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Essays on Credit Markets, Monetary Policy, and Macroeconomic Activity

DISSERTATION

submitted in partial satisfaction of the requirements
for the degree of

DOCTOR OF PHILOSOPHY

in Economics

by

Maximillian Barker Littlejohn

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2021

DEDICATION

To my parents for their relentless love, support, and commitment to my education and success.

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ABSTRACT OF THE DISSERTATION

Essays on Credit Markets, Monetary Policy, and Macroeconomic Activity

By

Maximillian Barker Littlejohn

Doctor of Philosophy in Economics

University of California, Irvine, 2021

Professor Guillaume Rocheteau, Chair

This dissertation consists of three chapters which study the relationship between over-the-counter (OTC) markets for credit, monetary policy, and macroeconomic activity. In the first chapter, I introduce an OTC credit default swap (CDS) market into a search model of credit and unsecured debt. The model is developed to study the effects of CDS trading on liquidity, risk sharing, and corporate borrowing costs in an economy where agents are matched trilaterally to trade capital and issue debt. With firms subject to an exogenous default risk, risk averse lenders have the opportunity to purchase a CDS from insurers as protection against their underlying bonds. The model raises implications for corporate credit and investment demand; borrowing costs that vary with the degree of default risk; and potential issues pertaining to moral hazard, empty creditors, and financial sector stability. In addition, its theory corroborates empirical evidence regarding the relationship between corporate debt markets and derivatives trading.

The second chapter investigates the implications of sector-specific credit supply shocks on real economic activity in the United States from 1952 to 2018. These sectors include private households, non-financial corporations, and banks. Within a structural vector autoregression (VAR) framework, I employ a novel sign-restriction strategy to identify one monetary policy shock, two aggregate macroeconomic shocks, and three sector-specific credit supply shocks.

I find evidence that credit supply shocks not only vary by the sectors in which they arise, but also by their consequences for business cycle dynamics. Credit shocks originating in the banking sector can explain up to 25% of output fluctuations while those arising in the household and corporate sectors can each explain up to 15%. In addition, household and bank credit shocks may hold long-run consequences for inflation explaining up to 15% of its fluctuations. Within a historical context, the model identifies several periods when credit supply has been a significant driver of GDP. With respect to the 2008 financial crisis, I find a smaller role for credit shocks relative to aggregate supply shocks than is typically found in the literature. This supports recent empirical evidence suggesting that the early stages of the crisis were more reminiscent of an oil price shock recession.

The third chapter analyzes the lending channel of monetary policy exclusively through credit card and small business bank loans. A time-varying parameter VAR is estimated providing evidence that the direction and strength in which credit card and small business loans respond to monetary policy are time-dependent. To investigate these findings analytically, I develop a general equilibrium model of consumer credit card and small business lending. Households and firms use a combination of monetary assets and bank loans to finance random consumption and investment opportunities. In accordance with conventional theory, when borrowers are sufficiently constrained, a monetary tightening reduces lending through the balance sheet subchannel of monetary policy. However, when borrowers are less constrained, a monetary tightening raises unsecured debt limits through a second subchannel and lending expands. This second subchannel, operating solely through unsecured credit, offers a new theory to justify an expansionary loan response to tightened monetary policy that the traditional lending channel literature has yet to address.

Chapter 1

Credit Default Swaps and Unsecured Debt

1.1 Introduction

While the effects from credit default swaps (CDS) within corporate and sovereign debt markets have been studied extensively, the evidence regarding their impact on credit access, borrowing costs, and lending relationships is mixed. Many critics blame the size, complexity, lack of transparency, and over-the-counter (OTC) nature of CDS markets for contributing to the severity of the recent financial crisis. In this paper, I develop a search model to study the relationship between CDS trading and lending in a corporate debt market. The market is characterized by unsecured debt, limited commitment to repayment, and risk averse lenders. I exploit the mechanism through which CDS can serve as a tool for hedging risk and examine its effects on credit and liquidity in an OTC debt market with varying levels of default risk. The model raises critical implications for corporate borrowing and provides a theoretical foundation to help explain some of the existing empirical evidence in the literature.

A CDS is a type of credit derivative used to transfer default risk between two agents. It is an insurance contract that provides protection against default for an underlying debt security. In a standard CDS, the protection buyer agrees to pay periodic premiums up until the maturity of the contract in exchange for a promise from the protection seller to pay the notional amount of the underlying debt to the buyer if default occurs. According to the Bank for International Settlements, outstanding notional for CDS was $60 trillion in 2007 and is around $9 trillion today. Duffie et al. (2010) assign responsibility for this reduction not to a decrease in CDS trading activity, but to the process of "portfolio compression" involving the elimination of redundant trade positions as required by regulators. Generally speaking, the market is still considered highly active and large enough to pose threats if not well-understood.

A handful of empirical studies have drawn connections between CDS trading and various aspects of the financial system and economy. Several specific areas of interest are the impact of CDS on debt market liquidity, corporate borrowing costs, and lending relationships. However, the evidence is often mixed. The purpose of this paper is to introduce CDS trading into a search model where agents are matched bilaterally to trade capital and issue unsecured debt. More specifically, the paper studies how the borrowing costs, investment behavior, and surplus of firms posing some level of default risk are affected by providing lenders with the opportunity to purchase CDS as insurance against their debt holdings. I take a New Monetarist approach by incorporating microfoundations into the exchange process to expose the mechanism through which CDS can influence the dynamics of OTC trade. I focus on the bargaining process between agents through which debt and CDS contracts are negotiated within the framework developed by Lagos and Wright (2005) and surveyed by Lagos et al. (2017). However, I depart from their framework with several modifications. These include the elimination of money to create a pure credit economy, dropping the infinite horizon structure so agents are short-lived, and modeling the standard decentralized market as a corporate debt, capital, and CDS market.

It would be difficult to motivate a role for CDS in a model without the presence of default risk and agents characterized by risk averse preferences. It would also be challenging to study and discuss the effects of CDS in a non-corporate setting, as CDS are instruments used solely by large financial institutions to hedge against risk on corporate bonds and loans. Considering this, the paper incorporates these features along with a distinctive approach to modeling the decentralized bargaining process. This approach entails having firms, lenders, and insurers enter the decentralized market to form two match types: uninsured and insured. Standard bilateral bargaining between firms and lenders occurs for uninsured matches to determine the terms of unsecured debt contracts. For insured matches, insurers are introduced leading to a trilateral bargaining setting. That is, all three agent types meet and simultaneously bargain over the terms of a debt and CDS contract. This dual match-type setting is critical for analyzing the role played by CDS within a debt market characterized by these features.

To establish a benchmark, I first study the equilibrium trade dynamics in the uninsured match when CDS are not available. I find that the combination of firms' inability to commit to future debt payments, lenders' risk aversion, and decentralized trade creates a potential inefficiency regarding the levels of capital lending and investment that arise in equilibrium. Once CDS are introduced, there is an expansion of credit for all levels of default risk. In the model, I consider a credit expansion to be any increase in lending regardless of the change in borrowing costs when comparing across matches. This suggests a potential role for CDS in maintaining liquidity in debt markets during periods of tight credit, rating downgrades, or economic downturns. In addition, a key result regarding the composition of the equilibrium CDS contract arises from a full insurance condition. That is, lenders secure an equal payout across the default and non-default states through the purchase of a CDS by successfully hedging against all counterparty default risk.

The model also raises implications for the relationship between CDS, capital lending, and the level of default risk. For insured matches, an increase in the repayment rate, or decrease in default risk, expands lending and investment while reducing borrowing costs which leads to a reduction in corporate leverage ratios. However, this result does not necessarily hold for insured matches. Depending on the level of risk aversion lenders display relative to their bargaining power, less default risk can lead to a lending expansion or contraction. If risk aversion and bargaining power are high, then lenders increase their total insurance coverage through the CDS while decreasing credit to firms. CDS trading enables lenders to capitalize on reduced risk by negotiating more favorable CDS terms at the expense of firms' credit access and investment. As a result, an expanding economy or periods of positive growth can potentially be associated with a tightening of credit conditions.

Comparative statics with respect to changes in the model's remaining parameters such as bargaining power are also studied. One key result is that while firms' bargaining power influences both the amount of lending and cost of borrowing when CDS are not present, their bargaining power can only influence borrowing costs and insurance premiums when CDS are present. A critical trade-off arises between the bargaining powers of firms and insurers directly impacting debt and CDS contracts in a manner that can potentially tighten credit conditions among firms with strengthened bargaining positions. Furthermore, in contrast to uninsured lenders, I find that raising the bargaining power of insured lenders can potentially decrease both investment and borrowing costs while increasing insurance coverage. This result suggests that CDS may become lenders' preferred instrument for securing future consumption instead of purchasing bonds or extending loans.

The model's analysis concludes with the introduction of a social planning decision to compare and contrast the decentralized equilibrium with that of one designated to maximize social welfare. A key insight from the planner's decision is that when default risk is high, uninsured and insured investment levels can both be inefficiently low. Following an increase in the repayment rate, equilibrium investment, borrowing costs, and CDS trading terms can either move closer toward or further away from social efficiency. These effects are contingent on the levels of risk aversion and default risk characterizing the economy, and depending on the allocation of bargaining powers, equilibrium may never simultaneously be efficient within both matches.

The remainder of this paper is organized as follows. Section 1.2 reviews the related empirical and theoretical literature. Section 1.3 details the model's features and environment. Section 1.4 introduces value functions and derives the model's decentralized equilibrium. Section 1.5 provides an analysis of the match-specific bargaining contracts and studies comparative statics. Section 1.6 presents the planner's problem and places the model's equilibrium within a social welfare context while Section 1.7 concludes.

1.2 Related Literature

The model developed in this paper is closely related to the literature on monetary search models. In particular, models where agents meet and trade in decentralized and centralized markets following Lagos and Wright (2005). This papers describes a model in a two-period static environment where credit is the only feasible means of payment. The first papers to introduce and study pairwise credit transactions in search models were Diamond (1987a,b, 1990). These models study credit transactions that are settled through the payment of goods, but differ in terms of repayment enforcement and punishment. Two earlier models addressing the enforcement of repayment are Kehoe and Levine (1993) and Alvarez and Jermann (2000) where agents are punished with the threat of losing future credit in the event of default. I take an approach similar to Rocheteau and Nosal (2017) and assume that if an agent is able to repay his debt he does so, ruling out the possibility of strategic default. In an effort to capture a similar notion of default risk, I model exogenous default with an investment opportunity for firms that yields a positive return at some exogenous rate.

This paper has roots in the strand of literature involving search models and generalized OTC financial markets. The first to explore this area was Duffie et al. (2005) in which investors trade assets in a decentralized market for risk hedging and liquidity purposes. They examine the relationship between investor bargaining power and marketmakers' bid-ask spreads. In a similar spirit, Lagos and Rocheteau (2009) study trading frictions and liquidity in an OTC asset market but allow for unrestricted asset holdings. Lagos and Zhang (2013) extend these frameworks by allowing agents to use fiat money for purchasing assets and analyze the transmission of monetary policy through OTC trade. In a less general OTC approach, the financial market in my model departs from these with agents entering in search of borrowing capital to focus on the role CDS trading plays in managing risk and alleviating trade frictions. Similar to these models, implications regarding the liquidity in OTC markets are raised.

A number of papers utilize search models to investigate the relationship between credit markets and limited commitment. Gu et al. (2013) study a pure credit model of limited commitment that leads to endogenous borrowing constraints resulting in nonstationary equilibria and fluctuations in credit conditions. Many models also allow for the coexistence of money and credit. One earlier paper would be Shi (1996) which studies money and credit as competing mediums of exchange and allows agents to use collateral in an effort to elude problems of credit and commitment. In Sanches and Williamson (2010) money and credit coexist in an environment with limited commitment and credit relationships that are inhibited by imperfect memory and the threat of theft. Several other papers to address issues of limited commitment in search frameworks include Venkateswaran and Wright (2014), Bethune et al. (2015), and Carapella and Williamson (2015) with applications for government debt and Ricardian Equivalence. In relation to this literature, my model incorporates a mechanism for alleviating issues of limited commitment within credit markets, however, I take an alternative approach with the inclusion of CDS.

Common to the monetary search literature is a setting in which households enter markets to finance private consumption expenditures. This model presents a corporate financial environment where firms seek to finance investment opportunities by issuing debt. Rocheteau et al. (2018) study the transmission of monetary policy in a model where entrepreneurs can finance random investment opportunities using money, bank liabilities, or trade credit extended by capital suppliers. My model relates with respect to firms demanding capital for investment and there only being a fraction of decentralized matches where bank credit is available. A key departure is that I model banks as insurers underwriting CDS contracts instead of loans and include an endogenous entry decision.

Another literature in which this paper draws close connections to would be that involving insurance markets. Specifically, studies that analyze the role between risk averse agents and risk neutral insurance underwriters. Kihlstrom and Roth (1982) examine the effects of risk aversion on the terms of insurance contracts determined by the Nash bargaining solution. Viaene et al. (2002) take this study a step further and develop a non-cooperative bargaining game with alternating offers and perfect information. When bargaining with risk neutral insurers, these papers show that more risk averse agents pay higher premiums for less insurance than less risk averse agents. This result is consistent with my analysis of how risk aversion impacts the terms of CDS contracts and insurer surplus. My paper also relates to Kihlstrom and Roth (1982) through the use of Nash bargaining for negotiating equilibrium insurance contracts.

Transitioning to the CDS aspect of the model, there is a vast financial literature on derivatives trading, risk sharing, and credit markets. Within the search theory framework, Atkeson et al. (2015) employ a search model to study OTC market entry and exit where banks subject to aggregate default risk can trade CDS to protect their risky portfolios. Sambalaibat (2013) develop a search model with an OTC bond and CDS market to explain the impact of naked CDS bans on bond market liquidity. In contrast to these frameworks, agents in my model bargain over CDS and debt contracts simultaneously, requiring lenders to hold the underlying debt. These papers also fix the notional principal of CDS contracts prior to bargaining while I endogenize their price and principal. In a general equilibrium setting, Darst and Refayet (2015) analyze the effects of covered and naked CDS trading on firms' borrowing costs and investment decisions. They introduce CDS into a setting where investors trade debt on firms subject to aggregate productivity shocks and find covered CDS to have a positive impact on investment, but negative impact on borrowing costs. Their results are reversed for naked CDS trading. In addition, the decrease in investment from naked CDS lowers the default risk of a firm suggesting a trade-off between the two. Bolton and Oehmke (2011) develop a model where firms issue debt to finance investment projects with a stochastic return across two periods. If default occurs, creditors can only liquidate firms up to a fraction of their realized return. They find investment increases after introducing CDS, but also that creditors choose to overinsure themselves leading to an empty creditor problem with inefficient debt restructuring and bankruptcy rates. Uncovered CDS contracts are ruled out in my model.

Implications regarding moral hazard and monitoring incentives raised in this paper are shared with those from the empirical CDS and finance literature as well. Hakenes and Schnabel (2010) argue that CDS risk sharing for corporate debt leads to an increase in credit supplied to risky borrowers, while Hirtle (2009) finds limited evidence of such an increase for the average firm, but does for long-term loans that is offset by higher spreads and borrowing costs. A similar result is found in Ashcraft and Santos (2009) where the average borrower receives no reduction in borrowing costs while safe and transparent firms do. Subrahmanyam et al. (2014) discuss the adverse effects of increased credit supply from CDS on the likelihood of bankruptcy among North American firms. Using S&P 500 data, Saretto and Tookes (2011) find a positive relationship between credit expansion and CDS resulting from increased firm leverage and debt maturity. Darst and Refayet (2015) develop a debt financing partial equilibrium model, calibrate it with Compustat data, and propose that CDS lead to increased firm investment and welfare through a reduction in borrowing costs.

1.3 Environment

Time is discrete and divided into two periods. The first period features a debt market, denoted DM, and the second features a settlement market, denoted CM. The CM is centralized and frictionless while the DM is decentralized and subject to trading and matching fictions discussed in detail below. A capital good is traded in the DM for investment while a numéraire good is traded and consumed in the CM. The set of agents is comprised of a [0,1] continuum of firms and lenders and a large continuum of insurers. All agents wish to consume the numéraire good but are unable to produce it through labor. Lenders have access to a technology allowing them to produce capital in the DM while firms have an investment opportunity which transforms capital into the numéraire.

The lifetime utility of a firm and insurer is $u(c)$ and for a lender it's $U(c) - c(k)$, where c is consumption of the numéraire and k is the quantity of capital produced and borrowed. Firms and insurers have linear utility over consumption while lenders can produce capital at linear cost which leads to $u(c) = c$ and $c(k) = k$. Lenders are risk averse in the CM with CRRA utility, $U(c) = c^{1-\sigma}/(1-\sigma)$, where $\sigma > 0$ is the coefficient of relative risk aversion. Moreover, I assume $U(0) = 0$, $U'(0) = \infty$, $U'(c) > 0$, and $U''(c) < 0$.

In the DM, all firms enter in search of lenders while lenders enter in search of firms and insurers. Firms and lenders enter at zero cost and are matched with certainty. However, only a measure n of insurers enter by paying a disutility cost ϵ. Introducing an entry decision for insurers generates a matching friction and leads to the existence of two types of DM matches: uninsured and insured. Uninsured matches consist only of a firm and lender while insured matches feature a firm, lender, and insurer. The measure of insured matches is provided by a matching function, $\mathcal{M}$, that is strictly increasing, twice continuously differentiable in both inputs, and homogenous of degree one. The probability a lender matches with an insurer is $\alpha_L(n) = \mathcal{M}(L, I)/L$, while the probability an insurer matches with a lender is

$\alpha_I(n) = \mathcal{M}(L, I)/I$. Due to constant returns to scale of the matching function, the match probabilities are functions of the ratio I/L. With I/L defined as CDS market tightness and the measure of lenders normalized to one, market tightness is captured by n. It is also implied from $\mathcal{M}$ that $\alpha(0) = 0$, $\alpha(\infty) = 1$, $\alpha'(n) > 0$, $\alpha''(n) < 0$, and $\alpha(n) \leq \min\{1, n\}$.

Due to the absence of money, the only feasible means of payment is credit. All transactions in the DM are made on credit and settled the following period with the numéraire. However, firms borrow capital from lenders to finance an investment opportunity that yields $f(k)$ units of numéraire with probability δ. This means that with probability δ, the investment pays off and firms repay their debt in full, eliminating the possibility of strategic default. Thus, with complementary probability $1 - \delta$ they are unable to produce the numéraire and default entirely on their debt. The repayment rate δ is exogenous and publicly known to all agents.

The combination of risk averse lenders with firms unable to commit to repayment creates the role for CDS and insurer entry in the DM. A lender who is matched with an insurer has the opportunity to simultaneously extend credit to a firm while purchasing a CDS from an insurer as protection against default. In contrast to capital being physically traded between firms and lenders, a CDS contract only serves as an agreement to future payments in the CM between lenders and insurers. Considering this, the question of commitment arises with respect to fulfilling CDS obligations. In contrast to firms, the model assumes that both parties can credibly and fully commit to making their CDS payments. The assumption of zero CDS counterparty risk is critical for isolating and analyzing the role CDS plays within this economy.

1.4 Equilibrium

1.4.1 Value Functions

This section begins by characterizing the value functions for agents entering the CM. Since there are two types of DM matches, lenders and firms enter the CM as either uninsured or insured. The value functions for an uninsured and insured lender are

$$W^L(b) = \delta U(b); \tag{1.1}$$

$$\hat{W}^L(\hat{b}) = \delta U(\hat{b} + s_1) + (1 - \delta)U(s_2), \tag{1.2}$$

where b and $\hat{b}$ are the bonds, or debt, issued by firms in an uninsured and insured DM match, respectively. The CDS contract negotiated between a lender and insurer specifies an insurance payment to be made in both possible states. The first state is that of full repayment occurring with probability δ and its respective insurance payment is s_1. The second is the default state occurring with complementary probability $1 - \delta$ and s_2 as its payment. To keep the CDS contract terms as general as possible and free to be determined in equilibrium, I place no restrictions on the signs of insurance payments. The composition of each payment will be discussed in greater detail later when I describe the CDS bargaining process. Since the CM is the final period, there is no continuation value and lenders consume everything they receive from their debt or CDS payment. Also, while the event of default or repayment by firms is not realized until agents enter the CM, a lender's value function must be expressed in terms of expected utility to reflect this uncertainty.

Now consider firms who enter the CM upon receiving capital and issuing debt in the DM. Even though a firm itself is not insured with a CDS, they would have been matched with either an uninsured or insured lender in the DM, and therefore their CM value functions reflect match type as they do for lenders. For ease of notation and clarity, I present the firm's value function from an insured match only:

$$\hat{W}^F(\hat{k}, -\hat{b}) = \delta\big[f(\hat{k}) - \hat{b}\big], \tag{1.3}$$

where $\hat{k}$ is the quantity of capital borrowed from an insured lender. As discussed earlier, the assumption that firms commit to repayment upon a positive investment return is reflected in $\hat{W}^F$. Also, since the investment either pays off in full or yields zero output, $\hat{W}^F$ only depends on one state.

Insurers enter the CM to settle their CDS obligations and consume the numéraire. These obligations include collecting premiums and making insurance payments. Even though all insurers are assumed to belong to a single bank and their match surplus can be multiplied by the measure $\alpha(n)$ of insured matches to represent their bank's surplus, I present the value function for a single insurer as

$$W^I = -\delta s_1 - (1-\delta)s_2. \tag{1.4}$$

For an insured lender, the CDS payments s_1 and s_2 enter their value function with a positive sign. Since the payments represent a transfer of the numéraire between lenders and insurers, they must enter (1.4) with the opposite sign. Notice that this does not state which agent s_1 or s_2 is paid to, just that they represent a transfer between the two agents.

Having the CM value functions fully characterized, I move back one period to the DM where all agents enter simultaneously. For uninsured matches, lenders produce and deliver k units of capital to firms in exchange for debt b. The quantity of debt b can be interpreted as a one-period, zero-coupon bond with face value b. The terms (k, b) from this exchange constitute the uninsured debt contract. For insured matches, a CDS contract is negotiated in addition to a debt contract. The CDS will specify the terms (s_1, s_2) to be settled in the following period. Further details regarding the mechanism through which the contracts from each match are determined are discussed in Sections 1.4.2 and 1.4.3.

Prior to entering the DM, all lenders begin as uninsured. Using (1.1) and (1.2), the DM value function for a representative lender takes the following form:

$$V^L = \alpha(n)\big[-\hat{k} + \delta U(\hat{b} + s_1) + (1-\delta)U(s_2)\big] + \big(1-\alpha(n)\big)\big[-k + \delta U(b)\big]. \tag{1.5}$$

The first term on the right is the expected surplus from an insured match while the second term is the expected surplus from an uninsured match. Their expected surplus from each match consists of their consumption in the CM net of their capital production cost. Due to capital's linear production technology, a lender's cost of producing k units of capital is $-k$. Similar to lenders, firms enter insured and uninsured matches at the same rate. While firms do not enter the DM in search for an insurer, their value function does depend on whether or not an insurer is present in their match. A firm's DM value function is

$$V^F = \alpha(n)\delta\big[f(\hat{k}) - \hat{b}\big] + \big(1-\alpha(n)\big)\delta\big[f(k) - b\big]. \tag{1.6}$$

For both matches, a firm's surplus is the expected utility they receive from their investment project's output net of their debt payment. The final value function left to characterize is that of an insurer. With only a measure n of insurers entering the DM in search for lenders, their probability of matching is $\alpha(n)/n$. Their value function is written as

$$V^I = -\epsilon + \frac{\alpha(n)}{n}\Big[-\delta s_1 - (1-\delta)s_2\Big], \tag{1.7}$$

where ϵ is the disutility cost of entering. If matched, an insurer's surplus is their expected utility in the CM net of the entry cost. If unmatched, an insurer still incurs the entry cost but receives zero additional surplus and does not participate in the CM.

1.4.2 Uninsured Bargaining

This section describes the bargaining process used to determine the DM terms of trade. First, consider an uninsured match between a firm and lender. The optimal terms of trade for a debt contract (k, b) are obtained through the generalized Nash bargaining solution. In order for both agents to participate in the bargaining process, the debt contract must satisfy each of their participation constraints. Thus, each agent must be expected to receive a non-negative match surplus. From the DM value functions, define $S^L = \delta U(b) - k$ as a lender's match surplus and $S^F = \delta\big[f(k) - b\big]$ as a firm's, with threat points k and b, respectively. Drawing upon the functional form of $U(c)$, the set of incentive feasible allocations can be defined as $\Omega = \Big\{(k, b) : \big(\frac{1-\sigma}{\delta}k\big)^{\frac{1}{1-\sigma}} \leq b \leq f(k)\Big\}$. With $\eta \in [0, 1]$ as the firm's bargaining power, the uninsured bargaining problem is

$$\max_{k,b} \quad \big[\delta f(k) - \delta b\big]^{\eta}\big[-k + \delta U(b)\big]^{1-\eta}. \tag{1.8}$$

The following lemma summarizes the solution determined by the first order conditions to (1.8).

Lemma 1.1. *The terms of the uninsured debt contract (k, b) as the solution to (1.8) are nonzero, unique, and satisfy*

$$f'(k) = \frac{1}{\delta U'(b)}; \tag{1.9}$$

$$U'(b) = \frac{\eta\big[\delta U(b) - k\big]}{(1-\eta)\delta\big[f(k) - b\big]}. \tag{1.10}$$

Proof in Appendix A.

With risk averse lenders, the optimal quantity of capital traded, k, is dependent on their marginal cost and utility in the CM. According to (1.9), k is chosen such that the firm's marginal product of capital is equal to the lender's marginal cost of production divided by their marginal utility. Given that both $f'(k)$ and $U'(b)$ are decreasing with k, there must be an inverse relationship between the two for a unique and positive k to be chosen.

1.4.3 Insured Bargaining

I now turn to the bargaining problem for an insured match where all three agents simultaneously bargain to determine a debt contract $(\hat{k}, \hat{b})$ and CDS contract (s_1, s_2). As for uninsured bargaining, the terms of each contract must provide agents with a non-negative match surplus for them to participate. Thus, the surplus for insured lenders and insurers is defined as $\hat{S}^L = \delta U(\hat{b} + s_1) + (1-\delta)U(s_2) - \hat{k}$ and $S^I = -\delta s_1 - (1-\delta)s_2$, respectively. Provided that firms have the same participation constraint in both matches, the set of incentive feasible allocations is $\hat{\Omega} = \left\{(\hat{k}, \hat{b}) : \left[\frac{1-\sigma}{\delta}\left(\hat{k} - (1-\delta)U(s_2)\right)\right]^{\frac{1}{1-\sigma}} - s_1 \leq \hat{b} \leq f(\hat{k})\right\}$ for debt contracts and $\hat{\Gamma} = \left\{(s_1, s_2) : \left[\frac{1-\sigma}{\delta}\left(\hat{k} - (1-\delta)U(s_2)\right)\right]^{\frac{1}{1-\sigma}} - b \leq s_1 \leq \frac{-(1-\delta)}{\delta} s_2\right\}$ for CDS contracts. The primary insight from comparing the sets of feasible allocations is that for $\hat{\Omega}$, the terms from the CDS contract will directly impact those of the debt contract. Also, σ will be a key determinant of CDS terms and therefore insured debt terms. The insured trilateral bargaining problem is presented as

$$\max_{\hat{k}, \hat{b}, s_1, s_2} \quad \left[\delta f(\hat{k}) - \delta \hat{b}\right]^{\gamma_1} \left[\delta U(\hat{b} + s_1) + (1-\delta)U(s_2) - \hat{k}\right]^{\gamma_2} \left[-\delta s_1 - (1-\delta)s_2\right]^{\gamma_3}, \quad (1.11)$$

where $\gamma_1 \in (0,1)$, $\gamma_2 \in (0,1)$, and $\gamma_3 = 1 - \gamma_1 - \gamma_2$ are the bargaining powers of the firm, lender, and insurer, respectively. The following lemma summarizes the solution determined by the FOCs to (1.11).

Lemma 1.2. *The terms of the insured debt contract* $(\hat{k}, \hat{b})$ *and CDS contract* (s_1, s_2) *as the* *solution to (1.11) are nonzero, unique, and satisfy*

$$f'(\hat{k}) = \frac{1}{\delta U'(\hat{b} + s_1)}; \tag{1.12}$$

$$\hat{b} = \frac{\gamma_3 \delta f(\hat{k}) + \gamma_1 s_1}{\gamma_3 \delta + \gamma_1 (1 - \delta)}; \tag{1.13}$$

$$U'(\hat{b} + s_1) = U'(s_2) = \frac{\gamma_3 \big[\delta U(\hat{b} + s_1) + (1 - \delta) U(s_2) - \hat{k}\big]}{\gamma_2 \big[- \delta s_1 - (1 - \delta) s_2\big]}. \tag{1.14}$$

Proof in Appendix A.

Similar to uninsured matches, $\hat{k}$ is chosen by equating marginal product of capital to the inverse of the lender's marginal utility in the repayment state. However, a key difference is that here the CDS terms enter the lender's utility and influence the amount of capital borrowed. $U'(\hat{b} + s_1)$ and $U'(s_2)$ determine the optimal CDS payments by equating the lender's marginal utility of insurance in each state with the ratio between the insurer's surplus and their own, weighted by bargaining power. Recall that the state-dependent CDS payments, s_1 and s_2, specify a net payment between a lender and insurer that are determined by two components. The first is the CDS premium paid to the insurer and the second is the insurance payment paid to the lender. Considering s_1 and s_2 both enter the lender's utility function with a positive coefficient sign, it follows that in equilibrium, a positive sign indicates the insurance payment is owed to the lender upon realization of the relevant state. If negative, then a premium is owed to the insurer without any payment to the lender.

The equivalence between $U'(\hat{b} + s_1)$ and $U'(s_2)$ presented in (1.14) implies $\hat{b} + s_1 = s_2$. This result is consistent with the expected utility and insurance contract literature. When a risk averse agent seeks insurance from a risk neutral insurer, the optimal insurance contract will equate each agent's marginal rate of substitution between the two states. That is, the point where the slopes of their indifference curves are equalized implying equal payouts or full insurance across states.[1] This full insurance condition raises two critical implications. The first is that insurers only make a payment to lenders in the default state. In the non-default state, insurers simply collect their premiums and consume the numéraire. This is consistent with the structure of a standard CDS. The second is that given full coverage across states, $\hat{b}+s_1 = s_2$, lenders are never insured for more than $\hat{b}$. Hence, $s_1 < 0$, $s_2 > 0$, and $\hat{b} > s_2$. This also rules out the possibility of naked CDS trading in equilibrium. As for the implications from δ in Lemma 1.2, if $\delta = 0$, a debt contract is not formed and $\hat{b} = 0$. This would also imply $s_1 = s_2 = 0$. There must be an underlying debt contract for a CDS to be purchased, otherwise, naked CDS trading would occur.

Using $\hat{b} + s_1 = s_2$, (1.13) and (1.14) can be rewritten as

$$\hat{b} = \frac{\gamma_3 \delta f(\hat{k}) + \gamma_1 s_2}{(1-\gamma_2)\delta}; \tag{1.15}$$

$$U'(s_2) = \frac{(1-\gamma_2)\left[U(s_2) - \hat{k}\right]}{\gamma_2\left[\delta f(\hat{k}) - s_2\right]}, \tag{1.16}$$

where $U(s_2) - \hat{k} > 0$ and $\delta f(\hat{k}) - s_2 > 0$ always hold in equilibrium. Beginning with (1.15), while $\hat{b}$ is positively related to both $\hat{k}$ and s_2, intuition for these relations arises from different sources. Intuition for the former is straightforward as lenders charge more to cover their costs when they produce larger quantities of capital. For the latter, when lenders are able to secure a higher guaranteed payoff, they are more willing to extend higher quantities of credit allowing firms to issue more debt. As for (1.16), notice that s_2 and therefore $\hat{k}$ only depend on the lender's bargaining power γ_2. This differs from uninsured matches where firms' bargaining power impacts both b and k. This effect will be revisited and discussed further in Section 1.5.

[1] See Kihlstrom and Roth (1982) for a thorough discussion regarding the full insurance result.

