Testing by Competitors in Enforcement of Product Standards

Erica L. Plambeck and Terry A. Taylor

Graduate School of Business, Stanford University

Haas School of Business, University of California, Berkeley

June 2015

Firms have an incentive to test competitors' products to reveal violations of safety and environmental standards, in order to have competitors' products blocked from sale. This paper shows that testing by a regulator crowds out testing by competitors, and can reduce firms' efforts to comply with the product standard. Relying on competitor testing (i.e., having the regulator test only to verify evidence of violations provided by competitors) is effective for industries with a moderate number of competitors, each with a strong brand and high quality, and for product standards that greatly benefit consumers (such as safety and energy-efficiency performance standards). Under those conditions, firms' compliance efforts and the detection-probability for a violation tend to be high. In contrast, firms with low quality do not draw testing from competitors, and so do not comply. Enforcing a product standard through competitor testing encourages entry by such low-quality, noncompliant firms and reduces quality investment by incumbents, if the standard provides negligible consumer benefit (i.e., is motivated by public welfare or environmental concerns). Stripping offending products of labels (such as "Energy Star"), instead of blocking them from the market, eliminates the problem of entry by low-quality, noncompliant firms, but may reduce incumbents' compliance efforts.

1. Introduction

This paper derives insights from a game-theoretic model of firms' efforts to comply with a product standard, and to test competitors' products for violations.

The model is motivated by the important role of competitor testing in enforcing Restrictions on Hazardous Substances (RoHS), energy efficiency performance standards, and product safety standards. Since 2002, the E.U. has restricted the use of lead, mercury, cadmium and several other hazardous substances in electronics. In 2002, Dutch authorities halted the sale of Sony PlayStation consoles because a peripheral cable contained cadmium, reputedly in response to a tip from one of Sony's competitors (Hess 2006), causing Sony to miss $110 million in revenue (Shah and Sullivan 2002). Since that incident, testing by competitors has become increasingly common (Green Supply Line 2006, Smith 2008). Since 2004, the E.U. has restricted the use of a growing number of hazardous substances in an increasingly wide variety of products, and has required

1

products to be labeled with the manufacturer's identity, which facilitates testing by competitors and helps regulators prevent the sale of products that violate safety standards; these regulations have substantially increased the annual number of products blocked from E.U. markets (Croft and Strongman 2004, European Commission 2012). In consumer products industries in the U.S. as well as the E.U., manufacturers commonly test competitors' products and report safety violations (Ross 2007, Kapoor 2012). Historically, the U.S. and E.U. have relied to large extent on testing by competitors in enforcement of energy efficiency performance standards (Weil and McMahon 2005, Gaffigan 2007, Department of Energy 2010).

An important feature of these motivating examples is that, through expenditure and effort, a firm can reduce the likelihood that its product will violate a standard, but not to zero. Risk of a violation arises from operational challenges, notably the difficulties of supply chain management to ensure that all components of a product meet design specification; see the first paragraph of the literature review below. Risk of a violation may also arise from uncertainty regarding the interpretation of a product standard.[1] Therefore, in the model, a firm cannot be certain that its product is compliant; each firm chooses how much to spend to reduce its risk of a violation.

For government regulatory authorities, testing to detect a violation is costly and difficult because there are very many potential failure modes (specific ways in which a product might fail to meet the standard) to be tested.[1] For example, to determine whether or not one personal computer is RoHS-compliant would require disassembly and testing of approximately 3,000 constituent materials for each of the six restricted substances, which would cost regulatory authorities as much as $200,000 (Bruschia 2008).[2]

In contrast, when a firm identifies *how* a competitor's product violates a standard and reports that information to the regulator, the regulator can cheaply and reliably verify that the product is noncompliant (Bruschia 2008, Smith 2008). Regulators follow up on credible reports that specify precisely how a product violates the standard and are well-supported by testing data (Bruschia 2008, Smith 2008). Why aren't regulators flooded with false or nonspecific reports of violations? Only a correct, specific report enables the regulator to verify noncompliance. Providing fraudulent information to a regulatory authority in order to damage a competitor is illegal in the E.U., U.S.

---

[1]U.S. energy efficiency standards require a product to have power consumption below a specified threshold in a variety of different operating modes and ambient conditions. A firm can identify additional failure modes–not addressed by those detailed specifications–that cause a competitor's product to have poor energy efficiency in actual use; Whirlpool did so, motivating the U.S. Department of Energy to prevent its competitor LG from selling certain models of refrigerator-freezers under the Energy Star label (Vestel 2009, GAO 2010, Brown 2012).

[2]The E.U.'s REACH regulation is expanding the number of restricted substances–from six under RoHS to 161 under REACH as of June 2015. In addition, REACH requires that all chemicals in a product be clearly identified and registered. Thus, REACH is multiplying the number of possible failure modes for a product. Whereas RoHS applies to electronics, REACH applies to all products.

and many other countries; in the U.S., for example, the penalties for doing so include fines and imprisonment for up to five years (see Subsection 18 of U.S. Code § 1001(a)). Therefore, in the model, each firm decides how much to spend on testing each competitor's product. A firm submits evidence to the regulator that a competitor's product is noncompliant if and only if its testing produces that evidence. The evidence characterizes the mode by which the product fails to meet the standard and, upon obtaining that evidence, the regulator verifies that the product is noncompliant and prevents its sale.

In testing a product to detect a violation of a product standard, competitors tend to be more efficient than the regulator, for the following reasons. Firms that provide products typically have equipment and trained staff for testing the compliance of their own products, and so can test competitors' products at little additional cost (Wiel and McMahon 2005, Gaffigan 2007, Smith 2008). They often purchase competitors' products to evaluate other quality characteristics, which reduces their procurement cost to test compliance (Day 2007). Through their own compliance efforts, firms develop a better understanding than the regulator of when and how a competitor's product is most likely to be noncompliant, and thus can better target their testing efforts (Hess 2006). For example, a firm's understanding of a competitor's suppliers' reputation and capabilities can provide insight into which components or constituent materials are likely to cause noncompliance. Unlike firms, government regulatory authorities lack a profit motive for efficiency in testing, and their budget for testing must be raised through taxes that distort the economy and reduce social welfare (Polinsky 1980).

Literature

Conformance-quality (i.e., product conformance to design specification) effort and inspection effort are key operational decisions, which reduce the probability a product is defective. An extensive operations management literature addresses variants of conformance-quality and inspection effort including, for example, statistical process control (Porteus and Angelus 1997) and contractual incentives for suppliers in conjunction with inspection of suppliers' output (Baiman et al. 2000, Balachandran and Radhakrishnan 2005, Babich and Tang 2012), auditing of suppliers' conformance-quality capability (Hwang et al. 2006), and investment in suppliers' conformance-quality capability (Zhu et al. 2007). The stylized model in this paper is most similar to those in (Baiman et al. 2000, Balachandran and Radhakrishnan 2005, Hwang et al. 2006, Babich and Tang 2012), wherein the supplier's entire output is either defective or conforming, and inspection effort increases the probability of detecting defective output. "Compliance" effort in this paper represent all actions, including conformance-quality and inspection effort, which reduce the probability a product vio-

lates the standard. This paper extends the operations management literature–which focuses on inspection of one's own or one's suppliers' products–by incorporating inspection of *competitors'* products.

Literature surveyed in (Cohen 1999) models how a *regulator* chooses effort to detect a violation, in game theoretic equilibrium with a potential violator; here, we highlight works most closely related to this paper. In (Mookherjee and Png 1992), the probability a victim reports a violation increases with its severity, which is chosen by the violator. The regulator chooses the likelihood that she will verify a victim's report and prosecute the violator. The regulator also chooses costly effort to directly detect a violation. Mookherjee and Png find that the regulator should *only* engage in verification (exert zero detection effort) when the cost of verification is low and the victim reports the violation with high probability. Boyer et al. (2000) examine how the stringency of a standard affects equilibrium compliance effort and detection effort. In contrast to (Mookherjee and Png 1992), wherein the regulator is the first mover and seeks to maximize social welfare, in Boyer et al. (2000) and other more recent works, the regulator moves simultaneously with the potential violator and maximizes its own utility, e.g., the expected fine less cost of effort to detect violation. Complementing this literature, this paper analyzes the equilibrium in firms' compliance and competitor-testing efforts, considering the regulator's testing effort as a parameter for sensitivity analysis. One may interpret that parameter as an initial commitment as in (Mookherjee and Png 1992) or as the level of regulator testing anticipated by the firms.

The literature on whistleblowing models whistleblowing by firms in cartels (see Spagnolo 2008, Bigoni et al. 2012, and papers surveyed therein) and by employees (Austen-Smith and Feddersen 2008, Ting 2008). Whereas that literature focuses on a potential whistleblower's decision *whether or not to report* a violation to a regulator, this paper focuses on firms' *efforts to detect* competitors' violations; in the setting of this paper, reporting a detected violation is optimal. The whistleblowing literature and this paper are consistent in assuming that any report of a violation is correct, and that the regulator always acts in response to the report.

Other related literature considers reporting to *consumers* rather than to a regulator. Voldman and Wiseman (2007) model costly effort by an activist nongovernmental organization to accurately inform consumers about the quality of a firm's product. In (Li and Peeters 2014), one firm chooses whether or not to incur a cost to learn the quality-level of a competitor's product and whether or not to report that information to consumers. Like the whistleblowing literature and this paper, Li and Peeters (2014) assume that any report is correct. However, reporting that the competitor's product is defective would motivate the competitor to set a lower price, which may deter a firm

from doing so. The "labeling" version of our model in §4 also represents a situation in which firms test the quality of competitors' products and, upon finding a defect, report the defect to consumers; a firm always benefits from reporting a competitor's defect because firms' quantities are set prior to that reporting, unlike in the price-competition-with-unlimited-capacity model in (Li and Peeters 2014).

Overview of Main Results

The first part of §3 shows that testing by a regulator crowds out testing by competitors and can reduce compliance. Specifically, a low level of testing by the regulator fails to increase firms' compliance efforts or the detection probability for a noncompliant product (i.e., the probability that, in the event that a product violates the standard, the regulator or a competitor detects that violation). It simply causes the firms to do less testing. A high level of testing by the regulator causes firms not to test, and can strictly reduce firms' compliance efforts. These results, combined with the observation that firms can test competitors' products at lower cost and more effectively than the regulator, suggest that social welfare might be improved by relying on competitor testing.

Therefore, the second part of §3 identifies conditions under which competitor testing alone is effective in enforcing a product standard. The detection probability for a noncompliant product initially increases and then decreases with the number of competitors. Each firm's compliance effort decreases with the number of competitors, and increases with its product quality and with consumers' benefit from compliance. Firms with low quality do not draw testing from competitors, and so do not comply.

§4 shows that these results hold in extensions of the model, with endogenous production quantities, fixed costs of testing, and alternative penalties for noncompliance. In addition, §4 shows that enforcing a product standard through competitor testing encourages entry by low-quality, noncompliant firms. It reduces quality investment by incumbents, if consumers' benefit from compliance is low (as for environmental, as opposed to safety standards). Informing consumers that a product violates the standard, instead of blocking the product from the market, eliminates the problem of entry by low-quality, noncompliant firms, but reduces incumbents' compliance efforts. Increasing fines stimulates compliance effort but, above a threshold, reduces the detection probability for a noncompliant product.

2. Model

N firms compete in a market, governed by a product standard. Each firm $n \in \mathcal{N} \equiv \{1, .., N\}$ makes an effort $e_n \in [0, \overline{e})$ to comply with the standard and incurs a cost $c_n(e_n)$ that is positive, strictly increasing and strictly convex with $\lim_{e_n \uparrow \overline{e}} [c_n(e_n)] = \infty$, where $\overline{e} \in (0, 1)$. It produces

quantity q_n of a product, which is compliant with probability e_n. The firm does not know with certainty whether its product is actually compliant; instead the firm knows this likelihood e_n. Then each firm n tests the product of competitor-firm m at level $t_{nm} \geq 0$, for $m \in \mathcal{N}\backslash m$, and incurs a cost $\Sigma_{m\in\mathcal{N}\backslash m}t_{nm}$. The regulator tests the product of firm n at level $t_{Rn} \geq 0$ for $n \in \mathcal{N}$, and incurs a cost $\Sigma_{n\in\mathcal{N}}t_{Rn}$. Let $\mathbf{t}_n = <t_{1n}, t_{2n}, .., t_{n-1,n}, t_{n+1,n}, .., t_{Nn}, t_{Rn}>$ denote the vector of testing levels. If firm n's product is noncompliant, then with probability $d_n(\mathbf{t}_n)$ testing provides evidence of this noncompliance to at least one party that tested firm n at a strictly positive level. A firm submits evidence to the regulator that a competitor's product is noncompliant if and only if its testing produces that evidence. The evidence characterizes the mode by which the product fails to meet the standard and, upon obtaining that evidence, the regulator confirms that the product is noncompliant and prevents its sale. Thus, with probability

$$s\left(e_n, \mathbf{t}_n\right) \equiv 1 - d\left(\mathbf{t}_n\right)\left(1 - e_n\right)$$

firm n successfully brings quantity q_n to market; with probability $d\left(\mathbf{t}_n\right)\left(1 - e_n\right)$ firm n sells nothing. That is, the sales quantity for firm $n \in \mathcal{N}$ is the random variable

$$\underline{q}_n \equiv \begin{cases} q_n & \text{with probability } s\left(e_n, \mathbf{t}_n\right) \\ 0 & \text{with probability } 1 - s\left(e_n, \mathbf{t}_n\right). \end{cases}$$

The detection probability for a noncompliant product $d_n(\mathbf{t}_n)$ is componentwise strictly increasing, continuously differentiable and satisfies $d_n(0, .., 0) = 0$, $d_n(\mathbf{t}_n) \leq \overline{d}$ for $\mathbf{t}_n \in R_+^{N+1}$, and $\lim_{t_{Rn}\to\infty} d_n(\mathbf{t}_n) = \overline{d}$, where $\overline{d} \in (0,1)$. Furthermore, to the extent that the detection probability for firm n is already high, additional testing is less effective: for $m \in \mathcal{N}\cup R$ and $n \in \mathcal{N}\backslash m$,

$$(\partial/\partial t_{mn})d_n(\mathbf{t}_n) < (\partial/\partial t_{mn})d_n(\mathbf{t}'_n) \text{ for } \mathbf{t}_n, \mathbf{t}'_n \in R_+^{N+1} \text{ such that } d_n(\mathbf{t}_n) > d_n(\mathbf{t}'_n). \tag{1}$$

The product of firm n has vertically-differentiated quality u_n, which may also represent brand strength. The market contains a unit mass of consumers with quality valuation parameter α uniformly distributed on $[0,1]$. A consumer with quality valuation α who purchases product n at price p_n has utility

$$\alpha u_n - p_n \tag{2}$$

in the event that the product is not compliant with the product standard, and has additional utility

$$\alpha u_n \Delta \tag{3}$$

in the event that the product is compliant, where $\Delta \geq 0$. We refer to Δ as a consumer's benefit from compliance.

Consumers form expectations regarding firm n's compliance effort $\widetilde{e}_n$, the testing applied to firm n by its competitors and the regulator $\widetilde{\mathbf{t}}_n$, and the resulting probability $\widetilde{e}_n/s\left(\widetilde{e}_n, \widetilde{\mathbf{t}}_n\right)$ that firm n's product is compliant given that it is available for sale in the market. Hence, the expected utility of a consumer with valuation parameter α from purchasing firm n's product is

6

$\alpha u_n \left[1 + \Delta \widetilde{e}_n / s \left(\widetilde{e}_n, \widetilde{\mathbf{t}}_n \right) \right] - p_n$. We focus on rational expectations equilibria that are consistent, i.e., satisfy $\widetilde{e}_n = e_n$ and $\widetilde{\mathbf{t}}_n = \mathbf{t}_n$ for all $n \in \mathcal{N}$.

A consumer will buy the product on the market that generates maximum expected utility or, in case expected utility would be negative from purchase of any product, will buy no product. Therefore, under the standard condition $\Sigma_{n \in \mathcal{N}} q_n < 1$, the unique market equilibrium price per unit for each firm n's product is

$$p_n = \widetilde{u}_n - \sum_{m \in \mathcal{N}} \min(\widetilde{u}_n, \widetilde{u}_m) \underline{q}_m, \tag{4}$$

where $\widetilde{u}_m$ reflects consumers' expected utility from consumption of firm m's product

$$\widetilde{u}_m \equiv u_m \left[1 + \Delta \widetilde{e}_m / s \left(\widetilde{e}_m, \widetilde{\mathbf{t}}_m \right) \right] \text{ for } m \in \mathcal{N}; \tag{5}$$

(4) is derived in the appendix.

