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A MICROSCOPIC DESCRIPTION OF INELASTIC SCATTERING. APPLICATION TO NICKEL ISOTOPES*

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Detailed calculations of inelastic α-α' scattering on even single-closed-shell nuclei are in progress. The ingredients are the distorted wave Born approximation, a finite range interaction between the projectile and the nucleons of the target, and a microscopic description of the states of the nucleus. In the usual method of analyzing inelastic alpha scattering one adopts the Bohr-Mottelson description of the nuclear states[1)]. We shall refer to this as the macroscopic picture. On the other hand excitation of states differing only in the coordinates of a single nucleon has also been considered[2)]. In recent years progress has been made towards a unified microscopic description of all nuclear states (collective or not) and the present work uses the corresponding nuclear wave functions[3-4)]. We recall briefly that they were calculated from a finite range interaction (of gaussian shape and with an exchange term) whose effects were taken into account in two steps. First the Bogolyubov-Valatin canonical transformation was calculated from this interaction to extract its pairing effects. In the second step the residual interaction between the quasi-particle (q.p.), responsible for the collective effects, was taken into account by diagonalization of the same interaction in

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the sub-space of two q.p. configurations corresponding to the major (here neutron) unfilled shell (Tam-Dancoff two q.p. approximation). In fact, at the second step, the more complicated equations of the random phase approximation[3] were also solved but their solutions did not differ significantly from the two q.p. diagonalization. In other words (for single closed shell nuclei and for the parameters used) the correlations taken into account by the RPA were quite negligible and the ground state was found to be simply the BCS vacuum of pairs. However, by solving the RPA (including the so-called exchange terms) one is able to isolate the spurious 0+ state introduced by the non-conservation of the particle number. This separation turns out to be crucial for the 0+ states. In the case of double closed shell nuclei (where the B.V. transformation is not necessary) Sanderson and Wall[5] have already used Gillet's wave functions[6] to calculate the alpha excitation to the 3- levels of Ca^{40}.

The results presented here concern the nickel isotopes. From the theoretical point of view it has already been stressed[4] that these isotopes are not the most favorable for a precise calculation and indeed the fit with the experimental spectra is not very good. Among other uncertainties the neglect of the inner shells (particularly the $1f_{7/2}$ sub-shell) is here certainly a rather crude approximation. On the other hand the experimental study of the Ni isotopes by alpha scattering has been intensive[7] (and very fruitful) so we can compare our results to experimental ones and also to some other theoretical calculations[8] assuming a vibrational description of the nuclear states.

In the distorted wave (first) Born approximation the inelastic scattering amplitude for excitation of the state J of an even nucleus (J denotes all the necessary quantum numbers except M) is given by the matrix element between the distorted waves in the incident and outgoing channels of the quantity

$$(2J+1)^{1/2}\langle\psi_J^M(A)|V(\vec{r}\cdot A)|\psi_0(A)\rangle \equiv \mathcal{F}_J(r)Y_J^{M*}(r), \quad (1)$$

where r refers to the projectile Equation (1) defines the form factor $\mathcal{F}_J(r)$, once a nuclear model and an interaction V between the projectile and the nucleons of the target have been chosen. In the macroscopic picture the interaction of the projectile with the nucleus is through a deformed one-body potential. The corresponding form factors $\mathcal{F}_J(r)$ for λ-pole deformations are then proportional to

$$\beta_\lambda R \frac{\partial V}{\partial r} \quad \text{and} \quad \frac{1}{2}(\beta_\lambda R)^2 \frac{\partial^2 V}{\partial r^2} \quad (2)$$

for one and two phonons states respectively. (β_λ is the so-called deformation parameter.) Both refer to the direct excitation of the final state.

In the present microscopic description the incident particle is assumed to act through a gaussian force $V_0 \exp(-\beta r^2)$ with the nucleons of the nucleus. The nuclear ground state is the BCS vacuum of pairs and, as already mentioned, <u>all</u> the excited states are considered as two q.p. configuration mixings (with amplitudes η^J_{ab}). The resulting form factor is

$$\mathcal{F}_J(r) = -\sum_{a\leq b} \eta^J_{ab} \frac{u_a v_b + (-)^J u_b v_a}{(1+\delta_{ab})^{1/2}} \mathcal{F}_J^{(a,b)}(r) \quad (3)$$

where the index "a" stands for "$n_a \ell_a j_a$." Only such terms appear which satisfy parity and angular momentum conservation between a, b and J. The coefficients of the B-V canonical transformation u_a and v_a are such that $u_a^2 + v_a^2 = 1$ and $u_a v_a (-)^{\ell_a} > 0$. Here $\mathcal{F}_J^{(a,b)}$ is the form factor for the single particle transition $b \rightarrow a$. Following a method developed earlier[2], a simple closed form can be obtained for it when the bound states are described by harmonic oscillator functions as is usual in nuclear structure calculations. Consequently we obtain, where $\nu = m\omega/\hbar$

$$\mathcal{F}_J(r) = -V_0 \exp\left(\frac{-\beta\nu r^2}{\beta + \nu}\right) \sum_{m=0}^{2N} d_m \left(\frac{\beta r}{\sqrt{\nu}}\right)^{2m+J} \qquad (4)$$

Here N is the oscillator quantum number of the highest shell used in the structure calculation. We will describe how to get the coefficients d in a full report. The closed form of (4) is of course very useful in the calculation of the cross section where it is involved in many radial integrals.

