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Trip Scheduling and Economic Analysis of Transportation Policies

DISSERTATION

submitted in partial satisfaction of the requirements for the degree of

DOCTOR OF PHILOSOPHY

in Economics

with a Concentration in Transportation Economics

by

Xuehao Chu

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1993

The dissertation of Xuehao Chu is approved,

and is acceptable in quality and form

for publication on microfilm:

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University of California, Irvine

1993

DEDICATION

To

Yan Shen

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ABSTRACT OF THE DISSERTATION

Trip Scheduling and Economic Analysis of Transportation Policies

by

Xuehao Chu

Doctor of Philosophy in Economics

University of California, Irvine, 1993

Professor Gordon J. Fielding, Co-Chair
Professor Kenneth A. Small, Co-Chair

The dissertation seeks to understand how urban commuters adjust their schedules and modes to congestion, as well policy implications of this adjustment. An equilibrium simulation model of commuting traffic on a hypothetical, urban highway corridor is developed. The demand side is a discrete choice model of mode and time of day, estimated with data from the San Francisco Bay Area. The supply side is a speed-flow function that predicts travel time from flows leaving the corridor.

The research has three objectives: to simulate the effects of capacity expansion, optimal toll, and six other pricing policies; to test hypotheses relating to schedule shifts in response to congestion and policy changes; and to estimate biases in policy effects when

schedule shifts are ignored. An iterative procedure is developed to compute optimal tolls that vary with time of day.

Policies are examined from five perspectives: welfare (consumer surplus, toll revenue, and total benefits), peaking (traffic counts and share in the peak 15-minute period), congestion (average and peak 15-minute travel delays), schedule delay (average variable schedule delay), and mode mix (mode shares, average occupancy, and total traffic).

Five results emerge. First, although an optimal toll can achieve substantial benefits, savings in travel delay are accompanied by increases in schedule delay. Second, a toll equal to the marginal social externalities of an additional trip at different times of day at a base case can achieve benefits equivalent to those of an optimal toll, which is equal to the marginal social externalities of an additional trip at different times of day at an social optimum. Third, schedule delay has variable and constant components. The constant component is the equilibrium level at a base case when travel is free-flow. The variable component changes with congestion and policies. Fourth, urban commuters shift their schedules in response to congestion and policy changes. Heavy congestion forces people away from the peak; capacity expansion attracts people back to the peak; an optimal toll discourages people driving alone in the peak. Fifth, the benefits of capacity expansion and an optimal toll are substantially overestimated if trip scheduling is ignored.

INTRODUCTION

Congestion pricing, as a method to allocate road space efficiently, has long been

advocated by economists. Scarcity of road funds in 1990s has revived the interest (Small,

1992c). A national conference was held in Washington, D.C. in 1993, and a private toll road

is under construction in Southern California that will demonstrate congestion pricing

(Fielding, 1993). Yet transportation researchers cannot answer basic questions about how

peak-period traffic would adjust itself to alternative trip making once congestion pricing is

adopted (Small, 1992a, p. 156).

Congestion pricing imposes charges that vary widely by time of day. If adopted, it

could bring about dramatic changes in travel behavior. Some travelers might shift to

alternative routes, modes; others may change residential and employment locations, while

others might forego discretionary trips. Trip schedules would change in many different ways.

 Analysis of congestion pricing has been limited, however, either to models that ignore trip

scheduling, or models of trip scheduling that oversimplify travelers' characteristics.

This dissertation aims to fill a gap in our understanding of how urban commuters

adjust their schedules and modes to congestion, as well as the policy implications of this

adjustment. It does so by developing a realistic, equilibrium simulation model of commuting

traffic on a hypothetical, urban highway corridor. The demand side is a discrete choice model

for both mode and time of day, estimated with a sample of commuters from the San Francisco

Bay Area. The

supply side is a speed-flow function that predicts travel time from flows leaving the corridor.

An iterative procedure is developed to calculate optimal tolls that vary with time of day.

The research relates mainly to three literature themes about commuting behavior and

transportation policy analysis. The first theme is equilibrium analysis of urban transportation

policies, using mode choice models. Small (1983) and Viton (1983, 1986) are the major

contributors of this work, though the models they develop ignore trip scheduling.

The second theme is abstract modeling of trip scheduling. There are three major

models: the Vickrey (1969) model, the Henderson (1974, 1977) model, the Mahmassani and

Herman (1984) model. All three focus on the journey to work for a fixed number of

commuters traveling on the same highway between home and work. Commuters are specified

as identical or slightly different in work start times or values of time. These models do

provide insights to scheduling behavior, but they are too simple to provide useful policy

guidelines.

The third theme is econometric modeling of trip scheduling. This theme includes

Small (1982), Abkowitz (1980), and Hendrickson and Plank (1984). However, Small (1982)

does not include mode choice; Abkowitz (1980) does not specify any variables measuring

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schedule delays; and Hendrickson and Plank (1984) do not include socioeconomic variables.

The dissertation consists of this introduction, two parts, and conclusions. Part I focuses on the theme of abstract modeling, while Part II applies a reformulated, abstract model from Part I to data from the San Francisco Bay Area. Part I makes four contributions: First, it shows that the Henderson and the Mahmassani and Herman models lack equilibrium. This is a subtle, but easily missed problem. It is subtle because conclusions drawn from non-equilibrium solutions can be erroneous; it is easily missed because trip cost is constant at the solutions to these two models. This condition of constant trip cost is necessary for equilibrium, but is often treated as sufficient.

As a second contribution, the Henderson model is reformulated. The reformulated model differs from the original Henderson model by assuming that, travel time for any commuter is determined by the arrival flow he arrives with at work rather than by the departure flow he departs with from home. The reformulated model eliminates the problem of lacking equilibrium in the original Henderson model.

Third, Part I compares the reformulated model with the Vickrey model both analytically and using simulations. The two models are found to result in the same buildup and decay of congestion in equilibrium. The behavior of the Vickrey model is found to become identical to that of the reformulated model as the elasticity of travel delay becomes bigger. The Vickrey model has been widely used in the literature, while the Henderson model is rarely used. This limiting relationship links the two models.

Fourth, Part I examines modeling of hyper-congestion in the context of the Mahmassani and Herman model. Hyper-congestion describes a situation where average flow is decreased because more vehicles are forced into a roadway; it happens when the density exceeds the level that gives the maximum flow. Hyper-congestion is inefficient. But to analyze it properly, a model that allows hyper-congestion is required. The Mahmassani and Herman model is the only such model, but can fail to achieve equilibrium. Therefore, it is useful to explore the behavior of the Mahmassani and Herman model to discover whether the model can be improved.

Part II applies the reformulated Henderson model to data from the San Francisco Bay Area, and simulates the equilibrium effects of eight transportation policies. The policies include (1) an optimal toll, (2) an incremental capacity expansion, and six alternative pricing policies: (3) a base-externality toll, (4) a piecewise-linear toll, (5) a coarse toll, (6) a uniform toll, (7) an optimal toll with bus and carpool users exempted, (8) an optimal toll with an incremental capacity expansion.

Part II also estimates biases in these policy effects when schedule shifts are ignored. Conventional models of peak-period congestion assume away scheduling behavior by using constant demand over a predetermined period. One way to measure these biases is to compare two simulations of a given policy: one with scheduling shifts allowed and the other with schedule shifts constrained.

Policies are simulated from five perspectives: (i) welfare (consumer surplus, toll

revenue, and total benefits), (ii) peaking (traffic counts and share in the peak 15-minute period), (iii) congestion (average and peak 15-minute travel delays), (iv) schedule delay (average schedule delay and ratio of schedule to travel delay), and (v) mode mix (mode shares, average occupancy, and total traffic).

An optimal toll is the first best solution, but can be difficult to calculate and implement. Therefore, it is useful to consider alternative pricing policies that are easier to calculate and implement. If these simple tolls can achieve a substantial proportion of the benefits of an optimal toll, they could be worthwhile. Capacity expansion is also examined for three purposes: first is to contrast the effects of an incremental expansion with those of an optimal toll, second to examine the miscalculation of effects from an incremental expansion, and to compare this miscalculation with that of an optimal toll, and third to see how the benefits of an incremental expansion compare with and without an optimal toll.

The original contributions of this dissertation are in Part II. First, the indirect utility function to be used in discrete choice models of trip scheduling is specified, based on utility maximization. This function is often specified without a theoretical foundation. Small (1982) is one exception. He extends models of time allocation (Gary Becker, 1965) by making travel cost and time dependent on scheduling. Small's setup has two shortcomings, however. First, duration of work, but not duration at the work site, enters the time budget. The gap is either time early or time late for work. This gap is the focus of those discrete choice models of trip scheduling. Second, Small's setup does not lead to an operational specification of the utility

function.

Second, Part II establishes a marginal-cost pricing rule in the context of a discrete

choice model of mode and schedule. It shows that a time-varying toll that maximizes social

welfare, the sum of consumer welfare and toll revenue, is a marginal-cost toll. Such

marginal-cost pricing rules have only been established in the context of simple continuous

choice models without scheduling considerations (Walter, 1961).

Third, Part II is the first effort in the literature to empirically calculate an optimal

time-varying toll based on a realistic model of commuting traffic on urban highways. An

optimal toll measures the marginal externality of an additional trip made at a given time at a

social optimum. Small (1992a, pp. 121-122) identifies two approaches to calculate an optimal

toll, but neither works with the more realistic model here. The approach used is to start from

the definition of marginal externality of travel at a given time, using an iterative procedure.

Fourth, Part II is the first effort in the literature to test hypotheses and questions about

trip scheduling that are of policy and methodological interest. The hypotheses concern how

people adjust their mode and schedule choices in response to congestion and policy changes.

Part II examines the following hypotheses:

(h1) Whether congestion deters people to alternative schedules.

(h2) Whether capacity expansion leads to further peaking.

(h3) Whether optimal congestion pricing leads to peak spreading.

The questions concern the effects of optimal pricing and other forms of pricing policies, and

miscalculation of the effects of optimal pricing and capacity expansion when schedule shifts are ignored:

(q1) What are the welfare effects of an optimal toll?

(q2) Does an optimal toll affect peaking, congestion, schedule delay, and mode mix?

(q3) How do the total benefits of other pricing policies compare with those of an optimal toll?

(q4) Does an incremental capacity expansion affect peaking, congestion, schedule delay, and mode mix?

(q5) Are the effects of an optimal toll biased when schedule shifts are ignored?

(q6) Are the effects of a ten-percent expansion biased when schedule shifts are ignored?

Part I consists of chapters 1 through 4. Chapter 1 shows the lack of equilibrium of the original Henderson model (Henderson 1974, 1977), and extends it for lateness. Chapter 2 reformulates the Henderson model, and solves it analytically. Chapter 3 compares the reformulated model with the Vickrey model. Chapter 4 explores the behavior of the Mahmassani and Herman model. It does not solve its lack of equilibrium. It does, however, give the features of the model that Mahmassani and Herman (1984) do not explore, and aspects that need to be improved.

Part II consists of chapters 5 through 8. Chapter 5 specifies the indirect utility function for discrete choice models of mode and time of day based on utility maximization,

and establishes the marginal-cost pricing rule. Chapter 6 estimates the discrete choice model

of mode and time of day with data from the San Francisco Bay Area. Chapter 7 assembles a

simulation model by combining a supply model of the form in chapter 2 with the demand

model from chapter 6. Sample enumeration is used to connect the supply and demand sides

(Ben-Akiva and Lerman, 1985). Chapter 7 also describes an iterative procedure to calculate

optimal tolls that vary with time of day. Chapter 8 reports simulation results on equilibrium

characteristics of a base case, the effects of the eight policies, and miscalculations of these

effects when schedule shifts are ignored. Chapter 8 also tests hypotheses and questions

formulated above.

PART I

ABSTRACT MODELS OF PEAK-PERIOD CONGESTION

CHAPTER 1

THE HENDERSON APPROACH REEXAMINED

This chapter examines the original Henderson approach and illustrates its problems. Section 1.1 reviews the original Henderson model that prohibits lateness (Henderson, 1977). An example is used to show that it lacks equilibria. Section 1.2 extends the original Henderson model for lateness. The example is used again to show that the extended Henderson model does have equilibria, but contains two peculiar features: 1) commuters can arrive earlier by starting later at the priced equilibrium; 2) the limits of the equilibria are no longer equilibria as the unit cost of being late goes to infinity.

1.1 Original Henderson Model: Lateness Prohibited

Consider the journey to work, where a fixed number of identical commuters, N, one per vehicle, travel on the same road m miles to work. All must arrive at work no later than a common work-start time t^*. Each chooses a home-departure time, t, to minimize the private trip cost, $C(t)$, which includes three parts. The first part is the travel time cost, $\alpha[m/S(t)]$, where α is the unit cost of travel time, and $m/S(t)$ is the travel time. The second part is the schedule delay cost, $\beta[t^*- t - m/S(t)]$, where β is the unit cost of schedule delay early--time wasted waiting for work to start, and $t + m/S(t)$ is the arrival time at work. The third part is

$$C(t) = \alpha \frac{m}{S(t)} + \beta \left[t^* - t - \frac{m}{S(t)} \right] + \tau(t).$$

the toll, $\tau(t)$, if any is imposed. So

Let R be the road capacity, S_{max} the free-flow speed, and $F(t)$ the departure rate at time t.

$$\frac{1}{S(t)} = \frac{1}{S_{max}} + \left[\frac{F(t)}{R} \right]^{\gamma},$$

Henderson (1977) assumes a power speed-flow function given by

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following Vickrey (1965). That is, the travel time for any commuter who departs at t is determined solely by the departure flow at the same time t. This supply model requires that departure flows at different times be independent. The second term of (1.2) measures the travel delay associated with departure flow $F(t)$. The parameter γ measures the elasticity of this travel delay with respect to $F(t)$.

Unpriced Solution. Equilibrium obtains when no commuter can reduce his trip cost by altering his departure time unilaterally. With identical commuters, it is necessary that the

$$\frac{dC(t)}{dt} = (\alpha - \beta)\frac{d}{dt}\left[\frac{m}{S(t)}\right] - \beta = 0,$$

private trip cost be constant across departure times, or

$$\frac{d}{dt}\left[\frac{m}{S(t)}\right] = \frac{\beta}{\alpha - \beta} > 0.$$

which, given $\alpha > \beta$, implies

Let C be the constant private trip cost. Those departing first at i travel at the free-

$$C = \alpha\, T_f + \beta\,(t^* - i - T_f),$$

flow speed: $S(i) = S_{max}$; using (1.1),

where $T_f \equiv m/S_{max}$, the free-flow travel time. Solving (1.4) with condition (1.5) yields the equilibrium travel time function given by

$$\frac{m}{S(t)} = T_f + \frac{\beta}{\alpha - \beta}\,(t - i)\,.$$

$$t^* - n = T_f + \frac{\beta}{\alpha - \beta}\,(n - i)\,.$$

$$\int_i^n F(t)\,dt = N\,.$$

Let n be the on-time departure time: $n + m/S(n) = t^*$, or

All N commuters depart in $[i, n]$:

To solve for i, n, and C analytically, (1.2) and (1.6) are used to write $F(t)$ in terms of t and i, and the resulting $F(t)$ is substituted into (1.8). (1.7) and (1.8) are then used to

$$\Phi = \left(\frac{N}{R}\frac{1+\gamma}{\gamma}\frac{\beta}{\alpha - \beta}\,m^{\frac{1}{\gamma}}\right)^{\frac{\gamma}{1+\gamma}},$$
$$i = t^* - T_f - \frac{\alpha}{\beta}\,\Phi\,,$$
$$n = t^* - T_f - \Phi\,,$$
$$C = \alpha\,T_f + \alpha\,\Phi\,,$$

solve for i and n, and (1.5) is used for C. The solution is

where Φ is the maximum travel delay, which occurs at n.[1]

Total variable cost of travel TVC, total cost of travel delay TCC, and total cost of

$$TCC = \int_i^n \alpha\, F(t) \left[\frac{m}{S(t)} - T_f \right] dt = \alpha\, N\, \Phi\, \frac{1+\gamma}{1+2\,\gamma},$$

$$TSC = \int_i^n \beta\, F(t) \left[t^* - t - \frac{m}{S(t)} \right] dt = \alpha\, N\, \Phi\, \frac{\gamma}{1+2\,\gamma},$$

$$TVC = TCC + TSC = \alpha\, N\, \Phi.$$

schedule delay TSC can be calculated as:

Optimally Priced Solution. To minimize the total cost of transporting N commuters

---

[1]Henderson (1977, p. 175) solves for i, n, and C numerically.

$$\int_i^n F(t) \left[\alpha \frac{m}{S(t)} + \beta \left(t^* - t - \frac{m}{S(t)} \right) \right] dt$$

to work, the traffic planner chooses $F(t)$, $t \in [i, n]$, to minimize

$$\_ = \int_i^n F(t) \left[\alpha \frac{m}{S(t)} + \beta \left(t^* - t - \frac{m}{S(t)} \right) \right] dt + \lambda \left[N - \int_i^n F(t) \, dt \right],$$

subject to (1.8). The Lagrangian of this optimal control, given i and n, is

where λ is the Lagrangian Multiplier of (1.8). The first order condition with respect to $F(t)$

$$\lambda = \alpha \frac{m}{S(t)} + \beta \left[t^* - t - \frac{m}{S(t)} \right] + (\alpha - \beta) F(t) \frac{d}{dF(t)} \left[\frac{m}{S(t)} \right].$$

is given by [2]

Henderson (1977) interprets λ as the social cost of transporting the marginal traveller

on the road at any departure time t; so (1.15) requires that this marginal social cost be equal

across all departure times. Commuters privately incur the first two terms in (1.15); the

optimal toll is equal to the third term in (1.15), or

---

[2]Henderson (1977) ignores the derivative of the schedule delay term with respect to $F(t)$ and results in an error in his first-order condition, which replaces $(\alpha - \beta)$ by α in the second term of (1.15). This amounts to calculating the extra travel delay caused by the marginal commuter but failing to offset its cost by the change in schedule delay.

$$\tau(t) = (\alpha - \beta) \, F(t) \frac{d}{dF(t)} \left[\frac{m}{S(t)} \right].$$

$$\tau(t) = (\alpha - \beta) \gamma \left[\frac{m}{S(t)} - T_f \right].$$

Using (1.1), this can be written as

$$C(t) = \alpha \frac{m}{S(t)} + \beta \left[t^* - t - \frac{m}{S(t)} \right] + (\alpha - \beta) \gamma \left[\frac{m}{S(t)} - T_f \right].$$

With this toll imposed, the private trip cost becomes

Equation (1.5) also holds at the optimally priced solution. Differentiating (1.18) with

respect to t, and solving the resulting equation with condition (1.5) yields the travel time

$$\frac{m}{S(t)} = T_f + \frac{1}{1+\gamma} \frac{\beta}{\alpha - \beta} \, (t - i).$$

function under optimal pricing

$$t^* - n = T_f + \frac{1}{1+\gamma} \frac{\beta}{\alpha - \beta} \, (n - i).$$

The equation that defines n changes from (1.7) to

Equations (1.2) and (1.18) are used to solve for $F(t)$, and the resulting $F(t)$ is substituted

into (1.8). Equations (1.8) and (1.20) are then used for i and n, and (1.5) is used for C.

$$i = t^* - T_f - \Phi \frac{\alpha}{\beta} \left[1 + \gamma \left(1 - \frac{\beta}{\alpha} \right) \right] \left(\frac{1}{1+\gamma} \right)^{\frac{\gamma}{1+\gamma}},$$

$$n = t^* - T_f - \Phi \left(\frac{1}{1+\gamma} \right)^{\frac{\gamma}{1+\gamma}},$$

$$C = \alpha T_f + \alpha \Phi \left[1 + \gamma \left(1 - \frac{\beta}{\alpha} \right) \right] \left(\frac{1}{1+\gamma} \right)^{\frac{\gamma}{1+\gamma}},$$

The solution is

$$TCC = \int_i^n \alpha F(t) \left[\frac{m}{S(t)} - T_f \right] dt = \alpha N \Phi \frac{1+\gamma}{1+2\gamma} \left(\frac{1}{1+\gamma} \right)^{\frac{\gamma}{1+\gamma}},$$

$$TSC = \int_i^n \beta F(t) \left[t^* - t - \frac{m}{S(t)} \right] dt$$

$$= \alpha N \Phi \frac{\gamma}{1+2\gamma} \left[1 + \gamma \left(1 - \frac{\beta}{\alpha} \right) \right] \left(\frac{1}{1+\gamma} \right)^{\frac{\gamma}{1+\gamma}},$$

$$TVC = TCC + TSC = \alpha N \Phi \left[1 + \frac{\gamma^2}{1+2\gamma} \left(1 - \frac{\beta}{\alpha} \right) \right] \left(\frac{1}{1+\gamma} \right)^{\frac{\gamma}{1+\gamma}}.$$

where Φ is given by (1.9). The aggregate costs can be calculated as

Lack of Equilibrium. Using the example given in Table 1.1, this section illustrates the lack of equilibria of the original Henderson approach when lateness is prohibited.

The supply model (1.2) assumes that the travel time for any commuter is determined solely by the departure flow he departs with. This can lead to overtaking, i.e., arriving earlier by starting later. For example, a group departing a bit later than a larger group would arrive earlier. As Henderson (1977) notes, by assuming $\alpha > \beta$, no overtaking can happen during the period of departures from i to n. A problem comes after the period of departures, however; commuters can reduce their private trip cost by unilaterally shifting departure to after n. The example given in Table 1.1 is used to illustrate the problem.

Table 1.1. Parameter Values for Abstract Models[a]

Demand side	Supply side
$N = 1000$	$R = 3817$ vehicles/hour
$\alpha = \$ 6.40$/hour	$\gamma = 4.08$
$\beta = \$ 3.90$/hour	$S_{max} = (60/2.48)$ miles/hour
$v = \$15.21$/hour	$m = 15$ miles
$t^* = 8{:}00$ A.M.	$T_f \equiv m/S_{max} = 0.62$ hours $= 37.2$ minutes

[a] Arnott et al. (1990) use the same values for α, β, and v. Parameters for the supply side are based on Small (1992a, p. 70). Using data from Dewees' (1978) simulation experiments on city arterials, Small estimates a speed-flow curve of $T = 2.48 + 0.254(V/1000)^{4.08}$, where T (travel time) and V (traffic flow) are measured in minutes per mile and vehicles per hour,

respectively. The coefficient 0.254 is converted into the denominator in the parentheses to get $R = 3817$ vehicles per hour. S_{max} is $60/2.48$ miles per hour, and γ is 4.08. The trip distance is set at 15 miles, the number of commuters at 1000, and the common work start time at $8:00$ A.M.

Panels a and b of Figure 1.1 show the cumulative departures and arrivals for the unpriced and priced solutions, respectively.[3] The slopes of the cumulative curves measure rates of departure and arrival, respectively; the horizontal distance between the two cumulative curves measures travel time if no overtaking happens; and the horizontal distance between the cumulative arrival curve and a vertical line at t^* measures schedule delay. The first group travels with free-flow speed (the slopes of the cumulative departure curves are zero at i), but incurs maximum schedule delay; the last group incurs no schedule delay, but travels with maximum travel delay; everyone departs during departure period.

At both solutions, no one has an incentive to shift unilaterally either across i because

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[3]If we let t' be the arrival time associated with departure time t, $CF(t)$ the cumulative departures, and $CA(t')$ the cumulative arrivals, then the following relationships hold:

$$t' = t + \frac{m}{S(t)}, \quad CA(t') = CF(t) = \int_i^t F(x)\, dx .$$

those departing at i travel at the free-flow speed, or among times between i and n because

private trip cost is constant during this period. But if

one shifts unilaterally across n, one can travel almost at the free-flow speed, and overtake the

group departing at n. Instead of spending the time between n and t^* in congestion as the

group departing at n does, one spends the time waiting for work to start. Since travel delay

is more costly than schedule delay, this unilateral shift lowers one's private trip cost. In fact,

one could unilaterally depart shortly before $t^* - m/S_{max}$ and suffer neither travel nor schedule

delay. Thus, the solutions of the original Henderson model are not equilibria.

What contributes to this lack of equilibria? The assumptions of both no lateness and

travel time being determined by departure flow play a role. While assuming travel time being

determined by departure flow makes overtaking possible, prohibiting lateness leads to a speed

at n that is below the free-flow level. To improve the original Henderson model, the

assumption of no lateness is relaxed next, and chapter 2 reformulates the assumption of travel

time being determined by departure flow.

Figure 1.1

1.2 Extended Henderson Model: Lateness Allowed

To extend the original Henderson model for lateness, define t as an early departure time and $t^* - t - m/S(t)$ as schedule delay early if $t + m/S(t) - t^*$ is negative. Define t as a late departure time and $t + m/S(t) - t^*$ as schedule delay late if $t + m/S(t) - t^*$ is positive. Define n such that

$$n + \frac{m}{S(n)} - t^* = 0.$$

Let v be the unit cost of schedule delay late and u the last departure time. The private trip

$$C(t) = \alpha \frac{m}{S(t)} + v \left[t + \frac{m}{S(t)} - t^* \right] + \tau(t).$$

cost is (1.2) for early departures; for late departures it is

Unpriced Solution. Those who depart at u do not incur travel delay:

$$C(u) = \alpha T_f + v (u + T_f - t^*).$$

$S(u) = S_{max}$, or

The equilibrium travel time function is (1.6) for early departure times. For late departure

times, differentiating (1.26) with respect to t, and solving the resulting equation with

$$\frac{m}{S(t)} = T_f + \frac{v}{\alpha + v} (u - t).$$

condition (1.27) yields

The private trip costs at i and u must be equal, and all N commuters depart

between i and u. That is,

$$C(i) = C(u),$$

$$\int_i^u F(t)\, dt = N.$$

Equations (1.2), (1.26), (1.6), and (1.28) are used to solve for $F(t)$, and the resulting $F(t)$ is

substituted into (1.29). Equations (1.7) and (1.29) are then used for i, n, and u, and (1.5) is

$$\Psi = \left(\frac{N}{R} \frac{1+\gamma}{\gamma} \frac{\delta}{\alpha} m^{\frac{1}{\gamma}} \right)^{\frac{\gamma}{1+\gamma}},$$

$$i = t^* - T_f - \Psi \frac{\alpha}{\beta},$$

$$n = t^* - T_f - \Psi,$$

$$u = t^* - T_f + \Psi \frac{\alpha}{v},$$

$$C = \alpha T_f + \alpha \Psi,$$

used for C. The solution is

where $\delta = \beta v / (\beta + v)$, and Ψ is the maximum travel delay, which occurs at n. The

$$TCC = \int_i^u \alpha F(t) \left[\frac{m}{S(t)} - T_f \right] dt = \alpha N \Psi \frac{1+\gamma}{1+2\gamma},$$

$$TSC = \int_i^n \beta F(t) \left[t^* - t - \frac{m}{S(t)} \right] dt + \int_n^u F(t) \left[t + \frac{m}{S(t)} - t^* \right] dt$$

$$= \alpha N \Psi \frac{\gamma}{1+2\gamma},$$

$$TVC = TCC + TSC = \alpha N \Psi.$$

aggregate costs can be calculated as:

Optimally Priced Solution. If there is a marginal-cost toll to support the social

optimum where the total variable cost of transporting N commuters to work is minimized,

$$\int_i^n F(t)\left[\alpha\frac{m}{S(t)}+\beta\left(t^*-t-\frac{m}{S(t)}\right)\right]dt+\int_n^u F(t)\left[\alpha\frac{m}{S(t)}+v\left(t+\frac{m}{S(t)}-t^*\right)\right]dt$$

the traffic planner needs to choose $F(t)$ to minimize

subject to the second constraint of (1.29). Let λ be the Lagrangian Multiplier of this

$$\_=\int_i^n F(t)\left[\alpha\frac{m}{S(t)}+\beta\left(t^*-t-\frac{m}{S(t)}\right)\right]dt$$

$$+\int_n^u F(t)\left[\alpha\frac{m}{S(t)}+v\left(t+\frac{m}{S(t)}-t^*\right)\right]dt+\lambda\left[N-\int_i^u F(t)\,dt\right].$$

constraint; the Lagrangian for this optimal control, given i and u, is

The first order condition with respect to $F(t)$ is (1.15) for early departure time;

$$\lambda=\alpha\frac{m}{S(t)}+v\left(t+\frac{m}{S(t)}-t^*\right)+(\alpha+v)\,F(t)\frac{d}{dF(t)}\left[\frac{m}{S(t)}\right].$$

for late departure time, it is

The constant λ can be interpreted as the marginal social cost of departing at any time

t; so (1.15) and (1.36) requires that this cost be equal across all departure times. Commuters

privately incur the first two terms in (1.15) for early departure times, and in (1.36) for late

departure times. The optimal toll is (1.17) for early departure times; for late departure times,

$$\tau(t) = (\alpha + \nu)\gamma\left[\frac{m}{S(t)} - T_f\right].$$

it is

$$C(t) = \alpha\frac{m}{S(t)} + \nu\left[t + \frac{m}{S(t)} - t^*\right] + (\alpha + \nu)\gamma\left[\frac{m}{S(t)} - T_f\right].$$

The private trip cost is (1.18) for early departures; for late departures it is

The travel time function for early departure times is (1.19); for late departure times, differentiating (1.38) with respect to t, and solving the resulting equation with condition

$$\frac{m}{S(t)} = T_f + \frac{1}{1+\gamma}\frac{\nu}{\alpha + \nu}(u - t).$$

(1.27) yields

$$t^* - n = T_f + \frac{1}{1+\gamma}\frac{\beta}{\alpha - \beta}(n - i).$$

Equations in (1.29) still hold; but the equation that defines n changes to

Equations (1.18), (1.38), (1.19), and (1.39) are used to get $F(t)$, and the resulting $F(t)$ is substituted into (1.29). Equations (1.29) and (1.40) are then used for i, n, and u, and (1.5) is used for C. The solution is

$$\theta = \frac{1 + \gamma \dfrac{\alpha - \beta}{\alpha + v}}{1 + \gamma}, \quad \Gamma = \left(\frac{1}{1+\gamma}\right)^{\frac{\gamma}{1+\gamma}} \left[\frac{\alpha/\delta}{\dfrac{\alpha-\beta}{\beta} + \dfrac{\alpha+v}{v}\theta^{\frac{1+\gamma}{\gamma}}} \right]^{\frac{\gamma}{1+\gamma}},$$

$$i = t^* - T_f - \frac{\alpha}{\beta}\Gamma\Psi\left[1 + \gamma\left(1 - \frac{\beta}{\alpha}\right)\right],$$

$$n = t^* - T_f - \Gamma\Psi,$$

$$u = t^* - T_f + \frac{\alpha}{v}\Gamma\Psi\left[1 + \gamma\left(1 - \frac{\beta}{\alpha}\right)\right],$$

$$C = \alpha T_f + \alpha\Gamma\Psi\left[1 + \gamma\left(1 - \frac{\beta}{\alpha}\right)\right],$$

$$TCC = \alpha\ N\ \Psi\ \Gamma\ \Gamma_1\ \frac{1+\gamma}{1+2\gamma},$$

$$TSC = \alpha\ N\ \Psi\ \Gamma\left[\left(1 - \frac{\gamma}{1+2\gamma}\Gamma_1\right) + \gamma\left(1 - \frac{\beta}{\alpha}\right)\left(1 - \frac{1+\gamma}{1+2\gamma}\Gamma_2\right)\right],$$

$$TVC = \alpha\ N\ \Psi\ \Gamma\left[1 + \gamma\left(1 - \frac{\beta}{\alpha}\right)\left(1 - \frac{1+\gamma}{1+2\gamma}\Gamma_2\right)\right],$$

$$\Gamma_1 = \frac{\dfrac{\alpha-\beta}{\beta} + \dfrac{\alpha+v}{v}\theta^{\frac{1+2\gamma}{\gamma}}}{\dfrac{\alpha-\beta}{\beta} + \dfrac{\alpha+v}{v}\theta^{\frac{1+\gamma}{\gamma}}}, \quad \Gamma_2 = \frac{\dfrac{(\alpha-\beta)^2}{\beta} + \dfrac{(\alpha+v)^2}{v}\theta^{\frac{1+2\gamma}{\gamma}}}{\dfrac{(\alpha-\beta)^2}{\beta} + \dfrac{(\alpha+v)(\alpha-\beta)}{v}\theta^{\frac{1+\gamma}{\gamma}}}.$$

where Ψ is given by (1.30). The aggregate costs can be calculated as

where Γ_1 and Γ_2 are given by

Overtaking and Limiting Solution. This section first examines whether the above

solutions are equilibria, and if they are, whether overtaking can happen. Supply model (1.2)

requires that traffic flows that depart at different times from home be independent. But

overtaking violates this independence. It then examines the limit of each equilibrium as the

unit cost of being late goes infinity. One would expect that such limits would still be

equilibria if there is no problem within the model.