1.4.4 Entry of Insurers

Having established the bargaining problems and solutions from which the terms of trade are determined in the DM, I move to the entry decision of insurers. An insurer chooses to enter the debt market as long as $V^I \geq 0$. From (1.7), this condition can be written as

$$\frac{\alpha(n)}{n}\Bigg[-\delta s_1 - (1-\delta)s_2\Bigg] - \epsilon \geq 0. \tag{1.17}$$

where $s_1 < 0$ and $s_2 > 0$. The insurer's expected surplus net of their entry cost must be non-negative to enter. The value to an insurer from searching in the DM depends on the CDS contract terms from bargaining and the probability of a match. Thus, the entry constraint will loosen with more favorable CDS terms and increased match probability. To solve for the optimal measure of insurers, it must be true that $V^I = 0$ in equilibrium. This is the free entry condition which implies zero profits such that (1.17) holds at equality. With the insurer's expected revenue equal to their costs, (1.17) can be rewritten as

$$\frac{\alpha(n)}{n}\Bigg[-\delta s_1 - (1-\delta)s_2\Bigg] = \epsilon. \tag{1.18}$$

Let $\underline{k}$ denote the level of capital that satisfies (1.18) for $n = 0$. As a necessary condition for $n > 0$, it must be true that $\hat{k} > \underline{k}$ which implies $\epsilon < -\delta s_1 - (1-\delta)s_2$, where $\hat{k}$ is determined by (1.12). However, if $\gamma_3 = 0$, insurers receive zero surplus and $-\delta s_1 - (1-\delta)s_2 = 0$. Thus, when $\epsilon > 0$, $\gamma_3 > 0$ must be true to guarantee $\hat{k} > 0$ and $n > 0$. Otherwise, an entry cost of zero would be required to encourage participation among insurers with no bargaining power. Definition 1.1 formally defines the model's complete decentralized equilibrium.

Definition 1.1. A unique equilibrium exists and is a list $(k, b, \hat{k}, \hat{b}, s_1, s_2, n)$ such that

(i) the uninsured debt contract (k, b) solves (1.9) and (1.10);

(ii) the insured debt contract $(\hat{k}, \hat{b})$ solves (1.12) and (1.13);

(iii) the CDS contract (s_1, s_2) solves (1.14); and

(iv) the measure of insurers n solves (1.18).

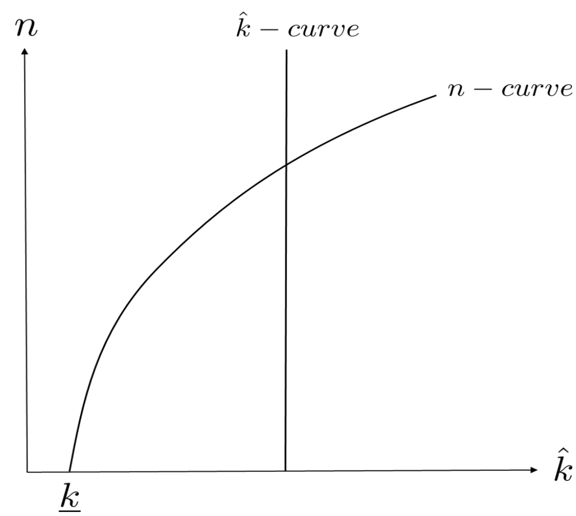
Equilibrium within each match follows a recursive structure. First, (1.10) and (1.14) determine the lender's payout in the non-default state for uninsured and insured matches, respectively. For uninsured matches, this payout is equivalent to the firm's debt payment. Next, (1.9) and (1.12) determine the firm's level of capital investment and completes equilibrium for uninsured matches as there are only two unknowns. For insured matches, (1.13) or (1.15) determine the firm's debt payment which simultaneously pins down the lender's insurance premium. Finally, (1.18) equates the insurer's expected revenue with their entry cost to determine the equilibrium measure of participating insurers.

Since the insurer's entry decision only depends on the terms of the CDS contract, equilibrium in $(\hat{k}, n)$ space is depicted in Figure 1.1. The n-curve is defined by (1.18) while the $\hat{k}$-curve is defined by (1.12). The $\hat{k}$-curve is linear and fixed because n has no effect on the terms of debt and CDS contracts. This is a direct consequence of bargaining not taking place until after agents are informed of their match type. While intersecting the $\hat{k}$-axis at $\underline{k}$ for $n = 0$, the n-curve is an increasing function of $\hat{k}$ as insurers' expected surplus is greater for increased levels of lending and therefore, increased demand for CDS coverage. As a result, credit expansions induce insurer entry.

1.5 Equilibrium Lending and CDS

This section begins by comparing the levels of k and $\hat{k}$ across matches to study the effects from introducing CDS on lending, debt, and investment. Comparative statics are then analyzed with particular emphasis on changes in δ as a repayment rate less than unity, or a positive risk of default, is instrumental in motivating demand for CDS in this environment.

Figure 1.1: Equilibrium in $(\hat{k}, n)$ Space



Proposition 1.1. *Assume* $\delta \in (0,1)$, $\sigma > 0$, *and* $1-\eta \geq \gamma_2 \in (0,1)$. *CDS trading leads to an expansion of investment such that* $\hat{k} > k$ *and* $s_2 < b$. *Its effect on the debt payment* $\hat{b}$ *is indeterminate.*

Proof in Appendix A.

Contingent on lenders being risk averse and firms possessing a positive repayment rate, Proposition 1.1 shows that firms can finance increased levels of capital and investment when CDS are traded referencing their debt. Also, notice from (1.12) that $\hat{k} > k$ implies $s_2 < b$, but does not imply $\hat{b} < b$. The response from the firm's debt payment is ambiguous as the subcomponents of s_2, $\hat{b}$ and s_1, do not display a linear relationship with $\hat{k}$ and s_2. Equation (1.15) demonstrates this by showing $\hat{b}$ is increasing in both $f(\hat{k})$ and s_2. Also notice that these results no longer hold if $\sigma = 0$ or $\delta = 0$. If lenders are risk neutral, there is no gain in expected utility from insuring with a CDS, rendering the quantity of lending in each match equivalent. This is because when $\sigma = 0$, (1.9) and (1.12) collapse to $f'(k) = 1/\delta$ and $k = \hat{k}$. If $\delta = 0$, firms default with certainty and therefore lenders don't lend capital or purchase insurance to avoid negative surplus. As discussed in Section 1.4.3, it's critical for there to be an underlying debt contract for a CDS to reference so naked CDS trading can be ruled out in equilibrium. Regarding the assumption Proposition 1.1 makes on bargaining power, it's designed to represent the most reasonable distribution of bargaining power when insurers enter the match. That is, either the insurer receives no bargaining power or it comes entirely from the firm's share such that $1 - \eta = \gamma_2$. Otherwise, a portion or all of $\gamma_3 > 0$ can come from the lender's bargaining power implying $1 - \eta > \gamma_2$.[2]

Having characterized the effects of CDS on equilibrium investment more generally, I turn to analyzing the role CDS play with respect to changes in the model's parameters. Beginning with the repayment rate, Proposition 1.2 outlines the effects from increasing δ on uninsured and insured corporate investment levels.

Proposition 1.2. *Assume $\delta \in (0,1)$, $\sigma > 0$, and η, γ_1, $\gamma_2 \in (0,1)$. Uninsured investment k increases, while uninsured debt b decreases with the repayment rate δ.*
(i) If $\sigma > \gamma_2^{-1}$, then $\hat{k}$ and $\hat{b}$ decrease, while s_1, s_2, and n increase with δ.
(ii) If $\sigma < \gamma_2^{-1}$, then $\hat{k}$, s_2, and n increase, while $\hat{b}$ and s_1 may increase or decrease with δ.

Proof in Appendix A.

The effects from an increase in the repayment rate on the uninsured debt contract are intuitive and can be inferred directly from (1.9) and (1.10). Low risk firms receive more favorable lending terms by borrowing increased levels of capital at reduced costs. In contrast, this result does not necessarily hold for insured matches. From (1.16), an increase in δ leads to an increase in s_2 for any given $\hat{k} > 0$. The availability of CDS allows lenders to increase their guaranteed payoff considering the full insurance condition, $\hat{b} + s_1 = s_2$, always holds. However, $\hat{k}$ is positively related to $\delta U'(s_2)$ which leads to a non-monotonic response since $\partial U'(s_2)/\partial \delta < 0$. According to Proposition 1.2, when $\sigma > \gamma_2^{-1}$, lenders with sufficiently

[2]Although the details underlying the mechanism and intuition for the allocation of bargaining power across matches is not critical, the specific allocation itself is important for equilibrium and comparative statics. However, $1 - \eta \geq \gamma_2$ serves as a useful benchmark for analyzing comparative statics and will be relaxed on occasion throughout the analysis.

high degrees of risk aversion and bargaining power leads to reduced lending among more trustworthy firms. When σ, γ_2, or both are high, lenders use their bargaining power to negotiate debt contracts that reduce their production costs while increasing their insurance coverage and future consumption. It also enables them to negotiate reduced CDS premiums.

The result from Part (i) of Proposition 1.2 reverses for $\sigma < \gamma_2^{-1}$ when lenders are still able to increase s_2, but only at the expense of producing more capital and potentially having to pay higher premiums. Considering $\partial s_2/\partial\delta > 0$ is always true, $\partial\hat{b}/\partial\delta$ and $\partial s_1/\partial\delta$ become ambiguous for $\sigma < \gamma_2^{-1}$ and $\partial\hat{k}/\partial\delta > 0$. Recalling (1.15), the debt payment is positively related to the firm's output and lenders payoff s_2, but inversely related to the repayment rate. Consequently, even when $\hat{k}$ and s_2 increase with δ, $\hat{b}$ may still decrease from the downward pressure δ exerts on the denominator in (1.15). Given this nonlinearity, consider the special case where $\sigma = \gamma_2^{-1}$ and $\partial\hat{k}/\partial\delta = 0$. Now, the debt payment response depends on the strength of $\partial s_2/\partial\delta$ relative to the increase in δ. The proof for Proposition 1.2 in Appendix A shows that for $\sigma \geq \gamma_2^{-1}$, $\partial\hat{b}/\partial\delta < 0$ always holds. Hence, whether $\hat{k}$ decreases or remains constant, the increase from δ dominates that of s_2 and the reduction in risk allows firms to obtain lower or equivalent levels of capital while issuing less debt. Since s_2 is higher, s_1 must increase so lenders pay lower premiums on more coverage. These effects are most pronounced when $\sigma > \gamma_2^{-1}$ and $\partial\hat{k}/\partial\delta < 0$.

The heterogeneous effects from a change in the repayment rate across insured and uninsured debt contracts raise a key implication regarding firms' leverage from the availability of CDS in this economy. To connect with previous literature on the relationship between CDS and corporate leverage, let $L = b/f(k)$ denote the firm's leverage ratio representing their total liabilities as a share of their total equity, or output with respect to their CM investment returns. Without CDS, the reduction in lending risk allows firms to effectively deleverage themselves as the ratio of their debt to equity unambiguously falls. Once CDS are introduced, this result can still hold as long as lenders' risk aversion or bargaining power are not too

Table 1.1: Summary of Comparative Statics

	k	b	$\hat{k}$	$\hat{b}$	s_1	s_2	n
δ	$+$	$-$	$+/-$	$+/-$	$+/-$	$+$	$+$
η	$+$	$-$	0	0	0	0	0
γ_1	0	0	0	$-$	$+$	0	$-$
γ_2	0	0	$-$	$+/-$	$+/-$	$+$	$-$
σ	$+$	$-$	$+$	$+/-$	$+/-$	$-$	$+$

high. However, if they are high, then the reduced risk suppresses investment and leverage may increase as lenders shift their portfolio away from bonds and lending towards CDS.

The final effect to consider from a change in δ is the positive entry response from insurers, or $\partial n/\partial \delta > 0$. Substituting $s_1 = s_2 - \hat{b}$ and (1.15) into $S^I = -\delta s_1 - (1-\delta)s_2$, the insurer's match surplus can be rewritten as

$$S^I = \frac{\gamma_3\big[\delta f(\hat{k}) - s_2\big]}{1-\gamma_2}, \tag{1.19}$$

where $\partial S^I/\partial \delta > 0$ is always true. Although (1.19) is decreasing in δ through s_2, its positive marginal effect on $\delta f(\hat{k})$ dominates regardless of whether investment rises or falls. Any increase in the repayment rate raises the likelihood of insurers collecting a positive CDS premium without having to make payments to lenders in the default state. As an insurer, selling CDS becomes more attractive and profitable in an economy characterized by low default risk and trustworthy corporate borrowers.

Table 1.1 summarizes comparative statics for the repayment rate, degree of risk aversion, and set of bargaining power parameters. As previously examined, the effects of δ on the uninsured debt contract and CDS premium are non-monotonic and depend on σ and γ_2. As for η when CDS are not available, an improved bargaining position among firms allows them to negotiate more favorable terms resulting in a lending expansion and reduction in debt and leverage. There is no change in entry as the CDS contract is independent of k and b.

Turning to the insured bargaining powers, Table 1.1 highlights several less intuitive implications. The first is with respect to investment's independency from the firm's bargaining power. Increasing γ_1 while leaving γ_2 fixed lowers γ_3. Recalling (1.15) and $\delta f(\hat{k}) > s_2$, this leads to a decrease in the numerator of $\hat{b}$. However, since (1.16) is independent of both γ_1 and γ_3, there is no change in s_2 and $\hat{k}$. While higher bargaining power allows firms to reduce their total borrowing cost, investment remains fixed in contrast to uninsured lending. Without changing γ_2, lenders are still able to maintain the same amount of CDS coverage, but at a lower cost as insurers are now in a weaker bargaining position when negotiating premiums. Since $\hat{k}$ is tied to s_2 and not its components, simply rebalancing $\hat{b}$ and s_1 has no effect on insured lending and investment.

These effects change for γ_2 as lenders use their bargaining power to increase their overall consumption regardless of whether that is through increased lending or reduced premiums. Consequently, $\hat{k}$ falls with s_2 but the effects on $\hat{b}$ and s_1 are nonlinear.[3] This provides critical insight into how CDS impact the terms of trade. The availability of CDS in the DM provides lenders with an alternative instrument for securing consumption in the CM. Due to the limited commitment of repayment associated with debt contracts, the CDS becomes lenders' preferred contract as it presents zero counterparty risk. When the bargaining position of insured lenders is improved, instead of lending less capital and increasing debt as is the response from uninsured lenders, they decrease lending while expanding CDS protection. Understanding that there must be underlying debt to purchase a CDS, lenders still produce capital for firms but have an incentive to keep $\hat{k}$ and $\hat{b}$ low as larger debt contracts lead to higher CDS premiums. This result suggests potential implications may exist for CDS

[3] The proof for Proposition 1.2 in Appendix A provides a reformulation for s_1 as a function of $\hat{k}$ and s_2 only. From this, one can show that $\partial s_1/\partial\gamma_2 > 0$ when $\gamma_1 = (1-\gamma_2)\delta$. In this case, the marginal effect of s_2 on s_1 is minimized while $\hat{k}$ maintains a strong negative effect. The decrease in $\hat{k}$ dominates the increase in s_2 and CDS premiums fall with γ_2.

and the empty creditor problem as discussed in Bolton and Oehmke (2011). Finally, with $\gamma_3/(1-\gamma_2)$ in (1.19) decreasing with γ_1 and γ_2, any reduction in γ_3 accompanied with or without a decrease in $\hat{k}$ or increase in s_2 leads to a lower equilibrium measure of insurers. This is again intuitive as raising γ_2 or γ_1 while leaving the other fixed weakens insurers' bargaining position.

The last row of Table 1.1 presents the effects from changes in the level of risk aversion among lenders. For uninsured matches, lenders with higher risk aversion are more willing to produce higher quantities of capital for lower expected debt payments. As a single unit of future consumption becomes more valuable with σ, the more lenders are willing to give up or produce today in order to secure it. This result also holds for the insured match except lenders are willing to accept an overall smaller payout s_2, and not necessarily $\hat{b}$. The ambiguous effects from σ on $\hat{b}$ and s_1 are shown by the positive impact of $\hat{k}$ and s_2 on $\hat{b}$ in (1.15). However, as found in Kihlstrom and Roth (1982), risk averse agents pay more for less coverage as higher levels of risk aversion is associated with lower certainty equivalents. This would be indicative of a decrease in s_1, but since s_2 is always lower, $\hat{b}$ may still increase or decrease.

1.6 Planner's Problem

To discuss welfare and efficiency in this economy, I now consider the problem of a social planner who chooses the efficient debt and CDS contract terms in an effort to maximize social welfare. This will provide a useful benchmark for discussing the efficiency of the debt and CDS terms determined in equilibrium via Nash bargaining. Provided there is a measure

$\alpha(n)$ of insured matches, the welfare function of the economy can be written as

$$\mathcal{W} = -n\epsilon + \alpha(n)\Big\{ - \hat{k} + \delta\big[f(\hat{k}) - \hat{b} + U(\hat{b} + s_1) - s_1\big] + (1-\delta)\big[U(s_2) - s_2\big]\Big\}$$
$$+ \big(1 - \alpha(n)\big)\Big\{ - k + \delta\big[f(k) - b + U(b)\big]\Big\}. \tag{1.20}$$

With all parameters of the model publicly known, the planner wishes to maximize $\mathcal{W}$ with respect to k, b, $\hat{k}$, $\hat{b}$, s_1, s_2, and n subject to each agent's participation constraint. The following lemma summarizes the planner's solution.

Lemma 1.3. *An efficient debt contract* (k^*, b^*) *and measure of insurers* n^* *solve the social planner's welfare maximization problem and are given by*

$$f'(k^*) = \frac{1}{\delta}; \tag{1.21}$$

$$U'(b^*) = 1; \tag{1.22}$$

$$\alpha'(n^*)\big(1-\delta\big)\big[U(b^*) - b^*\big] = \epsilon, \tag{1.23}$$

where $k^* = k = \hat{k}$ *and* $b^* = b = \hat{b} + s_1 = s_2$.

Proof in Appendix A.

Similar to conditions (1.9) and (1.12), the efficient quantity of capital equates firms' expected marginal product of capital to lenders' marginal cost of production weighted by the lender's marginal utility from CM consumption. The key difference is that the efficient debt repayment equates lenders' marginal utility to firms' marginal cost of repayment such that $U'(b^*) = 1$. Notice that Lemma 1.3 implies $\hat{b} > b^*$ since $\hat{b} + s_1 = b^*$ and $s_1 < 0$. Although $\hat{k} = k = k^*$, the efficient debt payment for insured matches exceeds that for uninsured matches to accommodate the lender's CDS cost and insurance premium. This would also imply an efficiently higher leverage ratio among firms with CDS traded in reference to their bonds. As for entry, (1.23) shows that the efficient measure of insurers equates the entry cost to their marginal social welfare contribution. This contribution occurs in the default state and is equal to the difference between the lenders utility from consuming their CDS payoff and the insurer's disutility from CM production to deliver that payment. Without insurers, CM social welfare in the default state would be zero.

Having established the optimal debt and CDS terms chosen by the social planner, Proposition 1.3 provides conditions on the repayment rate through which efficient trade can be achieved within each match. In doing so, the functional form $f(k) = \nu^{-1}k^{\nu}$ is assumed to improve tractability and allow for the derivation of closed-form expressions. This simplifies analytics for changes in the repayment rate as k^* is a function of δ through $f'(k^*) = 1/\delta$.

Proposition 1.3. *Assume $\eta, \gamma_1, \gamma_2 \in (0,1)$, $f(k) = \nu^{-1}k^{\nu}$, and $\sigma \in (0, \sigma^*)$ where $\nu \in (0,1)$ and $\sigma^* = \gamma_2(1-\nu)/\left[\nu + \gamma_2(1-2\nu)\right] < 1$. There exists a pair $(\delta^*, \hat{\delta}^*) \in (0,1)$ such that the terms of the uninsured and insured equilibrium contracts are socially efficient when either $\delta = \delta^*$ or $\delta = \hat{\delta}^*$, respectively. Furthermore, if $\gamma_2 \leq (1-\eta)$, then $\hat{\delta}^* > \delta^*$ and the complete decentralized equilibrium is never efficient.*

Proof in Appendix A.

According to Proposition 1.3, efficiency can be reached in either match when $\sigma \in (0, \sigma^*)$, but can never occur simultaneously if $\gamma_2 \leq (1-\eta)$. These assumptions on risk aversion and bargaining power hold two critical implications for efficiency. First, $\sigma < \sigma^*$ is required to guarantee that both δ^* and $\hat{\delta}^*$ lie within the unit circle. For example, if $\sigma > 1$ and lenders are too risk averse, then neither match can achieve a socially efficient allocation as investment levels become too high. Recalling Table 1.1, investment increases with risk aversion as lenders become willing to produce more capital in the DM for less consumption in the CM. Second, $\gamma_2 \leq (1-\eta)$ is a reasonable assumption on the bargaining power distribution across matches and also guarantees that a higher repayment rate is required for efficient insured match allocations. For $\hat{\delta}^* < \delta^*$ to be feasible, it must be true that $\gamma_2 > (1-\eta)$ which is more difficult to justify theoretically and provide strategic foundations for. However, to consider the potential scenario of lenders capturing more bargaining power after introducing insurers, suppose $\gamma_2 > (1-\eta)$ and $\hat{\delta}^* < \delta^*$. From Table 1.1, s_2 is increasing with γ_2 and δ. As a result, higher levels of δ could result in s_2 becoming too high from an efficiency perspective. This is from high bargaining power lenders weighting their portfolios away from bonds and more heavily on CDS. The higher levels of insurance coverage may also be associated with higher debt payments and leverage. The lower $\hat{\delta}^*$ would then be required to ensure efficiency before s_2 becomes too large relative to b^* leading to the possibility of an inefficiently high $\hat{b}$ and leverage ratio.

These implications can also be analyzed within the context of the conditions introduced in Proposition 1.2. Considering $\sigma^* < 1$, Proposition 1.3 implicitly implies that $\sigma < \gamma_2^{-1}$ with k and $\hat{k}$ both increasing with δ. If $\delta^* < \delta < \hat{\delta}^*$, uninsured investment would be inefficiently high while insured investment would be inefficiently low. Raising δ would increase $\hat{k}$ closer to the socially efficient level while increasing k further away from k^*. If the repayment rate continues to rise and surpasses $\hat{\delta}^*$ such as during an economic expansion, then investment in both matches would exceed k^*.

While the results from Proposition 1.3 only pertain to efficiency among the terms of the debt and CDS contracts, conditions can also be derived to ensure an efficient equilibrium measure of insurers consistent with (1.23). Substituting ϵ from (1.23) in for that of the free entry condition given by (1.18), one can obtain

$$\frac{\alpha'(n)n}{\alpha(n)} = \frac{\gamma_3}{\gamma_2}\left[\frac{U(b^*)-k^*}{(1-\delta)\big[U(b^*)-b^*\big]}\right], \tag{1.24}$$

where $n = n^*$ simultaneously satisfies free entry and (1.23). The left side of (1.24) represents the elasticity of the matching function with respect to n. The right side is the ratio between the insurer's share of the lender's lifetime utility and the lender's share of the additional CM surplus provided by CDS in the default state. Intuition for this ratio comes from considering the compositions of (1.14), (1.18), and (1.23). From (1.14), the optimal level of CDS coverage equates the lender's marginal CM utility with the ratio between the lender's and insurer's total surplus, weighted by bargaining power. The free entry condition then demonstrates that n is determined through the zero-profit condition when the insurer's expected surplus is equal to their entry cost ϵ. In contrast, the efficient level of entry, n^*, equates ϵ to the insurer's marginal social welfare contribution, $\alpha'(n^*)(1-\delta)\big[U(b^*) - b^*\big]$. Notice that the right side of (1.24) is identical to (1.14) for $\hat{k} = k^*$ and $s_2 = b^*$, but with the efficient entry component of $(1-\delta)\big[U(b^*) - b^*\big]$ representing the insurer's contribution to welfare in place of S^I.

1.7 Conclusion

A pure credit, two-period model is developed to study the dynamics of decentralized trade with risk averse lenders, unsecured debt, and CDS. An OTC market is introduced where capital can be borrowed and CDS can be purchased as protection against default. A distinctive feature of the model is the inclusion of a trilateral bargaining problem in which lenders can simultaneously bargain over the terms of a debt and CDS contract with firms and insurers, respectively. The contractual terms arising from trilateral bargaining matches are then compared against those from bilateral matches between lenders and firms without the presence of insurers. To illustrate, once CDS become available, lenders increase credit to firms and use CDS to fully insure themselves against all default risk. While raising the repayment rate leads to a lending expansion with reduced borrowing costs for uninsured matches, doing so can lead to either a lending expansion or contraction for insured matches. If lenders are sufficiently risk averse or have high bargaining power, then lending contracts while insurance coverage increases as they capitalize on reduced risk by negotiating more favorable CDS terms at the expense of investment. Consequently, the availability of CDS can potentially associate an expanding economy with a tightening of credit conditions.

The model also raises implications for social welfare and efficiency. The combination of positive default risk and risk averse lenders in a decentralized trade setting can lead to inefficiently low or high levels of capital lending and debt. When default risk is high, uninsured and insured investment can be inefficiently low. As a result, changes in risk can move equilibrium investment, debt, and insurance levels closer to or further away from social efficiency. However, depending on the allocation of bargaining power across matches, equilibrium efficiency within both matches may never be feasible simultaneously. Lastly, when analyzing changes in default risk, bargaining power, and risk aversion, the model's results can be considered within the context of corporate finance issues such as empty creditors, moral hazard, and financial market stability.

Chapter 2

The Effects of Sector-Specific Credit Supply Shocks on the U.S. Economy

2.1 Introduction

In recent years, the importance of credit markets for economic activity and their connection with business cycle dynamics have gained considerable attention. In accordance with key economic indicators such as consumption, investment, and aggregate output, credit markets also experience various disruptions and cyclic fluctuations. Between 1952 and 2018, the U.S. economy witnessed many episodes of credit supply expansions and contractions within the private sector. While some of these episodes can be linked to common observables such as stock market fluctuations and policy rate changes by the Federal Reserve, others are associated with less obvious exogenous disturbances. These disturbances can take the form of changes in financial regulation, credit risk aversion, bank liquidity funding costs, or new technology and innovation.[1]

[1]According to the October 2019 Senior Loan Officer Opinion Survey, banks cited concerns over borrower repayment ability, a less favorable economic outlook, and a general decrease in risk tolerance as most

Credit supply and bank lending growth are typically regarded as critical drivers for economic activity and development. Unexpected financial market disruptions can interfere with the flow of credit to borrowers and hold significant economic consequences. This was recently witnessed by the 2008 financial crisis with the collapse of the mortgage-backed security market and subsequent, prolonged credit crunch that ensued. Depending on their timing, magnitude, and severity, credit supply fluctuations can interfere with an economy's ability to properly allocate its financial resources directly impacting GDP growth. However, when investigating the relationship between aggregate credit and growth for developed countries, recent studies find nuanced results (e.g., Rioja and Valev 2004; Arcand et al. 2015). Others suggest that instead of using broad measures of total private sector credit, one may need to study credit cycles at the sector-specific level to determine the credit-growth relationship (e.g., Beck et al. 2014; Sassi and Gasmi 2014).

While many credit sectors exist that can be decomposed by geographic region, borrower demographics, or the credit instrument traded, this paper focuses on the following three: households, non-financial corporations, and banks. Considering these sectors each serve a unique economic role, changes to their credit supply would likely hold heterogeneous consequences for real activity. For example, a sudden contraction in credit supply to households may elicit a stronger impact on aggregate consumption than a contraction in corporate credit supply. The converse might be true for an adverse corporate credit shock and investment. In turn, these shocks would likely not hold the same consequences for GDP and should be studied separately.

The primary objective of this paper is twofold: to accurately identify credit supply shocks based upon the sectors in which they arise, and to assess their implications and relative importance for real economic activity. Within a structural vector autoregression (SVAR) framework, I employ a novel set of sign restrictions to identify an aggregate demand, ag-

important for their reduced willingness to approve new loans.

gregate supply, monetary policy, and three credit supply shocks: household, corporate, and bank. Household and corporate credit refers to the credit supplied to households and firms from all lenders, while bank credit refers to the credit supplied only by banks to those same borrowers. The purpose of including bank credit is to separate shocks inherent to the lending sector from those inherent to the borrowing sectors. When a credit supply contraction is observed in the data, the underlying shock could be a sudden change in borrower net worth or in bank capital requirements. However, regardless of the source, each shock's impact on credit supply may be indistinguishable in the data while their transmission to GDP may vary. It is reasonable to assume a banking sector shock could suggest fundamentally different economic conditions than a household sector shock. Hence, the shock source should be accounted for to properly investigate credit supply disruptions.

A baseline model is estimated initially that includes only the household and corporate credit shocks. This serves as a benchmark to study the economic implications from any type of change in household and corporate credit supply. The model is then extended with bank credit to determine if credit supply's economic impact is contingent on the sector in which the shock originates. The paper makes two key contributions to the literature. The first is from an empirical and methodological standpoint with respect to the strategy employed to identify three credit supply shocks within the same VAR. In the literature, using aggregate measures of private credit to identify a single credit supply shock is common practice. However, if credit cycles can vary by sector, then estimating a model with shocks to aggregate credit supply could provide misleading results and obfuscate the true relationship between credit and the macroeconomy. To address this, I decompose private credit and study its implications at the sector-specific level. The empirical strategy entails assigning sign restrictions on the impulse responses for a vector of macroeconomic indicators, interest rates, spreads, and credit ratios to identify six mutually exclusive shocks. The ratios, known as "mix" variables, are employed specifically to distinguish the three credit supply shocks.

The paper's second contribution comes from the estimation results and empirical analysis. Throughout the 66-year sample, I find credit supply shocks have not only varied by sector, but also by their consequences for GDP and inflation. The results show that in one sector, an adverse credit supply shock suppressing GDP growth can be accompanied by credit expansions in another promoting growth. The model estimates that bank credit shocks can explain up to 25% of unforecasted GDP fluctuations while household and corporate credit shocks can explain up to 15% on a one-year horizon. In addition, the model suggests household and bank credit shocks may hold long-run consequences for inflation and explain up to 15% of its fluctuations. However, the direction in which inflation responds to the shocks is sector-dependent. While theoretical models often disagree on the inflation response to financial shocks due to opposing demand and supply channel effects, some SVAR studies assign a procyclical inflation response to credit shocks (e.g., Busch et al. 2010; Furlanetto et al. 2017; Mumtaz et al. 2018). I take an agnostic stance and leave the inflation response unrestricted.[2]

Further evidence provided by historical decompositions identifies several periods throughout the sample where credit shocks have been significant drivers of GDP forecast errors. These periods include the early 1950s and 60s; the 1973, early 1980s and 90s recessions; the 2000s housing bubble; and the last several years. When assessing the model's empirical validity, many credit supply disturbances identified by the SVAR can be matched with well-known credit events documented in the literature. Examples include the Carter Administration's 1980 credit controls and the rapid expansion in mortgage lending during the early 2000s housing bubble. Also, in contrast to previous studies emphasizing the importance of credit markets during the 2008 financial crisis and downturn, I find only a minor role for credit supply shocks during the onset of the crisis. This suggests the recessionary effects from credit market distress beginning in late 2007 may have been more predictable than previously thought. Instead, adverse supply shocks are most responsible for the large contractions in

[2]Robustness with respect to price level restrictions is analyzed and discussed in Appendix B.

GDP during this period supporting evidence in Hamilton (2009) finding the recession to be highly reminiscent of an oil price shock recession. Additionally, historical decomposition analysis for inflation provides evidence on the possible role for credit supply in explaining the "missing disinflation" observed during the crisis.

The paper is organized as follows. Section 2.2 reviews the related empirical and theoretical literature. Section 2.3 details the econometric methodology and identification strategy. Section 2.4 presents results from the model's baseline and extended specifications. Section 2.5 interprets the results within a historical context and Section 2.6 concludes.

2.2 Related Literature

The ratio of total private sector credit-to-GDP is commonly used to measure a country's level of financial development or depth. King and Levine (1993) is one of the first studies finding the ratio to be a strong predictor for economic growth. Using a variety of econometric techniques, Beck et al. (2000a,b) also identify a positive relationship between financial development and GDP. More recently, SVAR frameworks have gained popularity as a tool to further investigate the credit-growth relationship. One area of the literature applies recursive techniques to identify credit shocks separating the time series into macro and financial blocks. This approach relies on the assumption that macroeconomic disturbances such as aggregate demand and supply shocks impact financial variables contemporaneously while financial shocks impact macro variables with a one-period lag. Following this methodology, Gilchrist and Zakrajšek (2012) isolate the excess bond premium from corporate bond spreads and find that exogenous shocks to the premium can lead to significant contractions in bank lending, equity valuations, and real activity. Using various measures of credit, other papers studying this relationship within recursive SVAR frameworks include Lown and Morgan (2006), Gilchrist and Zakrajšek (2011), Bassett et al. (2014), and Boivin et al. (2020).