For a given vector of the firms' compliance and testing efforts, we assume that $\underline{q}_n$ and $\underline{q}_m$ for $m \neq n$ are independent. Therefore, firm n's expected profit (gross of fixed production costs) is

$$\pi_n = \left[\widetilde{u}_n(1 - q_n) - \sum_{m \in \mathcal{N} \backslash n} \min(\widetilde{u}_n, \widetilde{u}_m) s(e_m, \mathbf{t}_m) q_m \right] s(e_n, \mathbf{t}_n) q_n - c_n(e_n) - \Sigma_{m \in \mathcal{N} \backslash n} t_{nm}. \tag{6}$$

A firm's compliance and testing decisions are not observable by consumers or competitors. Therefore, for purposes of analysis, the game consists of two stages. In the first stage, each firm $n \in \mathcal{N}$ chooses its compliance e_n and testing of competitors t_{nm} for $m \in \mathcal{N} \backslash n$ to maximize (6), given its beliefs about competitors' testing and compliance decisions. In the second stage, the firms that have not been blocked bring their products to market and receive revenue according the prices (4). We focus on pure strategy Nash equilibria in compliance and testing by the firms, and treat the regulator's testing as a parameter for sensitivity analysis.

Whereas §4 discusses how the results extend when firms choose their production quantities $\{q_n\}_{n \in \mathcal{N}}$ in the first stage of the game, for simplicity, the $\{q_n\}_{n \in \mathcal{N}}$ are fixed in the base model. This is equivalent to assuming that firms produce at capacity, as in (Chod and Rudi (2005), Anand and Girotra (2007), and Swinney et al. (2011)), in which case q_n represents the capacity of firm n. That assumption is reasonable under a new product standard because firms' compliance efforts and the potential for blocking increase the expected selling price, which tends to strengthen a firm's incentive to produce at capacity.

3. Results

Testing by the Regulator

What is the impact of regulator testing on the firms' testing and compliance efforts? Proposition 1 shows that regulator testing, at a small level, reduces firms' testing and has no impact on compliance; at a large level, it prevents firms' testing and can reduce compliance. For Proposition 1a,

consider an initial equilibrium in compliance and testing by the firms $\{\widehat{e}_n, \widehat{t}_{mn}\}_{n \in \mathcal{N}, \, m \in \mathcal{N} \backslash n}$ when the regulator does not test ($t_{Rn} = 0$ for $n \in \mathcal{N}$) and define

$$\tau_n \equiv \arg\max\{t_{Rn} : d_n(0, ..., 0, t_{Rn}) \leq d_n(\widehat{t}_{1n}, ..., \widehat{t}_{Nn}, 0)\} \quad \text{for } n \in \mathcal{N}. \tag{7}$$

Proposition 1a establishes that increasing regulator testing from 0 to a "small" level $t_{Rn} \leq \tau_n$ for $n \in \mathcal{N}$ crowds out testing by the firms and has no impact on their compliance. "Small" is a relative term, meaning that the probability that the regulator would detect a noncompliant product if competitors did not test, $d_n(0, ..., 0, t_{Rn})$ in (7), is smaller than the initial probability that the competitors detect a noncompliant product when the regulator does not test, $d_n(\widehat{t}_{1n}, ..., \widehat{t}_{Nn}, 0)$ in (7). Insofar as the regulator is less effective than firms in testing, and insofar as firms are motivated to test their competitors, each τ_n is *large*. The proofs of the results in this section are in the appendix.

Proposition 1 (*a.*) *A "small" amount of testing by the regulator ($t_{Rn} \leq \tau_n$ for $n \in \mathcal{N}$) reduces the equilibrium testing by each firm and has no impact on equilibrium compliance and the detection probability for a noncompliant product. (b.) There exists a threshold $\overline{\tau}$ such that if $t_{Rn} > \overline{\tau}$ for $n \in \mathcal{N}$, then no firm tests. Furthermore, there exist parameters with $t_{Rn} > \overline{\tau}$ for $n \in \mathcal{N}$ such that an increase in the regulator's testing of firm n, t_{Rn}, strictly reduces firm n's equilibrium compliance e_n, or a reduction in the regulator's testing to $t_{Rn} = 0$ for all firms $n \in \mathcal{N}$ strictly increases equilibrium compliance e_n for all firms $n \in \mathcal{N}$.*

The rationale is that testing by the regulator reduces the marginal probability that a firm can "knock out" a competitor through its own testing effort, and so discourages competitors from testing one another. Hence as the regulator increases testing of firm n from a "small" level t_{Rn}, the firms reduce their testing of firm n accordingly, such that the equilibrium compliance and detection probability for a noncompliant product remain unchanged (Proposition 1a). At sufficiently large levels of regulator testing, the firms cease to test (Proposition 1b).

In the latter scenario, additional testing of a firm by the regulator will obviously increase that firm's detection probability. Surprisingly, that increase in detection probability may backfire by strictly *reducing* the firm's compliance effort. The proof of Proposition 1b constructs an example with a unique equilibrium, wherein the regulator increases its testing of firm n while keeping its testing of all other firms unchanged, and compliance by firm n strictly decreases. The intuition is that increasing regulator testing of firm n makes the market more attractive to firm n's competitors because it is more likely that firm n will be blocked from the market. Consequently, firm n's competitors respond by increasing compliance. Because firm n's competitors are less likely to be

blocked from the market, the market is less attractive to firm n, so firm n responds by reducing compliance.

To prove that eliminating regulator testing can strictly *increase* compliance effort by all firms in the market, the proof of Proposition 1b constructs a simple example with two firms that are symmetric, except that firm 1 is more efficient in testing than firm 2. Initially, the regulator tests both firms at a high level that causes the firms not to test. Eliminating that regulator testing motivates each firm to test its competitor. In the resulting equilibrium, the detection probability for firm 1 is reduced from the initial setting with regulator testing. As firm 1 becomes more likely to bring its product to market, firm 1 is more likely to benefit from knocking out its competitor, and therefore firm 1 is more highly motivated to test its competitor's product. Firm 1 is also efficient in testing. Hence the equilibrium detection probability for firm 2 is higher than in the initial setting with regulator testing. That motivates firm 2 to adopt higher compliance effort. The increase in detection probability for firm 2 also motivates firm 1 to adopt higher compliance effort, because knocking out firm 2 increases the market price, and hence the incentive for firm 1 to successfully bring its product to market.

In summary, a regulator intent on increasing firms' compliance should understand that doing a small amount of testing will be ineffectual, and doing a large amount can be counterproductive.

The policy implication of Proposition 1 is that relying on competitor testing (i.e., regulator testing $t_{Rn} = 0$ for $n \in \mathcal{N}$) may be socially optimal–especially when doing so results in a large detection probability for a noncompliant product and large compliance effort by firms. A relatively "small" level of testing by the regulator reduces social welfare by increasing the total social cost of testing (crowding out the presumably more efficient testing by firms) without improving detection or compliance, according to Proposition 1a. To potentially increase detection or compliance, the regulator must incur an even higher testing cost, which is not worthwhile insofar as the detection probability and firms' compliance efforts would be large without regulator testing.

Therefore, in the remainder of this section, we assume that the regulator does not test, and identify conditions under which the firms' equilibrium compliance and testing efforts, and the resulting detection probability for a noncompliant product, are large.

Testing by Competitors

Focusing on the case that the firms are symmetric, Proposition 2 characterizes the impact of the number of firms, their quality, and consumers' benefit from compliance on the firms' equilibrium compliance and testing efforts, and the resulting detection probability for a noncompliant product. To ensure existence of a unique symmetric equilibrium, in which each firm exerts compliance effort

$\widehat{e}$ and tests each competitor at level $\widehat{t}$, Proposition 2 assumes that the compliance cost function is sufficiently convex, the detection function is sufficiently concave, and the slope of the detection function is finite, for $e \in [0, \overline{e})$

$$c''(e) > u\Delta/(1 - \overline{d})^2 \tag{8}$$

$$(\partial^2/\partial t_{nm}^2)d(\mathbf{t}_m) < -\max\{1/[d(\mathbf{t}_m) + \Delta], -4N(1 + \Delta)u(\partial/\partial t_{nm})d(\mathbf{t}_m)^2$$
$$\times [(1 - \overline{d})^2 c''(e) - u\Delta]^{-1}\}(\partial/\partial t_{nm})d(\mathbf{t}_m)^2 \tag{9}$$

$$(\partial/\partial t_{nm})d(\mathbf{t}_m) < \infty. \tag{10}$$

Proposition 2 *Suppose the firms are symmetric and* (8)-(10) *hold. In the unique symmetric equilibrium, firms test* $(\widehat{t} > 0)$ *if and only if*

$$u > 1/[q^2(\partial/\partial t_{nm})d(\mathbf{t}_m)|_{\mathbf{t}_m=\mathbf{0}}]. \tag{11}$$

The detection probability for a noncompliant product increases with the number of firms N for $N \leq \overline{N}$ and decreases with N for $N \geq \overline{N}$. Compliance $\widehat{e}$ increases with the firms' quality levels u and a consumer's benefit from compliance Δ, and decreases with the number of firms N.

The basic reason that equilibrium compliance effort $\widehat{e}$ increases with the firms' quality levels u, with a consumer's benefit from compliance Δ, and with market concentration (a smaller number N of firms supplying the market) is that those exogenous factors drive up the market price (4). Hence each firm has greater value from bringing products to market, and correspondingly stronger incentive for compliance effort to increase the probability of doing so.

However, equilibrium testing $\widehat{t}$ and the resulting detection probability for a noncompliant product are not monotonic in those factors. Equilibrium testing $\widehat{t}$ increases with the number of firms N for $N \leq \overline{N}$ and decreases with N for $N \geq \overline{N}$ because reduced compliance strengthens a firm's incentive for testing when compliance is high (as is the case with few firms $N \leq \overline{N}$) and weakens this incentive when compliance is low (as is the case with many firms $N \geq \overline{N}$). Reduced compliance has two countervailing effects.

First, a competitor's reduced compliance strengthens a firm's incentive for testing because testing is more likely to reveal noncompliance. This testing-incentive-strengthening effect is strong when the testing firm's compliance is high because then the firm is likely to bring products to market, so the firm's incentive for testing is sensitive to its competitor's reduced compliance.

Second, a firm's reduced compliance weakens its incentive to test because the firm is more prone to being blocked from the market. This testing-incentive-weakening effect is weak when the competitor firm's compliance is high because then it is unlikely that testing will be effective in blocking the competitor firm's products from the market, so the incentive for testing is insensitive

to the reduction to the testing firm's reduced compliance.

Together, Proposition 2 (see inequality (11)) and Proposition 3, for the general asymmetric case, highlight the critical importance of quality in stimulating competitor testing and hence compliance.

Proposition 3 *If firm m's quality is sufficiently low*

$$u_m \leq 1/\{(1+\Delta)q_m \max_{n \in \mathcal{N} \setminus m}[q_n(\partial/\partial t_{nm})d_m(\mathbf{t}_m)|_{\mathbf{t}_m=\mathbf{0}}]\}, \tag{12}$$

then firm m draws no testing ($t_{nm} = 0$ for all $n \in \mathcal{N} \setminus m$) and does not comply ($e_m = 0$).

The rationale for (11) and (12) is that the increase in market price (4) from knocking out firm m increases with firm m's quality u_m, which stimulates competitors to test firm m's product. When u_m is low, competitors do not do so, so firm m does not comply. The threshold quality level on the right hand side of (12) strictly decreases with a consumer's benefit from compliance Δ. That serves to rule out equilibria in which firm m exerts compliance effort and consumers believe that its product is likely to be compliant, which motivates greater testing by competitors and hence the compliance effort by firm m. In any setting with a unique equilibrium, a weaker condition ((12) with 0 substituted for Δ) implies that firm m draws no testing and does not comply; in the symmetric case, with 0 substituted for Δ, the right hand side of (12) simplifies to that of (11).

As a corollary to Proposition 3, enforcement of a product standard through competitor testing strictly improves the profitability of firms with low quality. The counterfactual scenario has no product standard or, equivalently, a standard that is not enforced, so that firm n's expected profit is $(u_n - \Sigma_{m=1}^{N} \min(u_n, u_m)q_m)q_n$.

Corollary *A product standard, enforced through competitor testing, increases the expected profit of firms with low quality (12), which do not comply with the standard.*

The rationale is that a low-quality firm spends nothing on compliance, may choose not to test, and sells at a higher price when its higher-quality competitors knock each other out of the market.

4. Extensions

Endogenous Quality: Entrants

The Corollary to Proposition 3 suggests that a product standard, enforced through competitor testing, will cause entry by low-quality or weakly-branded firms with little incentive for compliance. Proposition 4 confirms that such entry occurs, in the following, extended version of our model. Suppose that a firm may enter the market with capacity q and a product of low quality u^l at cost k^l, where

$$u^l \leq 1/[(1+\Delta)q^2 \max_{m \in \mathcal{N} \setminus n}(\partial/\partial t_{mn})d_n(\mathbf{t}_n)|_{\mathbf{t}_n=\mathbf{0}}]. \tag{13}$$

Alternatively, a firm may enter the market with capacity q and a product of high quality $u^h > u^l$ at cost k^h. A firm enters if doing so yields nonnegative expected profit. Initially (prior to adoption of the product standard), the market is in an equilibrium with more than two high-quality firms; a sufficient condition for existence of such an equilibrium is that their entry cost $k^h \leq \min(u^h(1 - 3q)q, k^l u^h / u^l)$. Similarly, assume that $k^l < u^l(1 - 2q)q$ so a low-quality firm might enter. Finally, assume that testing costs are sufficiently low that, after adoption of the product standard, at least one incumbent firm does some testing. The proofs of the results in this section are in the online supplement.

Proposition 4 *A product standard, enforced through competitor testing, causes entry by at least one firm with low quality u^l (the type of firm that does not comply) and no firms with high quality u^h if detection is sufficiently easy*

$$(\partial / \partial t_{nm}) d_m(\mathbf{t}_m) \geq \gamma \text{ if } d_m(\mathbf{t}_m) \leq \underline{d}, \tag{14}$$

and compliance is sufficiently costly

$$c_n(e_n) \geq \underline{c} \text{ for } e_n \geq \underline{e},$$

for $n \in \mathcal{N}$, where $\gamma \in (0, \infty)$, $\underline{d} < 1$, $\underline{c} \in (0, \infty)$ and $\underline{e} \in (0, \overline{e})$.

Endogenous Quality: Incumbents

We now turn to the impact of a product standard, enforced through competitor testing, on the quality chosen by incumbent firms. Suppose that each firm $n \in \mathcal{N}$ privately chooses a quality level $u_n \in \{u^1, u^2, .., u^M\}$ simultaneously with compliance e_n. The cost to firm n of choosing quality u^i is k_n^i.

Proposition 5 shows that enforcement of a product standard through competitor testing reduces quality. The quality reduction occurs for all firms in the symmetric case, and, with asymmetric firms, occurs for ones with highest equilibrium quality in the counterfactual scenario. As in the above Corollary and Proposition 4, the counterfactual scenario has no product standard or, equivalently, a standard that is not enforced. Proposition 5 focuses on a product standard motivated by public welfare or environmental concerns, such as RoHS. RoHS aims to prevent the release of hazardous substances to the environment when e-waste is recycled or disposed, and to protect the health of workers who manufacture or recycle electronics. In contrast, a consumer has little or no health benefit from purchasing a RoHS-compliant product, i.e., Δ is negligible.

Proposition 5 *Suppose that a consumer's benefit from compliance $\Delta = 0$. A product standard, enforced through competitor testing, reduces the quality of firms with highest equilibrium quality. If the firms are symmetric, the standard, enforced through competitor testing, reduces quality in any symmetric equilibrium.*

In reality, firms may observe competitors' investments in quality (or advertising expenditures, or other costly efforts to strengthen a brand) before investing in compliance and testing. A model with observable initial investment in quality is analytically intractable, but the fact that having lower quality reduces the incentive for testing by competitors suggests that a new product standard, enforced through competitor testing, will result in even greater reductions in quality when quality investments are observable.