Panels a and b of Figure 1.2 show the cumulative departures and arrivals for the

unpriced and priced solutions, respectively, using parameters in Table 1.1. The dashed lines

in panel b are explained below. Private trip costs are the same across departure times;

speeds are at the free-flow level at both ends of the departure period. So no one has an

incentive to shift unilaterally either within or outside the period; both solutions are equilibria.

Figure 1.2

To examine the possibility of overtaking in equilibrium, one examines arrival times t + $m/S(t)$. Using the equilibrium travel time functions in (1.6) and (1.28) for the unpriced equilibrium, and in (1.19) and (1.39) for the priced equilibrium, one gets $d[t + m/S(t)]/dt > 0$ within the periods of early and late departures, respectively. That is, in each of the periods before and after the on-time departure, the change in travel delay for those traveling later is not enough for them to catch up given the difference in departure times. So no overtaking happens within each period.

No overtaking within each period, however, does not rule out overtaking across the

two periods. Whether overtaking happens depends on whether travel

delay drops suddenly across the two periods. At the unpriced equilibrium, the only cost for

those departing at and immediately after n is travel time. Given identical unit costs of travel

time, the condition of equal private trip costs requires travel times to be equal at and

immediately after n. This rules out overtaking at the unpriced equilibrium.

At the priced equilibrium, however, the condition of equal private trip costs does not

require equal travel times across n because the toll imposed can make up the difference. In

fact, the toll given by (1.17) and (1.37) jumps discontinuously across n. The result is a

discontinuous drop in both toll and departure rate across n. Figure 1.3 shows the toll

schedule and departure rates. So overtaking happens across n at the priced equilibrium.

The two dashed lines in panel b of Figure 1.2 are explained as follows. Let n'

be the arrival time for those who depart just after n, n the

Figure 1.3

departure time for those who depart before n and arrive at n', and m_2 the departure time for those who depart after n and arrive at t^*. Those who depart between n and m_2 overtake those who depart between m_1 and n. The two dashed lines show the cumulative arrivals for the two segments of the curve of cumulative departures from m_1 to n and from n to m_2 respectively. The real curve of cumulative arrivals between n' and t^* is obtained by adding the number of overtaking vehicles, i.e., those departing between n and m_2 and arriving

between n' and t^*, to the lower dashed line.

The limits of the equilibria are examined now. When the unit cost of schedule delay late v approaches infinity, δ approaches β; Ψ and the last departure time at both solutions

$$\Psi = \left(\frac{N}{R} \frac{1+\gamma}{\gamma} \frac{\beta}{\alpha} m^{\frac{1}{\gamma}} \right)^{\frac{\gamma}{1+\gamma}},$$

$$u = t^* - T_f.$$

become:

Figure 1.4 shows the cumulative departures and arrivals of the limits. In the limit of the unpriced equilibrium, departures occur after the on-time departure. Those who depart after the on-time departure also arrive on time, but incur different travel delays. This cannot be true in equilibrium. This is why the unpriced solution shown in Figure 1.1a looks entirely different from this limit.

The limit of the priced equilibrium in Figure 1.4b is not an equilibrium. Compare the private trip costs at i and u, at which there is no travel delay or toll. But departure at i incurs schedule delay as well as the free-flow travel time, while departure at u incurs only the free-flow travel time.

Figure 1.4

To explain the two dashed lines, let n' be the arrival time for those who depart just after n and n_1 the departure time for those who depart before n and arrive at n'. Those who depart between n and u overtake those who depart between n_1 and n. The two

dashed lines show the cumulative arrivals for the two segments of the curve of cumulative departures from n_1 to n and from n to u respectively. The real curve of cumulative arrivals between n' and t^* is obtained by adding the number of overtaking vehicles, i.e., those departing between n and u and arriving between n' and t^*, to the lower dashed line.

1.3 Summary

The original Henderson approach assumes that travel time for any commuter is determined solely by the departure flow he departs with from home; and that traffic flows that depart at different times are independent. The section shows that the original Henderson approach has the following problems: a) the original Henderson model that prohibits lateness lacks equilibria; b) equilibria do exist in the extended Henderson model that allows lateness, but have two peculiar features: 1) commuters can arrive earlier by starting later at the priced equilibrium; 2) their limits are no longer equilibria as the unit cost of being late goes to infinity.

CHAPTER 2

THE HENDERSON APPROACH REFORMULATED

The original Henderson approach assumes that travel time for any commuter is determined solely by the departure flow he departs with from home; and that traffic flows that depart at different times are independent. Chapter 1 shows that the original Henderson approach has problems: a) the original Henderson model that prohibits lateness lacks equilibria; b) equilibria do exist in the extended Henderson model that allows lateness, but has two peculiar

features: 1) commuters can arrive earlier by starting later at the priced equilibrium; 2) their limits are no longer equilibria as the unit cost of being late goes to infinity.

The assumption of travel time being determined by departure flow plays a key role in the existence of these problems: it makes overtaking possible. When lateness is prohibited, the speed at the end of the departure period is below the free-flow level. This low speed at the end of the departure period and the possibility of overtaking after the departure period create an incentive to shift schedule across the end of the departure period. When lateness is allowed, on the other hand, overtaking that occurs in the priced equilibrium violates the assumption of independent traffic flows.

This chapter reformulates the original Henderson approach by assuming that the travel time for any commuter is determined solely by the arrival flow he

arrives with at work. This reformulation no longer requires that traffic flows that depart at different times from home are independent.

This new formulation is just as plausible as that in Henderson (1977): both are approximations to reality. In fact, it was used, without comments, for a different scheduling problem of commuters by Henderson (1981, 1985) to model production effects of staggered work hours, and by Henderson (1992) to investigate the biases inherent in cost-benefit analyses of capacity expansion that ignores scheduling behavior.

So the reformulated Henderson approach modifies congestion technology (1.2) so that

$$\frac{1}{s(t')} = \frac{1}{S_{max}} + \left[\frac{f(t')}{R}\right]^{\gamma},$$

travel speed, $s(t')$, is determined by arrival flow, $f(t')$, through

where t' is any arrival time at work. The elasticity γ may not be the same as in (1.2). Since the solution method for the reformulated Henderson approach is the same as for the original Henderson approach, the same numbers will be used with the first digit changed from 1 to 2. For example, (2.2) indicates that it is similar to (1.2).

2.1 Reformulated Henderson Model: Lateness Prohibited

$$c(t') = \alpha \frac{m}{s(t')} + \beta \ (t^* - t') + p(t'),$$

Each commuter chooses an arrival time t', no later than t^*, to minimize

where $p(t')$ is the toll schedule if any is imposed.

Unpriced Equilibrium. Equilibrium requires travel time to change at the following

$$\frac{d}{dt'}\left[\frac{m}{s(t')}\right] = \frac{\beta}{\alpha} > 0.$$

rate:

$$c = \alpha T_f + \beta (t^* - i').$$

The private trip cost of arriving first at i' is given by

The equilibrium travel time function is given by

$$\frac{m}{s(t')} = T_f + \frac{\beta}{\alpha}(t' - i').$$

$$\int_{i'}^{t^*} f(t') \, dt' = N.$$

The last group arrives at t^*, and all N commuters arrive in $[i', t^*]$:

Equations (2.2) and (2.6) are used to get $f(t)$, and the resulting $f(t)$ is substituted into (2.8).

Equation (2.8) is then used for i', and (2.5) is used for c. The solution is given by

$$\Phi' = \left(\frac{N}{R} \frac{1+\gamma}{\gamma} \frac{\beta}{\alpha} m^{\frac{1}{\gamma}} \right)^{\frac{\gamma}{1+\gamma}},$$

$$i' = t^* - \frac{\alpha}{\beta} \Phi',$$

$$c = \alpha T_f + \alpha \Phi',$$

where Φ' is the maximum travel delay, which occurs at t^*. For later comparison, the first

$$i = i' - T_f = t^* - T_f - \frac{\alpha}{\beta} \Phi',$$

$$n = t^* - T_f - \Phi'.$$

and last departure times, i and n, are given by

$$TCC = \int_{i'}^{t^*} \alpha \, f(t') \left[\frac{m}{s(t')} - T_f \right] dt' = \alpha N \Phi' \frac{1+\gamma}{1+2\gamma},$$

Once i' and c are determined, the aggregate costs can be calculated as

$$TSC = \int_{i'}^{*} \beta \; f(t')(t^{*} - t') \, dt' = \alpha \; N \; \Phi' \frac{\gamma}{1 + 2\gamma},$$

$$TVC = TCC + TSC = \alpha \; N \; \Phi'.$$

It is interesting to compare the unpriced solution of the reformulated Henderson approach, given by (2.9)-(2.12), with that of the original Henderson approach, given by (1.9)-(1.12), assuming that the elasticities of travel delay of the two approaches in (1.2) and (2.2) are the same. The maximum travel delay Φ', given by (2.9), is less than Φ, given by (1.9). Given the parameter values in Table 2, $\Phi' = 0.466$ and $\Phi = 0.991$. Comparing (1.9) with (2.9), the period of departures both starts and ends later in the reformulated Henderson approach. Comparing (1.10)-(1.12) with (2.10)-(2.12), aggregate costs are smaller in the reformulated Henderson approach.

One possible explanation for this difference between the original and reformulated Henderson approaches is that at the unpriced solution of the original Henderson approach, commuters can lower their private trip cost by shifting departures later. Without a full adjustment of departures to reach an equilibrium, the period of departures both starts and ends too early; aggregate costs are too high.

$$\int_{i'}^{*} f(t') \left[\alpha \frac{m}{s(t')} + \beta \; (t^{*} - t') \right] dt'$$

Optimally Priced Equilibrium. The traffic planner chooses $f(t)$ to minimize subject to (2.8). Let λ be the Lagrangian Multiplier of (2.8); then the Lagrangian for this

$$_- = \int_{t'_i}^{t^*} f(t') \left[\alpha \frac{m}{s(t')} + \beta(t^* - t') \right] dt' + \lambda \left[N - \int_{t'_i}^{t^*} f(t') dt' \right].$$

optimal control problem, given t', is

$$\lambda = \alpha \frac{m}{s(t')} + \beta(t^* - t') + \alpha f(t') \frac{d}{df(t')} \left[\frac{m}{s(t')} \right].$$

The first order condition with respect to $f(t)$ is

The constant λ can be interpreted as the marginal social cost of arriving at any time t; (2.15) requires that this cost be equal across all arrival times. The optimal toll is the third

$$p(t') = \alpha f(t') \frac{d}{df(t')} \left[\frac{m}{s(t')} \right].$$

term, or

$$p(t') = \alpha \gamma \left[\frac{m}{s(t')} - T_f \right].$$

Using (2.2), this can be written as

$$c(t') = \alpha \frac{m}{s(t')} + \beta(t^* - t') + \alpha \gamma \left[\frac{m}{s(t')} - T_f \right].$$

With this toll imposed, the private trip cost of arriving at t becomes

The equilibrium travel time function becomes

$$\frac{m}{s(t')} = T_f + \frac{1}{1+\gamma}\frac{\beta}{\alpha}\,(t'-i')\,.$$

Equations (2.2) and (2.19) are used to solve for $f(t)$, and the resulting $f(t)$ is

$$i' = t^* - \Phi'\frac{\alpha}{\beta}(1+\gamma)^{\frac{1}{1+\gamma}}\,,$$

$$c = \alpha\,T_f + \alpha\,\Phi'(1+\gamma)^{\frac{1}{1+\gamma}}\,,$$

substituted into (2.8). Equations (2.8) and (2.5) are then used for i and c. The solution is

$$i = i' - T_f = t^* - T_f - \Phi'\frac{\alpha}{\beta}(1+\gamma)^{\frac{1}{1+\gamma}}\,,$$

$$n = t^* - T_f - \Phi'\left(\frac{1}{1+\gamma}\right)^{\frac{\gamma}{1+\gamma}}.$$

where Φ' is given by (2.9). For later comparison, i and n are given by

$$TCC = \int_{i'}^{t^*} \alpha\,f(t')\left[\frac{m}{s(t')} - T_f\right]dt' = \alpha\,N\,\Phi'\,\frac{1}{1+2\gamma}\,(1+\gamma)^{\frac{\gamma}{1+\gamma}}\,,$$

$$TSC = \int_{i'}^{t^*} \beta\,f(t')\,(t^*-t')\,dt' = \alpha\,N\,\Phi'\,\frac{\gamma}{1+2\gamma}\,(1+\gamma)^{\frac{1}{1+\gamma}}\,,$$

The aggregate costs can be calculated as

$$TVC = TCC + TSC = \alpha N \ \Phi' \frac{1+\gamma}{1+2\gamma} \ (1+\gamma)^{\frac{1}{1+\gamma}}.$$

It is interesting to compare the priced solutions of the original and reformulated

Henderson approaches in (1.21)-(1.24) and (2.21)-(2.24), respectively, assuming the same

elasticities of travel delay in the two approaches. Since Φ' is less than Φ, the reformulated

approach gives a departure period that ends later (using (1.21) and (2.21)); it also gives a

smaller total cost of travel delay (using (1.22) and (2.22)).

The relative values in the first departure time, total cost of schedule delay, and total

variable cost of travel all depend on the relative values of $(1+\gamma)\Phi'$ and $[1+\gamma(1 - \beta/\alpha)]\Phi$.

Given the parameters in Table 1.1, $(1+\gamma)\Phi' = 2.365$ and $[1+\gamma(1 - \beta/\alpha)]\Phi = 2.569$. This

leads to the same result as for the unpriced solutions: the period of departures starts and ends

later, and aggregate costs are smaller in the reformulated Henderson approach than in the

original approach.

2.2 Reformulated Henderson Model: Lateness Allowed

If one arrives after $\overset{*}{t}$, one is late by an amount $t - \overset{*}{t}$. $t > \overset{*}{t}$ will be refereed to as

late arrival times. The private cost is (2.1) for $t \le \overset{*}{t}$; for $t > \overset{*}{t}$, it is

$$c(t') = \alpha \frac{m}{s(t')} + v \, (t' - t^*) + p(t').$$

Unpriced Equilibrium. Those who arrive at the end, u', incur no travel delay: $s(u') =$

$$c(u') = \alpha \, T_f + v \, (u' - t^*).$$

S_{max}, or

The equilibrium travel time function is (2.6) for $t \leq t^*$. For $t > t^*$, differentiating (2.26)

$$\frac{m}{s(t')} = T_f + \frac{v}{\alpha}(u' - t').$$

with respect to t', and solving the resulting equation with condition (2.27) yields

The private trip costs at i' and u' must be equal, and all N commuters arrive

$$c(i') = c(u'),$$
$$\int_{i'}^{u'} f(t') \, dt' = N.$$

between i' and u'. That is,

Equations (2.1) and (2.28) are used to solve for $f(t')$, and the resulting $f(t')$ is substituted

into (2.29); equation (2.29) is used for i' and u', and (2.5) is used for c. The solution is

given by

$$i' = t^* - \Psi \frac{\alpha}{\beta},$$

$$u' = t^* + \Psi \frac{\alpha}{v},$$

$$c = \alpha T_f + \alpha \Psi,$$

where Ψ, given by (1.30), is the maximum travel delay, which occurs at $\overset{*}{t}$. For later

$$i = i' - T_f = t^* - T_f - \Psi \frac{\alpha}{\beta},$$

$$n = t^* - T_f - \Psi,$$

$$u = u' - T_f = t^* - T_f + \Psi \frac{\alpha}{v}.$$

comparison, the first, on-time, and last departure times, i, n, and u, are

$$TCC = \int_{i'}^{u'} \alpha f(t') \left[\frac{m}{s(t')} - T_f \right] dt' = \alpha N \Psi \frac{1+\gamma}{1+2\gamma},$$

$$TSC = \int_{i'}^{t^*} \beta f(t')(t^* - t') dt' + \int_{t^*}^{u'} v f(t')(t' - t^*) dt' = \alpha N \Psi \frac{\gamma}{1+2\gamma},$$

$$TVC = TCC + TSC = \alpha N \Psi.$$

The aggregate costs can be calculated as

Again, one can compare the unpriced solutions of the original and reformulated

Henderson approaches given by (1.30)-(1.33) and (2.30)-(2.33), respectively. The unpriced

solutions are identical if the elasticities of travel delay are the same. This is no surprise

because unlike the unpriced solution without lateness, the unpriced solution with lateness in

the original Henderson approach is a real equilibrium without overtaking.

One can also compare the aggregate costs of the reformulated Henderson approach at the unpriced equilibria with and without lateness, given by (2.31)-(2.33) and (2.10)-(2.12) respectively. Allowing lateness saves aggregate costs by a fraction of $(\Phi' - \Psi)/\Phi' = 1 - (v/(\beta+v))^{\gamma/(1+\gamma)}$, where Ψ and Φ' are given by (1.30) and (2.9) respectively. The larger the elasticity of travel delay γ, or the larger the relative unit costs of being early and late, the larger is this saving. Given the parameter values in Table 1.1, this is about a 17 percent saving.

$$\int_{i'}^{t^*} f(t') \left[\alpha \frac{m}{s(t')} + \beta (t^* - t') \right] dt' + \int_{t^*}^{u'} f(t') \left[\alpha \frac{m}{s(t')} + v (t' - t^*) \right] dt'$$

Optimally Priced Solution. The traffic planner chooses $f(t)$ to minimize subject to the second constraint in (2.29). Let λ be the Lagrangian Multiplier of this

$$\underline{\quad} = \int_{i'}^{t^*} f(t') \left[\alpha \frac{m}{s(t')} + \beta (t^* - t') \right] dt'$$
$$+ \int_{t^*}^{u'} f(t') \left[\alpha \frac{m}{s(t')} + v (t' - t^*) \right] dt' + \lambda \left[N - \int_{i'}^{u'} f(t') dt' \right].$$

constraint; then the Lagrangian for this optimal control problem, given i' and u', is

The first order condition with respect to $f(t)$ for $t \leq t^*$ is (2.15); for $t > t^*$, it is

$$\lambda = \alpha \frac{m}{s(t')} + v(t' - t^*) + \alpha f(t') \frac{d}{df(t')} \left[\frac{m}{s(t')} \right].$$

The constant λ can be interpreted as the marginal social cost of arriving at any time t. Commuters privately incur the first two terms in (2.15) for $t \leq \hat{t}$, and in (2.36) for $t > \hat{t}$; the optimal toll is given by (2.16), the third term in (2.15) or (2.36).

$$c(t') = \alpha \frac{m}{s(t')} + v(t' - t^*) + \alpha \gamma \left[\frac{m}{s(t')} - T_f \right].$$

The private trip cost then is (2.18) for $t \leq \hat{t}$; when $t > \hat{t}$, it is

$$\frac{m}{s(t')} = T_f + \frac{1}{1+\gamma} \frac{v}{\alpha} (u' - t').$$

The equilibrium travel time function for $t \leq \hat{t}$ is (2.19); for $t > \hat{t}$, it is

$$i' = t^* - \frac{\alpha}{\beta} \Psi (1+\gamma)^{\frac{1}{1+\gamma}},$$

$$u' = t^* + \frac{\alpha}{v} \Psi (1+\gamma)^{\frac{1}{1+\gamma}},$$

$$c = \alpha T_f + \alpha \Psi (1+\gamma)^{\frac{1}{1+\gamma}},$$

Solving (2.29) for i' and u', and using (2.5) for c yields where Ψ is given in (1.30), with γ being the elasticity of travel delay with respect to arrival flow of the reformulated Henderson approach. The first, on-time, and last departure times, i, n, and u, are given by

$$i = i' - T_f = t^* - T_f - \frac{\alpha}{\beta} \Psi \ (1 + \gamma)^{\frac{1}{1+\gamma}},$$

$$n = t^* - T_f - \Psi \left(\frac{1}{1+\gamma} \right)^{\frac{\gamma}{1+\gamma}},$$

$$u = u' - T_f = t^* - T_f + \frac{\alpha}{v} \Psi \ (1 + \gamma)^{\frac{1}{1+\gamma}}.$$

$$TCC = \int_{i'}^{u'} \alpha \ f(t') \left[\frac{m}{s(t')} - T_f \right] dt' = \alpha \ N \ \Psi \ \frac{1}{1+2\gamma} \ (1+\gamma)^{\frac{1}{1+\gamma}},$$

$$TSC = \int_{i'}^{t^*} \beta \ f(t')(t^* - t') dt' + \int_{t^*}^{u'} v \ f(t')(t' - t^*) dt'$$

$$= \alpha \ N \ \Psi \ \frac{\gamma}{1+2\gamma} (1+\gamma)^{\frac{1}{1+\gamma}},$$

$$TVC = TCC + TSC = \alpha \ N \ \Psi \ \frac{1+\gamma}{1+2\gamma} \ (1+\gamma)^{\frac{1}{1+\gamma}}.$$

The aggregate costs can be calculated as

Cumulatives and Limiting Equilibria. This section uses the example in Table 1.1 to

illustrate that the reformulated Henderson approach is free of the problems noted earlier in the

original Henderson approach.

Panels a and b of Figure 2.1 show the cumulative arrivals and departures for the

unpriced and priced solutions, respectively, without lateness. Arrivals start at i' and end at t^*

. Private costs are the same across arrival times; there is no incentive for any commuter to

change arrival time within $[t', \hat{t}]$. Commuters arriving at t' travel at the free-flow speed;

arriving earlier than t' is worse off. So is arriving later than $\hat{t}$. So both solutions in Figure

2.1 are equilibria.

Panels a and b of Figure 2.2 show the unpriced and priced solutions with lateness.

Private cost associated with any arrival time between t' and u' is the same; nobody can do

better by changing arrival time within this period of arrivals. Neither can anybody outside the

period because travel associated with the start and end of the period is at the free-flow speed.

The equilibria with lateness converge to those without lateness as the unit cost of

being late v goes to infinity; therefore the curves of cumulative departures and arrivals for

the limiting equilibria are not shown separately.

Figure 2.1

Figure 2.2

2.3 Summary

This chapter reformulates the original Henderson approach by assuming the number of travelers arriving together at their destination, instead of departing together at their origin, determines their travel time. This chapter shows doing so eliminates all the problems of the original Henderson approach investigated in chapter 1.

The chapter also finds that giving travelers the flexibility of being late for activities at the destination can result in substantial savings. The larger the elasticity of travel delay with respect to arrival flow, or the larger the relative unit costs of being early and being late, the larger are these savings. For typical values of the elasticity and unit costs of schedule delays, these savings are about twenty percent.

CHAPTER 3

A COMPARISON WITH THE VICKREY APPROACH

This chapter compares the behavior of the reformulated Henderson approach of

chapter 2 with the Vickrey approach, focusing on: a) pattern of travel; b) five known results of

the Vickrey approach. Section 3.1 reviews the Vickrey approach, based on ADL (1990);

section 3.2 presents the comparison. ADL (1990) formalize the model in Vickrey (1969), and

focus on the characteristics of equilibrium with various pricing schemes. Since ADL (1990)

is followed closely in reviewing the Vickrey model, the term "Vickrey-ADL" model will be

used in detailed discussions. Models with lateness are used.

3.1 The Vickrey Model: Lateness Allowed

ADL (1990) set up the journey to work as follows. Travel is not congested except at a single segment of the road (the bottleneck) through which at most k vehicles can pass per unit of time; if the departure rate exceeds k, a queue develops at the bottleneck. The length

$$Q(t) = \int_{\hat{t}}^{t} \left[F(u) - k \right] du ,$$

of queue for those leaving home at t is

where the low limit is the last departure time before t when there was no queue. $Q(t)/k$ is their queuing delay. Let $T(t)$ be the travel time ; then

$$T(t) = T_f + \frac{Q(t)}{k}.$$

$$C(t) = \begin{cases} \alpha\,T(t) + \beta\,(\,t^* - t - T(t)\,) + \tau(t) \ \text{ for } t \leq n, \\ \alpha\,T(t) + \nu\,(\,t + T(t) - t^*\,) + \tau(t) \ \text{ for } t > n. \end{cases}$$

Each commuter chooses a departure time t to minimize the private cost

where $\tau(t)$ is the toll schedule if any is imposed.

Unpriced Equilibrium. As ADL (1990) show, all commuters except the first and last

$$F(t) = \begin{cases} \dfrac{\alpha\,k}{\alpha - \beta} \ \text{ for } t \leq n, \\[4mm] \dfrac{\alpha\,k}{\alpha + \nu} \ \text{ for } t > n. \end{cases}$$

experience congestion, and they depart home at a piecewise constant rate given by

The equilibrium travel-time function is given by

$$T(t) = \begin{cases} T_f + \dfrac{\beta}{\alpha - \beta}(t - i) & \text{for } t \leq n, \\[3mm] T_f + \dfrac{v}{\alpha + v}(u - t) & \text{for } t > n. \end{cases}$$

$$N = (n - i)\frac{\alpha k}{\alpha - \beta} + (u - n)\frac{\alpha k}{\alpha + v},$$

$$\alpha T_f + \beta(t^* - i - T_f) = \alpha T_f + v(u + T_f - t^*),$$

$$t^* - n = T_f + \frac{\beta}{\alpha - \beta}(n - i).$$

The first, one-time, and last departure times, i, n, u, can be solved with

The first states that all N commuters leave home between i and u; the second specifies

that the private costs are the same at i and u; and the last defines n.

$$i = t^* - T_f - \frac{\delta}{\beta}\frac{N}{k},$$

$$n = t^* - T_f - \frac{\delta}{\alpha}\frac{N}{k},$$

$$u = t^* - T_f + \frac{\delta}{v}\frac{N}{k},$$

Solving (3.6) yields

where $(\delta/\alpha)(N/k)$ is the maximum queuing delay, which occurs at n. Substituting i into

$$C = \delta \frac{N}{k}.$$

(3.6) yields the constant private trip cost

$$TCC = \int_i^u \alpha F(t) \left[T(t) - T_f \right] dt = \frac{\delta}{2} \frac{N^2}{k},$$

$$TSC = \int_i^n \beta F(t) \left[t^* - t - T(t) \right] dt + \int_n^u v F(t) \left[t + T(t) - t^* \right] dt = TCC,$$

$$TVC = TCC + TSC = \delta \frac{N^2}{k}.$$

The aggregate costs are calculated as

At the unpriced equilibrium, total cost of schedule delay is half of total variable cost of travel; total variable cost of travel is independent of the unit cost of travel time α.

Optimally Priced Equilibrium. ADL (1990) show that at the social optimum at which total variable cost of travel is minimized, there should be no queuing. It follows that at the social optimum, $F(t) = k$ for $t \in [i, u]$, and speed of travel is constant at m/T_f.

$$\tau(t) = \begin{cases} \delta \dfrac{N}{k} - \beta \left[t^* - t - T_f \right] \text{ for } t \le n, \\ \delta \dfrac{N}{k} - v \left[t + T_f - t^* \right] \text{ for } t > n. \end{cases}$$

This social optimum can be decentralized by a time-varying toll over $[i, u]$: With

$$\tau(t) = \begin{cases} \beta (t - i) \text{ for } t \le n, \\ v (u - t) \text{ for } t > n. \end{cases}$$

(3.7), this toll can be written alternatively as

As ADL (1990) show, this optimal toll does not change the period of arrivals, the

private trip cost, or total cost of schedule delay; but it eliminates queuing and thereby cuts

total variable cost of travel in half. That is, i and u are the same as in (3.7), C is the same

$$n = t^* - T_f .$$

as in (3.8), but n changes to

The aggregate costs are $TCC = 0$, $TSC = \delta N^2 / 2k$, and $TVC = \delta N^2 / 2k$.

3.2 Comparison

The comparison focuses on: a) pattern of travel; b) five known results of the Vickrey-

ADL model. Travel patterns are compared numerically. The five results are examined

analytically as well as numerically.

The five results of the Vickrey-ADL model are: 1) total cost of schedule delay is half

of total variable cost of travel at the unpriced equilibrium; 2) the optimal toll saves 100

percent of total cost of travel delay, 0 percent of total cost of schedule delay, and 50 percent

of total variable cost of travel; 3) the optimal toll does not change the period of arrivals; 4) the

optimal toll does not change the equilibrium private trip cost; and 5) total variable cost of

travel is independent of the unit cost of travel time at both the priced and unpriced equilibria.

Analytical. In the reformulated Henderson model, the five results of the Vickrey-ADL

model do not hold for any finite value of γ:

1) With (2.32)-(2.33), the ratio between total cost of schedule delay and total variable

$$SDR = \frac{\gamma}{1 + 2\gamma} < \frac{1}{2}.$$

cost of travel (SDR) at the unpriced equilibrium is given by

2) With (2.31)-(2.33) and (2.42), the fractional savings in total cost of travel delay

(STCC), total cost of schedule delay (STSC), and total variable cost of travel (STVC) due to

optimal pricing are given respectively by

$$STCC = 1 - \left(\frac{1}{1+\gamma}\right)^{\frac{\gamma}{1+\gamma}} < 1,$$

$$STSC = 1 - (1+\gamma)^{\frac{1}{1+\gamma}} < 0,$$

$$STVC = 1 - \frac{1+\gamma}{1+2\gamma} (1+\gamma)^{\frac{1}{1+\gamma}} < \frac{1}{2}.$$

STSC is negative, but both *STCC* and *STVC* are positive. The increase in total cost of

schedule delay due to optimal pricing is the result of travel being spread over a wider interval;

but it is more than offset by the saving in total cost of travel delay.

3) With (2.30) and (2.41), the fractional lengthening of the period of arrivals due to

$$\Delta (u' - i') = (1+\gamma)^{\frac{1}{1+\gamma}} - 1 > 0.$$

optimal pricing is given by

The optimal toll lengthens the period of arrivals by forcing the first arrival earlier and the last

one later. The changes in the first and last arrival times due to optimal pricing are given

$$\Delta i' = \Psi \frac{\alpha}{\beta} \left[1 - (1+\gamma)^{\frac{1}{1+\gamma}} \right] < 0,$$

$$\Delta u' = \Psi \frac{\alpha}{\nu} \left[(1+\gamma)^{\frac{1}{1+\gamma}} - 1 \right] > 0.$$

respectively by

4) With (2.30) and (2.41) again, the fractional change in the private trip cost due to

optimal pricing is given by

$$\Delta c = (1 + \gamma)^{\frac{1}{1+\gamma}} - 1 > 0.$$

The optimal toll increases the equilibrium private trip cost.

5) With (1.30), (2.33), and (2.42), total variable cost of travel depends on the unit cost

of travel time α, with a factor of $\alpha^{1/(1+\gamma)}$, at both the priced and unpriced equilibria. It also

depends on the schedule-delay parameters β and v through δ, just as in the Vickrey-ADL

model.

The five results hold, however, in the limit as γ goes to infinity. This is because as

γ goes to infinity, Ψ goes to $(\delta/\alpha)(N/R)$ and $(1+\gamma)^{1/(1+\gamma)}$ goes to unity. In the limit, the

equilibria of the reformulated Henderson model become exactly the same as those of the

Vickrey-ADL model, with k replaced by R. The intuition behind this limiting behavior of

the reformulated Henderson model is that the speed-flow function (2.2) approaches a shape as

$\lrcorner$, which is exactly the relationship implicitly assumed in the Vickrey-ADL model.

Numerical. The models are specified to compare the patterns of travel between the

two models, and to examine to what extent the five results of the Vickrey-ADL model hold in

the reformulated Henderson model.

Model Specification. One difficulty of this specification is to find appropriate values

of supply-side parameters of the two models so that they are comparable; only then is

comparison between the two approaches helpful. The parameter values in Table 1.1 are for

the reformulated Henderson model. The value for the bottleneck capacity is determined for

the Vickrey-ADL model so that the two models yield the same total variable cost at the unpriced equilibrium. The same free-flow travel time is used in the two models.

With (1.30), (2.33), and (3.11), the condition of equal total variable cost at the

$$k = \frac{\delta}{\alpha \Psi} N \,,$$

unpriced equilibrium yields the following level of capacity for the Vickrey-ADL model:

where Ψ is given by (1.30). k defined above approaches R as γ approaches infinity.

Model Comparison. Results using the base set of parameter values just described are reported in Table 3.1 and Figures 2.2 and 3.1. Table 3.1 presents equilibrium characteristics of both models. Figure 2.2 presents cumulative departure and arrivals for both models. Figure 3.1 presents departure and arrival rates for the two models. The travel patterns are discussed first.

At the unpriced equilibrium the two models yield the same travel time function. With the total variable costs of travel being equal, the first and last arrival times, as well as private trip cost, are all the same (see Table 3.1). This is verified by comparing equations (2.6) and (3.5). The difference between the equilibria of the two models is in departure and arrival rates (see Figure 3.1).

