## Title
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# Custodial Symmetry, Flavor Physics, and the Triviality Bound on the Higgs Mass

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The triviality of the scalar sector of the standard one-doublet Higgs model implies that this model is only an effective low-energy theory valid below some cut-off scale $\Lambda$. We show that the experimental constraint on the amount of custodial symmetry violation implies that the scale $\Lambda$ must be greater than of order 7.5 TeV. The underlying high-energy theory must also include flavor dynamics at a scale of order $\Lambda$ or greater in order to give rise to the different Yukawa couplings of the Higgs to ordinary fermions. This flavor dynamics will generically produce flavor-changing neutral currents. We show that the experimental constraints on the neutral $D$-meson mass difference imply that $\Lambda$ must be greater than of order 21 TeV. For theories defined about the infrared-stable Gaussian fixed-point, we estimate that this lower bound on $\Lambda$ yields an upper bound of approximately 460 GeV on the Higgs boson's mass, independent of the regulator chosen to define the theory. We also show that some regulator schemes, such as higher-derivative regulators, used to define the theory about a different fixed-point are particularly dangerous because an infinite number of custodial-isospin-violating operators become relevant.

## 1 Triviality and the Standard Model[1]

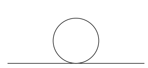
In the standard Higgs model, one introduces a fundamental scalar doublet:

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} , \tag{1}$$

with potential:

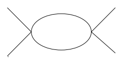
$$V(\phi) = \lambda \left( \phi^\dagger \phi - \frac{v^2}{2} \right)^2 . \tag{2}$$

While this theory is simple and renormalizable, it has a number of shortcomings. First, while the theory can be constructed to accommodate the breaking of electroweak symmetry, it provides no *explanation* for it – one simply assumes that the potential is of the form in eqn. (2). In addition, in the absence of supersymmetry, quantum corrections to the Higgs mass are naturally of order the largest scale in the theory



$$\Rightarrow m_H^2 \propto \Lambda^2 , \tag{3}$$

leading to the hierarchy and naturalness problems.[2] Finally, the $\beta$ function for the self-coupling $\lambda$



$$\Rightarrow \beta = \frac{3\lambda^2}{2\pi^2} > 0 \ , \tag{4}$$

leading to a "Landau pole" and triviality.[3]

The hierarchy/naturalness and triviality problems can be nicely summarized in terms of the Wilson renormalization group. Define the theory with a fixed UV-cutoff:

$$\begin{aligned} \mathcal{L}_\Lambda = \ & D^\mu\phi^\dagger D_\mu\phi + m^2(\Lambda)\phi^\dagger\phi + \tfrac{\lambda(\Lambda)}{4}(\phi^\dagger\phi)^2 \\ & + \tfrac{\hat{\kappa}(\Lambda)}{36\Lambda^2}(\phi^\dagger\phi)^3 + \ldots \end{aligned} \tag{5}$$

Here $\hat{\kappa}$ is the coefficient of a representative irrelevant operator, of dimension greater than four. Next, integrate out states with $\Lambda' < k < \Lambda$, and construct a new Lagrangian with the same *low-energy* Green's functions:

$$\begin{aligned} \mathcal{L}_\Lambda & \Rightarrow \mathcal{L}_{\Lambda'} \\ m^2(\Lambda) & \rightarrow m^2(\Lambda') \\ \lambda(\Lambda) & \rightarrow \lambda(\Lambda') \\ \hat{\kappa}(\Lambda) & \rightarrow \hat{\kappa}(\Lambda') \end{aligned} \tag{6}$$

The low-energy behavior of the theory is then nicely summarized in terms of the evolution of couplings in the infrared.[a] A three-dimensional representation of this flow in the infinite-dimensional space of couplings shown in Figure 1.

