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DC SQUID AS A TUNED RADIOFREQUENCY AMPLIFIER

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September 1984

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Abstract

The design criteria for a tuned radiofrequency amplifier based on a dc SQUID are briefly discussed. A practical amplifier involving a planar dc SQUID with a 4-1/4-turn spiral input coil and operated at 4.2K had a gain of 18.6 ± 0.5 dB and a noise temperature of 1.7 ± 0.5 K at 93 MHz. These results are in satisfactory agreement with predicted values.

Introduction

In an earlier paper,¹ we described a radiofrequency amplifier involving an untuned input circuit coupled to a dc Superconducting QUantum Interference Device (SQUID). However, for certain applications, notably nuclear magnetic resonance (NMR) and nuclear quadrupole resonance (NQR), it is advantageous to tune the input circuit. In this paper, we describe briefly the design and operation of a tuned radiofrequency amplifier based on a dc SQUID.

Theory

Figure 1 shows a dc SQUID in a tuned amplifier configuration. The SQUID has an inductance L , and is biased with a current I . Each Josephson junction has a critical current I_0 , a self-capacitance C , and a shunt resistance R . There is a voltage $V(t)$ across the SQUID and a current $J(t)$ circulating around the SQUID loop. The magnetic flux threading the SQUID is Φ . The SQUID is coupled to an input coil with inductance L_1 via a mutual inductance $M_1 = \alpha(LL_1)^{1/2}$. In series with the input coil is a resistor, R_1 , a capacitor, C_1 , and a pick-up coil with an inductance L_p . A voltage V_1 may be applied to the input circuit; alternatively, a signal may be generated by means of a changing magnetic flux in the pick-up coil, as would be the case, for example, for NMR or NQR. Not shown in Fig. 1 is the parasitic capacitance between the SQUID and the input coil which, in some cases, may influence the behavior of the SQUID quite significantly.²

Figure 1. Configuration of tuned radiofrequency amplifier.

In general, the impedance of the input circuit will be modified by the presence of the SQUID, while the parameters of the SQUID will be modified by the presence of the input circuit.³⁻⁵ We first discuss the effect

of the dynamic impedance of the SQUID on the input circuit. The flux-to-current transfer function of the SQUID, $J_\Phi \equiv (\partial J / \partial \Phi)_I$, is related to the dynamic inductance, $\mathcal{L}$, and dynamic resistance, $\mathcal{R}$, by the relation⁶

$$J_\Phi = -\frac{1}{\mathcal{L}} - \frac{j\omega}{\mathcal{R}}, \quad (1)$$

where $\omega/2\pi$ is the signal frequency. The transfer function is a property of the SQUID at the Josephson frequency, $\omega_J/2\pi$, and one observes a time-averaged value at the signal frequency, which is assumed to be small compared with $\omega_J/2\pi$. In general, J_Φ will be modified by the presence of the input circuit.^{3,4} The total inductance of the input is easily shown to be⁵

$$L_p + L_1' = L_p + L_1 + \Delta L_1 = L_p + L_1(1 - \alpha^2 \mathcal{L}/L). \quad (2)$$

We note that $\mathcal{L}$ is a strong function of the current and flux biases, and can be of either polarity. In the same way, the dynamic resistance of the SQUID induces a resistance in the input circuit. However, for the SQUID that we use in our experiments, there is a parasitic capacitance between the SQUID and the input coil, and the change in the input resistance turns out to be dominated by capacitive feedback from the SQUID output to the input circuit. If we approximate the distributed capacitance between the SQUID and the input coil by a single capacitor, C_p , a simple model predicts⁵

$$R_1' = R_1 + \Delta R_1 = R_1 - \omega_0^2 M_1^2 V_\Phi C_p L_1. \quad (3)$$

where $f_0 = \omega_0/2\pi$ is the resonant frequency of the input circuit. For certain designs of amplifier, these corrections to the input impedance can be substantial. Since both ΔL_1 and ΔR_1 depend on the flux in the SQUID, they vary with the signal amplitude, thereby introducing nonlinearities in the gain and bandwidth of the input circuit, and increasing the noise temperature above the optimum value. Thus, it is important to design the tuned amplifier so as to minimize these effects.

The presence of the input circuit implies that the transfer functions V_Φ and J_Φ and the spectral densities of the low frequency noise $S_V(f) = 2Y_V k_B T R$, $S_J(f) = 2Y_J k_B T / R$ and $S_{VJ}(f) = 2Y_{VJ} k_B T$ may differ from their values for the bare SQUID;^{6,7} here $S_V(f)$ and $S_J(f)$ are the spectral densities of $V(t)$ and $J(t)$, and $S_{VJ}(f)$ is the cross-spectral density of $V(t)$ and $J(t)$. It can be shown that the noise voltage across the SQUID coupled to the input circuit of Fig. 1 takes the form⁸

$$V_N^R(\omega) = V_H^R(\omega) + \frac{V_\Phi^R \alpha_{eff}^2 L J_N^R(\omega) (R_1 + 1/j\omega C_1)}{R_1 + j\omega(L_1 + L_p) + 1/j\omega C_1 - J_\Phi^R \alpha_{eff}^2 L (R_1 + 1/j\omega C_1)}. \quad (4)$$

Here, $V_N^R(\omega)$ and $J_N^R(\omega)$ refer to the voltage and current noises of a SQUID with inductance reduced to $L(1 - \alpha_{eff}^2)$, and V_Φ^R and J_Φ^R are the corresponding transfer functions;⁸ we define $\alpha_{eff}^2 = \alpha^2/(1 + L_p/L_1)$. The nu-

erator of the second term on the right hand side was obtained previously by Tesche* for the case $L_p = 0$.