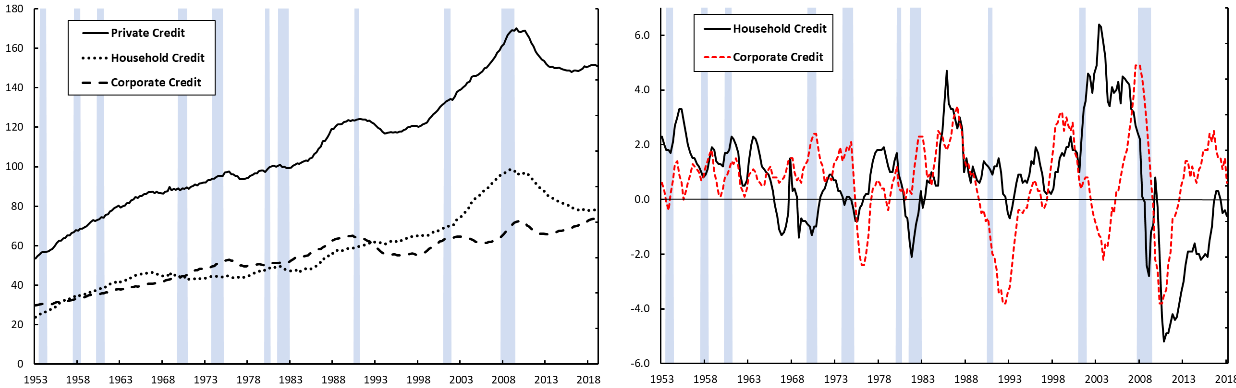
When using data at a quarterly frequency, the timing restrictions imposed for recursive identification may not be entirely plausible in all contexts. Alternative identification strategies such as sign restrictions attempt to circumvent this issue by taking a more agnostic approach. This entails assigning restrictions on impulse response functions to uniquely identify various shocks as in Faust (1998) and Uhlig (2005). Recent studies employing variations of this methodology in constant Bayesian VAR frameworks with credit shocks include Busch et al. (2010), Helbling et al. (2011), Hristov et al. (2012), Meeks (2012), Fornari and Stracca (2013), Eickmeier and Ng (2015), Abbate et al. (2016), Fadejeva et al. (2017), Furlanetto et al. (2017), and Mumtaz et al. (2018). Related models modify the sign identification strategy by using combinations of sign, timing, and zero restrictions as in Bean et al. (2010), Peersman (2012), Houssa et al. (2013), Barnett and Thomas (2014), Moccero et al. (2014), Peersman and Wagner (2015), and Duchi and Elbourne (2016). Despite the variation across identification methods, a procyclic relationship between credit shocks and output growth is consistent across these studies, especially in the short-run. This result serves as a critical component for justifying several of the within-quarter sign restrictions used in this study.

Considering SVARs require a priori assumptions regarding the behavior of the endogenous variables, their imposed restrictions should be supported both in theory and empirically. In particular, evidence from DSGE models have provided theoretical foundations upon which sign restriction-based approaches to credit shock identification are grounded. Gerali et al. (2010) estimate a DSGE model with financial frictions and identify credit supply shocks with loan-to-value ratios and lending spreads for households and firms. Using Euro Area data, they find credit shocks were most responsible for the 2008 economic downturn and recession. Curdia and Woodford (2010) assess the implications of a modified Taylor Rule that adjusts to fluctuations in credit spreads and aggregate credit volumes for mitigating the contractionary economic effects of credit shocks. Gilchrist and Zakrajšek (2012) perform a similar analysis but with a Taylor Rule incorporating their excess bond premium as a proxy for credit supply conditions. Additional theoretical contributions investigating these relationships include Christensen and Dib (2008), De Graeve (2008), Gertler and Karadi (2011), Carlstrom et al. (2014) and Christiano et al. (2014).

While it is well established throughout the empirical literature that credit growth fosters future economic growth in less developed economies, the same is not necessarily true for advanced economies. Among others, Rioja and Valev (2004) and Arcand et al. (2015) suggest there may exist a financial development threshold at which the credit-growth relationship turns negative. Arcand et al. (2015) find the relationship is positive and statistically significant until private credit-to-GDP reaches 72% before turning negative once exceeding 110%. While a range of explanations for this nonlinearity exists, the one of particular interest in this study pertains to borrowing sector composition. Within the context of credit supply shocks, a sudden contraction in private credit may be concentrated among one specific type of borrower or multiple. Therefore, aggregate credit measures may be too broad and should be decomposed by sector to properly explore credit supply's economic consequences. When private credit is split between households and non-financial corporations, empirical studies find nuanced results regarding the credit-growth relationship. According to Sassi and Gasmi (2014), more financially developed countries exhibit higher shares of household credit and the credit-growth relationship depends on borrower composition.

Figure 2.1a plots the time series for total private credit, household credit, and non-financial corporate credit as shares of GDP in percentage points. While each series displays an overall upward trend throughout the sample, household and corporate credit growth appear to follow distinct cycles. Corporate credit made up the larger share of GDP during the early 1950s, 70s, and 80s, while the late-50s, 60s, and last 25 years have witnessed a larger share of household credit. A more detailed display of these cycles is provided in Panel b which plots the year-on-year change for the household and corporate credit shares. While the time series vaguely follow similar trends on occasion, they vary substantially throughout the majority of the sample. Focusing on the 1990 and 2001 stock market crash periods,

Figure 2.1: Private Sector Credit Supply



(a) (b)

Notes: Panel (a) is total private, household, and non-financial corporate credit as shares of GDP. Panel (b) is the year-on-year change. Y-axis units are percentage points.

corporate credit growth slows significantly while household credit maintains positive growth. Household credit then experiences a rapid expansion during the early 2000s housing bubble with corporate credit expanding soon after until the recession hits and growth contracts in both sectors.

Within the empirical literature, Mian et al. (2017) find that growth in household and non-financial corporate credit predict a negative credit-to-GDP relationship in the long-run, while household credit is expansionary on shorter horizons. The predictive power of corporate credit for growth is not robust in the long-run. Using data on a panel of 143 countries, Leon (2016) also finds the relationship to be negative. In contrast, Beck et al. (2014) and Sassi and Gasmi (2014) find corporate credit growth to be expansionary with household credit eliciting contractionary or insignificant effects. Attempts to alleviate this ambiguity involve identifying the specific channels through which credit markets impact real activity such as consumption behavior, savings decisions, and human capital investment. A list of these studies include Galor and Zeira (1993), Jappelli and Pagano (1994), De Gregorio (1996), Levine (1997), Borio et al. (2015), Gopinath et al. (2017), and Charles et al. (2018). However, explaining the channels through which credit shocks may propagate and transmit to real activity is beyond the scope of this paper.

The literature most closely related to this study involves the identification of sector-specific credit shocks within VAR frameworks. Using a recursive SVAR, Walentin (2014) studies the impact of mortgage spread shocks on consumption, residential investment, and output. While he does focus on shocks to the non-corporate private sector, the model does not include measures for credit supply other than rates, spreads, and outstanding mortgage debt. Brunnermeier et al. (2018) include measures of business, real estate, and consumer loans along with various credit spreads and time-variation across shock volatilities. They identify ten independent shocks of which several closely resemble shocks to household credit, corporate credit, inter-bank stress, and monetary policy. Another study decomposing credit supply by sector but applied to the Dutch economy is Duchi and Elbourne (2016). Using sign and zero restrictions, they use total private lending with corporate bond spreads before estimating two separate VARs to analyze the impact of credit supply shocks on consumption, investment, and inflation. Following a credit shock, they find investment responds and recovers quickest while household lending and consumption respond less but with greater persistence. The key distinction between their approach and mine is that I identify multiple credit shocks within one VAR without imposing zero restrictions or isolating the components of GDP.

2.3 Empirical Methodology

2.3.1 The Model

The econometric analysis begins with estimating the reduced-form of the VAR model. This involves regressing each dependent variable at time t on its own lags and lags of the remaining dependent variables. Consider the model's reduced-form in vector notation:

$$y_t = c_a + \sum_{i=1}^{p} A_i y_{t-i} + u_t, \tag{2.1}$$

where y_t is an $(n \text{ x } 1)$ vector containing n endogenous variables, c_a is an $(n \text{ x } 1)$ vector of constants, A_i with $i = 1, \cdots, p$ represents the $(n \text{ x } n)$ coefficient matrices, and u_t is the $(n \text{ x } 1)$ reduced-form vector of residuals with $u_t \sim N(0, \Sigma)$ and Σ denoting the $(n \text{ x } n)$ residual variance-covariance matrix. The number of lags is denoted by p. The reduced-form of the model is sufficient for performing forecast analysis, but due to correlation across the error terms in u_t, structural identification is required to isolate the exogenous structural innovations, ϵ_t. The vector of residuals can then be expressed as a linear combination of the structural innovations, $u_t = B_0^{-1}\epsilon_t$, where B_0 is a non-singular parameter matrix and $\epsilon_t \sim N(0, I_n)$ where I_n is an $(n \text{ x } n)$ identity matrix. The structure of the variance-covariance matrix is $B_0^{-1}B_0^{-1'} = \Sigma$ which implies symmetry. The structural form can now be written as

$$B_0 y_t = c_b + \sum_{i=1}^{p} B_i y_{t-i} + \epsilon_t. \tag{2.2}$$

Given the symmetry of the variance-covariance matrix, $n(n-1)/2$ restrictions are required to derive B_0 and identify the SVAR. One of the most common ways for imposing the necessary restrictions to identify B_0 is the Cholesky decomposition which leaves the parameter matrix with a lower triangular structure. This implies a recursive ordering of the variables with zero restrictions to separate the fast and slow-moving variables. However, theory typically provides a stronger consensus on the directions and comovement between variables than the amount of time it takes for one variable to respond to others, especially for macro-finance SVARs. Following Uhlig (2005), I take a more theoretically consistent approach to identification by imposing sign restrictions on the impulse responses. While the sign restriction approach will not identify B_0 exactly, it will restrict B_0 to a credible range from which informative estimates can be derived from. Further details and support regarding the choice of sign over zero restrictions are provided in Section 2.3.3.

To impose the sign restrictions, I employ the methodology developed in Arias et al. (2018) which allows for combinations of sign, zero, and magnitude restrictions to be assigned on impulse responses in any given period. Remaining consistent with the recommendation outlined in Canova and Paustian (2011), all restrictions are imposed only on impact. The algorithm works as follows. The first step is to draw a vector β of reduced-form coefficients for $A_1, A_2, \cdots, A_p$ and a residual variance-covariance matrix Σ from their posterior distributions. From this, the reduced-form of the model in (2.1) can be recovered. The next step is to form combinations of the structural innovations derived from the recursively identified model. This is achieved by drawing a random orthogonal matrix Q from a uniform distribution such that $B_0^{-1}QQ'B_0^{-1'} = \Sigma$ holds. To successfully obtain matrix Q, an (n x n) random matrix X is drawn containing entries drawn from an independent standard normal distribution. A QR decomposition of X is then performed so that $Q = XR$ where R is an upper triangular matrix. Now, candidate impulse response functions are generated from BQ and A_i for $i = 1, 2, \cdots, p$ and checked to determine if they satisfy the restrictions. If they are satisfied, the proposed matrix Q is kept. If not, Q is discarded, a new matrix X is drawn, and the procedure is iterated over until a valid matrix Q is obtained.[3]

2.3.2 Data and Estimation

The model's baseline specification includes the demand, supply, monetary, household, and corporate credit shocks to establish a benchmark before introducing bank credit. A sample of quarterly U.S. data beginning in 1952 Q1 and ending in 2018 Q1 is used. The sample begins in 1952 due to the availability of private sector credit data. The vector of endogenous

[3]Additional details regarding the estimation procedure and strategy using sign restrictions can be found in Dieppe et al. (2016) and Arias et al. (2018).

variables include real GDP, the GDP price deflator, the federal funds rate, the spread between the bank prime lending rate and 3-month Treasury bill rate, the non-financial private credit mix, and the household credit mix. A list of the data series used in all estimation exercises along with their sources can be found in Table B.1 of Appendix B.

The private credit mix is the ratio between total credit to the non-financial private sector and GDP.[4] Private credit includes credit to households, non-profit institutions serving households, and all non-financial corporations public and privately owned. It's measured as the sum of all non-government loans and debt securities outstanding, which includes the market value of bonds and short-term commercial paper. Trade credit is excluded from the corporate credit measure due to globally poor underlying data. The lenders include domestic and foreign banks, residents, governments, and all other credit-providing sectors such as credit unions, pension funds, and various financial institutions. The household credit mix is the ratio between total household credit and non-financial private credit. It follows that the sum of household and non-financial corporate credit comprises total private credit. Section 2.3.3 provides a detailed discussion on the implementation and significance of mix variables for shock identification.

As for the policy rate, since the zero lower bound (ZLB) period is included in the sample, data for the shadow federal funds rate from Wu and Xia (2016) is used from 2009 Q1 through 2015 Q4. Since the shadow funds rate maintains a similar relationship with macro variables as the federal funds rate does historically, it provides a more accurate measure for the monetary policy stance during the ZLB period. The shadow rate is also appealing with respect to its low volatility and absence of sharp drops in accordance with the Federal Reserve's gradualist approach to monetary policy adjustments.[5]

[4]According to Mian et al. (2017), when measuring private debt fluctuations, it's important to normalize debt by GDP, because it's the growth in debt relative to the size of the economy that matters. Without normalizing, periods of real debt growth may appear large from a small base without being economically significant.

[5]During a 2004 speech at an economics luncheon in Seattle, Ben Bernanke detailed the Federal Reserve's gradualist approach to policy rate adjustments and its benefits for financial stability.

The endogenous variables enter the VAR in terms of their natural logarithm except for the interest rates and spreads which enter in levels. As suggested by the Bayesian Information Criterion (BIC), each equation of the model includes two lags and a constant term.[6] With the intent of improving estimation accuracy and efficiency, the reduced-form is estimated using Bayesian methods which requires specifying a prior distribution. Following the methodology introduced by Banbura et al. (2010), the dummy observation prior is used which performs particularly well when dealing with large VARs and many identified shocks. While matching the moments of the Minnesota prior via pseudo observations instead of placing direct restrictions on Σ, the prior allows covariance between the VAR coefficients and makes for a more tractable computation procedure. As is common throughout the literature, hyperparameter values are chosen such that the prior distribution is sufficiently informative. Robustness to prior specification is assessed in Appendix B.[7] All estimations are conducted using the European Central Bank's Bayesian Estimation, Analysis, and Regression (BEAR) toolbox in MATLAB developed by Dieppe et al. (2016).

2.3.3 Identification Strategy

There exists a strong consensus within the theoretical and empirical literature regarding the response of output, inflation, and the short-term interest rate to aggregate demand, supply, and monetary policy shocks. For demand shocks, output, prices, and the policy rate should move in the same direction, while for monetary policy shocks, output and prices move in the opposite direction as the policy rate. Supply shocks are identified with output and prices moving in opposite directions. The policy rate's response is less clear as it depends on whether monetary policy reacts stronger to output or inflation fluctuations. This ambiguity

[6] The BIC suggests three lags for the baseline and two lags for the extended model. For comparability I estimate both models with two lags, however, the paper's primary results are robust to the choice of lag length. All robustness exercises can be found in Appendix B.

[7] See Banbura et al. (2010), Dieppe et al. (2016), Miranda-Agrippino and Ricco (2019), and Section B.2 of Appendix B for more detailed discussions on the dummy observation prior.

arises from DSGE theory and Taylor rules that assign a negative interest rate response to rises in inflation and the output gap. Due to this uncertainty, I remain agnostic and leave the federal funds rate unrestricted for supply shocks.

Assigning credible restrictions to identify credit supply shocks is less straightforward. While choices for credit market indicators vary in the literature, there is consensus on the response of credit volume and price. Within the context of credit supply shocks, credit price, typically measured with lending rates or spreads relative to risk-free government rates, is assumed to move in the opposite direction as volume. While for credit demand, price and volume are expected to move in identical directions. However, the significance and frequency of credit demand shocks in the data are more difficult to justify than credit supply and are therefore not included in the VAR.[8]

One attractive feature of credit spreads is that they do not rule out periods of non-price credit rationing. Since credit spreads may increase with a rise in the lending rate or decline in the risk-free rate, credit supply shocks can be identified absent any changes to lending rates. Instead, higher spreads may result from looser monetary policy in response to depressed GDP following a credit contraction. They may also result from a flight-to-quality as banks shift their portfolios toward more government debt during periods of increased uncertainty, tight money, or liquidity shortages as discussed in Bernanke and Blinder (1988).[9] When using lending rates alone, they are restricted to rise with adverse supply shocks, making it difficult to identify periods of non-price rationing. I use the spread between the prime lending rate and 3-month Treasury yield as a proxy for credit price and the general willingness to bear private sector risk. The credit mix is used to measure credit volume. While corporate

[8] According to Bernanke and Blinder (1988), "we find it difficult to think of or identify major shocks to credit demand, that is, sharp increases or decreases in the demand for loans at given interest rates and GNP. But shocks to credit supply are easy to conceptualize and find in actual history." Wojnilower et al. (1980) finds credit demand to be inelastic with respect to the general level of interest rates and credit growth to be supply-determined.

[9] Andolfatto and Spewak (2018) find Treasury debt holdings increase in response to recent regulatory changes with respect to the shadow banking sector.

bond spreads such as the BAA-10 year Treasury are often used, I argue in favor of using the prime rate for studying household and corporate credit, considering banks often use it as a benchmark for charging both their household and corporate customers.

The strategy used to identify sector-specific credit supply shocks is a leading contribution of this paper. Mix variables were introduced in Kashyap et al. (1993) to identify a bank lending channel for the transmission of monetary policy, and have been employed in other credit VAR frameworks by Ludvigson (1998), Iacoviello and Minetti (2008), Milcheva (2013), and Halvorsen and Jacobsen (2014). In a monetary policy context, a drop in the ratio of bank lending-to-total lending following a monetary contraction would indicate a reduction in credit supply through a bank lending channel. In contrast, a decrease in overall credit supply not exclusive to banks, should have a larger impact on the ratio's denominator and either increase it or leave it unaltered. Following this intuition, the household mix is used to disentangle household and corporate credit supply shocks. It's restricted to respond negatively to adverse household shocks and positively to adverse corporate shocks.

A common approach taken to disentangle credit supply from macro and monetary shocks is through recursive identification and the Cholesky decomposition. This requires the block of credit or financial variables to be labeled as fast-moving and is placed second after the macro-variable block. The ordering restricts shocks originating in financial markets from impacting the macro variables contemporaneously, while macro shocks impact all variables within the period. This assumption may be plausible when using monthly or weekly data, but becomes significantly more difficult to justify using quarterly data. Additionally, the specific variable orderings within each block matters for contemporaneous relations, and therefore identification becomes ambiguous when using multiple fast-moving financial variables. In this study, it would be a stretch to assume that household consumption habits remain unchanged following household credit shocks for an entire quarter.

Within the sign-restriction literature, many models use a combination of sign and zero restrictions while others have imposed magnitude restrictions to separate macro and financial shocks. Consider the task of separating an adverse aggregate demand and credit supply shock. Both shocks would likely decrease GDP and credit growth within the quarter. However, the demand shock should have a stronger impact on GDP, while the credit shock should have a stronger impact on credit. Therefore, restrictions can be placed such that the initial response of one variable must be smaller in magnitude than the other. Furlanetto et al. (2017) use the ratio of private sector credit-to-real estate value for separating credit and housing sector shocks. In line with this intuition, I use the credit mix to separate the macro and credit shocks as in Eickmeier and Ng (2015). Following an adverse demand or supply shock, the mix is restricted to increase on impact with the opposite response imposed for adverse credit shocks. This assumption can be supported by the time series in Figure 2.1b. During many of the recessions, household and corporate credit as shares of GDP rise before dropping. This suggests a stronger reduction in GDP initially following the aggregate-level disturbances associated with demand and supply shocks during recessions, before credit markets begin to contract.

The final restriction separates the monetary policy and credit shocks. Identifying these to be mutually exclusive and justifiable is not trivial considering exogenous changes in the federal funds rate may have an immediate impact on credit markets. I exploit the direction in which the prime spread initially responds to differentiate these shocks. The spread is restricted to decrease for contractionary monetary shocks and increase for contractionary credit shocks. Considering the prime and federal funds rates are highly correlated, these restrictions assume the prime rate reacts quicker and stronger to credit market disruptions than it would to policy rate shocks. Theoretical and empirical studies find imperfect pass through between policy and lending rates.[10] Other studies explicitly applying this assumption to identify credit

[10] Using the financial accelerator model developed by Christiano et al. (2014), Cesa-Bianchi and Sokol (2017) show how the spread between loan and policy rates responds stronger than the policy rate to financial shocks. The opposite holds for aggregate demand shocks. Leaving the response of credit spreads unrestricted, Furlanetto et al. (2017) find that financial shocks generate countercyclical movements in credit spreads.

Table 2.1: Baseline Model Sign Restrictions

	Demand	Supply	Monetary	Household	Corporate
GDP	−	−	−	−	−
Prices	−	+	−		
Policy Rate	−		+		
Prime Spread			−	+	+
Credit Mix	+	+		−	−
Household Mix				−	+

shocks using spreads and sign restrictions include Helbling et al. (2011), Eickmeier and Ng (2015), Fadejeva et al. (2017), and Bäurle and Scheufele (2019).

Restrictions for the five shocks in the baseline model are summarized in Table 2.1.[11] All restrictions are imposed on impact and last one quarter. Each shock is specified as contractionary with a negative sign placed on GDP. However, the VAR estimation identifies both the contractionary and expansionary form of each shock.

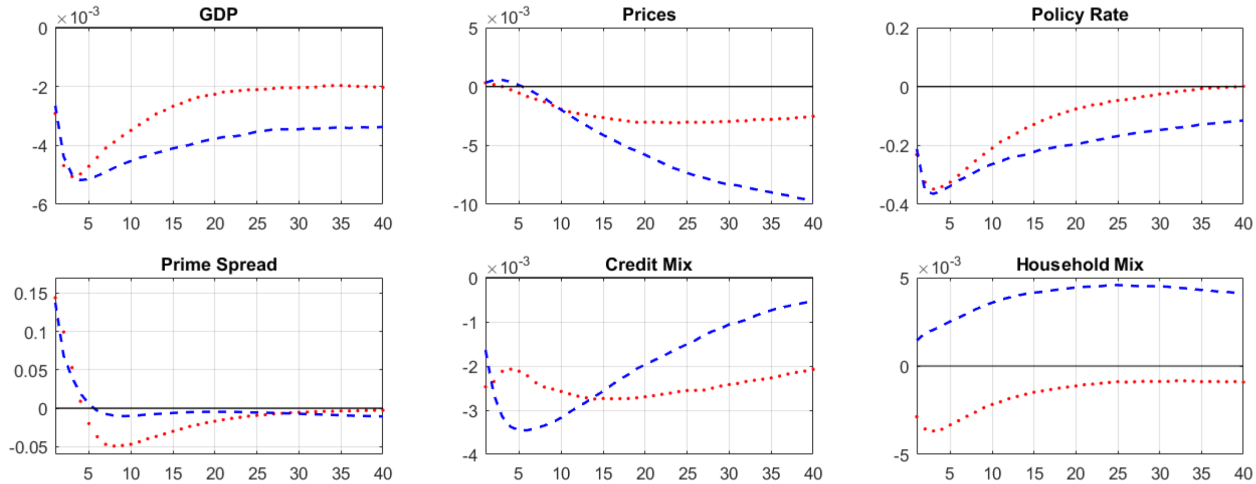
2.4 Results

2.4.1 Baseline Model

Results for the baseline estimation are presented first to study the implications of household and corporate credit supply for real activity. While the paper's focus pertains to credit supply, results for the demand, supply, and monetary policy shocks are referenced for comparison and evaluation of the model's performance. Figure 2.2 displays the median impulse responses for the six endogenous variables to a one-standard deviation adverse household

[11] While identifying a total of n shocks for an n-variable SVAR is common practice, the sixth shock is left unidentified. This serves as a residual shock and picks up any remaining disturbances unaccounted for. The residual shock is not included in Table 2.1.

Figure 2.2: Baseline Model Credit Supply Shock Impulse Responses



Notes: Impulse responses to a one-standard deviation adverse household (dotted line) and corporate credit supply shock (dashed line) for the baseline model. Estimates based on median response.

(dotted line) and corporate (dashed line) credit shock.[12] For both shocks, GDP declines to about -0.5% before beginning its recovery after three quarters. The drop is more persistent following the corporate shock with GDP at -0.2% and -0.35% after ten years for the household and corporate shocks, respectively. Impulse responses are similar yet negligible for inflation within the first two years before prices drop and level out for household credit, but decline further to -1% for corporate credit. While this suggests credit shocks may matter more for long-run price levels, Bean et al. (2010), Busch et al. (2010), Ciccarelli et al. (2010), and Gambetti and Musso (2017) find procyclical responses on shortened horizons. Analyzing loan supply shocks for the Netherlands, Duchi and Elbourne (2016) document a near-zero inflationary response. As for the policy rate, it declines by about 35 basis points following both shocks in accordance with the drops in output and inflation, but is more persistent for corporate credit. This complies with Taylor rules and the interest rate reaction to inflation and output gap fluctuations. The stronger responses by output and inflation following the corporate shock likely account for the policy rate's slower recovery.

[12]See Appendix B.1 for median impulse responses and corresponding credible intervals.

Moving to the financial block, the prime spread rises nearly 15 basis points and returns to steady state within five quarters for both shocks. However, it levels out slightly below steady state for corporate credit yet continues to decline another year for household credit. Around the same horizon, a divergence develops with the response of the credit mix. Corporate credit induces a larger decline with the mix reaching -0.35% after five quarters. However, the drop in the mix following the household shock is much more persistent and rests around -0.2% after ten years. The quick recovery following corporate shocks supports evidence regarding the alternative sources of corporate financing options not available to households during economic downturns.[13]

Comparing with the impulse responses for the macro and monetary shocks in Figure B.2 of Appendix B, credit shocks produce the largest contractions in GDP. Demand shocks elicit the weakest impact at -0.25% that fully recovers after three years. The supply and monetary shocks are stronger and more persistent with GDP dropping to -0.3% after two years and remaining depressed for the following eight. The inflation response to supply and monetary shocks is minimal, while for demand it resembles the corporate shock with prices gradually decreasing to almost -0.9%. The policy rate is also more responsive to demand shocks while remaining near its steady-state level for supply. This complies with the literature's uncertainty regarding the policy rate reaction to supply shocks and gives credence to leaving it unrestricted.

Table 2.2 reports the forecast error variance decomposition (FEVD) for each endogenous variable. The FEVD provides the share of forecast error variation explained by a given shock. Estimates are based on the median draw satisfying the sign restrictions at the 1, 4, 16, and 32-quarter horizons. For comparability, the decompositions are rescaled so they sum to one at each horizon.[14] Supply and credit shocks serve as the leading drivers of

[13]Giesecke et al. (2014) find banks increase their lending to firms shortly after bond default crises. With respect to the 2008 Lehman Brothers collapse, Ivashina and Scharfstein (2010) discuss how corporations increased borrowing by drawing on unused bank credit lines.

[14]According to Fry and Pagan (2011), using the median values from impulse responses to compute the

Table 2.2: Baseline Model Forecast Error Variance Decomposition

	Horizon	Demand	Supply	Monetary	Household	Corporate	Residual
GDP	1	0.15	0.21	0.14	0.21	0.18	0.12
	4	0.09	0.13	0.11	0.28	0.28	0.11
	16	0.07	0.14	0.14	0.22	0.31	0.11
	32	0.09	0.15	0.16	0.19	0.30	0.11
Prices	1	0.17	0.23	0.14	0.13	0.16	0.17
	4	0.23	0.24	0.10	0.13	0.15	0.16
	16	0.40	0.12	0.07	0.12	0.18	0.12
	32	0.38	0.09	0.06	0.10	0.26	0.10
Policy Rate	1	0.21	0.11	0.18	0.17	0.17	0.16
	4	0.30	0.08	0.09	0.19	0.22	0.12
	16	0.34	0.09	0.07	0.18	0.22	0.11
	32	0.31	0.10	0.08	0.16	0.24	0.11
Prime Spread	1	0.16	0.11	0.22	0.19	0.17	0.16
	4	0.16	0.14	0.16	0.20	0.16	0.17
	16	0.24	0.14	0.11	0.20	0.15	0.16
	32	0.26	0.13	0.11	0.19	0.15	0.16
Credit Mix	1	0.22	0.22	0.12	0.19	0.08	0.17
	4	0.24	0.16	0.12	0.13	0.20	0.15
	16	0.18	0.11	0.13	0.17	0.26	0.15
	32	0.16	0.12	0.13	0.22	0.21	0.15
Household Mix	1	0.12	0.12	0.13	0.40	0.10	0.14
	4	0.12	0.11	0.14	0.37	0.12	0.13
	16	0.17	0.11	0.14	0.18	0.27	0.13
	32	0.16	0.13	0.12	0.12	0.34	0.12

output fluctuations within the first quarter, while credit shocks dominate at all horizons beyond the first year. After one year, both credit shocks explain 28% of GDP fluctuations with corporate credit as the leading long-run driver explaining 30-31% after the fourth. Demand shocks make significant contributions to output on impact, but explain very little variation beyond one quarter. The short-lived decline in output following demand shocks reinforces this result. As for prices, demand shocks make modest contributions in the short-run but become increasingly more important explaining up to 40% of their fluctuations after four years. Supply shocks make similar contributions for prices as they do for output, but household and corporate shocks contribute less.

The FEVD for credit shocks are broadly in line with those found for various financial sector

FEVD combines information across different models allowing the decompositions to not necessarily sum to one. For clearer interpretation, the values can be rescaled so the variance is exhaustively accounted for without sacrificing quantitative significance.

shocks, but contribute more than what is often found for credit supply shocks. The literature typically assigns around 20-40% of GDP variation to financial shocks and 10-20% to credit supply shocks. As for inflation, studies assign an even wider range between 10-40% to credit shocks at various horizons. The FEVD for prices and the macro shocks in this study also broadly agree with the literature finding supply shocks to matter most for short-run fluctuations while demand shocks dominate in the long-run. However, I refrain from comparing my estimates closely with this literature as it varies widely with respect to variable choice, identification methods, and datasets. Also, many models identify financial shocks as disturbances exclusive to the financial or banking sectors. Considering this, the household and corporate credit shocks here would not necessarily be considered financial shocks since they may be responding to financial or non-financial conditions. To further address this issue, I now extend the model with bank credit to determine whether the effects from credit supply shocks arising in the banking sector differ from those in the household and non-financial corporate sectors.

2.4.2 Extended Model

In the baseline specification, household and corporate credit shocks may arise in response to credit conditions among borrowers or lenders. If perceived credit risk associated with a specific group of borrowers rises, lenders would likely respond by restricting credit to that group regardless of lender type. Alternatively, if the shock is concentrated among a specific lending sector such as a bank balance sheet disturbance, then bank lenders would be most likely to respond with tighter credit initially. To circumvent this potential ambiguity, I disentangle credit shocks propagating in response to household and corporate borrowers from those arising in the banking sector. The key assumption is that borrowing sector shocks reflect credit supply contractions from all lenders (bank and non-bank) due to borrower-specific reasons.

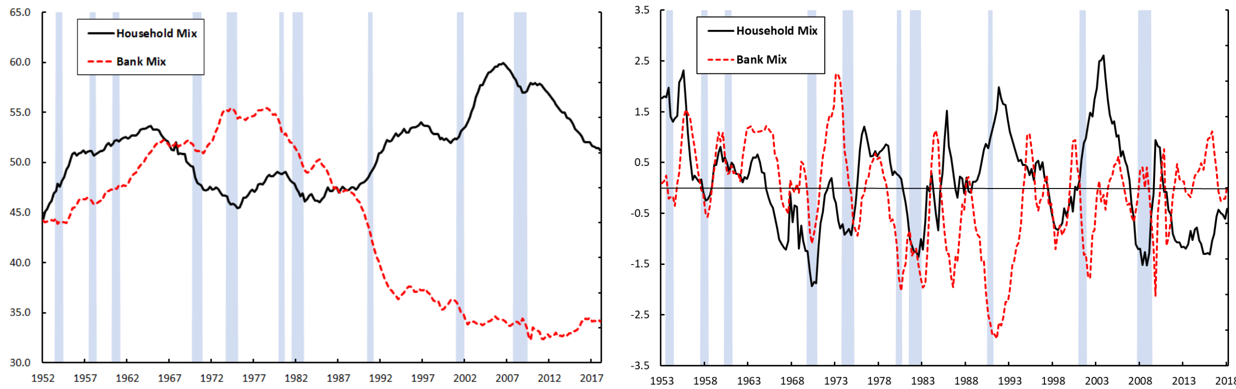
Banking sector shocks may arise as exogenous changes in risk preference, regulation, liquidity funding costs, or new financial technology and innovation. Consider a regulatory change that raises capital requirements forcing banks to increase their liquid assets. Depending on the costs and availability of short-term liquidity, their lending behavior may be adversely affected. Since banks possess superior knowledge, skills, and technology for assessing risk, changes in their willingness to extend credit may suggest more severe issues underlying the economy. Hence, bank and non-bank credit shocks displaying similar effects on aggregate credit in the data don't necessarily imply similar levels of health and risk in the economy. For these reasons, bank credit shocks may elicit a GDP response heterogeneous to those observed from the baseline estimation.