From Figure 1, we see that as we scale to the infrared the coefficients of irrelevant operators, such as $\hat{\kappa}$, tend to zero; *i.e.* the flows are attracted to the finite dimensional subspace spanned (in perturbation theory) by operators of dimension four or less; this is the modern understanding of *renormalizability.* On the other hand, the coefficient of the only *relevant* operator (of dimension 2), $m^2$, tends to infinity. This leads to the naturalness/hierarchy problem.[2] Since we want $m^2 \propto v^2$ at low energies we must adjust the value of $m^2(\Lambda)$ to a precision of

$$\frac{\Delta m^2(\Lambda)}{m^2(\Lambda)} \propto \frac{v^2}{\Lambda^2} \ . \tag{7}$$

Central to our discussion here is the fact that the coefficient of the only marginal operator $\lambda$ tends to 0, because of the positive $\beta$ function. If we try to

[a] For convenience, we ignore the corrections due to the weak gauge interactions. In perturbation theory, at least, the presence of these interactions does not qualitatively change the features of the Higgs sector.

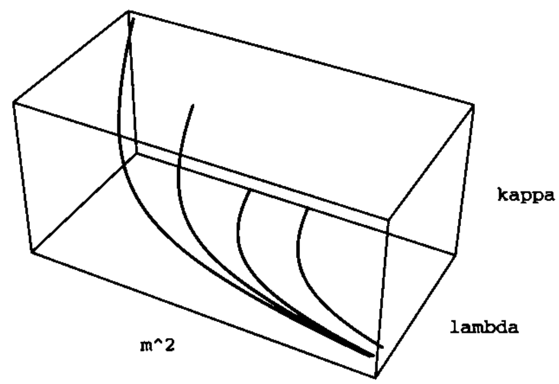



Figure 1: Renormalization group flow of Higgs mass $m^2$, Higgs self-coupling $\lambda$, and the coefficient of a representative irrelevant operator $\hat{\kappa}$. The flows go from upper-left to lower-right as one scales to the infrared.

take the continuum limit, $\Lambda \to +\infty$, the theory becomes free or trivial.[3] The triviality of the scalar sector of the standard one-doublet Higgs model implies that this theory is only an effective low-energy theory valid below some cut-off scale $\Lambda$. Physically this scale marks the appearance of new strongly-interacting symmetry-breaking dynamics. Examples of such high-energy theories include "top-mode" standard models[4] and composite Higgs models.[5] As the Higgs mass increases, the upper bound on the scale $\Lambda$ decreases. An estimate of this effect can be obtained by integrating the one-loop $\beta$-function, which yields

$$\lambda(m_H) \lesssim \frac{2\pi^2}{3\log\frac{\Lambda}{m_H}} \,. \tag{8}$$

Using the relation $m_H^2 = 2\lambda(m_H)v^2$ we find

$$m_H^2 \ln\left(\frac{\Lambda}{m_H}\right) \leq \frac{4\pi^2 v^2}{3} \,. \tag{9}$$

Hence a lower bound[6,7] on $\Lambda$ yields an upper bound on $m_H$. We must require that $M_H/\Lambda$ in eqn. (9) be small enough to afford the effective Higgs theory some range of validity (or to minimize the effects of regularization in the context of a calculation in the scalar theory). Quantitative[8] studies on the lattice using analytic and Monte Carlo techniques result in an upper bound on the Higgs mass of approximately 700 GeV. The lattice Higgs mass bound is potentially ambiguous because the precise value of the bound on the Higgs boson's mass depends on the (arbitrary) restriction placed on $M_H/\Lambda$. The "cut-off" effects arising from the regulator are not universal: different schemes can give rise to different effects of varying sizes and can change the resulting Higgs mass bound.

In this talk we show that, for models that reproduce the standard one-doublet Higgs model at low energies, electroweak and flavor phenomenology provide a lower bound on the scale $\Lambda$ of order $10 - 20$ TeV that is regularization-independent (i.e. independent of the details of the underlying physics). Using eqn. (9) we estimate that this gives an *upper* bound of $450 - 500$ GeV on the Higgs boson mass.

