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Generalized Storage Equations for Flood Routing with Nonlinear Muskingum Models

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Abstract

The nonlinear Muskingum model is a leading method for hydrologic routing. The efficiency of the nonlinear Muskingum model for routing of hydrograph outflow has been improved in recent years. This study introduces four Muskingum models with improved, generalized, nonlinear storage equations. The proposed models provide more degrees of freedom in fitting observed hydraulic data than other corresponding nonlinear Muskingum models and they have better predictive skill for river flow than other nonlinear Muskingum models. The accuracy of the proposed Muskingum models is herein demonstrated with examples.

Keywords Hydrologic routing method · Nonlinear Muskingum · Outflow hydrograph · Storage equation

1 Introduction

Floods are hazards that cause substantial losses worldwide. Flood control measures include riverine modifications, flood zoning, reservoir storage, flood forecasting, and prioritizing flood prone areas according to projects and flood sensitivity index maps. Flood routing is an important tool of flood forecasting (Singh and Scarlatos 1987; Tewolde and Smithers 2006).

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Flood routing methods can be divided into two main groups (1) hydraulic and (2) hydrological methods. The first method employs unsteady flow equations. High accuracy of flood estimation is an advantage of this method. On the other hand, hydraulic routing requires detailed and extensive information related to the geometry and other riverine characteristics and implies a substantial computational burden (Samani and Shamsipour 2004). In contrast, hydrological routing relies on the continuity and storage equations applied to a river reach. The Muskingum model is one of the hydrological routing methods which used for the first time for flood control in Muskingum River in Ohio, United States by McCarthy (1938).

Calibration and prediction are two steps in the application of the Muskingum method (Das 2004). In the first step the parameters of the Muskingum model are determined with application of observed data from the inflow and outflow hydrographs. In the second step the outflow hydrograph is obtained based on the inflow hydrograph and the Muskingum routing equations. There are linear and nonlinear variants of the Muskingum method. Yoon and Padmanabhan (1993) indicated a nonlinear Muskingum model is pertinent for flood routing whenever a nonlinear relation between storage, inflow, and outflow exits.

Several authors have reported nonlinear formulations of the Muskingum model. The first (NL1), second (NL2), third (NL3), and fourth (NL4) formulations of the nonlinear Muskingum model were respectively introduced by Chow (1959), Gill (1978), Easa (2014), and Bozorg-Haddad et al. (2015a). The main difference between these models concerns the parameterization of the storage equation. The number of parameters in the storage equations in the NL1, NL2, NL3, and NL4 models are three, three, four, and seven, respectively. The degrees of freedom of the NL4 model exceeds those of other models, which improves its predictive skill of observed flows based on performance criteria such as the sum of the squared deviations (*SSD*), the sum of the absolute value of the deviations (*SAD*), and difference between the peak of routed and actual flows (*DPO*) relative to the predictive skill of other routing methods.

The parameters of the nonlinear Muskingum models can be estimated by various techniques. The first of these estimates the parameters based on mathematical fitting algorithms such as the segmental-Least squares (S-LSQ) (Gill 1978), the nonlinear-Least squares (N-LSQ) (Yoon and Padmanabhan 1993), the Broyden Fletcher Goldfarb Shannon BFGS (Geem 2006), the Lagrange multiplier (LM) (Das 2004), the Nelder-Mead simplex (NMS) (Barati 2011), the Generalized Reduced Gradient (GRG) search (Barati 2013 and Hamedi et al. 2014). Globally optimal estimates is a key advantage of these methods (Geem 2011), provided that the initial guess of the optimal solutions are close enough to the global optimal solutions (Geem 2011).

The second class estimates the parameters relying on phenomenon-pattern algorithms. Those include pattern search (PS) (Tung 1985), the genetic algorithm (GA) (Mohan 1997), harmony search (HS) (Kim et al. 2001), particle swarm optimization (PSO) (Chu and Chang 2009), parameter-setting-free HS (PSF-HS) (Geem 2011), differential evolution (DE) (Xu et al. 2012), simulated annealing (SA) and shuffled frog leaping algorithm (SFLA) (Orouji et al. 2013), the modified honey bee mating optimization (MHBMO) (Niazkar and Afzali 2015), the hybrid of bat algorithm and particle swarm optimization (HBSA) (Ehteram et al. 2018), and Improved bat algorithm (IBA) (Farzin et al. 2018). These methods reach the global optimal solutions by means of a random search, although they are beset by the slow convergence to global optima by the computationally intensive nature of random search (Barati 2013).

A third class estimates the parameters with combination of the first and second class, such as hybrid GA and NMS (GA-NMS) (Barati 2013), GA and GRG (GA-GRG) (Easa 2013), HS with BFGS (HS-BFGS) (Karahan et al. 2013), and SFLA combination with NMS (SFLA-NMS) (Bozorg-Haddad et al. 2015b). Among these parameter-estimation methods those in the third class (i.e., combination methods) appear to be the most efficient in solving parameter estimation with the nonlinear Muskingum models (Easa 2014; Karahan et al. 2013; Barati 2013; Bozorg-Haddad et al. 2015b).

