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GENERALIZED ANALYSIS OF PHASE-SENSITIVE DETECTION
CIRCUIT OPERATING CHARACTERISTICS AT THE SIGNAL
DETECTION IN THE PRESENCE OF NOISE

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June 1971

Previous analysis of phase-sensitive detection circuits^(1, 2) have been based on the assumption that the additive noise is non-existent in the detector reference channel. However, in contemporary experimental research instrumentation and phase-lock systems, a significant amount of noise can be present in the reference channel.⁽³⁾ Such case will be discussed, assuming that the input signal and the reference wave are in the presence of independent, stationary, and additive Gaussian noise.

The generalized detection circuit under consideration is shown in Fig. 1. The circuit input consists of a signal $s^*(t)$ superimposed on a broad-band noise $n_s^*(t)$. After filtering, the sum $s_i(t) = s(t) + n_s(t)$ is applied to the balanced phase-sensitive detector. The detector reference input consists of a reference wave $r^*(t)$ superimposed on a broad-band noise $n_r^*(t)$. After filtering, the sum $r_i(t) = r(t) + n_r(t)$ is applied to the detector and differential circuit. The generalized and normalized form of the detector operating characteristics is $S_o / \eta_d (N_r^2 + N_s^2)^{1/2} = (\pi/2)^{1/2} \Delta {}_1F_1(x_s, \mu_r, \psi, n)$, where S_o represents the the output signal; η_d is the detection efficiency; N_r and N_s are rms

values of the narrow-band noise in the reference channel and signal channel, respectively; $x_s = S/N_s$ is the input signal-to-noise ratio; $\mu_r = R/N_r$ is the reference wave-to-noise ratio; $n = N_s/N_r$ is the signal channel noise-to-reference channel noise ratio; ${}_1F_1(x_s, \mu_r, \psi, n)$ denotes a confluent hypergeometric function of the form ${}_1F_1(-1/2; 1; x_s, \mu_r, \psi, n)$; Δ signifies the difference of the hypergeometric functions; ψ is the phase angle between the input signal and reference wave. Detector characteristics nonlinearity $N_A^*(x_s)$ relating to the input signal-to-noise ratio is described by a generalized equation $N_A^*(x_s) = 1 - [(n - n^{-1}) \Delta {}_1F_1(x_s, \mu_r, n)] / [x_s \mu_r {}_1F_1(\mu_r, n)]$. The terms ${}_1F_1(x_s, \mu_r, n)$ and ${}_1F_1(\mu_r, n)$ denote confluent hypergeometric functions of the form ${}_1F_1(-1/2; 1; x_s, \mu_r, n)$ and ${}_1F_1(1/2; 2; \mu_r, n)$, respectively. $N_A^*(x_s)$ as a function of x_s is calculated and plotted in Figs. 2-3. From the curves it can be seen that $N_A^*(x_s)$ increases with increasing x_s , for given μ_r and n . Also, $N_A^*(x_s)$ decreases when the ratio of μ_r/x_s increases, particularly when $\mu_r \geq 1$. Furthermore, when the ratio n is large, the minimization of $N_A^*(x_s)$ requires a very high value of the ratio μ_r/x_s . For a fixed ratio n and phase angle ψ , the proper choice of μ_r and x_s minimizes $N_A^*(x_s)$ to an acceptable amount. This is important in a wide dynamic range, wide-band detection application using solid state circuit components where the ratio μ_r is often close in value to the ratio x_s , resulting in a large nonlinearity. Curves in Fig. 3 show that for $n = 1$ and $x_s = 1$, the increase of the μ_r/x_s ratio by one order of magnitude will decrease the nonlinearity by almost three orders of magnitude. Furthermore, $N_A^*(x_s)$ is reduced even more for larger amounts of x_s .

For $\mu_T = 10$, increasing μ_T by one order of magnitude decreases the nonlinearity by more than four orders of magnitude.

Detector characteristics nonlinearities $N_B^*(\psi)$ and $N_C^*(\psi)$ relating to the phase angle ψ for operating points $\psi_B = (2m+1)\pi/2$ and $\psi_C = 2m\pi$, where $m = 0, 1, 2, \dots$, are described by equations

$$N_B^*(\psi) = 1 - [(n+n^{-1}) {}_1F_1(x_S, \mu_T, \psi, n)] / [\mu_T x_S (\pi/2 - \psi) {}_1F_1(x_S, \mu_T, n)] \text{ and}$$

$$N_C^*(\psi) = 1 - [\Delta {}_1F_1(x_S, \mu_T, \psi, n)] / [\Delta {}_1F_1(x_S, \mu_T, n)]$$

Hypergeometric functions ${}_1F_1(x_S, \mu_T, \psi, n)$ and ${}_1F_1(x_S, \mu_T, n)$ are of the same form as in the expression for $N_A^*(\psi)$; Δ signifies the difference of functions of the form ${}_1F_1(x_S, \mu_T, n)$. The nonlinearity $N_B^*(\psi)$ depends strongly upon ψ , x_S , μ_T , and n , as shown in Fig. 4. To obtain the most information about $N_B^*(\psi)$ behavior, the nonlinearity is calculated and plotted as a function of x_S . The curves show that for $n = 10^{-3}$ and $\mu_T = 1$, the nonlinearity $N_B^*(x_S)$ is practically independent of x_S over a wide dynamic range. On the contrary, $N_B^*(x_S)$ has a strong dependence on ψ , n , and μ_T . From the curves it follows that the proper choice of μ_T for a given n and x_S minimizes the nonlinearity. Any other value of μ_T can be considered as nonoptimum.

The curves for the nonlinearity $N_C^*(x_S)$ in Fig. 5 show that particular values of μ_T minimize the nonlinearity. For this purpose it is important to avoid the regions where $N_C^*(x_S)$ curves have maximums. Analysis and the curves show that the presence of the reference channel noise has a dominant role in the detector optimum operating conditions.

From a comparison of the nonlinearity of $N_B^*(x_S)$ and $N_C^*(x_S)$ curves it can be seen that nonoptimum values of μ_T , x_S , and n have far more influence on the behavior of $N_C^*(x_S)$ than on $N_B^*(x_S)$. In general, the

nonoptimum μ_T , x_S , and n values maximally increase $N_B^*(x_S)$ approximately one-half order of magnitude in the worst case. However, $N_C^*(x_S)$ nonoptimum values of μ_T , x_S , and n increase $N_C^*(x_S)$ two orders of magnitude. Consequently, the phase angle $\psi_B = (2m+1)\pi/2$ should be chosen as the operating point wherever possible.

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FOOTNOTE AND REFERENCES

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FIGURE CAPTIONS

- Fig. 1 Generalized phase-sensitive detection circuit under consideration.
- Fig. 2. Nonlinearity $N_A^*(x_s)$ versus signal-to-noise ratio x_s , for the signal channel noise-to-reference channel noise ratio $n = 10^{-3}$ and 10^3 .
- Fig. 3 Nonlinearity $N_A^*(s)$ versus signal-to-noise ratio x_s , for the signal channel noise-to-reference channel noise ratio $n = 1$.
- Fig. 4 Nonlinearity $N_B^*(x_s)$ versus signal-to-noise ratio x_s , for the signal channel noise-to-reference channel noise ratio $n = 10^{-3}$ and 10^3 .
- Fig. 5 Nonlinearity $N_C^*(x_s)$ versus signal-to-noise ratio x_s , for the signal channel noise-to-reference channel noise ratio $n = 10^{-3}$ and 10^3 .

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Fig. 1

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Fig. 2

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Fig. 3

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Fig. 4

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Fig. 5

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