The discussion we have presented is based on perturbation theory and is valid in the domain of attraction of the "Gaussian fixed point" ($\lambda = 0$). In principle, however, the Wilson approach can be used *non-perturbatively*, even in the presence of nontrivial fixed points or large anomalous dimensions. In a conventional Higgs theory, neither of these effects is thought to occur.[8] We return to the issue of the possible existence of other, potentially non-trivial, fixed points in section 4 below.

## 2 Dimensional Analysis

We will analyze the effects of the underlying physics by estimating the sizes of various operators in a low-energy effective lagrangian containing the (presumably composite) Higgs boson and the ordinary gauge bosons and fermions. Since we are considering theories with a heavy Higgs field, we expect that the underlying high-energy theory will be strongly interacting. Borrowing a technique from QCD we will rely on dimensional analysis[9] to estimate the sizes of various effects of the underlying physics.

A strongly interacting theory has no small parameters. As noted by Georgi,[10] a theory [b] with light scalar particles belonging to a single symmetry-group representation depends on two parameters: $\Lambda$, the scale of the underlying physics, and $f$ (the analog of $f_\pi$ in QCD), which measures the amplitude for producing the scalar particles from the vacuum. Our estimates will depend on the ratio $\kappa = \Lambda/f$, which is expected to fall between 1 and $4\pi$.

Consider the kinetic energy of a scalar bound-state in the appropriate low-energy effective lagrangian. The properly normalized kinetic energy is

$$\partial^\mu \phi^\dagger \partial_\mu \phi = \Lambda^2 f^2 \left(\frac{\partial^\mu}{\Lambda}\right) \left(\frac{\phi^\dagger}{f}\right) \left(\frac{\partial_\mu}{\Lambda}\right) \left(\frac{\phi}{f}\right) \ , \tag{10}$$

[b] These dimensional estimates only apply if the low-energy theory, when viewed as a scalar field theory, is defined about the infrared-stable Gaussian fixed-point. We return to potentially "non-trivial" theories below.

where, because the fundamental scale of the interactions is $\Lambda$, we ascribe a $\Lambda$ to each derivative and an $f$ to each $\phi$ since $f$ measures the amplitude to produce the bound state. This tells us that the overall magnitude of each term in the effective lagrangian is $\mathcal{O}(f^2\Lambda^2)$. We can next estimate the "generic" size of a mass term in the effective theory:

$$m^2\phi^\dagger\phi = \Lambda^2 f^2 \left(\frac{\phi^\dagger}{f}\right)\left(\frac{\phi}{f}\right) \Rightarrow m^2 \propto \Lambda^2 \ . \tag{11}$$

This is a reproduction of the hierarchy problem. In the absence of some other symmetry not accounted for in these rules, fine-tuning [c] is required to obtain $m^2 \ll \Lambda^2$. Next, consider the size of scalar interactions. From the simplest interaction

$$\lambda(\phi^\dagger\phi)^2 \Rightarrow \lambda \propto \left(\frac{\Lambda}{f}\right)^2 = \kappa^2 \ , \tag{12}$$

we see that $\kappa$ will determine the size of coupling constants. Similarly, for a higher-dimension interaction such as the one in eqn. (5) we find

$$\frac{\hat\kappa}{\Lambda^2}(\phi^\dagger\phi)^3 \Rightarrow \hat\kappa \propto \kappa^4 \ . \tag{13}$$

These rules are easily extended to include strongly-interacting fermions self-consistently. Again, we start with the properly normalized kinetic-energy

$$\bar\psi\partial\!\!\!/\psi = \Lambda^2 f^2 \left(\frac{\bar\psi}{f\sqrt{\Lambda}}\right)\left(\frac{\partial\!\!\!/}{\Lambda}\right)\left(\frac{\psi}{f\sqrt{\Lambda}}\right) \ , \tag{14}$$

and learn that $f\sqrt{\Lambda}$ is a measure of the amplitude for producing a fermion from the vacuum. Next, consider a Yukawa coupling of a strongly-interacting fermion to our composite Higgs,

$$y(\bar\psi\phi\psi) \Rightarrow y \propto \kappa \ . \tag{15}$$

And finally, the natural size of a four-fermion operator is

$$\frac{\nu}{\Lambda^2}(\bar\psi\psi)^2 \Rightarrow \nu \propto \kappa^2 \ . \tag{16}$$