Most researchers have focused on improving the calculation of the storage equation in the nonlinear Muskingum model. Several researches have focused recently on altering the structure of the storage equation of the nonlinear Muskingum model with the aim of introducing greater flexibility in fitting observed hydrograph data. These efforts intended to improve the predictive skill of the nonlinear Muskingum routing model. Four types of nonlinear storage equations have been reported. This study introduces four new nonlinear Muskingum models that generalize structure of the previously proposed nonlinear storage equations. The next sections describe the new nonlinear Muskingum models with applications to three case studies.

2 Nonlinear Muskingum Models Equations

McCarthy (1938) proposed continuity and storage equations for flood routing. The continuity equation is:

$$\frac{dS}{dt} \approx \frac{\Delta S}{\Delta t} = I - O \tag{1}$$

In this equation, t denotes time, S, I, and O represent storage volume, inflow, and outflow in a river reach, respectively. The storage equation is as follows:

$$S = K[XI + (1-X)O] \tag{2}$$

Where K is a time-storage factor for a reach of river, X denotes the weight given to the inflow and outflow in the routing. It ranges between 0 and 0.3 for rivers (Mohan 1997; Geem 2006). The two parameters *(K* and *X)* must be determined. These parameters are obtained from observed inflow and outflow data in a river reach (Yoon and Padmanabhan 1993).

Chow (1959) proposed Eqs. (3) through (6) for the NL1 model relating the water depth (y) to inflow (I), outflow (O), and storage (S) in a river reach:

$$I = ay^n \tag{3}$$

$$O = ay^n \tag{4}$$

$$S_{in} = by^m \tag{5}$$

$$S_{out} = by^m \tag{6}$$

in which a, b, m, and n are model coefficients to be estimated. S_{in} and S_{out} denote respectively the storage at the upstream and downstream sections of a river reach. Equations (3)-(6) are combined to yield that the following expressions:

$$S_{in} = b\left(\frac{I}{a}\right)^{m/n} \tag{7}$$

$$S_{out} = b\left(\frac{O}{a}\right)^{m/n} \tag{8}$$

Chow (1959) introduced the Muskingum model NL1 described by Eq. (10) assuming S is calculated with Eq. (9):

$$S = XS_{in} + (1-X)S_{out} \tag{9}$$

$$S = K[XI^{\alpha} + (1-X)O^{\alpha}] \tag{10}$$

In Eq. (10) $K = \frac{b}{a^{\alpha}}$ and $\alpha = \frac{m}{n}$. Chow (1959) reported that the value of a for uniform flow in rectangular channels equals 0.6 with n and m respectively being equal to 5.3 and 1. In natural channels the value of a exceeds 0.6.

Gill (1978) applied a parameter β to Eq. (2) to create the model NL2, given by Eq. (11):

$$S = K[XI + (1-X)O]^{\beta} \tag{11}$$

β is a parameter for considering the effects of non-linearity between weighted flow and storage volume.

Easa (2014) introduced the Muskingum model NL3 described by Eq. (12):

$$S = K[XI^{\alpha} + (1-X)O^{\alpha}]^{\beta} \tag{12}$$

in which α and K are given by $\alpha = \frac{m}{n}$ and $K = \left(\frac{b}{a^{\alpha}}\right)^{\beta}$, respectively.

Bozorg-Haddad et al. (2015a) rewrote S_{in} and S_{out} as follows:

$$S_{in} = b\left(\frac{I}{a_1}\right)^{m/n_1} \tag{13}$$

$$S_{out} = b\left(\frac{O}{a_2}\right)^{m/n_2} \tag{14}$$

in which a_1 and n_1 represent the flow-depth profile at the upstream section of a river reach, and a_2 and n_2 represent the flow-depth profile at the downstream section of the river reach. Equation (15) describes the nonlinear model NL4:

$$S = K[X(C_1 I^{\alpha_1}) + (1-X)(C_2 O^{\alpha_2})]^{\beta} \tag{15}$$

where

$$K = b^{\beta}, \alpha_1 = \frac{m}{n_1}, \alpha_2 = \frac{m}{n_2}, C_1 = \left[\left(\frac{1}{a_1}\right)^{\frac{m}{n_1}}\right]^{\beta} \text{ and } C_2 = \left[\left(\frac{1}{a_2}\right)^{\frac{m}{n_2}}\right]^{\beta} \tag{16}$$

The NL4 model becomes the Muskingum model NL3 when $C_1 = C_2 = 1$, and $\alpha_1 = \alpha_2$. The Muskingum model NL4 has seven parameters (C_2, C_1, β, α_2, α_1, X, K) and, thus, more degrees of freedom than the other nonlinear Muskingum models.