In contrast to Proposition 5, enforcement of a product standard to protect consumers' health or safety, for which Δ is large, might increase quality.

Other Penalties for Noncompliance

In the event that a firm's product is shown to be noncompliant, instead of blocking the sale of the product, a regulator could simply notify consumers that the product is noncompliant. Notification could be accomplished by publishing a list of noncompliant products or preventing a firm from labeling its product as compliant. For example, the U.S. Department of Energy employs this "labeling" approach in enforcement of the Energy Star standard. In this labeling scenario, firm n always sells q_n units, but at a reduced price in the event that its product is shown to be noncompliant. The unique market equilibrium price per unit for each firm n's product is

$$p_n = \underline{u}_n - \sum_{m \in \mathcal{N}} \min(\underline{u}_n, \underline{u}_m) q_m,$$

where $\underline{u}_m$ is the random variable

$$\underline{u}_m \equiv \begin{cases} \widetilde{u}_m & \text{with probability } s(e_m, \mathbf{t}_m) \\ u_m & \text{with probability } 1 - s(e_m, \mathbf{t}_m), \end{cases}$$

with $\widetilde{u}_m$ defined in (5). For a given vector of the firms' compliance and testing efforts, we assume that $\underline{u}_n$ and $\underline{u}_m$ for $m \neq n$ are independent. Therefore, firm n's expected profit (gross of fixed production costs) is

$$\pi_n = \left[\widetilde{u}_n(1 - q_n) - \sum_{m \in \mathcal{N} \setminus n} \{ s(e_m, \mathbf{t}_m) \min(\widetilde{u}_n, \widetilde{u}_m) + [1 - s(e_m, \mathbf{t}_m)] \min(\widetilde{u}_n, u_m) \} q_m \right] s(e_n, \mathbf{t}_n) q_n$$

$$+ \left[u_n(1 - q_n) - \sum_{m \in \mathcal{N} \setminus n} \{ s(e_m, \mathbf{t}_m) \min(u_n, \widetilde{u}_m) + [1 - s(e_m, \mathbf{t}_m)] \min(u_n, u_m) \} q_m \right]$$

$$\times [1 - s(e_n, \mathbf{t}_n)] q_n - \sum_{m \in \mathcal{N} \setminus n} t_{nm}. \tag{15}$$

Another motivation for this labeling scenario is that, in the absence of a government-imposed standard, firms may test competitors' products in order to detect and inform consumers of a defect or false claim. For example, competitor testing alone serves to enforce voluntary product labeling standards, devised by industry and other non-governmental organizations, for "natural" and "organic" personal-care products (Story 2008, Struck 2008).

Suppose that when a firm's product is shown to be noncompliant, the firm also incurs a fine $f \geq 0$. That introduces the term $-f[1 - s(e_n, \mathbf{t}_n)]$ into the objective function for firm n in (6)

for the scenario with blocking and in (15) for the scenario with labeling. That "fine" f may also incorporate costs associated with civil lawsuits, criminal penalties, long-term reputational damage, or (with blocking) the cost to safely destroy and dispose of a product shown to be noncompliant.

The propositions in §3 extend with the fine $f \geq 0$ and either blocking or labeling. Proposition 1 holds. Proposition 3 holds when $\mathbf{I}_L \min_{i \in \mathcal{N}}\{u_i\}/(1 + \Delta)$, where $\mathbf{I}_L = 1$ in the labeling scenario and $\mathbf{I}_L = 0$ in the blocking scenario, is added to the right hand side of (12). In the extension of Proposition 2, the symmetric equilibrium compliance increases with the fine f, and there exists $\overline{f}$ such that the symmetric equilibrium detection probability increases with f for $f \leq \overline{f}$ and decreases with f for $f \geq \overline{f}$. For the labeling scenario, the comparative statics in Proposition 2 hold for the symmetric equilibrium with the largest compliance level; nonuniqueness arises because of existence of an equilibrium with zero compliance and testing.

Should a regulator strip offending products of labels (such as "Energy Star"), instead of blocking them from the market? Doing so reduces compliance effort by incumbent firms, in the context of Proposition 2. Specifically, for any fine $f \geq 0$, any symmetric equilibrium compliance under labeling is lower than the unique symmetric equilibrium compliance under blocking. However, with asymmetric firms, the extended Proposition 3 implies that the switch to labeling might cause firms with low–but not too low–quality to draw testing by a competitor and exert some compliance effort. Labeling reverses the results in the Corollary and Proposition 4: A product standard, enforced through competitor testing and labeling, *decreases* the expected profit of firms that do not comply and, in the setting of Proposition 4, does *not* cause entry by low-quality, noncompliant firms. To summarize, the switch to labeling eliminates the problem of entry by low-quality, noncompliant firms, but may reduce incumbents' compliance efforts.

The larger, more general point is that the choice of penalty (e.g., blocking vs. notifying consumers, the magnitude of the fine) influences competitor testing and hence, interactively, firms' compliance efforts.

Endogenous Production Quantities

Suppose that each firm $n \in \mathcal{N}$ chooses its quantity q_n, unobserved by its competitors, at cost $C_n(q_n)$ strictly increasing in q_n. The lead time for production is sufficiently long that a firm must commit to its quantity prior to observing which products are blocked.[3] For purposes of analysis, this is captured by amending the the two-stage game described in §2 so that firms make quantity, in addition to compliance and testing, decisions in the first stage. The propositions in §3 are

---

[3]Alternatively, if the firms had unlimited capacity and extremely short lead times, they could instead engage in price competition after observing which products are blocked. Each firm would choose its price, and firms' production quantities would be determined by the market equilibrium demand, i.e., by inverting (4).

robust in this extension. Specifically, Proposition 1 and Proposition 3 hold, with (12) simplified to $u_m \leq 1/\{(1 + \Delta) \max_{n \in \mathcal{N} \backslash m}[(\partial/\partial t_{nm})d_m(\mathbf{t}_m)|_{\mathbf{t}_m = \mathbf{0}}]\}$. The online supplement provides numerical results consistent with Proposition 2. Furthermore, the Corollary to Proposition 3 is robust for product standards motivated by public welfare or environmental concerns, like RoHS. We prove the Corollary for the case that consumers do not benefit from compliance $\Delta = 0$ and assuming, for analytic tractability, linear production costs and two types of firms: of high and low quality. In contrast, when Δ is large, the product standard, enforced through competitor testing, can reduce the expected profit of a noncompliant low-quality firm, by stimulating production by high-quality firms.

Fixed Costs for Testing

Suppose that each firm $n \in \mathcal{N}$ must incur a fixed cost $\phi_n \geq 0$ in order to test the product of any competitor-firm $m \in \mathcal{N} \backslash n$ at a strictly positive level $t_{nm} > 0$. This captures the setting in which firm n needs to acquire equipment and/or technical expertise to test its competitors' products; ϕ_n represents the fixed cost associated with obtaining this competency. The setting of §2 wherein $\phi_n = 0$ represents the case where the firm already has this competency, for example, due to prior investments in its own product development process (Wiel and McMahon 2005). Alternatively, $\phi_n = 0$ may arise because firm n relies on an outside laboratory to conduct tests. Similarly, suppose the regulator incurs a fixed cost $\phi_R \geq 0$ if $t_{Rm} > 0$ for any $m \in \mathcal{N}$.

The propositions in §3 extend. Proposition 1a holds, with a more complex expression for τ_n reflecting that a marginal increase in regulator testing can motivate a discontinuous drop in competitor testing from a high level to zero. Proposition 1b holds. In fact, a stronger version of Proposition 1b holds: There exist parameters and a threshold $\overline{\tau}$ such that increasing regulator testing from $t_{Rn} = 0$ to $t_{Rn} > \overline{\tau}$ for $n \in N$ strictly reduces the compliance effort by every firm, strictly reduces the detection probability for a noncompliant product for every firm, and causes the firms not to test, in equilibrium. Proposition 3 and its Corollary hold. Proposition 2 holds provided that the fixed cost of testing is not too large.

5. Concluding Remarks

This paper is the first to study firms' testing of competitors' products to reveal violations of a product standard (to the best of our knowledge and according to (Li and Peeters 2014)).

Insights from the stylized model in this paper suggest that testing by government regulatory authorities to detect violations of a product standard may be detrimental to social welfare. With a small budget–as is common in practice (Bruschia 2008, Smith 2008)–regulator testing fails to improve firms' compliance efforts and fails to increase the detection probability for a noncompliant

product. With a large budget, regulator testing can strictly reduce firms' compliance efforts and strictly reduce the detection probability for a noncompliant product. The underlying phenomenon is that testing by a regulator crowds out testing by competitors. That crowding out may be detrimental to social welfare simply because a regulator is less efficient than competitors in detecting violations, for the reasons documented in §1.

Relying on competitors to detect violations is most effective for standards that benefit consumers and for industries with a moderate number of competitors, each with strong brands and high quality. In our stylized model, each firm's compliance effort increases with its product quality and brand strength, and with a consumer's benefit from compliance. The detection probability for a noncompliant product is maximized with a moderate number of competitors. Compliance effort decreases with the number of competitors. In practice, however, in an industry with a small number of competitors, those firms might collude to prevent competitor testing. According to (Bruschia 2008, Smith 2008), regulatory authorities encourage competitor testing by offering anonymity for firms that report competitors' violations, but suspect that this is ineffective when the number of competitors is small. Having a common reputation (as in some food industries (Winfree and McClusky 2005)), common noncompliant suppliers, or network effects (as in (Wang and Xie 2001)) also might deter firms from testing competitors.

When relying on competitor testing to detect violations, stronger penalties may be detrimental. Increasing the fine increases compliance effort, but can reduce competitor testing and the detection probability for a noncompliant product. Blocking the sale of a product shown to be noncompliant– rather than simply publicizing that the product is noncompliant–increases compliance effort by incumbent firms, but may cause entry by low-quality, noncompliant firms that do not draw testing from competitors. This paper has assumed that the regulator always imposes one of those penalties (blocking or at least labeling a product as noncompliant) for a detected violation. However, the regulator could do so with probability less than 1, as in (Mookherjee and Png 1992), to induce less testing and compliance effort from firms, in case the firms would otherwise exert more testing and compliance effort than is socially optimal.

For a voluntary standard like Energy Star, relying on competitor testing is effective only if consumers have favorable beliefs. In a pernicious equilibrium, consumers think that products are noncompliant, firms have no incentive to test competitors' products and–without testing–have no incentive to comply with the standard. Indeed, the U.S. relied on competitor testing for enforcement of Energy Star and consumers trusted the Energy Star label–until a General Accountability Office report and related publicity of violations damaged that trust. In response, the U.S. Department of

16

Energy instituted a new requirement for firms to pay for third-party testing of their products in order to use the Energy Star label (Gaffigan 2007, GAO 2010, Rosner 2010).

Third-party testing has qualitatively the same impacts on firms' compliance and testing efforts as does regulator testing. Specifically, in the model in this paper, one may interpret the parameter t_{Rn} as the level of third-party testing. Proposition 1 holds with "third-party" substituted for "regulator." A fiscally-constrained government might impose higher t_{Rn} with third-party testing, by requiring firms to pay for the third-party testing. However, if the cost of third-party testing is too high, a firm will choose not to bring its product to market, or not to label its product as compliant, rather than pay for the third-party testing. (For example, the new requirement for third-party testing is causing some firms to opt out of the Energy Star program (Brown 2012).) The testing level t_{Rn} or effective detection probability function $d_n()$ also might differ because a regulator and third-party have different selfish incentives for testing effort, disclosure, and corruption (Mookherjee and Png 1995, Boyer et al. 2000, Farhi et al. 2013). Therefore, endogenizing the strategic choice of testing level by a regulator or third-party is an important direction for future research.

References

Anand, K. S., K. Girotra. 2007. The strategic perils of delayed differentiation. *Management Science* **53**(5) 697-712.

Austen-Smith, D., T. J. Feddersen. 2008. Public disclosure, private revelation, or silence: Whistle-blowing incentives and managerial policy. Northwestern University Working Paper.

Babich V., C. Tang. 2012. Managing opportunistic supplier adulteration: Deferred payments, inspection, and combined mechanisms. *Manufacturing & Service Operations Management* **14**(2) 301-314.

Baiman, S., P. E. Fischer, M. V. Rajan. 2000. Information, contracting, and quality costs. *Management Science* **46**(6) 776-789.

Balachandran, K. R., S. Radhakrishnan. 2005. Quality implications of warranties in a supply chain. *Management Science* **51**(8) 1266-1277.

Bigoni, M., S.-O. Fridolfsson, C. Le Coq, G. Spagnolo. 2012. Fines, leniency, and rewards in antitrust. *RAND Journal of Economics* **43**(2) 368-390.

Boyer, M., T.R. Lewis, W.L. Liu. 2000. Setting standards for credible compliance and law enforcement. *Canadian Journal of Economics* **33** 319-340.

Brown, R. 2012. Interview with Richard Brown, Research Scientist, Environmental Energy Technologies Division, Lawrence Berkeley National Laboratory. August 8, 2012.

Bruschia, R. August 1-25, 2008. Personal interview and email exchange with Robert Bruschia, Research Scientist, California EPA's Department of Toxic Substances Control.

Cohen, M. 1999. Monitoring and enforcement of environmental policy. In T. Tietenberg and H. Folmer, eds., *The International Yearbook of Environmental and Resource Economics 1999/2000* Northampton, MA: Edward Elgar Publishers.

Chod, J., N. Rudi. 2005. Resource flexibility with responsive pricing. *Operations Research* **53**(3) 532-548.

Croft, S. L., Strongman, J. 2004. The revised General Product Safety Directive: A practical look. *Global Counsel Dispute Resolution Handbook* 27-32.

Day, G. 2007. *The Market Driven Organization: Understanding, Attracting, and Keeping Valuable Customers* New York: Free Press.

Department of Energy. March 19, 2010. Press Release: EPA, DOE Announce New Steps to Strengthen Energy Star.

European Commission. 2012. http://ec.europa.eu/consumers/safety/rapex/stats_reports_en.htm.

Farhi, E., J. Lerner, J. Tirole. 2013. Fear of rejection? Tiered certification and transparency. *RAND Journal of Economics* **44**(4) 610-631.

Gaffigan, M. September 26, 2007. Energy efficiency: Opportunities exist for federal agencies to better inform household consumers. Statement of Mark Gaffigan, Acting Director, Natural Resources and Environment, *General Accountability Office Report GAO-07-116.* 1-10.

GAO 2010. Energy Star Program: Covert testing shows the Energy Star Program certification process is vulnerable to fraud and abuse. *General Accountability Office Report GAO-10-470.*

Green Supply Line. November 27, 2006. RoHS police are busy at work. *Green Supply Line.* United Business Media. http://www.greensupplyline.com/showArticle.jhtml?articleID=196500146.

Hess, R. April 19, 2006. Ensuring compliance: A content perspective. Presentation at Stanford Global Supply Chain Management Forum, Stanford University.

Hwang, I., S. Radhakrishnan, L. Su. 2006. Vendor certification and appraisal: Implications for supplier quality. *Management Science* **52**(10) 1472 - 1482.

Kapoor, A. 2012. Focus on product identification: What applies now that the new Product Safety Act has come into effect? *PSI Journal Online.*

http://www.psi-network.org/interview_mit_rechtsanwalt_dr_arun_kapoor_1-2374.html.

Li, X., R. Peeters. 2014. Rivalry information acquisition and disclosure. Maastricht University Working Paper.

Mookherjee, D., I. P. L. Png. 1992. Monitoring vis-á-vis investigation in enforcement of law. *American Economic Review* **82**(3) 556-565.

Mookherjee, D., I. P. L. Png. 1995. Corruptible law enforcers: How should they be compensated? *The Economic Journal* **105**(428) 145-159.

Polinsky, A.M. 1980. Private versus public enforcement of fines. *The Journal of Legal Studies* **9** 105-127.

Porteus, E. L., A. Angelus. 1997. Opportunities for improved statistical process control *Management Science* **43**(9) 1214-1228.

Rosner, S. 2010. Energy Star certification now tougher. *Mother Earth News* August/September 2010.

Ross, D. December 13, 2007. Personal interview with Dale Ross, former Mars, Inc. executive.

Shah, J. B., L. Sullivan. September 30, 2002. Firms warned to brace for environmental fallout – New laws threaten to disrupt the unprepared. *Electronic Buyer's News* 1-2.