We have computed the form factors of many excited states of all the even isotopes of nickel from mass 58 to 66. For any variation of ν and β (within reasonable limits at least) the qualitative features of the form factors remain unchanged. In fact this has also been found to be the case for the tin isotopes. The qualitative shape and the magnitude of the form factors depend very much on the configuration mixing of the corresponding state; more precisely on the coherence (or incoherence) of this mixing for the process involved [see eq. (3)]. In the figure we give a sample of our results. The form factor of the lowest 2+ level is always larger than that of any of the other levels. Its shape is roughly that corresponding to the macroscopic one phonon shape. The differences between our description of the 2_2^+, 4_1^+, and 0_2^+ states (0_1^+ denotes the ground state) and the vibrational one have already been emphasized[4]. In spite of this, the 2_2^+ form factor has roughly the two-phonon shape. The 0_2^+ has a form factor with two nodes having no precise counterpart in the macroscopic picture; still it corresponds crudely to the two-phonons shape. But the 4_1^+ level has a very different form factor corresponding more to a single four-pole phonon state than to a state of two quadrupole phonons. The macroscopic form factors are much sharper than those shown in the figure, and are centered at r=6.35.

While there is an interesting qualitative similarity between the form factors of the collective states obtained in the macroscopic and microscopic descriptions respectively, we find little quantitative agreement. Therefore it appears that caution should be exercised when interpreting experimental results in terms of the transition moment, β.

As to an exploration of the details of the form factors exhibited in the figure, alpha particles are not suitable projectiles because of their strong absorption. For example, the calculated angular distributions for exciting the 2_1 and 2_2 levels of Ni^{62} are the same within the accuracy of experimental measurements, although the form factors are completely different inside $r \simeq 6$ F. Beyond this radius they are roughly proportional to each other accounting for the similar angular distributions. The magnitudes of the cross section are in the ratio of the square of the proportionality constant. That the scattering of alpha particles is sensitive only to the outside region accounts for the observed absence of variety in the angular distributions. For this reason the alpha scattering experiments are very useful in determining the parities, and often the spins of excited states.

By way of contrast, nucleons are much less strongly absorbed, and therefore inelastic nucleon scattering will depend more sensitively on the detailed structure of the form factors. Consequently it seems that such experiments may be very useful in investigating correlations in the nucleus which express themselves in the variety of shapes possessed by the form factors.

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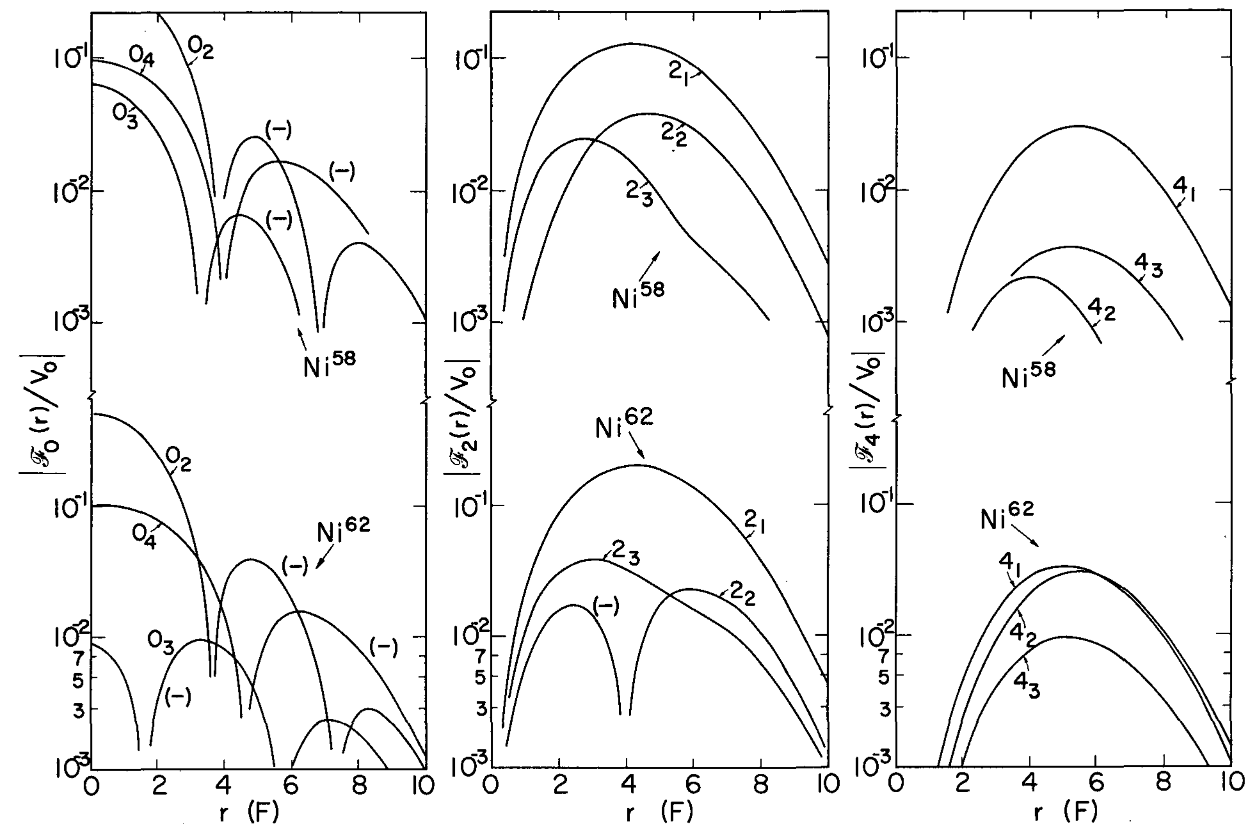
Figure Caption

Fig. 1. Form factors for α-scattering on some levels of $Ni^{58,62}$. Those for nucleon scattering are similar except they do not extend to quite so large radii, and are a little sharper in their details. Some of the form factors oscillate but since the plot is logarithmic we show this by a (-) sign.



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