We will rely on these estimates to derive bounds on the scale $\Lambda$. By way of justification, we note that these estimates work in QCD for the chiral-Lagrangian,[9] with $f \to f_\pi$, $\Lambda \to 1$ GeV, and $\kappa \approx \mathcal{O}(4\pi)$. For example, four

[c] We will not be addressing the hierarchy problem here; we will simply assume that some other symmetry or dynamics has produced the appropriate light scalar state.

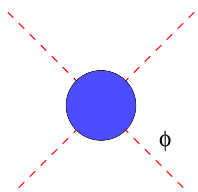
nucleons operators of the form shown in eqn. (16) arise in the vector channel from $\rho$-exchange and we obtain $\Lambda = m_\rho$ and $\kappa = g_\rho \approx 6$. In a QCD-like theory with $N_c$ colors and $N_f$ flavors one expects[11] that

$$\kappa \approx \min\left(\frac{4\pi a}{N_c^{1/2}}, \frac{4\pi b}{N_f^{1/2}}\right) , \tag{17}$$

where $a$ and $b$ are constants of order 1. In the results that follow, we will display the dependence on $\kappa$ explicitly; when giving numerical examples, we set $\kappa$ equal to the geometric mean of 1 and $4\pi$, *i.e.* $\kappa \approx 3.5$.

## 3 Isospin Violation and Bounds[13] on $m_H$

Because of the $SU(2)_W \times U(1)_Y$ symmetry of the low-energy theory, all terms of dimension less than or equal to four respect custodial symmetry.[12] The leading custodial-symmetry violating operator is of dimension six[14,15] and involves four Higgs doublet fields $\phi$. According to the rules of dimensional analysis, the operator




$$\Rightarrow \frac{\kappa^2}{\Lambda^2}(\phi^\dagger D^\mu \phi)(\phi^\dagger D_\mu \phi) , \tag{18}$$

should appear in the low-energy effective theory with a coefficient of order one.[15] Such an operator will give rise to a deviation

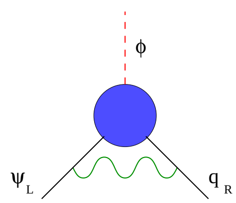
$$\Delta\rho_* = -\mathcal{O}\left(\kappa^2 \frac{v^2}{\Lambda^2}\right) , \tag{19}$$

where $v \approx 246$ GeV is the expectation value of the Higgs field. Imposing the constraint[18,19] that $|\Delta\rho_*| \leq 0.4\%$, we find the lower bound

$$\Lambda \gtrsim 4\,\mathrm{TeV} \cdot \kappa . \tag{20}$$

For $\kappa \approx 3.5$, we find $\Lambda \gtrsim 14$ TeV.

Alternatively, it is possible that the underlying strongly-interacting dynamics respects custodial symmetry. Even in this case, however, there must be custodial-isospin-violating physics (analogous to extended-technicolor[16,17] interactions) which couples the $\psi_L = (t,\ b)_L$ doublet and $t_R$ to the strongly-interacting "preon" constituents of the Higgs doublet in order to produce a top quark Yukawa coupling at low energies and generate the top quark mass. If, for simplicity, we assume that these new weakly-coupled custodial-isospin-violating interactions are gauge interactions with coupling $g$ and mass $M$, dimensional analysis allows us to estimate the size of the resulting top quark Yukawa coupling. The "natural size" of a Yukawa coupling (eqn. (15)) is $\kappa$ and that of a four-fermion operator (eqn. (16)) is $\kappa^2/\Lambda^2$; the ratio $(g^2/M^2)/(\kappa^2/\Lambda^2)$ is the "small parameter" associated with the extra flavor interactions and we find




$$\Rightarrow \frac{g^2}{M^2}\frac{\Lambda^2}{\kappa}\bar{q}_R\phi\psi_L \ . \tag{21}$$