Smith, C. September 4, 2008. Personal interview with Chris Smith, Technical Manager of the United Kingdom's RoHS Enforcement Authority and Secretary of the European RoHS Enforcement Network.

Spagnolo, G. 2008. Leniency and Whistleblowers in Antitrust. in P. Buccirossi, ed., Handbook of Antitrust Economics. Cambridge, Massachusetts: MIT Press.

Story, L. January 6, 2008. Can Burt's Bees turn Clorox green? *New York Times* C1.

Struck, H. May 1, 2008. Dr. Bronner's organic discord. *Forbes.com.* http://www.forbes.com/2008 /05/01/bronner-organic-soap-faces-markets-cx_hs_0501autofacescan01.html.

Swinney, R., G. Cachon, S. Netessine. 2011. Capacity investment timing by start-ups and established firms in new markets. *Management Science* 57(4) 763-777.

Ting, M. M. 2008. Whistleblowing. *American Political Science Review* **102** 249-267.

Vestel, L. B. 2009. Energy Star gets tough on LG Electronics. *New York Times.* December 30, 2009.

Volden, C., A. E. Wiseman. 2007. A theory of government regulation and self-regulation with the specter of nonmarket threats. Ohio State University Working Paper.

Wang, Q., J. Xie. 2001. Will consumers be willing to pay more when your competitors adopt your technology? The impacts of the supporting-firm base in markets with network effects. *Journal*

of Marketing **75**(5) 1-17.

Wiel, S. and J.E. McMahon. 2005. Energy-efficiency labels and standards: a guidebook for appliances, equipment and lighting, 2nd edition. Collaborative Labeling and Appliance Standards Program (CLASP) Publication,Washington, D.C..

Winfree, J. A., J. J. McCluskey. 2005. Collective reputation and quality. *American Journal of Agricultural Economics* **87**(1) 206-213

Zhu, K., R. Q. Zhang, F. Tsung. 2007. Pushing quality improvement along supply chains. *Management Science* **53**(3) 421-436.

Appendix

Derivation of Equilibrium Prices in Equation (4): Index the firms such that $\widetilde{u}_n$ increases with n. In any market equilibrium, prices p_n also increase with n, consumers with the lowest quality valuation $\alpha \in [0, 1 - \Sigma_{m=1}^N \underline{q}_m)$ do not purchase, and the marginal consumer with $\alpha = 1 - \Sigma_{m=1}^N \underline{q}_m$ purchases the product with lowest price and value, from the firm with index $l = \arg\min\{\widetilde{u}_m : \underline{q}_m > 0\}$, which implies that $p_l = \widetilde{u}_l(1 - \Sigma_{m=1}^N \underline{q}_m)$, establishing (4)) for $n = l$. For every other product in the market, $n > l$ with $\underline{q}_n > 0$, the marginal buyer for product n has $\alpha = 1 - \Sigma_{m=n}^N \underline{q}_m$ and is indifferent between the purchase of product n or the product with next-highest price and value, indexed $j = \arg\max\{\widetilde{u}_m : \underline{q}_m > 0, m < n\}$, so

$$\widetilde{u}_n(1 - \Sigma_{m=n}^N \underline{q}_m) - p_n = \widetilde{u}_j(1 - \Sigma_{m=n}^N \underline{q}_m) - p_j. \tag{16}$$

Having already established that (4) holds for the case that $j = l$, we proceed by induction, assuming that (4) holds for firm j, i.e.,

$$p_j = \widetilde{u}_j - \sum_{m \in \mathcal{N}} \min(\widetilde{u}_j, \widetilde{u}_m)\underline{q}_m. \tag{17}$$

Together, (16) and (17) imply (4) for product n.

Proof of Proposition 1: (a.) Consider the introduction of regulator testing $\mathbf{t}_R \neq \mathbf{0}$, where $\mathbf{t}_R = <t_{R1}, t_{R2}, .., t_{RN}>$ denotes the vector of regulator testing levels, that satisfies $t_{Rn} \leq \tau_n$ for $n \in \mathcal{N}$. We will construct best response testing levels for the firms $\{\widehat{t}'_{mn}\}_{n \in \mathcal{N}, \, m \in \mathcal{N} \setminus n}$, and show that the initial compliance and detection probability for a noncompliant product for every firm $n \in \mathcal{N}$ are preserved in equilibrium with those (lowered) testing levels. We adopt the vector notation $\widehat{\mathbf{t}}_n = <\widehat{t}_{1n}, \widehat{t}_{2n}, .., \widehat{t}_{n-1,n}, \widehat{t}_{n+1,n}, .., \widehat{t}_{Nn}>$ for the initial equilibrium testing of firm n by other firms and $\widehat{\mathbf{t}}'_n = <\widehat{t}'_{1n}, \widehat{t}'_{2n}, .., \widehat{t}'_{n-1,n}, \widehat{t}'_{n+1,n}, .., \widehat{t}'_{Nn}>$ for their best response testing of firm n after the introduction of regulator testing. The construction of τ_n in (7) implies that $d_n(\mathbf{0}, t_{Rn}) \leq d_n(\widehat{\mathbf{t}}_n, 0)$ for $n \in \mathcal{N}$. If $t_{Rn} = 0$, then set $\widehat{\mathbf{t}}'_n = \widehat{\mathbf{t}}_n$. If $t_{Rn} > 0$, then $0 < d_n(\mathbf{0}, t_{Rn}) \leq d_n(\widehat{\mathbf{t}}_n, 0)$, so at least one competitor tested firm n in the initial equilibrium. Let N_n denote the number of firms that tested firm n in the initial equilibrium. Suppose that the firms are indexed so that firms $1, .., N_n$ are doing

that testing, i.e., $\widehat{t}_{mn} > 0$ for $m = 1, .., N_n$ and $\widehat{t}_{mn} = 0$ for $m > N_n$. Let $\mathcal{T}_1(t, \mathbf{t}_n)$ denote the vector of competitor testing levels for firm n with first component t and second through N^{th} components identical to those of $\mathbf{t}_n$. In other words, $\mathcal{T}_1(t, \mathbf{t}_n)$ transforms a vector $\mathbf{t}_n$ by substituting t for its first component. Set

$$\widehat{t}_{1n}' = \min\{t \geq 0 : d_n(\mathcal{T}_1(t, \widehat{\mathbf{t}}_n), t_{Rn}) \geq d_n(\widehat{\mathbf{t}}_n, 0)\}.$$

As d_n is continuous and componentwise strictly increasing, one of the following two cases must occur. In the first case, $d_n(\mathcal{T}_1(\widehat{t}_{1n}', \widehat{\mathbf{t}}_n), t_{Rn}) = d_n(\widehat{\mathbf{t}}_n, 0)$. In the second case, $d_n(\mathcal{T}_1(\widehat{t}_{1n}', \widehat{\mathbf{t}}_n), t_{Rn}) > d_n(\widehat{\mathbf{t}}_n, 0)$ and $\widehat{t}_{1n}' = 0$. In the first case, set $\widehat{\mathbf{t}}_n' = \mathcal{T}_1(\widehat{t}_{1n}', \widehat{\mathbf{t}}_n)$. In the second case, sequentially reduce the testing of firm n by firms $m = 2, 3, ...$ in the same manner, until the detection probability for a noncompliant product by firm n falls to the initial level $d_n(\widehat{\mathbf{t}}_n, 0)$. Specifically, for $m \in \mathcal{N} \backslash n$, let $\mathcal{T}_m(t, \mathbf{t}_n)$ denote the vector of competitor testing levels for firm n with components 1 through $m - 1$ set to zero, component m set to t, and all other components identical to those of $\mathbf{t}_n$. In other words, $\mathcal{T}_m(t, \mathbf{t}_n)$ transforms a vector $\mathbf{t}_n$ by substituting t for its mth component, and 0 for components 1 through $m - 1$. Iteratively, starting with $m = 2$, set

$$\widehat{t}_{mn}' = \min\{t \geq 0 : d_n(\mathcal{T}_m(t, \widehat{\mathbf{t}}_n), t_{Rn}) \geq d_n(\widehat{\mathbf{t}}_n, 0)\}.$$

If $d_n(\mathcal{T}_m(\widehat{t}_{mn}', \widehat{\mathbf{t}}_n), t_{Rn}) = d_n(\widehat{\mathbf{t}}_n, 0)$, then stop with $\widehat{\mathbf{t}}_n' = \mathcal{T}_m(\widehat{t}_{mn}', \widehat{\mathbf{t}}_n)$. Otherwise, $d_n(\mathcal{T}_m(\widehat{t}_{mn}', \widehat{\mathbf{t}}_n), t_{Rn})$ $> d_n(\widehat{\mathbf{t}}_n, 0)$ and $\widehat{t}_{mn}' = 0$. Increment $m = m + 1$, until $d_n(\mathcal{T}_m(\widehat{t}_{mn}', \widehat{\mathbf{t}}_n), t_{Rn}) = d_n(\widehat{\mathbf{t}}_n, 0)$ and then stop with $\widehat{\mathbf{t}}_n' = \mathcal{T}_m(\widehat{t}_{mn}', \widehat{\mathbf{t}}_n)$. Observe that $d_n(\mathcal{T}_m(\widehat{t}_{mn}', \widehat{\mathbf{t}}_n), t_{Rn}) = d_n(\widehat{\mathbf{t}}_n, 0)$ is achieved for $m \leq N_n$ because d_n is continuous and component-wise strictly increasing, and $d_n(\mathbf{0}, t_{Rn}) \leq d_n(\widehat{\mathbf{t}}_n, 0)$. Follow this process to construct $\widehat{\mathbf{t}}_n'$ for each $n \in \mathcal{N}$, with the properties that $d_n(\widehat{\mathbf{t}}_n', t_{Rn}) = d_n(\widehat{\mathbf{t}}_n, 0)$ and $\widehat{\mathbf{t}}_n' \leq \widehat{\mathbf{t}}_n$. Property (1) and the assumption that d_n is continuously differentiable and component-wise strictly increasing imply that d_n is a strictly concave function of t_{mn} and, with $d_n(\widehat{\mathbf{t}}_n', t_{Rn}) = d_n(\widehat{\mathbf{t}}_n, 0)$, imply that $(\partial/\partial t_{mn})d_n(\widehat{\mathbf{t}}_n', t_{Rn}) = (\partial/\partial t_{mn})d_n(\widehat{\mathbf{t}}_n, 0)$, for $m \in \mathcal{N}$ and $n \in \mathcal{N} \backslash m$. Therefore, given regulator testing $\mathbf{t}_R$, $\{\widehat{e}_n, \widehat{t}_{mn}'\}_{n \in \mathcal{N}, \, m \in \mathcal{N} \backslash n}$ constitutes an equilibrium in compliance and testing for the firms, in which the detection probability for a noncompliant product for each firm $n \in \mathcal{N}$ is unchanged from the initial equilibrium. A sufficient condition for τ_n to be strictly positive is that firm n draws testing from competitors in the initial equilibrium. (b.) First, we establish that if t_{Rn} is large enough for $n \in \mathcal{N}$, in any equilibrium the firms do not test. Let $\overline{\tau}_{mn} = 0$ if $(\partial/\partial t_{mn})d_n(0, .., 0) \leq (\max_{i \in \mathcal{N}} u_i(1 + \Delta))^{-1}$, and otherwise let $\overline{\tau}_{mn}$ denote the unique solution to

$$(\partial/\partial t_{mn})d_n(0, ..0, t_{Rn})|_{t_{Rn} = \overline{\tau}_{mn}} = (\max_{i \in \mathcal{N}} u_i(1 + \Delta))^{-1}.$$

Existence of such a solution follows from the assumptions that $d_n(\mathbf{t}_n)$ is componentwise strictly increasing, continuously differentiable and satisfies $d_n(\mathbf{t}_n) \leq \overline{d} \in (0, 1)$ for $\mathbf{t}_n \in R_+^{N+1}$ and $\lim_{t_{Rn} \to \infty} d_n(\mathbf{t}_n) = \overline{d}$, which imply that $\lim_{t_{Rn} \to \infty} (\partial/\partial t_{mn})d_n(0, .., 0, t_{Rn}) = 0$. Let

$\overline{\tau} = \max_{i \in \mathcal{N}, \, j \in \mathcal{N} \setminus i} \overline{\tau}_{ij}$, and consider the case where $t_{Rn} > \overline{\tau}$ for $n \in \mathcal{N}$. For $m \in \mathcal{N} \setminus n$,

$$(\partial/\partial t_{mn})\pi_m|_{t_{mn} > 0} = \min(\widetilde{u}_n, \widetilde{u}_m)s\left(e_m, \mathbf{t}_m\right) q_n q_m (1 - e_n)(\partial/\partial t_{mn})d_n(\mathbf{t}_n) - 1$$

$$\leq \max_{i \in \mathcal{N}} u_i (1 + \Delta)(\partial/\partial t_{mn})d_n(\mathbf{t}_n) - 1$$

$$\leq \max_{i \in \mathcal{N}} u_i (1 + \Delta)(\partial/\partial t_{mn})d_n(0, .., 0, t_{Rn}) - 1, \tag{18}$$

where (18) follows from $d_n(\mathbf{t}_n)$ being componentwise strictly increasing and (1). Therefore, $t_{Rn} > \overline{\tau}_{mn}$ implies that $(\partial/\partial t_{mn})\pi_m|_{t_{mn} > 0} < 0$, so in any equilibrium $t_{mn} = 0$. Because $t_{Rn} > \overline{\tau}$ for $n \in \mathcal{N}$, in any equilibrium $t_{mn} = 0$ for $n \in \mathcal{N}$ and $m \in \mathcal{N} \setminus m$.

Second, we provide an example in which higher regulator testing of firm 1 reduces firm 1's equilibrium compliance. Let $N = 2$, $\Delta = 0$, $u_1 = u_2 = 1$, $q_1 = q_2 = 0.48$, $\overline{e} = \overline{d} = 0.99$, $c_1(e) = \max(e^2/2, (2e - \overline{e})/[100(\overline{e} - e)])$, $c_2(e) = c_1(e)/4$, and $d_i(t_{ji}, t_{Ri}) = 2\overline{d}\left(\sqrt{(t_{ji} + t_{Ri})^2 + t_{ji} + t_{Ri}} - t_{ji} - t_{Ri}\right)$ for $i \in \{1, 2\}$ and $j \neq i$. Then, under regulator testing $(t_{R1}, t_{R2}) = (0.266, 5.70)$, the unique equilibrium has zero testing by the firms and compliance $(e_1, e_2) = (0.0691, 0.580)$. Under $(t_{R1}, t_{R2}) = (5.70, 5.70)$, the unique equilibrium has zero testing by the firms and compliance $(e_1, e_2) = (0.0460, 0.866)$. Third, we provide an example with $N = 2$ in which a reduction in the regulator's testing to $t_{Rn} = 0$ for $n \in \{1, 2\}$ strictly increases equilibrium compliance e_n for $n \in \{1, 2\}$. Let $\Delta = 0$, $u_1 = u_2 = 1$, $q_1 = q_2 = 0.48$, $\overline{e} = \overline{d} = 0.99$, $c_i(e) = 0.15 \times \max(e^2, (2e - \overline{e})/[50(\overline{e} - e)])$ and $d_i(t_{ji}, t_{Ri}) = \min\left(\sqrt{\beta_i(t_{ji} + t_{Ri})}, 2\overline{d}\left(\sqrt{[300(t_{ji} + t_{Ri})]^2 + 300(t_{ji} + t_{Ri})} - 300(t_{ji} + t_{Ri})\right)\right)$ for $i \in \{1, 2\}$ and $j \neq i$, where $\beta_1 = 8$ and $\beta_2 = 2$. Then, under regulator testing $(t_{R1}, t_{R2}) = (0.1200, 0.0004)$, the unique equilibrium has zero testing by the firms and compliance $(e_1, e_2) = (0.835, 0.0213)$. Under $(t_{R1}, t_{R2}) = (0, 0)$, the unique equilibrium has testing by the firms $(t_{21}, t_{12}) = (0.07850, 0.00188)$ and compliance $(e_1, e_2) = (0.866, 0.0380)$. ∎

For the proof of Proposition 2, see the proof of the more expansive result Proposition 2A below. Proposition 2A generalizes Proposition 2 to include the fine f and the labeling penalty for noncompliance, which are described in §4. We generalize the right hand side of (9) by replacing $(1 + \Delta)u$ with $(1 + \Delta)(u + f)$.