Bank credit is measured as the sum of all domestic bank claims on the private non-financial sector. This consists of all outstanding loans and debt securities provided by the sectoral balance sheets of U.S. depository institutions, excluding the Federal Reserve.[15] Following the same approach for household and corporate credit, I use a bank credit mix variable to separate bank and non-bank sector shocks. The bank mix is the ratio of total bank-issued credit-to-total private credit. Using this mix, Halvorsen and Jacobsen (2014) identify a VAR with sign restrictions to compare the effects between bank credit supply and monetary policy shocks on real activity in the UK and Norway. Iacoviello and Minetti (2008) isolate bank credit shocks within a recursive VAR to study their impact on GDP and prices, but within the context of mortgage lending and the housing market. Adverse credit shocks exclusive to banks should lower the bank mix while adverse household or corporate shocks should raise it.

The bank mix is plotted alongside the household mix in Figure 2.3a with its year-on-year

[15] One issue for bank credit data is securitization. Derecognized securitized loans are not reported on banks' balance sheets under traditional accounting rules and should not be included in the measure even though banks often support their loan portfolios with off-balance sheet claims as demonstrated during the 2008 financial crisis. However, total private credit is unaffected by this issue since it covers credit from all sectors including special purpose vehicles to which banks sell their loan portfolios. Additional details are provided in Dembiermont et al. (2013).

Figure 2.3: Bank and Non-Bank Private Sector Credit Supply



(a) (b)

Notes: Panel (a) is total credit to households and credit supplied by banks as shares of total credit to the non-financial private sector. Panel (b) is the year-on-year change. Y-axis units are percentage points.

change in Figure 2.3b. Early in the sample, both mixes follow upward trends until the mid-1960s when corporate credit begins to dominate the household mix while bank credit continues expanding relative to non-bank lenders. This pattern begins to change around the mid-1970s and completely reverses by 1990. Upon further inspection, Figure 2.3b reveals specific episodes such as the late-1970s expansion and early 2000s housing bubble during which positive growth in one may not be accompanied by positive growth in the other. This suggests a nonlinearity as to whether the sources for household and corporate credit fluctuations are driven by factors akin to the banking or borrowing sectors.

The full set of sign restrictions for the extended model are presented in Table 2.3 and the impulse responses for credit shocks are displayed by Figure 2.4. The extended model's FEVD and non-credit impulse responses can be found in Appendix B.1. The impulse responses for demand and supply shocks follow similar patterns as in the baseline, except now GDP recovers quicker and prices respond less for demand. Monetary policy shocks elicit larger drops in inflation and now account for 24% of long-run GDP fluctuations making it the largest non-credit shock contributer. From Figure 2.4, bank credit shocks (solid line) cause the largest GDP contraction at -0.43% while the impact from household and corporate shocks

Table 2.3: Extended Model Sign Restrictions

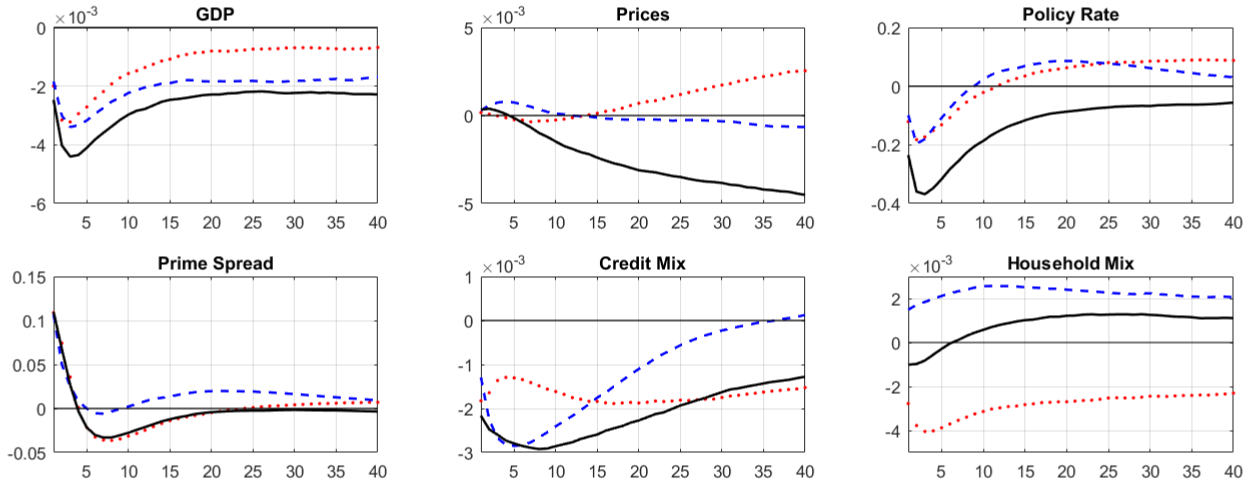
	Demand	Supply	Monetary	Household	Corporate	Bank
GDP	−	−	−	−	−	−
Prices	−	+	−			
Policy Rate	−		+			
Prime Spread			−	+	+	+
Credit Mix	+	+		−	−	−
Household Mix				−	+	
Bank Mix				+	+	−

are around -0.32%. In response to depressed output, the policy rate displays the largest and most persistent decline following the bank shock at -35 basis points after three quarters. Unlike the baseline, the policy rate now only drops by 18 basis points before turning positive after three years following the household and corporate shocks.

The inflation response is negligible initially, but household and bank shocks appear to matter for the long-run, while corporate shocks leave prices almost unchanged in line with Christiano et al. (2010) regarding the low levels of inflation observed during U.S. stock market booms.[16] These results depart from the baseline which predicted a 0.25% and 1% long-run price decline following household and corporate shocks, respectively. Also, household credit shocks now lead to a persistent price increase instead of decrease. This result is not likely due to the loose monetary response as the policy rate's path shows little variation following the household and corporate shocks. Mian et al. (2017) find significant evidence of boom-bust cycles for real estate prices following household credit expansions. Although household shocks appear to be non-inflationary in the short-run, this evidence would agree with the price level's long-run response. In addressing the literature's ambiguity regarding credit supply shocks and prices, these results suggest their inflationary effects may be sector-dependent and emphasize the potential limitations of using aggregate credit supply measures.

[16] Christiano et al. (2010) discuss how inflation was relatively low during all 18 of the stock market booms that occurred in the U.S. throughout the last two centuries. This was also observed for Japan during their 1980s stock market boom.

Figure 2.4: Extended Model Credit Supply Shock Impulse Responses



Notes: Impulse responses to a one-standard deviation adverse household (dotted line), corporate (dashed line), and bank (solid line) credit supply shock for the extended model. Estimates based on median response.

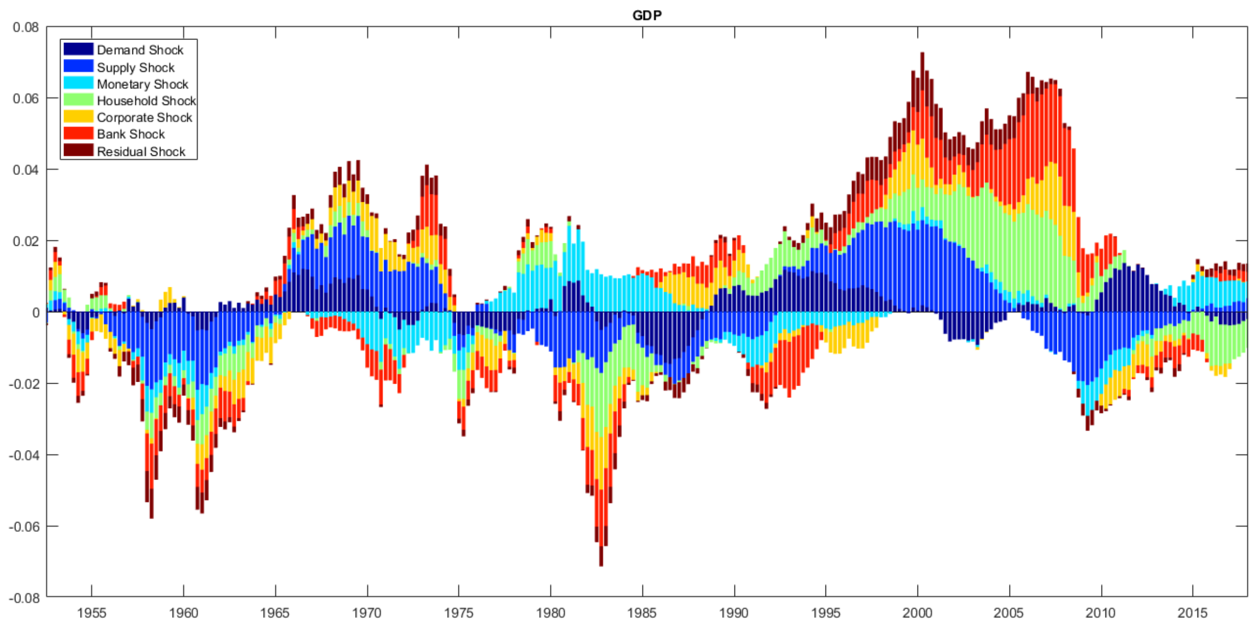
Transitioning to the financial block, the prime spread rises by about ten basis points for all three shocks, but appears to be least sensitive to corporate credit. This is shown by the spread hovering around steady-state after the first year for the corporate shock while dipping below steady-state for the others. Corporate and bank shocks produce the largest drop in the credit mix reaching -0.28% after six quarters with only -0.15% for household shocks. Consistent with the baseline, the credit mix recovers quickest following corporate shocks and remains around -0.15% after ten years for household and bank credit. The larger declines in GDP and credit growth following bank shocks provide evidence that the shock's source may be critical for identifying the real effects from credit supply disruptions. During the 2008 crisis, financial contagion became an important channel for the recession's size and transmission onto a global scale. While an exogenous disturbance in the banking sector would initially hit bank balances sheets hardest, it would likely spread to other non-bank lending institutions further suppressing credit and output.

Table B.2 in Appendix B.1 displays the extended model's FEVD. Bank shocks are the largest contributer for GDP while household and corporate shocks are significantly less important when compared with the baseline. From explaining close to 30% of GDP errors, they now explain up to 15% along with demand shocks within the first year. At the one-year horizon, bank shocks explain 25% of the error variance while monetary shocks now explain 24% instead of 16% after eight years. For inflation, credit shocks are nearly equivalent with each contributing to almost 15% of its short-run variation. However, bank shocks matter more for long-run price fluctuations as do the demand and monetary shocks. The policy rate now appears to be most sensitive to demand and bank shocks instead of demand and corporate shocks. This corroborates the corporate shock's limited impact and role for price fluctuations in Figure 2.4.

2.5 Historical Analysis

This section places the model's results within a historical context to assess their empirical validity and relevance. I draw upon evidence presented by the historical decomposition (HD) for GDP and the credit shock volatilities displayed by Figures 2.5 and 2.6, respectively. The HD summarizes the individual contributions for each structural innovation to the total forecast error of a variable at each point in time. It's computed by transforming the reduced-form residuals into a set of structural innovations and calculating the cumulative impact each innovation makes on that period's forecast error since the beginning of the sample. This allows me to answer questions such as, What share of GDP's forecast errors in 2007 Q4 can be attributed to all current and past household credit supply shocks? As for Figure 2.6, the volatility for each credit innovation is represented by its quarterly standard deviation from steady-state. Since all restrictions are imposed to identify shocks as contractionary, positive values correspond to negative GDP growth. While the dataset spans 66 years, I focus on four periods in which credit shocks make significant contributions to output relative to the non-credit shocks. These periods cover the early 1950s, the 1970s, the early 1980s and 90s

Figure 2.5: GDP Historical Decomposition



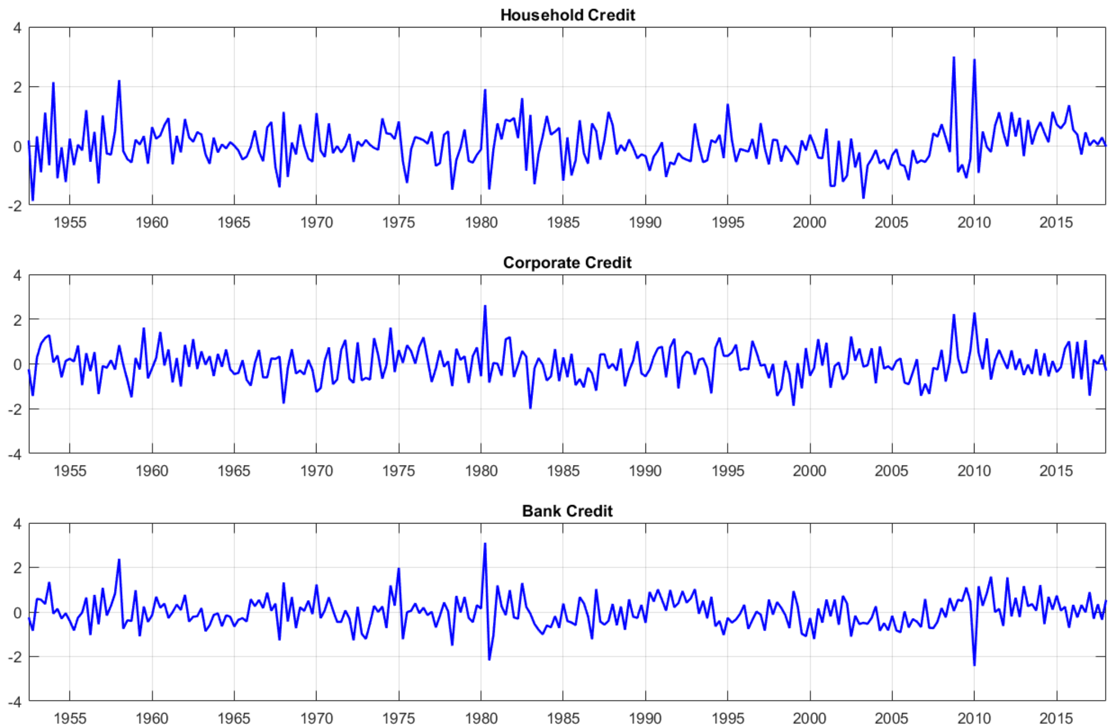
Notes: Historical decomposition of GDP forecast errors for the extended model. The green bars correspond to the household credit supply shock, orange bars for corporate credit supply, and red bars for bank credit supply.

recessions, and the pre-2008 financial crisis period.

2.5.1 Early 1950s

Beginning with the early 1950s, all three credit shocks contribute to positive GDP growth. This coincides with the introduction of the Diners Club Card that is often regarded as the first modern day credit card. Surrounding the 1953 recession, adverse corporate and bank credit shocks play the largest roles in driving down GDP. During this period, short-term Treasury rates rose while the rates banks could offer on savings and time deposits were bound by Regulation Q. Without the ability to attract customers and funds by raising deposit rates, disintermediation ensued, banks became liquidity constrained, and credit contracted. In addition, new 30-year Treasury bonds were issued offering high coupon rates contributing to a corporate debt sell-off which may account for the adverse corporate credit shocks. Although

Figure 2.6: Credit Supply Shock Innovations



Notes: Median credit supply shock innovations for the extended model. Y-axis units are standard deviations.

household credit plays a negligible role during this period, Figure 2.6 shows high volatility with adverse shocks rising above two standard deviations in 1953. This pattern is reversed near 1955 with a series of expansionary household and bank shocks. Wojnilower et al. (1980) documents an expansion in bank lending with no-down-payment mortgage advertisements.

2.5.2 1970s

The 1970s begin with a pattern of adverse bank and expansionary corporate shocks while household credit contributes little. Credit shocks account for nearly 50% of output fluctuations. Around 1971, bank credit turns expansionary coinciding with the agriculture commodity boom and large dollar depreciation in response to the gold standard abandonment. Credit expansions begin reversing by 1974 with bank shocks reaching two positive standard deviations and household credit becoming the largest contributor. According to Owens and Schreft (1995), rising short-term rates in 1973 led to an increase in the prime rate with banks only lending to the most credit-worthy firms. A housing production slowdown is also documented at this time which may account for the household credit contractions. Later in the decade, all three credit shocks turn expansionary with household credit again as the most significant. Wojnilower et al. (1980) discusses a bank credit expansion impacting the most credit-sensitive sectors further fueling inflation and house prices in 1976. Kuhn et al. (2017) argue that the period witnessed rising household debt on the intensive margin operating through the credit-supply channel.

2.5.3 1980-1990 Recessions

Significant adverse credit shocks reaching at least two standard deviations begin during the early 1980s recessions. By 1982, credit shocks account for more than 50% of GDP fluctuations. While these recessions are typically associated with Paul Volcker's disinflation policies, the Carter Administration also imposed a set of credit controls in hopes of slowing inflation. These placed direct restrictions on marginal reserves, unsecured consumer lending, and loans for mergers and acquisitions. While this effectively inhibited credit supply to both household and corporate borrowers, Wojnilower (1985) claims the credit crunch was concentrated in mortgage credit with reduced consumption expenditures accounting for nearly 80% of the GDP decline.

The 1990 recession is associated with a real estate and stock market crash. From Figure 2.5, bank and corporate credit shocks turn negative while household shocks remain positive from the 1980s mortgage market expansions. According to Bernanke and Lown (1991), a New England real estate crash triggering a bank credit crunch was responsible for the reductions in credit supply. Owens and Schreft (1995) argue in favor of a credit supply shock concentrated in the commercial real estate market: a "sector-specific credit crunch that prevented commercial real estate developers and business borrowers using real estate as collateral from obtaining credit at any price." While these episodes support the adverse corporate and bank shocks, explanations for the household credit expansions remain less clear. However, the recession was followed by regulatory changes in 1992 entailing reduced loan documentation and appraisal requirements for mortgages. This may have counteracted the negative effects from the real estate crash on household credit.

2.5.4 Pre-Financial Crisis

Aggregate demand, supply, and corporate credit shocks depress output during the 2001 recession while household credit expands. This is reasonable considering the recession was triggered by the technology stock crash absent a real estate crisis. During the early-mid 2000s, the large and positive GDP fluctuations from bank and household credit supply are consistent with the rapid credit expansions associated with the pre-crisis housing bubble. This period is characterized by historically low policy rates, changes to financial regulation and lending standards, and increased government initiatives to extend homeownership. Prior to 2003, supply and credit shocks are equally responsible for the vast majority of output fluctuations until GDP is driven almost entirely by bank and household credit. This supports the extensive empirical literature studying the pivotal role of financial and credit market shocks throughout the period.

With respect to the 2008 financial crisis and ZLB period, my results depart from what is typically found in the literature. Many studies find financial market shocks to be among the largest contributers to declining GDP during the crisis, with some FEVD estimates reaching 50% and higher. Mumtaz et al. (2018) suggest the GDP drop in 2009 would have been reduced by 50% in the absence of credit supply shocks. The importance of financial markets for the recession is also well documented throughout the DSGE and theoretical literature. According to Figures 2.5 and 2.6, large contractions in household and corporate credit begin at the onset of the crisis, however, credit shocks still maintain a net positive impact on output from the previous expansion. Their impact doesn't turn negative until 2010. Instead, supply shocks are most responsible for driving down GDP which is consistent with Hamilton (2009) who argues that the early stages of the recession resembled an oil price shock recession due to one of the largest oil price shocks on record. He associates the 2007 rise in oil prices with a combination of increasing demand and stagnating production leading to a collapse in consumer spending, especially on automobiles. Absent the shock, he believes the economy would have been characterized by slow growth, but not a recession. According to Figure 2.5, the minor contributions from credit shocks during the crisis suggest that the effects from deteriorating credit markets on GDP may have been more predictable than previously thought.

Before concluding this analysis, results from the historical decomposition for inflation during this period are worth discussing. According to Figure B.7 in Appendix B.1, the non-credit shocks explain most price variation for the initial two-thirds of the sample until household and bank credit become significantly more important during the Great Recession. However, bank credit shocks contribute to positive price fluctuations while household shocks elicit a negative impact. From Figure 2.4, the impulse responses suggest a possible long-run procyclical and countercyclical price response to bank and household credit shocks, respectively. Considering the housing bubble and pre-crisis period witnessed expansionary household and bank credit shocks, their counteracting impact on the recession's inflation errors could be their long-run implications from that period. Furlanetto et al. (2017) associate the non-inflationary effects found for financial shocks with the observed "missing disinflation" period surrounding the crisis. In contrast, Abbate et al. (2016) uncover a countercyclical inflation response to credit supply and attribute the missing disinflation to a combination of adverse credit and non-financial shocks. Figure B.7 may partially reconcile these claims by suggesting pre-crisis household credit shocks were responsible for driving inflation down while aggregate supply and bank credit shocks drove inflation up. These opposing effects would have contributed to a more subdued inflation response during the crisis. Similar to the argument made for the credit-growth relationship's dependency on credit composition, the ambiguity surrounding credit growth and inflation may also hinge on a sector-specific relationship.

2.6 Conclusion

This paper identifies sector-specific credit supply shocks and analyzes their implications for real economic activity. Using a SVAR model, it employs a novel strategy for identifying one monetary policy shock, two macroeconomic shocks, and three sector-specific credit supply shocks via sign restrictions. While the imposed restrictions are consistent with a large body of theoretical and empirical literature, I remain as agnostic as possible regarding the relationships among the macro and financial variables. Using Bayesian methods and U.S. data from 1952 to 2018, a baseline model is estimated for only household and corporate credit shocks to study the general relationship between private-sector credit supply and economic activity. I then extend the baseline with the addition of bank credit to investigate the importance of controlling for the underlying shock source.

I find that credit supply shocks have not only varied by sector, but their implications for GDP and inflation are also sector-dependent. An adverse credit supply shock may arise in one sector simultaneously with a credit expansion in another rendering an ambiguous net effect on total credit supply and real activity. This reinforces the claim that not all credit cycles are the same and should be studied at the sector-specific level. FEVD analysis suggests that bank credit supply shocks may explain up to 25% of GDP fluctuations which is consistent with the literature on financial sector shocks and business cycles. Household and corporate credit shocks also play a significant role in explaining up to 15% of GDP fluctuations. For inflation, household and bank credit shocks may hold long-run consequences contributing up to 15% of its fluctuations. Within a historical context, the model captures many credit supply episodes documented in the literature and provides a possible explanation for the "missing disinflation" observed during the 2008 financial crisis.

Chapter 3

The Credit Card and Small Business Lending Channels of Monetary Policy

3.1 Introduction

According to conventional macroeconomic theory, a monetary tightening should be followed by a contraction in lending. This is the basic premise upon which the lending channel of monetary policy operates, but when taken to the data, its empirical relevance becomes elusive. This is especially true for time series data on consumer and business bank lending. Following a monetary tightening, consumer loans are typically found to contract, while business loans expand. However, consumer and business loans are considerably broad categories and if their subcomponents (e.g., credit card, auto, and small business) respond heterogeneously to policy, then only analyzing the aggregate variable's dynamics could render inaccurate conclusions and misleading implications for the lending channel (den Haan et al., 2007). Heeding this potential risk, this study investigates the bank lending channel of monetary policy by focusing on two of its major subcomponents: consumer credit card and small business loans.

Among the many types of bank loans that can be analyzed, credit card and small business loans are selected for several reasons. First, their high agency costs require the skills and technology inherent to large financial intermediaries leaving them heavily reliant on banks as their primary funding source. According to Bernanke and Blinder (1988), for a lending channel to exist, borrowers must display a high degree of bank-dependency with the inability to frictionlessly substitute between bank loans and alternative forms of credit. In addition, credit card and small business occupy relatively large shares of banks' loan portfolios. In the U.S. today, credit cards and small businesses account for around 50% and 57% of consumer and business bank lending, respectively.[1] Considering their bank-dependent nature and large portfolio shares, credit card and small business loans should not be overlooked as they may hold key implications for the transmission of monetary policy.

This paper investigates the two lending channels from an empirical and theoretical perspective. First, I estimate a time-varying parameter vector autoregression (TVP-VAR) model to document the lending response to a monetary tightening between 1970 and 2007. The importance of allowing for time-variation during this period is emphasized by Brady (2008, 2011) which provide evidence of structural breaks and potential changes in the credit card lending channel since the 1980s. If time-variation is present in the data, then estimating a constant parameter VAR within this context could provide misleading and uninformative results. Furthermore, in light of the many changes to financial sector regulation, innovation, and monetary policy regimes documented over the last half century, I introduce stochastic volatility to allow for possible heteroskedastic residuals (Cogley and Sargent, 2005; Koop et al., 2009).

The TVP-VAR's impulse responses provide evidence supporting a time-dependent lending channel through credit card and small business loans. In particular, both the direction and

[1] Source: Federal Reserve Board of Governors. Consumer loans include credit cards and other revolving plans; auto loans; student loans; and personal loans. Business loans include all corporate and noncorporate commercial and industrial (C&I) loans.

strength in which each loan responds to a monetary tightening has varied between 1970 and 2007. For example, credit card lending primarily contracts prior to the mid-1980s and after 2001. The business loan response remains negative throughout the sample other than the early 1970s, 1980s, and mid-2000s. Regarding the transmission channels' strength, each loan's interest rate semi-elasticity remains relatively low throughout the 1980s, fluctuating between -1.5% and 1%. This differs dramatically from the early 1970s when business loan growth reaches nearly 4%, as well as the early and mid-2000s when consumer loan growth falls to around -3%.

Following the empirical analysis, I develop a general equilibrium model of consumer and business lending to analytically study the underlying mechanisms that may be driving the channels' observed time-variation and nonlinearities. The model's baseline environment extends the entrepreneurial finance framework of Rocheteau et al. (2018) with the inclusion of an unsegmented consumer and business lending market. Here, households and firms can supplement their liquidity holdings with over-the-counter bank loans to finance random consumption and investment opportunities. The loan market is OTC in the sense that borrowers must meet a bank and have their loan applications approved before bargaining over their loan terms. As a result, the costs associated with searching for and obtaining a loan creates an extensive margin (ease of obtaining a loan) in conjunction with an intensive margin (loan size) for credit.[2]

Regarding the enforcement of debt repayment, borrowers lack commitment giving rise to the possibility of strategic default. However, banks possess technology to track the identity and repayment history of borrowers enabling them to refuse credit to those who have defaulted on past debts. Similar to the limit on a credit card or line of credit, banks' record

[2]The uncertainty regarding loan approvals captures the idiosyncratic risk of lending to a specific type of borrower. The rate at which borrowers meet banks captures a more general or systematic friction inherent to the financial services sector that all borrowers are exposed to. This is consistent with recent survey data pertaining to the reported wait times and congestion involved with loan applications and credit market access (Federal Reserve Banks, 2019).

keeping technology allows for the imposition of debt limits on unsecured lending that are based on borrowers' default history. Following Kehoe and Levine (1993) and Alvarez and Jermann (2000), debt limits are determined endogenously according to incentive compatibility constraints. In addition, firms can pledge a share of the future output their investments generate to partially secure loans. Thus, consumer credit card lending is entirely unsecured while small business lending may be secured, unsecured, or both. The partial pledgeability of firms' output is motivated more by issues of moral hazard and serves a key role in capturing the secured and unsecured nature of small business credit (e.g., business installment loans and unsecured lines of credit).

The model's theory builds upon previous literature emphasizing two separate subchannels through which monetary policy operates through lending. The first is the balance sheet subchannel involving the effects of monetary policy on the net worth and assets of liquidity-constrained, bank-dependent borrowers. The second is the bank lending subchannel which emphasizes the role monetary policy plays in changing banks' reserves, liquidity, wholesale funding costs, and ultimately, their loan supply.[3] While my model captures the former via the relationship between monetary policy, liquidity holdings, and business collateral, it makes a key departure from the literature with respect to the latter. Given the development and structural changes that have occurred among interbank and wholesale markets throughout the last several decades, the relevance of the bank lending subchannel's theory for explaining bank loan supply has received criticism and scrutiny from Romer and Romer (1990) and others.[4] In this study, I do not explicitly model reserves or banks' liquidity and capital cost structure for capturing their loan supply. Instead, I focus on the effects from an interaction

[3]Some confusion may arise here when referencing the balance sheet and bank lending subchannel theories from the literature since this study focuses specifically on two types of bank loans. Therefore, I refer to these theories as "subchannels" operating within the more broadly defined lending channel of monetary policy.

[4]Criticism of the bank lending subchannel has typically focused on the assumption that banks cannot easily recover shortfalls in their loanable funds. See Romer and Romer (1990), Gertler and Gilchrist (1993a), and Kashyap et al. (1993) for discussions on this issue. However, these criticisms are irrelevant for the balance sheet subchannel.

between monetary policy, unsecured lending, and debt limits.[5]

Steady-state equilibrium is characterized by the level of nominal interest rates and the types of loans banks are willing to extend. While secured business loans are available at any positive interest rate, unsecured lending only emerges when interest rates are sufficiently high to ensure borrowers value credit enough to repay their debts. The direction in which lending responds to monetary policy, and whether a lending channel would be considered active, depends on whether borrowers are constrained by their liquidity, debt limits, or pledgeable output. If unconstrained by all three, a monetary tightening raises liquidity costs and lending expands as external finance becomes more attractive. Once borrowers are constrained (a precondition for an active lending channel), the lending response depends on the interaction between the balance sheet and debt limit subchannels. When the latter is turned off, lending contracts with tighter monetary policy as borrowers' wealth and collateral fall with their liquidity holdings. However, when both subchannels become active, lending expands as tighter policy raises debt limits allowing borrowers to hold less liquidity while increasing their overall financing capacity.

The transmission of monetary policy through the debt limit subchannel provides a new theory to justify an expansionary loan response to an increase in interest rates not previously addressed in the literature. Otherwise, a monetary tightening operates solely through balance sheets and lending contracts as predicted by conventional theory. The model also provides insight on the relationship between the nominal interest rate, pledgeability, and the strength of the transmission channel. When lending is unconstrained, higher interest rates and pledgeability are associated with low liquidity demand and large shares of external finance. As a consequence, liquidity demand is relatively inelastic and the monetary transmission mechanism is weak. When constrained, pledgeability still weakens transmis-

[5] Using credit bureau data from Equifax, Fulford and Schuh (2018) find credit card limit volatility to be several times higher than that of income. Furthermore, average credit card limits increase by more than 700% from ages 20–40. After age 40, limits continue to increase, but at a slightly slower rate.

sion through large principals and low liquidity, but the nominal interest rate strengthens transmission. This is a direct implication of an active debt limit subchannel as interest rates increase both the debt limit and its elasticity. Thus, a high interest rate environment with low liquidity demand can be characterized by a strong or weak transmission channel depending on whether lending is constrained or unconstrained.

The remainder of this paper proceeds as follows. Section 3.2 reviews the literature. Section 3.3 details the econometric methodology and empirical results. Section 3.4 introduces the general equilibrium model and derives its equilibrium, while Section 3.5 provides the paper's theoretical analysis. Section 3.6 concludes.

3.2 Related Literature

The first branch of literature this study relates to is that on the broad credit channel of monetary policy. Bernanke and Gertler (1995) were the first to separate the credit channel into its bank lending and balance sheet subchannels.[6] The basic theory underpinning the bank lending channel follows that of Bernanke and Blinder (1988) suggesting that tighter monetary policy drains reserves and deposits from the banking system, reducing the supply of loans to bank-dependent borrowers. However, Romer and Romer (1990) among others criticize this theory by arguing that banks can frictionlessly recover any loss in deposits through alternative fundings sources (e.g., wholesale and federal funds) effectively shielding their loan supply from monetary policy. The balance sheet subchannel's theory is typically associated with Bernanke and Blinder (1988) and described in Bernanke et al. (1996) as a reduction in credit availability following a monetary tightening due to the increased agency costs from a deterioration of borrowers' net worth and financial positions.

[6] See Bernanke and Gertler (1995) for an in-depth discussion of the balance sheet and bank lending subchannels.

Early empirical studies on the credit channel have typically used measures of aggregate bank loans without attempting to differentiate the bank lending and balance sheet subchannels. A list of these includes King (1986), Bernanke and Blinder (1992), Kashyap et al. (1993), Gertler and Gilchrist (1994), Kashyap et al. (1994), and Morgan (1998). However, due to a non-robust and statistically insignificant response from aggregate bank loans to a monetary policy tightening, many struggle to find convincing evidence to reconcile the lending channel's basic theory (Morris and Sellon, 1995).[7] As summarized by Gertler and Gilchrist (1993a), "Conventional wisdom holds that tightening of monetary policy should reduce bank lending. It is surprisingly difficult, however, to find convincing time series evidence to support this basic prediction of macroeconomic theory."