Table 3.1 Equilibria for Alternative Approaches and Pricing Schemes[a]

	Vickrey			Henderson			
	Unpriced	Priced	%	Unpriced	Priced	%	
First arrival time		7:22	7:22	NA[b]	7:22	7:07	NA[b]
Last arrival time		8:10	8:10	NA[b]	8:10	8:13	NA[b]
Peak length: minutes	48	48	0%	48	66	+38%	Private
trip cost: $/trip	2.48	2.48	0%	2.48	3.42	+38%	Average Toll:
$/trip	0	1.24	NA[b]	0	1.52	NA[b]	Total cost of travel
delay: $	1240	0000	-100%	1375	373	-73%	Total cost of schedule
delay: $	1240	1240	0%	1105	1522	+23%	Total variable cost of
Travel: $	2480	1240	-50%	2480	1895	-24%	

[a] Demand-side parameters for both approaches, and supply-side parameters for the reformulated Henderson approach are given in Table 1.1. For the Vickrey approach, supply-side parameter T_f is the same as m/S_{max} in the reformulated Henderson approach; parameter k is determined by setting its total cost of travel at the unpriced equilibrium equal to that of the reformulated Henderson approach:

$$k = 1251 \text{ vehicles/hour}$$

[b] Percentage changes do not apply here.

Figure 3.1

Panels a and c of Figure 2.2 show the cumulative departures and arrivals in the

unpriced equilibrium for both models. Panels a and c of Figure 3.1 show the corresponding

departure and arrival rates. In the Vickrey-ADL model, traffic flows enter the road at a

constant rate of 3203 vehicles per hour, which is 2.6 times larger than the bottleneck capacity

(1251 vehicles per hour). A queue grows steadily and speeds decline continuously until the

on-time departure time

n. Thereafter, traffic flows enter at a constant rate of 373 vehicles per hour, far

below the bottleneck capacity. The queue shrinks and speed increases until the end u.

Traffic flows exit at the constant rate of the bottleneck capacity.

In the reformulated Henderson model, however, commuters enter the road at an

increasing rate from 0 at the beginning i to 3988 vehicles per hour at the on-time departure

time n, and at a decreasing rate thereafter from 461 at n to 0 vehicles per hour at the end u.

 Unlike in the Vickrey-ADL model, speed is determined by arrival flows, which exit at an

increasing rate from 0 at i' to 1558 vehicles per hour at t^* , and at a decreasing rate from

1558 at t^* to 0 vehicles per hour at the end u'.

Panels b and d of Figure 2.2 depict the cumulative departures and arrivals, and

those of Figure 3.1 depict the rates of departure and arrival at the priced equilibrium for both models. With the optimal toll, the first and last arrival times are no longer the same between the two models. Neither are their travel time functions.

Specifically, the departure rate in the Vickrey-ADL model reduces to the level of the bottleneck capacity, resulting in free-flow travel for everyone. The period of arrivals is not changed since each commuter now pays the optimal toll instead of queuing at the bottleneck.

In the presence of the optimal toll in the reformulated Henderson model, commuters leave home still at an increasing rate before the on-time departure, and at a decreasing rate thereafter. The arrive rate is smaller than at the unpriced equilibrium, increasing continuously from 0 at the beginning to 1131 vehicles per hour at t^*, and decreasing continuously to 0 at the end. The result is less severe congestion throughout. Unlike in the Vickrey-ADL model, the period of arrivals lengthens with the first arrival earlier and the last one later.

With an understanding of the equilibria of both models, it is ready to numerically examine the five results listed earlier. Only the first four are considered. This discussion is based on Table 3.1.

The first result is that total cost of schedule delay is 50 percent of total variable cost of travel at the unpriced equilibrium in the Vickrey-ADL model. Using parameter values in Table 1.1, total cost of schedule delay is about 44 percent of total variable cost of travel in the reformulated Henderson model.

The second result is that the optimal toll saves 100 percent in total cost of travel delay,

0 percent in total cost of schedule delay, and 50 percent in total variable cost of travel in the Vickrey-ADL model. Again using the base set of parameter values in Table 1.1, the optimal toll saves about 73 percent in total cost of travel delay, minus 23 percent in total cost of schedule delay, and about 24 percent in total variable cost of travel in the reformulated Henderson model.

The third result is that the optimal toll leaves the period of arrivals unchanged in the Vickrey-ADL model. In the reformulated Henderson model with the parameter values of Table 1.1, however, the period of arrivals lengthens by 38 percent.

The fourth result is that the optimal toll leaves private trip cost unchanged in the Vickrey-ADL model. With parameter values of Table 1.1, private trip cost increases by about 38 percent in the reformulated Henderson model.

As emphasized above, these percentages in the reformulated Henderson model are based on the base set of parameter values in Table 1.1, while those in the Vickrey-ADL model are independent of any of its parameters. One natural question is: do these percentages in the reformulated Henderson model vary with its parameters; and, if so, how? The answer is that these percentages in the reformulated Henderson model depend only on the elasticity of travel delay with respect to arrival flows, γ, as shown in equations (3.15)-(3.19) and (3.21).

How do these percentages in the reformulated Henderson approach vary with γ? The variation of these percentages with γ is shown in Figures 3.2-3.4 with γ ranging from 1 to 30. These percentages converge to those of the Vickrey-ADL model as γ approaches ∞

(i.e., infinity). The base value for γ is 4.08 , which lies between 2.5 and 5 , a range

suggested for γ (Small, 1992a).

Figure 3.2 shows the percentage ratio between total cost of schedule delay and total

variable cost of travel at the unpriced equilibrium. This ratio reaches 40 and 45 percent as γ

= 2 and 5 respectively. After 5, it levels off, and converges to 100 percent (the Vickrey-ADL

level) as $\gamma = \infty$.

Figure 3.3 shows the percentage savings in aggregate costs. With $\gamma = 4.08$,

percentage savings in total cost of travel delay, total cost of schedule delay, and total variable

cost of travel are 73 , -23 , and 24 , respectively; as $\gamma = 5$, they are 88 , -35 , and 39 ; they

converge to 100 , 0 , and 50 (the Vickrey-ADL levels) as $\gamma = \infty$.

Figure 3.4 shows the percentage lengthening of the period of arrivals. It is 38 percent

as $\gamma = 4.08$, 35 percent as $\gamma = 5$, and 0 (the Vickrey-ADL level) as $\gamma = \infty$. The percentage

increase in the private trip cost with respect to γ is not separately shown because it follows

the same pattern as the percentage lengthening of the period of arrivals (see (3.19) and (3.21)).

Some comments are in order on the cusps in Figures 3.3 and 3.4 in the lengthening of

the period of arrivals and in the savings of total cost of schedule delay. These cusps actually

occur at $\gamma = e - 1 \approx 1.72$, where e is the base of natural logarithm. It is intuitive to have the

largest increases in total cost of schedule delay and in the period of arrivals occur together

because it is the spreading of arrivals that causes total cost of schedule delay to increase. But

it is not intuitive that these largest increases occur at

$\gamma \approx 1.72$.

Figure 3.2

Figure 3.3

Figure 3.4

3.3 Summary

This chapter compares the behavior of the reformulated Henderson approach of

chapter 2 with that of the Vickrey approach both analytically and using simulations. It finds

that the behavior of the reformulated Henderson approach varies with its elasticity of travel

delay with respect to arrival flow at destination, while the Vickrey approach lacks such a

flexibility; that the behavior of the Vickrey approach is the limit of that of the reformulated

Henderson approach as the elasticity of travel delay goes to infinity; and that the behaviors of

the two approaches are not close when the elasticity of travel delay varies between 2.5 and 5 ,

the range suggested in the literature.

CHAPTER 4

ON MODELING OF HYPER-CONGESTION

This chapter explores the behavior of the Mahmassani and Herman model and shows a subtle, but easily missed problem. It does not solve the problem. It does give the features of the model that Mahmassani and Herman (1984) do not explore, and aspects that need to be improved.

Data for midtown Manhattan on a typical weekday in 1983 show that traffic density exceeds that for the maximum flow from 11 A.M. to 7 P.M (Vickrey, 1991). As a result, the more vehicles forced into the street network, the lower is the average flow. This phenomenon is often called hyper-congestion in the economics literature, or congested flow condition in the engineering literature. Given this pattern of travel in midtown Manhattan, the marginal social cost of traveling two-mile at 11 A.M., as Vickrey (1993) claims, can be around $1000. Hyper-congestion is inefficient because the same flow can be carried at a much lower cost.

To achieve a socially optimal travel pattern, an optimal toll based on the marginal social cost *in equilibrium* needs to be imposed. Vickrey's claim presumes no change in driver behavior; but the whole point of the toll is to modify travel patterns. How can we compute the marginal social cost of traveling in a street network like midtown Manhattan not under current conditions as Vickrey tries to do but in equilibrium?

To address this properly, a structural model of peak-period congestion for a street

network is required. The model should explicitly treat not only travelers' scheduling decisions, but also allows hyper-congestion. The model should also be tractable for economic analysis.

Most structural models of peak-period congestion either do not allow hyper-congestion or are too complex for economic analysis. The bottleneck model (Vickrey, 1969; Hendrickson and Kocur, 1981) is tractable but does not allow hyper-congestion. When a queue exists at the bottleneck, traffic leaves the bottleneck always at the rate of the bottleneck capacity; when there is no queue, traffic leaves at the rate as it arrives at the bottleneck. The Henderson model (Henderson, 1977; Chu, 1993) is also tractable but does not allow hyper-congestion either; traffic flows entering a roadway at different times do not interfere throughout the trip. Instead of treading a roadway homogeneous as in Mahmassani and Herman (1984) by using average density, speed, and flow over the whole road, Newell (1988) treats a roadway heterogeneous and works with location-specific density, speed, and flow in a theoretical model. So do Chang, Mahmassani, and Herman (1985) with computer simulation models. But tractability is lost in both cases. The model by Mahmassani and Herman (1984) is the exception: it is tractable and allows hyper-congestion.

Newell (1988) criticizes the Mahmassani and Herman model precisely for the possibility of hyper-congestion on an isolated section of a roadway. Newell correctly states that the density in the isolated section can not exceed that for its maximum flow. If it did, the wave velocity for this density would be negative; the density perturbation would propagate

further upstream, causing it to back out of the isolated section. The critique does not hold, however, if the Mahmassani and Herman model is adapted to non-isolated roadways that have downstream bottlenecks. A city street network such as midtown Manhattan is full of such bottlenecks.

Is it reasonable to adapt the Mahmassani and Herman model to a street network? For detailed studies of traffic operations, either the computer simulation approach or Newell's theoretical approach would be more appropriate because both treat a street network as heterogeneous. For economic analysis, however, they are too complex. Instead, we may consider the entire area under study as a homogeneous mass of streets over which some traffic relationships are assumed to hold (Vickrey, 1993). This is the approach by Mahmassani and Herman (1984).

4.1 Model Review

The notation follows Mahmassani and Herman. A fixed number N of identical

commuters desire to go from home to work along a single road. The common desired arrival

$$c(t) = \alpha_1 \tau(t) + \alpha_2 [A - t - \tau(t)]$$

time is A. Every commuter chooses a time t to enter the road to minimize trip cost

where $\tau(t)$ and $A - t - \tau(t) > 0$ are the travel time and schedule delay, respectively; α_1 and

α_2 are their unit costs.

The road has two distinct sections: the upstream section has l miles; the downstream

section has m miles. Congestion can only occur in the upstream section, refereed later as the

congestable section. Let $k(t)$ and $v(t)$ be the average density and speed, respectively, along

the congestable section at time t. The average flow along the congestable section at time t,

$$q(t) = k(t) \ v(t).$$

$q(t)$, is defined as:

Mahmassani and Herman make three additional assumptions:

1. Greenshields' linear relationship between the average speed and average density

$$v(t) = v_m \left[1 - \frac{k(t)}{k_j} \right], k(t) \le k_j$$

along the congestable section holds at any time t:

where v_m is the free-flow speed and k_j the jam density.

2. Let $\lambda(t)$ be the number of commuters entering the congestable section at time t (the input flow at time t). The following conservation equation holds on the congestable

$$l \frac{dk(t)}{dt} = \lambda(t) - q(t).$$

section:

$$\tau(t) = \frac{l}{v(t)} + \tau_f$$

3. The travel time $\tau(t)$ for a commuter leaving home at time t is

where $l/v(t)$ is the travel time on the congestable section; $\tau_f \equiv m/v_m$, the travel time on the non-congestable section. Equation (4.5) uses the average speed prevailing on the congestable section at the time when a commuter enters the section to compute his travel time. The average speed, however, varies with the average density of the congestable section while one travels. If the average density increases during one's journey, one's travel time would be underestimated; if it decreases, one's travel time would be overestimated.

Evidence from Austin and Dallas, Texas shows that (4.2) holds in a street network

(Ardekani and Herman, 1987). The Greenshilds linear relationship (4.3) is a special case of

those in Ardekani and Herman (1987) estimated for traffic in street networks. For street

networks, input flow would include not only traffic entering the area but also traffic originated

from parking lots within the area. Similarly, output flow would include not only traffic

leaving the area but also traffic terminated at parking lots within the area.

Mahmassani and Herman solve the model as follows. It is necessary in equilibrium

$$\frac{d\tau(t)}{dt} = \frac{\alpha_2}{\alpha_1 - \alpha_2}$$

that the user cost be constant for all departure times. This leads to

$$\frac{d\tau(t)}{dt} = \frac{\tau_m}{k_j} \left[1 - \frac{k(t)}{k_j} \right]^{-2} \frac{dk(t)}{dt}$$

for $\alpha_1 > \alpha_2$. Differentiating (4.5) and using (4.3) yields

where $\tau_m \equiv l/v_m$, the free-flow travel time on the congestable section. Combining (4.6) and

(4.7) and rearranging yields a differential equation for the normalized average density, $K(t) \equiv$

$k(t)/k_j$, along the congestable section:

$$\frac{dK(t)}{dt} = \frac{1}{\tau_m} \frac{\alpha_2}{\alpha_1 - \alpha_2} \left[1 - K(t)\right]^2.$$

$$K(t) = 1 - \frac{1}{1 + \frac{1}{\tau_m} \frac{\alpha_2}{\alpha_1 - \alpha_2} (t - t_0)}.$$

$$\tau(t) = \tau_m + \frac{\alpha_2}{\alpha_2 - \alpha_2} (t - t_0) + \tau_f$$

Define departure time t_0 such that $K(t_0) = 0$. The solution for $K(t)$ is

Substituting (4.9) into (4.3) and using (4.5) yields the travel time function given by

where the second term measures travel delay. The average flow function is obtained from

(4.9) through (4.2) and (4.3). The departure rate function is obtained by substituting (4.2) and

$$\lambda(t) = l \ k_j \ \frac{dK(t)}{dt} + v_m \ k_j \ K(t)\left[1 - K(t)\right].$$

(4.3) into (4.4) and rearranging as follows:

$$t_f + \tau(t_f) = A$$

Define departure time t_f such that

where $\tau(t_f)$ is given by (4.10). All N commuters depart between t_0 and t_f, or

$$N = \int_{t_0}^{t_f} \lambda(t)\, dt$$

where $\lambda(t)$ is given by (4.11). Equations (4.9) through (4.13) can be used to determine t_0 and t_f numerically. The period of departures is defined by t_0 and t_f.

4.2 Model Behavior

It is crucial to understand how the Mahmassani and Herman model behaves not only within but also after the period of departures. Neither Mahmassani and Herman nor Newell (1988) examine the latter. This section first solves the model for the period of no departures. It then identifies the condition under which hyper-congestion may occur and the condition under which overtaking may occur. At the end, it shows that the Mahmassani and Herman

model lacks equilibrium.

After the period of departures, $\lambda(t) \equiv 0$ and (4.6) no longer applies. From (4.11), the

$$\frac{dK_n(t)}{dt} = -\frac{1}{\tau_m} K_n(t)[1 - K_n(t)]$$

normalized average density $K_n(t)$ on the congestable section satisfies

where the subscript, n , indicates no departures. Using the initial condition that

$$K_n(t) = 1 - \frac{1}{1 + \frac{K(t_f)}{1 - K(t_f)} \exp\left[-\frac{t - t_f}{\tau_m}\right]}$$

$K_n(t_i) = K(t_i)$ and solving equation (4.14) yields

where $K(t_i)$ is given by (4.9) and exp represents the exponential function. Substituting

$$\tau_n(t) = \tau_m \left\{ 1 + \frac{K(t_f)}{1 - K(t_f)} \exp\left[-\frac{t - t_f}{\tau_m}\right] \right\} + \tau_f .$$

(4.15) into (4.3) and using (4.5) yields the travel time function after the period of departures

The average flow along the congestable section is obtained from (4.15), using (4.2) and (4.3).

Figures 4.1 - 4.3 show the behavior of the model both before and after the period of

departures for three different lengths of the congestable section. The parameter values chosen

are listed at the top of each figure. The following parameters are the same for all three

figures: $N = 500$, $A = 8{:}00$, $\alpha_1 = 6.4$, $\alpha_2 = 3.9$, $k_j = 220$, $v_m = 30$, and $m = 10$ miles.

Mahmassani and Herman use the same values for A and k_j . Arnott, de Palma, and Lindsey

(1990) use the same values for α_1 and α_2 . The length of the congestable section, l, is 2

miles in Figure 4.1; 4 miles in Figure 4.2 ; and 1 miles in Figure 4.3 .

In each figure, the five variables $\lambda(t)$, $k(t)$, $v(t)$, $q(t)$, and $\tau(t)$ are plotted in the

five boxes labelled as "Departure Rate," "Density," "Speed," "Flow," and "Travel Time,"

respectively. In all boxes, the horizontal axis measures clock time, scaled from t_0 to A .

Variables on the vertical axis vary with boxes. Notice that a horizontal line is also drawn in

the Density box at half of the jam density, at which flow reaches its maximum flow, q_m, as

labeled in the Flow box. The system behaves differently within and after the period of

departures. For example, average density and travel time both increase with time within the

period of departures, but both decrease with time after the period of departures.

Four features of the Mahmassani and Herman model are explored here. First, the

condition under which hyper-congestion may occur is identified. Hyper-congestion occurs in

both Figures 4.1 and 4.3 , but not in Figure 4.2 . In fact, hyper-congestion occurs if and only

if $K(t_0) > 1/2$, or, from (4.9),

$$\frac{\alpha_2}{\alpha_1 - \alpha_2}(t_f - t_0) > \tau_m.$$

This is equivalent to $\tau(t_l) - \tau_f > 2\tau_m$. That is, when the parameters are such that the travel

delay, $\tau(t_l) - \tau_m - \tau_f$, for the last departure at t_l is greater than the free-flow travel time along

the congestable section, hyper-congestion occurs within the period of departures.

Second, the condition under which one can arrive earlier by starting later--a

phenomenon called overtaking is identified. Given $\alpha_1 > \alpha_2$, overtaking can not occur within

the period of departures because arrival time $t + \tau(t)$ increases with departure time t. But

overtaking can occur after the period of departures. To see this, one can examine the

difference in arrival times between those who depart at t_l and any hypothetical commuter

$$\Delta(t) \equiv [t + \tau_o(t)] - [t_f + \tau(t_f)]$$
$$= (t - t_f) + \frac{\alpha_2}{\alpha_1 - \alpha_2}(t_f - t_0)\left[\exp\left(-\frac{t - t_f}{\tau_m}\right) - 1\right]$$

who would depart at some time t after t_l:

The function in (4.18) is plotted in the three figures, labeled as "Overtaking."

The purpose is to see whether there is any set of parameters at which $\Delta(t) < 0$ for

some time $t > t_l$. Since $\Delta(t)$ is zero at t_l and its second derivative is positive after t_l,

$$\frac{d \Delta(t)}{dt} = 1 - \frac{1}{\tau_m}\frac{\alpha_2}{\alpha_1 - \alpha_2}(t_f - t_0)\exp\left[-\frac{t - t_f}{\tau_m}\right]$$

overtaking will occur if the first derivative of (4.18) given by

has a root larger than t_i. In fact, (4.19) is zero for some $t > t_i$ if and only if (4.17) holds. Unlike hyper-congestion, however, overtaking occurs only after the period of departures.

Thus, neither or both overtaking and hyper-congestion occur for a given set of parameters. To see this in the figures, note hyper-congestion occurs when the average density exceeds the horizontal line at half the jam density in the Density box; overtaking occurs when the curve in the Overtaking box is below the horizontal line at zero. They occur in Figures 4.1 and 4.3; but neither occurs in Figure 4.2.

Third, the overtaking problem lessens when the congestable section is shorter. Figure 4.2 has the shortest congestable section but no overtaking; Figure 4.3 has the longest congestable section but the most serious problem of overtaking.

Lastly, Mahmassani and Herman's model lacks user equilibrium for those sets of parameters that lead to hyper-congestion. This is a direct result of the overtaking problem. When a commuter can overtake those departing at t_i by unilaterally departing after t_i, he can reduce his trip cost by the schedule shift. By definition, the corresponding solution is not a user equilibrium.

Figure 4.1

Figure 4.2

Figure 4.3

4.3 Summary

The Mahmassani and Herman model treats a roadway homogeneous and allows hyper-congestion to occur. Its framework can be useful for considering marginal-cost pricing in city street networks where hyper-congestion are likely to occur. In fact, Vickrey (1991) does just that for midtown Manhattan. But their formulation of the framework can lead to lack of user equilibrium for parameters that result in hyper-congestion. Is this a general problem of assuming homogeneity in traffic or just a result of their particular formulation? If the later is the case, can we modify their formulation so that it retains tractability and the possibility of hyper-congestion, and at same time it admits equilibrium? It is hoped that the findings on the behavior of their model will help stimulate further research on their framework and lead to answers to these questions.

PART II   AN APPLIED MODEL OF PEAK-PERIOD CONGESTION

CHAPTER 5

THEORETICAL FRAMEWORK

This chapter develops a theoretical framework for an equilibrium simulation model of

peak-period congestion. The demand side is a discrete choice model of both mode and time

of day. The indirect utility function of the choice model is specified analytically with utility

maximization. The supply side is a bottleneck model in discrete form or a speed-flow

function of chapter 2. This chapter derives equilibrium conditions and a marginal-cost pricing

rule.

The indirect utility function to be used in discrete choice models of trip scheduling is

often specified without a theoretical foundation. Variables on time being early or late for

destination activities are arbitrarily added. Small (1982) is one exception. He extends models

of time allocation (Becker, 1965) by making travel cost and time dependent on scheduling.

Small's setup has two shortcomings, however. First, duration of work, but not duration at the

work site, enters the time budget. The gap is either time early or time late for work. This gap

is the focus of those discrete choice models of trip scheduling. Second, Small's setup does not

lead to an operational specification of the utility function.

Marginal-cost pricing rules have only been established in the context of simple

continuous models without scheduling considerations (see for example, Walter, 1961). This

chapter analytically establishes a marginal-cost pricing rule in the context of a discrete choice

model of mode and schedule. It shows that an optimal toll that maximizes social welfare, the

sum of consumer welfare and toll revenue, is a marginal-cost toll.

Section 5.1 extends Train and McFadden (1978) to include both mode and time of

day. Section 5.2 describes alternative models of supply. Equilibrium conditions and the

marginal-cost pricing rule are derived in section 5.3.

5.1 Demand

Train and McFadden (1978) show how to specify an indirect utility function for a mode choice model based on utility maximization. Their formulation given in Small (1992a, pp. 40-43) is extended to time-of-day choice.

Systematic Utility

Consider a sample of N commuters. Commuter i ($i = 1$,..., N) has daily unearned income Y^i, wage rate w^i, total time available T^i, and work-start time s^i. Each commuter consumes some amount of a numeraire commodity denoted by Z, and spends time L in pure leisure, H in working, and t_{ms} in commuting with mode m and schedule s. Define schedule delay early as $SDE_s = s^i - s$ if $s < s^i$, and schedule delay late as $SDL_s = s - s^i$ if $s \geq s^i$. Train and McFadden (1978) assume that time spent in working or commuting enters utility with an adjustment to L, so that utility depends on Z and 'effective leisure' $\_$:

$$\overline{L} = L + \alpha_h H + \alpha_e \ SDE_s + \alpha_l \ SDL_s + \sum_{m,s} \alpha_{ms} \ t_{ms}$$

Adjustments for SDE_s and SDL_s have been added for the extension here.

Conditional on mode m and schedule s, each commuter chooses Z and _ to

$$U^i_{ms} = \ln Z + b \ \overline{L} + u^i_{ms} \equiv W^i_{ms} + u^i_{ms}$$

maximize direct utility, U_{ms}, given by

$$Z + (\ c_{ms} + \tau_{ms} \ ) = Y^i + w^i \ H$$

subject to a money budget

$$L + H + SDE_s - SDL_s + t_{ms} = T^i$$

and a time budget

where $\ln$ represents natural logarithm; b is a parameter; c_{ms} and τ_{ms} are non-toll travel cost

and toll, respectively, to and from work with mode m and schedule s. Only one of the two

schedule delay variables has an effect on the time budget; which one is in effect depends on

whether one is early or late for work. The first two terms, W_{ms}, in (5.2) comprise the direct,

systematic utility. The last term in the right hand side of (5.2) is an idiosyncratic taste

constant, which measures the unobserved attraction of mode m and schedule s to the

commuter. Figure 5.1 shows an example of the time budget when $s < s^i$.

An additional term, SDE_s or SDL_s, is added to the time budget to take into account

the time the commuter waits for work to start in the morning, SDE_s, if $s < s^i$ or to subtract

the time the commuter is late for work, SDL_s, from work hours H if $s > s^i$. This is to put

the duration at the work site, not the duration of work, in the time budget. Commuters are

assumed to leave the work place in the afternoon right at the work-end time $s^i + H$. Thus,

travel time for the afternoon commute is independent of schedule s.

$$\overline{L} = T^i - (1 - \alpha_h) H - (1 - \alpha_e)\ SDE_s - (1 + \alpha_l)\ SDL_s - (1 - \alpha_{ms})\ t_{ms}$$

Using (5.1), the time budget can be rewritten as

Solving constraints (5.3) and (5.5) for Z and _ , substituting them into (5.2), maximizing

with respect to H, and substituting the solved H into (5.2), yields the indirect, systematic

$$W^i_{ms} = - b(1 - \alpha_h)\frac{c_{ms} + \tau_{ms}}{w^i}$$
$$- b(1 - \alpha_e)\ SDE_s - b(1 + \alpha_l)\ SDL_s - b(1 - \alpha_{ms})\ t_{ms}$$

utility W^i_{ms} conditional on mode m and schedule s:

$$\lambda^i \equiv \frac{\partial W^i_{ms}}{\partial (c_{ms} + \tau_{ms})} = - \frac{b(1 - \alpha_h)}{w^i}$$

This specification implies a marginal utility of income given by

a marginal value of commuting time with mode m and schedule s given by

$$(v_t^i)_{ms} \equiv \frac{\partial W_{ms}^i / \partial t_{ms}}{\partial W_{ms}^i / \partial c_{ms}} = \frac{1 - \alpha_{ms}}{1 - \alpha_h} \, w^i$$

$$v_e^i \equiv \frac{\partial W_{ms}^i / \partial SDE_s}{\partial W_{ms}^i / \partial c_{ms}} = \frac{1 - \alpha_e}{1 - \alpha_h} \, w^i$$

$$v_l^i \equiv \frac{\partial W_{ms}^i / \partial SDL_s}{\partial W_{ms}^i / \partial c_{ms}} = \frac{1 + \alpha_l}{1 - \alpha_h} \, w^i$$

a marginal value of schedule delay for early schedule s given by

and a marginal value of schedule delay for late schedule s given by
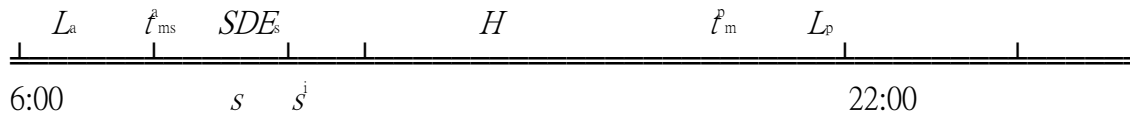
Figure 5.1. An Example of Time Allocation for a Commuter who Arrives at Work Early. He

sleeps between 22:00 P.M. and 6:00 A.M. He spends $L = L_a + L_p$ in leisure. He arrives at

work at s, being early for work by $SDE_s = s^i - s$. He spends $\hat{t}_{ms}$ and $\hat{t}_m$ for A.M. and P.M.

commutes, respectively. $t_{ms} = \overset{a}{t}_{ms} + \overset{p}{t}_{m}$. He works H hours.

<u>Discrete Choice</u>

Suppose the morning peak-period is divided into S discrete intervals, and there are M modes available in each of the intervals. A commuter will choose mode $m\,(m = 1\,,...,\,M)$

$$P_{ms}^{i} = \text{Prob}\,\left[W_{ms}^{i} + u_{ms}^{i} \geq W_{m's'}^{i} + u_{m's'}^{i}\,;m' = 1\,,...,\,M\,,s' = 1\,,...,\,S\right]$$

and schedule $s\,(s = 1\,,...,\,S)$ with a probability given by

such that $\sum_{ms} P_{ms}^{i} = 1$. Here s is used to index any of the discrete time intervals. Each interval is d minutes long.

Suppose u_{ms}^{i} are identically distributed across modes and time intervals and independent across modes but correlated across time intervals, according to the nested logit model (McFadden, 1978). The choice probabilities are:

$$P^i_{ms} = P^i_{s|m} \bullet P^i_s \,,$$

$$P^i_{s|m} = \frac{\exp\left(W^i_{ms} / \rho_m\right)}{\exp\left(I^i_m\right)}\,,$$

$$P^i_m = \frac{\exp\left(\rho_m\right)}{\sum_{m'}\exp\left(\rho_{m'}\right)}\,,$$

$$I^i_m = \ln \sum_s \exp\left(W^i_{ms} / \rho_m\right).$$

where I_m in (5.12) is the inclusive value for schedule choice conditioned on mode m; $1 - \rho_m$

is a parameter, measuring correlation of u^i_{ms} across schedules.

5.2 Supply

Speed-flow curves with a maximum flow (Walter, 1961, for example) could be useful

for representing hyper-congestion (or congested flow conditions) in urban street networks, as

argued in chapter 4. It may also be satisfactory for representing moderate levels of congestion

in off-peak periods or long-tern analysis (Keeler and Small, 1977). It is unsatisfactory,

however, for describing demand conditions where demand for highway use exceeds highway

capacity.

One way to represent extreme demand conditions is to use a deterministic queuing

model, a continuous form of which is given by (3.1)-(3.2). For discrete choice analysis, it is

useful to write the queuing model in a discrete form. To do this, let V_s be the flow in

passenger car units per hour arriving at the bottleneck in time interval s, Q_s and q_s be the

queue and queuing delay for those arriving at the bottleneck in time interval s, and T_s be the

$$Q_1 = \mathrm{Max}\{0, V_1 - C\},$$
$$Q_s = \mathrm{Max}\{0, Q_{s-1} + (V_s - C)\},$$
$$T_s = T_f + \frac{Q_s}{C},$$

sum of queuing delay and free-flow travel time, T_f. For $s = 2, \cdots, S$,

where C is the bottleneck capacity.

An alternative is a Henderson-type supply model that has no maximum flow as given in (1.2) or (2.2). This supply model is static in itself, and has been widely used in static analysis of traffic congestion (Small, 1992a, p. 70). But as demonstrated in the first three chapters, it can also be satisfactorily used in dynamic analysis of traffic congestion.