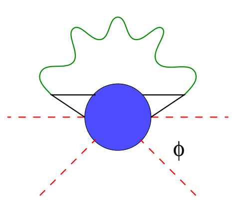
In order to give rise to a quark mass $m_q$, the Yukawa coupling must be equal to

$$\frac{\sqrt{2}m_q}{v} \tag{22}$$

where $v \approx 246$ GeV. This implies

$$\Lambda \gtrsim \frac{M}{g}\sqrt{\sqrt{2}\kappa\frac{m_q}{v}} \ . \tag{23}$$

These new gauge interactions will typically also give rise to custodial-isospin-violating 4-preon interactions [d] which, at low energies, will give rise to an operator of the same form as the one in eqn. (18). Using dimensional analysis, we find




$$\Rightarrow \left[\frac{g^2}{M^2}\left(\frac{\kappa^2}{\Lambda^2}\right)^{-1}\right]\frac{\kappa^2}{\Lambda^2}(\phi^\dagger D^\mu\phi)(\phi^\dagger D_\mu\phi) \ , \tag{24}$$

which results in the bound $M/g \gtrsim 4$ TeV. From eqn. (23) with $m_t \approx 175$ GeV we then derive the limit

$$\Lambda \gtrsim 4\,\mathrm{TeV}\cdot\sqrt{\kappa} \ . \tag{25}$$

For $\kappa \approx 3.5$, we find $\Lambda \gtrsim 7.5$ TeV.

[d] These interactions have previously been considered in the context of technicolor theories.[20]

## 4 Non-Trivial Scaling

Dimensional analysis was crucial to the discussion given above. If the low-energy Higgs theory does not flow toward the trivial Gaussian fixed-point in the infrared limit, the scaling dimensions of the fields and operators can be very different than naively expected. In this case the bounds given above do not apply.

A nice example of a scalar theory with non-trivial behavior has been given by Jansen, Kuti, and Liu.[21] They consider a theory defined by an $O(4)$-symmetric Lagrange density with a modified kinetic-energy

$$\mathcal{L}_{kin} = -\frac{1}{2}\phi^\dagger(\Box + \frac{\Box^3}{\mathcal{M}^4})\phi \ . \tag{26}$$

In the large-$N$ limit, this higher-derivative kinetic term is sufficient to eliminate all divergences. A lattice simulation of this theory[22] indicates that this approach can be used to define a non-trivial Higgs theory with a Higgs boson mass as high as 2 TeV, while avoiding any noticeable effects from the (complex-conjugate) pair of ghosts which are present because of the higher derivative kinetic-energy term.

As shown by Kuti,[23] in the infrared this higher-derivative theory flows to a non-trivial fixed point on an infinite dimensional critical surface, which corresponds to a continuum field theory with an infinite number of relevant operators. The reason there are an infinite number of relevant operators is that, if the continuum limit is taken so that the scale $\mathcal{M}$ remains finite as required in order to flow to a non-trivial theory, the scaling dimension[23] of the Higgs doublet field $\phi$ is -1 instead of the canonical value of +1!

If one could impose an exact $O(4)$ symmetry on the symmetry breaking sector, this would lead to a strongly-interacting electroweak symmetry-breaking sector without technicolor[22]. However, as argued above, custodial isospin violation in the flavor sector must couple to the symmetry-breaking sector to give rise to the different top- and bottom-quark masses. Furthermore, if the scaling dimension of the Higgs field is -1, there is an infinite class of custodial-isospin-violating operators (including the operator in eqn. (18)) which are relevant. Since these operators are relevant, even a small amount of custodial isospin violation coming from high-energy flavor dynamics will be amplified as one scales to low energies, ultimately contradicting the bound on $\Delta\rho_*$. We therefore conclude that these non-trivial scalar theories cannot provide a phenomenologically viable theory of electroweak symmetry breaking.