To address this ambiguity, studies have disaggregated loan variables into their subcomponents (e.g., consumer and business loans) to reveal counterintuitive responses. One of the more noteworthy and puzzling findings is the decrease in consumer lending accompanied by an increase in business lending following tighter policy discussed in Gertler and Gilchrist (1993b), Gertler and Gilchrist (1994), Bernanke and Gertler (1995), Kashyap and Stein (1995), Ludvigson (1998), and more recently, den Haan et al. (2007). However, the most relevant branch of this literature pertains to those addressing the relationship between monetary policy, credit card, and small business lending. Upon further disaggregation, studies that find a lending contraction for small firms, but expansion for large firms include Gertler and Gilchrist (1994), Kashyap et al. (1994), Oliner and Rudebusch (1996), and Kandrac (2012). Looking at disaggregated consumer loans, studies document evidence of an active lending channel among various sectors such as auto loans (Ludvigson, 1988) and credit cards (Brady, 2011). However, the key distinction between this literature and my analysis is the separation of credit card and small business loans within the same model, and introducing time-variation and stochastic volatility among its parameters. Within the context of mone-

[7] See Black and Rosen (2011, 2016) and Black et al. (2012) for overviews of the empirical literature regarding these studies.

tary policy shocks, this strand of literature includes, but is not limited to, Primiceri (2005), Benati and Mumtaz (2007), Canova and Gambetti (2009), Nakajima (2011a,b), Bijsterbosch and Falagiarda (2015), and Gambetti and Musso (2017). Also, Koop et al. (2009) and Cogley and Sargent (2005) argue against constant-volatility models due to the high likelihood the monetary transmission mechanism has undergone periods of structural change.

Regarding this paper's theoretical counterpart, a relevant group of studies are those pertaining to monetary search models. The benchmark models in this literature along with their many varieties and extensions have been recently surveyed by Lagos et al. (2017). More specifically, this paper relates to search models featuring money, credit, and bank lending. It directly builds upon the benchmark framework introduced in Lagos and Wright (2005) and extended by Rocheteau and Wright (2005). While these earlier studies focus on settings in which consumers finance their consumption expenditures via internal finance, this study focuses on borrowers' internal and external financing decisions.

This paper closely relates to the framework and setting established in Rocheteau et al. (2018) to analyze the relationship between corporate finance and monetary policy. Their model features an OTC credit market where entrepreneurs can supplement their cash holdings with a combination of both trade and bank credit to finance random investment opportunities. Their paper primarily focuses on the relationship between corporate financing decisions, investment, and monetary policy with respect to the pass-through of nominal policy rates to lending rates. In this study, I focus on how unsecured and secured loan volumes respond to changes in nominal interest rates. A key feature consistent across these models relates to the dual role of holding fiat money for borrowers. While it serves as an insurance tool to hedge against credit market frictions, it also plays a strategic role in improving borrower bargaining position. Other related papers on corporate liquidity demand and financial investment opportunities include Bates et al. (2009) and Sánchez and Yurdagül (2013).

A key focus of this paper's theory involves the implications from borrowers' external and internal financing decisions which draws connections to search models featuring the trade-off between fiat money and credit more generally. Several early papers in this literature include Shi (1996), Kocherlakota and Wallace (1998), and Cavalcanti and Wallace (1999). More recent studies modeling the coexistence of money and credit include Sanches and Williamson (2010), Gomis-Porqueras and Sanches (2013), Liu et al. (2019), Gu et al. (2016), Berentsen et al. (2018), and Telyukova and Wright (2008) for consumer credit and an application to the credit card debt puzzle. Among the variety of credit frictions addressed in the literature, this study focuses on those related to debt repayment, and more specifically, limited commitment and pledgeability. To mitigate the challenges imposed by borrowers' inability to promise repayment, debt limits are endogenized in the spirit of Kehoe and Levine (1993) and Alvarez and Jermann (2000). That is, lenders do not extend credit beyond the amount that renders borrowers indifferent between repayment and default. Following this approach within the monetary search literature, Bethune et al. (2018) study its implications in a dynamic, pure-credit environment under alternative bargaining solutions and non-stationary equilibria that generate credit cycles. Bethune et al. (2015) apply it within a setting of consumer credit card debt accompanied with liquid, non-monetary assets to analyze its role for unemployment. In a more general theoretical setting, Lotz and Zhang (2016) allow lenders to invest in record-keeping technologies to allow for future punishment if borrowers renege. The model features strategic complementarities between endogenous debt limits and record-keeping decisions from which they draw upon to assess the consequences of changes in the nominal interest rate for welfare. Other search models raising implications for the role of limited commitment and pledgeability include, but are not limited to, Lagos (2010), Venkateswaran and Wright (2014), Gu et al. (2016), and Berentsen et al. (2018).

3.3 Empirical Methodology

3.3.1 Model Specification

A structural vector autoregression with time-varying parameters and stochastic volatility is specified to perform the econometric analysis. This introduces time variation among the VAR coefficients and residual covariance matrix. Before estimating the structural form of the baseline model, the reduced-form is estimated initially. Consider the reduced-form expressed in vector notation as

$$y_t = c_t + A_{1,t}y_{t-1} + A_{2,t}y_{t-2} + \cdots + A_{p,t}y_{t-p} + u_t, \qquad t = 1, \cdots, T \tag{3.1}$$

where y_t is an (n x 1) vector of endogenous variables, c_t is an (n x 1) vector of time-varying intercepts, $A_{1,t}, A_{2,t}, \cdots, A_{p,t}$ are (n x n) matrices of time-varying coefficients, and u_t is an (n x 1) vector of residuals distributed according to

$$u_t \sim N(0, \Sigma_t). \tag{3.2}$$

The model's stochastic volatility is accounted for by the time subscript and period-specific residual covariance matrix Σ_t. More compactly, for a given period the model can be written as

$$y_t = \overline{X}\beta_t + u_t \tag{3.3}$$

where

$$\overline{X}_t = \underbrace{I_n \otimes X_t}_{nxq} \quad , \qquad X_t = \underbrace{(y'_{t-1} \;\; y'_{t-2} \;\; \cdots \;\; y'_{t-p})}_{1xk} \tag{3.4}$$

and

$$\beta_t = \underbrace{vec(B_t)}_{qx1} \quad , \quad B_t = \underbrace{(A'_{1,t} \;\; A'_{2,t} \;\; \cdots \;\; A'_{p,t})'}_{kxn}. \tag{3.5}$$

With regards to the evolution of the coefficients, they are assumed to follow an autoregressive process such that

$$\beta_t = \beta_{t-1} + \nu_t \quad , \quad \nu_t = N(0, \Omega) \tag{3.6}$$

where Ω is the variance-covariance matrix for the shocks on the autoregressive process and is a random variable determined by the VAR endogenously. This process can also be interpreted as a random walk.

Turning to the time dependency of Σ_t, it can be decomposed as $\Sigma_t = F\Lambda_t F'$, where F is a lower triangular matrix with ones on its diagonal and Λ_t is a time-varying diagonal matrix such that $diag(\Lambda_t) = \big(\bar{s}_1 exp(\lambda_{1,t}), \cdots, \bar{s}_n exp(\lambda_{n,t})\big)$ where $\bar{s}_1, \bar{s}_2, \cdots, \bar{s}_n$ are known scalers and $\lambda_{1,t}, \lambda_{2,t}, \cdots, \lambda_{n,t}$ govern the dynamic processes generating the heteroskedastic residuals. These are assumed to be autoregressive taking the form:

$$\lambda_{i,t} = \gamma\lambda_{i,t-1} + \upsilon_{i,t} \quad , \quad \upsilon_{i,t} \sim N(0, \phi_i). \tag{3.7}$$

Furthermore, the F and Λ_t matrices can be expressed as follows:

$$F = \underbrace{\begin{pmatrix} 1 & 0 & \cdots & \cdots & 0 \\ f_{21} & 1 & \ddots & & \vdots \\ f_{31} & f_{32} & 1 & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 \\ f_{n1} & f_{n2} & \cdots & f_{n(n-1)} & 1 \end{pmatrix}}_{n \, x \, n} \tag{3.8}$$

$$\Lambda_t = \underbrace{\begin{pmatrix} \overline{s}_1 exp(\lambda_{1,t}) & 0 & \cdots & \cdots & 0 \\ 0 & \overline{s}_2 exp(\lambda_{2,t}) & \ddots & & \vdots \\ \vdots & \ddots & \overline{s}_3 exp(\lambda_{3,t}) & \ddots & \vdots \\ \vdots & & \ddots & \ddots & 0 \\ 0 & \cdots & \cdots & 0 & \overline{s}_n exp(\lambda_{n,t}) \end{pmatrix}}_{n \, x \, n} \tag{3.9}$$

This concludes the outline for estimating the model's reduced-form. To summarize, the principal parameters I seek to estimate each period are the VAR coefficients β_t for $t = 1, \cdots, T$; the shock covariance matrix Ω; the elements f_i^{-1} relating to matrix F for $i = 2, \cdots, n$; the dynamic coefficients $\lambda_{i,t}$ for $i = 1, \cdots, n$ and $t = 1, \cdots, T$; and the parameters ϕ_i capturing heteroskedasticity for $i = 1, \cdots, n$.

3.3.2 Data and Estimation

Among the macro-financial VAR literature pertaining to credit markets, standard specifications include variables capturing real activity, the price level, the monetary policy stance, and a measure for credit activity. To remain consistent, I include the following five endogenous variables in the baseline specification: real GDP, the GDP price deflator, the federal funds rate, real outstanding credit card loans, and real outstanding small business loans.

While the credit measure typically takes the form of aggregate or disaggregate outstanding balances, others have incorporated alternative indicators such as indexes or survey data. Those interested in the credit channel of monetary policy more broadly may utilize loan data from multiple lending sources. Considering this analysis concerns the lending channels among two bank-dependent borrowers, I only include lending data from banks.[8]

The credit card data series falls under outstanding consumer revolving credit which is primarily comprised of bank credit card loans, but also includes prearranged overdraft plans. As for business loans, the data is classified as outstanding bank loans to nonfinancial, noncorporate businesses. Considering bank lines of credit and installment loans are both critical financing sources for small businesses, this series encompasses revolving and non-revolving loans that may be secured or unsecured. The businesses include all sole proprietorships, partnerships, and government enterprises operating in the retail trade, accommodation, and food services sectors.[9] While not all of these firms may be considered "small" in terms of their revenue, capital, or employment; they do lack access to the expansive non-bank credit sources available to corporations via public debt and equity markets. This is the population of bank-dependent businesses I seek to target. A description for each data series accompanied by their sources can be found in Table C.1 of Appendix C.

Each variable enters the VAR in the first-difference of its natural logarithm except for the policy rate which enters in the first-difference of its level. This allows the log-transformed variables to be interpreted in terms of their quarter-on-quarter growth rates. Furthermore, each non-interest rate series is seasonally adjusted where relevant.[10]

[8]One minor inconsistency worth noting is that the lenders associated with this data are classified as depository institutions by the Federal Reserve. This includes all U.S.-chartered commercial banks and savings institutions.

[9]Government agencies such as the U.S. Postal Service and local transit authorities comprise government enterprises.

[10]The non-adjusted credit card series displays substantial seasonality around December and includes a significant break in 2010. While not displaying obvious seasonality, the business loan series is still adjusted for consistency and any subtle or non-visible trends that may be present. All adjustments are performed using the U.S. Census Bureau's X-13ARIMA-SEATS program.

The model is estimated with two lags as suggested by the Bayesian Information Criterion (BIC). This decision is relatively standard as many TVP-VAR models are limited to two lags due to their high dimensionality and computational intensity (Casalis, 2020). With the intent of improving estimation accuracy and efficiency, the reduced-form is estimated using Bayesian methods which require the imposition of prior information. A normal prior distribution is specified for β_t, F, and Λ_t; while an inverse Gamma distribution is specified for Ω and ϕ_i. In addition, I set the autoregressive coefficient on the residual variance γ to 0.85.[11]

The estimation sample consists of quarterly U.S. data from 1970 Q1 through 2007 Q3. Although data is available from 1968 through today, I begin the sample in 1970 due to large outliers and explosive impulse responses among the credit series during the late 1960s which could bias the estimation results. I end the sample at the onset of the Great Recession to avoid the zero lower bound period and additional explosive steady-states throughout the 2010s. Finally, to estimate the posterior distribution, 5,000 iterations of the Gibbs sampler are performed with the first 2,000 iterations dropped to achieve convergence. Every 5th iteration is then kept to eliminate potential autocorrelation resulting in a final estimation based on 600 iterations.

3.3.3 Identification Strategy

While reduced-form VARs are sufficient for performing forecast analysis, they cannot be used to compute impulse response functions for isolating individual shocks due to correlation across the error terms in u_t. Hence, structural identification must be implemented to identify the independent innovations which entails assigning a set of credible restrictions on their relationships to identify the parameters of matrix F. Given the symmetry of the variance-

[11]The estimation is performed using the European Central Bank's BEAR toolbox in MATLAB. All hyper-parameters are parameterized in accordance with Dieppe et al. (2016). See Dieppe et al. (2016) for further technical details.

covariance matrix, $n(n-1)/2$ restrictions are required to derive β_t and identify the SVAR.

One of the most common ways for imposing the necessary restrictions to identify β_t is the Cholesky decomposition which leaves the parameter matrix with a lower triangular structure. This implies a recursive ordering of the variables with zero restrictions to separate the fast and slow-moving variables. However, strategies vary widely with respect to recursive assumptions especially when working with quarterly time series. For example, restrictions that eliminate all contemporaneous effects appear reasonable when using monthly data. At the quarterly frequency, this assumption becomes less clear and many studies follow the recommendation outlined in Canova and Paustian (2011) and impose all restrictions only on impact. This assumes that financial markets and other interest rates respond quickly enough to influence the consumption and investment behavior of households and firms within the quarter. An additional issue surrounding recursive assumptions involves the use of time series containing historical data vintages that are subject to revision. That is, when the Fed responds contemporaneously to changes in current output and price data, these observations or estimates can differ from the final revised values contained in the researcher's data. Rudebusch (1998) argues that this subjects a VAR's estimated coefficients to potential biases and inconsistencies.

Given these criticisms surrounding the recursive literature, theory typically provides a stronger consensus on the directions and comovement between variables than the amount of time it takes for one variable to respond to another, especially for macro-finance SVARs. Following Uhlig (2005), I take a more theoretically consistent approach to identification by imposing sign restrictions on the impulse responses. While the sign restriction approach will not identify β_t exactly, it will restrict β_t to a credible range from which informative estimates can be derived from. To impose the sign restrictions, I employ the methodology developed in Arias et al. (2018) which allows for combinations of sign, zero, and magnitude restrictions to be assigned on impulse responses in any given period. Remaining consistent with the recommendation outlined in Canova and Paustian (2011), all restrictions are imposed only on impact. The reduced-form estimation and sign identification algorithms then follow the same steps detailed in Section 2.3 of Chapter 2.[12]

Since this analysis is concerned with the effects from monetary policy, the final step is to impose a set of credible sign restrictions to accurately identify the monetary policy shock. A consistent theme throughout both the sign and recursive VAR literatures is their adherence to Bernanke and Blinder (1992) in assuming innovations in the federal funds rate as the relevant indicator for monetary policy shocks. In addition, the sign-restriction literature widely agrees that in response to a monetary policy shock, output and prices should move in the opposite direction as the policy rate. However, by requiring GDP and the price level to respond on impact, potential monetary policy shocks that move the federal funds rate without GDP or prices would not be identified. Taking a more agnostic and comprehensive approach, I identify a contractionary monetary shock as an increase in the federal funds rate accompanied by a non-positive response from GDP and inflation. These restrictions are imposed on impact lasting one quarter.

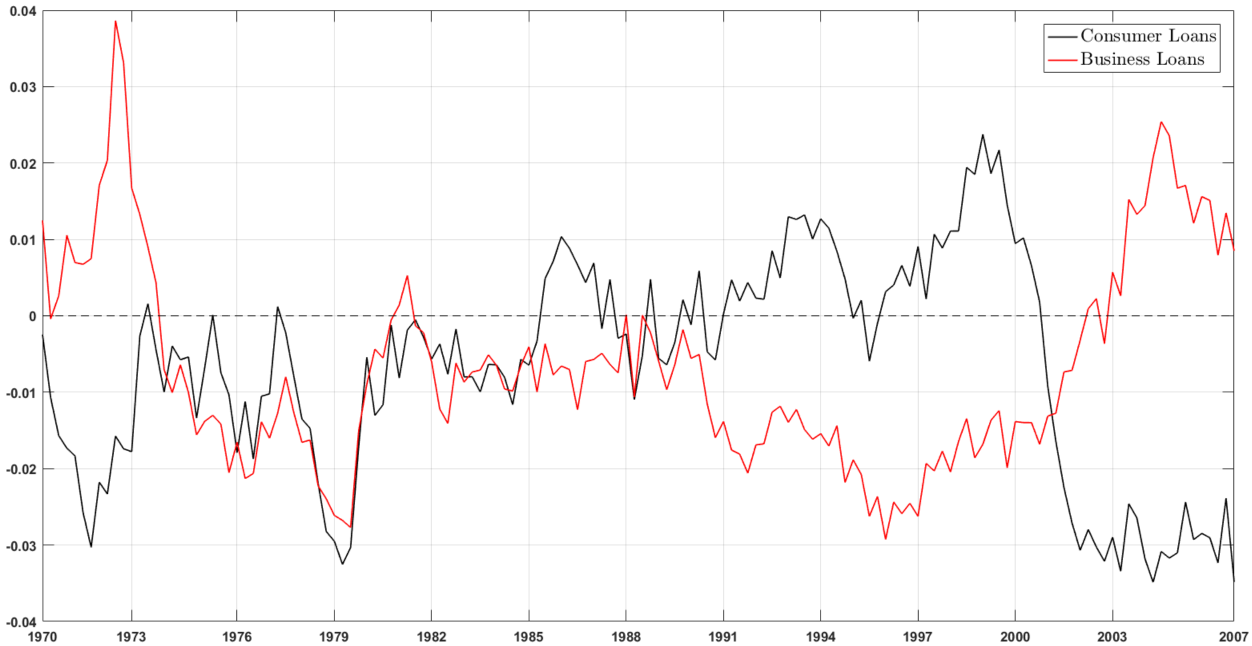
3.3.4 Empirical Results

Following a one standard deviation contractionary monetary policy shock, impulse responses are computed for every period of the sample out to a horizon of 33 quarters, or 8 years.[13] Although useful for analyzing the lending response to policy more generally, without normalizing the policy rate's response each period, it reveals little regarding the elasticity or strength of the lending response relative to the federal funds rate. For example, recursive VAR studies typically compute impulse responses by normalizing the policy rate's response

[12] Additional details regarding the estimation procedure and strategy using sign restrictions can be found in Dieppe et al. (2016) and Arias et al. (2018).

[13] This is done to ensure each variable's response is tracked long enough so it can reach its new steady-state. However, this horizon could be shortened as each variable returns to steady-state after 3-4 years.

Figure 3.1: Consumer and Small Business Lending Impulse Responses



Notes: Long-run interest rate semi-elasticities of consumer credit card (black) and small business (red) bank lending. Estimates based on median impulse responses.

to a one standard deviation or 100 basis point increase for clear comparison across studies. In light of this, I compute quarterly semi-elasticities by dividing the accumulated responses from each loan series by the accumulated responses from the federal funds rate in every period. This allows me to not only analyze the direction in which each lending channel operates, but to also assess their sensitivity to the federal funds rate.

Figure 3.1 plots the interest rate semi-elasticities for credit card and small business loans.[14] To clarify interpretation, consider 2005 Q3. The data implies a one percentage point increase in the federal funds rate decreases the volume of credit card lending by about 3% and increases business lending volume by about 1.75%. Figure 3.1 presents two key observations. The first is the overall time-varying response present among both loan series. Credit card lending either contracts or remains constant until the mid-1980s when it begins to expand

[14]Estimates are based on the median long-run impulse response. The full set of impulse response functions and corresponding 68% credible intervals are available upon request.

before contracting again during the early 2000s. This provides evidence in favor of an active lending channel through credit cards prior to 1985 and during the early-mid 2000s. Until around 2001, this finding confirms that of Brady (2011) who finds the consumer lending channel through revolving and non-revolving loans to no longer be active after the mid-1980s. As for small business lending, its response remains primarily negative throughout the sample other than the early 1970s, 1980s, and mid-2000s. Compared to credit cards, this suggests that both channels were most likely to be active between the mid-1970s and early 1980s, while the business channel remained active until the early 2000s around the time the credit card channel became active again.

The second important insight from Figure 3.1 concerns the time-dependency among transmission strength. Throughout the 1980s both channels' elasticities remain relatively low fluctuating between -1.5% and 1%. This differs dramatically from 1972 when small business loans increase by nearly 4% and the mid-2000s when consumer loans decrease by about 3% throughout the real estate bubble period. The years between the late 1990s and mid-2000s are also worth noting when the credit card channel switches from expansionary to contractionary while maintaining relatively high transmission strength. This pattern is essentially identical for small business lending but in the opposite direction.

To summarize, the results from this analysis provide evidence in favor of an active and time-dependent consumer and business lending channel through credit card and small business loans. However, little can be said from the estimation regarding the why or how this time-dependent nature exists in the data. To gain further insight into what mechanisms and fundamentals may be responsible for driving these results, I now transition to the paper's theoretical counterpart.

3.4 The Model

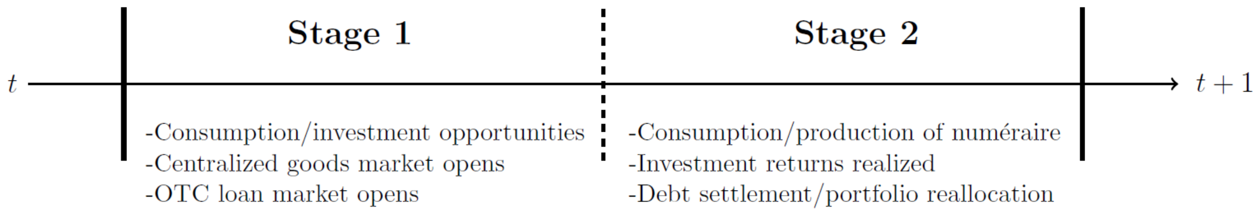
3.4.1 Environment

Time is discrete and continues forever. Each period $t = 1, 2, ...$ is divided into two stages as depicted in Figure 3.2. In the first stage, there is a market for bank credit and a centralized market for a good that can either be consumed or invested as capital. In the second stage, there is a frictionless market where agents trade final goods, assets, and settle debts. There are three types of agents, $j \in \{h, f, b\}$: households, firms, and banks, each of unit measure. In Stage 2, all agents have linear preferences $U(c, h) = c - h$ over a numéraire good, where c denotes consumption and h denotes hours of work. In Stage 1, households enjoy utility $u(q)$ from consuming, where $u(0) = 0, u'(0) = \infty$, $u'(\infty) = 0$, and $u'(q) > 0 > u''(q) \;\; \forall\, q > 0$. The discount rate between periods is $\beta = 1/(1 + \rho)$ with $\rho > 0$.

At the beginning of Stage 1, households and firms face three forms of idiosyncratic uncertainty. The first is with respect to preferences and investment opportunities while the other two relate to bank credit access. With probability $\sigma \in [0, 1]$, households wish to consume in Stage 1. With complementary probability $1 - \sigma$, households produce the Stage 1 good according to a linear production technology such that $q, k = h$. Firms receive investment opportunities with probability λ allowing them to transform k units of capital into $f(k)$ units of c, where $f(0) = 0$, $f'(0) = \infty$, $f'(\infty) = 0$, and $f'(k) > 0 > f''(k) \;\; \forall\, k > 0$.

The credit market in Stage 1 is described as an over-the-counter market where banks and borrowers form bilateral relationships. The market is unsegmented providing households and firms with equal probability α of meeting a bank. All matches are short-lived and do not continue beyond the period during which they are formed. Contingent on meeting a bank, borrowers are approved for a loan with probabilities μ_h and μ_f, respectively. Assuming independence, the measure of households that wish to consume, have access to bank credit,

Figure 3.2: Timing of Events



and are approved for a loan is $\sigma\alpha\mu_h$. The measure of firms with an investment opportunity, access to bank credit, and loan approval is $\lambda\alpha\mu_f$.

With respect to the enforcement of debt, households and firms lack commitment and may choose to renege on their debt in Stage 2. In the tradition of Kehoe and Levine (1993) and Alvarez and Jermann (2000), banks have access to a private record keeping technology allowing them to punish defaulters with permanent exclusion from future unsecured credit access. With this threat, limits on unsecured debt are determined endogenously and by the degree to which borrowers value future credit access. The record keeping technology is unique to financial intermediaries and is not available to producers rendering credit infeasible between producers and borrowers. In addition, firms can pledge a fraction χ of the Stage 2 output from their Stage 1 investment projects for securing loans.[15] The partial pledgeability of firms' output allows for the existence of both secured and unsecured lending between banks and firms. Since households consume in Stage 1 and don't receive investment opportunities, they have no future output to pledge and therefore all household lending is unsecured.

The limitations to commitment and pledgeability promote demand for outside liquidity by households and firms. Outside liquidity in the form of fiat money is storable across periods and evolves according to $M_{t+1} = (1+\pi)M_t$, where M_t is the current period's money supply and π is the inflation rate. The money supply grows (or shrinks) through lump sum transfers, $T = \pi M_t$, provided to households and firms in Stage 2. One unit of money can buy $p_{m,t}$

[15]This fraction captures inherent properties of capital and output such as portability, resale value, industry characteristics, and legal systems.

units of the numéraire, where $Z_t = p_{m,t}M_t$ denotes aggregate real balances. Since I focus on steady-state equilibria where real balances are constant, $p_{m,t} = (1+\pi)p_{m,t+1}$ must hold in any monetary equilibrium. This implies that the real gross rate of return on money is $1 + r_m = p_{m,t+1}/p_{m,t} = 1/(1+\pi)$. Regarding the credit market and non-monetary assets, banks issue intra-period liabilities, or IOUs, that they credibly commit to redeem in Stage 2. Bank IOUs are normalized to be worth one unit of numéraire and resemble non-interest bearing notes or demand deposits.

3.4.2 Preliminaries

To begin, consider a household entering Stage 2 with financial wealth ω denominated in units of numéraire. An agent's financial wealth is comprised of their real balance holdings z and lump sum transfers T, net of debt obligations. The household's lifetime expected utility is

$$W^h(\omega) = \max_{c,h,z'_h} \{c - h + \beta \mathbb{E} V^h(z'_h)\} \qquad s.t. \quad c + \frac{z'_h}{1+r_m} = h + \omega \tag{3.10}$$

where $V^h(z'_h)$ is the continuation value with next period's choice of real balances z'_h. Using the budget constraint to eliminate $c - h$, the household's lifetime utility becomes

$$W^h(\omega) = \omega + \max_{z'_h} \left\{ -\frac{z'_h}{1+r_m} + \beta \mathbb{E} V^h(z'_h) \right\}. \tag{3.11}$$

Since W^h is linear in wealth, the household's choice of z'_h is independent of their current real balances.

Carrying their portfolio into the next period, a household that receives a consumption shock has the following value function at the beginning of Stage 1:

$$V^h(z'_h) = \mathbb{E}\left[u(q) + W^h(z'_h - p_q q - \phi_h) \right], \tag{3.12}$$

where p_q is the price of the Stage 1 good and ϕ_h is the household's interest payment to the bank in numéraire. Expectations are with respect to whether or not the household meets a bank and is approved for a loan. If so, the household borrows and ϕ_h is positive; otherwise, $\phi_h = 0$ and q is financed internally using only their real balances.

Similarly, the value function for a household that becomes a producer in Stage 1 is

$$V^h(z_h') = \max_{q,k} \Big\{ -(q+k) + W^h\big(z_h' + p_q q + p_k k\big) \Big\}. \tag{3.13}$$

Given that q and k are the same good, but used differently by households and firms, it follows that total Stage 1 production is $q+k$ and $p_q = p_k = p$. Since producers do not demand credit, the expectations operator can be eliminated. Using the linearity of W^h, the solution to (3.13) is $p = 1$ where marginal revenue equals marginal cost. Now the household's Stage 1 value function before the realization of the consumption shock can be written as

$$V^h(z_h) = \sigma \Big\{ \alpha\mu_h \big[u(q) + W^h(\omega - q - \phi_h)\big] + (1-\alpha\mu_h)\big[u(q^m) + W^h(\omega - q^m)\big] \Big\} \tag{3.14}$$

$$+ (1-\sigma) W^h(\omega), \tag{3.15}$$

where q and q^m denote the quantities of externally and internally financed consumption for a banked and unbanked household, respectively. Combining (3.13) with the linearity of W^h, the household's value function reduces to

$$V^h(z_h) = \sigma \Big\{ \alpha\mu_h \big[u(q) - q - \phi_h\big] + (1-\alpha\mu_h)\big[u(q^m) - q^m\big] \Big\} + \omega + \beta V^h(z_h). \tag{3.16}$$

Substituting $V^h(z_h)$ into (3.11), the household's portfolio choice can be rewritten as

$$\max_{z_h} \left\{ -iz_h + \sigma\alpha\mu_h\big[u(q) - q - \phi_h\big] + \sigma(1-\alpha\mu_h)\big[u(q^m) - q^m\big] \right\}, \tag{3.17}$$

where the nominal interest rate is defined as $i \equiv (1+\pi)(1+\rho)-1$. This can also be interpreted as the nominal interest rate on an illiquid bond that cannot be used as a medium of exchange in Stage 1.

The problem for a firm in Stage 2 is identical to that of households, except a firm enters holding k units of capital that are transformed into $f(k)$ units of output:

$$W^f(k,\omega) = f(k) + \omega + \max_{z_f'} \left\{ -\frac{z_f'}{1+r_m} + \beta V^f(z_f') \right\}. \tag{3.18}$$

Following the same line of reasoning as for (3.12), upon receiving an investment opportunity a firm's Stage 1 value function is

$$V^f(z_f') = \mathbb{E}W^f(z_f' - k - \phi_f), \tag{3.19}$$

where $k = pk$ from the producer's problem. The remaining results and derivations follow the same methodology outlined for households in (3.12)-(3.17) and are identical except with $f(k)$ replacing $u(q)$ and all h subscripts replaced with f.

As for banks, their Stage 2 value function is analogous to (3.12) with b in place of h. It follows that their Stage 1 value function can be written as

$$V^b(z_b') = \mathbb{E}W^b(z_b' + \Phi), \tag{3.20}$$

where Φ is the bank's intra-period profits collected from all interest payments.

3.4.3 Stage 1

Having characterized preliminary equilibrium results related to agents' value function optimization and Stage 1 good pricing, I now turn to equilibrium bargaining. Both borrowers enter Stage 1 upon realization of either a consumption shock for households, or investment opportunity for firms. In contrast to households who seek credit for financing consumption, firms seek credit to purchase and invest the Stage 1 good as capital yielding output in Stage 2. This provides firms with a form of collateral for securing loans not available to households. Considering this is the sole distinction between the two borrowers, and to avoid redundancy, I solve the remainder of the model within the context of firms while providing details and interpretations for households where necessary.

Consider a firm entering Stage 1 with an investment opportunity. Without access to a bank, the firm relies on internal finance to purchase directly from producers in the frictionless capital market. Their surplus is

$$\Delta(z_f) = f(k^m) - k^m \qquad \text{where} \qquad k^m = \min\{z_f, k^*\}, \tag{3.21}$$

where z_f is real balances, k^m is the quantity of internally financed capital, and k^* is the first best quantity of capital solving $f'(k) = 1$. That is, the efficient level of investment obtains when the firm's marginal revenue is equated to marginal cost. For all $z_f < k^*$, $\Delta(z_f)$ is increasing and concave. Now consider a firm that meets a bank and is approved for a loan. External finance is only incentive compatible for $z_f < k^*$, otherwise, k^* is purchased directly from producers leaving no possible gains from using credit. A loan contract between a firm and bank is a pair (ℓ_f, ϕ_f) that specifies a principal, $\ell_f = k - d$, comprised of bank-financed capital k and down payment d; as well as an interest payment ϕ_f. Individual match surpluses are

$$S^f \equiv W^f(k, \omega - k - \phi_f) - W^f(k^m, \omega) = f(k) - k - \phi_f - \Delta(z_f); \tag{3.22}$$

$$S^b \equiv W^b(w + \phi_f) - W^b(w) = \phi_f, \tag{3.23}$$

implying a total match surplus of $S^f + S^b = f(k) - k - \Delta(z_f)$. Since borrowers have the option to transact with producers at any time, $\Delta(z_f)$ serves as the firm's disagreement point, or outside option, when bargaining with a bank.