5.3 Social Optimum

The purpose is to drive an optimal pricing rule within the context of a discrete choice

$$g^i_{ms}(\overline{V}) = c_{ms} + v^i_e \ SDE_s \ + \ v^i_l \ SDL_s \ + \ (v^i_t)_{ms} \ t_{ms}$$

demand model. Let g^i_{ms} denote the generalized travel cost; so

where $SDE_s = d(s^i - s)$ if $s < s^i$; $SDL_s = d(s - s^i)$ if $s \geq s^i$; and d is the duration of each

time interval. Rewrite systematic utility in (5.6) as

$$W_{ms}^{i} = - \lambda^{i} \left(g_{ms}^{i} + \tau_{ms} \right)$$

where the toll per person τ_{ms} may be written as the ratio of the toll per passenger car unit and

the passenger car occupancy of mode $m : \tau_s / r_m$.

$$CW(\overline{g(\overline{V})}, \overline{\tau}) \equiv \sum_{i} \frac{\alpha^{i}}{\lambda^{i}} \ln \sum_{m} \exp \left(\rho_{m} I_{m}^{i} \right)$$

Following Small (1992a, p. 28), the following

measures aggregate consumer welfare. Its change equals a change in consumer's surplus. The

'bar' in (5.16) indicates a vector. The factor α^{i} in (5.16) is the weight applying to commuter

i ; this weight equals the number of travelers commuter i represents in a population under

$$R(\overline{V}, \overline{\tau}) = \frac{d}{60} \sum_{s} \tau_s V_s$$

study who are identical to commuter i on observed characteristics. Toll revenue is

where the factor $d/60$ adjusts flow to number of vehicles traveling in an interval of duration

d minutes. The objective is to maximize social welfare as follows:

$$\text{Max } SW(\overline{V}, \overline{\tau}) \equiv CW(\overline{g}(\overline{V}), \overline{\tau}) + R(\overline{V}, \overline{\tau})$$
$$\overline{V}, \overline{\tau}$$

$$\frac{\partial SW(\overline{V}, \overline{\tau})}{\partial \tau_s} = V_s - \frac{60}{d} \sum_i \sum_m \alpha^i \frac{P^i_{ms}}{r_m} = 0 \text{ for } s = 1, \dots, S$$

$$\frac{\partial SW(\overline{V}, \overline{\tau})}{\partial V_s} = \tau_s - \frac{60}{d} \sum_i \sum_{m,k} \alpha^i P^i_{mk} \frac{\partial g^i_{mk}(\overline{V})}{\partial V_s} = 0 \text{ for } s = 1, \dots, S$$

The first order conditions are:

These two sets of conditions state that the optimal toll and traffic flow in each time interval should be set such that the marginal increase in toll revenue is equal to the marginal decrease in consumer welfare.

The first set of conditions (5.19) are conditions for a stochastic equilibrium (Anas, 1990): a stochastic equilibrium results when the travel time in each interval is such that the expected number of commuters choosing to travel in the interval equals the traffic counts in that interval that gives rise to the travel time in that interval.

$$\tau_s = \frac{60}{d} \sum_i \sum_{m,k} \alpha^i \ P^i_{mk} \frac{\partial \ g^i_{mk}(\overline{V}\ )}{\partial \ V_s} \quad \text{for s} = 1,\dots,\text{S}$$

The second set of conditions give the optimal pricing rule:

The right hand side of this pricing rule is the marginal increase in the expected generalized

$$TC(\overline{V}\ ) \equiv \sum_i \sum_{m,k} \alpha^i \ P^i_{mk} \ g^i_{mk}(\overline{V}\ )$$

travel cost

from an increase in traffic counts in a given time interval while holding P_{mk} constant. So

(5.21) is a marginal-cost pricing rule that maximizes social welfare.

CHAPTER 6

ESTIMATION OF DEMAND MODEL

This chapter empirically specifies and estimates the discrete choice model of mode

and time-of-day proposed in chapter 5. Data used in estimation are from the Urban Travel

Demand Forecasting Project, University of California-Berkeley.

Both Abkowitz (1980) and Hendrickson and Plank (1984) attempt to estimate discrete

choice models of mode and time-of-day. Neither is satisfactory for the current research for

reasons reviewed later, and a new demand model must be estimated. The data are described

in section 6.1. The demand model is specified in section 6.2, and results are reported in

section 6.3.

6.1 Data

Data are for a sample of 991 commuters in the San Francisco Bay Area for the Urban Travel Demand Forecasting Project (UTDFP) at the University of California at Berkeley. Details are in Johnson (1976) and Reid (1977).

Behavioral and Socio-economic Data.

The UTDFP staff compiled behavioral and socioeconomic data for this sample of 991 commuters from two personal surveys, Work Travel Study (WTS) and BART Impact Travel Study-1 (BITS). Each survey included a home interview and a mailback questionnaire following the home interview.

WTS was conducted in the Spring of 1972. During this survey, BART had not yet begun operation. Respondents consisted of "potential transit commuters" from the East Bay Area who lived within feasible range of BART service in the East Bay, and worked in Oakland, Berkeley, San Francisco, Daly City, or Emeryville. There were 213 completed home interviews and 319 completed mailback questionnaires.

BITS was conducted in the Winter of 1973. During this survey, BART offered daytime service in San Francisco and the East Bay; trans-bay service connecting the two

portions of the system was not yet available. There were 1724 completed home interviews and 3447 completed mailback questionnaires.

Under both surveys, the study area was divided into a number of geographic strata and then each stratum was sampled by multistage area probability sampling method (selecting, in order, study areas, blocks within selected areas, and houses within selected blocks) such that each household had equal probability of being selected. At each household selected, screening questions were used to identify eligible respondents, i.e., those who worked in the centrally located cities identified for WTS and those who were eighteen years of age or older for BITS. Only one person was interviewed per household; if a household contained more than one eligible person, a random procedure was used to select a respondent. Questionnaires were limited to persons sixteen years of age or older from each household sampled for the home interviews.

The home interviews determined the following information for each respondent: usual travel mode, work-start time, regular arrival time at work for the morning trip, and other standard socioeconomic characteristics such as household income, respondent's age, sex, occupation, etc. Work-quit time was also reported but no departure time for the afternoon trip was reported.

The questionnaires for WTS requested a coded description of every trip made on the first weekday following the home interview. The questionnaires for the BITS requested a coded description of every trip made on the Tuesday following the home interview. The

description included trip purpose, origin and destination, time started and ended, method of travel, number of blocks walked at start of the trip. For car trips, the description also included number of people in the car, method of parking at the trip end, and cost of parking. For transit trips the description also included number of transfers.

Trip Time and Cost Data.

Trip time and cost data were prepared by the Urban Travel Demand Forecasting Project. The data reflect travel conditions in the San Francisco Bay Area in mid-1972.

A. Data Source. The core data source was the regional planning network data files maintained by the Metropolitan Transportation Commission for the San Francisco Bay Area as updated to represent conditions in January 1972. This data source included a highway network file and a transit network file.

The highway network file contained, for each link in the network, the node numbers at each end, the distance, and morning peak and midday travel times. In addition, various characteristics of the link such as road classifications are coded. The morning peak network values were defined to be the average travel time values for the peak AM hour of travel.

The transit network file contained headways at each boarding or transferring point and line-haul times for all transit lines in the region for the morning peak, midday periods, and the afternoon peak. Average walk access times within zones of the transit network were also coded.

The other major source was external travel-time data for the morning peak and the afternoon peak, respectively, from floating-car runs on major Bar Area freeways between 1971 and 1973. The data cover most freeways subject to congestion, usually in the congested direction only, and usually over a long enough period between 2.5 to 3 hours to completely include the buildup and decay of congestion for one peak. Typically the data for one stretch of freeway consist of 20 to 40 runs taken over the same 2.5 to 3 hour period on two or three different days. The data were collected by the Highway Operations Group of the Caltrans at San Francisco.

Other data sources included origin and destination locations from the personal surveys, transit timetables for the region from transit operators representing 1972 conditions, an interzonal bus fare matrix from MTC representing late 1971 conditions, data on automobile operating and maintenance costs as a function of road type and speed from Keeler and Small (1975), and a survey of all off-street fee parking in the study area in 1972.

B. Automobile Trip Time. Three procedures were used to compute automobile travel times: a path-finding procedure using network travel times, a congestion curve-fitting procedure using external congestion data, and a time-calculating procedure using the fitted congestion curves.

The path-finding procedure first calculated an impedance for each link in the highway network file. The impedance approximates operating and maintenance costs plus a cost of time. The cost of time was the link travel time times value of time at $1.68 per hour. This

value of time was inferred from McFadden (1974). The procedure then ran a path algorithm and produced one path for each work-to-home trip (using midday network times) and two paths for each home-to-work trip (using morning peak and midday network times, respectively) in the sample of 991 commuters.

The congestion curve-fitting procedure fitted a congestion curve for each link in the highway network file. If a link represents a freeway for which floating-car run data were available, two piecewise-linear curves were fitted to the floating-car run data, one for the morning peak and the other for the afternoon peak. If a link represents a non-freeway or a freeway for which floating-car run data were unavailable, an approximation was used. This approximation step uses the congestion curve for a nearby representative freeway stretch in the same area of a given link to determine the shape and the timing of congestion variation for that link.

The travel time calculation procedure calculated travel times for 12 alternative arrival times for each home-to-work trip. These 12 arrival times resulted from dividing the arrival time window for each sample commuter between 42.5 minutes early and 17.5 minutes late from his work-start time. The procedure calculated travel time for each of these arrival time by starting from the work end of the trip and simulating the trip link-by-link toward the home end. At each link, travel time was adjusted using the morning congestion curve if the work arrival time was before noon or using the afternoon congestion curve otherwise, and was cumulated to define a clock time for the next link. For the work-to-home trip, travel time was

calculated using the only path. For the each of the 12 alternative arrival times for the morning

trip, both paths were adjusted in this way, impedances were re-calculated, and the one with

lower impedance was chosen. These two paths, one for the home-to-work and one for the

work-to-home trip, were used for computing automobile operating and maintenance costs.

C. Transit Trip Time Components. The zone-average walk access times in the transit network

file were thought inappropriate for UTDFP. Instead, descriptions of residential and workplace

locations or their nearest cross streets were used to derive spatially more disaggregated data.

Calculated walk access times then were substituted for the zonal values in the transit network

file. Access time to transit by automobile was computed similarly.

Other transit trip attributes were computed with a path-finding procedure and an

attribute-computing procedure. The path-finding procedure first calculated an impedance for

each path using transit trip attributes in the transit network file for each transit line in the

network and for the morning peak, midday, and the afternoon peak, respectively, and using

attribute weights. These weights were inferred from a mode choice model based on the WTS

sample. The procedure then ran a path algorithm using the origin and destination information

for each trip in the sample, and determined a path for each trip.

The calculation began with finding two schedule times for each line and each

boarding or transferring point on the line from operators' timetables to determine which

service (morning peak, midday, or afternoon peak) was available and its probable headways

for each transit trip in the sample. These two schedule times were then compared with the

work-end schedule and its adjustment for the elapsed time to other points on the path. This procedure produced first headways, transfer headways, and line-haul times. Number of transfers were from personal surveys; access times by walk or automobile were computed independently from the path-finding procedure.

D. Auto Operating and Parking Costs. Operating costs were defined as the sum of gasoline, oil, and maintenance costs for the average auto in the region in 1972. Since the particular road classifications and speeds were available, they were used to identify the cost of using each highway link in the trip time adjusting programs. Trip operating costs were the sums of those link operating costs. Tolls were also separately accumulated for those trips that crossed the only toll bridge in the study area. The commuter discount toll was used.

The source of the parking cost data was a survey of all off-street fee parking in the study area in 1972. Parking costs were associated with each round trip, if there was at least one off-street fee lot in the traffic zone where each survey respondent worked, and if the respondent noted in the project's household surveys that his employer did not provide free parking. Fee parking in a zone was presumed to imply that no other free street parking was available. Street meters were assumed inapplicable to commuters.

E. Carpool Alternative Trip Attributes. An average occupancy of 2.5 persons per carpool, evenly dividing costs, was uniformly applied for the trips. The extra time involved with picking up and discharging passengers was assumed to be a uniform 6.25 minutes per trip.

Where priority lane time or cost savings were applicable, these were subtracted from the network line-haul times or costs for those trips.

Data Reduction

The reduction of data from the two personal surveys to the sample of 991 cases was carried out by the UTDFP. The BITS included work trips and non-work trips, but the non-work trips excluded from the 991 cases.

For the estimation sample, the following criteria are used for data reduction:

1) incomplete data on the variables used;

2) usual work arrival time outside the one-hour interval between 42.5 minutes early and 17.5 minutes late relative to the work start time;

3) no fixed work start time.

Criteria 1) leaves 783 cases for the estimation of marginal mode choice models; criteria 2) and 3) leave 569 cases for the estimation of the conditional schedule choice models.

6.2 Specification

Data on travel times are available for 12 arrival times for auto modes and available for

work start time for bus modes. As a result, the choice set on scheduling for auto users is

limited to 12 relative intervals. For bus users, only the on-time relative interval is available.

Each relative interval is 5 minutes long, centered around one of the 12 arrival times. The on-

time relative interval for a given individual is centered around his work start time. Figure 6.1

shows a nested logit structure for mode and schedule choices.

The nested structure in Figure 6.1 assumes that unobserved preferences for the

scheduling alternatives and for the mode alternatives are independent. Two patterns of

correlation other than independence have been hypothesized for unobserved preferences for

the scheduling alternatives (Brownstone and Small, 1989, p. 70). One correlation pattern

occurs among nearby alternatives, induced by an ordering of the scheduling alternatives, in

which case an ordered generalized extreme value structure for the alternative schedules is

more appropriate (Small, 1987). The other correlation pattern occurs among alternatives within each of the three groups: arriving early (alternative 1-8), on-time (alternative 9), and arriving late (alternatives 10-12), in which case a nested logit structure for the alternative schedules is more appropriate.

There is evidence, however, indicating that neither of the two hypothesized correlation patterns is statistically significant. Small (1987, Table II) reports results from the maximum likelihood method that fail (at a ten percent level using a one-sided test) to reject the independence hypothesis against the

correlation pattern that would be consistent with an ordered generalized extreme value structure. Brownstone and Small (1989, Table 1) report a maximum likelihood estimate of 0.807 for the coefficient of inclusive values with a standard error of 0.178, which would fail (at the same significance level using a

one-sided test) to reject the independence hypothesis against the correlation pattern that would be consistent with a nested logit structure.

Auto mode is seperated into carpool and drive-alone to examine policies that affect carpool differently from drive-alone.[4] Bus mode is seperated into bus-with-walk-access and bus-with-auto-access because previous studies based on the same sample have reported more

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[4]Carpool is defined as autos with two or more occupants. Small (1983b, p. 47) reports a somewhat better fit with carpool defined as autos with three or more occupants. The data set in its current form does not allow constructing a dummy for carpool in this way. It is unclear which definition is more useful for policy analysis because occupancy requirement for high-occupancy-vehicle lanes in the United States is equally divided between the two (Transportation Research Board, 1990, p. 8).

plausible estimates with this separation.

With these separations of modes, it is possible that there is a closer correlation among the unobserved preferences for the two auto modes and the two bus modes respectively. For the easiness of model estimation and equilibration grouping of modes is not considered.

Since buses normally are operated on fixed schedules, bus users would have much less choice of arrival time at work than auto users. Given this being the case the choice structure in Figure 6.1 may not be as limited as it may first appear. Bus users will have different schedules if their work-start times differ.

Figure 6.1

To empirically specify the systematic utility in (5.6), it is decomposed into two

$$W^i_{mj} = \_m(X^i_m, S^i; \theta) + \_{mj}(Z^i_{mj}, S^i; \phi),$$

components as follows:

where $\_m(X^i_m, S^i; \theta)$ is a function of the observed attributes X^i_m associated with mode m

and socioeconomic characteristics S^i, $\_{mj}(Z^i_{mj}, S^i; \phi)$ is a function of the observed attributes

Z^i_{mj} associated with mode m and relative interval j and S^i. θ and ϕ are two vectors of

parameters to be estimated. For ease of reference, the two components in (6.1) are further

decomposed as follows:

$$\_m(X_m^i, S^i; \theta) = \alpha_m + B_m(S^i; \beta) + D_m(X_m^i, \omega^i; \delta),$$
$$\_{mj}(Z_{mj}^i, S^i; \phi) = R_j(\eta) + G_{mj}(TIM_{mj}^i, S^i; \xi) + F_{mj}(SD_j, S^i; \psi),$$

where X_m^i and S^i are defined as in (6.1), and in addition, α_m is a constant, a scaler

component of θ; β and δ are subvectors of θ; η, ξ, and ψ are subvectors of ϕ; ω^i is

wage rate, a scaler component of S^i; TIM_{mj} is the on-vehicle time associated with relative

interval j and mode m; SD_j is the schedule delay associated with relative interval j; j = 1,

···, 12 indexing the twelve relative intervals; m = 1 for auto-alone, 2 for bus with walk

access, 3 for bus with auto access, and 4 for carpool.

The mode-choice literature shows that transportation variables such as monetary costs,

on-vehicle time, walk time, and wait time, are key determinants of mode-choice. In this

research, monetary costs include operating, maintenance, parking, and tolls for auto-alone; for

carpool, costs are those for auto-alone divided by 2.5, the average occupancy of carpool in

the sample. On-vehicle time, walk time, and wait time are entered as separate variables.

$$D_m(X_m^i, \omega^i; \delta) = \delta_1 \left(\frac{c_m^i}{\omega^i} \right) + \left(\delta_2 TWK_m^i + \delta_3 TWT_m^i + \delta_4 TXF_m^i \right) \Delta^{23},$$

Overall,

where c_m^i is monetary costs in cents associated with mode m; TWK_m^i is walk time in

minutes associated with mode m; $TWT_m^i = 5 + H/4$ if $H > 10$; $H/2$ otherwise; H is

headway for the first bus in minutes;[5] TXR_m^i is cumulated transfer time in minutes associated

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[5]Thus $TWAIT_m^i$, first-wait time, is set at one-half the headway up to 10 minutes, plus

with mode m ; $\Delta^{23} = 1$ if m = 2, 3 ; 0 otherwise.

In addition to transportation variables, socioeconomic characteristics such as age, occupation, household income, sex, and presence of young children are often reported as determinants of taste for modes and trip schedules in the literature. One problem with certain socioeconomic characteristics is that they are often endogenous, causing biases in the coefficients for variables with which they are jointly determined. Examples include automobile ownership and job and residential locations.

Another issue with socioeconomic variables is how they should be specified. One way is to treat them as utility-shift variables, specified as dummies interacted with alternative-specific dummies. The other way is to treat them as parameter-shift variables, specified as

one-fourth the increment in headway beyond 10 minutes, following Train and McFadden (1975) and Small (1983b).

dummies interacted with transportation variables. For mode-choice, household income,

presence of children under age seventeen, length of residence, age, and sex are included;

except for sex, Small (1983b) has been followed. They are specified as utility-shift variables

$$B_m(S^i;\beta) = \beta_1 Y^i + \beta_2 BA^i + \beta_3 CLD^i + \beta_4 A 45^i + \beta_5 MAL^i \Delta^1$$

as follows:

for $m = 1, 4$, and $B_m(S^i;\beta) = 0$ for $m = 2, 3$. In (6.4), β_i ($i = 1, 2, 3, 4, 5$) are components

of vector β; Y^i is the household income of commuter i in thousands per year with a ceiling

of ten;[6] BA^i is the length of residence of commuter i in the Bay Area in years; $CLD = 1$ if

commuter i has children under 17 present; 0 otherwise; $A45 = 1$ if commuter i is 45 or

older; 0 otherwise; $MAL^i = 1$ if commuter i is a male; 0 otherwise; $\Delta^1 = 1$ for auto-alone;

0 otherwise.

For time-of-day choice, occupation (professional and managerial) and age (forty-five

years and older) are included as indicators of taste. They are specified as parameter-shift

variables interacting with schedule delay variables only, hypothesizing that these commuters

value schedule-delay savings differently from others. This results in

---

[6]I have followed Small (1983a, 1983b) in using the truncated family income. I myself tried actual family income, and the truncated family income seems work better.

$$G_{mj}(TIM^i_{mj}, S^i; \xi) = \xi \bullet TIM^i_{mj},$$

$$F_{mj}(SD_j, S^i; \psi) = \psi_1 SDE_j + \psi_2 SDE_j \bullet A45^i + \psi_3 SDE_j \bullet CP^i + \psi_4 SDL_j$$
$$+ \psi_5 SDL_j \bullet PF^i + \psi_6 D1L_j + \psi_7 D1L_j \bullet PF^i$$

and

for m = 1, 4, and $F_{mj}(SD_j, S^i; \psi) = 0$ for m = 2, 3, where S^i, ξ, and $\psi = (\psi_1, \cdots, \psi_7)$ are

defined as in (6.2), $A45^i$ is defined as in (6.4); $CP^i = 1$ for carpool; 0 otherwise; $PF^i = 1$ for

professional and managerial workers; 0 otherwise; $SDE_j = \text{Max } \{0, 5(9-j)\}$, a measure of

schedule-delay early;

$SDL_j = \text{Max } \{0, 5(j-9)\}$, a measure of schedule-delay late; $D1L_j = 1$ if j > 8; 0 otherwise.

TIM^i_{mj} in (6.5), the on-vehicle time associated with auto mode m and relative interval j,

is measured by the sum of on-vehicle times of the afternoon trip at work-quit time and of the

morning trip in relative interval j; TIM^i_{mj}, for bus modes, is measured by the sum of on-

vehicle times of the morning trip at work-start and of the afternoon trip at work-quit time. In

doing so the following assumption is made explicit: the mode-choice decision is for the round

trip but the scheduling decision is for the morning trip only. Under fixed work-start and

work-quit times, it is reasonable to assume that the arrival-time choice for the morning trip

and the leaving time choice for the afternoon trip are independent decisions.

In reality, though, dependence of the two scheduling decisions could occur, and occurs usually for commuters with flexible work hours.[7] For example, a commuter who wants to make the afternoon trip in the shoulders of an extremely wide peak period may make the morning trip extremely early or late. In doing so, the commuter takes into account travel costs at alternative schedules for the morning trip as well as for the afternoon trip.

Considering either the scheduling decision for the afternoon trip or flexible work hours would require data that are unavailable. Although it is possible to model flexible work hours with the concept of *work* schedule delay by setting schedule delay at zero for the alternatives within the flexible period, this approach would increase the number of time intervals so that the equilibration procedure may become infeasible.

The specification in (6.7) assumes that the two auto modes have the same coefficients for all variables except that for schedule-delay early: carpoolers are hypothesized to be less averse to arriving at work early than commuters driving alone. The dummy for professionals and managerial workers interacts with $D1L_j$ and SDL_j, hypothesizing that professionals and

---

[7]Under flexible work hours, an employee typically chooses his work-start time within the flexible period, a fixed range of two hours, for example, early or late from a fixed point of time in the morning, and leaves his work place in the afternoon after a fixed

managerial workers are less averse to late arrival. The dummy for commuters forty-five years

and older

interacts with SDE_i, hypothesizing that workers at least forty-five years old are

number of hours of work.

less averse to early arrival either because they are older or because they have less family-related schedule constraints.[8]

The specification above, which follows Small (1982) closely, catches the trade-off between travel time and schedule delay, and allows inferring values of schedule-delay savings. These inferred values of schedule-delay savings can vary with socioeconomic groups such as occupation, age and will be extremely useful in measuring benefits of policy-induced schedule shifts.

Abkowitz (1980, p. 164) does not include any measure of schedule delay, a factor that has been identified as the key to understand scheduling behavior and hence traffic-peaking, which greatly limits the usefulness of his model in measuring benefits of policy-induced schedule shifts. This is one reason why a new demand model is needed. Hendrickson and

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[8]Small (1982) does not try any age variable, but both Abkowitz (1980, p. 154) and Moore et al. (1984, p. 153) report that older workers are more likely to arrive early at work.

Small (1982, p. 475) also includes reported flexibility as a taste indicator for time-of-day choice despite the recognized endogeneity problem created by doing so. In addition, Small (1982, p. 474) has family status (single or non-single) interact with both travel time and schedule-delay early; this leads to an extremely small and insignificant coefficient on travel time for non-single commuters and a wrong sign on travel time for single commuters.

Plank (1984, Table 2) do not include any socioeconomic characteristics and do not estimate a

significant coefficient for travel time.

As Small (1982, p. 472) reports, commuters tend to round off their reported times to

the nearest five-minute as well as to the nearest ten- or fifteen-minute. The five-minute

rounding is eliminated with the five-minute intervals describing scheduling alternatives, and

the ten- or fifteen-minute rounding, following Small (1982), is accounted for with reporting

$$R_j(\eta) = \eta_1 R15_j + \eta_2 R10_j,$$

error variables:

where η_1 and η_2 are components of η ; $R15_j$ is 1 for $j = 3, 6, 9, 12$; 0 otherwise; $R10_j$ is

1 for $j = 1, 3, 5, 7, 9, 11$; 0 otherwise.

6.3 Estimation

Tables 6.1 and 6.2 show estimation results for the model specified above. The

sequential approach is used.[9] Table 6.1 shows results for the conditional time-of-day choice;

Table 6.2 shows results for the marginal mode choice.

The time-of-day choice model is estimated first, conditioned on an auto mode chosen;

since each bus mode only has the on-time alternative attached, bus users drop out of this

stage. The inclusive values for auto modes and bus modes are computed, respectively, as

follows:

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[9]The two-stage approach has computational convenience but gives underestimated
standard errors in the second stage. The maximum likelihood (ML) method was tried,
some coefficients are unable to be estimated probably because of the large number of

$$I_m = \ln \sum_j \exp \left(\_{mj} / \rho_m \right), m = 1, 4$$

$$I_m = \hat{\xi} \; TIM^i_{m9}, m = 2, 3$$

where $\hat{}$ indicates the estimate of a parameter. The marginal mode-choice model is then

estimated with the computed inclusive values as an additional variable. The model is

estimated both restricting the inclusive variable to have the same coefficient for all four modes

and allowing it to have different coefficients for auto modes and bus modes. A log likelihood

test rejects the generic specification for the inclusive variable.

Table 6.1 Sequential Estimation Results for Conditional Time-of-Day Choice[a]

Independent Variable[b]	Coefficient Symbol	Coefficient Estimate	Error	Asymptotic[c] Standard
Reporting Error:				
R15	η_1	1.1992		0.1090
R10	η_2	0.4120		0.1140
Travel Time:				
TIM	ξ	-0.0857		0.0346
Early Arrival:				
SDE	Ψ_1	-0.0821		0.0084
SDE·AGE45	Ψ_2	0.0244	0.0081	

parameters.

SDE·CP	Ψ_3	0.0176	0.0083

Late Arrival:

SDL	Ψ_4	-0.1993	0.0285
SDL·PF	Ψ_5	0.0605	0.0344
D1L	Ψ_6	-0.9389	0.2121
D1L·PF	Ψ_7	0.6928	0.1890

Sample Size: 569
Log-Likelihood: -1118 at convergence
-1414 with all coefficients zero

Notes to Table 6.1:

[a] Dependent variable: relative interval chosen.

[b] Definition of independent variables: *R15* = 1 for j = 3, 6, 9, 12 ; 0 otherwise. *R10* = 1 for j = 1, 3, 5, 7, 7, 9, 11 ; 0 otherwise. *TIM* = on-vehicle time in minutes. *SDE* = Max {0, 5(9-j)}. *SDL* = Max {0, 5(j-9)}. *D1L* = 1 if j > 8 ; 0 otherwise. *AGE45* = 1 if individual age is 45 years or older ; 0 otherwise.
CP = 1 for carpool; 0 otherwise. *PF* = 1 for professional and managerial workers; 0 otherwise.

[c] These standard errors are estimated in the first stage and are consistent.

Table 6.2 Sequential Estimation Results for Marginal Mode Choice[a]

Independent Variable[b]	Coefficient Symbol	Coefficient Estimate	Error	Asymptotic[f] Standard
Inclusive value for auto	ρ_{14}	0.6842	0.1133	
Inclusion value for bus	ρ_{23}	0.2242	0.0841	
Cost/post-tax wage (min)	δ_1	-0.0270	0.0056	
Walk time (min)[c]	δ_2	-0.0722	0.0109	
First wait time (min)[c]	δ_3	-0.0738	0.0239	
Transfer wait time (min)	δ_4	-0.0469	0.0155	
Family income ($1000)[d]	β_1	0.1643	0.0465	

Children under 17 (0-1)[d]	β2	-0.7755	0.2561
Length of residence (years)[d]	β3	0.0127	0.0028
45 years and older (0-1)	β4	-0.3723	0.2425
Male respondent[e]	β5	0.7188	0.1634
Mode 1 dummy (0-1)	α1	-4.3017	0.5810
Mode 3 dummy (0-1)	α2	-2.3838	0.2971
Mode 4 dummy (0-1)	α3	-5.1715	0.5782

Sample Size	783
Log-Likelihood	-698 at convergence
	-1086 with all coefficients zero

Notes to Table 6.2:

[a] Dependent variable: mode chosen.
Modes: 1 = drive alone; 2 = bus-with-walk-access;
          3 = bus-with-auto-access; 4 = carpool.

[b] Transportation variables are measured for the round trip.

[c] The variable is specified on mode 2 and 3 only.

[d] The variable is specified on mode 1 and 4 only.

[e] The variable is specified on mode 1 only.

[f] These standard errors are estimated in the second stage. They are inconsistent and downward biased.

CHAPTER 7

THE SIMULATION MODEL

This chapter develops a simulation model of commuting on a hypothetical stretch of limited-access urban highway. The demand side determines the number of commuters, by mode and time of day in a given period, as a function of travel time and schedule delays

associated with alternative times of day. The supply side predicts travel times for a given

number of commuters on each mode in each clock time interval in a given period. The

sample enumeration connects the demand and supply sides. Section 7.1 describes the supply

side. Section 7.2 specifies an enumeration sample. Section 7.3 completes the simulation

model. Section 7.4 shows the existence of an solution using Brouwer's Fixed Point Theorem.

Section 7.5 describes how the simulation model can be used to calculate optimal tolls that

maximize social welfare, the sum of consumer welfare and toll revenue.

7.1 The Supply Side

A ten-mile long, limited-access highway connects a residential area where all

commuters live and a work location where all commuters work. Every commuter enters the

highway from the home end and leaves the highway at the work end. No mid-entry or exit is

allowed. The highway has three lanes in each direction with a directional capacity of 5310 vehicles per hour. This number of 5310 is taken from Small (1983b, p. 32) for a stretch of the Eastshore Freeway in the San Francisco Bay Area. Traffic on the highway is governed by the power function (2.2); its parameters are determined as follows. A free-flow speed is assumed at sixty miles per hour, which implies $T^0 = 1$. $T^1 = 0.15$, following U.S. Bureau of Public Roads (1964). Since there is no evidence on the value for γ, two values, 1 and 2.5 are used. These are summarized in Table 7.1.

The literature on trip scheduling has three alternative forms of supply model to the power function in (2.2). One is the power function (1.2) used in the original Henderson approach. Another one is used in Mahmassani and Herman (1984). As discussed in chapters 2 and 4, both of these two supply models can lead to lack of equilibrium. My option is the bottleneck model used in the Vickrey approach: a continuous form is in (3.1) and (3.2); a discrete form is in (5.13). The power function is chosen for two reasons.

First, the bottleneck model leads to difficulty in convergence. The bottleneck model requires traffic flows entering the highway to predict travel times. But the demand model predicts traffic flows leaving the highway. Rounding errors in the conversion between these two forms of traffic flow seem to create the convergence difficulty for the bottleneck model. The power function (2.2), however, does not require this conversion because it directly uses traffic flows from the demand model. This reason is tentative; future research needs to investigate further the convergence properties of the bottleneck model.

Second, chapter 3 shows that the bottleneck model and the power function (2.2) could result in almost identical equilibria when the power parameter is large. The two models always predict the same rate of build-up and decay in congestion in unpriced equilibria. This can be seen by comparing the equilibrium travel time functions from the bottleneck model in (3.5) and from the power function (2.2) in (2.6) and (2.28). The equilibrium travel time function from (2.2) needs to be converted from time leaving the highway to time entering the highway.

Table 7.1 The Supply Side

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$$T_s = l \left[T^0 + T^1 \left(\frac{V_s}{C} \right)^{\gamma} \right]$$

T_s = Travel time in minutes

V_s = passenger car units per hour leaving the highway

l = 10 miles

T^0 = 1 minute per mile

T^1 = 0.15 minutes per mile

C = 5310 passenger car units per hour

γ = 1, 2.5

_____

7.2 Equilibration

Demand for arriving on each mode and at relative interval can be obtained using the

sample enumeration method. Sample enumeration uses a sample of the population of interest to represent the entire population. Let the size of a population of morning commuters be fixed at N, an exogenous parameter to be varied. Let the size of an enumeration sample be N_f. Each sample individual will be assumed to represent N/N_f commuters in the population. The choice probability P_{mj} for commuter i in (5.12) represents $P_{mj}*(N/N_f)$ number of commuters arriving at work on mode m at relative interval j. The subscript s for the choice probability in (5.12) is replaced with j because of a change in choice set noted in chapter 6. These weighted choice probabilities for relative intervals can then be aggregated over the enumeration sample with assumed passenger car occupancy rate for each mode to obtain traffic flows leaving or entering the highway at absolute intervals.