Proposition 2A *Suppose the firms are symmetric and (8)-(10) hold. The unique symmetric equilibrium compliance effort in the blocking scenario is higher than the compliance effort in any symmetric equilibrium in the labeling scenario. In the labeling scenario, an equilibrium in compliance and testing $(\widehat{e}, \widehat{t}) = (0, 0)$. In the blocking scenario, in the unique symmetric equilibrium, firms test $(\widehat{t} > 0)$ if and only if (11). The unique symmetric equilibrium in the blocking scenario has the following properties: The detection probability for a noncompliant product increases with the number of firms N for $N \leq \overline{N}$ and decreases with N for $N \geq \overline{N}$. The detection probability*

for a noncompliant product increases with the fine f for $f \leq \overline{f}$ and decreases with f for $f \geq \overline{f}$. Compliance $\widehat{e}$ increases with the firms' quality levels u, a consumer's benefit from compliance Δ and the fine f, and decreases with the number of firms N. For the labeling scenario the symmetric equilibrium with the largest compliance level also has these properties.

Lemma 1 is useful in the proof of Proposition 2A. Let $s_0(e,t)$ denote $s(e,\mathbf{t})$, $d_0(t) = d(\mathbf{t})$, $d_1(t) = (\partial/\partial t_{nm})d(\mathbf{t})$, $d_{11}(t) = (\partial^2/\partial t_{nm}t_{km})d(\mathbf{t})$ and $d_2(t) = (\partial^2/\partial t_{nm}^2)d(\mathbf{t})$ for $n \in \mathcal{N}$, $k \in \mathcal{N} \backslash n$ and $m \in \mathcal{N} \backslash (n \cup k)$, where $\mathbf{t} = <t,t,..t,0>$ denotes the vector wherein each firm exerts testing effort t and the regulator does not test. Let

$$f_1(e,t) = u[\mathbf{I}_B s_0(e,t) + \Delta e]q^2(1-e)d_1(t) - 1$$

$$f_2(e,t) = \{u[\mathbf{I}_B + \Delta e/s_0(e,t)](1 - [1+(N-1)s_0(e,t)]q)q + f\}d_0(t) - c'(e),$$

where $\mathbf{I}_B = 1$ in the blocking scenario and $\mathbf{I}_B = 0$ in the labeling scenario. $f_1(e,t)$ is the first derivative of firm n's profit function with respect to t_{nm} and $f_2(e,t)$ is the first derivative with respect to e_n, when each firm $m \in \mathcal{N}$ chooses compliance $e_m = e$ and testing $t_{mk} = t$ for $k \in \mathcal{N} \backslash m$. Define for $t > 0$,

$$\underline{\mathbf{e}}_0(t) = \left(\mathbf{I}_B[2d_0(t)-1] + \Delta - \sqrt{(\mathbf{I}_B+\Delta)^2 - \frac{4[\mathbf{I}_B d_0(t)+\Delta]}{ud_1(t)q^2}}\right) \Big/ (2[\mathbf{I}_B d_0(t)+\Delta])$$

$$\overline{\mathbf{e}}_0(t) = \left(\mathbf{I}_B[2d_0(t)-1] + \Delta + \sqrt{(\mathbf{I}_B+\Delta)^2 - \frac{4[\mathbf{I}_B d_0(t)+\Delta]}{ud_1(t)q^2}}\right) \Big/ (2[\mathbf{I}_B d_0(t)+\Delta]),$$

and let $\underline{\mathbf{e}}_0(0) = \lim_{t\downarrow 0} \underline{\mathbf{e}}_0(t)$ and $\overline{\mathbf{e}}_0(0) = \lim_{t\downarrow 0} \overline{\mathbf{e}}_0(t)$. If $\lim_{t\downarrow 0} d_1(t) < 4\Delta/[uq^2(\mathbf{I}_B+\Delta)^2]$, then $f_1(e,t) < 0$ for $(e,t) \in [0,\overline{e}) \times [0,\infty)$. Suppose instead that $\lim_{t\downarrow 0} d_1(t) \geq 4\Delta/[uq^2(\mathbf{I}_B+\Delta)^2]$ and let $\overline{t}$ denote the unique solution to

$$[\mathbf{I}_B d_0(t) + \Delta]/d_1(t) = uq^2(\mathbf{I}_B+\Delta)^2/4.$$

If $t > \overline{t}$, then no value of e satisfies $f_1(e,t) = 0$; otherwise, $f_1(e,t) = 0$ has two roots in e : $\underline{\mathbf{e}}_0(t)$ and $\overline{\mathbf{e}}_0(t)$. Note that $f_2(e,t)$ is strictly increasing in t and strictly decreasing in e with $\lim_{e\uparrow\overline{e}} f_2(e,t) < 0$. Let

$$\underline{t} = \inf_{t \geq 0}\{t : f_2(0,t) > 0\}.$$

If $t > \underline{t}$, then let $\breve{e}_0(t)$ denote the unique solution to

$$f_2(e,t) = 0,$$

and note that

$$\breve{e}_0(t) > 0;$$

otherwise, let $\breve{e}_0(t) = 0$. Let $\underline{\mathbf{e}}(d) = \underline{\mathbf{e}}_0(t)|_{t \text{ such that } d_0(t)=d}$, $\overline{\mathbf{e}}(d) = \overline{\mathbf{e}}_0(t)|_{t \text{ such that } d_0(t)=d}$, $\breve{e}(d) = \breve{e}_0(t)|_{t \text{ such that } d_0(t)=d}$, $\overline{\overline{d}} = d_0(\overline{t})$ and $\underline{d} = d_0(\underline{t})$. Our assumption that $d(\cdot)$ is continuously differentiable and component-wise strictly increasing implies existence of $\underline{\mathbf{e}}(d)$ and $\overline{\mathbf{e}}(d)$ for $d \in [0, \overline{\overline{d}}]$.

Lemma 1 *Suppose the firms are symmetric and (8)-(10) hold. (a.) In the blocking scenario, if (11) is violated, then the unique symmetric equilibrium in compliance and detection probability $(\widehat{e}, \widehat{d}) = (0, 0)$. Otherwise, the unique symmetric equilibrium $(\widehat{e}, \widehat{d})$ has $\widehat{d} \in (0, \overline{d}]$ and one of the following:*

$$\widehat{e} = \underline{\mathbf{e}}(\widehat{d}) = \breve{e}(\widehat{d}) \tag{19}$$

$$\widehat{e} = \overline{\mathbf{e}}(\widehat{d}) = \breve{e}(\widehat{d}). \tag{20}$$

Further, $\underline{\mathbf{e}}(\cdot)$ is continuous and strictly increasing and $\overline{\mathbf{e}}(\cdot)$ is continuous and strictly decreasing; $\breve{e}(d)$ is continuous on $d \in [0, \overline{d})$ and increasing in d, strictly so on $d \in (\underline{d}, \overline{d})$. Finally, $\breve{e}(0) = 0$; (11) implies $\underline{\mathbf{e}}(0) < 0 < \overline{\mathbf{e}}(0)$. (a.) In the labeling scenario, a symmetric equilibrium in compliance and detection probability $(\widehat{e}, \widehat{d}) = (0, 0)$. The following are necessary and sufficient conditions for a symmetric equilibrium $(\widehat{e}, \widehat{d})$ with $\widehat{d} \in (0, \overline{\overline{d}}]$: any $(\widehat{e}, \widehat{d})$ which satisfies (19) or (20) is a symmetric equilibrium; any symmetric equilibrium $(\widehat{e}, \widehat{d})$ with $\widehat{d} \in (0, \overline{\overline{d}}]$ satisfies (19) or (20).

Proof of Lemma 1: The proof proceeds in eight steps. The first three steps consider both the blocking and labeling scenarios. First, we establish necessary conditions for a symmetric equilibrium. Second, we show that these conditions are sufficient. Third, we establish properties of the functions $\breve{e}(d)$, $\underline{\mathbf{e}}(d)$ and $\overline{\mathbf{e}}(d)$.

Steps 4 to 7 address the blocking scenario. Fourth, we show that $f_1(\breve{e}_0(t), t)$ is strictly decreasing in t. Fifth, we show that if (11) is violated, then the unique symmetric equilibrium has zero compliance and testing. Sixth, we show that if (11) is satisfied, then a symmetric equilibrium must satisfy (19) or (20). Seventh, we show that if (11) is satisfied, then a unique symmetric equilibrium exists.

Step 8, addresses the labeling scenario, establishes that $(\widehat{e}, \widehat{d}) = (0, 0)$ is an equilibrium, and establishes necessary and sufficient conditions for an equilibrium with $\widehat{d} > 0$.

First, we establish necessary conditions for a symmetric equilibrium. In a symmetric equilibrium (e, t), each firm $m \in \mathcal{N}$ chooses compliance $e_m = e$ and testing $t_{mk} = t$ for $m \in \mathcal{N}$ and $k \in \mathcal{N} \backslash m$. If firm n anticipates that the remaining firms $m \in \mathcal{N} \backslash n$ will choose compliance $e_m = e$ and testing $t_{mk} = t$ for $m \in \mathcal{N}$ and $k \in \mathcal{N} \backslash m$, then for compliance $e_n = e$ and testing $t_{nm} = t$ for $m \in \mathcal{N}$ to be a best response for firm n, the following first order conditions must be satisfied

$$(\partial/\partial t_{nm})\pi_n|_{e_i=\tilde{e}_i=e,\ t_{ij}=\tilde{t}_{ij}=t \text{ for } i\in\mathcal{N} \text{ and } j\in\mathcal{N}\backslash i} = f_1(e, t) \leq 0 \tag{21}$$

$$(\partial/\partial e_n)\pi_n|_{e_i=\tilde{e}_i=e,\ t_{ij}=\tilde{t}_{ij}=t \text{ for } i\in\mathcal{N} \text{ and } j\in\mathcal{N}\backslash i} = f_2(e, t) \leq 0, \tag{22}$$

where (21) must hold with equality if $t > 0$ and (22) must hold with equality if $e > 0$.

Second, we establish that any solution to (21)-(22) is a symmetric equilibrium. If firm n anticipates that the remaining firms $m \in \mathcal{N} \backslash n$ will choose compliance $e_m = e$ and testing $t_{mk} = t$

24

for $m \in \mathcal{N}$ and $k \in \mathcal{N}\backslash m$, then any solution to the first order conditions for firm n must for $m \in \mathcal{N}\backslash n$ have $t_{nm} = \tau$ for some $\tau \geq 0$. We can write firm n's expected profit under compliance e_n and testing level τ as

$$\pi_n(e_n, \tau) = (1 - \mathbf{I}_B)u(1 - Nq)q + [\tilde{u} - (1 - \mathbf{I}_B)u]\{1 - [1 + (N-1)s_o(e, t, \tau)]q\}$$
$$\times qs_0(e_n, t) - c(e_n) - (N-1)\tau, \tag{23}$$

where $s_o(e, t_a, t_b) = 1 - d_o(t_a, t_b)(1 - e)$, where $d_o(t_a, t_b)$ denotes the detection probability when all firms but one chooses testing level t_a, one firm chooses testing level t_b and the regulator does not test, $d_o(t_a, t_b) = d(t_a, ..t_a, t_b, t_a, ..t_a, 0)$. The assumptions that $c(\cdot)$ is strictly convex and $(\partial^2/\partial t_{nm}^2)d(\mathbf{t}_m) < -(N-1)u(\mathbf{I}_B + \Delta)(\partial/\partial t_{nm})d(\mathbf{t}_m)^2/[(1-\overline{d})^2 c''(e)]$ imply that π_n is jointly strictly concave in (e_n, τ). Therefore, if (21)-(22) are satisfied, then firm n's best response is $(e_n, \tau) = (e, t)$.

Third, we establish properties of the functions $\breve{e}(d)$, $\underline{e}(d)$ and $\overline{e}(d)$. Because $(\partial^2/\partial t_{nm}^2)d(\mathbf{t}_m) < -\mathbf{I}_B N(\partial/\partial t_{nm})d(\mathbf{t}_m)^2/[\mathbf{I}_B d_0(t) + \Delta]$, $\underline{e}_0(t)$ is strictly increasing and $\overline{e}_0(t)$ is strictly decreasing in t. Therefore, because $d(\cdot)$ is component-wise strictly increasing and continuously differentiable, $\underline{e}(d)$ is continuous and strictly increasing in d and $\overline{e}(d)$ is continuous and strictly decreasing in d. Because $f_2(\cdot, \cdot)$ is continuous, $\breve{e}_0(t)$ is continuous on $t \in [0, \infty)$. By the implicit function theorem, $\breve{e}_0(t)$ is strictly increasing in t for $t \in (\underline{t}, \infty)$

$$\frac{\partial \breve{e}_0(t)}{\partial t} = \frac{(\partial/\partial t)f_2(e, t)}{-(\partial/\partial e)f_2(e, t)}$$
$$= \{u\left[\Delta\{(N-1)s_0(e,t)^2 q - (1-q)[1 - d_0(t)]\}d_0(t)q + \mathbf{I}_B(N-1)q^2 s_0(e,t)^2 d_0(t)^2\right]$$
$$+ s_0(e,t)^2 c''(e)\}^{-1} \times [uq\{[\mathbf{I}_B s_0(e,t)^2 + \Delta e][1 - Nq + (N-1)(1-e)d_0(t)q]$$
$$+ [\mathbf{I}_B s_0(e,t)^2 + \Delta e s_0(e,t)](1-e)(N-1)qd_0(t)\} + s_0(e,t)^2 f](N-1)d_1(t) \tag{24}$$
$$> 0,$$

where the inequality follows from (8). Therefore, because $d(\cdot)$ is component-wise strictly increasing and continuously differentiable, $\breve{e}(d)$ is continuous and strictly increasing in $d \in (\underline{d}, \overline{d})$. Let $f_i(e, 0)$ denote $\lim_{t \downarrow 0} f_i(e, t)$ for $i = 1, 2$.

Steps 4 to 7 address the blocking scenario. Fourth, we establishing that $f_1(\breve{e}_0(t), t)$ is strictly decreasing in t. Note that

$$(\partial/\partial t)f_1(\breve{e}_0(t), t) = u\{-[1 - \breve{e}_0(t)]^2(N-1)d_1(t)^2 - \{(1 - 2d_0(t)[1 - \breve{e}_0(t)]) - \Delta[1 - 2\breve{e}_0(t)]\}d_1(t)\breve{e}'(t)$$
$$+ [s_0(\breve{e}_0(t), t) + \Delta\breve{e}_0(t)][1 - \breve{e}_0(t)][d_2(t) + (N-2)d_{11}(t)]\}q^2. \tag{25}$$

Using (24) and (25), with some effort it is possible to show that $(\partial^2/\partial t_{nm}^2)d(\mathbf{t}_m) < -4N(1+\Delta)(u + f)(\partial/\partial t_{nm})d(\mathbf{t}_m)^2/[(1-\overline{d})^2 c''(e) - u\Delta]$ for $e \in [0, \overline{e})$ implies that $(\partial/\partial t)f_1(\breve{e}_0(t), t) < 0$.

Fifth, suppose (11) is violated. Then $f_1(0, 0) \leq 0$ and $f_2(0, 0) \leq 0$, so $(\widehat{e}, \widehat{t}) = (0, 0)$ is an equilibrium; it remains to show that it is unique. From the analysis in Step 1, a symmetric

25

equilibrium with $t > 0$ must have $(e,t) = (\breve{e}_0(t), t)$, where t satisfies $f_1(\breve{e}_0(t), t) = 0$. Because $f_1(0,0) = f_1(\breve{e}_0(0), 0) \leq 0$ and $f_1(\breve{e}_0(t), t)$ is strictly decreasing in t, $f_1(\breve{e}_0(t), t) < 0$ for $t > 0$. Therefore, any equilibrium must have $t = 0$. Because $f_2(\cdot, 0)$ is strictly decreasing, $f_2(e, 0) < 0$ for $e > 0$; this implies that any equilibrium with $t = 0$ must have $e = 0$. Therefore, $(\widehat{e}, \widehat{d}) = (0, 0)$ is the unique symmetric equilibrium.