3 Generalized Nonlinear Muskingum Models' Storage Equations

Nonlinear Muskingum models assume the volume of storage in the i^{th} time interval depends only on S_{in} and S_{out} at the i^{th} time interval. The calculation of the storage volume at the i^{th} time interval by the models NL1, NL2, NL3, and NL4 requires inflow and outflow values at the i^{th} time interval, and one must estimate 3, 3, 4, and 7 parameters for the aforementioned models, respectively.

There is some dependence between the storages at the i^{th} and $i + 1$st time intervals. For this reason, this study considers the interdependence of S_{in} at the i^{th} and $i + 1$st time intervals to construct generalized nonlinear Muskingum models seeking to improve the accuracy of the calculated storage and outflow hydrograph. These modified nonlinear Muskingum models are called the generalized NL1 model (GNL1), generalized NL2 model (GNL2), generalized NL3 model (GNL3), and generalized NL4 model (GNL4).

The GNL1 model writes storage in the routing reach at the time i, S_i, as a weighted combination of the storage at the upstream and downstream sections of the reach as follows:

$$S_i = X_1 S_{in,i} + X_2 S_{in,i+1} + (1 - X_1 - X_2) S_{out,i} \tag{17}$$

where:

$$S_{in,i} = b\left(\frac{I_i}{a}\right)^{m/n} \tag{18}$$

$$S_{in,i+1} = b\left(\frac{I_{i+1}}{a}\right)^{m/n} \tag{19}$$

$$S_{out,i} = b\left(\frac{O_i}{a}\right)^{m/n} \tag{20}$$

Substituting Eqs. (18)-(20) into Eq. (17) yields the GNL1 model:

$$S_i = K\left[X_1 I_i^{\alpha} + X_2 I_{i+1}^{\alpha} + (1 - X_1 - X_2) O_i^{\alpha}\right] \quad \text{(GNL1)} \tag{21}$$

X_1 and X_2 are weighting factors reflecting the importance of the inflows in the i^{th} and $i + 1$th time intervals, respectively. Calculation of S_i for the GNL1 model requires the values of I_i, I_{i+1}, O_i and four parameters.

The storage equations of the GNL2, GNL3, and GNL4 models are respectively given by Eqs. (22), (23), and (24):

$$S_i = K[X_1 I_i + X_2 I_{i+1} + (1 - X_1 - X_2) O_i]^{\beta} \quad \text{(GNL2)} \tag{22}$$

$$S_i = K\left[X_1 I_i^{\alpha} + X_2 I_{i+1}^{\alpha} + (1-X_1-X_2)O_i^{\alpha}\right]^{\beta} \quad (\text{GNL3}) \tag{23}$$

$$S_i = K\left[X_1\left(C_1 I_i^{\alpha_1}\right) + X_2\left(C_1 I_{i+1}^{\alpha_1}\right) + (1-X_1-X_2)\left(C_2 O_i^{\alpha_2}\right)\right]^{\beta} \quad (\text{GNL4}) \tag{24}$$

The storage volumes calculated with the GNL2, GNL3, and GNL4 models at the i^{th} and $i+1$st time intervals with Eqs. (22), (23), and (24), respectively, requires I_i, I_{i+1}, O_i and estimation of four, five and eight parameters in the GNL2, GNL3, and GNL4 models, respectively.

4 Estimation of the Generalized Nonlinear Muskingum Models Parameters

Simulation and optimization processes are applied to estimate the parameters of the proposed generalized nonlinear Muskingum models. The next sections present the simulation and optimization processes.

5 Simulation Process of the Generalized Nonlinear Muskingum Models

This study employs the flood-routing method of Tung (1985) to simulate the flood routing with the generalized nonlinear Muskingum models. This method was applied by Geem (2006). The observed inflow data, calculated outflow data, and calculated storage at the i^{th} time interval are denoted by I_i, $\hat{O}_i$, and S_i, respectively, where $i = 0, 1, \ldots, I$ indicates the simulation time intervals. The steps of GNL1 model are as follows (similar steps apply to the other three generalized nonlinear Muskingum models).

Step 1: Estimate the hydrologic parameters (K, X_1, X_2, and α).