To obtain these traffic flows at absolute intervals, one need to determine the range of clock time to be considered. This range is determined by the chosen window of relative intervals for the simulation and the range of work-start times in the enumeration sample. If we let t_p and $t_{p'}$ be the mid-points of the first and last absolute intervals, respectively, the

$$S = \frac{t_{p'} - t_p}{d} + 1$$

number of absolute intervals will be

where d is duration of these intervals. One can choose t_p and $t_{p'}$ such that the right hand side results in an integer. An absolute interval is indexed with s,

$s = 1, \cdots, S$.

Second, one needs a one-to-one correspondence for each individual in the enumeration sample between his relative intervals and the absolute intervals. Let E^i be the work-start time for commuter i in the enumeration sample, and T_s the travel time at absolute interval s. Let d be the duration of a relative or absolute interval, and j_o be the index for the on-time relative interval. Then $E^i - d(j_o - j)$ is the midpoint of relative interval j in clock time leaving the highway; $t_p + d(s - 1) + T_s$ is the midpoint of absolute interval s, in clock time leaving the highway. The correspondence between absolute interval s and relative interval j is created by placing $E^i - d(j_o - j)$ in one of the absolute intervals. Let s_j^i be the absolute interval where relative interval j locates for individual i. The correspondence between any

$$s_j^i \equiv s$$

$$\text{if } \left[E^i - d \bullet (j_o - j) \right] \text{is within the } \frac{d}{2} \text{-radius of } \left[t_p + d \bullet (s - 1) + T_s \right]$$

combination of s and j for a given individual is given by

The interval correspondence (7.2) is for the Vickrey or original Henderson model which requires flows entering the highway. For the reformulated Henderson model, which requires flows leaving the highway, T_s in (7.2) is set to zero.

With the interval correspondence (7.2), the choice probabilities in (5.12) from the demand model, P_{mj}, can be aggregated in the following way. The probability for commuter i to choose an auto mode m (m = 1, 4) at absolute interval s is

$$P_m^{is} = P_{mj}^i \text{ for } s = s_j^i\,(\,j=1, \bullet\bullet\bullet, J\,);\, 0 \text{ otherwise}.$$

where J is the total number of relative intervals used in the simulation model. The number

of commuters driving alone N_1, carpooling N_4, and taking bus N_{23} in absolute interval s,

$$N_1^s \equiv \sum_i \frac{N}{N_f} P_1^{is},$$

$$N_4^s \equiv \sum_i \frac{N}{N_f} P_4^{is},$$

$$N_{23}^s \equiv \sum_i \frac{N}{N_f}\left(1 - P_1^{is} - P_4^{is}\right)\Delta_s^i,$$

respectively, are

where $\Delta_s^i = 1$ if $s = s_{jo}^i$; 0 otherwise; Δ_s^i appears because bus modes are assumed available

only in the on-time relative interval for each commuter. Traffic flow in passenger-car-units

$$V_s = \frac{60}{d}\left(N_1^s + \frac{N_{23}^s}{b} + \frac{N_4^s}{c}\right),$$

per hour leaving the highway at absolute interval s is

where c and b are the occupancy rates of carpool and bus, respectively, in passenger-car-

units. Carpools are assumed to have a passenger occupancy of

$c = 2.5$, which was originally used in creating cost variables for carpools in the estimation

sample. Buses are assumed to have a passenger occupancy of 40 and a factor of passenger-

car-units of 3. This implies $b = 40/3$.

Once traffic flows leaving the highway are obtained for all absolute intervals, the

power function specified in Table 7.1 can be used to predict

on-vehicle travel time T_s for these intervals. This predicted travel time T_s for a absolute

$$TIM^i_j = T_s \text{ for } s = s^i_j (j = 1, \bullet\bullet\bullet, J).$$

interval s then needs to be converted to that for a relative interval j for commuter i :

to be used in the demand model for the next iteration.

One need not use the same values of j_o, d, and J in estimating the demand model

for the simulation model. The following will be used: $d = 15$ minutes; $J = 5$; and $j_o = 4$. So

the window for relative intervals is 75 minutes; maximum schedule delay early is 45 minutes;

maximum schedule delay late is 15 minutes. It is a good idea to keep the durations of relative

and absolute intervals equal. Wider absolute intervals have two effects: they smooth peak

without changing the distribution of work start times; and they reduce the number of absolute

intervals for a given commute window to speed up computation. These changes in interval

definition require to redefine schedule delay variables accordingly.

In addition to modifying interval definitions, the definition of transportation variables

for the simulation model is also changed from measuring for the round trip to measuring for

the morning commute only. These variables include auto operating costs, bus walk and

waiting times, and bus fares. Since not enough information is available to separate auto

operating costs, bus walk and waiting times and bus fares for the morning commute, half of

their round trip values is used.

Most work on empirical analysis of mode choice uses round trip measures. It is believed that travelers choose modes taking into account factors of both trips. Also some of the costs associated with a particular mode are joint costs for both trips. Most work on trip scheduling, however, has assumed away the afternoon trip and deals with the morning trip only. Henderson (1992) is one of few exceptions that deals with the afternoon trip. Henderson assumes perfect symmetry between the two trips. His model examines how workers choose their work start times relative to a most desired work start time. The symmetry assumption assumes that workers start working once they arrive at work, and they leave immediately after a fixed number of work hours. This symmetry assumption is inappropriate when one is examining how workers choose their daily commuting schedule relative to a given work start time. When work start time and number of work hours are fixed, workers do not necessarily start working once arriving at work; they do not all leave after the same number of work hours because they may start working at different times even if their work-start times are the same.

Chapter 5 assumes that each commuter leaves the work place in the afternoon right at the work-end time. As a result, travel time for the afternoon commute is independent of the morning schedule. To get round-trip travel time for any given commuter, the morning travel time for the on-time schedule is added to all his schedules. It seems reasonable given that workers have fixed number of work hours and can choose when to arrive at work but not when to start work. But empirically, this assumption can lead to paradoxes when work start

times are very peaked. I choose to follow the literature by focusing on the morning commute

only, assuming that any bias from this assumption on mode choices, which we understand

well, is much less serious than biases from assumptions on scheduling behavior for the

afternoon trip, which we do not understand. Focusing on the morning trip requires to modify

all transportation variables measured for round trip in chapter 6.

7.3 The Enumeration Sample

The enumeration sample overlaps with the sample used in estimating the demand

model of chapter 6. Only those whose work-start time is between 390 and 570 minutes after

midnight are selected to narrow the overall period to be equilibrated. No mode-captive

individuals are excluded because information on these individuals is unavailable. Ideally only

those who travel on an expressway for part of their commute are selected as in Small

(1983b).[10] Since information on expressway use is unavailable for the enumeration sample,

expressway and non-expressway users are not distinguished. There are 641 commuters in the

enumeration sample.

Table 7.2 presents summary statistics for some of the characteristics of the

enumeration sample that will be used in the simulation model. Figure 7.1 presents the

distribution of work-start times of the enumeration sample. Over 40 percent of sample

individuals have a work start time at 8:00 A.M. This distribution will lead to traffic narrowly

peaked around 8:00 A.M. for the simulations because all individuals are assumed to work

right off the highway. In reality, however, a narrowly peaked distribution of work start times

can still result in a smooth peak on major expressways because people normally travel further

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[10]The sample Small (1983b) uses is a subsample of the one used here.

once they get off a particular expressway for different length of time. Figure 7.2 presents the

cumulative distributions of post-tax wage rates and household income.

Table 7.2 Summary Statistics of the Enumeration Sample

Variable	Mean	Standard Deviation
Transportation		
Drive-Alone Round Trip		
Distance (miles)	9.9	7.1
On-vehicle time (minutes)	40.0	22.9
Operating costs (cents)	138.0	102.0
Car-Pool Round Trip		
Distance (miles)	11.3	6.5
On-vehicle time (minutes)	57.3	20.6
Operating costs (cents)	71.7	45.1
Bus-Walk Round Trip		
Distance (miles)	7.2	6.1
On-vehicle time (minutes)	53.1	25.2
Walking time (minutes)	18.4	12.1
Initial Headway (minutes)	13.8	10.8
Fare (cents)	103.0	58.0
Bus-Auto Round Trip		
Distance (miles)	18.4	7.8
On-vehicle time (minutes)	94.6	32.3
Walking time (minutes)	5.8	3.8
Initial Headway (minutes)	16.6	15.6
Fare (cents)	167.6	58.0
Other		
Family income (10,000 dollars)	16.7	12.1

Post-tax wage rate (cents per minute)	8.3	6.6

Value of on-vehicle time as a proportion of post-tax wage rate for
Auto 2.1667
Bus 0.7116

Table 7.2 continues

Table 7.2 continued

Marginal rates of substitution between on-vehicle time and
  Schedule delay early (minutes/minute)
    Age 45 and older   0.6733
    Carpooler   0.7526
    Carpooler and age 45 and older0.4679
    Others   0.9580

  Schedule delay late (minutes/minute)
    Professional   1.6196
    Others   2.3256

  Not being early (minutes)
    Professional   2.8716
    Others   10.9557

Percentage of Sample in Categories

Professional   39.6%
Age 45 and older   33.5%
Having children under 18 years older   20.6%
Male   59.1%

Family income < 10   32.5%
Family income ≥ 10 and < 20   45.2%
Family income ≥ 20   22.3%

Wage rate < 5   28.6%

Wage rate ≥ 5 and < 10   51.5%
Wage rate ≥ 10   20.0%
_____

Figure 7.1

Figure 7.2

7.4 Solution Method and Existence of Equilibrium

Fixed-point algorithms have been developed (Scarf and Shoven, 1984; Todd, 1976) to

solve equilibrium models that have a large number of variables. These algorithms have been

widely used in applied general equilibrium analysis for international trade and tax policies

(Scarf and Shoven, 1984). They have also been used in urban and transportation analyses:

Arnott and MacKinnon (1977) on allocating consumers to spatial locations in an urban area;

MacFadden, Talvitie, and Associates (1977) and Talvitie and Hasan (1980) on allocating

commuters to modes and routes along a corridor. They have also been proposed to solve for

unpriced equilibrium in a structural model of peak-period traffic congestion (Richter, Griffin,

and Arnott, 1990).

Fixed-point algorithms have disadvantages. First, their computational time increases

as least with the cube of the number of variables (Richter, Griffin, and Arnott, 1990, p. 1).

Second, the intuition one gets from a simple Cobweb procedure is lost in these fixed-point

algorithms. Third, it is unknown how these fixed-point algorithms would perform when an

endogenous congestion toll is being calculated. Viton (1983) and Small (1983b) are followed

to use an algorithm based on a Cobweb procedure, which is described below.

Letting V, P, and T denote the vectors $\{V_s\}$, $\{P^s_m\}$, and $\{T_s\}$, respectively, the

$$P = p(T), V = v(P), T = t(V),$$

equilibrium model can be represented as

which can be re-written as

$$f(V) = V$$

with some substitution. The equilibrium solution to (7.7) is a vector V^* which satisfies (7.8)

and can be interpreted as a fixed point of the transformation $f(\cdot)$.

Fixed-point algorithms will not be used for equilibration. But McFadden, Talvitie,

and Associates (1977) show that the idea can be useful to prove the existence of a solution.

The transformation $f(\cdot)$ is continuous because all the functions in (7.7) are continuous. For a

continuous transformation whose domain and range are on a simplex (a set of vectors whose

components are non-negative and sum to one), Brouwer's Fixed Point Theorem guarantees the

existence of a fixed point. To limit the domain and range of $f(\cdot)$ to a simplex, one creates a

$$X_s = \frac{V_s}{S * V_{max}}, \quad s = 1, \bullet\bullet\bullet, S$$

$$X_{S+1} = 1 - \sum_{s=1}^{S} X_s$$

new vector X

and a new transformation $x(X)$

$$x(X_s) = \frac{f(V_s)}{S * f(V_{max})}, \quad s = 1, \bullet\bullet\bullet, S$$

$$x(X_{S+1}) = 1 - \sum_{s=1}^{S} x(X_s)$$

V_{max} is the maximum flow possible. Equations (7.9)-(7.10) satisfy both the continuity and the

simplex conditions and result in a linear relation between V and X. Once a fixed point X^*

$$V_s = X_s * S * V_{max} \quad, \quad s = 1, \bullet\bullet\bullet, S$$

for $x(\cdot)$ is found, a fixed point for $f(\cdot)$ is calculated as follows:

7.5 Calculating Optimal Toll

How an Optimal Toll Should be Calculated

This section describes an algorithm to calculate optimal tolls. The issue is: what is the

correct measure of the marginal social cost of an extra trip at any time? Is it the increase in

social cost of putting an extra user at that time, without announcement? Or is it the increase

in social cost, allowing other users to modify their choices in response to the added congestion

caused by the extra user?

The answer depends on what equilibrium the marginal social cost is measured at. If

measured at an optimum, marginal social cost is correctly computed either way because of the

envelop theorem, which guarantees that marginal shifts on the part of users have only second-

order effects on social cost. On calculating marginal social cost at the on-toll equilibrium in

the Vickrey (1969) model, Arnott, de Palma, and Lindsey (1993, p. 166) comment:

> Marginal social cost should be computed mutatis mutandis [allowing other
>
> users to adjust their behavior], not ceteris paribus [not allowing other users to
>
> adjust their behavior]; that is, marginal social cost is computed incorrectly if
>
> one adds a commuter and computes the increase in social cost from doing so,
>
> without allowing other drivers to adjust their departure times. The reason why
>
> computing marginal social cost ceteris paribus is incorrect is that the envelope
>
> theorem does not hold. Because there is no toll, prices are distorted.
>
> Consequently, the adjustments that commuters make in their departure times
>
> in response to the added driver alter the deadweight loss from unpriced
>
> congestion.

A researcher, however, normally does not know a priori the optimum before

calculating the optimal toll, except cases of simplified theoretical models. One example of

such exceptions is the Vickrey model. Small (1992a, p. 125) first finds the Pareto optimum, and then calculates the marginal social externality at the optimum ceteris paribus. This is one of the two ways that Small (1992a, pp. 121-122) identifies to determine optimal tolls in a model of trip scheduling.

The other way is "to determine the short-run marginal cost (SRMC) of adding an additional traveler under prescribed conditions, define the fee formula that brings each traveler's perceived price equal to this SRMC, then solve for the new equilibrium with that fee formula applied." (p. 122) This is what is done with the reformulated Henderson model in chapter 2. The fee formula is given by (2.17) for early arrivals. This fee then is added to individual trip cost to give (2.18). A new equilibrium is solved to give (2.19), using (2.18).

$$p^o(t') = \frac{\gamma}{1+\gamma} \beta (t' - i'^o)$$

Substituting (2.19) into (2.17) gives the optimal fee for early arrivals as follows

where i'^o is the first arrival time at the optimum given by (2.21a).

Chapter 5 establishes that the optimal toll that maximizes total welfare, the sum of consumer welfare and toll revenue, is a marginal-cost toll, as given by the pricing rule (5.21). But (5.21) holds at an optimum only. Thus, neither of the two approaches discussed above applies to the more realistic simulation model developed here. The only approach applicable to this case is to start from the definition of marginal externality of travel at a given time, using an iterative procedure.

The strategy is the following. Each iteration starts with a set of tolls for all the absolute intervals. Given these tolls, the procedure iterates until a base convergence. An aggregate travel cost is calculated at this base convergence, measuring the total cost for existing traffic *before* an extra vehicle is added to any of the absolute intervals. For any given absolute interval, the procedure iterates again while a small arbitrary number of extra vehicles, *MV*, are added to that interval, increasing its flow rate by MV•(60/d). At convergence, another aggregate travel cost is calculated, measuring the total cost for existing traffic *after* the extra vehicles are added and existing traffic has adjusted its behavior. The difference between the two aggregate costs for existing traffic measures the marginal externality of traveling at that given absolute interval.

How an Optimal Toll can be Calculated Numerically

Optimal tolls are calculated with four subroutines: DEMAND(τ,T; P,V), SUPPLY(V; T), PRICE(τ ; τ), and TCOST(T,P; TC), and three types of equilibration loops: GLOBAL, BASE, and TOLL. The symbols following a subroutine name are major input and output variables, expressed in vectors; input variables are ahead of the semi-colon. Figures 7.3, 7.5, and 7.6 give the flow charts for the GLOBAL loop and the BASE and TOLL loops, respectively. Rectangles with one side open represent comments. Circles represent connections. Diamond shaped boxes represent decision making.

Subroutines are described first. DEMAND(τ,T; P,V) takes toll, τ , and travel time, T

, as given and calculates choice probabilities, P, and traffic flow, V. For the reformulated

Henderson approach, this traffic flow measures arrivals; for the Vickrey approach, this traffic

flow measures departures. It does so by first converting toll and travel time for absolute

intervals from last iteration to relative intervals according to (7.2) and (7.6). Again, this

conversion depends on which supply model is used, as explained below (7.2). SUPPLY(V;

T) takes traffic flow as given and calculates travel time for each absolute interval according to

the supply model chosen. Again, traffic flow measures arrivals for the reformulated

Henderson approach and departures for the Vickrey approach. The power function in Table

7.1. is used for this analysis. PRICE(τ ; τ) takes the toll from last iteration as given and

calculates a new toll for each absolute interval. It does so by computing two aggregate travel

costs for existing traffic to measure the marginal externality of an added vehicle to that

absolute interval. PRICE(τ ; τ) is described in detail below as the loops are described.

TCOST(T,P; TC) takes travel time and choice probabilities as given and calculates an

aggregate travel cost, TC, a scalar, including congestion costs, schedule delay costs,

operating and maintenance costs, costs of waiting and walking times for bus users, and bus

agency costs.

The three loops are described as follows. The GLOBAL loop, shown in Figure 7.3,

governs the equilibration of the optimum. The DEMAND(τ,T; P,V), SUPPLY(V; T), and

PRICE(τ ; τ) subroutines interact with one another. It terminates when both toll and traffic

flow change less than a chosen tolerance level for each of the absolute intervals. The toll in

convergence gives the optimal one. This GLOBAL convergence (or the optimum) depends on the number of commuters and the demand and supply models.

An arbitrary damping procedure is used in GLOBAL to prevent tolls to jump back and forth between iterations. This damping procedure is shown in a box labeled as DAMPING in the GLOBAL loop; its detail is shown in Figure 7.4. The damping only applies to τ^p, the peak toll, because for the scenarios considered tolls at other times are much less volatile than the peak one. Damping is not applied to flow either because it is toll that creates fluctuation in flow. This damping procedure is developed through trial and error.

The damping works as follows. Each time a new toll is calculated, its peak value, τ^p, is checked to see whether it is within bounds, τ_l and τ_u. If it is, new tighter bounds are created, depending on whether τ^p is larger than the peak toll from the last iteration, τ^p_o. In either case, two new bounds are created with a damping parameter, β, which goes to zero as the GLOBAL loop iterates. Depending on whether τ^p is larger than τ^p_o, a new low bound is created by weighing τ_l and τ^p or τ^p_o with β; a new up bound is created by weighing τ_u and τ^p or τ^p_o with β. DEMAND($\tau, T; P, V$) then is applied in the GLOBAL loop. If the current peak toll is out of bounds, however, up to five new damping parameters, ranging from 0+ to 1-, are used to get a new starting value for peak toll that falls in the bounds. Each time one such damping is used by weighing the bounds, the PRICE(τ ; τ) subroutine is used again to compute a new peak toll. If five applications of damping cannot create a new peak

toll that falls within bounds, one of the bounds is expanded, and the loop goes to the

DEMAND(τ, T; P, V) subroutine in the GLOBAL loop.

Both the BASE and TOLL loops appear in the PRICE(τ ; τ) subroutine. The BASE

loop governs the equilibration of a base equilibrium against which tolls are calculated. In the

BASE loop, the DEMAND(τ, T; P, V) and SUPPLY(V; T) subroutines interact with each

other, holding constant the tolls from the last GLOBAL iteration. It terminates when traffic

flow changes less than a chosen tolerance level for each of the absolute intervals. At

convergence, the

TCOST(T, P; TC) subroutine calculates the aggregate travel cost. This BASE convergence

depends only on the toll from the last GLOBAL iteration.

For any given absolute interval, the TOLL loop calculates the expected travel costs for

existing commuters after they have adjusted their choices for some fixed extra vehicles being

added to that interval. These travel costs are then subtracted by those calculated using the

BASE loop to give the increase in travel costs on existing commuters due to the presence of

the added vehicles. The new toll for that absolute interval obtains by dividing this increase in

travel costs by the number of added vehicles with the units appropriately adjusted.

The TOLL loop achieves this with the DEMAND(τ, T; P, V) and SUPPLY(V; T)

subroutines interacting with each other, while the same number of extra vehicles, MV, are

being added to the flow V_s of a given interval s at each iteration and the toll from the last

GLOBAL iteration is being held constant. The TOLL loop terminates for interval s when

traffic flow, V , including the extra ones, change less than a chosen tolerance level for each of

the absolute intervals. At convergence, the TCOST(T,P ; TC) subroutine calculates the

aggregate travel cost again for existing commuters, excluding the extra vehicles.

The TOLL loop is done for each of the absolute intervals. The new toll for any given

absolute interval is the change in the expected travel costs from the BASE convergence to the

TOLL convergence divided by the fixed number of extra vehicles and the unit is adjusted by

$60/d$. This gives a new set of tolls to be used in the next GLOBAL iteration.

Figure 7.3

Figure 7.4

Figure 7.5

Figure 7.6

7.6 Summary

The purpose of this chapter has been to put together an equilibrium simulation model.

The demand model is estimated in chapter 6; the supply side is specified in this chapter. Sample enumeration method is used to connect the demand and supply sides. The enumeration sample is a subsample of that used to estimate the demand model. Using Brouwer's Fixed Point Theorem, it show the existence of an solution to the simulation model. More importantly, it develops a computer procedure to calculate equilibrium and marginal-cost tolls. Chapter 8 will use the model developed in this chapter to simulate the effects of eight capacity expansion and congestion pricing policies.

CHAPTER 8

POLICY EFFECTS AND THEIR MISCALCULATION

This chapter reports simulation results on the effects of eight pricing and capacity

expansion policies, and the miscalculation of these effects from ignoring schedule shifts.

These policies include (1) an optimal toll, (2) a base-externality toll, (3) a piecewise-linear

toll, (4) a one-step toll, (5) a uniform toll, (6) an optimal toll with bus and carpool users

exempted from paying toll, (7) an optimal toll with a ten-percent capacity expansion, and (8) a

ten-percent capacity expansion. Policy (7) will be referred to as an optimal toll with HOV

exemption.

The effects of these policies are simulated from five perspectives: (i) welfare

(consumer surplus, toll revenue, and total benefits), (ii) peaking (traffic counts and share in

the peak 15-minute period), (iii) congestion (average and peak 15-minute travel delays), (iv)

schedule delay (average variable schedule delay and ratio of variable schedule to travel delay

for auto users), and (v) mode mix (mode shares, average occupancy, and total traffic).

Miscalculation of policy effects from ignoring schedule shifts are examined for an optimal toll

and the ten-percent capacity expansion.

The array of simulations has three dimensions: elasticity of congestion with respect to

arrival flow on the supply side, total number of commuters on the demand side, and policies.

Each combination of congestion elasticity and total number of commuters represents a

separate scenario. The effects of a given policy for a given scenario are obtained by comparing two simulations for that scenario, one after introducing the given policy and the other without that policy--a base simulation.

The chapter is divided into four parts. Section 8.1 introduces the eight policies and the base case, hypothesizes on equilibrium characteristics of the base case and miscalculation of policy effects, and formulates questions on the effects of the eight policies.

Section 8.2 presents characteristics of the base case. This is done with the same values of congestion elasticity and total number of commuters to be used for evaluating policy effects. Also presented are traffic counts by mode and time of day for the most congested scenario. These characteristics provide the benchmarks to evaluate the policy effects in sections 8.3 and 8.4.

Section 8.3 presents the effects of the eight policies. The effects of an optimal toll and the ten-percent capacity expansion are presented as percentage changes from the base case except for welfare measures. The effects of other six policies are presented as percentage differences from the effects of an optimal toll. Also presented are changes in traffic counts by mode and time of day due to each of the policies for the most congested scenario.

Section 8.4 presents the miscalculation of effects of an optimal toll and the ten-percent capacity expansion that would occur if schedule shifts are analytically constrained. Conventional models of peak-period congestion assume away scheduling behavior by using constant demand over a predetermined period. The question is, how biased are predictions of

policy effects by these conventional models? One way to answer this question is to compare

two simulations of a given policy: one with scheduling shifts and the other with schedule

shifts analytically constrained.

8.1 Questions and Hypotheses

This section introduces the base case and eight policies, hypothesizes on equilibrium characteristics of the base case and miscalculation of policy effects, and formulates questions on the effects of the eight policies.

Base Case

The base case provides conditions before a policy is introduced. The following are some of the questions about the characteristics of the base case:

(b1) What is the cost of congestion?

(b2) What is the social cost of a marginal traveler?

(b3) Does the amount of congestion affect average occupancy?

(b4) Do travelers shift their schedules as congestion worsens?

(b5) How do schedule and travel delays compare with each other?

One hypothesis on question (b4) is that travelers shift their schedules away from the peak as congestion worsens. This peak spreading would create one form of latent demand for

traveling at the peak. If a policy is such that commuters are better off by traveling in the peak than off-peak, this latent demand will return back to the peak. This return of latent demand to the peak has two effects in opposite directions: it increases congestion at the peak; it decreases schedule delay and congestion off-peak.

One hypothesis on question (b5) is that schedule delay is smaller, but comparable to travel delay. Accepting this hypothesis would imply two types of bias from models of peak-period congestion that ignore scheduling behavior. The first type of bias is underestimation of total travel costs. The second type of bias is changes in schedule delay resulting from a policy that induces schedule shifts.

How does the amount of congestion affect variable schedule delay relative to travel delay? The literature on trip scheduling has not hypothesized how the ratio of variable schedule to travel delay might vary with the amount of congestion. Travel delay increases with congestion. If peak spreads as congestion worsens, does travel delay increase faster than variable schedule delay? If the answer is yes, one would expect that the ratio reduces as congestion worsens. One question then is: do conventional models result in small biases in cases of heavy congestion?

Eight Policies

For the capacity expansion policy, the same ten-percent applies to all scenarios considered. The percentage is chosen arbitrarily. Capacity expansion is also considered for

three purposes. First is to contrast qualitatively the effects of an incremental expansion with those of an optimal toll. Second is to examine the miscalculation of effects from an incremental expansion, and to compare this miscalculation qualitatively with that of an optimal toll. Third is to see how the effects of an incremental expansion compare with and without an optimal toll.

An optimal toll measures the marginal increases in total travel costs due to an additional trip made at different times of day. It is a function of time. These travel costs are measured at the optimum where total welfare, sum of consumer welfare and toll revenue, is maximized. Consumer welfare is measured by (5.16). Travel costs include costs of on-vehicle travel time, costs of bus walk and waiting time, costs of schedule delay, and auto operating and maintenance costs and bus agency costs.[11] Individual travel costs are aggregated by weighting them with choice probabilities and enumeration sample weights as in (5.22). Chapter 7 describes how an optimal toll and the corresponding optimum are calculated.

Calculating and implementing an optimal toll can be difficult. A researcher often does not know what an optimum looks like, let alone the marginal externality of traveling at different times of day at the optimum. Even if one does know the optimum and calculates the corresponding optimal toll, one still has difficulty in implementing it, although the technology

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[11]I use existing fares in the enumeration sample as bus agency costs. This makes two assumptions. First, fares reflect bus agency costs. Second, pricing policies considered here would not affect bus agency costs.

of Automated Toll Collection (ATC) can reduce this difficulty to some extent.

Given the difficulties in calculating and implementing an optimal toll, it is important to consider alternative forms of toll that are easier to calculate or to implement or both. Four alternatives are considered: a base-externality toll, a piecewise-linear toll, a one-step toll, and a uniform toll.

A base-externality toll measures the marginal increases in total travel costs due to an additional trip at different times of day at the base equilibrium. These marginal externalities are calculated after existing travelers have adjusted their behavior in response to the additional trip. Calculating this toll is easier because a current condition can often be considered as a base equilibrium. This toll, however, is as fine-tuned as an optimal toll, and would not reduce the difficulty of implementation.

Figure 8.1 shows structures of a piecewise-linear toll and a one-step toll. The vertical axis measures level of toll; the horizontal axis measures clock time. t_b and $t_{b'}$ are the mid-points of the first and last absolute intervals used in the simulation model. The piecewise-linear toll starts with zero at t_i, increases linearly until at 8:00 , then decreases linearly to zero at $t_{i'}$. Its start and end points are chosen by examining the timing of an optimal toll, which would start sometime between t_b and t_i, and end sometime between $t_{i'}$ and $t_{b'}$. Its slope (the same on each side) is chosen semi-endogenously through screening with a small increment to maximize total welfare. The start and end points, once chosen, apply to all scenarios. The slope could vary with scenarios.

The one-step toll starts at t_b, stays constant, then ends at t_e. Its start and end points are chosen by examining the timing of an optimal toll. Its level is also chosen semi-endogenously through screening with a small increment to maximize total welfare. As with a piecewise-linear toll, the start and end points apply to all scenarios; the level could vary with scenarios.

A one-step toll is easier to implement. Simple theoretical models predict that an optimal one-step toll can achieve more than half the benefits of the optimal toll for the same scenario (Arnott, de Palma, and Lindsey, 1993, Table 1). It is interesting to see if a one-step toll can do as well using a more realistic model.

Figure 8.1

A uniform toll applies from 5:30 to 10:00. Its level is also chosen semi-endogenously through screening with a small increment to maximize total welfare. The same table in Arnott, de Palma, and Lindsey (1993) indicates that an optimal uniform toll can achieve 12 to 30 percent of the benefits of the optimal toll, depending on demand elasticity.

How do the effects of a pricing policy compare when bus users and carpoolers pay versus when they are exempted from paying the toll? This is one of the frequently asked questions when a pricing policy is being proposed. This chapter considers such a policy with an optimal toll recalculated conditional on exempting bus users and carpoolers.

The last pricing policy considered is an optimal toll with a ten-percent capacity expansion. Optimal tolls are recalculated. The ten-percent increment is arbitrarily chosen, and applies to all scenarios. The purpose is to examine how the benefits of an incremental expansion compare with and without the presence of an optimal toll.

Questions on the effects of these policies may be grouped in three categories: positive, normative, and methodological. Positive ones concern effects on traffic peaking, congestion, schedule delay, and mode mix:

(p1) Does an optimal toll affect traffic peaking, congestion, schedule delay, and mode mix?

(p2) Does the ten-percent capacity expansion affect traffic peaking, congestion, schedule delay, and mode mix?

(p3) How much of a change in traffic peaking due to an optimal toll or the capacity expansion comes from mode and schedule shifts, respectively?

(p4) How do the effects of other pricing policies compare with those of an optimal toll?

One hypothesis on the effects of capacity expansion on peaking is that an expansion leads to more peaking. Normative questions concern welfare effects of these policies:

(n1) What are the total benefits of an optimal toll?

(n2) How do the total benefits of other pricing policies compare with those of an

optimal toll?

(n3) How do the total benefits of the ten-percent expansion compare with and without an optimal toll?

If simple toll forms can achieve a substantial proportion of the benefits of an optimal toll, they could be worthwhile. One hypothesis on the benefits of capacity expansion is that the benefits of an incremental expansion are smaller when an optimal toll is present.

Methodological questions concern the miscalculation of policy effects from ignoring schedule shifts. This miscalculation is examined for an optimal toll and the ten-percent capacity expansion:

(m1) Are the effects of an optimal toll biased when schedule shifts are constrained?

(m2) Are the effects of an optimal toll biased when schedule shifts are constrained?

(m3) How do the miscalculations for an optimal toll and capacity expansion compare qualitatively?

The literature on trip scheduling suggests that miscalculations for an incremental expansion can go either way, depending on the model used (Small, 1992a; Henderson, 1992). Abstract models have been exclusively used for this examination, which is made possible because capacity enters these models as an exogenous parameter. Miscalculations for an incremental expansion are examined by doing two comparative static analyses on an aggregate measure (total benefits, for example): one with fixed schedule and the other with variable schedule.

This comparative static approach, however, does not apply to examining miscalculations for an optimal toll. The reason is analytical complexity: an optimal toll not only is endogenous, but also is a function of time. This may explain why no one has rigorously examined miscalculations for an optimal toll.