In the limit of a high Q of present interest, at the resonant frequency $\omega_0 = [C_1(L_1 + L_p)]^{-1/2}$ Eq. (3) simplifies to

$$V_N^C(\omega) = V_N^R(\omega) - \frac{j\omega_0 M_1^2 V_\phi^R R_N^R(\omega)}{R_1^R}, \quad (5)$$

where, for a SQUID in which capacitive feedback effects dominate, the reflected resistance R_1^R is given by Eq. (3). As we will demonstrate in due course, the optimal operation of the amplifier requires $\alpha_{eff}^2 Q = 1$. Hence, in the high Q limit, $\alpha_{eff}^2 \ll 1$ and the reduced SQUID parameters resume their bare values.

Thus, in the high Q limit, Eq. (3) reduces to the result of Clarke, Tesche, and Giffard* except for the modification of the input resistance. [In ref. 7, the minus sign in Eq. (5) was erroneously replaced by a plus sign; this did not affect any of the results for the tuned amplifier.] We can write the noise temperature of the amplifier at the resonant frequency in the form*

$$T_N(f_0) = \frac{\omega_0^2 L_1 R_T}{2\alpha^2 L R_1 V_\phi^2} \left[\gamma_V \left(\frac{R_1^R}{\omega L_1} \right)^2 + \gamma_J \left(\frac{\alpha^2 L V}{R} \right)^2 \right]. \quad (6)$$

At resonance, $T_N(f_0)$ is independent of γ_V . Minimizing Eq. (6) with respect to R_1 to obtain the optimal source resistance, we find

$$R_1^{opt} = [(R_{10}^{opt})^2 + (\Delta R_1)^2]^{1/2}, \quad (7)$$

where

$$R_{10}^{opt} = (\gamma_J / \gamma_V)^{1/2} \omega_0^2 M_1^2 V_\phi / R \quad (8)$$

is the optimal source impedance when the effects on the SQUID on the input resistance can be neglected. The optimized noise temperature is

$$T_N^{opt} = (\gamma_V \gamma_J)^{1/2} \omega_0 T / V_\phi. \quad (9)$$

Equation (8) has an important consequence. To a reasonable approximation,* $(\gamma_J / \gamma_V)^{1/2} = 1$ and $V_\phi = R/L$, so that

$$R_{10}^{opt} = \alpha^2 \omega L_1. \quad (10)$$

As a result, the quality factor of the tuned input circuit neglecting contributions from the SQUID, $Q = \omega_0(L_1 + L_p)/R_1$, takes the optimal value

$$Q = (1 + L_p/L_1)/\alpha^2 \quad (11)$$

or

$$Q\alpha_{eff}^2 = 1, \quad (12)$$

where $\alpha_{eff}^2 = \alpha^2(1 + L_p/L_1)$. Thus, it becomes evident that a reasonably high value of Q (say 100) implies a small value of α_{eff}^2 (say 0.01). If α^2 is close to unity, as is the case for the present SQUID, one must introduce a large series inductance, L_p , to satisfy the optimization condition, Eq. (11).

We now examine the influence of ΔL_1 and ΔR_1 on the input circuit in the light of Eq. (11). The fractional change in the resonant frequency due to ΔL_1 is

$$\left| \frac{\Delta f_0}{f_0} \right| = \frac{1}{2} \left| \frac{\Delta L_1}{L_1 + L_p} \right| = \frac{1}{2\alpha^2 Q} \left| \frac{\Delta L_1}{L_1} \right| = \frac{1}{2Q} \left| \frac{L}{L'} \right|. \quad (13)$$

This fractional change is independent of frequency, and inversely proportional to Q. The fractional change in Q is

$$\left| \frac{\Delta Q}{Q} \right| = \left| \frac{\Delta R_1}{R_1^{opt}} \right| = \frac{\omega_0 M_1 |V_\phi| C_p}{\alpha^2} \quad (14)$$

and scales as ω_0 . In the next section, we will give numerical estimates of $|\Delta f_0/f_0|$ and $|\Delta Q/Q|$.

Experiment

We used a planar, thin-film SQUID with a 4-1/4-turn spiral input coil.^{1,2} The parameters had the approximate values $L = 400$ pH, $I_0 = 10$ μ A, $\beta = 2LI_0/\Phi_0 = 4$, $R = 8$ Ω , $C = 0.5$ pF, $\beta_0 = 0.8$, $L_1 = 5.6$ nH, $M_1 = 1$ nH and $\alpha^2 = 0.6$. The measured value of V_ϕ was about 3×10^{10} s⁻¹ (≈ 1.5 R/L). The self-resonance of the input coil was at about 450 MHz with a Q higher than 100, so that one could use this amplifier at frequencies up to about 400 MHz.