Step 2: Calculate the initial storage (S_0) with using Eq. (25) which assumes that the initial outflow equals the initial inflow ($\hat{O}_0 = I_0$):

$$S_i = K\left[X_1 I_0^{\alpha} + X_2 I_1^{\alpha} + (1-X_1-X_2)\hat{O}_0^{\alpha}\right] \quad i = 0 \tag{25}$$

Step 3: Calculate the rate of change in the storage volume at the i^{th} time interval, ($\frac{\Delta Si}{\Delta t}$) with Eq. (26):

$$\frac{\Delta Si}{\Delta t} = I_i - \left\{\left[\frac{1}{(1-X_1-X_2)}\right]\left(\frac{S_i}{K}\right) - \left[\frac{X_1}{(1-X_1-X_2)}\right]I_i{}^{\alpha} - \left[\frac{X_2}{(1-X_1-X_2)}\right]I_{i+1}{}^{\alpha}\right\}^{\frac{1}{\alpha}} i = 0, 1, \ldots, I-1 \tag{26}$$

Step 4: Calculate the storage at the i^{th} time interval, (S_i), with Eq. (27):

$$S_i = S_{i-1} + \Delta t\left(\frac{\Delta S_{i-1}}{\Delta t}\right) \quad i = 1, 2, \ldots, I \tag{27}$$

Step 5: Calculate the river-reach outflow at the i^{th} time interval, ($\hat{O}_i$) with Eq. (28):

$$\hat{O}_i = \left\{\left[\frac{1}{(1-X_1-X_2)}\right]\left(\frac{S_i}{K}\right)-\left[\frac{X_1}{(1-X_1-X_2)}\right]I_i^{\alpha}-\left[\frac{X_2}{(1-X_1-X_2)}\right]I_{i+1}{}^{\alpha}\right\}^{\frac{1}{\alpha}} \quad i = 1, 2, \ldots, I \tag{28}$$

In previous studies the application of I_{i-1} was more common than I_i in Eq. (28).

Step 6: Repeat steps (3) to (5) until the completing the simulation.

Equation (29) was chosen as the objective function to estimate the parameters of the four generalized nonlinear Muskingum models.

$$Min.SSD = \sum_{i=1}^{N}\left(O_i-\hat{O}_i\right)^2 \quad i = 0, 1, 2, \ldots, I \tag{29}$$

where SSD is sum of the square deviations between the observed and calculated outflow at the i^{th} time interval, and O_i is the observed outflow at the i^{th} time interval. Other objective functions herein considered minimize the sum of the absolute deviations (SAD) between the observed and calculated outflow at the i^{th} time interval, or minimize the difference between the routed and observed peak (DPO) stream flow. These are given by:

$$SAD = \sum_{i=1}^{I}|O_i-\hat{O}_i| \quad i = 0, 1, 2, \ldots, I \tag{30}$$

$$DPO = |O_P-\hat{O}_P| \tag{31}$$

where O_P is the value of peak in the observed outflow; and $\hat{O}_P$ is the value of peak in the routed outflow. Thus, the SSD is the main objective function in parameter estimation of the generalized nonlinear Muskingum models, and the SAD and DPO are alternative objective functions whose minima are also found.

Equation (32) calculated the absolute deviation (AD) between the observed and calculated outflows for i^{th} time interval.

$$AD = |O_i-\hat{O}_i| \tag{32}$$

6 Optimization Algorithm

This study implements a hybrid SFLA-NMS method introduced by Bozorg-Haddad et al. (2015b) for the parameters estimation of all models. The SFLA-NMS method involves two stages. The first stage of the SFLA calculates the parameters that are applied to the nonlinear

Muskingum models. The optimal parameters are estimated with the nonlinear Muskingum model in the second stage relying of the predictions made by the nonlinear Muskingum model in stage one.

7 Results and Discussions of the Generalized Nonlinear Muskingum Models

The proposed models were applied with three different case studies (smooth single peak hydrograph, real routing example, and multi modal example). Some features of the routing examples are: (1) considering nonlinear relationship between *S* and [*XI* + *(1-X) O*] in all three case studies, (2) using the different nonlinear Muskingum models based on the previous studies, and (3) comparison of the proposed models' performances with the previous results obtained from these examples.

8 Case Study 1: Smooth Single-Peak Hydrograph

The inflow and outflow hydrographs of Wilson (1974) were applied as a first case study, in which the number of time steps, *I,* and the duration of the simulation time step, Δt, equal 21 and 6 h, respectively. This case study is experimental example and is used extensively in the literature as a benchmark problem. Figure 1 depicts a comparison between the observed and calculated hydrographs with the models: (a) GNL1, (b) GNL2, (c) GNL3 and (d) GNL4 from the first case study. In this Figure the calculated hydrographs of nonlinear model are also presented. Results