Arnott, de Palma, and Lindsey (1990) is the only one that has touched on the issue. They use the Vickrey model (Vickrey, 1969). As reviewed in chapter 3, the optimal toll in the Vickrey model eliminates queuing by charging a toll equal to the cost of queuing delay travelers incur at base equilibrium. As a result, this optimal toll collects a toll revenue equal to half the total travel costs at the base equilibrium and leaves travelers indifferent. So total benefits are half the total travel costs at the base. They then compare these benefits with those from unrelated studies of congestion tolling that ignore scheduling behavior. The small percentages of total benefits estimated from those studies lead them to suggest that ignoring schedule shifts can substantially underestimate the benefits of an optimal toll. But these studies also differ in other aspects; one can not be sure the small savings from congestion tolling in these studies are due to ignoring scheduling flexibility.

The simulation model developed in chapter 7 provides a unique opportunity to examine miscalculations of policy effects from ignoring schedule flexibility. The strategy is to compare two simulations of a given policy: one with schedule shifts allowed and the other with schedule shifts constrained. Constraining schedule shifts is achieved by equating the conditional probability of schedule choice at its base value in simulating the effects of that

given policy. In this way, the effects of the same policy with and without constraining

schedule shifts are compared with the same base case.

8.2 Base Case

This section presents characteristics of the base case from the five perspectives

mentioned above: welfare, peaking, congestion, schedule delay, and mode mix. Also

presented are traffic counts by mode and time of day for the most congested scenario.

Table 8.1 presents characteristics of an base case as well as on marginal externality.

All variables are measured for the morning trip. Marginal externality is in cents per passenger

car unit per morning trip, in 1972 prices. These characteristics are tabulated with two values

of congestion elasticity and three values of total number of commuters to give a range of

baseline congestion. The two values of congestion elasticity are 1 and 2.5, representing

single interaction and multiple interaction, two of the six classifications of congestion by

Vickrey (1969, p. 251).

Under either single or multiple interaction, three numbers of total commuters are

tabulated, representing a range of baseline congestion for a given congestion elasticity. The

three tabulated numbers are 12000, 24000, and 48000, listed in columns (1), (2), and (3),

respectively. If the spread of work start times is measured by four standard deviations of the

distribution (132 minutes), these numbers are equivalent to the following ratios of number of

passengers to capacity over that period: 1.03, 2.05, and 4.11. That is, using the middle

number, if all 24000 commuters drove alone and spread out evenly over this 132-minute

period, the volume would be 2.05 times of capacity. Column (4) gives differences between

columns (3) and (1). For variables measured in absolute units, these differences in column (4)

are in percentages; for variables measured in percentage, these differences in column (4) are

in percentage points.

Change in consumer surplus measures change in average consumer welfare given by

(5.16). This change is in cents per passenger for the morning commute, 1972 prices. It is

measured relative to column (1).

The externality of traveling at a particular time measures the marginal increase in total

travel costs at a base case. Components of travel costs include costs of on-vehicle travel time,

costs of bus walk and waiting times, costs of schedule delay, auto operating and maintenance

costs and bus agency costs. Externality for a base case is measured with peak externality and

average externality. Peak refers to the 15-minute period centered around 8:00 A.M.

Externality is in cents per passenger car unit for the morning trip, 1972 prices.

Peaking is measured with peak traffic and peak share of traffic. Peak traffic is the

number of passenger car units arriving at work within the peak 15-minute period centered

around 8:00 A.M. Peak share of traffic is the proportion of peak traffic out of total number of

passenger car units over the whole period considered: from 5:30 to 10:00 A.M. A change in

peak share of traffic could result from mode shifts only, schedule shifts only, or a combination

of them. A change in peak traffic could result not only from mode or schedule shifts, but also

from a change in total number of commuters. Peak share of traffic is probably a better

measure for peaking.

Table 8. Characteristics of Base Case
for Selected Amounts and Elasticities of Congestion

		Number of Commuters[a]			%Difference
		12000	24000	48000	(1)-(3)
Single Interaction ($\gamma = 1.0$)[a]	(1)	(2)	(3)	(4)	
Welfare (cents/passenger)					
(01) Change in Consumer Surplus[b]	0.00	-10.25	-29.43	NA[g]	

Externality (cents/pcu)

(02) Peak Externality[c,e]	58.29	88.16	127.68	119.04
(03) Average Externality	26.18	44.99	67.41	157.49

Peaking

(04) Peak Traffic (pcu)[c]	1403.48	2701.66	5058.68	260.44
(05) Peak Share of Traffic (%)	25.30	24.56	23.38	-1.92[d]

Congestion (minutes)

(06) Free-Flow Travel Time	10.00	10.00	10.00	0.00
(07) Peak Travel Delay[c]	1.59	3.05	5.72	259.75
(08) Average Travel Delay	0.90	1.76	3.37	274.44

Schedule Delay

(09) Constant Schedule Delay of Auto Users (minutes)[f]	10.93	10.93	10.93	0.00
(10) Variable Schedule Delay of Auto Users (minutes)	0.15	0.29	0.53	253.33
(11) Ratio of Variable Schedule to Travel Delay for Auto Users (%)	18.86	18.32	17.59	-1.27[d]

Mode Mix (measured with pcu's)

(12) Drive-Alone Share of Traffic (%)	80.13	80.06	79.99	-0.14[d]
(13) Bus Share of Traffic (%)	7.99	8.06	8.13	0.14[d]
(14) Carpool Share of Traffic (%)	11.88	11.88	11.87	-0.01[d]
(15) Average Occupancy	2.16	2.17	2.18	0.93
(16) Total Traffic	5547.54	8285.39	11000.20	98.29

(Table 8.1 Continued)

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	Number of Commuters[a]			%Difference
	12000	24000	48000	(1)-(3)
Multiple Interaction ($\gamma = 2.5$)[a]	(1) (2) (3)			(4)

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Welfare (cents/passenger)

(17) Change in Consumer's Surplus[b]	0.00	-57.25	-95.14	NA[g]

Externality (cents/pcu)

(18) Peak Externality[c,e]	95.27	298.66	564.74	492.78
(19) Average Externality	33.60	113.66	255.23	659.61

Peaking

(20) Peak Traffic (pcu)[c]	1378.86	2363.98	3562.06	158.33
(21) Peak Share of Traffic (%)	24.77	21.62	17.32	-7.45[d]

Congestion (minutes)

(22) Free-Flow Travel Time	10.00	10.00	10.00	0.00
(23) Peak Travel Delay[c]	1.65	6.35	17.69	972.12
(24) Average Travel Delay	0.67	2.74	8.91	1229.85

Schedule Delay

(25) Constant Schedule Delay of Auto Users (minutes)[f]	10.93	10.93	10.93	0.00
(26) Variable Schedule Delay of Auto Users (minutes)	0.21	0.76	1.94	823.81
(27) Ratio of Variable Schedule to Travel Delay for Auto Users (%)	39.09	34.52	27.47	-11.62[d]

Mode Mix (measured with pcu's)

(28) Drive-Alone Share of Traffic (%)	80.18	79.86	78.80	-1.38[d]
(29) Bus Share of Traffic (%)	7.92	8.24	9.38	1.46[d]
(30) Carpool Share of Traffic (%)	11.90	11.90	11.82	-0.08[d]
(31) Average Occupancy	2.16	2.19	2.33	7.87
(32) Total Traffic	5567.76	10935.88	20560.34	269.27

Notes to Table 8.1:

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[a] γ is a supply parameter measuring elasticity of travel delay with respect to arrival flow in the supply model (Table 7.1). Numbers of commuters are exogenously chosen to give a range of baseline congestion.

[b] It is change in consumer welfare given by equation (5.16) relative to column (1).

[c] Peak refers to the 15-minute period centered around 8:00 A.M.

[d] These are absolute changes.

[e] Externality measures the marginal increase in total travel costs due to an additional trip after existing travelers have adjusted their choices. Total costs include costs of on-vehicle travel time, costs of bus walk and waiting time, costs of schedule delay, auto operating and maintenance costs and bus agency costs. It is in cents per passenger car unit for the morning commute, 1972 prices.

[f] Constant schedule delay is the average schedule delay of auto users in equilibrium when travel is free-flow. Thus, given their unobserved characteristics, auto users choose to arrive early or late for 11 minutes on average even when there is no congestion.

[g] Difference does not apply here.

Congestion is measured with peak and average travel delays. Schedule delay is measured with average variable schedule delay of auto users and ratio of variable schedule to travel delay for auto users. Schedule delay is measured only for auto users because bus users are assumed to arrive on time. The schedule delay tabulated is the variable component of average schedule delay; the constant component is the average schedule delay in equilibrium when travel is free-flow. Thus, given their unobserved characteristics, auto users choose to arrive early or late for about 11 minutes on average even when there is no congestion.

Mode mix is measured with drive-along share, bus share, and carpool share of traffic, average occupancy, and total traffic in passenger car units. Carpool and bus occupancies are assumed at 2.5 and 40/3 per pcu, respectively. So the mode shares presented are by passenger car unit not by passenger. Average occupancy is the ratio of total number of commuters and the corresponding total traffic.

(b1) <u>What is the cost of congestion</u>? An average commuter's welfare drops by almost one dollar when average travel delay jumps from below one minute to nine minutes under multiple interaction. An average commuter's welfare reduces by about 30 cents when average travel delay increases from just below one minute to over three minutes under single interaction. These welfare losses are measured only for the morning trip in 1972 prices.

(b2) <u>What is the social cost of a marginal traveler</u>? The marginal externality for traveling in the peak 15-minute period ranges from 58 to 565 cents per passenger car unit for

all the scenarios considered. The range for average externality is from 26 to 255 cents per passenger car unit. These costs are for the morning commute only, in 1972 prices. Under multiple interaction, peak externality becomes six times bigger and average externality becomes more than seven times bigger when the amount of congestion becomes more than 10 times bigger. Under single interaction, peak and average externalities increase by 119 and 157 percent, respectively, when the amount of congestion more than triples. Marginal externality increases with the amount of congestion as well as elasticity of congestion.

(b3) <u>Does the amount of congestion affect average occupancy</u>? Barely. As the amount of congestion becomes 10 times bigger under multiple interaction, average occupancy increases by 7.87 percent from 2.16 to 2.33. This percentage reduces to 2.78 under single interaction. In both cases increase in occupancy results from minor shifts from drive-alone to bus. Carpool share of traffic remains unchanged as congestion worsens. Congestion elasticity appears to increase average occupancy slightly.

(b4) <u>Do travelers shift their schedules as congestion worsens</u>? Congestion leads to peak spreading: the heavier congestion is the lower the share of peak traffic. The extent of this spreading depends on congestion elasticity: the higher the elasticity the more spreading. This increase in peak share of traffic as a result of heavier congestion is consistent with the increase in variable schedule delay as the amount of congestion increases.

(b5) <u>How do schedule and travel delays compare with each other</u>? The percentage ratio of variable schedule to travel delay for auto users ranges between 18 to 39 for all the

scenarios considered. This ratio reduces as congestion worsens. It reduces by more than 12

percentage points under multiple interaction and 1 percentage point under single interaction.

This reduction in their relative values result from a faster increase in travel delay than in

schedule delay as congestion worsens. This relative value of schedule delay is not as high as

simple theoretical models predict (e.g., Arnott, de Palma, and Lindsey, 1990, predict a ratio of

100 percent in monetary values).

Table 8.2 presents traffic share and counts by mode and time of day for the most

congested scenario. Traffic is in passenger car units. The occupancies of carpool and bus are

assumed at 2.5 and 40/3 per pcu, respectively. So the peak 15-minute period gets 17.32

percent of total traffic (as presented in Table 8.1, row 21); the peak one-hour period gets 66.82

percent of total traffic.

Table 8.2 Traffic Counts by Mode and Schedule at Base Case[a]

($\gamma = 2.5$, Number of Commuters = 48000)[b]

Time (1)	Drive-Alone (2)	Bus[d] (3)	Carpool (4)	Total Traffic (5)	Share[c] (6)
6:00	33.87	0.00	5.61	39.48	0.19
6:15	148.80	0.00	24.77	173.57	0.84
6:30	273.23	27.64	40.55	341.42	1.66
6:45	488.05	3.90	82.72	574.67	2.80
7:00	837.61	97.76	120.45	1055.82	5.14
7:15	1316.90	7.23	250.09	1574.21	7.66
7:30	1944.64	119.64	326.89	2391.17	11.63
7:45	2531.06	67.47	397.98	2996.50	14.57
8:00	2412.21	851.07	298.78	3562.07	17.32
8:15	1995.50	126.18	298.02	2419.70	11.77
8:30	1762.86	358.67	248.87	2370.41	11.53
8:45	1021.58	11.83	152.93	1186.34	5.77
9:00	1052.54	229.50	126.95	1409.00	6.85
9:15	215.90	5.22	32.85	253.98	1.24
9:30	143.82	23.23	18.42	185.47	0.90
Total (1)	16201.49	1929.36	2429.49	20560.34	100.00
Share (2)[c]	78.80	9.38	11.82	100.00	

Notes to Table 8.2:

[a] Traffic is in passenger car units. The occupancies of carpool and bus are assumed at 2.5 and 40/3 per pcu, respectively.

[b] γ is a supply parameter measuring elasticity of travel delay with respect to arrival flow in the supply model (Table 7.1). Number of commuters is exogenously chosen to give certain level of baseline congestion. The most congested scenario is presented.

[c] The shares in column 6 are percentages of traffic (in pcu's) in the 15-minute intervals

centered around the clock times in column 1. The shares in row 2 are percentages of traffic by drive-alone, bus, and carpool, respectively.

[d] Bus users are assumed to arrive on time.

8.3 Policy Effects

This section compares the effects of the eight policies listed earlier. Each policy is simulated with two values of congestion elasticity and three numbers of commuters as used in the base case. For the optimal toll and capacity expansion polices, their effects are presented as percentage changes from the base case. For other policies, their effects are presented as percentage differences from the effects of the optimal toll policy. Changes in traffic counts by mode and time of day are also presented for the most congested scenario.

Table 8.3 presents the equilibrium effects of an optimal toll for all the scenarios considered. The optimal toll at a particular time is the marginal increase in total travel costs due to an additional passenger-car trip made at that time. This optimal toll can vary significantly with time and scenarios. Consumer surplus measures change in consumer welfare due to the optimal toll policy. Total benefits sum consumer surplus and toll revenue. Welfare measures are in cents per passenger, 1972 prices. Peak toll is the optimal toll for the peak 15-minute period centered around 8:00 A.M.. The tolls are in cents per passenger car unit, 1972 prices. Other variables are defined as in Table 8.1 in section 8.2; those followed by

"%" are in percentage changes from the base case; those not followed by "%" are in percentage points.

(p1) <u>Does an optimal toll affect traffic peaking, congestion, schedule delay, and mode mix</u>? In the most congested scenario, peak and average travel delays at the base case are about 18 and 9 minutes, respectively. An optimal toll reduces peak traffic and peak share of traffic by about 36 percent and 4 percentage points, respectively. It cuts peak travel delay by two-thirds and average travel delay by 57 percent. Both variable schedule delay and the ratio of schedule to travel delay increase by about three-fourths. An optimal toll also increases average occupancy by one fifth by shifting traffic from drive-alone to both bus and carpool. This increase in average occupancy is accompanied by a 16 percent reduction in total traffic.

The answer to question (p1) is qualitatively unchanged with other scenarios. Quantitatively, however, the effects can vary substantially. The effects on most measures increase with the amount and elasticity of congestion except for measures of schedule delay. The effects on both variable schedule delay and ratio of schedule to travel delay tend to decrease with congestion; the effects on variable schedule delay also tend to decrease with congestion elasticity.

Table 8.4 presents the effects of the ten-percent expansion for the same scenarios as in Table 8.3. The interpretation of variables, labels, and numbers remains the same as in Table 8.3. This expansion applies to all scenarios.

(p2) <u>Does the ten-percent capacity expansion affect traffic peaking, congestion,</u>

schedule delay, and mode mix? For the most congested scenario, peak traffic increases by 4.6 percent; its share increases by half a percentage point. Peak and average travel delays are reduced by about 12 and 14 percent, respectively. Variable schedule delay reduces by about 10 percent.

Table 8.3 Equilibrium Effects of An Optimal Toll[a]
for Selected Amounts and Elasticities of Congestion

	Number of Commuters[b]		
	12000	24000	48000
Single Interaction ($\gamma = 1.0$)[b]	(1)	(2)	(3)
Welfare (relative to base case; in cents/passenger)			
(01) Consumer Surplus[c]	-10.85	-17.23	-24.98
(02) Toll Revenue	11.09	18.14	27.74
(03) Total Benefits	0.24	0.90	2.76
Toll (cents/pcu)			
(04) Peak Toll	56.18	85.13	130.40
(05) Average Toll	24.71	41.58	66.53
Peaking (relative to base case)			
(06) Peak Traffic (%)[d]	-11.44	-14.26	-13.59
(07) Peak Share of Traffic	-0.77	-10.64	-9.49
Congestion (relative to base case)			
(08) Peak Travel Delay (%)[d]	-11.95	-14.16	-13.44
(09) Average Travel Delay (%)	-6.67	-9.70	-9.66
Schedule Delay (relative to base case)			

(10) Variable Schedule Delay for Auto Users[e] (%)		286.67	213.79	175.47
(11) Ratio of Variable Schedule to Travel Delay for Auto Users		57.76	45.72	38.55

Mode Mix (relative to base case; measured with pcu's)

(12) Drive-Alone Share of Traffic	-0.98		-1.65	-2.64
(13) Bus Share of Traffic		0.46	0.79	1.30
(14) Carpool Share of Traffic	0.52		0.86	1.35
(15) Average Occupancy (%)	3.24		5.05	8.11
(16) Total Traffic (%)	-2.90		-4.84	-7.53

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(Table 8.3 Continued)

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		Number of Commuters[b]		
		12000	24000	48000
Multiple Interaction ($\gamma = 2.5$)[b]	(1) (2)		(3)	

Welfare (relative to base case; in cents/passenger)

(17) Consumer Welfare[c]		-13.03	-26.50	-26.33
(18) Toll Revenue		14.17	35.20	64.66
(19) Total Benefits		1.14	8.71	38.32

Toll (cents/pcu)

(20) Peak Toll[d]	102.01		220.51	507.01
(21) Average Toll		31.67	85.10	180.72

Peaking (relative to base case)

(22) Peak Traffic (%)[d]	-19.88		-26.66	-35.86
(23) Peak Share of Traffic		-4.18	-4.16	-4.01

Congestion (relative to base case)

(24) Peak Travel Delay (%)[d]	-42.42		-54.02	-67.04
(25) Average Travel Delay (%)	-35.82		-45.99	-56.90

Schedule Delay (relative to base case)

(26) Variable Schedule Delay for Auto Users[e] (%)	309.52	138.16	78.35
(27) Ratio of Variable Schedule to Travel Delay for Auto Users	208.93	118.71	77.29

Mode Mix (relative to base case; measured with pcu's)

(28) Drive-Alone Share of Traffic	-1.24		-3.32		-6.69	
(29) Bus Share of Traffic		0.57		1.59		3.33
(30) Carpool Share of Traffic	0.67		1.73		3.37	
(31) Average Occupancy (%)	3.70		10.50		20.17	
(32) Total Traffic (%)	-3.61		-9.21		-16.48	

Notes to Table 8.3:

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[a] An optimal toll measures the marginal increases in total travel costs due to an additional passenger car trip at different times of day at the optimum where total welfare, sum of consumer welfare and toll revenue, is maximized. Travel costs include costs of on-vehicle travel time, costs of bus walk and waiting time, costs of schedule delay, and auto operating and maintenance costs and bus agency costs. Tolls are in 1972 prices.

[b] γ is a supply parameter measuring elasticity of travel delay with respect to arrival flow in the supply model (Table 7.1). Number of commuters is exogenously chosen to give a range of baseline congestion.

[c] Consumer welfare is measured by equation (5.16).

[d] Peak refers to the 15-minute period centered around 8:00 A.M.

[e] The average schedule delay tabulated is the variable component of average schedule delay; the constant component is above the variable component. The constant component is the average schedule delay of auto users in equilibrium when travel is free-flow.

Table 8.4 Equilibrium Effects of a Ten-Percent Capacity Expansion
for Selected Levels and Elasticities of Congestion

	Number of Commuters[b]		
	12000	24000	48000
	(1)	(2)	(3)
Single Interaction (γ = 1.0)[b]			
Welfare (relative to base case; in cents /passenger)			
(01) Total Benefits[c]	0.96	1.83	3.37
Peaking (relative to base case)			
(02) Peak Traffic[d] (%)	0.37	0.67	1.12
(03) Peak Share of Traffic	0.07	0.13	0.19

Congestion (relative to base case)

(04) Peak Travel Delay[d] (%)	-8.81	-8.52	-8.22
(05) Average Travel Delay (%)	-8.89	-8.52	-8.31

Schedule Delay (relative to base case)

(06) Variable Schedule Delay for Auto Users[e] (%)	-6.67	-6.90	-7.55
(07) Ratio of Variable Schedule to Travel Delay for Auto Users	0.07	0.09	0.12

Mode Mix (relative to base case; measured with pcu's)

(08) Drive-Alone Share of Traffic	0.01	0.03	0.05
(09) Bus Share of Traffic	-0.02	-0.03	-0.06
(10) Carpool Share of Traffic	0.01	0.00	0.01
(11) Average Occupancy (%)	0.00	0.00	-0.45
(12) Total Traffic (%)	0.08	0.15	0.29

(Table 8.4 Continued)

	Number of Commuters[b]		
	12000	24000	48000
	(1)	(2)	(3)
Multiple Interaction ($\gamma = 2.5$)[b]			

Welfare (relative to base case; in cents /passenger)

(13) Total Benefits[c]	1.46	5.24	14.11

Peaking (relative to base case)

(14) Peak Traffic[d] (%)	1.13	2.98	4.63

(15) Peak Share of Traffic	0.25	0.55	0.57

Congestion (relative to base case)

(16) Peak Travel Delay[d] (%)	-18.79	-15.28	-11.76
(17) Average Travel Delay (%)	-189.40	-16.42	-13.80

Schedule Delay (relative to base case)

(18) Variable Schedule Delay for Auto Users[e] (%)	-19.05	-14.47	-10.31
(19) Ratio of Variable Schedule to Travel Delay for Auto Users	0.38	0.85	1.00

Mode Mix (relative to base case; measured with pcu's)

(20) Drive-Alone Share of Traffic	0.01	0.02	0.22
(21) Bus Share of Traffic	-0.02	-0.07	-0.24
(22) Carpool Share of Traffic	0.08	0.00	0.02
(23) Average Occupancy (%)	-0.46	0.00	-1.29
(24) Total Traffic (%)	0.10	0.41	1.30

Notes to Table 8.4:

[a] Capacity expansion is exogenously chosen at ten percent of the base level.

[b] γ is a supply parameter measuring elasticity of travel delay with respect to arrival flow in the supply model (Table 7.1). Number of commuters is exogenously chosen to give a range of baseline congestion for a given value of congestion elasticity.

[c] Total benefits are changes in consumer welfare in equation (5.16).

[d] Peak refers to the 15-minute period centered around 8:00 A.M.

[e] The average schedule delay tabulated is the variable component of average schedule delay; the constant component is above the variable component. The constant component is the average schedule delay of auto users in equilibrium when travel is free-flow.

But ratio of schedule to travel delay increases by 1 percentage point--a result of faster reduction in travel delay than in schedule delay due to the capacity expansion. There are some mode shifts from bus to drive-alone.

The answer to question (p2) is not changed qualitatively for other scenarios. For the

least congested scenario, peak traffic is increased by only one third a percent; peak share is barely changed. Congestion is reduced by about 9 percent. Variable schedule delay is reduced by about 7 percent. Ratio of schedule to travel delay is barely increased. Minor mode shifts from bus to drive-alone and increase in total traffic still exits. Overall, the percentage reduction in both travel and schedule delays tend to decrease with the amount of congestion, which is likely because a fixed expansion apply to all scenarios.

How do the effects of an optimal toll and the ten-percent expansion compare? Because the capacity expansion is arbitrary, a meaningful comparison is in the direction of effects rather than the magnitudes of effects. Both polices reduce congestion and increase ratio of schedule to travel delay. But they have opposite effects on peaking and mode mix. An optimal toll leads to peak spreading and reduction in share of drive-alone, while an incremental expansion in capacity leads to further peaking and increase in share of drive-alone. The relative benefits of an optimal toll versus the capacity expansion are much more favorable under multiple interaction than under single interaction.

This difference in effects on peaking and mode mix between an optimal toll and the incremental expansion is better contrasted using changes from the base case in traffic counts by mode and time of day as presented in Tables 8.5 and 8.6 for the two policies, respectively. As in Table 8.2, only the most congested scenario is presented; traffic is in passenger car units. The bottom row gives changes in traffic counts for the three modes separated and all modes combined. Column (5)

gives changes in traffic counts for each of the 15-minute intervals for all modes combined.

The effects of an optimal toll and the incremental expansion on traffic differ as follows. The ten-percent expansion increases traffic by 358 pcu's during the peak one-hour period between 7:30 and 8:30 A.M. This traffic increase results partly from 51 more carpool pcu's and 74 less bus pcu's during the same period. The optimal toll, on the other hand, decreases traffic by 3302 pcu's during the peak one-hour period. This traffic reduction results partly from 253 more bus pcu's and 178 more carpool pcu's during the same period.

(p4) <u>How much of these changes in traffic during the peak one-hour period comes from mode and schedule shifts, respectively?</u> Schedule shifts account for about 8.4 percent of the reduction due to the optimal toll policy, and 15.4 percent of the increase due to the expansion policy. The following calculation uses 40/3 and 2.5 passengers per pcu for bus and carpool, respectively. It also assumes that mode shifts occur only between drive-alone and bus or drive-alone and carpool.

Under the optimal toll policy, the increased 229 bus pcu's would have been 229*40/3 = 3055 more drive-alone pcu's; shifts to bus mode reduces traffic by 3055 - 229 = 2826 pcu's. Similarly, the increased 132 carpool pcu's would have

Table 8.5 Change in Traffic Counts due to an Optimal Toll[a,b]
by Mode and Schedule: Schedule Shifts Allowed

$(\gamma = 2.5$, Number of Commuters = 48000)[c]

Time (1)	Drive-Alone (2)	Bus[d] (3)	Carpool (4)	Total Traffic (5)

6:30	8.81	0.03	1.48	10.33
6:45	39.84	0.01	6.29	46.14
7:00	-42.02	2.78	4.03	-35.22
7:15	16.10	0.42	18.46	34.98
7:30	-399.45	12.77	21.34	-365.33
7:45	-827.42	7.70	55.80	-763.93
8:00	-1446.74	141.52	28.01	-1277.20
8:15	-373.39	16.76	22.98	-333.65
8:30	-616.23	50.38	4.11	-561.74
8:45	2.02	0.81	12.22	15.06
9:00	-237.57	19.30	-3.06	-221.33
9:15	48.56	0.20	5.32	54.08
9:30	4.14	0.28	0.62	5.03
Total	-3818.97	252.98	178.38	-3387.61

Notes to Table 8.5

[a] An optimal toll measures the marginal increases in total travel costs due to an additional passenger car trip at different times of day at the optimum where total welfare, sum of consumer welfare and toll revenue, is maximized. Travel costs include costs of on-vehicle travel time, costs of bus walk and waiting time, costs of schedule delay, and auto operating and maintenance costs and bus agency costs. Tolls are in 1972 prices.

[b] Traffic is in passenger car units. The occupancies of carpool and bus are assumed at 2.5 and 40/3 per pcu, respectively.

[c] γ is a supply parameter measuring elasticity of travel delay with respect to arrival flow in the supply model (Table 7.1). Number of commuters is exogenously chosen to give certain level of baseline congestion. The most congested scenario is presented.

[d] Bus users are assumed to arrive on time.

Table 8.6 Change in Traffic Counts due to a Ten-Percent Expansion[a,b]
by Mode and Schedule: Schedule Shifts Allowed

($\gamma = 2.5$, Number of Commuters = 48000)[c]

Time (1)	Drive-Alone (2)	Bus[d] (3)	Carpool (4)	Total Traffic (5)
6:45	-7.36	0.00	-1.41	-8.77
7:00	-8.16	-0.17	-1.59	-9.92
7:15	-37.91	-0.01	-8.45	-46.37
7:30	22.94	-1.02	3.12	25.05
7:45	82.81	-0.74	14.78	96.85
8:00	158.27	-14.69	21.27	164.85
8:15	27.95	-2.41	5.71	31.26
8:30	39.34	-5.63	6.07	39.79
9:45	-18.31	-0.18	-2.57	-21.06
9:00	3.42	-1.30	0.46	2.59
9:15	-3.56	-0.01	-0.48	-4.05
Total	257.50	-26.16	36.53	267.87

Notes to Table 8.6:

[a] Capacity expansion is exogenously chosen at ten percent of the base level.

[b] Traffic is in passenger car units. The occupancies of carpool and bus are assumed at 2.5 and 40/3 per pcu, respectively.

[c] γ is a supply parameter measuring elasticity of travel delay with respect to arrival flow in the supply model (Table 7.1). Number of commuters is exogenously chosen to give certain level of baseline congestion. The most congested scenario is presented.

[d] Bus users are assumed to arrive on time.

been 132*2.5 = 330 less drive-alone pcu's; shifts to carpool reduces traffic by 330 - 132 = 198

pcu's. So mode shifts to bus and carpool reduce traffic by 198 + 2826 = 3024 pcu's, which accounts for 91.6 percent of the total reduction during the peak one-hour period. Schedule shifts account for the other 8.4 percent.

Under the expansion policy, the traffic increase during the peak one-hour period can be similarly decomposed. The reduced 24 bus pcu's would have been 24*40/3 = 327 less drive-alone pcu's; shifts to bus increases traffic by 327 - 24 = 303. The increased 51 carpool pcu's would have been 51*2.5 = 127 less drive-alone pcu's; shifts to carpool reduces traffic by 127 - 51 = 76 pcu's. Net mode shifts to bus and carpool increase traffic by 303 - 76 = 227 pcu's, which accounts for 84.7 percent of the total increase of 268 pcu's due to the expansion during the peak one-hour period. The rest, 15.3 percent, comes from schedule shifts.

Next the welfare effects of other policies are compared with those from an optimal toll, using Tables 8.7 through 8.10, and compare other pricing policies with an optimal toll on other effects, using Tables 8.11 through 8.18.

Table 8.7 compares total benefits per passenger. Total benefits of a policy sum toll revenue (for pricing policies) and the change in consumer's surplus from the base case due to the policy. These benefits are for the morning commute. The total benefits for an optimal toll are given in cents, 1972 prices; the total benefits for other policies are in ratio to those of an optimal toll. For example, the number in column 3 and row 09 is 38.32, indicating that the optimal toll for this scenario increases social welfare per passenger by about 38 cents for the morning commute, in 1972 prices.

Table 8.7 Comparison of Total Benefits per Passenger[a]
between an Optimal Toll and Other Polices[b]

	Number of Commuters[c]		
	12000 (1)	24000 (2)	48000 (3)

Single Interaction ($\gamma = 1.0$)[c]

(01) Optimal Toll (relative to base case) (cents/morning trip, 1972 prices)	0.24	0.90	2.76

Other Policies (in ratio to row 01)

(02) Base-Externality Toll	1.00	0.99	1.00
(03) Piecewise-Linear Toll	0.83	0.86	0.91
(04) One-Step Toll	0.63	0.62	0.69
(05) Uniform Toll	0.58	0.55	0.63
(06) Optimal Toll with HOV Exemption	0.88	0.90	0.94
(07) 10% Expansion	4.00	2.03	1.22
(08) Optimal Toll with 10 % Expansion	4.83	2.88	2.10

Multiple Interaction ($\gamma = 2.5$)[c]

(09) Optimal Toll (relative to base case) (cents/morning trip, 1972 prices)	1.14	8.71	38.32

Other Policies (in ratio to row 09)

(10) Base-Externality Toll	1.02[d]	0.99	1.02[d]
(11) Piecewise-Linear Toll	0.61	0.78	0.93
(12) One-Step Toll	0.38	0.58	0.77
(13) Uniform Toll	0.26	0.43	0.65
(14) Optimal Toll with HOV Exemption	0.89	0.93	0.93
(15) 10% Expansion	1.28	0.60	0.37

(16) Optimal Toll with 10% Expansion	1.99	1.38	1.22

Notes to Table 8.7:

[a] Total benefits sum toll revenue and change in consumer's surplus from the base case. Benefits for policies other than an optimal toll is in ratio to those of the optimal toll.

[b] An optimal toll measures the marginal increases in total travel costs due to an additional passenger car trip at a given time interval at the optimum where total welfare, sum of consumer welfare and toll revenue, is maximized. Travel costs include costs of on-vehicle travel time, costs of bus walk and waiting time, costs of schedule delay, and auto operating and maintenance costs and bus agency costs. The change in costs is calculated after existing travelers have adjusted their behavior in response to the added trip.