Sixth, suppose (11) holds. Because $f_1(0,0) > 0$, $(e,t) = (0,0)$ is not an equilibrium. Because $f_2(e,0) < 0$ for $e \in (0, \overline{e})$, an equilibrium cannot have $t = 0$. Thus, in any equilibrium (21) must hold with equality. Thus, a symmetric equilibrium must satisfy $\widehat{t} \in (0, \overline{t}]$, and $\widehat{e} = \underline{\mathbf{e}}_0(\widehat{t}) = \breve{e}_0(\widehat{t})$ or $\widehat{e} = \overline{\mathbf{e}}_0(\widehat{t}) = \breve{e}_0(\widehat{t})$. Therefore, a symmetric equilibrium must satisfy $\widehat{d} \in (0, \overline{\overline{d}}]$, and (19) or (20).

Seventh, suppose that (11) holds. From Step 6, this implies that a symmetric equilibrium (e,t) must have $t > 0$. From the analysis in Step 1, a symmetric equilibrium must have $(e,t) = (\breve{e}_0(t), t)$ where t satisfies

$$f_1(\breve{e}_0(t), t) = 0, \tag{26}$$

and, from Step 2, any such solution is an equilibrium. To establish that there exists an unique symmetric equilibrium it is sufficient to show that there exists a unique solution to (26). Assumption (11) implies $f_1(\breve{e}_0(0), 0) > 0$; further, $\lim_{t \uparrow \infty} f_1(\breve{e}_0(t), t) < 0$. Therefore, the existence of a unique solution to (26) follows from the fact that $f_1(\breve{e}_0(t), t)$ is strictly decreasing in t (as shown in Step 4). Finally, it is straightforward to verify that (11) implies $\underline{\mathbf{e}}_0(0) < 0 < \overline{\mathbf{e}}_0(0)$, which in turn implies $\underline{\mathbf{e}}(0) < 0 < \overline{\mathbf{e}}(0)$.

Step 8, addresses the labeling scenario, establishes that $(\widehat{e}, \widehat{d}) = (0,0)$ is an equilibrium, and establishes necessary and sufficient conditions for an equilibrium with $\widehat{d} > 0$. Inequality (10) implies that $f_1(0,0) \leq 0$ and $f_2(0,0) \leq 0$, so $(\widehat{e}, \widehat{t}) = (0,0)$ is an equilibrium; equivalently, $(\widehat{e}, \widehat{d}) = (0,0)$ is an equilibrium. Because $f_2(\cdot, 0)$ is strictly decreasing, $f_2(e, 0) < 0$ for $e > 0$; this implies that any equilibrium with $t = 0$ must have $e = 0$. From the analysis in Step 1, a symmetric equilibrium with $t > 0$ must have $(e,t) = (\breve{e}_0(t), t)$ where t satisfies (26) and, from Step 2, any such solution is an equilibrium. Therefore, a symmetric equilibrium with $\widehat{d} \in (0, \overline{\overline{d}}]$ must satisfy (19) or (20).∎

Proof of Proposition 2A: The proof proceeds in four parts. Part A establishes the properties of the symmetric equilibrium in the blocking scenario. Part B establishes the properties of the symmetric equilibrium in the labeling scenario. Part C establishes that the symmetric equilibrium compliance effort in the blocking scenario is higher than any symmetric equilibrium compliance effort in the labeling scenario.

Part A: From Lemma 1, in the blocking scenario, if (11) is violated, then the unique symmetric equilibrium in compliance and detection $(\widehat{e}, \widehat{d}) = (0,0)$. Therefore, in the remainder, we consider

the case where (11) holds. First, we demonstrate the comparative statics for the number of firms N. By the implicit function theorem, $\breve{e}(d)$ is decreasing in N. Let

$$\tilde{N} = \max_{N \in \{2,3,..\}} \left\{ N : \breve{e}(\overline{\overline{d}}) \geq 1 - [1 + \Delta]/[2(\overline{\overline{d}} + \Delta)] \right\}.$$

With some abuse of notation, let $(\widehat{e}(N), \widehat{d}(N))$ denote the unique symmetric equilibrium. If $N \leq \tilde{N}$, then $\widehat{d}$ is the unique solution to

$$\overline{e}(d) - \breve{e}(d) = 0.$$

Further,

$$\overline{e}(d) - \breve{e}(d) \geq 0 \text{ if and only if } d \in [0, \widehat{d}]. \tag{27}$$

For any $N_0 < N_1 \leq \tilde{N}$,

$$\begin{aligned}
0 &= \left[\overline{e}(\widehat{d}(N_0)) - \breve{e}(\widehat{d}(N_0)) \right] |_{N=N_0} \\
&\leq \left[\overline{e}(\widehat{d}(N_0)) - \breve{e}(\widehat{d}(N_0)) \right] |_{N=N_1},
\end{aligned}$$

where the inequality follows because $\breve{e}(d)$ is decreasing in N. This implies that

$$\widehat{d}(N_0) \leq \widehat{d}(N_1) \tag{28}$$

(from (27)). Thus,

$$\widehat{e}(N_0) = \overline{e}(\widehat{d}(N_0)) \geq \overline{e}(\widehat{d}(N_1)) = \widehat{e}(N_1), \tag{29}$$

where the inequality follows from (28) and the fact that $\overline{e}(\cdot)$ is decreasing. By similar argument, for any $\tilde{N} < N_2 < N_3$,

$$\widehat{d}(N_2) \geq \widehat{d}(N_3) \tag{30}$$

$$\widehat{e}(N_2) \geq \widehat{e}(N_3). \tag{31}$$

Further, for any $N_1 \leq \tilde{N} < N_2$,

$$\widehat{e}(N_1) \geq 1 - [1 + \Delta]/[2(\overline{\overline{d}} + \Delta)] \geq \widehat{e}(N_2). \tag{32}$$

Together (29), (31) and (32) imply that $\widehat{e}$ is decreasing in N. Together (28) and (30) imply that $\widehat{d}$ is increasing in N for $N \leq \overline{N}$ and decreasing in N for $N \geq \overline{N}$, where $\overline{N} = \arg\max_{N \in \{2,3,..\}} \{\widehat{d}\}$.

Second, we consider the comparative statics for the fine f. By the implicit function theorem, $\breve{e}(d)$ is increasing in f. By argument parallel to that for the comparative statics for N, $\widehat{e}$ is increasing in f and $\widehat{d}$ is increasing in f for $f \leq \overline{f}$ and decreasing in f for $f \geq \overline{f}$, where $\overline{f} = \arg\max_{f \in [0,\infty)} \{\widehat{d}\}$.

Third, we demonstrate the comparative statics for the quality level u. By the implicit function theorem, $\breve{e}(d)$ is increasing in u. With some effort, one can show that $\overline{e}(d)$ is increasing in u and that $\underline{e}(d)$ is decreasing in u for $d \in [0, \overline{\overline{d}}]$; recall that $\widehat{d} \leq \overline{\overline{d}}$ (by Lemma 1). Because $\underline{e}(\cdot)$ and $\breve{e}(\cdot)$ are continuous, $\underline{e}(0) < \breve{e}(0)$, $\underline{e}(d) < \overline{e}(d)$, and $\widehat{d}$ satisfies either $\underline{e}(\widehat{d}) = \breve{e}(\widehat{d})$ or $\overline{e}(\widehat{d}) = \breve{e}(\widehat{d})$,

$$\underline{e}(d) < \breve{e}(d) \text{ for } d \in [0, \widehat{d}). \tag{33}$$

27

With some abuse of notation, let $(\widehat{e}(u), \widehat{d}(u))$ denote the unique symmetric equilibrium under quality level u. From (33), for any $u_h > u_l$ and any $d < \widehat{d}(u_l)$

$$\underline{\mathbf{e}}(d)|_{u=u_h} \leq \underline{\mathbf{e}}(d)|_{u=u_l} < \breve{e}(d)|_{u=u_l} \leq \breve{e}(d)|_{u=u_h}. \tag{34}$$

If $\widehat{e}(u_h) = \underline{\mathbf{e}}(\widehat{d}(u_h))|_{u=u_h}$, then $\underline{\mathbf{e}}(\widehat{d}(u_h))|_{u=u_h} = \breve{e}(\widehat{d}(u_h))|_{u=u_h}$ and (34) together imply that $\widehat{d}(u_h) \geq \widehat{d}(u_l)$. Therefore,

$$\widehat{e}(u_h) = \breve{e}(\widehat{d}(u_h))|_{u=u_h} \geq \breve{e}(\widehat{d}(u_l))|_{u=u_h} \geq \breve{e}(\widehat{d}(u_l))|_{u=u_l} = \widehat{e}(u_l).$$

It remains to show that if $\widehat{e}(u_h) = \bar{\mathbf{e}}(\widehat{d}(u_h))|_{u=u_h}$, then $\widehat{e}(u_h) \geq \widehat{e}(u_l)$. Suppose $\widehat{e}(u_h) = \bar{\mathbf{e}}(\widehat{d}(u_h))|_{u=u_h}$ and $\widehat{e}(u_l) = \bar{\mathbf{e}}(\widehat{d}(u_l))|_{u=u_l}$. Let $D(u_a, u_b)$ denote the unique solution to

$$\bar{\mathbf{e}}(D)|_{u=u_a} - \breve{e}(D)|_{u=u_b} = 0. \tag{35}$$

Note that when $u_a = u_b = u_i$ for $i \in \{h, l\}$, there is only one solution to (35) and $\widehat{d}(u_i) = D(u_i, u_i)$. We will show that

$$\widehat{d}(u_l) \geq D(u_l, u_h). \tag{36}$$

The proof is by contradiction. Suppose that $\widehat{d}(u_l) < D(u_l, u_h)$. Then

$$\bar{\mathbf{e}}(D(u_l, u_h))|_{u=u_l} < \breve{e}(D(u_l, u_h))|_{u=u_l}$$

$$\leq \breve{e}(D(u_l, u_h))|_{u=u_h}, \tag{37}$$

where the first inequality holds because the continuity of $\underline{\mathbf{e}}(\cdot)$ and $\breve{e}(\cdot)$, $\bar{\mathbf{e}}(0) > \breve{e}(0)$, and the uniqueness of $(\widehat{e}, \widehat{d})$ imply $\bar{\mathbf{e}}(d) < \breve{e}(d)$ for $d > \widehat{d}$; the second inequality holds because $\breve{e}(d)$ is decreasing in u. Because (37) contradicts the definition of $D(u_l, u_h)$, we have established (36). By similar argument,

$$\widehat{d}(u_h) \geq D(u_l, u_h). \tag{38}$$

We conclude that

$$\widehat{e}(u_l) = \bar{\mathbf{e}}(\widehat{d}(u_l))|_{u=u_l} \leq \bar{\mathbf{e}}(D(u_l, u_h))|_{u=u_l} = \breve{e}(D(u_l, u_h))|_{u=u_h} \leq \breve{e}(\widehat{d}(u_h))|_{u=u_h} = \widehat{e}(u_h),$$

where the first inequality follows from (36) and $\bar{\mathbf{e}}(\cdot)$ being decreasing; the second inequality follows from (38) and $\breve{e}(\cdot)$ being increasing. By similar argument, if $\widehat{e}(u_h) = \bar{\mathbf{e}}(\widehat{d}(u_h))|_{u=u_h}$ and $\widehat{e}(u_l) = \underline{\mathbf{e}}(\widehat{d}(u_l))|_{u=u_l}$, then $\widehat{e}(u_l) \leq \widehat{e}(u_h)$.

Fourth, we consider the comparative statics for the consumers' valuation of compliance Δ. By the implicit function theorem, $\breve{e}(d)$ is increasing in Δ. With some effort, one can show that $\bar{\mathbf{e}}(d)$ is increasing in Δ and that $\underline{\mathbf{e}}(d)$ is decreasing in Δ for $d \in [0, \bar{\bar{d}}]$. By argument parallel to that for the comparative statics for u, $\widehat{e}$ is increasing in Δ.

Part B: From Lemma 1, in the labeling scenario, a symmetric equilibrium in compliance and detection probability $(\widehat{e}, \widehat{d}) = (0, 0)$. The proof that the comparative statics in part A apply to the maximal symmetric equilibrium in compliance and detection probability $(\widehat{e}_M, \widehat{d}_M)$ parallels the

proof in Part A. The primary change is that $(\widehat{e}, \widehat{d})$, which denotes the unique solution to (19) or (20) in the blocking scenario, is replaced by $(\widehat{e}_M, \widehat{d}_M)$, which denotes the maximal solution to (19) or (20) in the labeling scenario. In addition, the expression $1 - [1 + \Delta]/[2(\overline{\overline{d}} + \Delta)]$, which appears in the definition of $\tilde{N}$ and in (32) is replaced by $1/2$.

Part C: Because $f_2(e, t)$ is larger in the blocking scenario, $\breve{e}_0(t)$ and hence $\breve{e}(d)$ is larger in the scenario with blocking. Because $f_1(e, t)$ is larger in the blocking scenario, $\overline{e}_0(t)$ is larger and $\underline{e}_0(t)$ is smaller in the blocking scenario for $t \in [0, \overline{t}]$; thus, $\overline{e}(d)$ is larger and $\underline{e}(d)$ is smaller in the blocking scenario for $d \in [0, \overline{\overline{d}}]$. By argument parallel to that for the comparative statics for u in Part A, it follows that the symmetric equilibrium compliance effort in the blocking scenario is higher than the maximal symmetric equilibrium compliance effort in the labeling scenario.∎

Proof of Proposition 3: Any consistent rational expectations equilibrium must satisfy the following first order necessary condition for the testing by firm $n \in \mathcal{N} \backslash m$:

$$(\partial/\partial t_{nm})\pi_n = \min(\overline{u}_n, \overline{u}_m)s(e_n, \mathbf{t}_n)q_m q_n(1 - e_m)(\partial/\partial t_{nm})d_m(\mathbf{t}_m) - 1 \leq 0, \qquad (39)$$

wherein

$$\overline{u}_m \equiv u_m[1 + \Delta e_m/s(e_m, \mathbf{t}_m)] \text{ for } m \in \mathcal{N}$$

and the inequality in (39) must hold with equality if $t_{nm} > 0$. Expression (39) follows from differentiating the expected profit function (6) for firm n with respect to t_{nm} (with $\widetilde{u}_m$ constant, because consumers do not observe t_{nm}) and then substituting $\overline{u}_m$ for $\widetilde{u}_m$ to reflect the consistency of consumers' beliefs, i.e., that $\widetilde{e}_m = e_m$ and $\widetilde{\mathbf{t}}_m = \mathbf{t}_m$ for $m \in \mathcal{N}$.