We first measured the changes in the impedance of the input circuit due to its coupling to the SQUID at a frequency of 27 MHz. At this relatively low frequency we were able to connect a spectrum analyzer with a high input impedance across the high-Q input circuit without loading it substantially.³ The maxima of $|L/L'|$ occurred at $\phi = n\Phi_0/2$ and their value never exceeded 0.6 at any bias parameters, so that the maximum fractional change in input inductance, $|\Delta L_1/L_1| = \alpha^2 |L/L'|$, was never greater than 40%. In practice, one operates an amplifier with a flux bias near $(n \pm 1/4)\Phi_0$. Over the working range of the amplifier, say within $\pm \Phi_0/10$ of the operating point, $|L/L'|$ was always less than 0.2 and the fractional change in input inductance was correspondingly smaller, roughly 15%.

The change in input resistance, ΔR_1 , scales with V_ϕ and thus is a maximum near the operating point of the amplifier. At 27 MHz, we found the maximum value of $|\Delta R_1|$ to be about 30 m Ω . Hence at 100 MHz, we expect $|\Delta R_1|$ not to exceed 0.4 Ω , a correction of about 20% to the optimal input resistance ($\alpha^2 \omega L_1$) of about 2 Ω . Within $\pm \Phi_0/10$ of the operating point, however, ΔR_1 should not change by more than about 7%.

These preliminary measurements together with Eqs. (10), (13) and (14) allow us to make the following predictions: $|\Delta f_0/f_0| \leq 1/3Q$ and $|\Delta Q/Q| \leq 0.2$ for any bias condition, and $T_N^{opt} = 1.1$ K at 100 MHz and at 4.2 K, using the values $\gamma_V = 25$ and $\gamma_J = 6$ (obtained from an analog simulator with $\beta = 4$). The predicted power gain, $M_1^2 V_\phi^2 / R_1 R_D$, is about 17 dB for a SQUID dynamic output resistance, R_D , of 8 Ω and an input resistance, R_1 , of 2 Ω .

We tested the performance of our dc SQUID as a tuned radiofrequency amplifier at frequencies near 100 MHz. The input resistance, R_1 , was 2 Ω and the input capacitance, C_1 , was 20 pF. A cold, 50- Ω signal source was connected across the input resistance. The input circuit had a resonant frequency of 113 MHz and an average Q of about 36. The resonant frequency corresponds to a stray series inductance in the input circuit, L_p , of about 90 nH, so that $\alpha_{eff}^2 = 0.036$. Hence the value of $Q\alpha_{eff}^2$ was about 1.3, close to the optimal value. The maximum shift in the resonant frequency, f_0 , as the flux bias of the SQUID was varied over a flux quantum was ± 0.8 MHz or less than 1% of f_0 , as expected. The fractional change in Q between $\phi = (n + 1/4)\Phi_0$ and $\phi = (n - 1/4)\Phi_0$ was $\pm 25\%$, agreeing approximately with our prediction and furthermore verifying the fact that ΔR_1 scales with ω_0^2 . The associated change in power gain was ± 2.7 dB.

We then measured the gain and the noise temperature

of the tuned SQUID amplifier with the same values of R_1 and C_1 . For the measurements, we enclosed the input resistor in a vacuum can so that the temperature of the resistor could be raised above the bath temperature by means of a heater. By measuring the total noise out of the amplifier as a function of the temperature of R_1 , we were able to determine the noise temperature of the amplifier quite accurately.¹ However, in this configuration the stray inductance was increased somewhat, and the resonant frequency was reduced to 93 MHz. At a bath temperature of 4.2K, the measured gain was 18.6 ± 0.5 dB, while the noise temperature was 1.7 ± 0.5 K. Considering the uncertainty in the values of the SQUID parameters and, in particular, our neglect of the effects of the parasitic capacitance on the SQUID noise characteristics and transfer functions, we feel that the agreement between predicted and measured performance is quite satisfactory.

Concluding Remarks

We have seen that the optimum noise temperature for a tuned amplifier is obtained when the conditions $R_1 = a^2 \omega L_1$ and $Q a_{eff}^2 = 1$ are satisfied. When Q is reasonably large and a_{eff}^2 correspondingly small, the mutual influences of the SQUID and input circuit on each other become relatively unimportant. The requirement of low a_{eff}^2 would probably be met most advantageously by increasing the spacing between the SQUID and the input coil, thereby reducing the parasitic capacitance substantially, possibly to an insignificant level. In the present configuration, however, the coupling coefficient a^2 was relatively high (≈ 0.6), and a low effectively coupling coefficient $a_{eff}^2 = a^2 / (1 + L_p/L_1)$ was obtained by introducing a large series inductance L_p . The performance obtained with this amplifier at 93 MHz and 4.2K, namely a gain of 18.2 ± 0.5 dB and a noise temperature of 1.7 ± 0.5 K, was in satisfactory agreement with predictions.

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