A base-externality toll is the marginal increase in total travel costs at the base case as a result of an additional passenger car trip at a given time interval.

A piecewise-linear toll starts with zero at 7:00 A.M., increases linearly until 8:00 A.M., decreases linearly to zero at 9:00 A.M.; its start and end points are chosen exogenously; its slope (the same on each side) is chosen through screening at a 5-cent increment to maximize total welfare.

A one-step toll starts at 7:30 A.M., stays constant, ends at 8:30 A.M.; its start and end points are chosen exogenously; its level is chosen through screening at a 10-cent increment to maximize total welfare.

A uniform toll applies between 5:30 to 10:00 A.M.; its level is chosen through screening at a 10-cent increment to maximize total welfare.

An optimal Toll with HOV exemption is the marginal increase in total travel costs due to an additional passenger car trip at a given time interval with carpooler and bus users exempted.

Capacity expansion is exogenously chosen at 10 percent of the base level.

An optimal toll with expansion is the marginal increase in total travel costs due to an additional passenger car trip at a given time interval with the capacity expanded by 10 percent.

[c] γ is a supply parameter measuring elasticity of travel delay with respect to traffic flow in the supply model (Table 7.1). Number of commuters is exogenously   chosen to give a range of baseline congestion.

[d] These larger than unity ratios are likely due to numerical inaccuracy.

(n1) <u>What are the total benefits of an optimal toll?</u> Table 8.3 presents change in

consumer's surplus from the base case, toll revenue, and total benefits for the optimal toll

policy. These welfare measures are in cents, 1972 prices, for the morning commute. Its total

benefits range from one quarter a cent to 2 and three quarter a cent under single interaction.

Under multiple interaction, however, the range of total benefits increases to from 1 to 38

cents.

Two points should be made about the welfare effects of an optimal toll. First,

significant welfare transfers occur from commuters to the government. Under the least

congested scenario, implementing an optimal toll would reduce average consumer welfare by

10.85 cents for the morning trip, while the government collects 11.09 cents on each trip.

Under the most congested scenario, the average commuter loses 26 cents due to the optimal

toll policy, while the government collects 65 cents in toll revenue. Second, once costs of toll

collection are considered, it is probably not worthwhile to implement an optimal toll when

congestion is light or congestion elasticity is low. On the other hand, the aggregate benefits of

an optimal toll can be substantial when the baseline congestion is heavy. Under the most

congested scenario, for example, the total benefits of 38 cents in 1972 prices per passenger

per morning trip are equivalent to annual total benefits of more than 14.5 million dollars in

1992 prices for the 250 morning trips of 48000 commuters. A deflator of 3.1856 is used to convert dollars from 1972 to 1992 prices.

(n2) <u>How do the total benefits of other pricing policies compare with those of an optimal toll?</u> A base-externality toll does just as well as an optimal toll for all scenarios. (A ratio of 1.02 under two of the scenarios, however, are likely due to numerical inaccuracies.) Recall that a base-externality toll is equal to the marginal externality of an additional passenger car trip at a given time interval at the base case. This marginal externality is calculated after existing travelers have adjusted their behavior in response to the added trip, which should be much closer to the marginal externality at an optimum than the marginal externality calculated without allowing existing travelers to adjust their choices.

Under single interaction, a piecewise-linear toll achieves from 83 to 91 percent of the benefits of an optimal toll; this percentage ranges from 62 to 69 percent for a one-step toll, from 56 to 63 percent for a uniform toll, and from 88 to 94 percent for an optimal toll with HOV exemption. The amount of congestion does not seem to affect their total benefits much relative to those of an optimal toll. Except for the policy of an optimal toll with HOV exemption, these percentages vary widely with the amount of congestion for a given policy under multiple interaction. A piecewise-linear toll achieves about 61 to 93 percent of the benefits of an optimal toll; a one-step toll achieves about 38 to 77 percent; a uniform toll achieves about 26 to 65 percent. Congestion elasticity does not seem to affect the total benefits of an optimal toll with HOV exemption relative to those of an optimal toll alone.

These percentages for a one-step toll are in line with the estimates by Arnott, de Palma, and Lindsey (1993, Table 1). Their estimates for a uniform toll are smaller, however. Instead of allowing mode choice in addition to scheduling choice, they extend the Vickrey model (Vickrey, 1969) to allow elastic total demand. With demand elasticity ranging between 0.2 and 1, their estimate for an optimal one-step toll ranges from 58 to 62 percent; their estimate for an optimal uniform toll ranges from 12 to 31 percent. Their estimates are based on a base scenario with an average queuing delay of about 36 minutes, which doubles the average travel delay under the most congested scenario here.

(n3) <u>How do the total benefits of the ten-percent expansion compare with and without an optimal toll</u>? The answer is obtained by subtracting unity from rows 08 and 16 and comparing the resulting numbers with those in rows 07 and 15, respectively. With the most congested scenario under single interaction, the capacity expansion achieves total benefits of 10 and 22 percent more than the optimal toll with and without the presence of an optimal toll, respectively. With the most congested scenario under multiple interaction, the capacity expansion achieves 22 and 37 percent more with and without an optimal toll. So the presence of an optimal toll can reduce the level of optimal capacity substantially.

Tables 8.8 and 8.9 present peak and average tolls, respectively, for the pricing policies. Tolls are in cents per passenger car unit for the morning commute, 1972 prices. Table 8.10 lists the entire toll schedules for the optimal toll, base-externality toll, piecewise-linear toll, one-step toll, uniform toll, and optimal toll with HOV exemption under the most

congested scenario.

Table 8.8 Comparison of Average Tolls
between The Optimal Toll Policy and Other Pricing Polices[a]

(in cents per passenger car unit per morning trip, 1972 prices)

	Number of Commuters[b]		
	12000	24000	48000
	(1)	(2)	(3)
Single Interaction ($\gamma = 1.0$)[b]			
(01) Optimal Toll	25.	42.	67.
(02) Base-Externality Toll	25.	43.	64.
(03) Piecewise-Linear Toll	16.	47.	77.
(04) One-Step Toll	14.	35.	60.
(05) Uniform Toll	20.	40.	80.
(06) Optimal Toll with HOV Exemption	24.	33.	67.
(07) Optimal Toll with 10 % Expansion	23.	37.	66.
Multiple Interaction ($\gamma = 2.5$)[b]			
(08) Optimal Toll	32.	85.	181.
(09) Base-Externality Toll	30.	96.	224.
(10) Piecewise-Linear Toll	48.	121.	249.
(11) One-step Toll	28.	90.	176.
(12) Uniform Toll	40.	110.	290.
(13) Optimal Toll with HOV Exemption	29.	80.	145.

| (14) Optimal Toll with 10% Expansion | 22. | 67. | 187.[c] |

Notes to Table 8.8:

[a] An optimal toll measures the marginal increases in total travel costs due to an additional passenger car trip at a given time interval at the optimum where total welfare, sum of consumer welfare and toll revenue, is maximized. Travel costs include costs of on-vehicle travel time, costs of bus walk and waiting time, costs of schedule delay, and auto operating and maintenance costs and bus agency costs. The change in costs is calculated after existing travelers have adjusted their behavior in response to the added trip.

A base-externality toll is the marginal increase in total travel costs at the base case as a result of an additional passenger car trip at a given time interval.

A piecewise-linear toll starts with zero at 7:00 A.M., increases linearly until 8:00 A.M., decreases linearly to zero at 9:00 A.M.; its start and end points are chosen exogenously; its slope (the same on each side) is chosen through screening at a 5-cent increment to maximize total welfare.

A one-step toll starts at 7:30 A.M., stays constant, ends at 8:30 A.M.; its start and end points are chosen exogenously; its level is chosen through screening at a 10-cent increment to maximize total welfare.

A uniform toll applies between 5:30 to 10:00 A.M.; its level is chosen through screening at a 10-cent increment to maximize total welfare.

An optimal Toll with HOV exemption is the marginal increase in total travel costs due to an additional passenger car trip at a given time interval with carpooler and bus users exempted.

Capacity expansion is exogenously chosen at 10 percent of the base level.

An optimal toll with expansion is the marginal increase in total travel costs due to an additional passenger car trip at a given time interval with the capacity expanded by 10 percent.

[b] γ is a supply parameter measuring elasticity of travel delay with respect to traffic flow in the supply model (Table 7.1). Number of commuters is exogenously chosen to give a range of baseline congestion.

[c] The difference between this value and the corresponding optimal toll is within the tolerance level for convergence.

Table 8.9 Comparison of Peak Tolls[a]
between The Optimal Toll Policy and Other Pricing Polices[b]

(in cents per passenger car unit per morning trip, 1972 prices)

	Number of Commuters[c]		
	12000	24000	48000
	(1)	(2)	(3)
Single Interaction ($\gamma = 1.0$)[c]			
(01) Optimal Toll	56.	85.	130.
(02) Base-Externality Toll	58.	88.	128.
(03) Piecewise-Linear Toll	25.	75.	125.
(04) One-Step Toll	20.	50.	90.
(05) Uniform Toll	20.	40.	80.
(06) Optimal Toll with HOV Exemption	57.	65.	139.
(07) Optimal Toll with 10 % Expansion	51.	72.	129.
Multiple Interaction ($\gamma = 2.5$)[c]			

(08) Optimal Toll	102.	221.	507.
(09) Base-Externality Toll	95.	299.	565.
(10) Piecewise-Linear Toll	75.	200.	450.
(11) One-Step Toll	40.	140.	320.
(12) Uniform Toll	40.	110.	290.
(13) Optimal Toll with HOV Exemption	104.	227.	534.
(14) Optimal Toll with 10% Expansion	60.	182.	489.

Notes to Table 8.9:

[a] Peak refers to the 15-minute period centered around 8:00 A.M.

[b] An optimal toll measures the marginal increases in total travel costs due to an additional passenger car trip at a given time interval at the optimum where total welfare, sum of consumer welfare and toll revenue, is maximized. Travel costs include costs of on-vehicle travel time, costs of bus walk and waiting time, costs of schedule delay, and auto operating and maintenance costs and bus agency costs. The change in costs is calculated after existing travelers have adjusted their behavior in response to the added trip.

A base-externality toll is the marginal increase in total travel costs at the base case as a result of an additional passenger car trip at a given time interval.

A piecewise-linear toll starts with zero at 7:00 A.M., increases linearly until 8:00 A.M., decreases linearly to zero at 9:00 A.M.; its start and end points are chosen exogenously; its slope (the same on each side) is chosen through screening at a 5-cent increment to maximize total welfare.

A one-step toll starts at 7:30 A.M., stays constant, ends at 8:30 A.M.; its start and end points are chosen exogenously; its level is chosen through screening at a 10-cent increment to

maximize total welfare.

A uniform toll applies between 5:30 to 10:00 A.M.; its level is chosen through screening at a 10-cent increment to maximize total welfare.

An optimal Toll with HOV exemption is the marginal increase in total travel costs due to an additional passenger car trip at a given time interval with carpooler and bus users exempted.

Capacity expansion is exogenously chosen at 10 percent of the base level.

An optimal toll with expansion is the marginal increase in total travel costs due to an additional passenger car trip at a given time interval with the capacity expanded by 10 percent.

[c] γ is a supply parameter measuring elasticity of travel delay with respect to traffic flow in the supply model (Table 7.1). Number of commuters is exogenously chosen to give a range of baseline congestion.

Table 8.10 Comparison of Toll Schedules[a]
(in cents per passenger car unit per morning trip, 1972 prices)

(γ = 2.5 , Number of Commuters = 48000)[b]

Time (1)	Optimal Toll (2)	Base-nality Toll (3)	Piece-Linear Step Toll (4)	Toll (5)	Optimal HOV Exempted (6)		
6:30	1.91	2.62	0.00	0.00	1.93		
6:45	7.19	8.24	0.00	0.00	6.90		
7:00	35.96	49.67	90.00	0.00	31.25	7:15	58.48
8:00	507.01	564.74	450.00	320.00	534.40	8:15	157.78
9:00	115.03	170.55	90.00	0.00	110.29	9:15	0.44
9:30	0.91	1.29	0.00	0.00	0.92		

Notes to Table 8.10:

[a] An optimal toll measures the marginal increases in total travel costs due to an additional passenger car trip at a given time interval at the optimum where total welfare, sum of consumer welfare and toll revenue, is maximized. Travel costs include costs of on-vehicle travel time, costs of bus walk and waiting time, costs of schedule delay, and auto operating and maintenance costs and bus agency costs. The change in costs is calculated after existing travelers have adjusted their behavior in response to the added trip.

A base-externality toll is the marginal increase in total travel costs at the base case as a result of an additional passenger car trip at a given time interval.

A piecewise-linear toll starts with zero at 7:00 A.M., increases linearly until 8:00 A.M., decreases linearly to zero at 9:00 A.M.; its start and end points are chosen exogenously; its slope (the same on each side) is chosen through screening at a 5-cent increment to maximize total welfare.

A one-step toll starts at 7:30 A.M., stays constant, ends at 8:30 A.M.; its start and end points are chosen exogenously; its level is chosen through screening at a 10-cent increment to maximize total welfare.

An optimal Toll with HOV exemption is the marginal increase in total travel costs due to an additional passenger car trip at a given time interval with carpooler and bus users exempted.

[b] γ is a supply parameter measuring elasticity of travel delay with respect to traffic flow in the supply model (Table 7.1). Number of commuters is exogenously chosen to give a range of baseline congestion.

How do a base-externality toll, piecewise-linear toll, one-step toll, and uniform toll

compare with an optimal toll on their effects on peaking, congestion, schedule delay, and

mode mix? The answer is obtained using Tables 8.11 through 8.15. Variables are the same

as in Table 8.2; numbers now represent percentage differences in their effects relative to the

effects of the optimal toll for a given scenario. A positive number implies a larger effect in

absolute value from the policy of concern than the optimal toll; a negative smaller than 100 in

absolute

value implies a smaller effect in absolute value in the same direction as the optimal toll; a

negative larger than 100 in absolute value implies an effect in the opposite direction as the

optimal toll; and a zero implies the same effect as the optimal toll.

A base-externality toll, presented in Table 8.11, affects peaking, congestion, schedule

delay, and mode mix in the same direction as the optimal toll for a given scenario. The base-externality toll tends to have a larger effect in absolute value on all four aspects of travel under both single and multiple interactions. When the number commuters is 48000 under single interaction and 12000 under multiple interaction, however, its effects on the four aspects of travel are smaller than those of an optimal toll in absolute values.

A piecewise-linear toll, presented in Table 8.12, tends to have lower effects in absolute values on peaking, congestion, and schedule delay than the optimal toll policy. The exception occurs to its effects on mode mix. Except for the least congested scenario, a piecewise-linear toll has a larger effect in absolute value on mode mix than an optimal toll. For example, the piecewise-linear toll under the

Table 8.11 Equilibrium Effects of A Base-Externality Toll[a]
for Selected Amounts and Elasticities of Congestion

(in percentage difference from those of an optimal toll)[f]

| | Number of Commuters[b] | | |
| | 12000 | 24000 | 48000 |
Single Interaction ($\gamma = 1.0$)[b]	(1) (2)	(3)	
Welfare			
(01) Consumer Surplus (%)[c]	1.38	-17.23	-24.98
(02) Toll Revenue (%)	1.35	18.14	27.74
(03) Total Benefits (%)	0.00	0.90[g]	2.76[g]
Toll			
(04) Peak Toll (%)[d]	3.76	85.13	130.40
(05) Average Toll (%)	1.34	41.58	66.53
Peaking			

(06) Peak Traffic (%)[d]	4.85	-14.26	-13.59
(07) Peak Share of Traffic (%)	5.83	-10.64	-9.49

Congestion

(08) Peak Travel Delay (%)[d]	0.00	-14.16	-13.44
(09) Average Travel Delay (%)	16.67	-9.70	-9.66

Schedule Delay

(10) Variable Schedule Delay for Auto Users[e] (%)	4.65	213.79	175.47
(11) Ratio of Variable Schedule to Travel Delay for Auto Users (%)	4.76	45.72	38.55

Mode Mix (measured with pcu's)

(12) Drive-Alone Share of Traffic (%)	1.02	3.64	-3.79
(13) Bus Share of Traffic (%)	0.00	3.80	-4.62
(14) Carpool Share of Traffic (%)	1.92	3.49	-4.44
(15) Average Occupancy (%)	0.00	9.09	-5.56
(16) Total Traffic (%)	1.07	2.23	-3.70

(Table 8.11 Continued)

	Number of Commuters[b]		
	12000	24000	48000
Multiple Interaction ($\gamma = 2.5$)[b]	(1) (2)	(3)	

Welfare

(17) Consumer Welfare (%)[c]	-4.91	15.89	41.93
(18) Toll Revenue (%)	-4.38	11.68	18.40
(19) Total Benefits (%)	1.75[g]	-1.38	2.27[g]

Toll

(20) Peak Toll (%)[d]	-6.61	35.44	11.39
(21) Average Toll (%)	-4.52	12.94	23.83

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Peaking

(22) Peak Traffic (%)[d]	-6.24	34.88	4.60
(23) Peak Share of Traffic (%)	-6.70	49.04	-6.23

Congestion

(24) Peak Travel Delay (%)[d]	-4.29	24.49	3.12
(25) Average Travel Delay (%)	-4.17	19.84	8.68

Schedule Delay

(26) Variable Schedule Delay for Auto Users[e] (%)	-4.62	36.19	25.66
(27) Ratio of Variable Schedule to Travel Delay for Auto Users (%)	-7.40	42.98	42.70

Mode Mix (measured with pcu's)

(28) Drive-Alone Share of Traffic (%)	-4.84	12.05	25.86
(29) Bus Share of Traffic (%)	-3.51	13.21	28.23
(30) Carpool Share of Traffic (%)	-4.48	10.98	23.15
(31) Average Occupancy (%)	-12.50	13.04	25.53
(32) Total Traffic (%)	-4.48	11.26	22.18

Notes to Table 8.11:

[a] An base-externality toll measures the marginal increase in total travel costs due to an additional passenger car trip at a given interval at the base case. Travel costs include costs of on-vehicle travel time, costs of bus walk and waiting time, costs of schedule delay, and auto operating and maintenance costs and bus agency costs.

[b] γ is a supply parameter measuring elasticity of travel delay with respect to arrival flow in the supply model (Table 7.1). Number of commuters is exogenously chosen to give a range of baseline congestion.

^c Consumer's surplus is change in consumer welfare in equation (5.16).

^d Peak refers to the 15-minute period centered around 8:00 A.M.

^e The average schedule delay tabulated is the variable component of average schedule delay; the constant component is above the variable component. The constant component is the average schedule delay of auto users in equilibrium when travel is free-flow.

^f A positive number implies a larger effect in absolute value from the policy of concern than the optimal toll; a negative smaller than 100 in absolute
value implies a smaller effect in absolute value in the same direction as the optimal toll; a negative larger than 100 in absolute value implies an effect in the opposite direction as the optimal toll; and a zero implies the same effect as the optimal toll.

^g These are likely due to numerical inaccuracy.

Table 8.12 Equilibrium Effects of A Piecewise-Linear Toll^a
for Selected Amounts and Elasticities of Congestion

(in percentage difference from those of an optimal toll)^f

	Number of Commuters^b		
	12000	24000	48000
Single Interaction (γ = 1.0)^b	(1)	(2)	(3)

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Welfare

(01) Consumer Surplus (%)[c]	-34.65	14.22	16.53
(02) Toll Revenue (%)	-34.27	12.73	13.95
(03) Total Benefits (%)	-16.67	-14.44	-9.42

Toll

(04) Peak Toll (%)[d]	-55.50	-11.90	-4.14
(05) Average Toll (%)	-34.93	13.76	15.80

Peaking

(06) Peak Traffic (%)[d]	-67.17	-31.34	-22.65
(07) Peak Share of Traffic (%)	-79.37	-54.20	-47.44

Congestion

(08) Peak Travel Delay (%)[d]	-68.42	-31.11	-22.81
(09) Average Travel Delay (%)	-66.67	-22.22	-14.58

Schedule Delay

(10) Variable Schedule Delay for Auto Users[e] (%)	-76.74	-54.84	-52.69
(11) Ratio of Variable Schedule to Travel Delay for Auto Users (%)	-78.17	-53.46	-47.50

Mode Mix (measured with pcu's)

(12) Drive-Alone Share of Traffic (%)	-33.67	16.36	19.32
(13) Bus Share of Traffic (%)	-32.61	20.25	21.54
(14) Carpool Share of Traffic (%)	-34.62	12.79	16.30
(15) Average Occupancy (%)	-28.57	18.18	22.22
(16) Total Traffic (%)	-31.76	17.62	19.45

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(Table 8.12 Continued)

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Number of Commuters[b]		
12000	24000	48000

Multiple Interaction ($\gamma = 2.5$)[b]	(1)	(2)	(3)
Welfare			
(17) Consumer Welfare (%)[c]	54.18	54.19	77.25
(18) Toll Revenue (%)	46.72	35.43	27.05
(19) Total Benefits (%)	-38.60	-21.81	-7.41
Toll			
(20) Peak Toll (%)[d]	-26.48	-9.30	-11.24
(21) Average Toll (%)	50.08	41.90	-38.00
Peaking			
(22) Peak Traffic (%)[d]	-50.30	-28.99	-24.19
(23) Peak Share of Traffic (%)	-73.68	-66.59	-77.06
Congestion			
(24) Peak Travel Delay (%)[d]	-45.71	-24.49	-18.38
(25) Average Travel Delay (%)	-37.50	-16.67	-5.72
Schedule Delay			
(26) Variable Schedule Delay for Auto Users[e] (%)	-61.54	-49.52	-38.82
(27) Ratio of Variable Schedule to Travel Delay for Auto Users (%)	-64.20	-42.49	-0.57
Mode Mix (measured with pcu's)			
(28) Drive-Alone Share of Traffic (%)	55.65	47.89	47.98
(29) Bus Share of Traffic (%)	64.91	54.09	53.15
(30) Carpool Share of Traffic (%)	47.76	42.77	42.43
(31) Average Occupancy (%)	62.50	47.83	51.06
(32) Total Traffic (%)	58.13	45.21	40.16

Notes to Table 8.12:
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[a] A piecewise-linear toll starts with zero at 7:00, increases linearly until 8:00, decreases linearly to zero at 9:00; its start and end points are chosen exogenously; its slope (the same on each side) is chosen through screening at a 5-cent increment to maximize total welfare.

[b] γ is a supply parameter measuring elasticity of travel delay with respect to arrival flow in the supply model (Table 7.1). Number of commuters is exogenously chosen to give a range of baseline congestion.

[c] Consumer's surplus is change in consumer welfare in equation (5.16).

[d] Peak refers to the 15-minute period centered around 8:00 A.M.

[e] The average schedule delay tabulated is the variable component of average schedule delay; the constant component is above the variable component. The constant component is the average schedule delay of auto users in equilibrium when travel is free-flow.

[f] A positive number implies a larger effect in absolute value from the policy of concern than the optimal toll; a negative smaller than 100 in absolute
value implies a smaller effect in absolute value in the same direction as the optimal toll; a negative larger than 100 in absolute value implies an effect in the opposite direction as the optimal toll; and a zero implies the same effect as the optimal toll.

Table 8.13 Equilibrium Effects of A One-step Toll[a]
for Selected Amounts and Elasticities of Congestion

(in percentage difference from those of an optimal toll)[f]

| | Number of Commuters[b] | | |
| | 12000 | 24000 | 48000 |
Single Interaction ($\gamma = 1.0$)[b]	(1) (2)	(3)	
Welfare			
(01) Consumer Surplus (%)[c]	-41.66	-14.74	-6.65
(02) Toll Revenue (%)	-41.57	-15.93	-9.08
(03) Total Benefits (%)	-37.50	-37.78	-31.16
Toll			
(04) Peak Toll (%)[d]	-64.40	-41.27	-30.98
(05) Average Toll (%)	-42.25	-16.43	-9.45
Peaking			
(06) Peak Traffic (%)[d]	-79.25	-62.01	-51.56
(07) Peak Share of Traffic (%)	-92.83	-85.88	-80.13
Congestion			
(08) Peak Travel Delay (%)[d]	-78.95	-62.22	-51.75
(09) Average Travel Delay (%)	-66.67	-44.44	-33.33
Schedule Delay			
(10) Variable Schedule Delay for Auto Users[e] (%)	-79.07	-61.29	-51.61
(11) Ratio of Variable Schedule to Travel Delays for Auto Users (%)	-79.26	-61.53	-49.68
Mode Mix (measured with pcu's)			

	12000 (1)	24000 (2)	48000 (3)
(12) Drive-Alone Share of Traffic (%)	-41.84	-14.55	-6.82
(13) Bus Share of Traffic (%)	-41.30	-12.66	-5.38
(14) Carpool Share of Traffic (%)	-42.31	-16.28	-8.89
(15) Average Occupancy (%)	-42.86	-9.09	-5.56
(16) Total Traffic (%)	-39.22	-12.52	-4.91

(Table 8.13 Continued)

	Number of Commuters[b]		
	12000	24000	48000
Multiple Interaction ($\gamma = 2.5$)[b]	(1) (2)	(3)	

Welfare

(17) Consumer Welfare (%)[c]	-6.52	20.23	24.27
(18) Toll Revenue (%)	-10.94	4.89	-3.56
(19) Total Benefits (%)	-62.28	-41.91	-22.65

Toll

(20) Peak Toll (%)[d]	-60.79	-36.51	-36.88
(21) Average Toll (%)	-11.05	6.15	-2.78

Peaking

(22) Peak Traffic (%)[d]	-78.45	-56.21	-46.70
(23) Peak Share of Traffic (%)	-94.74	-91.83	-89.78

Congestion

(24) Peak Travel Delay (%)[d]	-75.71	-50.44	-38.62
(25) Average Travel Delay (%)	-66.67	-37.30	-20.91

Schedule Delay

(26) Variable Schedule Delay for Auto Users[e] (%)	-73.85	-39.05	-13.82
(27) Ratio of Variable Schedule to Travel Delay for Auto Users (%)	-79.36	-45.28	-12.08

Mode Mix (measured with pcu's)

(28) Drive-Alone Share of Traffic (%)	-8.06	10.54	2.84
(29) Bus Share of Traffic (%)	-3.51	13.84	5.41
(30) Carpool Share of Traffic (%)	-11.94	7.51	0.00
(31) Average Occupancy (%)	-12.50	13.04	4.26
(32) Total Traffic (%)	-4.77	11.78	4.04

Notes to Table 8.13:

[a] A one-step toll starts at 7:30 A.M., stays constant, ends at 8:30 A.M.; its start and end points are chosen exogenously; its level is chosen through screening at a 10-cent increment to maximize total welfare.

[b] γ is a supply parameter measuring elasticity of travel delay with respect to arrival flow in the supply model (Table 7.1). Number of commuters is exogenously chosen to give a range of baseline congestion.

[c] Consumer's surplus is change in consumer welfare in equation (5.16).

[d] Peak refers to the 15-minute period centered around 8:00 A.M.

[e] The average schedule delay tabulated is the variable component of average schedule delay; the constant component is above the variable component. The constant component is the average schedule delay of auto users in equilibrium when travel is free-flow.

[f] A positive number implies a larger effect in absolute value from the policy of concern than the optimal toll; a negative smaller than 100 in absolute
value implies a smaller effect in absolute value in the same direction as the optimal toll; a negative larger than 100 in absolute value implies an effect in the opposite direction as the optimal toll; and a zero implies the same effect as the optimal toll.

most congested scenario reduces drive-alone share by 48 percent more than the optimal toll, increases bus and carpool shares by 53 and 42 percent, respectively, more than the optimal toll. As a result, it increases average occupancy by 51 percent more and reduces total traffic by 40 percent more than the optimal toll.

A one-step toll, presented in Table 8.13, leads to a smaller effect than the optimal toll on peaking, congestion, and schedule delay. That is, a one-step toll reduces peaking and congestion by less than the optimal toll, but increases variable schedule delay by more than the optimal toll. With the two higher number of commuters, however, a one-step toll has a larger effect in absolute value on mode mix than an optimal toll. That is, when congestion is light, a one-step toll decreases drive-alone share and total traffic and increases bus and carpool shares and average occupancy by less than the optimal toll. But when congestion is heavier, the opposite occurs. This may be no surprise because its effects relative to an optimal toll tend increase with the amount of congestion.

The effects of a uniform toll, presented in Table 8.14, on peak share of traffic for all scenarios and on variable schedule delay for the most congested scenarios are in the opposite direction of an optimal toll. That is, a uniform toll tends to increase peak share of traffic but decrease variable schedule delay, while an optimal toll reduces peak share but increases variable schedule delay. On the other hand, it tends to result in smaller effects in absolute values on other variables and scenarios except for its effect on mode mix under the most congested scenarios. A uniform toll tends to induce more shifts from drive-alone to bus and carpool when congestion is heavy.

Table 8.14 Equilibrium Effects of A Uniform Toll[a]
for Selected Amounts and Elasticities of Congestion

(in percentage difference from those of an optimal toll)[f]

| | Number of Commuters[b] | | |
| | 12000 | 24000 | 48000 |
Single Interaction ($\gamma = 1.0$)[b]	(1) (2)	(3)	
Welfare			
(01) Consumer Surplus (%)[c]	-18.16	-1.74	23.82
(02) Toll Revenue (%)	-18.67	-3.86	17.74
(03) Total Benefits (%)	-41.67	-44.44	-36.96
Toll			
(04) Peak Toll (%)[d]	-64.40	-53.01	-38.65
(05) Average Toll (%)	-19.06	-3.80	20.25
Peaking			
(06) Peak Traffic (%)[d]	-81.40	-72.80	-61.84
(07) Peak Share of Traffic (%)	-104.04	-108.02	-115.38

Congestion

(08) Peak Travel Delay (%)[d]	-78.95	-73.33	-61.40
(09) Average Travel Delay (%)	-83.33	-61.11	-45.83

Schedule Delay

(10) Variable Schedule Delay for Auto Users[e] (%)	-95.35	-95.16	-95.70
(11) Ratio of Variable Schedule to Travel Delay for Auto Users (%)	-96.07	-94.38	-91.70

Mode Mix (measured with pcu's)

(12) Drive-Alone Share of Traffic (%)	-17.35	-1.21	25.00
(13) Bus Share of Traffic (%)	-15.2	2.53	28.46
(14) Carpool Share of Traffic (%)	-19.23	-4.65	20.74
(15) Average Occupancy (%)	-14.29	0.00	27.78
(16) Total Traffic (%)	-14.23	1.12	25.37

(Table 8.14 Continued)

	Number of Commuters[b]		
	12000	24000	48000
Multiple Interaction ($\gamma = 2.5$)[b]	(1)	(2)	(3)

Welfare

(17) Consumer Welfare (%)[c]	33.15	51.28	149.15
(18) Toll Revenue (%)	24.56	24.57	39.95
(19) Total Benefits (%)	-73.68	-56.83	-35.05

Toll

(20) Peak Toll (%)[d]	-60.79	-50.12	-42.80
(21) Average Toll (%)	26.30	29.26	60.47

Peaking

(22) Peak Traffic (%)[d]	-80.78	-69.96	-58.57

(23) Peak Share of Traffic (%)	-106.46	-126.68	-173.07

Congestion

(24) Peak Travel Delay (%)[d]	-78.57	-65.01	-50.59
(25) Average Travel Delay (%)	-70.83	-57.14	-33.53

Schedule Delay

(26) Variable Schedule Delay for Auto Users[e] (%)	-96.92	-102.86	-115.79
(27) Ratio of Variable Schedule to Travel Delays for Auto Users (%)	-96.29	-92.73	-80.08

Mode Mix (measured with pcu's)

(28) Drive-Alone Share of Traffic (%)	31.45	32.45	79.97
(29) Bus Share of Traffic (%)	40.35	42.77	91.29
(30) Carpool Share of Traffic (%)	23.88	30.64	68.25
(31) Average Occupancy (%)	37.50	39.13	85.11
(32) Total Traffic (%)	35.53	35.85	64.82

Notes to Table 8.14:

[a] A uniform toll applies between 5:30 to 10:00 A.M.; its level is chosen through screening at a 10-cent increment to maximize total welfare.

[b] γ is a supply parameter measuring elasticity of travel delay with respect to arrival flow in the supply model (Table 7.1). Number of commuters is exogenously chosen to give a range of baseline congestion.

[c] Consumer's surplus is change in consumer welfare in equation (5.16).

[d] Peak refers to the 15-minute period centered around 8:00 A.M.

[e] The average schedule delay tabulated is the variable component of average schedule delay; the constant component is above the variable component. The constant component is the

average schedule delay of auto users in equilibrium when travel is free-flow.

