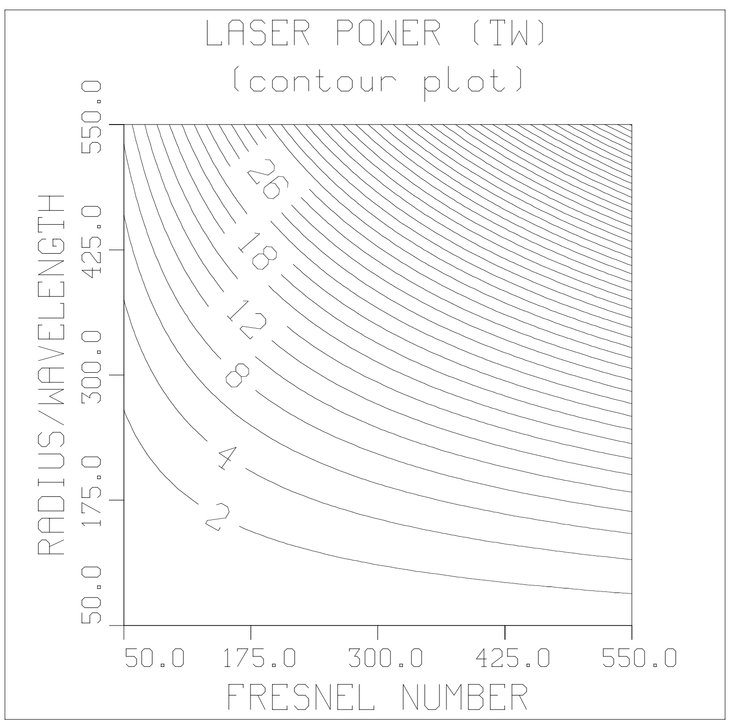

FIG. 4. P_0 vs. $\{N, a/\lambda\}$ at $\{\lambda = 1\mu m, E_e = 10GV/m\}$.

TABLES

TABLE I. Example Cases of Acceleration Parameters.

CASES	IA	IB	IIA	IIB
$\lambda(\mu m)$	1	1	10	10
a(mm)	0.5	0.25	1	0.5
L(mm)	1	0.125	2	1
N	250	500	50	25
E_a(GV/m)	0.54	1.5	0.6	0.84
ΔW_s(MeV)	227	161	100	35
L_s(cm)	69	17	28	7.2
P_0(TW)	34	17	7.1	0.92
I_{av}(PW/cm^2)	4.3	8.6	0.23	0.12
I_e(TW/cm^2)	13	13	3.3	3.3
ϕ_c(degree)	0.26	0.13	1.3	2.5
α_c(%)	0.019	0.0067	0.2	0.56
α_s(%)	12	8.8	25	33
Q(10^6)	33	12	0.62	0.11
Z_L(MΩ/m)	0.045	0.25	0.049	0.14
T_s	0.62	0.62	0.59	0.58
E_e(GV/m)	10	10	5	5