The loan terms are determined by the Kalai (1977) proportional bargaining solution which guarantees the bilateral trade to be Pareto efficient.[16] The firm's equilibrium loan contract as determined by the proportional bargaining solution is given by

$$\max_{k,d,\phi_f} \phi_f \quad s.t. \ f(k) - k - \phi_f - \Delta(z_f) \geq \frac{1-\theta_f}{\theta_f}\phi_f; \tag{3.24}$$

$$k - d + \phi_f \leq \chi f(k) + \bar{b}_f; \tag{3.25}$$

$$d \leq z_f, \tag{3.26}$$

where $\theta_f \in [0, 1]$ is the bank's bargaining power with firms. According to (3.24)-(3.26), the bank chooses the loan size and interest payment to maximize S^b subject to three constraints. The first is the surplus sharing rule which guarantees firms receive at least a share $1 - \theta_f$ of the total match surplus. While the sharing rule is represented as an inequality to guarantee existence, the strict proportional solution requires a strict inequality for (3.24). The second constraint is the firm's debt constraint restricting total debt $k - d + \phi_f$ from exceeding their pledged output $\chi f(k)$ and debt limit $\bar{b}_f$. For now, debt limits are taken as exogenous but will be endogenized according to incentive compatibility constraints in Section 3.4.4.

[16]The proportional bargaining solution has several desirable features that the generalized Nash bargaining solution does not always exhibit. One feature is that it ensures agents' value functions are concave. Another is that its solution is monotonic which is not consistent with generalized Nash as one agent can suffer a loss in surplus while total surplus is raised. For further details, see Aruoba et al. (2007) and Bethune et al. (2018) for a discussion of the proportional solution's normative properties.

The final constraint, $d \leq z_f$, is the liquidity constraint imposing that the down payment can be no larger than the firm's liquidity position. If (3.25) is slack, then k and ϕ_f are uniquely determined, but d is not. In this case, the highest solution is chosen implying $d = \min\{z_f, k^*\}$. The household's bargaining problem follows the same structure as (3.24)-(3.26), but with $\chi = 0$.

Before presenting the bargaining solution, I define two critical values. First, let $\chi^* \equiv \big[(1-\theta_f)k^* + \theta_f f(k^*)\big]/f(k^*)$ denote the minimum share of pledgeable output necessary to finance k^* absent unsecured debt and internal finance. Second, for $\chi < \chi^*$, define $b_f^* \equiv (1-\theta_f)k^* + (\theta_f - \chi)f(k^*)$ as the minimum quantity of unsecured debt in addition to $\chi f(k^*)$ required to finance k^* absent internal finance. The solution to (3.24) is summarized by the following lemma.

Lemma 3.1. *If $\chi < \chi^*$ and $\bar{b}_f < b_f^*$, there exists a z_f^* such that $z_f^* < k^*$ solves*

$$(1-\theta_f)z_f^* + \theta_f f(z_f^*) = b_f^* - \bar{b}_f. \tag{3.27}$$

Assume $\chi < \chi^*$ and $\bar{b}_f < b_f^*$. If $z_f \geq z_f^*$, the solution to (3.24) is $k = k^*$ and

$$\phi_f = \theta_f\big[f(k^*) - k^* - \Delta(z_f)\big]. \tag{3.28}$$

If $z_f < z_f^*$, the debt constraint binds and (k, ϕ_f) solves

$$(1-\theta_f)k + (\theta_f - \chi)f(k) = z_f + \bar{b}_f + \theta_f\Delta(z_f); \tag{3.29}$$

$$k + \phi_f = \chi f(k) + z_f + \bar{b}_f. \tag{3.30}$$

Proof in Appendix C.

Business lending is unconstrained when the firm's total borrowing capacity, $z_f + \bar{b}_f + \chi f(k)$, is sufficiently high to finance k^* with interest ϕ_f given by (3.28). This occurs when either

$\chi \geq \chi^*$, $\bar{b}_f \geq b_f^*$, or $z_f \in [z_f^*, k^*)$, where z_f^* is the minimum down payment necessary when the quantity of collateral and unsecured debt are insufficient. From (3.28), the interest fee is independent of χ while increasing with θ_f and decreasing with z_f. With more cash in hand, firms finance the same level of investment while reducing their principal and interest. However, when the firm's financing capacity does bind, $k < k^*$ and ϕ_f are determined by (3.29) and (3.30), respectively. For a given $\bar{b}_f$ and z_f, one can check that $\partial k/\partial \chi > 0$ and $\partial k/\partial \theta_f < 0$; while $\partial \phi_f/\partial \chi > 0$ and $\partial \phi_f/\partial \theta_f > 0$. With $\chi f(k) - k$ increasing in k, as discussed below, an increase in pledgeability allows banks to charge higher interest fees on larger loans. However, an increase in banks' bargaining power allows them to also charge higher interest fees, but on smaller loans.

Moving to the effects of internal finance on investment, by holding additional liquidity, firms can raise investment more than one-for-one. To see this, (3.29) can be differentiated with respect to z_f to obtain

$$\frac{dk}{dz_f} = \frac{(1-\theta_f) + \theta_f f'(z_f)}{(1-\theta_f) + (\theta_f - \chi) f'(k)} > 1. \tag{3.31}$$

This result relies on a financial multiplier effect created by the interaction between the firm's real balances, outside option, and pledgeability. A stronger liquidity position allows firms to make larger down payments while having the option to purchase more capital directly from producers which both promote higher levels of external finance. However, with $\chi > 0$, firm's pledgeable output increases simultaneously with k and is therefore also increasing with z_f. Thus, (3.31) is increasing with χ as pledgeability amplifies the firm's financial multiplier. Although households have no collateral to pledge, their financial multiplier operating solely through their down payment and outside option $\Delta(z_h)$ is still strong enough to increase consumption more than one-for-one as well.

Pledgeability in this setting also allows for the existence of a critical value that specifies a minimum level of investment. Suppose firms carry no cash into the credit market and are constrained such that $\Delta(z_f) = 0$. The maximum possible surplus an agent can receive is $\chi f(\underline{k}) - \underline{k} \leq f(\underline{k}) - \underline{k}$ where $\underline{k}$ is the solution to $\chi f'(\underline{k}) = 1$. This sets a lower bound on k as the surplus of both agents can be increased for all $k < \underline{k}$ maintaining compliance with Pareto efficiency. Therefore, $k \in [\underline{k}, k^*]$ is assumed and can be taken as a preliminary without any loss in generality.[17]

Lemma 3.2. *The firm's portfolio choice is a solution to*

$$\max_{z_f \geq 0} \Big\{ - i z_f + \lambda \alpha \mu_f (1 - \theta_f) \Delta(k) + \lambda \big[1 - \alpha \mu_f (1 - \theta_f) \big] \Delta(z_f) \Big\}, \tag{3.32}$$

where $\Delta(k) \equiv f(k) - k$ is the surplus from using external finance.

Proof in Appendix C.

According to (3.32), the firm's optimal choice of real balances maximizes their expected surplus from bringing cash into Stage 1 net of its opportunity cost i. If $z_f \geq k^*$, the firm can internally finance k^* without credit and $\Delta(k) = \Delta(z_f) = f(k^*) - k^*$ resulting in a pure-monetary equilibrium. However, this case can only emerge when $i = 0$. Otherwise, the marginal benefit of an additional unit of real balances is zero at k^* while the marginal cost is positive, so firms always hold $z_f < k^*$ when $i > 0$. Next, consider when $z_f \in [z_f^*, k^*)$. Firms can still finance k^*, but only by supplementing their liquidity with a bank loan. The FOC for an unconstrained firm's portfolio choice is

$$i = \lambda \big[1 - \alpha \mu_f (1 - \theta_f) \big] \Delta'(z_f), \tag{3.33}$$

where $\Delta'(z_f) = f'(z_f) - 1$. Notice that $z_f > 0$ holds for $\alpha \mu_f = 1$. Even when borrowers obtain bank loans with certainty, their improved bargaining position via $\Delta(z_j)$ lets them negotiate lower interest payments which incentivizes them to maintain positive liquidity.

[17] Restricting k to the range $[\underline{k}, k^*]$ also ensures the denominator in (3.31) never takes a negative value.

Now consider when (3.25) and (3.26) bind and business lending is constrained. With χ and $\bar{b}_f$ too low to finance the first best, i must be sufficiently high to ensure firms hold $z_f < z_f^*$. The firm's optimal portfolio choice then corresponds to

$$i = \lambda\alpha\mu_f(1-\theta_f)\Delta'(k)\big[1+\theta_f\Delta'(z_f)\big] + \lambda\big[1-\alpha\mu_f(1-\theta_f)\big]\Delta'(z_f), \tag{3.34}$$

where $\Delta'(k) = \big[f'(k)-1\big]/\big[(1-\theta_f)+(\theta_f-\chi)f'(k)\big]$. With $k < k^*$, investment and liquidity demand become functions of the debt limit. The following lemma summarizes how changes in the debt limit affect borrowers' liquidity and financing capacity.

Lemma 3.3. *For $j \in \{h, f\}$, assume $\chi < \chi^*$, $\bar{b}_j < b_j^*$, and $z_j < z_j^*$. Household and firms' real balances are decreasing, while their financing capacities are increasing with their debt limits.*

Proof in Appendix C.

In response to a higher debt limit, borrowers reduce their cash holdings less than one-for-one due to the the positive share of unbanked matches requiring internal finance only. This allows them to reduce their liquidity holding costs, without having to sacrifice their external financing capacity. This is the opposite result when $\bar{b}_j = 0$ and any reduction in z_j would further constrain lending, consumption, and investment. This result will be revisited in Section 3.5 when bank lending and liquidity demand are analyzed within the context of monetary policy.

3.4.4 Endogenous Debt Limits

I now turn to the determination of the unsecured debt limit, $\bar{b}_j$. To ensure repayment, banks must impose a credible punishment on defaulting borrowers. Provided that banks have access to a private record of repayment histories, they can punish defaulters with permanent exclusion from future credit.

While $\overline{b}_h$ and $\overline{b}_f$ are endogenized in the same manner, they differ with respect to how defaulters are punished. In the event of default, banks cannot recover the unsecured debt portion given by $\overline{b}_j$. In accordance with households, defaulting firms permanently lose access to future unsecured credit. However, with $\chi > 0$, there is no reason for banks to seize issuing secured loans when they can guarantee full recovery of the principal and interest. Hence, the firm's debt carried into the settlement market includes a recoverable portion $\chi f(k)$, and unrecoverable portion $\overline{b}_f$. This means firms who have previously defaulted can continue searching for banks in future periods as their output may still be pledged for securing loans. This is not the case for households as consumer lending is entirely unsecured forcing defaulters to solely rely on internally financed consumption.

Equilibrium debt limits are determined such that the borrower voluntarily repays their debt obligation. The firm's incentive compatibility constraint for voluntary repayment is

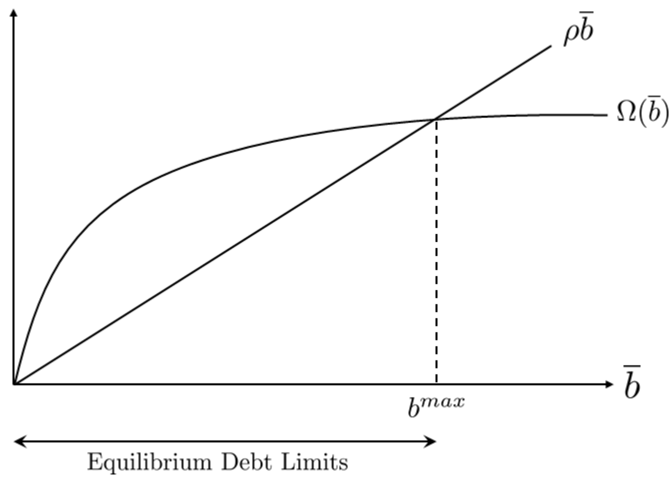
$$W^f(k, z_f - \overline{b}_f) \geq \hat{W}^f(k, z_f), \tag{3.35}$$

where $W^f(k, z_f - \overline{b}_f)$ is the value function of a firm who repays their debt and retains access to future unsecured credit, while $\hat{W}^f(k, z_f)$ is the value function of a firm with access to secured credit only. According to (3.35), firms repay their debt if the lifetime utility from repaying is higher than that from default. Using the linearity of W^f and $\hat{W}^f$ with respect to wealth and investment returns, the firm's borrowing constraint can be expressed as

$$b_f \leq \overline{b}_f \equiv W^f(0) - \hat{W}^f(0), \tag{3.36}$$

where b_f is total unsecured debt, $k - z_f + \phi_f - \chi f(k)$, and $\overline{b}_f$ is the endogenous debt limit. Absent wealth effects, the firm's debt limit is independent of their current capital, real balances, and is the solution to

Figure 3.3: Determination of Endogenous Debt Limits



$$
\begin{aligned}
\rho\bar{b}_f \leq & \max_{z_f \geq 0} \left\{ -iz_f + \lambda\alpha\mu_f(1-\theta_f)\Delta(k) + \lambda\big[1-\alpha\mu_f(1-\theta_f)\big]\Delta(z_f) \right\} \\
& - \max_{\hat{z}_f \geq 0} \left\{ -i\hat{z}_f + \lambda\alpha\mu_f(1-\theta_f)\Delta(\hat{k}) + \lambda\big[1-\alpha\mu_f(1-\theta_f)\big]\Delta(\hat{z}_f) \right\}; \\
& \equiv \Omega(\bar{b}_f),
\end{aligned} \tag{3.37}
$$

where $\hat{z}_f$ denotes the real balances of a defaulting firm. The equilibrium debt limit is any $\bar{b}_f \leq b_f^{max}$ satisfying (3.37) where b_f^{max} solves $\rho\bar{b}_f = \Omega(\bar{b}_f)$. The first term on the right side of (3.37) is identical to (3.32) representing the firm's optimization problem when $\bar{b}_f > 0$. The second term is the defaulting firm's optimization problem with $\hat{k}$ and $\Delta(\hat{k})$ as functions of $\hat{z}_f$ and χ. Since defaulters maintain secured credit access, it follows that $\Delta(\hat{k})$ is determined according to the same piecewise bargaining solution given by (3.28)-(3.30), except for $\bar{b}_f = 0$.

Figure 3.3 plots $\rho\bar{b}$ and $\Omega(\bar{b})$ as functions of a general limit debt $\bar{b}$. First, notice that $b \in \{0, b^{max}\}$ are the only values for $\bar{b}$ that satisfy (3.37) at equality. However, there is a continuum of debt limits that satisfy (3.37) and are consistent with a steady-state equilibrium indexed by $\bar{b} \leq b^{max}$. While any debt limit less than b^{max} can be sustained in equilibrium, it's self-fulfilling in the following sense. Even though it is incentive compatible for borrowers to repay their debt up to b^{max}, if they believe they will only be able to borrow up to $\bar{b} < b^{max}$ in future periods, then they will default with certainty on any quantity of debt in excess of $\bar{b}$ without sacrificing future credit access. Banks understand this and will only extend up to $\bar{b}$ of unsecured credit in the current period. The same intuition applies for an equilibrium without credit where (3.37) is satisfied at $\rho\bar{b} = \Omega(\bar{b}) = 0$ for the fixed point $\bar{b} = 0$.

I now characterize the conditions necessary to ensure unsecured business lending is incentive compatible. For $\bar{b}_f > 0$, defaulting firms must be constrained. This implies $\chi < \chi^*$ and $\hat{z}_f < z_f^*$ where z_f^* is the value of $\hat{z}_f$ such that the firm's liquidity constraint binds with $\hat{k} = k^*$, or $(1-\theta_f)z_f^* + \theta_f f(z_f^*) = (\chi^* - \chi)f(k^*)$. In addition, the slope of $\Omega(\bar{b}_f)$ at $\bar{b}_f = 0$ must exceed ρ. Differentiating the right side of (3.37) with respect to $\bar{b}_f$ gives $\Omega'(\bar{b}_f) = \lambda\alpha\mu_f(1-\theta_f)\Delta'(k)$. Evaluating $\Omega'(\bar{b}_f)$ at $\bar{b}_f = 0$ implies $k = \hat{k}$ and a necessary condition for $\bar{b}_f > 0$ is given by

$$\lambda\alpha\mu_f(1-\theta_f)\Delta'(\hat{k}) > \rho. \tag{3.38}$$

Unsecured business lending is feasible when the marginal surplus from granting defaulting firms a positive debt limit exceeds the rate of time preference. Intuitively, defaulting firms must be sufficiently constrained for unsecured credit to be valued enough to ensure repayment. Using $\Delta'(k)$ to solve for $f'(\hat{k})$, we have the following.

Lemma 3.4. *Assume $\chi < \chi^*$, $\hat{z}_f < z_f^*$, and $\lambda\alpha\mu_f(1-\theta_f) > \rho\theta_f$. There exists a $\bar{k} \in (\underline{k}, k^*)$ that solves*

$$f'(\bar{k}) = \frac{(1-\theta_f)[\rho + \lambda\alpha\mu_f]}{\rho(\chi - \theta_f) + \lambda\alpha\mu_f(1-\theta_f)}. \tag{3.39}$$

If $\hat{k} < \bar{k}$, then $b_f^{max} > 0$ exists and unsecured business lending is incentive compatible.

Proof in Appendix C.

Lemma 3.4 shows that $\overline{b}_f > 0$ can be supported whenever the defaulter's investment drops below $\overline{k}$. While independent of i, $\overline{k}$ is increasing in λ, α, μ_f, and χ; while decreasing in θ_f and ρ. The condition given by $\lambda\alpha\mu_f(1-\theta_f) > \rho\theta_f$ guarantees $\overline{k} > \underline{k}$, otherwise defaulters' investment levels would never fall low enough to support unsecured lending. Although $\overline{k}$ is independent of i, $\hat{z}_f$ and $\hat{k}$ are not. It follows that Lemma 3.4 can be reformulated within the context of the nominal interest rate. Let $\overline{z}_f$ denote the real balances held by defaulting firms when $\hat{k} = \overline{k}$. Replacing $f'(k)$ from (3.34) with (3.39), one can obtain

$$i > \bar{\imath}_f \equiv \rho + \Big\{\rho\theta_f + \lambda\big[1 - \alpha\mu_f(1-\theta_f)\big]\Big\}\Delta'(\overline{z}_f). \tag{3.40}$$

As soon as i rises above $\bar{\imath}_f$, $\hat{z}_f$ falls below $\overline{z}_f$ and $\hat{k} < \overline{k}$. Repeating for households, unsecured consumer credit emerges when $\sigma\alpha\mu_h(1-\theta_h) > \rho\theta_h$ and

$$i > \bar{\imath}_h \equiv \frac{\rho\sigma}{\sigma\alpha\mu_h(1-\theta_h) - \rho\theta_h}. \tag{3.41}$$

Without pledgeable output, household defaulters choose their real balances according to $u'(\tilde{z}_h) = 1 + i/\sigma$ allowing $\bar{\imath}_h$ to be expressed in closed form with respect to the rate of time preference, consumption demand, and the degree of credit market frictions. Similarly for firms, unsecured consumer lending emerges when households' patience, consumption demand, bargaining power, and likelihood of obtaining a loan are high. Furthermore, the opportunity cost of holding liquidity must be high enough to guarantee household defaulters are sufficiently constrained.

Before concluding this section, comparative statics are studied for debt limits set "not-too-tight," or when $\overline{b} = b^{max}$. Otherwise, when $\overline{b} < b^{max}$, debt limits are set "too-tight" and taken as exogenous.

Lemma 3.5. *For $j \in \{h, f\}$, assume $i > \bar{\imath}_j$ and $\overline{b}_j = b_j^{max}$. Comparative statics for $\overline{b}_j$ are summarized in the following table.*

Table 3.1: Debt Limit Comparative Statics

	σ	λ	α	μ_h	μ_f	θ_h	θ_f	χ	i	ρ
$\bar{b}_h$	+	0	+	+	0	−	0	0	+	−
$\bar{b}_f$	0	+	+	0	+	0	+/−	+/−	+	−

Proof in Appendix C.

For households, the positive effects from higher demand, match rates, loan approvals, and bargaining power on their debt limit is intuitive as these increase the value of future unsecured credit access explicitly at the extensive margin. However, this is not necessarily true for θ_f and χ with firms due to their intensive margin effects. Since defaulting firms still bargain with banks, changes in θ_f and χ disproportionately impact k and $\hat{k}$, creating an ambiguous net effect on default costs. Since the heterogeneity across investment responses will be negligible when the difference between k and $\hat{k}$ is small, $\bar{b}_f$ is more likely to increase with χ and decrease with θ_f when $k - \hat{k}$ is small, or when $\bar{b}_f$ is low.

As for the policy rate, an increase in i now has two direct effects in the model. The first is with respect to its increase in the opportunity cost of holding real balances which lowers z_j. The second pertains to the cost of default. Higher i punishes defaulters more as they hold higher real balances. The increase in their liquidity holding costs outweighs that of non-defaulters and raises the cost of default. This is a direct implication of Lemma 3.3 regarding the inverse relationship between the debt limit and liquidity demand.

3.4.5 Equilibrium Definition

Equilibrium in this economy follows a recursive structure. First, (3.37) determines debt limits as functions of defaulting and non-defaulting borrowers' future consumption and investment. After observing their debt limits, borrowers' choice of real balances are determined according to (3.33) and (3.34). Finally, (3.28)-(3.30) determine q and k given financing capacities $z_h+\bar{b}_h$ and $\chi f(k) + z_f + \bar{b}_f$, respectively.

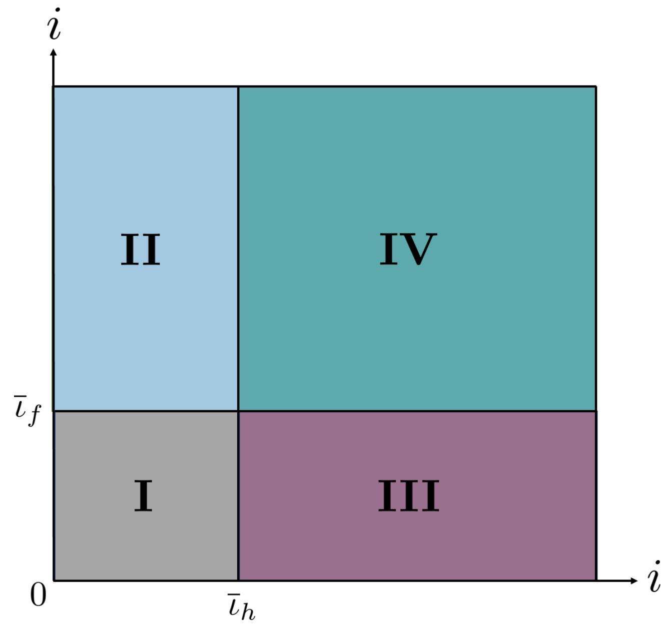
Definition 3.1. For $j \in \{h, f\}$, a steady-state equilibrium in the Stage 2 settlement market is a list $(q, k, \hat{k}, z_j, \tilde{z}_h, \hat{z}_f, \bar{b}_j)$ such that

(i) the household's loan $(q - z_h, \phi_h)$ and real balances z_h solve (3.28)-(3.30) and (3.32);
(ii) the firm's loan $(k - z_f, \phi_f)$ and real balances z_f solve (3.28)-(3.30) and (3.32);
(iii) the defaulting household's real balances $\tilde{z}_h$ solve $u'(\tilde{z}_h) = 1 + i/\sigma$;
(iv) the defaulting firm's loan $(\hat{k} - \hat{z}_f, \hat{\phi}_f)$ and real balances $\hat{z}_f$ solve (3.28)-(3.30) and (3.32) evaluated at $\bar{b}_f = 0$; and
(v) debt limits $\bar{b}_j \geq 0$ solve (3.37).

3.5 Steady-State Equilibria

To analyze comparative statics, I focus solely on steady-state equilibria for the remainder of the analysis. The nominal interest rate i is considered the relevant measure for monetary policy and is typically associated with the nominal yield on the 3-month Treasury bill in the literature. To guarantee a role for bank intermediation and the existence of monetary-credit equilibria, i must be strictly positive with $\sigma\big[1-\alpha\mu_h(1-\theta_h)\big] > 0$ and $\lambda\big[1-\alpha\mu_f(1-\theta_f)\big] > 0$. To demand credit, there must be a positive opportunity cost to holding cash. To hold cash, there must either be a positive share of non-credit matches or banks must have some bargaining power. That is, borrowers cannot expect to meet a bank and be approved for

Figure 3.4: Equilibrium Regions



a loan with certainty while capturing all trade gains. For now, assume both θ_h and θ_f are positive. These conditions will later be relaxed when transmission strength is analyzed in Section 3.5.3.

Considering this study concerns the lending channel of monetary policy, I focus on the steady-state relationship between i and the following variables of interest: household consumption q, business investment k, liquidity holdings z_j, loan principals ℓ_j, and interest fees ϕ_j for $j \in \{h, f\}$. To begin, I introduce four monetary-credit equilibrium regions characterized by the nominal interest rate and types of bank loans available. These regions are labeled Types I-IV and are displayed by Figure 3.4.[18] In a Type-I equilibrium, the interest rate is not high enough to support unsecured lending and only secured business loans are available. In a Type-II equilibrium, secured and unsecured business loans are available, while only unsecured consumer loans are available in a Type-III. Finally, all three loans are available

[18] In reference to the origin of Figure 3.4 when $i = 0$, equilibrium is characterized by a pure monetary steady-state as all agents hold sufficient liquidity to internally finance their first best allocation. Consequently, this leaves no role for bank credit or comparative statics to analyze.

in a Type-IV. Since Types I-III equilibria overlap with that of a Type-IV regarding loan availability, I focus the analysis on the latter to avoid repetition and redundancy. Type-IV equilibria will also be the most relevant for drawing connections between the paper's theoretical and empirical components in Section 3.5.4.

3.5.1 Consumer Lending

Type-IV equilibria emerge when the nominal interest rate is sufficiently high relative to the search and bargaining frictions of the credit market for unsecured consumer and business lending to be valued and incentive compatible. This corresponds to the top-right region of Figure 3.4 where $i > \bar{\iota}_f,\ \bar{\iota}_h$. To begin, consider the household. From Section 3.4.4, $\sigma\alpha\mu_h(1-\theta_h) > \rho\theta_h$ must hold for $\bar{\iota}_h > 0$ to exist. Hence, for unsecured consumer loans to be available, i, σ, α, and μ_h must be sufficiently high relative to ρ and θ_h. Proposition 3.1 summarizes the effects of monetary policy on unconstrained and constrained consumer lending.

Proposition 3.1 (*Consumer lending and monetary policy*)**.** *Assume $i > \bar{\iota}_h$ and $b_h^{max} < b_h^*$.*

(i) If $z_h \geq z_h^*$, then $q = q^*$, and ℓ_h and ϕ_h are increasing with i.

(ii) If $z_h < z_h^*$ and $\bar{b}_h < b_h^{max}$, then q and ℓ_h are decreasing, while ϕ_h is increasing with i.

(iii) If $z_h < z_h^*$ and $\bar{b}_h = b_h^{max}$, then q, ℓ_h, and ϕ_h are increasing with i.

Proof in Appendix C.

According to Part (i) of Proposition 3.1, when households hold sufficiently high liquidity, $q = q^*$ and consumer lending is unconstrained and independent of the debt limit. As long as $i > \bar{\iota}_h$, households can be unconstrained if either i and $\bar{b}_h$ are high enough allowing them to finance q^* regardless of their real balances, or when i is low enough such that $z_h \geq z_h^*$.

The results from Part (i) are derived explicitly from the households unconstrained liquidity premium given by

$$i = \sigma\big[1 - \alpha\mu_h(1-\theta_h)\big]\Delta'(z_h). \tag{3.42}$$

As i rises and cash becomes more costly to hold, households reduce their real balances and their wealth falls. As a consequence, lending increases as q is fixed and ℓ_h is decreasing with z_h. The household's wealth reduction and weakened bargaining position also allows banks to charge higher interest payments increasing their profitability. This can be seen from the firm's unconstrained bargaining solution. Within the context of households, (3.28) is written as $\phi_h = \theta_h\big[u(q^*) - q^* - \Delta(z_h)\big]$. Hence, when borrowers are unconstrained, a monetary tightening increases lending and bank profitability through liquidity demand and wealth. Although policy is operating solely through borrowers' balance sheets, the lending expansion should not be attributed to an active balance sheet subchannel as its theory explicitly predicts a lending contraction. Furthermore, the bank-dependency precondition for an active lending channel may not be satisfied among unconstrained equilibria. According to Gertler and Gilchrist (1994), the population of borrowers that are considered both bank-dependent and balance-sheet constrained overlap significantly. Considering this, the traditional balance sheet theory would not be applicable when lending is unconstrained.

Now consider Parts (ii) and (iii) of Proposition 3.1 when consumer lending is constrained and the transmission channel depends on $\bar{b}_h$. According to Part (ii), a monetary tightening reduces q and ℓ_h when the debt limit is tight and independent of i. Similar to unconstrained households, when $\bar{b}_h$ is fixed, a monetary tightening only affects lending through liquidity holdings. However, once lending is constrained and borrowing constraints bind, q responds to policy as the households bargaining solution is given by $(1-\theta_h)q + \theta_h u(q) = z_h + \theta_h\Delta(z_h) + \bar{b}_h$.

Differentiating q with respect z_h, the household's financial multiplier can be written as

$$\frac{dq}{dz_h} = \frac{1 - \theta_h + \theta_h u'(z_h)}{1 - \theta_h + \theta_h u'(q)} > 1, \tag{3.43}$$

which takes the same form as (3.31) for firms, but with $\chi = 0$. As for firms, the dual effects from households' liquidity holdings on their down payment and outside option creates a financial multiplier for consumption. Following the increase in i, q falls more than one-for-one with z_h resulting in $d\ell_h/di < 0$. As for $d\phi_h/di > 0$, the effects of monetary policy on interest payments depend on the marginal returns to external and internal finance, that is, $\Delta'(q)$ and $\Delta'(z_h)$. With $q > z_h$, the latter dominates and a monetary tightening allows banks to charge households higher interest, even though the decrease in consumption is stronger than that for liquidity. With households constrained in this setting, this result would be consistent with the traditional balance sheet subchannel and its theory described by Bernanke et al. (1996).

Considering Part (iii) of Proposition 3.1, comparative statics change once $\bar{b}_h = b_h^{max}$ and the debt limit subchannel becomes active. As established by Lemmas 3.3 and 3.5, a monetary tightening raises debt limits and allows borrowers to successfully hedge against higher liquidity costs while maintaining an overall larger financing capacity. Hence, the household's principal and interest payment display an unambiguous increase with the nominal interest rate when the balance sheet and debt limit subchannels are both active. This result takes a key departure from traditional bank lending theory which cannot account for an expansionary lending response to tightened policy. Without the availability of unsecured credit, an alternative subchannel through debt limits would not exist, and a monetary tightening would decrease consumer lending through the household's liquidity and wealth. Once it becomes active, then bank lending can display a countercyclical monetary policy response among constrained, bank-dependent households.

It's also worth noting that the debt limit subchannel has implications for z_h^* and the composition of consumer finance. From Lemma 3.1, z_h^* solves

$$(1-\theta_h)z_h^* + \theta_h u'(z_h^*) = b_h^* - \bar{b}_h, \tag{3.44}$$

where $b_h^* \equiv (1-\theta_h)q^* + \theta_h u(q^*)$. When debt limits don't respond to policy, z_h^* is fixed and raising i expands the gap between z_h and z_h^*, while q and ℓ_h decrease further. When debt limits are responsive, z_h^* decreases with i as $d\bar{b}_h/di > 0$ and $dz_h^*/d\bar{b}_h < 0$. From (3.44) , if i continues to rise, then $\bar{b}_h$ approaches b_h^* and z_h^* approaches zero. However, households will be unconstrained before z_h^* reaches zero as borrowers always hold positive liquidity. This implies that consumer lending may be unconstrained in a low interest rate environment with high liquidity, or in a high rate environment with low liquidity. Hence, the debt limit subchannel allows unconstrained lending to be characterized by either low interest rates and shares of external finance, or high interest rates and shares of external finance.

3.5.2 Business Lending

Turning to firms, Lemma 3.4 confirms that $\lambda\alpha\mu_f(1-\theta_f) > \rho\theta_f$ must hold for $\bar{\imath}_f > 0$ to exist. Furthermore, $\hat{k} < \bar{k}$ must hold to ensure defaulters are sufficiently constrained. Otherwise, default costs are too low and $\bar{b}_f = 0$. Proposition 3.2 summarizes the effects of monetary policy on unconstrained and constrained business lending.