[f] A positive number implies a larger effect in absolute value from the policy of concern than the optimal toll; a negative smaller than 100 in absolute
value implies a smaller effect in absolute value in the same direction as the optimal toll; a negative larger than 100 in absolute value implies an effect in the opposite direction as the optimal toll; and a zero implies the same effect as the optimal toll.

Finally the effects of an optimal toll with HOV exemption are discussed as presented in Table 8.15. Two patterns emerge from Table 8.14. First, an optimal toll with HOV exemption reduces consumer's surplus much less than an optimal toll alone. In fact, average consumer's surplus is increased under the most congested scenario considered. But the average toll revenue is less than half of that from the optimal toll under the same scenario. As a result, its total benefits are still 6.5 percent lower than those of the optimal toll. Second, it

tends to change mode mix more than an optimal toll alone.

Tables 8.16, 8.17, and 8.18 present changes in traffic counts from the base case by

mode and time of day for the optimal toll with HOV exemption, the one-step toll, and the

uniform toll, respectively, under the most congested scenario. The uniform toll reduces total

traffic more than the one-step toll by switching more drive-alone vehicles to bus and carpool.

The uniform toll reduces traffic in every 15-minute period. A one-step toll reduces traffic

only when the toll applies. Traffic increases before and after the toll. The optimal toll with

HOV exemption reduces traffic by 3370 pcu's, 2 percent more than under the optimal toll

alone. With a similar calculation as with Table 8.4, mode shifts account for 99.86 percent of

this traffic reduction. The figure for the optimal toll alone is 91.59 percent.

Table 8.15 Equilibrium Effects of An Optimal Toll with HOV Exemption[a]
for Selected Amounts and Elasticities of Congestion

(in percentage difference from those of an optimal toll)[f]

	Number of Commuters[b]		
	12000	24000	48000
Single Interaction ($\gamma = 1.0$)[b]	(1) (2)	(3)	
Welfare			
(01) Consumer Surplus (%)[c]	-24.61	-38.65	-27.06
(02) Toll Revenue (%)	-24.35	-37.27	-24.98

(03) Total Benefits (%)	-12.50	-10.00	-5.80

Toll

(04) Peak Toll (%)[d]	0.59	-23.70	6.56
(05) Average Toll (%)	-3.52	-19.82	0.93

Peaking

(06) Peak Traffic (%)[d]	-0.11	-21.35	3.05
(07) Peak Share of Traffic (%)	-4.93	-29.01	-5.45

Congestion

(08) Peak Travel Delay (%)[d]	0.00	-20.00	2.63
(09) Average Travel Delay (%)	0.00	-16.67	6.25

Schedule Delay

(10) Variable Schedule Delay for Auto Users[e] (%)	-11.63	-27.42	-15.05
(11) Ratio of Variable Schedule to Travel Delay for Auto Users (%)	-11.65	-28.54	-11.75

Mode Mix (measured with pcu's)

(12) Drive-Alone Share of Traffic (%)	44.90	20.00	57.58
(13) Bus Share of Traffic (%)	6.52	-12.66	10.77
(14) Carpool Share of Traffic (%)	78.85	51.16	101.48
(15) Average Occupancy (%)	14.29	0.00	22.22
(16) Total Traffic (%)	14.58	-5.52	19.56

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(Table 8.15 Continued)

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	Number of Commuters[b]		
	12000	24000	48000
Multiple Interaction ($\gamma = 2.5$)[b]	(1)	(2)	(3)

_____

Welfare

(17) Consumer Welfare (%)[c]	-29.16	-39.02	-102.96
(18) Toll Revenue (%)	-27.66	-31.16	-45.82
(19) Total Benefits (%)	-10.53	-7.35	-6.52

Toll

(20) Peak Toll (%)[d]	1.71	3.13	5.40
(21) Average Toll (%)	-7.04	-6.31	-19.59

Peaking

(22) Peak Traffic (%)[d]	-1.17	-2.11	-8.39
(23) Peak Share of Traffic (%)	-4.31	-10.10	-20.70

Congestion

(24) Peak Travel Delay (%)[d]	0.00	-1.75	-5.99
(25) Average Travel Delay (%)	0.00	0.79	-3.16

Schedule Delay

(26) Variable Schedule Delay for Auto Users[e] (%)	-15.38	-21.90	-45.39
(27) Ratio of Variable Schedule to Travel Delays for Auto Users (%)	-14.36	-16.17	-28.46

Mode Mix (measured with pcu's)

(28) Drive-Alone Share of Traffic (%)	48.39	56.63	55.01
(29) Bus Share of Traffic (%)	8.77	6.92	-3.60
(30) Carpool Share of Traffic (%)	82.09	102.31	112.46
(31) Average Occupancy (%)	12.50	17.39	8.51
(32) Total Traffic (%)	16.30	15.83	7.66

Notes to Table 8.15:

[a] An optimal Toll with HOV exemption is the marginal increase in total travel costs due to an

additional passenger car trip at a given time interval with carpooler and bus users exempted.

[b] γ is a supply parameter measuring elasticity of travel delay with respect to arrival flow in the supply model (Table 7.1). Number of commuters is exogenously chosen to give a range of baseline congestion.

[c] Consumer's surplus is change in consumer welfare in equation (5.16).

[d] Peak refers to the 15-minute period centered around 8:00 A.M.

[e] The average schedule delay tabulated is the variable component of average schedule delay; the constant component is above the variable component. The constant component is the average schedule delay of auto users in equilibrium when travel is free-flow.

[f] A positive number implies a larger effect in absolute value from the policy of concern than the optimal toll; a negative smaller than 100 in absolute
value implies a smaller effect in absolute value in the same direction as the optimal toll; a negative larger than 100 in absolute value implies an effect in the opposite direction as the optimal toll; and a zero implies the same effect as the optimal toll.

Table 8.16 One-step Toll and Change in Traffic Counts[a,b]

by Mode and Schedule: Schedule Shifts Allowed

$(\gamma = 2.5$, Number of Commuters = 48000$)^c$

Time (1)	Drive-Alone (2)	Bus[d] (3)	Carpool (4)	Traffic (5)
6:45	35.53	0.00	5.39	40.92
7:00	88.21	0.26	10.81	99.28
7:15	395.29	0.02	32.52	427.83
7:30	-1055.63	12.25	-21.06	-1064.44
7:45	-1035.79	9.10	58.33	-968.35
8:00	-886.80	147.16	58.94	-680.70
8:15	-914.51	24.99	1.26	-888.27
8:30	-842.12	67.38	-9.71	-784.45
8:45	208.43	0.59	15.69	224.71
9:00	48.65	4.74	4.65	58.04
9:15	6.82	0.03	0.82	7.66
Total	-3949.31	266.54	158.15	-3524.61

Notes to Table 8.16:

[a] A one-step toll starts at 7:30, stays constant, ends at 8:30; its start and end points are chosen exogenously; its level is chosen through screening at a 10-cent increment to maximize total welfare.

[b] Traffic is in passenger car units. The occupancies of carpool and bus are assumed at 2.5 and 40/3 per pcu, respectively.

[c] γ is a supply parameter measuring elasticity of travel delay with respect to arrival flow in the supply model (Table 7.1). Number of commuters is exogenously chosen to give certain level of baseline congestion. The most congested scenario is presented.

[d] Bus users are assumed to arrive on time.

Table 8.17 Uniform Toll and Change in Traffic Counts[a,b]
by Mode and Schedule: Schedule Shifts Allowed

($\gamma = 2.5$, Number of Commuters = 48000)[c]

Time (1)	Drive-Alone (2)	Bus[d] (3)	Carpool (4)	Traffic (5)
6:00	-13.85	0.00	0.41	-13.44
6:15	-62.61	0.00	0.89	-61.72
6:30	-118.59	8.31	0.64	-109.64
6:45	-229.70	0.64	-1.37	-230.43
7:00	-379.16	36.34	-0.37	-343.20
7:15	-629.21	2.61	-19.22	-645.82
7:30	-772.89	42.99	18.20	-712.40
7:45	-843.05	13.41	77.93	-751.70
8:00	-756.36	164.37	62.87	-529.12
8:15	-761.30	23.79	23.87	-713.64
8:30	-663.14	74.21	18.23	-570.70
9:45	-427.07	2.37	-1.99	-426.69
9:00	-400.92	53.57	6.21	-341.14
9:15	-91.17	1.51	2.78	-86.88
9:30	-47.16	6.12	1.18	-39.84
Total	-6203.61	429.54	190.54	-5583.52

Notes to Table 8.17:

[a] A uniform toll applies between 5:30 to 10:00 A.M.; its level is chosen through screening at a 10-cent increment to maximize total welfare.

[b] Traffic is in passenger car units. The occupancies of carpool and bus are assumed at 2.5 and 40/3 per pcu, respectively.

[c] γ is a supply parameter measuring elasticity of travel delay with respect to arrival flow in the supply model (Table 7.1). Number of commuters is exogenously chosen to give certain level of baseline congestion. The most congested scenario is presented.

Bus users are assumed to arrive on time.

Table 8.18 Optimal Toll with HOV Exemption and Change in Traffic Counts[a,b]
by Mode and Schedule: Schedule Shifts Allowed

(γ = 2.5 , Number of Commuters = 48000)[c]

Time	Drive-Alone	Bus[d]	Carpool	Traffic
(1)	(2)	(3)	(4)	(5)
6:30	5.93	0.04	0.77	6.74
6:45	23.30	0.01	2.73	26.03
7:00	-54.64	2.49	4.89	-47.25
7:15	-51.12	0.38	-12.63	-63.37
7:30	-493.61	11.43	42.48	-439.71
7:45	-986.96	6.04	189.95	-790.96
8:00	-1668.50	116.45	382.00	-1170.05
8:15	-464.45	10.45	57.02	-396.99
8:30	-692.48	36.74	83.13	-572.61
8:45	-28.98	0.53	10.11	-18.34
9:00	-260.05	15.92	17.71	-226.42
9:15	36.53	0.12	1.82	38.47
Total	-4628.50	200.78	780.58	-3647.14

Notes to Table 8.18:

[a] An optimal Toll with HOV exemption is the marginal increase in total travel costs due to an additional passenger car trip at a given time interval with carpooler and bus users exempted.

[b] Traffic is in passenger car units. The occupancies of carpool and bus are assumed at 2.5 and 40/3 per pcu, respectively.

[c] γ is a supply parameter measuring elasticity of travel delay with respect to arrival flow in the supply model (Table 7.1). Number of commuters is exogenously chosen to give certain level

of baseline congestion. The most congested scenario is presented.

[d] Bus users are assumed to arrive on time.

8.4. Miscalculation of Policy Effects

This section presents miscalculation of policy effects that would occur if schedule shifts are ignored. The policies of an optimal toll and a ten-percent capacity expansion are examined. The same scenarios are used as in preceding sections. This miscalculation for each policy is presented as percentage difference in its effects between constraining and not constraining schedule shifts.

Table 8.19 presents miscalculations of the effects of an optimal toll on average welfare, toll, peaking, congestion, schedule delay, and mode mix. Welfare measures and tolls are in cents for the morning commute, 1972 prices. Total benefits include toll revenue and change in consumer surplus. A positive 50 implies that ignoring schedule shifts overestimates the effect of an optimal toll in absolute value on the corresponding variable by 50 percent. Schedule shifts are constrained by equating the conditional probability of schedule choices to its base value at each iteration. Optimal tolls are recalculated.

(m1) <u>Are the effects of an optimal toll biased when schedule shifts are constrained</u>?

Constraining schedule shifts overestimates total benefits by a range from 92 to 371 percent

under single interaction, and from 42 percent to 108 percent under multiple interaction. This

significant overestimation on the benefits of an optimal toll contradicts ADL's (1990)

suggestion that conventional models of peak-period congestion substantially underestimate

total benefits of an optimal toll. This overestimation decreases with the amount of congestion

and elasticity of congestion.

Table 8.19 Percentage Miscalculation from Ignoring Schedule Shifts:[a]
Optimal Toll[b]

	Number of Commuters[c]		
	12000	24000	48000
	(1)	(2)	(3)
Single Interaction ($\gamma = 1.0$)[c]			
Welfare			
(01) Consumer Surplus	-19.82	-7.25	10.29
(02) Toll Revenue	-11.36	1.43	18.46
(03) Average Total Benefits[d]	370.83	168.89	92.03
Toll			
(04) Peak Toll[e]	-22.94	-1.72	21.53
(05) Average Toll	-11.78	1.25	19.51
Peaking			
(06) Peak Traffic[e]	-73.88	-63.32	-52.04
(07) Peak Share of Traffic	-94.17	-91.60	-90.38
Congestion			
(08) Peak Travel Delay[e]	-5.26	-13.33	-21.05
(09) Average Travel Delay	50.00	16.67	4.17

Schedule Delay

(10) Variable Schedule Delay for Auto Users[f]	-93.02	-93.55	-92.47
(11) Ratio of Variable Schedule to Travel Delay for Auto Users	-90.67	-87.49	-83.53

Mode Mix (measured with pcu's)

(12) Drive-Alone Share of Traffic	-14.29	-2.42	13.26
(13) Bus Share of Traffic	-15.22	-3.80	10.77
(14) Carpool Share of Traffic	-13.46	-1.16	14.81
(15) Average Occupancy	-14.29	0.00	11.11
(16) Total Traffic	-14.45	-3.77	10.80

(Table 8.19 Continued)

	Number of Commuters[c]		
	12000	24000	48000
	(1)	(2)	(3)

Multiple Interaction ($\gamma = 2.5$)[c]

Welfare

(17) Consumer Surplus	-15.50	13.28	81.96
(18) Toll Revenue	-5.50	24.77	58.38
(19) Average Total Benefits[d]	107.89	59.59	42.20

Toll

(20) Peak Toll[e]	-10.95	44.31	87.63
(21) Average Toll	-6.09	24.09	66.94

Peaking

(22) Peak Traffic[e]	-78.40	-57.22	-50.84
(23) Share of Peak Traffic	-91.15	-84.62	-100.50

Congestion

(24) Peak Travel Delay[e]	-35.57	-25.66	-23.95
(25) Average Travel Delay	-25.92	-16.67	-9.07

Schedule Delay

(26) Variable Schedule Delay for Auto Users[f]	-95.79	-94.29	-86.84
(27) Ratio of Variable Schedule to Travel Delay for Auto Users	-89.80	-74.55	-43.67

Mode Mix (measured with pcu's)

(28) Drive-Alone Share of Traffic	-16.13		-3.31		13.90	
(29) Bus Share of Traffic		-17.54		-6.29		7.51
(30) Carpool Share of Traffic	-14.93		-1.16		19.88	
(31) Average Occupancy		-25.00		-8.70		8.51
(32) Total Traffic		-17.29		-5.15		7.43

Notes to Table 8.19:

[a] Schedule shifts are constrained by equating the conditional probability of schedule choice (equation (5.12)) to its base value at each iteration.

[b] The optimal toll for a given case is the marginal increase in travel costs (including on-vehicle travel time, bus walk and waiting time, schedule delay, auto operating costs and bus fares) at the optimum where total welfare, sum of consumer welfare and toll revenue, is maximized.

[c] γ is a supply parameter measuring elasticity of travel delay with respect to traffic flow in the supply model (Table 7.1). Number of commuters is exogenously chosen to give a range of baseline congestion.

[d] Total benefits sum consumer surplus and toll revenue.

[e] Peak refers to the 15-minute period centered around 8:00 A.M.

[f] The average schedule delay tabulated is the variable component of average schedule delay; the constant component is above the variable component. The constant component is the

average schedule delay of auto users in equilibrium when travel is free-flow.

Constraining schedule shifts on an optimal toll tends to underestimate tolls when congestion is light, but overestimate tolls when congestion is heavy. Peak toll is underestimated by 23 percent under the least congested scenario; it is overestimated by 88 percent under the most congested scenario. Overestimation of an optimal toll is what one would expect from constraining schedule shifts because it is more costly to put an extra vehicle to travel at a given time when existing travelers are only allowed to change mode than when they are allowed to change schedule as well.

Constraining schedule shifts on an optimal toll substantially underestimates peaking spreading. Reduction in peak traffic is underestimated by a range from 51 to 78 percent.

Reduction in peak share is underestimated by a range from 85 to 100 percent. As a result of underestimating traffic reduction in the peak interval, savings in peak travel delay due to an optimal toll are underestimated by a range from 5 to 35 percent. Savings in average travel delay are overestimated under single interaction, but are underestimated under multiple interaction.

Constraining schedule shifts substantially underestimates effects of an optimal toll on schedule delay. This is what one would expect because an optimal toll induces peak spreading, which is prevented when schedule shifts are constrained.

Constraining schedule shifts underestimates total traffic and overestimates average occupancy by overestimating the shares of bus and carpool vehicles and underestimating the share of drive-alone. Because commuters can only shift modes, overestimating average occupancy is an expected result from constraining schedule shifts. For the least congested scenario, however, constraining schedule shifts results in overestimating total traffic; but the overestimation is small.

(m2) Are the effects of a ten-percent expansion biased when schedule shifts are constrained? The answer is obtained from Table 8.20, which presents similar information for the ten-percent expansion as Table 8.19 does for an optimal toll. The same expansion applies to all scenarios tabulated. Constraining schedule shifts overestimates total benefits under all scenarios; the overestimation ranges from 1 percent under the least congested scenario to 22 percent under the most congested scenario.

Constraining schedule shifts on an incremental expansion underestimates its effects on both peaking and variable schedule delay, but overestimates its effects on congestion and the ratio of schedule to travel delay. Since capacity expansion increases peaking, miscalculation predicts somewhat less peaking than really occurs. Since capacity expansion decreases travel delay, miscalculation predicts somewhat more travel delay than really occurs. The effect of constraining schedule shifts on mode mix is negligible, which could be because the ten-percent expansion has negligible effects on mode mix even when schedule shifts are not constrained (Table 8.4).

(m3) <u>What are the qualitative differences and similarities in these miscalculations between an optimal toll and an incremental expansion?</u> First, constraining schedule shifts overestimates total benefits for both policies. Second, constraining schedule shifts underestimates schedule delay for an optimal toll, but overestimates schedule delay for capacity expansion. Third, constraining schedule

Table 8.20 Percentage Miscalculation from Ignoring Schedule Shifts:[a]
Ten-Percent Expansion in Capacity[b]

	Number of Commuters[c]		
	12000	24000	48000
	(1)	(2)	(3)
Single Interaction ($\gamma = 1.0$)[c]			
Welfare			
(01) Average Total Benefits[d]	1.04	2.20	2.97

Peaking

(02) Peak Traffic[e]	-75.19	-73.31	-70.68
(03) Peak Share of Traffic	-100.00	-92.31	-100.00

Congestion

(04) Peak Travel Delay[e]	7.14	3.85	8.51
(05) Average Travel Delay	0.00	0.00	3.57

Schedule Delay

(06) Variable Schedule Delay for Auto Users[f]	-100.00	-100.00	-100.00
(07) Ratio of Variable Schedule to Travel Delay for Auto Users	2571.43	1900.00	1316.67

Mode Mix (measured with pcu's)

(08) Drive-Alone Share of Traffic	0.00		0.00		0.00
(09) Bus Share of Traffic		0.00		0.00	0.00
(10) Carpool Share of Traffic	0.00		0.00		0.00
(11) Average Occupancy		0.00		0.00	0.00
(12) Total Traffic		0.45		0.41	0.30

(Table 8.20 Continued)

	Number of Commuters[c]		
	12000	24000	48000
	(1)	(2)	(3)
Multiple Interaction ($\gamma = 2.5$)[c]			

Welfare

(13) Average Total Benefits[d]	6.85	16.41	22.06

Peaking

(14) Peak Traffic[e]	-86.73	-81.21	-69.13
(15) Peak Share of Traffic	-96.00	-96.36	-96.49

Congestion

(16) Peak Travel Delay[e]	12.90	31.96	56.25
(17) Average Travel Delay	7.69	22.22	32.52

Schedule Delay

(18) Variable Schedule Delay for Auto Users [f]	-100.00	-100.00	-105.00
(19) Ratio of Variable Schedule to Travel Delay for Auto Users	2607.89	897.67	486.00

Mode Mix (measured with pcu's)

(20) Drive-Alone Share of Traffic	0.00	14.29	13.64
(21) Bus Share of Traffic	0.00	14.29	4.17
(22) Carpool Share of Traffic	0.00	0.00	0.00
(23) Average Occupancy	0.00	0.00	0.00
(24) Total Traffic	3.83	6.00	4.96

Notes to Table 8.20:

[a] Schedule shifts are constrained by equating the conditional probability of schedule choice (equation (5.12)) to its base value at each iteration.

[b] Capacity expansion is exogenously chosen at 10 percent of base level. The same expansion applies to all scenarios.

[c] γ is a supply parameter measuring elasticity of travel delay with respect to traffic flow in the supply model (Table 7.1). Number of commuters is exogenously chosen to give a range of baseline congestion.

[d] Total benefits sum consumer surplus and toll revenue.

[e] Peak refers to the 15-minute period centered around 8:00 A.M.

[f] The average schedule delay tabulated is the variable component of average schedule delay; the constant component is above the variable component. The constant component is the average schedule delay of auto users in equilibrium when travel is free-flow.

shifts overestimates total traffic for an optimal toll, but does not miscalculate

mode mix for capacity expansion. Forth, constraining schedule shifts underestimates peaking

and congestion for capacity expansion, but overestimates them for an optimal toll under

multiple interaction and has mixed effects under single interaction.

8.5 Summary

This chapter has simulated the effects of eight capacity expansion and pricing policies. Two values of travel-delay elasticity with respect to arrival flow at work are considered. This section summarizes results for scenarios with travel-delay elasticity equal to 2.5. The corresponding range of baseline congestion is between one and nine minutes of travel delay.

I. Base Case

Urban commuters choose to arrive early or late for work by about 11 minutes even when there is no congestion. This result comes about in this model but not in abstract models because while abstract models allow difference in observed attributes such as travel time, this model allows difference in observed as well as unobserved attributes.

An average commuter's welfare drops by 57 to 95 cents in 1972 prices. This is equivalent to an annual loss of 454 to 756 dollars in 1992 prices per commuter for the morning commute alone. A significant portion of this welfare loss results from increases in schedule delay. Therefore, models that ignore schedule shifts underestimate the social costs of congestion.

II. Policy Effects

Optimal toll. The total benefits of an optimal toll can be substantial. With a baseline travel delay of nine minutes, the total benefits are 38 cents in 1972 prices per passenger for the morning commute alone. This is equivalent to annual total benefits of more than 14.5 million dollars in 1992 prices for a population of 48000 commuters for the morning commuting alone.

Significant welfare transfers also occur from commuters to the government. Forty-one percent of toll revenue comes from losses in consumer welfare when the baseline travel delay is nine minutes. The transfer becomes 91 percent when the baseline travel delay is one minute.

An optimal toll reduces peak one-hour traffic by about one-third when a baseline travel delay is nine minutes. Ninety-two percent of this reduction comes from mode shifts; the other 8.4 percent comes from schedule shifts. Overall, an optimal toll results in one-fifth increase in average occupancy and one-sixth reduction in total traffic with the same baseline congestion. What is notable is that an optimal toll cuts congestion by more than half, but at the same time increases schedule delay by three-fourths.

Other Pricing Policies. How do the total benefits of other pricing policies compare with those of an optimal toll? One surprise is that a base-externality toll achieves total benefits equivalent to those of an optimal toll. A base-externality toll is equal to the marginal externality of an additional trip at a given time at the base case, while an optimal toll measures the marginal externality at an optimum. An optimal toll

with HOV exemption achieves 89 to 93 percent of the total benefits of an optimal toll alone. Two features of this policy are worth pointing out. First, an optimal toll with HOV exemption reduces consumer welfare by much less than an optimal toll alone. In fact, average consumer welfare is even increased when the baseline travel delay is 9 minutes. Second, it encourages more carpooling than an optimal toll alone, but not bus use. This comes about because toll savings to bus users are much smaller than to carpoolers.

Relative to an optimal toll, a piecewise-linear toll achieves 63 to 91 percent of the benefits; a one-step toll achieves 38 to 77 percent of the benefits; a uniform toll achieves 26 to 66 percent of the benefits. These tolls result in less peak spreading and less congestion relief. These tolls, however, have bigger effects on mode mix than an optimal toll when congestion is heavy. That is, when congestion is heavy, these tolls shift more traffic from drive-alone to bus and carpool; average occupancy increases more, and total traffic is reduced more.

Capacity expansion also affects traffic peaking, schedule delay, and mode mix. The ten-percent incremental expansion leads to higher peak share, lower variable schedule delay, and higher share of drive-alone. These effects are just in the opposite of those from an optimal toll.

III. Miscalculation

Optimal toll. When schedule shifts are ignored, total benefits of an optimal toll are 42 to 108 percent higher. One factor of this overestimation is that increased costs of schedule

delay are excluded.

Ignoring schedule shifts also results in higher optimal tolls. Peak toll is 88 percent higher when the baseline congestion is nine minutes. This is because it is more costly to put an extra vehicle at a given time when only mode shift is allowed than when schedule shift is allowed as well.

Ignoring schedule shifts from an optimal toll results in less peak spreading. Reduction in peak traffic is 51 to 78 percent lower. Reduction in peak share is 85 to 100 percent lower.

Ignoring schedule shifts results in less congestion relief from an optimal toll. Savings in peak travel delay are 26 to 36 percent lower. Savings in average travel delay are 9 to 26 percent lower.

Ignoring schedule shifts from an optimal toll also results in lower total traffic and higher average occupancy by overestimating the shares of bus and carpool and underestimating the share of drive-alone.

Capacity expansion. Ignoring schedule shifts results in total benefits of an incremental capacity expansion 7 to 22 percent higher. This overestimation in total benefits comes from an underestimation in peaking but an overestimation in congestion relief. While reduction in peak share is 96 percent lower, savings in peak travel delay is 13 to 56 percent higher.

CONCLUSIONS

The dissertation seeks to understand how urban commuters adjust their schedules and

modes to congestion, as well as policy implications of this adjustment. The research has

focused on three objectives: (1) to simulate the effects of an optimal toll, capacity expansion,

and six other pricing policies; (2) to test hypotheses relating to schedule shifts in response to

congestion and policy changes; (3) to estimate biases in policy effects when schedule shifts

are ignored.

Each objective is considered separately. The chapter ends with suggestions for future

research.

I. Policy Effects

An optimal toll can achieve substantial benefits. With a baseline travel delay of nine

minutes, the total benefits are 38 cents per person in 1972 prices, for the morning commute.

This is equivalent to annual benefits of 14.5 million dollars in 1992 prices for the morning

commute of 48000 workers.

These benefits of an optimal toll, however, are accompanied by welfare transfers.

Forty-one percent of toll revenue comes from losses in consumer welfare when the baseline

travel delay is nine minutes. The transfer becomes 91 percent when the baseline travel delay

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is one minute.

An optimal toll reduces peak one-hour traffic by about one-third when the baseline travel delay is nine minutes. Mode shifts account for 92 percent of this reduction; schedule shifts account for the other 8.4 percent. Overall, an optimal toll results in one-fifth increase in average occupancy and one-sixth reduction in total traffic with the same baseline congestion. What is notable is that an optimal toll cuts congestion by more than half, but at the same time increases variable schedule delay by three-fourths.

How do the effects of other policies compare with those of an optimal toll? One surprise is that a base-externality toll achieves total benefits equivalent to those of an optimal toll. Both tolls measure the marginal externality of an additional trip at a given time. Both are calculated after existing traffic has adjusted its behavior in response to the added trip. The two tolls differ, however, in where the externalities are measured. A base-externality toll is measured at a base case where there is no policy. An optimal toll is measured at an optimum where the sum of consumer welfare and toll revenue is maximized.

An optimal toll with HOV exemption achieves 89 to 93 percent of the total benefits of an optimal toll alone when the baseline travel delay is one to nine minutes. Two features of this policy are worth pointing out. First, an optimal toll with HOV exemption reduces consumer welfare by much less than an optimal toll alone. In fact, average consumer welfare is even increased when the baseline travel delay is nine minutes. Second, it encourages more

carpooling than an optimal toll alone, but not bus use. This comes about because toll savings to bus users are much smaller than to carpoolers.

A ten-percent incremental expansion leads to higher peak share, lower variable schedule delay, and higher share of drive-alone. These effects are just the opposite of those from an optimal toll.

II. Hypotheses Testing

Schedule delay has variable and constant components. The constant component is the equilibrium level when there is no congestion or policy. The variable component changes with congestion and policies. The constant component exists in this model but not in abstract models. While abstract models allow differences in observed attributes such as travel time, this model allows differences in observed as well as unobserved attributes.

Urban commuters shift their schedules in response to congestion and policy changes. Peak share of traffic is reduced by 7.5 percent as average travel delay increases from one to nine minutes; that is, heavy congestion forces people away from the peak. The ten-percent capacity expansion reduces variable schedule delay by 19 and 10 percent when the baseline travel delay is one and nine minutes, respectively; that is, capacity expansion attracts people back to the peak. An optimal toll increases variable schedule delay by 310 and 78 percent when the baseline travel delay is one and nine minutes, respectively; that is, an optimal toll

discourages people from driving in the peak.

III. Miscalculation of Policy Effects

When schedule shifts are ignored, total benefits of an optimal toll are 42 to 108 percent higher. This overestimation results from two opposite biases. While schedule delay is downward biased, resulting in overestimation of benefits, travel delay is upward biased, resulting in underestimation of benefits.

Ignoring schedule shifts also results in higher optimal tolls. The peak toll is 88 percent higher when the baseline congestion is nine minutes, because an extra trip is more costly when only mode shift is allowed.

Ignoring schedule shifts from an optimal toll reduces peak spreading. With a baseline congestion of one to nine minutes, reduction in peak traffic is downward biased by 51 to 78 percent, and reduction in peak share is downward biased by 85 to 100 percent.

Ignoring schedule shifts results in less congestion relief from an optimal toll. With the same range of baseline congestion, savings in peak travel delay are downward biased by 36 to 24 percent, and savings in average travel delay are downward biased by 26 to 9 percent.

Ignoring schedule shifts from an optimal toll also results in lower total traffic and higher average occupancy by overestimating bus and carpool shares but underestimating the drive-alone share.

The total benefits of the ten-percent expansion are 7 to 22 percent higher when

schedule shifts are ignored, with a baseline travel delay of one to nine minutes. This overestimation in total benefits also results from two opposite biases. While reduction in variable schedule delay is downward biased by 100 percent, savings in peak travel delay are upward biased by 13 to 56 percent.

IV. Future Research

The simulation model has two major strengths. First, it allows behavioral differences not only in observed but also in unobserved characteristics. These unobserved characteristics lead to urban commuters choosing to arrive early or late even when there is no congestion. Second, it allows general distributions in individual values of times and in individual work-start times.

The simulation model, however, has a number of deficiencies that requires additional research. First, the key supply parameter, travel-delay elasticity with respect to arrival flow at work, is not empirically estimated. Effort should be made to collect data that relates travel delay on a segment of urban highway to flows leaving the segment. Second, the method used to calculate optimal tolls is ad hoc. It should be improved for efficiency and for robustness with higher values of travel-delay elasticities. Third, bus users should be allowed to shift schedules. Fourth, rather than assuming that commuters know the buildup and decay of congestion with certainty, future research should incorporate unreliability of schedules.

Equilibrium models of mode and trip scheduling with uncertainty would be useful to evaluate policies that provide traffic information to drivers.

Another deficiency in the current model is the exclusion of possible effects of mode shift on user inputs and agency costs. This deficiency has possibly caused an underestimation of the benefits of pricing policies relative to those of capacity expansion. The literature tends to show that increased use of buses lowers the sum of bus agency costs and costs of user inputs. But this conclusion does not take into account the possibility that increased operation in the peak period requires more bus drivers for the whole day. As a result, increased operation in the peak period may increase agency costs, and this increase may more than offset the savings in user costs. The current model should be improved to incorporate these possible effects. An improved model could be useful, for example, to evaluate a policy that allows a bus operator to use toll revenues from the same corridor to assist bus operation.

Recent interest in high-occupancy-vehicle buy-in (HOV buy-in) lanes creates another possibility. HOV buy-in lanes are tolled, but vehicles with certain level of occupancy are exempted (Fielding, 1993). Small's (1983b) treatment of HOV lanes can be adopted. The key is to allow low occupancy vehicles to choose between free and HOV buy-in lanes.

Another direction for future research would be to analyze the geographical effects of congestion pricing. What effects would congestion pricing have on land-use patterns? To answer this question would require the incorporation of congestion pricing into a spatial model that allows endogenous location of residence or employment or both. Such

geographical effects of congestion pricing are of great interest to transportation economists,

planners, and geographers.

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