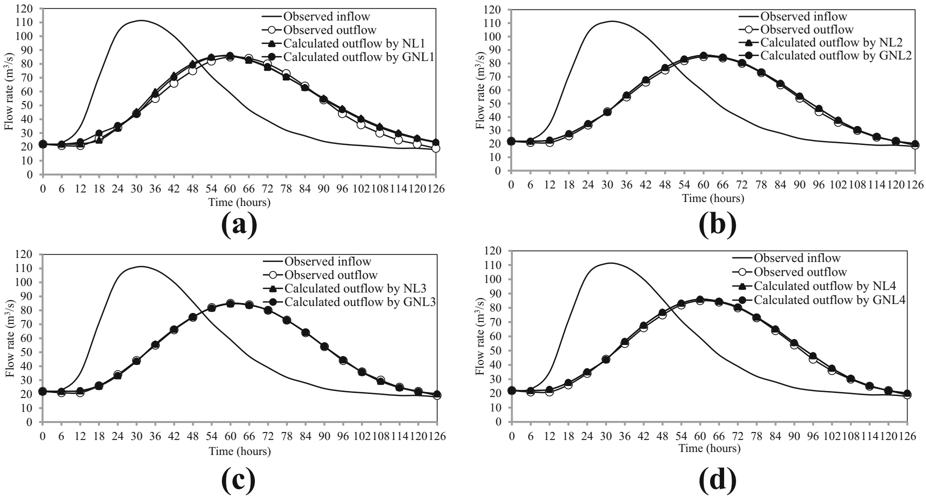

Fig. 1 Comparison of the observed and calculated hydrographs with the different models: **a** GNL1 and NL1, **b** GNL2 and NL2, **c** GNL3 and NL3, and **d** GNL4 and NL4 in the first case study

Table 1 Comparison of the optimal values of the *SSD*, *SAD*, and *DPO* calculated with the different models for the first case study

Model	K	X_1	X_2	α	α_1	α_2	β	C_1	C_2	*SSD*	*SAD*	*DPO*
NL1	0.46	0.229	–	1.50	–	–	–	–	–	245.58	60.69	0.55
GNL1	0.70	0.409	−0.154	1.43	–	–	–	–	–	183.34	54.84	1.05
NL2	0.52	0.287	–	–	–	–	1.865	–	–	36.77	23.47	0.90
GNL2	0.70	0.349	0.021	–	–	–	1.800	–	–	34.01	23.55	1.04
NL3	0.83	0.296	–	0.43	–	–	4.079	–	–	7.67	10.31	0.31
GNL3	0.91	0.322	−0.024	0.45	–	–	3.863	–	–	7.41	10.31	0.31
NL4	0.48	0.0833	–	–	0.70	0.425	3.822	0.619	0.735	5.44	6.69	0.05
GNL4	0.79	0.024	0.005	–	0.80	0.371	4.371	1.00	1.00	4.81	6.52	0.03

show the calculated hydrograph with the proposed models has a good accordance with the observed hydrograph.

Table 1 lists the values of the objective functions (*SSD*, *SAD*, and *DPO*) obtained with the nonlinear Muskingum (Bozorg-Haddad et al. 2015b), and the generalized nonlinear Muskingum models (this study). According to Table 1, the values of the *SSD* and *SAD* in the GNL1 model are, respectively, 25 and 10% lower (better) than those of the NL1 model (the GNL1 model has lower accuracy than the NL1 model with respect to DPO). The *SSD* value obtained with the GNL2 Model is 8% lower (better) than that of the NL2 model (the GNL2 model has less accurate than the NL2

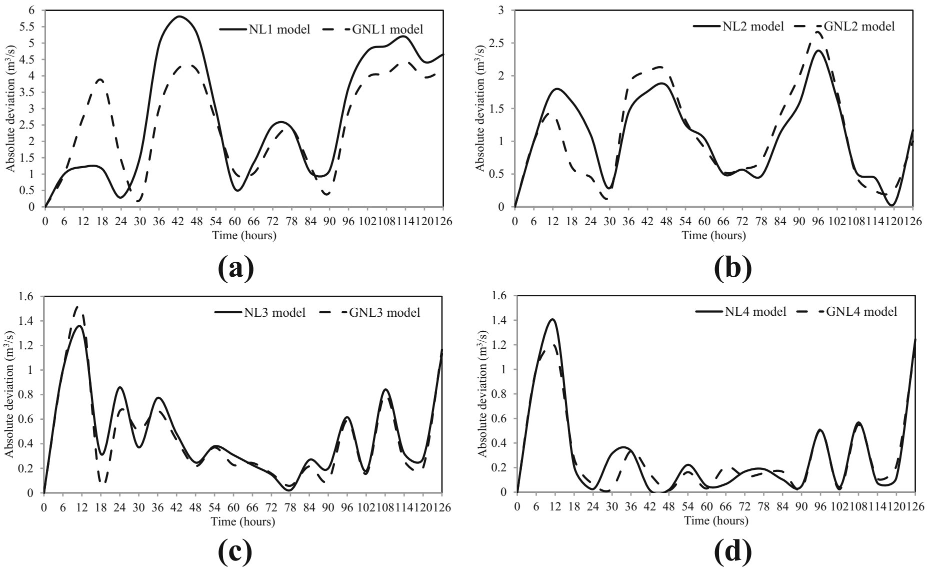

Fig. 2 Comparison of the *AD* values calculated with different models: **a** NL1 and GNL1, **b** NL2 and GNL2, **c** NL3 and GNL3, and **d** NL4 and GNL4 in the first case study