Proposition 3.2 (*Business lending and monetary policy*)**.** *Assume $i > \bar{\imath}_f$, $0 < \chi < \chi^*$, and $b_f^{max} < b_f^*$.*

(i) If $z_f \geq z_f^*$, then $k = k^*$, and ℓ_f and ϕ_f are increasing with i.

(ii) If $z_f < z_f^*$ and $\bar{b}_f < b_f^{max}$, then k and ℓ_f are decreasing, while ϕ_f is increasing with i for

$z_f < \tilde{z}_f$, where $\tilde{z}_f$ solves

$$f'(\tilde{z}_f) = 1 + \frac{i}{\lambda}.$$

(iii) If $z_f < z_f^*$ and $\overline{b}_f = b_f^{max}$, then k, ℓ_f, and ϕ_f are increasing with i.

Proof in Appendix C.

Similar to households, Proposition 3.2 shows that business lending and interest payments expand in response to tightened policy when firms are unconstrained or constrained with debt limits active. If constrained with $\overline{b}_f < b_f^{max}$ fixed, then k and ℓ_f fall. Since unsecured business loans are also available in a Type-II equilibrium, these dynamics hold for Type-II and IV firms. However, the comparative statics presented in Part (ii) can also be applied to Type-I firms when $\overline{b}_f = 0$ and business lending is entirely secured. The only difference is that Type-IV firms hold less liquidity as the nominal interest rate is higher to ensure $i > \overline{\iota}_f$ and $\overline{b}_f > 0$. Also, depending on how high i and $\overline{b}_f$ are, a Type-IV firm's investment k may or may not exceed $\hat{k}$ of a Type-I firm when $i \leq \overline{\iota}_f$.

In contrast to Proposition 3.1, notice that the firm's interest fee response is non-monotonic in Part (ii) of Proposition 3.2. This is a direct implication of the complementarity that arises between firms' pledgeability and liquidity demand, alongside the subsequent amplification effect χ imposes on their financial multiplier given by (3.31) and discussed in Section 3.4.3. In line with household defaulters, let $\tilde{z}_f$ denote the real balances held by defaulting firms when $\chi = 0$ (i.e., no credit market access). From (3.33), $z_f < \tilde{z}_f$ always holds at $k = k^*$. All else equal, it follows that $z_f > \tilde{z}_f$ occurs when i is sufficiently high and lending is relatively more constrained. Therefore, $d\phi_f/di < 0$ is most likely to arise when business investment, collateral, principals, and liquidity demand are low. From (3.31), the amplification effect from χ is stronger when the marginal increase in pledgeable output from additional investment is high, since dk/dz_f is increasing in $\chi f'(k)$. Following a monetary tightening, this elicits a stronger drop in k than would occur otherwise if $\chi f'(k)$ was low, and ϕ_f falls with ℓ_f and $\chi f(k)$.

This reasoning can be reversed for $z_f < \tilde{z}_f$ when k, ℓ_f, and z_f are higher. The higher levels of liquidity and pledgeable output dampen the financial multiplier rendering k and $\chi f(k)$ less elastic to changes in monetary policy. Although principals still decrease with the policy rate, banks can charge higher interest fees on firms displaying stronger collateral and balance sheet positions. As shown by Proposition 3.1, the potential for banks to lose profits after a monetary tightening vanishes when $\chi = 0$ for consumer lending.

3.5.3 Transmission Strength

To analyze the strength for the transmission of monetary policy through the lending channel analytically, comparative statics are performed with respect to the interest rate semi-elasticities of consumer and business lending. For drawing connections between the theory and the TVP-VAR estimation, let $L_h \equiv \sigma\alpha\mu_h\ell_h$ and $L_f \equiv \lambda\alpha\mu_f\ell_f$ denote aggregate consumer and business lending, respectively. To characterize the semi-elasticity of lending in a Type-IV equilibrium while retaining analytical tractability, suppose $\theta_h = \theta_f = 0$. Beginning with firms, the constrained bargaining solution becomes $k = \chi f(k) + z_f + \overline{b}_f$. From (3.33) and (3.34), z_f and $z_f^* = k^* - \chi f(k^*) - \overline{b}_f$ now solve

$$i = \lambda\alpha\mu_f \frac{(1-\chi)f'(k)}{1-\chi f'(k)} + \lambda(1-\alpha\mu_f)\big[f'(z_f) - 1\big]; \tag{3.45}$$

$$i_f^* = \lambda(1-\alpha\mu_f)\big[f'(z_f^*) - 1\big], \tag{3.46}$$

respectively. Business lending is constrained as long as $i > i_f^*$ and $z_f < z_f^*$. Corollary 3.1 summarizes comparative statics for the interest rate semi-elasticity of business lending.

Corollary 3.1 (*Business lending interest rate semi-elasticity*)**.** *Assume $\chi < \chi^*$, $\theta_f = 0$, and $\bar{b}_f < b_f^{max} \in (0, b_f^*)$.*

(i) If $i_f^* - i > 0$, but small, then $k = k^*$ and the strength of the transmission channel is decreasing with i and χ.

(ii) If $i - i_f^* > 0$, but small, then $k < k^*$ and the strength of the transmission channel is increasing with i, while decreasing with λ and χ.

Proof in Appendix C.

According to Corollary 3.1, when debt limits are fixed the strength of the lending channel depends on whether firms are constrained or unconstrained. Beginning with the latter, Part (i) establishes that the interest rate semi-elasticity for business lending is a positive and decreasing function of pledgeability and the policy rate. Intuition for these results can be gathered from considering loan size and the elasticity of liquidity demand dz_f/di. Given $i_f^* - i$ is positive but small, $z_f \approx z_f^*$ and the proof for Corollary 3.1 shows that ℓ_f is increasing with i and χ since firms hold less liquidity when either interest rates or pledgeability are high.[19] Furthermore, $|dz_f/di|$ is also low when χ is high as firms' liquidity holding costs are low. As a result, transmission is weaker when the share of externally financed investment is high while liquidity demand and its elasticity are low. This can also be viewed analytically with the interest rate semi-elasticity rewritten as $d \ln L_f/di \approx -\ell_f^{-1} dz_f/di > 0$.

Now consider Part (ii) of Corollary 3.1 when business lending is constrained. Recalling Proposition 3.2, when only the balance sheet subchannel is active, a high interest rate environment suppresses liquidity demand, investment, and principals. In contrast, k and ℓ_f are high when λ and χ are high as firms finance higher shares of their investment externally by

[19] When firms are unconstrained, pledgeability has no effect on the terms of the loan contract or liquidity premium. However, it does have implications for the semi-elasticity, because it influences the minimum level of liquidity, z_f^*, required for firms to be unconstrained. Hence, when $i_f^* - i$ is positive but small, $z_f \approx z_f^*$ and the firm's principal depends on z_f^* and therefore χ. From (3.27), $dz_f^*/d\chi = -f(k^*)$.

either holding more liquidity or pledging more collateral. The only difference is that high pledgeability leads to large principals as more capital can be financed at any given interest rate and level of liquidity, while high investment demand (i.e., high λ) raises principals via higher liquidity demand and therefore investment. Also, $|dz_f/di|$ is again decreasing with λ and χ associating weakened transmission with large principals and low liquidity demand elasticity.

As for the model's remaining parameters, α and μ_f, their implications for the transmission channel also vary and depend on ℓ_f and χ. Whether constrained or unconstrained, firms hold less liquidity when their likelihood of receiving a loan is high. However, when unconstrained, holding less liquidity leads to higher principals which puts downward pressure on $d\ln L_f/di$. It follows that for $d\ln L_f/di$ to be high in an environment with high α and μ_f, dz_f/di must be high to offset the large principal. The proof for Corollary 3.1 in Appendix C demonstrates that dz_f/di is increasing with α and μ_f as unconstrained borrowers are more willing to reduce their liquidity when their likelihood of receiving a large loan is high. This differs for constrained firms with $\bar{b}_f < b_f^{max}$, because now when α and μ_f are high, low liquidity promotes low principals. However, this does not necessarily imply $|dz_f/di|$ is high, because firms' liquidity holdings become more important when their external financing capacity is low. Since $|dz_f/di|$ is decreasing with χ, Section C.2.8 in Appendix C presents a necessary condition on χ to show that $|d\ln L_f/di|$ increases with α and μ_f if χ is sufficiently low.[20] Hence, the transmission channel strengthens for constrained businesses characterized by low levels of liquidity, collateral, and external financing.

For analytical tractability, Corollary 3.1 assumes debt limits are tight so z_f^* and i_f^* are independent of policy. When this is not the case and $\bar{b}_f = b_f^{max}$, comparative statics regarding

[20]This result may seem counterintuitive as $|dz_f/di|$ increases with α and μ_f when unconstrained firms are more willing to sacrifice liquidity if they are likely to receive large loans. However, intuition comes from considering the firm's financial multiplier. Increased pledgeability amplifies the multiplier by allowing a higher levels of investment to be financed from carrying an additional unit of real balances. As a consequence, liquidity holdings are valued more and become less elastic to policy.

transmission strength become significantly difficult to characterize due to the presence of many nonlinearities that arise when the balance sheet and debt limit subchannels operate simultaneously. For example, consider pledgeability and Lemma 3.5 where $d\bar{b}_f/d\chi$ is shown to be non-monotonic. While χ reduces z_f^* directly via $\chi f(k^*)$, it may raise z_f^* via lower $\bar{b}_f$ as pledgeability can reduce default costs and debt limits. As a result, the overall change in z_f^* and subsequent effects on transmission depend on which of these two effects dominate.

However, to gain some insight into the debt limit's role for transmission strength, consider the household when consumer lending is constrained and $\phi_h = 0$. Following the same methodology as Corollary 3.1, aggregate consumer lending and its semi-elasticity can be approximated by

$$L_h \approx \sigma\alpha\mu_h\bar{b}_h; \tag{3.47}$$

$$\frac{d\ln L_h}{di} \approx \frac{1}{\bar{b}_h}\frac{d\bar{b}_h}{di}, \tag{3.48}$$

respectively, where $\bar{b}_h = q^* - z_h^*$ and $d\bar{b}_h/di = (\tilde{z}_h - z_h)/\rho > 0$. If $i - \bar{\imath}_f$ and $i - i_h^*$ are both positive, but small, then $\tilde{z}_h \approx z_h^*$ and a first-order linear approximation of $\tilde{z}_h - z_h$ would yield $d\bar{b}_h/di \approx -\alpha\mu_h i/\rho\sigma(1-\alpha\mu_h)u''(z_h^*) > 0$. From (3.48), the interest rate semi-elasticity for consumer lending can be approximated by

$$\frac{d\ln L_h}{di} \approx \frac{-\alpha\mu_h i}{\rho\sigma(1-\alpha\mu_h)[q^* - z_h^*]u''(z_h^*)}. \tag{3.49}$$

The strength of the transmission channel is increasing in i, α, and μ_h, while decreasing in σ and ρ. The comparative statics presented for $\bar{b}_h$ in Lemma 3.5 and the debt limit's convexity with respect to i, α, and μ_h are responsible for their expansionary effects on L_h and its semi-elasticity. Recalling Proposition 3.1, with only the balance sheet subchannel active for consumers, low liquidity demand leads to small principals and low levels of consumption. In contrast, once the debt limit subchannel becomes active, low money demand is associated with high debt limits, principals, consumption, and stronger transmission. Comparing these results with those for constrained businesses, monetary transmission can be strong in a high interest rate environment with low credit frictions, but the direction in which it operates depends on which subchannels are active. Finally, notice that if $\bar{b}_h < b_h^{max}$, then $d \ln L_h/di \approx 0$ as $\bar{b}_h$ and z_h^* no longer respond to policy and (3.47) is linear in the debt limit. This suggests that banks must have some positive bargaining power to corroborate a strong transmission channel when consumer debt limits are tight.

3.5.4 TVP-VAR Connection

To complete this analysis, I now draw upon the model's theoretical predictions to discuss within the context of the results and patterns observed in the paper's empirical TVP-VAR analysis. Considering the composition of the loan data used for the estimation, Type-IV equilibria are the most relevant for drawing connections between the theory and empirics. From Figure 3.1 , consider the early 1970s and mid-2000s when business loans display their strongest expansionary responses while credit cards display strong, but negative responses. According to the theory, the consumer lending response implies that households are constrained with $\bar{b}_h < b_h^{max}$. Debt limits must be tight for credit card loans to contract following a monetary tightening. Regarding transmission strength, aggregate consumer lending is approximately linear in the debt limit when banks have no bargaining power and ℓ_h captures the entire share of $\bar{b}_h$. This result comes from (3.47) where $L_h \approx \sigma\alpha\mu_h\bar{b}_h$ for $\theta_h = 0$. Therefore, banks must have positive bargaining power to explain a strong, negative credit card semi-elasticity, otherwise the theory predicts $d \ln L_h/di \approx 0$.

The response from business loans can either be explained by unconstrained or constrained lending with debt limits set "not-too-tight." According to Corollary 3.1, transmission weakens with pledgeability and the policy rate when firms are unconstrained. Considering when transmission strength is strongest around 1972 and 2004, the federal funds rate sat near 4% and 1.5%, respectively. Historically, these were relatively low interest rate periods which could be consistent with a strong transmission channel among unconstrained firms. However, just comparing 1972 with 2004, transmission is relatively stronger during the higher policy rate period. This would suggest that more than just the policy rate is influencing transmission such as higher pledgeability during 2004, or that business lending is constrained with an active debt limit subchannel during at least one of these periods.

Now consider 1993 and 1999 from Figure 3.1 when the credit card lending channel displays its strongest expansionary responses. In this setting, credit card lending may be either unconstrained or constrained with $\bar{b}_h = b_h^{max}$. If unconstrained, then transmission strength follows Corollary 3.1 and weakens with i. However, transmission is stronger around 1999 when the federal funds rate was close to 5% versus 3% in 1993 suggesting that consumer lending was likely constrained during one of these periods. Now, suppose credit card lending was constrained during both. If $d \ln L_h / di$ is increasing with i, α, and μ_h, then the stronger 1999 response could be attributed to a higher nominal interest rate, match rate, loan approval rate, or a combination of all three. Even if lending was unconstrained during only one of these periods, the theory can potentially reconcile the heterogeneous elasticities via differences across σ, α, and μ_h.

For businesses, debt limits must be tight to elicit the negative response. From Corollary 3.1 Part (ii), the semi-elasticity of L_f strengthens with i and weakens with χ and λ. With transmission strength approximately equivalent during both periods, but a higher policy rate in 1999, the theory suggests that either χ or λ may have been higher in 1999 relative to 1993. However, if this were not the case and χ was low during both periods, then a lower α or μ_f in 1999 could potentially corroborate the equivalence observed across the channel's transmission strengths.

3.6 Conclusion

In light of the difficulties conventional macroeconomic theory finds in explaining an expansionary loan response following a monetary tightening, this paper investigates the lending channel of monetary policy exclusively through consumer credit card and small business bank loans. As a first step, I estimate a TVP-VAR model with stochastic volatility to document each loan's response to a contractionary monetary policy shock. The model's impulse response analysis provides evidence suggesting that the direction and strength in which credit card and small business loans respond to higher interest rates are time-dependent.

To investigate these findings analytically, I develop a general equilibrium model of consumer and small business lending to study the underlying mechanisms that may be driving the channels' nonlinearities. Households and firms use a combination of monetary assets and over-the-counter banks loans to finance random consumption and investment opportunities. In accordance with theory developed from earlier models of the bank lending channel, when borrowers are sufficiently constrained, a monetary tightening reduces lending through the traditional balance sheet subchannel of monetary policy. However, when borrowers are less constrained, a second subchannel arises and tighter monetary policy raises unsecured debt limits and lending expands. The transmission of monetary policy through the debt limit subchannel provides a new theory to explain a countercyclical lending response to monetary policy not previously addressed in the literature. Otherwise, monetary policy operates solely through the balance sheet subchannel and lending contracts with interest rates as predicted by conventional theory.

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Appendix A

A.1 Proofs and Derivations

A.1.1 Proof of Lemma 1.1

Taking the first order conditions of (1.8) with respect to k and b gives

$$\frac{\eta f'(k)}{f(k)-b} = \frac{1-\eta}{\delta U(b)-k};$$
$$\frac{\eta}{f(k)-b} = \frac{(1-\eta)\delta U'(b)}{\delta U(b)-k}.$$

Substituting either one of the above FOCs into the other and solving for $f'(k)$ yields (1.9) for determining k as a function of b. Rearranging and solving for b then gives

$$b = f(k) - \frac{\eta\big[\delta U(b)-k\big]}{(1-\eta)\delta U'(b)},$$

from which (1.10) can be obtained by solving for $U'(b)$.

To prove there exists a solution (k, b) to (1.9) and (1.10) that is both nonzero and unique,

$U'(b) = f'(k)/\delta$ can be substituted in for $U'(b)$ of (1.10) and rearranged to obtain

$$(1-\eta)\big[f(k) - b\big] = \eta f'(k)\big[\delta U(b) - k\big]. \tag{A.1}$$

First, notice that (A.1) is satisfied for $k = b = 0$. Next, consider when $k = 0$ and $b > 0$. the left side of (A.1) is negative while the right side is positive infinity. When taking the limit of each side as $k \longrightarrow \infty$, the left side converges to positive infinity while the right side converges to zero. Hence, there is a $k > 0$ that satisfies (A.1) for $b > 0$ and therefore, a unique (k, b) exists as a solution to (1.9) and (1.10). This same line of reasoning can be followed to prove uniqueness when starting with $k > 0$ and taking the limits of (A.1) as $b \longrightarrow \infty$. □

A.1.2 Proof of Lemma 1.2

Deriving the solution to (1.11) for the terms of the insured debt $(\hat{k}, \hat{b})$ and CDS (s_1, s_2) contracts follow the same methodology as that for the proof of Lemma 1.1. The first order conditions with respect to $\hat{k}$ and $\hat{b}$ yield (1.12) and (1.13). Those with respect to s_1 and s_2 yield (1.14) and the full insurance condition where $U'(\hat{b} + s_1) = U'(s_2)$. After substituting (1.12) and $\hat{b} + s_1 = s_2$ into (1.16) and rearranging, one can obtain

$$\gamma_2\big[\delta f(\hat{k}) - s_2\big] = (1-\gamma_2)\delta f'(\hat{k})\big[U(s_2) - \hat{k}\big]. \tag{A.2}$$

In contrast to the proof for Lemma 1.1, (A.2) can be used to first show a unique $\hat{k} > 0$ and $s_2 > 0$ exist which can then be substituted into (1.15) to solve for a unique $\hat{b} > 0$. Once again, (A.2) is satisfied for $\hat{k} = s_2 = 0$. Beginning with $\hat{k} = 0$ and some $s_2 > 0$, as $\hat{k} \longrightarrow \infty$, the left side of (A.2) increases from $-\gamma_2 s_2$ to positive infinity while the right side decreases from positive infinity to zero. Hence, a unique $\hat{k} > 0$ and $s_2 > 0$ exist solving (A.2). From (1.15), this result implies a unique $\hat{b} > 0$ exists which can then be plugged into (1.13) to solve for a unique s_1. Lastly, as a direct consequence of the full insurance condition, $\hat{b} + s_1 = s_2$, combined with the insurer's incentive compatibility constraint, $-\delta s_1 - (1-\delta)s_2$, it follows that $s_1 < 0$ and $\hat{b} > s_2$ must alway hold in equilibrium. □

A.1.3 Proof of Proposition 1.1

Proposition 1.1 assumes $\delta \in (0,1)$, $\sigma > 0$, and $1 - \eta \geq \gamma_2 \in (0,1)$. To prove that the introduction of CDS trading leads to an investment expansion where $\hat{k} > k$ given these conditions, (A.1) and (A.2) from the proofs for Lemmas 1.1 and 1.2, respectively, are used. With (1.9) determining k as a function of b and (1.12) similarly determining $\hat{k}$ as a function of s_2 in place of b, consider (A.1) and (A.2), but with $s_2 = b$. Condition (A.2) can then be rewritten as

$$\gamma_2\big[\delta f(\hat{k}) - b\big] = (1-\gamma_2)f'(\hat{k})\big[\delta U(b) - \delta\hat{k}\big]. \tag{A.3}$$

Comparing (A.1) with (A.3), notice that for $\hat{k} = k$ and $1 - \eta \geq \gamma_2$, the left side of (A.3) would be less than the left side of (A.1) while the right side of (A.3) would exceed that of (A.1). Considering the left and right sides of (A.3) are increasing and decreasing in $\hat{k}$, respectively, it must be true that $\hat{k} > k$ for (A.3) to hold when $s_2 = b$. From (1.9) and (1.12) where $f'(k) = 1/\delta U'(b)$ and $f'(\hat{k}) = 1/\delta U'(s_2)$, it follows that $s_2 < b$ must hold when $\hat{k} > k$. This is also consistent with (A.3) as its left side is decreasing in s_2 while its right side is increasing in s_2 through $\delta U(s_2)$. □

A.1.4 Proof of Proposition 1.2

The effects from δ on the terms of the uninsured debt contract can be derived visually by analyzing the composition of conditions (1.9) and (1.10). Beginning with (1.10), an increase in δ leads to an increase in the right hand side's numerator. This corresponds to an increase in $U'(b)$ and decrease in b. Due to the decreasing returns to scale assumption on $f(k)$, (1.9) shows an inverse relationship between $f'(k)$ and $\delta U'(b)$. As a result, the increase in $\delta U'(b)$ leads to an unambiguous decrease in $f'(k)$ and increase in k. Hence, $\partial k/\partial\delta > 0$ and $\partial b/\partial\delta < 0$.

Turning to the insured match, comparative statics are derived from conditions (1.12), (1.15), and (1.16). The derivations follow the same recursive methodology as for $\partial k/\partial\delta$ and $\partial b/\partial\delta$, except (1.16) determines the value for $s_2 = \hat{b} + s_1$, not $\hat{b}$. In contrast to (1.10), the right side of (1.16) and therefore $U'(s_2)$ is decreasing with δ such that $\partial s_2/\partial\delta > 0$. According to (1.12), the same inverse relationship between $\hat{k}$ and $\delta U'(s_2)$ holds as it does for k and $\delta U'(b)$, however, δ and $U'(s_2)$ are also inversely related. The response from $\hat{k}$ depends on whether the increase from δ or s_2 dominates. Holding $\hat{k}$ fixed and taking the derivative of $\delta U'(s_2)$ and (1.16) with respect to δ yields

$$\frac{\partial\delta U'(s_2)}{\partial\delta} = U'(s_2) + \delta U''(s_2)\frac{\partial s_2}{\partial\delta}; \tag{A.4}$$

$$\frac{\partial s_2}{\partial\delta} = \frac{f(\hat{k})}{\gamma_2^{-1} - \frac{U''(s_2)}{U'(s_2)}\left[\delta f(k) - s_2\right]} > 0, \tag{A.5}$$

where $U''(s_2)/U'(s_2) = -\sigma/s_2$ can be derived from the CRRA functional form of $U(\cdot)$. While $\partial s_2/\partial\delta > 0$ always holds for $\delta > 0$, the sign on $\partial\delta U'(s_2)/\partial\delta$ is indeterminate. After substituting (A.5) in for $\partial s_2/\partial\delta$ in (A.4) and simplifying, a condition that guarantees $\partial\delta U'(s_2)/\partial\delta > 0$ and $\partial f'(k)/\partial\delta < 0$ is $\sigma < \gamma_2^{-1}$. Hence, the sign on $\partial\hat{k}/\partial\delta$ is positive for $\sigma < \gamma_2^{-1}$ and negative for $\sigma > \gamma_2^{-1}$.

Having established the responses from s_2 and $\hat{k}$, the remaining effects to consider are those for $\hat{b}$, s_1, and n. Beginning with $\hat{b}$, the right side of (1.15) is increasing with $\hat{k}$ and s_2, but decreasing with δ. This allows for a potential nonlinearity regardless of the response from $\hat{k}$. To address this, I begin by analyzing the response in the s_2 component of (1.15) given by

$\gamma_1 s_2/(1-\gamma_2)\delta$. Dropping the bargaining powers and taking the derivative with respect to δ gives

$$\frac{\partial \delta^{-1} s_2}{\partial \delta} = -\delta^{-2} s_2 + \delta^{-1} \frac{\partial s_2}{\partial \delta}.$$

Using (A.5), it follows that $\partial \delta^{-1} s_2/\partial \delta < 0$ holds when $(1-\sigma)\delta f(\hat{k}) < (\gamma_2^{-1} - \sigma)s_2$. Given $\delta f(\hat{k})/s_2 > 1$, s_2/δ is decreasing when $\sigma \geq \gamma_2^{-1}$. With $\hat{k}$ either decreasing or constant when $\sigma \geq \gamma_2^{-1}$, it's straightforward to show $\partial \hat{b}/\partial \delta < 0$ and therefore $\partial s_1/\partial \delta > 0$, since $s_1 < 0$. Furthermore, if $1 \leq \sigma < \gamma_2^{-1}$, then s_2/δ is still decreasing, but now $\partial \hat{k}/\partial \delta > 0$ and $\partial \hat{b}/\partial \delta$ may be positive or negative. Lastly, if $\sigma < 1$, then s_2/δ increases and both $\partial \hat{k}/\partial \delta > 0$ and $\partial \hat{b}/\partial \delta > 0$ hold. However, the response from $s_1 = s_2 - \hat{b}$ is still indeterminate as it depends on whether the increase in $\hat{b}$ dominates that of s_2.

To derive a condition that determines the sign on $\partial s_1/\partial \delta$ when $\partial \hat{k}/\partial \delta > 0$, (1.15) can be rearranged by replacing $\hat{b}$ with $s_2 - s_1$ to obtain

$$(1-\gamma_2)s_1 = \left[(1-\gamma_2) - \gamma_1 \delta^{-1}\right] s_2 - \gamma_3 f(\hat{k}) < 0.$$

While s_1 is always decreasing in $\hat{k}$, it may be increasing or decreasing in s_2 depending on the sign of $(1-\gamma_2) - \gamma_1\delta^{-1}$. Consider the simple case when $(1-\gamma_2) - \gamma_1\delta^{-1} = 0$ such that $\gamma_2 = 1 - \gamma_1\delta^{-1}$ where $\delta > \gamma_1$. Now, s_1 is independent of s_2 and a function of $\hat{k}$ only. With $\sigma < \gamma_2^{-1}$ and $\partial \hat{k}/\partial \delta > 0$, this implies s_1 is decreasing with δ when $\sigma < \delta/(\delta + \gamma_1)$.

To determine the sign on $\partial n/\partial \delta$, differentiating S^I and $f'(\hat{k})$ yields

$$\frac{\partial S^I}{\partial \delta} = \frac{\gamma_3}{1-\gamma_2}\left[f(\hat{k}) + \delta f'(\hat{k})\frac{\partial \hat{k}}{\partial \delta} - \frac{\partial s_2}{\partial \delta}\right]; \tag{A.6}$$

$$\frac{\partial \hat{k}}{\partial \delta} = \frac{s_2^{\sigma-1}}{\delta^2 f''(\hat{k})}\left[\sigma\delta\frac{\partial s_2}{\partial \delta} - s_2\right]. \tag{A.7}$$

Substituting (A.7), (A.5), and $U''(s_2)/U'(s_2) = -\sigma/s_2$ into (A.6), the following condition ensures $\partial n/\partial\delta$ takes a positive sign:

$$f'(\hat{k})^2 - f(\hat{k})f''(\hat{k}) > \left[\frac{\sigma\delta f'(\hat{k})^2 - f''(\hat{k})s_2}{(\gamma_2^{-1} - \sigma)s_2 + \sigma\delta f(\hat{k})}\right] f(\hat{k})$$

With both sides of the above condition positive, one can show that it always holds for $\gamma_2^{-1} \geq 1$. Hence, n is increasing with δ. □

A.1.5 Proof of Lemma 1.3

Taking the first order conditions of (1.20) with respect to k, b, $\hat{k}$, $\hat{b}$, s_1, s_2, and n yields the following:

$$f'(k) = f'(\hat{k}) = \frac{1}{\delta};$$
$$U'(b) = U'(\hat{b} + s_1) = U'(s_2) = 1;$$
$$\epsilon = \alpha'(n)(1-\delta)\big[U(\hat{b} + s_1) - \hat{b} - s_1\big].$$

Let k^*, b^*, and n^* denote the efficient levels of capital lending, lenders' CM payment, and insurer entry chosen by the planner. It follows that $k = \hat{k} = k^*$ where k^* solves $f'(k^*) = 1/\delta$, $b = \hat{b} + s_1 = s_2 = b^*$ where b^* solves $U'(b^*) = 1$, and $n = n^*$ where n^* solves

$$\alpha'(n^*)(1-\delta)\big[U(b^*) - b^*\big] = \epsilon.$$

For $\delta \in (0,1)$, $\sigma > 0$, and $\epsilon \geq 0$, it holds in equilibrium that k^*, b^*, and n^* are each positive and unique. □

A.1.6 Proof of Proposition 1.3

The proof for Proposition 1.3 proceeds in two steps. First, thresholds for the repayment rate, δ^* and $\hat{\delta}^*$, are derived to ensure efficiency within the uninsured and insured match, respectively. Second, thresholds for the degree of risk aversion, σ^* and $\hat{\sigma}^*$, are derived to ensure that both repayment rate thresholds fall within the unit circle. Otherwise, an efficient match allocation would never be feasible.