To establish that firm m with sufficiently low u_m draws no testing, observe that our assumption that $d_m(\mathbf{t}_m)$ is componentwise strictly increasing, continuously differentiable, and satisfies (1) implies that $(\partial^2/\partial t_{nm}^2)d_m(\mathbf{t}_m) < 0$ for $\mathbf{t}_m \in R_+^{N+1}$ and, using the expression for $(\partial/\partial t_{nm})\pi_n$ in (39), that

$$(\partial/\partial t_{nm})\pi_n < u_m(1 + \Delta)q_m q_n(\partial/\partial t_{nm})d_m(\mathbf{t}_m)|_{\mathbf{t}_m = \mathbf{0}} - 1. \qquad (40)$$

Inequality (12) implies that the quantity on the right hand side of (40) is negative for all $n \in \mathcal{N} \backslash m$, which implies that $t_{nm} = 0$ for all $n \in \mathcal{N} \backslash m$ (i.e., firm m draws no testing). If firm m draws no testing, then regardless of her compliance choice $e_m \in [0, \overline{e})$, firm m is successful in bringing its product to market with probability $s(e_m, \mathbf{t}_m) = 1$; because compliance is costly, firm m's optimal compliance $e_m = 0$. Therefore, inequality (12) also implies that $e_m = 0$.∎

Online Supplement

Endogenous Quality: Entrants

Proof of Proposition 4: First consider the adoption of a product standard, enforced through competitor testing. We will refer to this as "adoption of a product standard." The proof proceeds in four parts. In the first part, we establish two properties of the equilibrium prior to the adoption of the product standard. Let $(\hat{N}^h, \hat{N}^l)$ denote the number of high- and low-quality firms prior to the adoption of the product standard. Prior to the adoption of the product standard, the profit of an entering high-quality firm is

$$[u^h(1 - \hat{N}^h q) - u^l \hat{N}^l q]q - k^h \tag{41}$$

and the profit of a low-quality firm is

$$u^l[1 - (\hat{N}^h + \hat{N}^l)q]q - k^l. \tag{42}$$

Because, by assumption, a high-quality firm enters, (41) must be nonnegative:

$$(u^h q - k^h)/(u^h q^2) \geq \hat{N}^h + (u^l/u^h)\hat{N}^l. \tag{43}$$

Similarly, if $\hat{N}^l > 0$, then (42) must be nonnegative:

$$(u^l q - k^l)/(u^l q^2) \geq \hat{N}^h + \hat{N}^l. \tag{44}$$

Furthermore, as (42) strictly decreases with the number of firms, and we have assumed that the potential to earn a nonnegative profit attracts entry, an additional low-quality firm cannot enter and achieve nonnegative profit:

$$u^l[1 - (\hat{N}^h + (\hat{N}^l + 1))q]q - k^l < 0. \tag{45}$$

In the second part, we show that there exist $\gamma \in (0, \infty)$, $\underline{d} < 1$, $\underline{c} \in (0, \infty)$ and $\underline{e} \in (0, \overline{e})$ such that after the adoption of the product standard, the success probability of any high-quality firm is strictly less than

$$\sigma \equiv \min(\sigma_1, \sigma_2, \sigma_3),$$

where $\sigma_1 \equiv k^h/[(1+\Delta)u^h(1-q)q]$, $\sigma_2 \equiv (\hat{N}^h - 2)/(\hat{N}^h - 1)$, and $\sigma_3 \equiv u^h(u^l q(1-2q) - k^l)/[u^l(u^h q(1-q) - k^h)]$, with the exception that if only one high-quality firm does testing, it will have a success probability of 1. Our assumptions that $\hat{N}^h > 2$ and $k^l < u^l(1 - 2q)q$ ensure that σ is strictly positive. Let $\underline{c} = (1 + \Delta)u^h(1 - q)q$, $\gamma = 1/[u^h(1 + \Delta)(1 - \underline{e})(1 - \overline{d})q^2]$, and select $\underline{d} < 1$, and $\underline{e} \in (0, \overline{e})$ such that $1 - \underline{d}(1 - \underline{e}) = \sigma$. Let n denote the index of an incumbent firm that tests competitors' products. Firm n must have high quality because (1) and (39), as in the proof of Proposition 3, imply that in equilibrium, for any low-quality firm L and any other firm $m \in \mathcal{N}\backslash L$,

$$(\partial/\partial t_{Lm})\pi_L < u^l(1 + \Delta)q^2(\partial/\partial t_{Lm})d_m(\mathbf{t}_m)|_{\mathbf{t}_m = \mathbf{0}} - 1,$$

wherein assumption (13) implies the right hand side is negative, so $t_{Lm} = 0$ (i.e., no low-quality firm will test a competitor). Let $m \neq n$ denote the index of any other high-quality firm. The proof

1

proceeds in three steps. First, we show, in equilibrium, that high-quality firm m chooses compliance $e_m < \underline{e}$. If the high-quality firm m chooses compliance $e_m \geq \underline{e}$, then because $c_m(e_m) \geq \underline{c}$, the firm's cost of compliance $c_m(e_m)$ exceeds its expected revenue. If instead the high-quality firm chooses compliance $e_m = 0$, its expected revenue exceeds its cost of compliance (because $c_m(0) = 0$). We conclude that, in equilibrium, $e_m < \underline{e}$. Second, we show that, in equilibrium, the probability that a high-quality firm's noncompliance is detected $d_m(\mathbf{t}_m) > \underline{d}$. Recall that m and $n \neq m$ denote the indices of high-quality firms. Because $d_n(\mathbf{t}_n) < \overline{d}$ and $e_m < \underline{e}$, condition (14) implies that firm n's expected profit is strictly increasing in t_{nm} if $d_m(\mathbf{t}_m) \leq \underline{d}$. We conclude that, in equilibrium, $d_m(\mathbf{t}_m) > \underline{d}$. Third, because $s\,(e_m, \mathbf{t}_m) < 1 - \underline{d}(1 - \underline{e}) = \sigma$ and because the choice of the index m was arbitrary among high-quality firms, we conclude that the success probability of any high-quality firm is strictly less than σ (with the exception that if only one incumbent firm does testing, it will have a success probability of 1).

In the third part, we show that no additional high-firms will enter following the adoption of the product standard. From the previous step, after the adoption of the product standard, the success probability of a high-quality entrant would be strictly less than σ, so its expected profit would be strictly less than

$$(1 + \Delta)u^h(1 - q)\sigma q - k^h; \tag{46}$$

$\sigma \leq \sigma_1$ implies that (46) is negative, so no additional high-firms will enter.

In the fourth part, we show that at least one additional low-quality firm will enter following the adoption of the product standard. From the previous step, $\sigma \leq \sigma_1$ implies that no additional high-quality firms will enter. Proposition 3 and our assumption (13) imply that low-quality firms will not comply, test, or draw testing from competitors. If Λ low-quality firms enter, the expected profit of an entering low-quality firm is strictly greater than $u^l q[1 - (\sigma(\hat{N}^h - 1) + 1 + \hat{N}^l + \Lambda)q] - k^l$. Therefore, to establish that at least one additional low-quality firm will enter following the adoption of the product standard, it is sufficient to show that

$$u^l[1 - (\sigma(\hat{N}^h - 1) + 1 + \hat{N}^l + 1)q]q - k^l \geq 0. \tag{47}$$

First, suppose that $\hat{N}^l > 0$. Together $\hat{N}^h > 2$ and $\sigma \leq \sigma_2$ imply that $\hat{N}^h + \hat{N}^l \geq \sigma(\hat{N}^h - 1) + 1 + \hat{N}^l + 1$. This, together with (44) implies (47). Second, suppose that $\hat{N}^l = 0$. Then $\sigma \leq \sigma_3$ and (43) imply (47).∎

Endogenous Quality: Incumbents

Proof of Proposition 5: Without loss of generality, order the quality levels such that $u^1 < u^2 < .. < u^M$. Let $\{\breve{u}_n\}_{n \in \mathcal{N}}$ denote equilibrium quality levels of the firms under no product standard. Let $u^j = \max_{n \in \mathcal{N}} \breve{u}_n$, i.e., the highest equilibrium quality level is u^j. Let H denote the index of a

firm with this quality level, $\check{u}_H = u^j$. If firm H anticipates that the remaining firms $m \in \mathcal{N} \backslash H$ will choose quality $u_m = \check{u}_m$, where $\check{u}_m \le u^j$, then for quality $u_H = u^j$ to be a best response for firm H it must be that firm H's profit under quality $u_H = u^j$ is greater than under quality $u_H = u^i$ for $i > j : [u^j(1 - q_H) - \Sigma_{m \in \mathcal{N} \backslash H} \check{u}_m q_m] q_H - k_H^j \ge [u^i(1 - q_H) - \Sigma_{m \in \mathcal{N} \backslash H} \check{u}_m q_m] q_H - k_H^i$, which implies

$$k_H^i - k_H^j \ge (u^i - u^j)(1 - q_H) q_H \text{ for all } i > j. \tag{48}$$

We will show that under a product standard enforced by competitor testing, in any equilibrium firm H's quality is reduced from its level under no standard. Let $\{\hat{u}_n, \hat{e}_n, \hat{t}_{nm}\}_{n \in \mathcal{N}, \, m \in \mathcal{N} \backslash n}$ denote equilibrium quality, compliance and testing levels of the firms under a product standard enforced by competitor testing. To establish that firm H's quality is reduced from its level under no standard $\hat{u}_H \le \check{u}_H$, suppose to the contrary that $\hat{u}_H = u^i$ where $i > j$. Under competitor testing, if firm H anticipates that the remaining firms $m \in \mathcal{N} \backslash H$ will choose quality $u_m = \hat{u}_m$, compliance $e_m = \hat{e}_m$ and testing $t_{mn} = \hat{t}_{mn}$ for $n \in \mathcal{N} \backslash m$, then for quality $u_H = u^i$, compliance $e_H = \hat{e}_H$ and testing $t_{Hn} = \hat{t}_{Hn}$ for $m \in \mathcal{N} \backslash H$ to be a best response for firm H it must be that

$$\left[u^i(1 - q_H) - \sum_{m \in \mathcal{N} \backslash H} \min(u^i, u_m) s(\hat{e}_m, \hat{\mathbf{t}}_m) q_m \right] s(\hat{e}_H, \hat{\mathbf{t}}_H) q_H - c_H(\hat{e}_H) - \sum_{m \in \mathcal{N} \backslash H} \hat{t}_{Hm} - k_H^i$$

$$\ge \max_{(e_H, t_{Hm})_{m \in \mathcal{N} \backslash H} \in [0, \bar{e}] \times R_+^{N+1}} \left\{ \left[u^j(1 - q_H) - \sum_{m \in \mathcal{N} \backslash H} \min(u^j, u_m) s(\hat{e}_m, \hat{\mathbf{t}}_{m-}, t_{Hm}) q_m \right] s(e_H, \hat{\mathbf{t}}_H) q_H \right.$$

$$\left. - c_H(e_H) - \sum_{m \in \mathcal{N} \backslash H} t_{Hm} \right\} - k_H^j$$

$$\ge \left[u^j(1 - q_H) - \sum_{m \in \mathcal{N} \backslash H} \min(u^j, u_m) s(\hat{e}_m, \hat{\mathbf{t}}_m) q_m \right] s(\hat{e}_H, \hat{\mathbf{t}}_H) q_H - c_H(\hat{e}_H) - \sum_{m \in \mathcal{N} \backslash H} \hat{t}_{Hm} - k_H^j, \tag{49}$$

where $\hat{\mathbf{t}}_{m-}, t_{Hm}$ denotes the vector $\hat{\mathbf{t}}_m$ wherein $\hat{t}_{Hm}$ is replaced by t_{Hm}. Inequality (49) implies

$$\left[(u^i - u^j)(1 - q_H) - \sum_{m \in \mathcal{N} \backslash H} [\min(u^i, u_m) - \min(u^j, u_m)] s(\hat{e}_m, \hat{\mathbf{t}}_m) q_m \right] s(\hat{e}_H, \hat{\mathbf{t}}_H) q_H \ge k_H^i - k_H^j. \tag{50}$$

It is straightforward to verify that the right hand side of (48) is strictly larger than the left hand side of (50). Consequently, inequality (48) implies that (50) is violated. We conclude that firm H's quality is reduced from its level under no standard $\hat{u}_H \le u^j = \check{u}_H$.

Suppose the firms are symmetric. Let $\check{u}$ denote a symmetric equilibrium quality level under no product standard. By parallel argument to that above, if $\check{u} = u^j$ is an equilibrium under no product standard, then

$$k^i - k^j \ge (u^i - u^j)(1 - q)q \text{ for all } i > j. \tag{51}$$

Let $(\hat{u}, \hat{e}, \hat{\mathbf{t}})$ denote symmetric equilibrium quality, compliance and testing levels under a product standard enforced by competitor testing. By parallel argument to that above, if a symmetric equilibrium under competitor testing has quality $\hat{u} = u^i$ for $i > j$, then

$$(u^i - u^j)[1 - q - (N - 1)s(\hat{e}, \hat{\mathbf{t}})q]s(\hat{e}, \hat{\mathbf{t}})q \ge k^i - k^j. \tag{52}$$

3

It is straightforward to verify that the right hand side of (51) is strictly larger than the left hand side of (52). Consequently, inequality (51) implies that (52) is violated. We conclude that the equilibrium quality under competitor testing is reduced from its level under no standard $\hat{u} \leq u^j = \check{u}$. ∎

Other Penalties for Noncompliance

Proposition 1. (a) The proof is unchanged. (b) For the first part, for the blocking scenario the proof is unchanged by the fine $f \geq 0$. For the labeling scenario, it is straightforward to show that $(\partial/\partial t_{nm})\pi_n \leq \max_{i \in \mathcal{N}} u_i(1 + \Delta)(\partial/\partial t_{nm})d_m(t_m) - 1$. The conclusion that in any equilibrium the firms do not test follows by the same argument as in the blocking scenario. For the second part, we provide an example in the labeling scenario in which higher regulator testing of firm 2 reduces firm 2's equilibrium compliance. Let $N = 2$, $\Delta = 12.8$, $u_1 = u_2 = 1$, $q_1 = q_2 = 0.48$, $\overline{e} = \overline{d} = 0.99$, $f = 0$, $c_1(e) = \max(e^2/2, (2e - \overline{e})/[100(\overline{e} - e)])$, $c_2(e) = 0.8 \times c_1(e)$, and $d_i(t_{ji}, t_{Ri}) = 2\overline{d}\left(\sqrt{(t_{ji} + t_{Ri})^2 + t_{ji} + t_{Ri}} - t_{ji} - t_{Ri}\right)$ for $i \in \{1, 2\}$ and $j \neq i$. Then, under regulator testing $(t_{R1}, t_{R2}) = (0.068, 0.035)$, the unique equilibrium has zero testing by the firms and compliance $(e_1, e_2) = (0.518, 0.844)$. Under $(t_{R1}, t_{R2}) = (0.068, 0.040)$, the unique equilibrium has zero testing by the firms and compliance $(e_1, e_2) = (0.591, 0.633)$.

Proposition 2. The proof is provided as the proof of Proposition 2A in the appendix.

Proposition 3. Proposition 3 holds when $\mathbf{I}_L \min_{i \in \mathcal{N}}\{u_i\}/(1 + \Delta)$, where $\mathbf{I}_L = 1$ in the labeling scenario and $\mathbf{I}_L = 0$ in the blocking scenario, is added to the right hand side of (12). The proof in the blocking scenario is unchanged by the fine $f \geq 0$. The proof in the labeling scenario follows that in the blocking scenario, where in (39) $\min(\overline{u}_n, \overline{u}_m)s(e_n, \mathbf{t}_n)$ is replaced by $[\min(\overline{u}_n, \overline{u}_m) - \min(\overline{u}_n, u_m)]$ and in (40) $u_m(1 + \Delta)$ is replaced by $[u_m(1 + \Delta) - \min_{i \in \mathcal{N}}\{u_i\}]$.

Corollary and Proposition 4. Labeling reverses these results: A product standard, enforced through competitor testing and labeling, *decreases* the expected profit of firms that do not comply and, in the setting of Proposition 4, does *not* cause entry by low-quality, noncompliant firms. The proof is as follows: Our assumption (13) and Proposition 3 imply that a low-quality firm L will not comply. Therefore, in any rational expectations equilibrium, $\widetilde{u}_L = u^l$ so firm L cannot obtain a higher selling price by testing and reporting on any competitor, its expected profit (15) would strictly decrease with any testing, and it does not test. Therefore firm L has the same profit with or without the product standard, expressed in (42). From (45), prior to the adoption of the product standard, entry of an additional low quality firm would result in negative profit for the low quality firm. That profit could only decrease due to entry by additional firms of high quality. Therefore, no firm of low quality will enter.

Endogenous Production Quantities

Proposition 1. The statement of the Proposition is unchanged, except that "any initial equilibrium in compliance and testing by the firms $(\widehat{\mathbf{e}}, \widehat{\mathbf{t}})$" is replaced by "any initial equilibrium in compliance, quantity and testing by the firms $(\widehat{\mathbf{e}}, \widehat{\mathbf{q}}, \widehat{\mathbf{t}})$." The proofs of part (a.) and the first part of (b.) are unchanged. For the second part of (b.), with the addition of $C_n(q_n) = 0$ for $q_n \leq 0.48$ and $C_n(q_n) = 100$ otherwise for $n = 1, 2$ to the numerical example, the proof holds.

Regarding Proposition 2, we present numerical results. For the 125 parameter combinations of $N \in \{2, 3, 4, 5, 6\}$, $\Delta \in \{0.0, 0.1, 0.2, 0.3, 0.4\}$, $u \in \{0.6, 0.8, 1.0, 1.2, 1.4\}$, $\overline{e} = \overline{d} = 0.999$, $c(e) = 0.01 \times \max(e^2, (2e - \overline{e})/[1000(\overline{e} - e)])$, and $d(\mathbf{t}_n) = \min\left(\sqrt{200\Sigma_{m \in \mathcal{N}\backslash n} t_{mn}}, 2\overline{d}\left(\sqrt{(200\Sigma_{m \in \mathcal{N}\backslash n} t_{mn})^2 + 200\Sigma_{m \in \mathcal{N}\backslash n} t_{mn}} - 200\Sigma_{m \in \mathcal{N}\backslash n} t_{mn}\right)\right)$, the results are consistent with Proposition 2: Equilibrium compliance $\widehat{e}$ increases with u and Δ, and decreases with N. The equilibrium detection probability increases and then decreases with N.