To construct a phenomenologically viable theory of a strongly-interacting Higgs sector it is not sufficient to construct a theory with a heavy Higgs boson;

one must also ensure that all potentially custodial-isospin-violating operators remain irrelevant.[e]

## 5 Flavor-Changing Neutral-Currents[25]

The high-energy flavor physics responsible for the generation of the quark-preon couplings *must* distinguish between different flavors so as to give rise to the different masses of the corresponding fermions. In addition to the Higgs-fermion coupling discussed above, the flavor physics will also give rise to flavor-specific couplings of ordinary fermions to themselves. These new current-current interactions among ordinary fermions generically give rise to flavor-changing neutral currents (as previously noted[16] for the case of ETC theories) that affect Kaon and $D$-meson physics. For instance, consider the interactions responsible for the $s$-quark mass. Through Cabibbo mixing, these interactions must couple to the $d$-quark as well. This will give rise to the interactions

$$\begin{aligned}\mathcal{L}_{eff} \;=\; & -(\cos\theta^s_L \sin\theta^s_L)^2 \frac{g^2}{M^2}(\overline{s}_L\gamma^\mu d_L)(\overline{s}_L\gamma_\mu d_L) \\ & -(\cos\theta^s_R \sin\theta^s_R)^2 \frac{g^2}{M^2}(\overline{s}_R\gamma^\mu d_R)(\overline{s}_R\gamma_\mu d_R) \\ & -\cos\theta^s_L \sin\theta^s_L \cos\theta^s_R \sin\theta^s_R \frac{g^2}{M^2}(\overline{s}_L\gamma^\mu d_L)(\overline{s}_R\gamma_\mu d_R)\ , \qquad (27)\end{aligned}$$

where the coupling $g$ and mass $M$ are of the same order as those in the interactions which ultimately give rise to the $s$-quark Yukawa coupling in eqn. (21), and the angles $\theta^s_L$ and $\theta^s_R$ represent the relation between the gauge eigenstates and the mass eigenstates. The operators in eqn. (27) will clearly affect neutral Kaon physics. Similarly, the interactions responsible for other quarks' masses will give rise to operators that contribute to mixing and decays of various mesons.

Since the operators responsible for generating quark masses and for causing flavor-changing neutral currents violate flavor symmetries differently,[26] in principle one could construct a theory with an approximate GIM symmetry.[26,27,28] In such models, flavor-changing neutral currents would be suppressed but different quarks would still receive different masses. A theory of this type which included a light scalar state (unlike previous examples[26,27,28]) would be able to evade the flavor-changing neutral current limits discussed here.

[e] This is also a concern in walking technicolor.[24]

*5.1 Flavor-Changing Neutral Currents: $\Delta S$*

To start, let us consider the four-fermion interactions in eqn. (27), which will alter the predicted value of the $K_L - K_S$ mass difference. Using the vacuum-insertion approximation,[29] we can estimate separately how much the purely left-handed (LL), purely right-handed (RR) and mixed (LR) current-current operators contribute. Requiring each contribution to be less than the observed mass difference $\Delta m_K$, we find the bounds

$$\left(\frac{M}{g}\right)_{\rm LL,RR} \gtrsim f_K \left(\frac{2 m_K B_K}{3\Delta m_K}\right)^{1/2} \cos\theta^s_{L,R} \sin\theta^s_{L,R} \qquad (28)$$

$$\approx 0.92\times 10^3\,{\rm TeV}\cos\theta^s_{L,R}\sin\theta^s_{L,R} \qquad (29)$$

from the first two operators in eqn. (27), and

$$\left(\frac{M}{g}\right)_{\rm LR} \gtrsim f_K \left\{\frac{m_K B'_K}{3\Delta m_K}\left[\frac{m_K^2}{(m_s+m_d)^2}-\frac{3}{2}\right]\right\}^{1/2} (\cos\theta^s_L\sin\theta^s_L\cos\theta^s_R\sin\theta^s_R)^{1/2}$$

