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It is commonly assumed that it is impossible to perform calculations with a local nonrenormalizable field theory. Some authors[1] have therefore postulated that nonrenormalizable interactions cannot have physical significance. We propose to show by means of a mathematical model that such pessimism may be unwarranted.

Consider, for example, the propagator for a boson field. By use of a spectral representation this may be expressed as[2]

$$\Delta_F'(p^2) = \frac{1}{p^2+\mu^2-i\epsilon} + \int_{m_0^2}^{\infty} \frac{dm^2\rho(m^2)}{p^2+m^2-i\epsilon} . \qquad (1)$$

The spectral density function can be computed by use of perturbation theory for either a renormalizable theory or a nonrenormalizable theory. For a renormalizable theory there will be contributions to $\rho(m^2)$ having an asymptotic behavior for large m^2 as $\frac{1}{m^2}$, $\frac{1}{m^2}\ln m^2$, $\frac{1}{m^2}(\ln m^2)^2$ etc.

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[1]See, for example, Hans Bethe and Frederic de Hoffman, Mesons and Fields Vol. II (Row, Peterson, Evanston, Ill., 1955), p. 29.

[2]H. Lehman, Nuovo cimento 11, 342 (1954).

When such contributions are substituted into Eq. (1) they yield finite results.

For a typical nonrenormalizable theory one finds for $\rho(m^2)$ a series of terms that behave asymptotically as 1, m^2, m^4, etc. When terms having this behavior are substituted into Eq. (1) the integrals encountered do not exist and because of this either such theories have been abandoned or cutoff functions have been introduced.

We shall now exhibit two functions whose power-series expansions (in g^2) have just the properties we have described. It is our point of view that these functions are to be considered as mathematical models for the exact propagators of the two kinds of field theories we have been discussing. Consider

$$\Delta'_{F_R}(p^2) = \frac{1}{p^2+\mu^2-i\epsilon} + g^2 \int_{m_0^2}^{\infty} \frac{dm^2}{p^2+m^2-i\epsilon} \, \frac{1}{m^2[(1-g^2 \ln \frac{m^2}{M^2})^2+g^4]} \tag{2}$$

$$\Delta'_{F_U}(p^2) = \frac{1}{p^2+\mu^2-i\epsilon} + \frac{g^2}{M^2} \int_{m_0^2}^{\infty} \frac{dm^2}{p^2+m^2-i\epsilon} \, \frac{1}{[(1-g^2 \frac{m^2}{M^2})^2+g^4]} \,. \tag{3}$$

Both these functions are well defined for real, nonzero g^2 (an expression having roughly the form of Eq. (2) has been derived by one of us[3] starting from perturbation theory). The first expression has the characteristic properties of a renormalizable theory and the second corresponds to a nonrenormalizable theory. Each expression is a singular function of g^2 in the complex g^2 plane for those values of g^2 which cause the denominator to vanish for some m^2 in the region of integration. In each case these "branch curves" extend to the origin of the g^2 plane. The crucial point is that the first of the two integrals admits of an asymptotic expansion about $g^2 = 0$ while the second one does not. We note in passing that the wave function renormalization[2]

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[3] Peter J. Redmond, (submitted to Physical Review).

$$Z^{-1} = \lim_{p \to \infty} p^2 \Delta_F^{\,2} (p^2) \qquad (4)$$

is finite for both models. In neither case, however, is a coupling-constant expansion possible.

It seems reasonable to conjecture that all nontrivial field theories yield functions that are singular for vanishing coupling constant[4] and that the difference between renormalizable and nonrenormalizable theories is that the former permit asymptotic expansions in the neighborhood of $g^2 = 0$ for renormalized quantities. Furthermore, we surmise that in both theories it is possible to obtain spectral density functions that yield finite propagators. The spectral functions would be calculated by summing appropriate infinite sets of Feynman diagrams. We further conjecture that when suitable integral representations are found for higher-order (i.e., many-particle) propagators and vertex functions a similar situation will obtain.

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[4] Freeman J. Dyson, Phys. Rev. 85, 631 (1952).