Proposition 3. With $u_m \leq 1/\{(1+\Delta)\max_{n \in \mathcal{N}\backslash m}[(\partial/\partial t_{nm})d_m(\mathbf{t}_m)|_{\mathbf{t}_m=\mathbf{0}}]\}$ in (12), the proof of Proposition 3 in the appendix establishes that for any given quantities $(q_m, q_n) \in [0,1]^2$, firm m does not draw testing from competitors so does not comply. In equilibrium, every firm $n \in \mathcal{N}$ chooses a quantity $q_n \in [0,1)$ because $C_n(q_n)$ strictly increases with q_n and if a firm were to supply $q_n \geq 1$ to the market, its price (4) would be zero.

Corollary A considers a setting with two types of firms. Let $\{h, l\} \in \mathcal{S}$ denote the sets of high and low quality firms, where $\mathcal{S}$ denotes the set of all firms. Firms of the same type have the same quality, compliance cost function, linear production cost and detection probability function: for $n \in S$ where $S \in \{h, l\}$, $u_n = u^S$, $c_n(e_n) = c^S(e_n)$, $C_n(q_n) = C^S q_n$ and $d_n(\mathbf{t}_n) = d^S(\mathbf{t}_n)$. Let $N^S = |S|$ for $S \in \{h, l\}$. In a symmetric equilibrium, firms of the same type have the same compliance, quantity and detection probability: for $n \in S$ where $S \in \{H, L\}$, $e_n = e^S$, $q_n = q^S$, and $d_n(\mathbf{t}_n) = d^S$. As in §2 we assume $\Sigma_{n \in \mathcal{N}} q_n < 1$.

Corollary A *Suppose firms are of two types, where type specifies the quality, compliance cost function, linear production cost and detection probability function, and a consumer's benefit from compliance $\Delta = 0$. Suppose that a product standard is enforced through competitor testing. In any symmetric equilibrium, that increases the expected profit of firms with low quality $u^l \leq 1/[\max_{m \in \mathcal{N}\backslash n}(\partial/\partial t_{mn})d^l(\mathbf{t}_n)|_{\mathbf{t}_n=\mathbf{0}}]$, which do not comply with the standard.*

Proof of Corollary A: By Proposition 3, in equilibrium, a low quality firm $n \in l$, does not draw testing, $t_{mn} = 0$ for all $m \in \mathcal{N}\backslash n$, does not comply $e_n = 0$, and is successful in bringing its product to market with probability $s(e_n, \mathbf{t}_n) = 1$. Let $\sigma = s(e_n, \mathbf{t}_n)$ denote the success probability for a high quality firm $n \in H$ in an equilibrium. If high quality firm $H \in h$ anticipates that

5

high quality firms have success probability σ, other high quality firms produce quantity $q^h(\sigma)$, and low quality firms produce quantity $q^l(\sigma)$, then firm H chooses q_H to maximize expected profit $\pi_H = [u^h(1 - q_H) - N^l u^l q^l(\sigma) - (N^h - 1)u^h \sigma q^h(\sigma)]\sigma q_H - C^h q_H$. Similarly, if low quality firm $L \in l$ anticipates that high quality firms have success probability σ and produce quantity $q^h(\sigma)$, and other low quality firms produce quantity $q^l(\sigma)$, then firm L chooses q_L to maximize expected profit

$$\pi_l = u^l[1 - q_L - (N^l - 1)q^l(\sigma) - N^h \sigma q^h(\sigma)]q_L - C^l q_L. \tag{53}$$

The first order conditions for the profit maximizing quantity for firm $H \in h$ and $L \in l$ imply that in a symmetric equilibrium, when the success probability of a high-quality firm is σ, the production quantities of the low- and high-quality firms are $q^l(\sigma) = \{u^l[N^h C^h + (2 - \sigma)u^h] - u^h C^l[2 + (N^h - 1)\sigma]\}/\{u^l[(N^l + 1)(2 + (N^h - 1)\sigma)u^h - N^h N^l \sigma u^l]\}$ and $q^h(\sigma) = \{\sigma(N^l C^l + u^h + N^l(u^h - u^l)) - (N^l + 1)C^h\}/\{\sigma[(N^l + 1)(2 + (N^h - 1)\sigma)u^h - N^h N^l \sigma u^l]\}$. Observe from (53) that the equilibrium expected profit of a low-quality firm is decreasing in $(N^l - 1)q^l(\sigma) + N^h \sigma q^h(\sigma)$ and that without a new product standard enforced through competitor testing, the success probability of a high quality firm $\sigma = 1$. Therefore, to show that the expected profit of a low quality firm increases as a result of the new product standard enforced through competitor testing, it is sufficient to show that

$$(N^l - 1)q^l(1) + N^h \sigma q^h(1) \geq [(N^l - 1)q^l(\sigma) + N^h \sigma q^h(\sigma)]|_{\sigma \in [0,1)}. \tag{54}$$

It is straightforward to show that

$$(\partial/\partial\sigma)[(N^l - 1)q^l(\sigma) + N^h \sigma q^h(\sigma)] = 2N^h Z/[(N^l + 1)(2 + (N^h - 1)\sigma)u^h - N^h N^l \sigma u^l]^2 \tag{55}$$

where $Z = 2u^h(N^l C^l + u^h + N^l(u^h - u^l)) + [(N^h - 1)(N^l + 1)u^h - N^h N^l u^l]C^h$. Because $q^h(1) \geq 0$, $N^l C^l + u^h + N^l(u^h - u^l) \geq (N^l + 1)C^h$. Therefore, $Z \geq [(N^h + 1)(N^l + 1)u^h - N^h N^l u^l]C^h$, which implies that (55) is positive, which in turn implies that (54) holds.∎

Fixed Costs for Testing

Proposition 1. (a.) Define $\widehat{\mathbf{t}} = <\widehat{\mathbf{t}}_1, ..., \widehat{\mathbf{t}}_N>$ and $\widehat{\mathbf{t}}'(\mathbf{t}_R) = <\widehat{\mathbf{t}}'_1, ..., \widehat{\mathbf{t}}'_N>$, where, as in the proof of Proposition 1a in the appendix, $\widehat{\mathbf{t}}_n$ represents the initial equilibrium testing of firm n by the other firms when the regulator does not test and $\widehat{\mathbf{t}}'_n$ is the candidate equilibrium testing of firm n by the other firms with regulator testing $\mathbf{t}_R$, where $\mathbf{t}_R = <t_{R1}, t_{R2}, .., t_{RN}>$. Recall that $\widehat{\mathbf{t}}'_n$ is constructed from $\widehat{\mathbf{t}}_n$ and t_{Rn} for each $n \in \mathcal{N}$. Let $\pi^0_m(\mathbf{t}_R)$ denote the expected profit for firm m, assuming: regulator testing $\mathbf{t}_R$; firms $n \in \mathcal{N}\backslash m$ test competitors according to $\widehat{\mathbf{t}}'(\mathbf{t}_R)$; firms $n \in \mathcal{N}\backslash m$ maintain their initial equilibrium compliance levels; firm m chooses not to test competitors (avoiding the fixed cost ϕ_m); and firm m chooses compliance e_m to maximize its expected profit. Define $\varepsilon_m = \widehat{\pi}_m - \pi^0_m(\mathbf{0})$ where $\widehat{\pi}_m$ denotes the expected profit of firm m in the initial equilibrium and,

by construction, $\pi_m^0(\mathbf{0})$ is the maximum expected profit of firm m if firm m chooses not to incur the fixed cost ϕ_m to test competitors, the regulator does not test, and the other firms $n \in \mathcal{N}\backslash m$ have the initial equilibrium testing and compliance levels. Extend definition (7) to define τ_n as the minimum of $\arg\max\{t_{Rn} : d_n(\mathbf{0}, t_{Rn}) \leq d_n(\widehat{\mathbf{t}}_n, 0)\}$ and

$$\arg\max\{\|\mathbf{t}_R\|_\infty : \ \pi_m^0(\mathbf{t}_R) - \pi_m^0(\mathbf{0}) \leq \varepsilon_m \text{ for } m \in \mathcal{N} \text{ such that } \widehat{t}_{mn} > 0 \text{ for any } n \in \mathcal{N}\}, \quad (56)$$

where $\|\mathbf{t}_R\|_\infty = \max\{|t_{R1}|, .., |t_{RN}|\}$. The proof of Proposition 1a in the appendix establishes that, with regulator testing $t_{Rn} \leq \tau_n$ for $n \in \mathcal{N}$, the (lowered) testing levels $\widehat{\mathbf{t}}'(\mathbf{t}_R)$ maintain the initial detection probability for a noncompliant product and, with the initial equilibrium compliance levels, constitute an equilibrium for the firms–*if no firm $m \in \mathcal{N}$ can achieve strictly greater expected profit with a unilateral deviation from testing competitors to not doing so (avoiding the fixed cost ϕ_m).* Constraining t_{Rn} to be smaller than (56) for $n \in \mathcal{N}$ ensures that $\widehat{\pi}_m \geq \pi_m^0(\mathbf{t}_R)$ for every firm m that was testing in the initial equilibrium, i.e., every firm m with $\widehat{t}_{mn} > 0$ for any $n \in \mathcal{N}$. That means that with a unilateral deviation to cease testing competitors, firm m has weakly lower expected profit $\pi_m^0(\mathbf{t}_R)$ than in the initial equilibrium without regulator testing, $\widehat{\pi}_m$. Clearly, firm m has greater expected profit in the candidate equilibrium with regulator testing than in the initial equilibrium without regulator testing. Therefore, no firm deviates to avoid the fixed cost of testing; the candidate equilibrium is indeed an equilibrium. A sufficient condition for τ_n to be strictly positive is that firm n draws testing from competitors in the initial equilibrium, and a firm $m \in \mathcal{N}$ chooses to incur the fixed cost ϕ_m only if, by doing so, the firm strictly increases its expected profit.

(b.) For the first part, the proof is unchanged. For the second part, we provide an example with $N = 2$ in which increasing regulator testing from $t_{Rn} = 0$ to $t_{Rn} > 0$ for $n \in \mathcal{N}$ strictly reduces the compliance effort for both firms, strictly reduces the detection probability for a noncompliant product for both firms, and causes the firms not to test, in equilibrium. Let $\Delta = \phi_R = 0$, $\phi_1 = \phi_1 = 0.01$, $u_1 = u_2 = 1$, $q_1 = q_2 = 0.48$, $\overline{e} = \overline{d} = 0.99$, $c_i(e) = 0.1 \times \max(e^2, (2e - \overline{e})/[50(\overline{e} - e)])$ and $d_i(t_{ji}, t_{Ri}) = \min\left(\sqrt{20(t_{ji} + t_{Ri})}, 2\overline{d}\left(\sqrt{[50(t_{ji} + t_{Ri})]^2 + 50(t_{ji} + t_{Ri})} - 300(t_{ji} + t_{Ri})\right)\right)$ for $i \in \{1, 2\}$ and $j \neq i$. Under regulator testing $t_{R1} = t_{R2} = 0$, the unique equilibrium has testing by the firms $t_{12} = t_{21} = 0.0235$, compliance $e_1 = e_2 = 0.583$ and detection probability $d_1 = d_2 = 0.686$. Then, under regulator testing $t_{R1} = t_{R2} = 0.0151$, the unique equilibrium has zero testing by the firms, compliance $e_1 = e_2 = 0.473$ and detection probability $d_1 = d_2 = 0.550$.

Proposition 2A. The result holds provided that the firms' fixed cost of testing $\phi < \overline{\phi}$, for some $\overline{\phi} > 0$. The proof of Proposition 2A is unchanged. The proof of Lemma 1 is modified by inserting the following immediately after Step 2: Steps 1 and 2 have established necessary and sufficient conditions for a symmetric equilibrium under $\phi = 0$. In this next step, we show that there exists

$\overline{\phi} > 0$ such that the necessary and sufficient conditions for a symmetric equilibrium under $\phi \in (0, \overline{\phi})$ are identical to the necessary and sufficient conditions under $\phi = 0$. Again suppose firm n anticipates that the remaining firms $m \in \mathcal{N} \backslash n$ will choose compliance $e_m = e$ and testing $t_{mk} = t$ for $m \in \mathcal{N}$ and $k \in \mathcal{N} \backslash m$. By the same argument in Step 2, under $\phi > 0$, any solution to the first order conditions for firm n must for $m \in \mathcal{N} \backslash n$ have $t_{nm} = \tau$ for some $\tau \geq 0$. We generalize the definition in (23) by adding the term $-\mathbf{I}_{\{\tau > 0\}} \phi$ to the right hand side. Let $\psi_n(e_n, \tau) = \pi_n(e_n, \tau) + \mathbf{I}_{\{\tau > 0\}} \phi$. Thus, $\psi_n(e_n, \tau)$ denotes firm n's expected profit under $\phi = 0$, and $\pi_n(e_n, \tau)$ denotes firm n's expected profit under $\phi > 0$. Step 2 establishes that under $\phi = 0$, there exists a unique best response for firm n, which we denote $(e_n, \tau) = (\hat{e}, \hat{\tau})$. Under $\phi > 0$, there exists a best response for firm n; to see this observe that there exists $\varsigma \in (0, \infty)$ such that if $\tau \geq \varsigma$, then $\pi_n(e_n, \tau) < 0$; therefore, in terms of maximizing $\pi_n(e_n, \tau)$ one can, without loss of generality, restrict attention to $(e_n, \tau) \in [0, \overline{e}) \times [0, \varsigma]$. Let $(e_n, \tau) = (\check{e}, \check{\tau})$ denote a best response for firm n under $\phi \in (0, \overline{\phi})$. We will show that there exists $\overline{\phi} > 0$ such that if $\phi \in (0, \overline{\phi})$, then

$$(\hat{e}, \hat{\tau}) = (\check{e}, \check{\tau}), \tag{57}$$

that is, there exists a unique best response for firm n under $\phi \in (0, \overline{\phi})$ and it is identical to the best response under $\phi = 0$. If $\hat{\tau} > 0$, then let

$$\overline{\phi} = \psi_n(\hat{e}, \hat{\tau}) - \max_{e_n \in [0, \overline{e})} \psi_n(e_n, 0),$$

and observe that $\overline{\phi} > 0$. First, suppose $\hat{\tau} > 0$ and that there exists a best response under $\phi \in (0, \overline{\phi})$ with $\check{\tau} = 0$. Then

$$\pi_n(\hat{e}, \hat{\tau}) = \psi_n(\hat{e}, \hat{\tau}) - \phi > \psi_n(\hat{e}, \hat{\tau}) - \overline{\phi} = \max_{e_n \in [0, \overline{e})} \psi_n(e_n, 0) = \max_{e_n \in [0, \overline{e})} \pi_n(e_n, 0) = \pi_n(\check{e}, \check{\tau}),$$

which contradicts that $(e_n, \tau) = (\check{e}, \check{\tau})$ is a best response under $\phi \in (0, \overline{\phi})$. Therefore, if $\hat{\tau} > 0$, then any best response under $\phi \in (0, \overline{\phi})$ must have $\check{\tau} > 0$, which implies that the best response is unique and that it satisfies (57). Second, suppose $\hat{\tau} = 0$ and that there exists a best response under $\phi \in (0, \overline{\phi})$ with $\check{\tau} > 0$. Then

$$\pi_n(\hat{e}, \hat{\tau}) = \pi_n(\hat{e}, 0) = \psi_n(\hat{e}, 0) > \psi_n(\check{e}, \check{\tau}) > \psi_n(\check{e}, \check{\tau}) - \phi = \pi_n(\check{e}, \check{\tau}),$$

which contradicts that $(e_n, \tau) = (\check{e}, \check{\tau})$ is a best response under $\phi \in (0, \overline{\phi})$. Therefore, if $\hat{\tau} = 0$, then any best response under $\phi \in (0, \overline{\phi})$ must have $\check{\tau} = 0$, which implies that the best response is unique and that it satisfies (57). We conclude that there exists $\overline{\phi} > 0$ such that the necessary and sufficient conditions for a symmetric equilibrium under $\phi \in (0, \overline{\phi})$ are identical to the necessary and sufficient conditions under $\phi = 0$.

Proposition 3. The proof is unchanged.