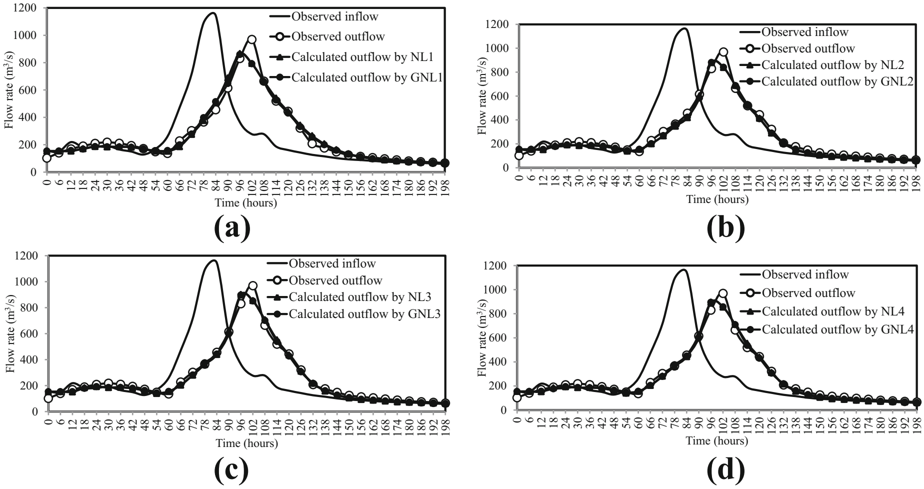

Fig. 3 Comparison of the observed and calculated hydrographs with the different models: **a** GNL1 and NL1, **b** GNL2 and NL2, **c** GNL3 and NL3, and **d** GNL4 and NL4 in the second case study

model with respect to the SAD and DPO). The values of the *SSD*, *SAD*, and *DPO* of the GNL3 model are respectively, 3, 7, and 27% lower (better) than those of the NL3 model, and the values of the *SSD*, *SAD* and *DPO* of the GNL4 model are, respectively, 11, 2, and 40% lower (better) than those of the NL4 model. Results show the GNL4 model produced the best values of the objective functions. By increasing the degrees of freedom in the model the objective functions were reduced, a desirable trait.

The absolute deviation (*AD*) was determined between the observed and calculated outflows is depicted in Fig. 2 for i^{th} time interval. Figure 2 displays the superior fitting capacity of the generalized nonlinear Muskingum models compared to that those of the corresponding nonlinear Muskingum models (NL1, NL2, NL3, NL4). Based on Fig. 2 the generalized nonlinear Muskingum models estimates the *AD* values more accurately than those of the corresponding nonlinear Muskingum models.

Table 2 Comparison of the optimal values of the *SSD*, *SAD*, and *DPO* calculated with the different models for the second case study

Model	K	X_1	X_2	α	α_1	α_2	β	C_1	C_2	*SSD*	*SAD*	*DPO*
NL1	0.52	0.347	–	1.30	–	–	–	–	–	55,548	839	97
GNL1	2.14	0.415	−0.091	0.34	–	–	–	–	–	49,559	773	109
NL2	0.08	0.415	–	–	–	–	1.59	–	–	34,789	793	90
GNL2	0.69	0.465	−0.046	–	–	–	1.52	–	–	33,911	759	87
NL3	0.44	0.404	–	1.20	–	–	1.33	–	–	32,299	743	76
GNL3	0.64	0.465	0.052	0.587	–	–	1.35	–	–	30,090	716	72
NL4	0.60	0.609	–	–	1.056	1.16	1.40	1.00	1.00	30,894	732	73
GNL4	1.56	0.656	−0.091	–	0.99	1.13	1.37	1.04	0.621	28,853	705	73