$$\approx 1.4\times 10^3\,{\rm TeV}\,(\cos\theta^s_L\sin\theta^s_L\cos\theta^s_R\sin\theta^s_R)^{1/2} \qquad (30)$$

from the last operator in eqn. (27). In evaluating these expressions, we have used the values $f_K \approx 113$ MeV, the "bag" factors $B_K, B'_K \sim 0.7$, and $m_s + m_d \sim 200$ MeV. In order to produce the observed $d-s$ mixing, we expect that at least one of the angles $\theta^s_L$, $\theta^s_R$ is of order the Cabibbo angle, $\theta_C$. Then we find from any one operator that

$$\frac{M}{g} \gtrsim 200\,{\rm TeV}\ . \qquad (31)$$

From eqn. (23) it follows that

$$\Lambda \gtrsim 6.8\,{\rm TeV}\sqrt{\kappa\left(\frac{m_s}{200\,{\rm MeV}}\right)}\ . \qquad (32)$$

For $\kappa \approx 3.5$, this yields a lower bound of approximately 13 TeV on $\Lambda$.

Typically, in addition to the operators in eqn. (27) there will be flavor-changing operators which are products of color-octet currents[f]. At least in the vacuum-insertion approximation, the matrix elements of products of color-octet currents are enhanced relative to those shown in (27) by a factor of 4/3 for

[f] Note that it is likely that color must be embedded in the flavor interactions in order to avoid possible Goldstone bosons[16] and large contributions to the $S$ parameter.[30]

the LL and RR operators and a factor of approximately 7 for the LR operator. Furthermore, because left-handed quarks are weak doublets it is possible that flavor physics associated with the $c$-quark mass also contributes to $\Delta S = 2$ interactions. If so, one would replace $m_s$ with $m_c$ in eqn. (32), yielding a lower bound on $\Lambda$ of order $20\sqrt{\kappa}$ TeV. For these reasons, the bounds given above may be conservative.

### *5.2 Flavor-Changing Neutral Currents: $\Delta C$*

Usually, the strongest constraints on nonstandard physics from flavor-changing neutral currents come from processes involving Kaons, like those considered above. In the present case, however, the constraints from $D^0 - \overline{D}^0$ mixing are also important because the $c$-quark is heavier than the $s$-quark, while the $u-c$ mixing is as large as the $d-s$ mixing.

Again, there are contributions to $D$-meson mixing from the color-singlet products of currents analogous to those in eqn. (27). The purely left-handed or right-handed current-current operators yield

$$\left(\frac{M}{g}\right)_{\rm LL,RR} \gtrsim f_D \left(\frac{2m_D B_D}{3\Delta m_D}\right)^{1/2} \cos\theta^c_{L,R} \sin\theta^c_{L,R} \approx 120\,{\rm TeV}\ , \tag{33}$$

where we have used the limit[18] on the neutral $D$-meson mass difference, $\Delta m_D \lesssim 1.4\times 10^{-10}$ MeV, and $f_D\sqrt{B_D} = 0.2$ GeV, $\theta^c_{L,R} \approx \theta_C$. The bound on the scale of the underlying strongly-interacting dynamics follows from eqn. (23):

$$\Lambda \gtrsim 11\,{\rm TeV}\sqrt{\kappa\left(\frac{m_c}{1.5\,{\rm GeV}}\right)}\ , \tag{34}$$

so that $\Lambda \gtrsim 21$ TeV for $\kappa \approx 3.5$.