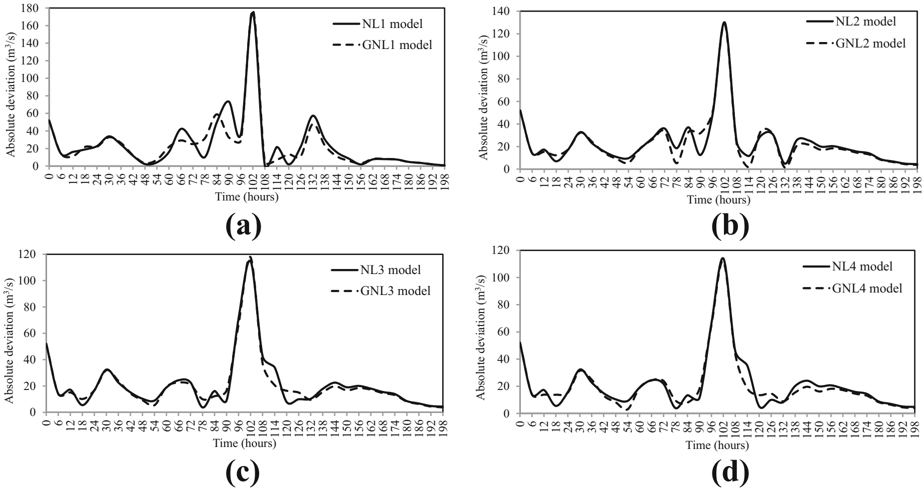

Fig. 4 Comparison of the *AD* values calculated with different models: **a** NL1 and GNL1, **b** NL2 and GNL2, **c** NL3 and GNL3 and **d** NL4, and GNL4 in the second case study

9 Case Study 2: Non-smooth Single-Peak Hydrograph

The second case study is about a 1960 flood in the Wye River in the United Kingdom (1 [Natural Environment Research Council (NERC), 1975]. This reach concerned was treated as a single reach despite its considerable length (O'Donnell et al. 1988). The 69.75-km reach of the RiverWye from Erwood to Belmont has no tributaries and very small lateral inflow (Karahan et al. 2013). This case study features $I = 33$ and $\Delta t = 6$ h. The comparison of the observed and calculated hydrographs with the generalized nonlinear Muskingum models and nonlinear Muskingum models of this case study is depicted in Fig. 3, which shows the calculated hydrographs with the generalized nonlinear Muskingum models are in good agreement with the observed hydrograph.

Table 2 lists the values of the objective functions obtained with the different models. It is clear that the use of the generalized nonlinear Muskingum models improves the fit to the observed outflows than to the corresponding nonlinear Muskingum models. The values of the *SSD* obtained with the GNL1, GNL2, GNL3 and GNL4 models are 11, 3, 7, and 7% lower than those calculated with the NL1, NL2, NL3 and NL4 models, respectively. Also, the value of the *SAD* of the GNL1 model is 8% lower (better) than that of the NL1 model (the GNL1 has lower accuracy than the NL1 model with respect to the DPO). The values of the *SAD* and *DPO* of the GNL2 model are respectively, 4 and 3% lower (better) than those from the NL2 model. Moreover, the values of the *SAD* and *DPO* from the GNL3 model are 4 and 5% lower (better) than those calculated with the NL3 model, respectively. The calculated *SAD* value of the GNL2 model is 4%lower (better) than that of the NL4 model (*DPO* estimates with the GNL4 and NL4 models are almost equal). In addition, it is evident from Table 2 the GNL4 model exhibits the best (smallest) values of the objective functions in the second case study.

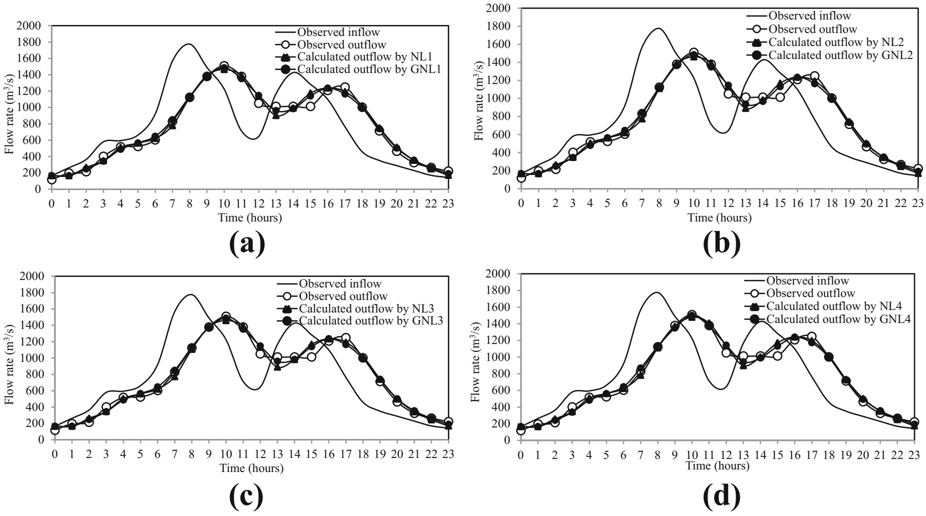

Fig. 5 Comparison of the observed and calculated hydrographs with the different models: **a** GNL1 and NL1, **b** GNL2 and NL2, **c** GNL3 and NL3, and **d** GNL4 and NL4 in the third case study

The generalized nonlinear Muskingum models' ability to fit the data better than the corresponding nonlinear Muskingum models is illustrated with a comparison between the *AD* values calculated with different models: (a) NL1 and GNL1, (b) NL2 and GNL2, (c) NL3 and GNL3 and (d) NL4 and GNL4 displayed in Fig. 4. These values demonstrate that the generalized nonlinear Muskingum models achieved better data fitting than the corresponding nonlinear Muskingum models.