The $\Delta C = 2$, LR product of color-singlet currents gives a weaker bound than eqn. (34) but the LR product of color-octet currents,

$$\mathcal{L}_{eff} = -\cos\theta^c_L \sin\theta^c_L \cos\theta^c_R \sin\theta^c_R \frac{g^2}{M^2} (\overline{c}_L\gamma^\mu T^a u_L)(\overline{c}_R\gamma_\mu T^a u_R)\ , \tag{35}$$

where $T^a$ are the generators of $SU(3)_C$, gives a stronger bound:

$$\left(\frac{M}{g}\right)_{\rm LR} \gtrsim \frac{4f_D}{3(m_c+m_u)} \left(\frac{m_D^3 B'_D}{\Delta m_D}\right)^{1/2} (\cos\theta^c_L \sin\theta^c_L \cos\theta^c_R \sin\theta^c_R)^{1/2} \tag{36}$$

$$\approx 240\,{\rm TeV}\left(\frac{1.5\,{\rm GeV}}{m_c}\right)\ , \tag{37}$$

corresponding to

$$\Lambda \gtrsim 22\,\mathrm{TeV}\sqrt{\kappa\left(\frac{1.5\,\mathrm{GeV}}{m_c}\right)}\ . \tag{38}$$

## 6 Higgs Mass Limits

Because of triviality, a lower bound on the scale $\Lambda$ yields an upper limit on the Higgs boson's mass. A rigorous determination of this limit would require a nonperturbative calculation of the Higgs mass in an $O(4)$-symmetric theory subject to the constraint on $\Lambda$. Here we use eqn. (9) to provide an estimate of this upper limit by naive extrapolation of the lowest-order perturbative result.[g] The bound $\Lambda \gtrsim 13$ TeV given by the contribution of the $\Delta S = 2$ product of color-singlet currents to the $K_L - K_S$ mass difference, eqn. (32), in the case $\kappa \approx 3.5$, results in the limit[h] $m_H \lesssim 490$ GeV. The bound $\Lambda \gtrsim 21$ TeV, given by the contribution of the $\Delta C = 2$ , LL or RR product of color-singlet currents to the neutral $D$-meson mass difference, eqn. (34), yields $m_H \lesssim 460$ GeV. Limits from the contributions of color-octet currents or from the relationship between $m_c$ and $\Delta m_K$ would be even more stringent.

## 7 Conclusions

Because of triviality, theories with a heavy Higgs boson are effective low-energy theories valid below some cut-off scale $\Lambda$. We have shown that the experimental constraint on the amount of custodial symmetry violation implies that the scale $\Lambda$ must be greater than of order 7.5 TeV. The underlying high-energy theory must also include flavor dynamics at a scale of order $\Lambda$ or greater in order to produce the different Yukawa couplings of the Higgs to ordinary fermions. This flavor dynamics will generically give rise to flavor-changing neutral currents. In this note we showed that satisfying the experimental constraints on extra contributions to $\Delta m_K$ and $\Delta m_D$ requires that the scale of the associated flavor dynamics exceed certain lower bounds. At the same time, the new physics must provide sufficiently large Yukawa couplings to give the quarks their observed masses. In order to give rise to a sufficiently large $s$-quark Yukawa coupling, we showed that $\Lambda$ must be greater than of order 13 TeV, while in the case of the $c$-quark the bound is even more stringent, $\Lambda \gtrsim 21$ TeV.

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[g] The naive perturbative bound has been remarkably close to the non-perturbative estimates derived from lattice Monte Carlo calculations.[8]

[h] If $\kappa \approx 4\pi$, $\Lambda$ would have to be greater than 24 TeV, yielding an upper limit on the Higgs boson's mass of 450 GeV. If $\kappa \approx 1$, $\Lambda$ would be greater than 6.8 TeV, yielding the upper limit $m_H \lesssim 570$ GeV.

For theories defined about the infrared-stable Gaussian fixed-point, we estimated that this lower bound on $\Lambda$ yields an upper limit of approximately 460 GeV on the Higgs boson's mass, independent of the regulator chosen to define the theory. We also showed that some regulator schemes, such as higher-derivative regulators, used to define the theory about a different fixed-point are particularly dangerous because an infinite number of custodial-isospin-violating operators become relevant.

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