10 Case Study 3: Multiple-Peak Hydrograph

The third case study is a flood hydrograph with multiple peak introduced by Viessman and Lewis (2003). This case study employs $I = 23$ and $\Delta t = 1$ h. The input hydrographs of the inflow and outflow are shown in Fig. 5. Figure 5 depicts a comparison between the observed and calculated hydrographs by the models: (a) GNL1, (b) GNL2, (c) GNL3 and (d) GNL4 in the third case study. In this

Table 3 Comparison of the optimal values of the *SSD*, *SAD*, and *DPO* calculated with the different models for the third case study

Model	K	X_1	X_2	α	α_1	α_2	β	C_1	C_2	*SSD*	*SAD*	*DPO*
NL1	0.070	0.153	–	1.46	–	–	–	–	–	74,307	1042	55
GNL1	1.922	0.519	−0.277	1.25	–	–	–	–	–	55,338	929	23
NL2	0.077	0.167	–	–	–	–	1.45	–	–	73,399	1037	56
GNL2	2.218	0.544	−0.272	–	–	–	1.23	–	–	56,670	953	25
NL3	0.077	0.167	–	0.92	–	–	1.57	–	–	73,379	1033	56
GNL3	0.326	0.525	−0.277	1.23	–	–	1.02	–	–	55,331	930	23
NL4	0.007	5*E-6	–	–	3.12	1.42	1.00	1.00	1.00	69,861	934	51
GNL4	0.473	0.063	−0.033	–	1.80	1.18	1.004	0.083	0.874	52,469	890	10

Figure the calculated hydrographs of nonlinear model are also presented. As shown, the calculated hydrographs have a good accordance with the observed hydrograph.

A comparison of the values of the objective functions obtained from corresponding original and generalized models in this case study is presented in Table 3, where it is read that the objective functions calculated with the generalized models are better than those calculated with the corresponding original models.

The *AD* values calculated with the two set of models (i.e., original nonlinear and generalized nonlinear) are shown in Fig. 6. Results demonstrate the generalized models achieved better data fitting than the corresponding original models.

11 Concluding Remarks

This study introduced the generalized nonlinear Muskingum models with more parameters than previous nonlinear storage equations. The generalized models involve a more complex calibration process than the corresponding nonlinear Muskingum models. Yet, the further complexity is compensated by a significant improvement in the quality of flood routing. A hybrid estimation method can decrease the further complexity of the model calibration. Based on this study, the SFLA-NMS method can be successfully used to estimate optimal parameter values with different generalized nonlinear Muskingum models. The performances of the generalized nonlinear Muskingum models were compared with those of the corresponding nonlinear Muskingum models by means of three case studies. A comparison of the calculated results demonstrated that overall superior data-fitting capacity of the generalized nonlinear Muskingum models for flood routing relative to the original nonlinear Muskingum models.

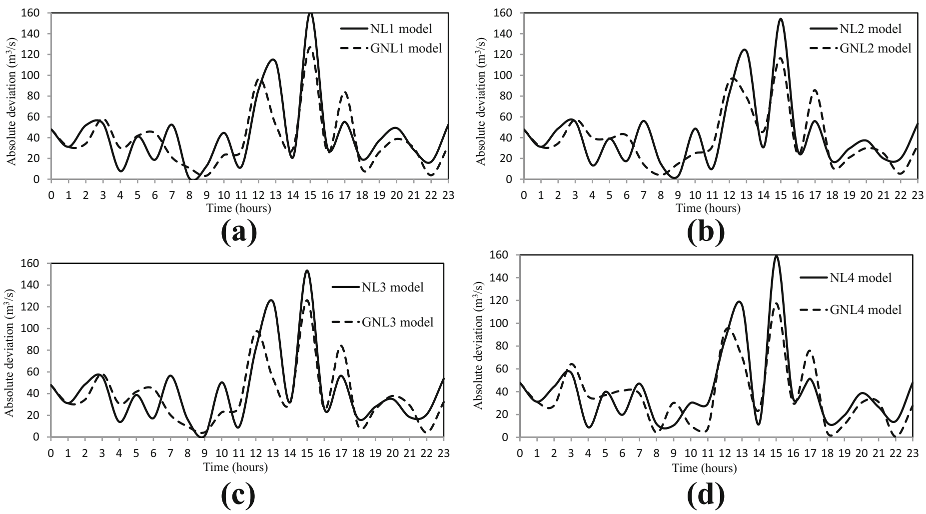

Fig. 6 Comparison of the *AD* values calculated with different models: **a** NL1 and GNL1, **b** NL2 and GNL2, **c** NL3 and GNL3, and **d** NL4 and GNL4 in the third case study

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Compliance with Ethical Standards

Conflict of Interests None.

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