Setting $k = \hat{k} = k^*$ and $U'(b) = U'(s_2) = U'(b^*) = 1$, (1.10) and (1.16) can be rewritten as

$$(1-\eta)\delta\big[f(k^*) - b^*\big] = \eta\big[\delta U(b^*) - k^*\big]; \tag{A.8}$$

$$\gamma_2\big[\delta f(k^*) - s_2\big] = (1-\gamma_2)\big[U(b^*) - k^*\big]. \tag{A.9}$$

Assuming the functional forms $U(b) = b^{1-\sigma}/(1-\sigma)$ and $f(k) = \nu^{-1}k^{\nu}$, (A.8) and (A.9) then become

$$(1-\eta)\delta\big[\nu^{-1}\delta^{\nu/(1-\nu)} - 1\big] = \eta\big[\delta(1-\sigma)^{-1} - \delta^{1/(1-\nu)}\big];$$
$$\gamma_2\big[\delta\nu^{-1}\delta^{\nu/(1-\nu)} - 1\big] = (1-\gamma_2)\big[(1-\sigma)^{-1} - \delta^{1/(1-\nu)}\big],$$

where $U(b^*) = (1-\sigma)^{-1}$ and $k^* = \delta^{1/(1-\nu)}$. By solving for δ in each of the above equalities, one can obtain the following repayment rate thresholds for an uninsured and insured match, respectively:

$$\delta^* = \left[\frac{\nu\big[1-\sigma(1-\eta)\big]}{(1-\sigma)\big[1-\eta(1-\nu)\big]}\right]^{(1-\nu)/\nu}$$
$$\hat{\delta}^* = \left[\frac{\nu(1-\sigma\gamma_2)}{(1-\sigma)\big[\gamma_2 + (1-\gamma_2)\nu\big]}\right]^{(1-\nu)/\nu}$$

where $\hat{\delta}^* > \delta^*$ for $(1-\eta) \geq \gamma_2$. Having established both δ^* and $\hat{\delta}^*$ in closed form, conditions must be derived to ensure they are both less than unity. Restricting the right sides of δ^* and

$\hat{\delta}^*$ to be less than one, σ can be isolated to one side of each inequality to obtain

$$\sigma^* = \frac{(1-\eta)(1-\nu)}{(1-\eta)(1-\nu)+\eta\nu};$$
$$\hat{\sigma}^* = \frac{\gamma_2(1-\nu)}{\nu+\gamma_2(1-2\nu)},$$

where $\sigma^* < 1$ and $\hat{\sigma}^* < 1$. One can then show that $\sigma^* > \hat{\sigma}^*$ when $(1-\eta) > \gamma_2$ which is the same condition that ensures $\hat{\delta}^* > \delta^*$ as stated in Proposition 1.3. Hence, for $\sigma < \hat{\sigma}^*$ and $\gamma_2 \leq (1-\eta)$, it's implicitly guaranteed that $\sigma < \sigma^*$ and $\delta^* < \hat{\delta}^* < 1$. $\square$

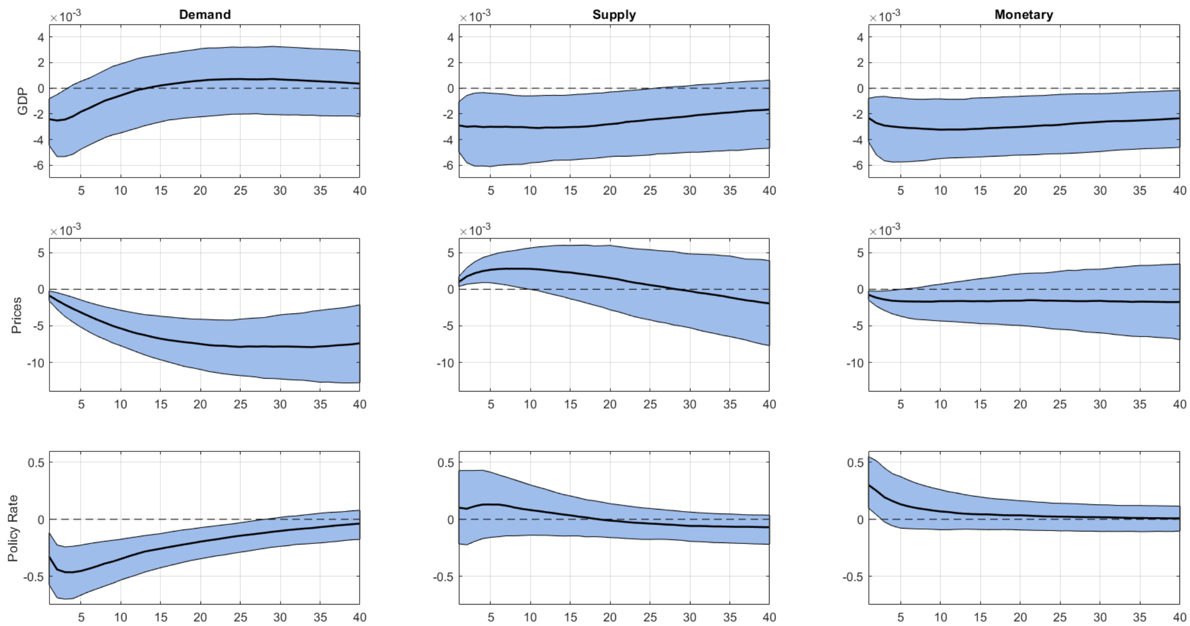
Appendix B

B.1 Supplementary Tables and Figures

Table B.1: Chapter 2 Data and Sources

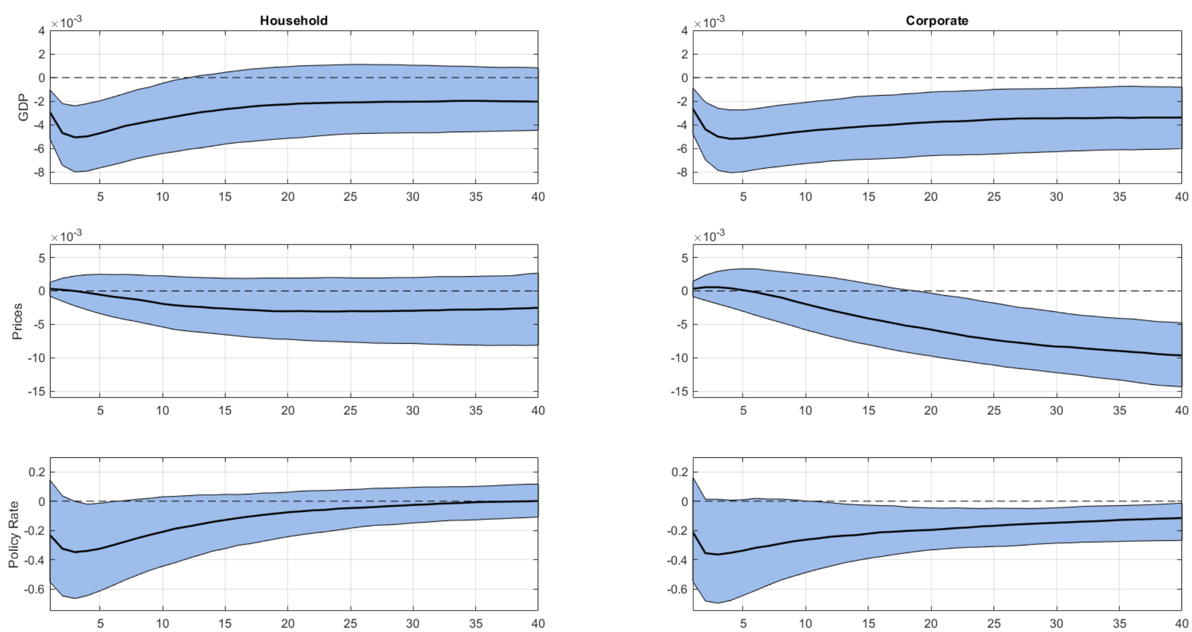
Variable	Description	Source
GDP	Log of real GDP	Federal Reserve Bank of St. Louis
GDP deflator	Log of GDP price index	Federal Reserve Bank of St. Louis
Policy rate	Shadow federal funds rate	Board of Governors of the Federal Reserve System, Wu and Xia (2016)
Credit spread	Bank prime loan rate - 3 month Treasury bill	Board of Governors of the Federal Reserve System
Private credit	Loans and debt securities to non-financial private sector	Federal Reserve Bank of St. Louis
Household credit	Loans to households and non-profit institutions serving households	Federal Reserve Bank of St. Louis
Corporate credit	Loans and debt securities to non-financial corporations	Federal Reserve Bank of St. Louis
Bank credit	Loans and debt securities to non-financial private sector provided by domestic banks	Federal Reserve Bank of St. Louis

Figure B.1: Baseline Model Macro and Monetary Policy Shock Impulse Responses



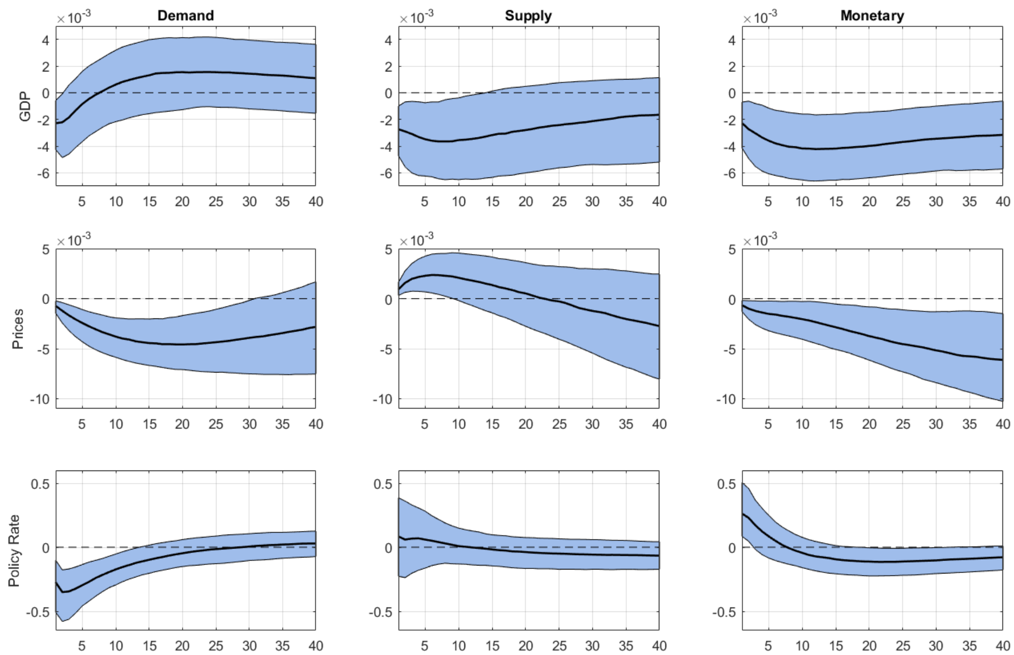
Notes: Impulse responses of GDP, prices, and the policy rate to a one-standard deviation adverse demand, supply, and monetary policy shock for the baseline model. The solid line depicts the posterior median response at each horizon while the shaded area indicates the 68% posterior probability region.

Figure B.2: Baseline Model Household and Corporate Credit Shock Impulse Responses



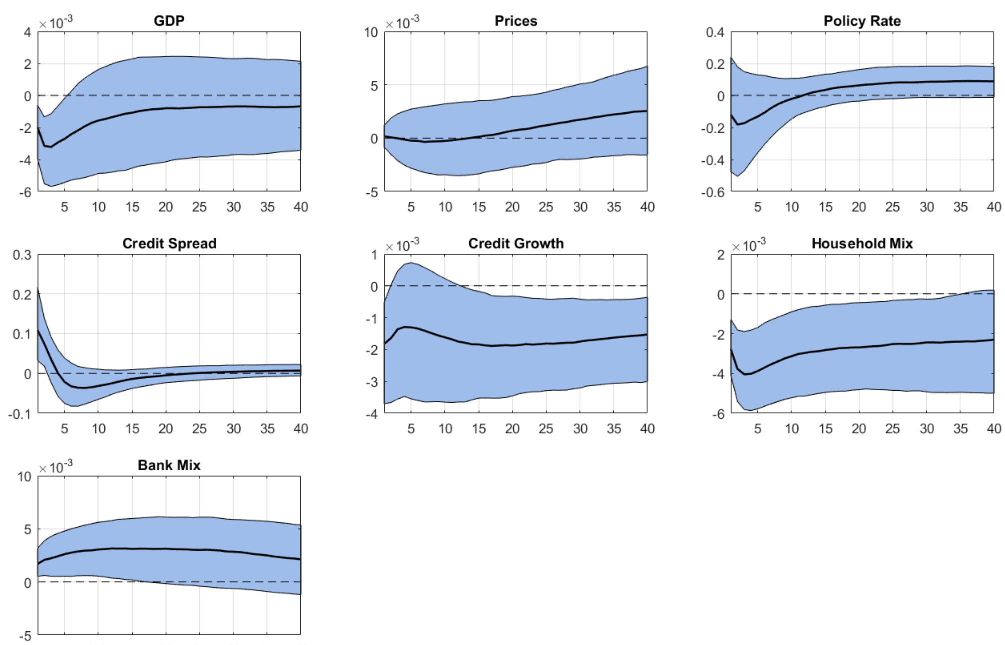
Notes: Impulse responses of GDP, prices, and the policy rate to a one-standard deviation adverse household and corporate credit supply shock for the baseline model. The solid line depicts the posterior median response at each horizon while the shaded area indicates the 68% posterior probability region.

Figure B.3: Extended Model Macro and Monetary Policy Shock Impulse Responses



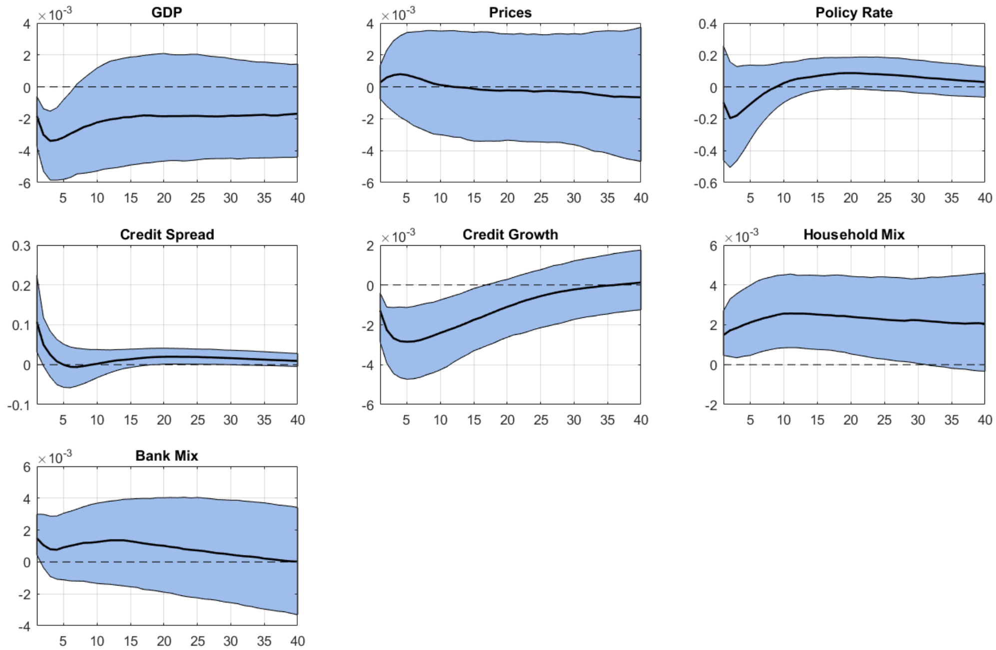
Notes: Impulse responses to a one-standard deviation adverse demand, supply, and monetary policy shock for the extended model and macro variables only. The solid line depicts the posterior median response at each horizon while the shaded area indicates the 68% posterior probability region.

Figure B.4: Extended Model Household Credit Shock Impulse Responses



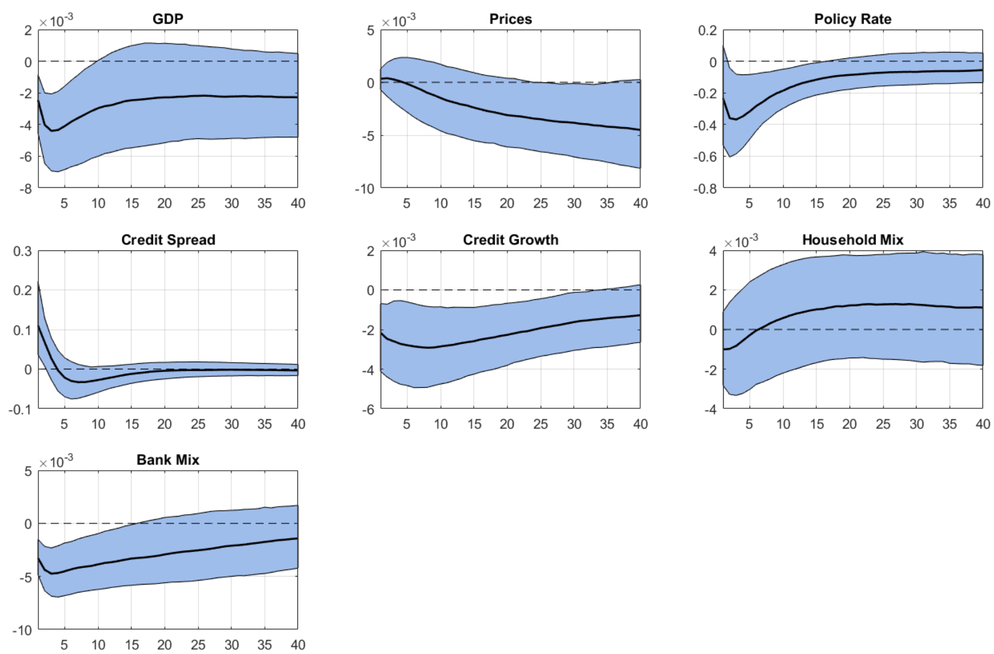
Notes: Impulse responses to a one-standard deviation adverse household credit supply shock for the extended model. The solid line depicts the posterior median response at each horizon while the shaded area indicates the 68% posterior probability region.

Figure B.5: Extended Model Corporate Credit Shock Impulse Responses



Notes: Impulse responses to a one-standard deviation adverse corporate credit supply shock for the extended model. The solid line depicts the posterior median response at each horizon while the shaded area indicates the 68% posterior probability region.

Figure B.6: Extended Model Bank Credit Shock Impulse Responses

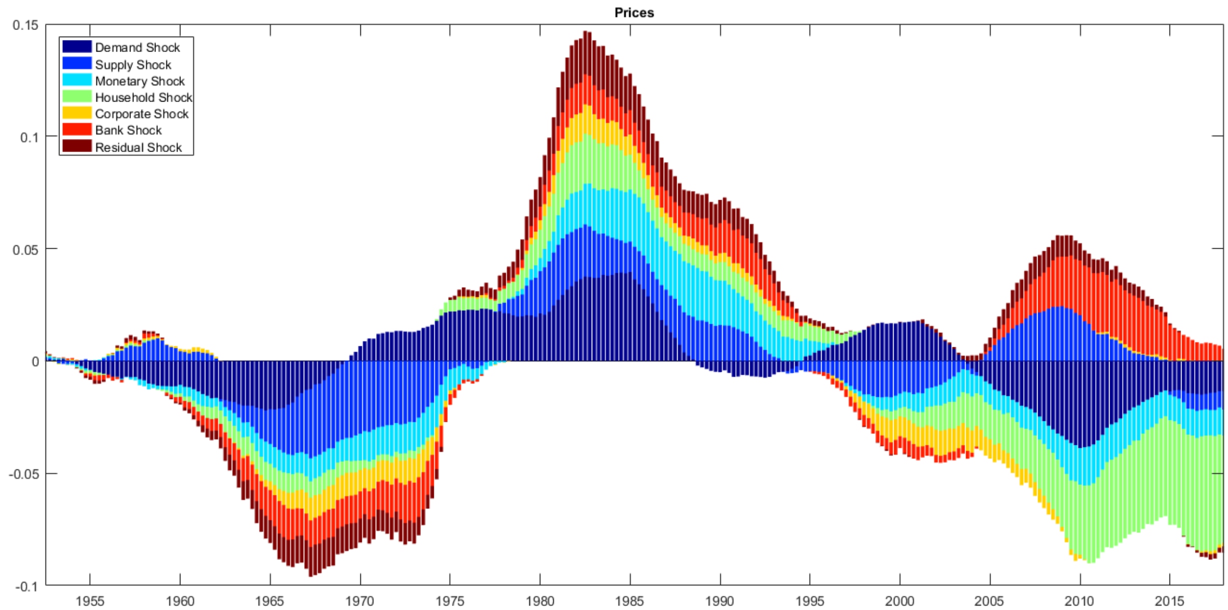


Notes: Impulse responses to a one-standard deviation adverse bank credit supply shock for the extended model. The solid line depicts the posterior median response at each horizon while the shaded area indicates the 68% posterior probability region.

Table B.2: Extended Model Forecast Error Variance Decomposition

	Horizon	Demand	Supply	Monetary	Household	Corporate	Bank	Residual
GDP	1	0.15	0.21	0.14	0.11	0.09	0.17	0.13
	4	0.07	0.15	0.14	0.14	0.15	0.25	0.11
	16	0.07	0.18	0.22	0.11	0.13	0.17	0.12
	32	0.08	0.17	0.23	0.10	0.13	0.16	0.12
Prices	1	0.13	0.22	0.11	0.13	0.15	0.13	0.13
	4	0.16	0.21	0.09	0.13	0.14	0.12	0.14
	16	0.29	0.12	0.11	0.11	0.12	0.13	0.12
	32	0.24	0.12	0.19	0.10	0.09	0.14	0.12
Policy Rate	1	0.16	0.10	0.15	0.15	0.15	0.17	0.13
	4	0.21	0.08	0.09	0.13	0.13	0.24	0.11
	16	0.22	0.09	0.10	0.11	0.12	0.24	0.11
	32	0.19	0.11	0.12	0.11	0.12	0.22	0.12
Prime Spread	1	0.16	0.11	0.22	0.12	0.11	0.12	0.15
	4	0.15	0.14	0.15	0.14	0.13	0.13	0.16
	16	0.20	0.12	0.11	0.15	0.12	0.14	0.15
	32	0.19	0.12	0.13	0.14	0.14	0.13	0.14
Credit Mix	1	0.22	0.21	0.11	0.11	0.05	0.15	0.14
	4	0.24	0.13	0.11	0.07	0.14	0.16	0.14
	16	0.21	0.09	0.11	0.09	0.17	0.21	0.13
	32	0.18	0.10	0.12	0.13	0.13	0.21	0.14
Household Mix	1	0.11	0.10	0.09	0.38	0.11	0.10	0.11
	4	0.12	0.09	0.09	0.41	0.09	0.09	0.10
	16	0.15	0.11	0.09	0.30	0.14	0.10	0.11
	32	0.14	0.15	0.10	0.24	0.14	0.11	0.12
Bank Mix	1	0.10	0.10	0.09	0.10	0.08	0.40	0.12
	4	0.10	0.10	0.11	0.10	0.04	0.41	0.14
	16	0.10	0.11	0.15	0.15	0.06	0.29	0.14
	32	0.12	0.13	0.14	0.17	0.08	0.22	0.14

Figure B.7: Price Level Historical Decomposition

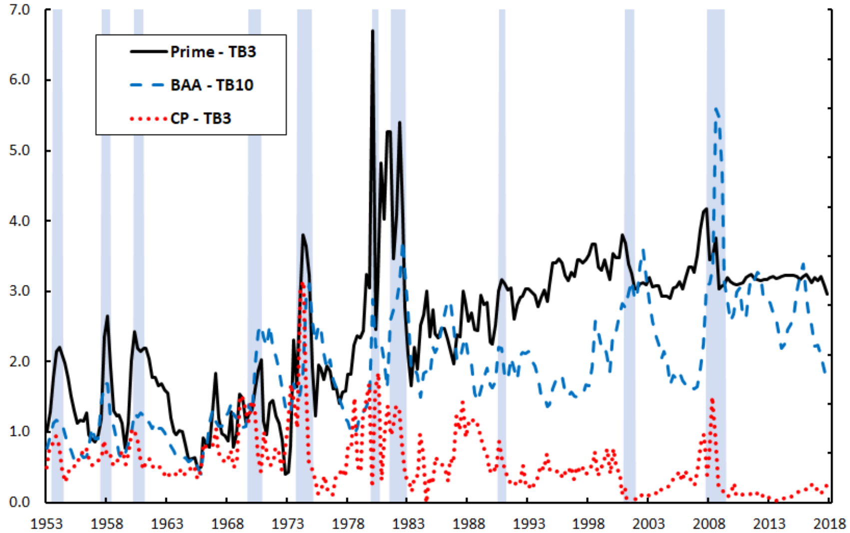


Notes: Historical decomposition of price level forecast errors for the extended model. The green bars correspond to the household credit supply shock, orange bars for corporate credit supply, and red bars for bank credit supply.

B.2 Robustness

In this section, a series of robustness exercises are discussed to further assess the validity of the model's econometric methodology and results. The baseline model's framework is altered in the following four ways to test its robustness: (i) replacing the prime rate spread with a corporate bond and commercial paper spread, (ii) estimating the model under two alternative identification strategies, (iii) extending the lag length out to three, four, and five quarters, and (iv) specifying two alternative priors. Keeping in line with the paper's primary interest, the robustness analysis will focus on the credit shocks and their consequences for GDP and inflation. Since the estimation time for the extended model with two lags exceeds one week, all robustness estimations are performed for the baseline specification with two lags and five shocks identified according to Table 2.1 in the main text.

Figure B.8: Alternative Credit Spreads



Notes: The prime (solid line), BAA corporate bond (dashed line), and AA commercial paper (dotted line) spreads. Y-axis units are percentage points.

B.2.1 Credit Spreads

In the baseline specification, the spread between the prime loan rate and 3-month Treasury rate is used as a proxy for measuring the relative price of credit in the private sector. When analyzing both household and corporate credit shocks within a single VAR, the prime spread is a useful indicator, because it's closely tied to the interest rates bank charge their corporate and non-corporate customers. However, in the VAR literature dealing with credit supply and financial sector shocks, corporate bond yields and spreads are commonly used as the proxy for private sector risk. Figure B.8 plots the prime spread, the corporate BAA bond and 10-year Treasury spread, as well as the 3-month AA commercial paper and 3-month Treasury spread. While these spreads do rise and fall together on occasion, such as during the recessions in 1973, 1980, and 2008, the magnitude and timing of their movements vary substantially for the majority of the sample. For example, the prime spread was highly active during the early 1950s-1960s while the corporate spreads still fluctuated but with less volatility. In contrast, the bond spread displays higher volatility during the 1969 and

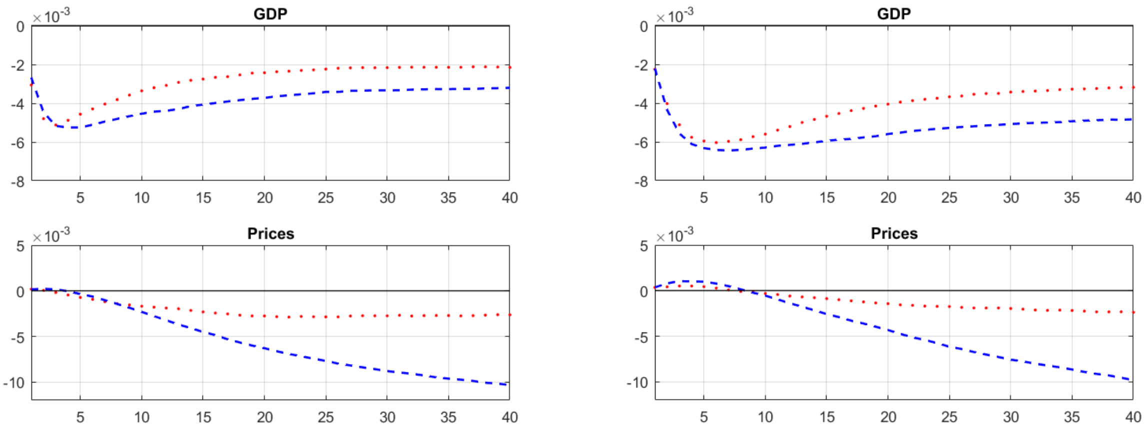
2008 recessions. Overall, the time series of these spreads suggest they may be responding to different types of risks arising in the private sector.

While many studies only identify a single aggregate credit or financial shock, arguments have been made in defense of using bond spreads to measure the relative risk of lending to households as well. When studying the effects of credit supply shocks to households and firms within two separate VARs, Duchi and Elbourne (2016) use the corporate bond spread in both VARs as a general proxy for the willingness of banks to bear risk and supply credit to the private sector. They argue that since banks play a major role as the market-makers for corporate bonds, bond yields can reflect their overall risk-taking appetite for lending to all customers.

As a robustness check, I run two separate estimations of the baseline model: one with the commercial paper (CP) spread and one with the corporate bond (CB) spread. The impulse responses for GDP and inflation are displayed by Figure B.9. For the CB spread, the GDP and inflation responses are nearly identical to those of the baseline. The only subtle difference is that inflation rises marginally on impact for the baseline. However, the GDP response for the CP spread does depart from the baseline in terms of a smaller initial drop, but deeper and more persistent contraction. Both shocks lower GDP to around -0.6% before recovery begins after eight quarters. This is compared to a drop of -0.5% and recovery after three quarters for the prime and CB spreads.

Results from the CB spread's FEVD are also similar to the baseline for GDP and inflation and differ by only 1-2 percentage points at all horizons. However, the CP spread's FEVD shows that corporate credit shocks explain considerably more GDP variation than in the baseline. Compared to 28% and 30% at horizons of one and eight years, respectively, corporate shocks explain 33% and 40% of GDP under the CP spread. At those horizons, household credit also contributes more than the baseline with 29% and 26% compared to 28% and 19%. The FEVD for inflation changes marginally with household credit explaining 3% more after one

Figure B.9: Alternative Credit Spread Impulse Responses



(a) BAA - TB10 (b) CP3 - TB3

Notes: Impulse responses to a one-standard deviation adverse household (dotted line) and corporate (dashed line) credit supply shock with alternative credit spreads. Panel (a) is for the corporate bond spread and Panel (b) is for the commercial paper spread. Estimates based on median response.

quarter and corporate credit explaining 6% less after eight years.

Overall, these exercises suggest the median contributions of credit shocks for GDP might be overstated using the CP spread. Looking at Figure B.8, the prime spread displays more volatility than the CP spread during some of the non-recessionary periods in the sample. Also, the CP spread appears to follow a relatively flat long-run trend while the prime spread follows an overall upward trend beginning around 1970. More frequent spikes in the prime spread may be associated with more small-scale GDP fluctuations than those for the CP spread. This suggests the prime spread might be more sensitive to minor credit market disruptions and could possibly account for its lower median contributions.

B.2.2 Identification

As discussed in Section 2.3.3 of the main text, the identification strategy imposes no timing restrictions and remains agnostic to the inflation and policy rate response to credit shocks.

Even though more ambiguity surrounds the theoretical literature on credit and timing restrictions, many SVAR studies do impose them to delineate macro and credit shocks. Studies such as Ciccarelli et al. (2010), Peersman (2012), Barnett and Thomas (2014), Peersman and Wagner (2015), and Duchi and Elbourne (2016) assign zero restrictions on the response of GDP and inflation, while Busch et al. (2010) and Halvorsen and Jacobsen (2014) assign them on inflation only. Within the theoretical literature, results for the response of inflation to credit supply shocks vary. Christiano et al. (2010), Curdia and Woodford (2010), Gertler and Karadi (2011), and Gilchrist and Zakrajšek (2011) find a procyclical inflation response while Atta-Mensah and Dib (2008) and Gerali et al. (2010) find a countercyclical response. From the baseline and extended specifications, the impulse responses for prices in Figures 2.2 and 2.4 do show a near-zero response to credit shocks on impact. Taking these results into account, one alternative identification strategy to test for robustness is assigning zero restrictions on the inflation response. While I argue that leaving inflation unrestricted when confronted with ambiguous theory is a superior approach, imposing zero restrictions on its response is more justified than for GDP or the policy rate.

By setting the price response to zero for the first quarter only, the credit supply shocks are already exclusively identified and no additional sign restrictions are required to separate them from the non-credit shocks. Imposing as few restrictions as possible without compromising accuracy, I remove the restrictions on the credit mix for demand and supply shocks, as well as those on the prime spread for the monetary shock. However, the restrictions on the credit mix and spread for credit supply remain as the price and volume response following credit supply shocks is theoretically consistent. Otherwise, the model may identify shocks not exclusive to credit supply disturbances.

In addition to zero restrictions, I also check robustness with a second identification strategy that imposes a negative policy rate response to credit shocks on impact. While not fully consistent in theory due to the uncertain inflation response, this restriction assumes the central bank can accommodate the observed GDP contraction following credit shocks within the quarter. The impulse responses for the baseline and extended models displayed by Figures B.2 and B.4-B.6 also show that while the median policy rate response is negative on impact, the 68% probability region still spans positive space. In fact, the probability region never lies below zero for household and corporate credit in the extended model. Studies taking this identification approach include Peersman (2012), Halvorsen and Jacobsen (2014), Gambetti and Musso (2017), and Mumtaz et al. (2018).

While not displayed, the impulse responses under both these identifications show little deviation from the baseline. The only subtle differences are that for household credit, GDP and inflation are marginally higher than the baseline after 40 quarters. For corporate credit, the GDP recovery doesn't begin until closer to the sixth quarter instead of the fifth, and the inflation response dips slightly below -1% after 40 quarters.

The FEVD for GDP shows that the two credit shocks contribute less than they do under the baseline specification. For zero restrictions, credit shocks explain between two and six percentage points less than the baseline in the short and long-run, respectively. For the policy rate restriction, credit shocks explain even less ranging between 2% and 7% in the short- and long-run. Considering the policy rate falls on impact, the heightened monetary accommodation is likely responsible for credit shocks playing a more subdued role for GDP fluctuations. However, since the paper's primary results regarding the contributions from credit supply shocks fall inside the one-year horizon, they remain within two percentage points of those under both alternative identifications.

I'll now briefly discuss the FEVD for inflation. As would be expected under zero restrictions, credit shocks explain almost no price variation within the first year. After four years, household credit accounts for only 7% of the variation which is 5% less than in the baseline. Interestingly, corporate credit accounts for 5% less after four years, but 4% more after eight years. For the policy rate restriction, both household and corporate credit explain marginally more price variation in the short-run, and corporate credit explains 5% more than the baseline after eight years. Under both alternative strategies, these results for corporate credit reiterate the paper's finding that credit supply shocks may hold long-run implications for inflation. Recalling the IRFs for the extended model, credit shocks from the banking sector accounted for the majority of the long-run price decline that was originally associated with corporate credit in the baseline.

While it's understood that price and interest rate movements can be critical for business cycles, the FEVD's sensitivity to both alternative identification strategies is not surprising. However, zero restrictions and recursive orderings can potentially generate significant bias for credit supply shocks as discussed in Gambetti and Musso (2017). Considering identification strategies should have strong theoretical underpinnings, the sign restrictions in this paper's baseline and extended specifications are appealing by assigning more weight to consensus and remaining agnostic where the theory is weak. Also, since the FEVD results at the one-year horizon vary little from the paper's primary conclusions, I argue in favor of leaving inflation and the policy rate unrestricted for the credit shocks.

B.2.3 Lag Length

While the Bayesian, or Schwarz Information Criterion (SIC), suggests three lags for the baseline model and two lags for the extended model with bank credit, two lags are used in the paper for comparability. However, alternative information criteria including the Akaiki, Final Prediction Error, and Hannan-Quinn suggest up to five lags for the baseline. As robustness checks, the baseline model is estimated with lag lengths of three, four, and five.

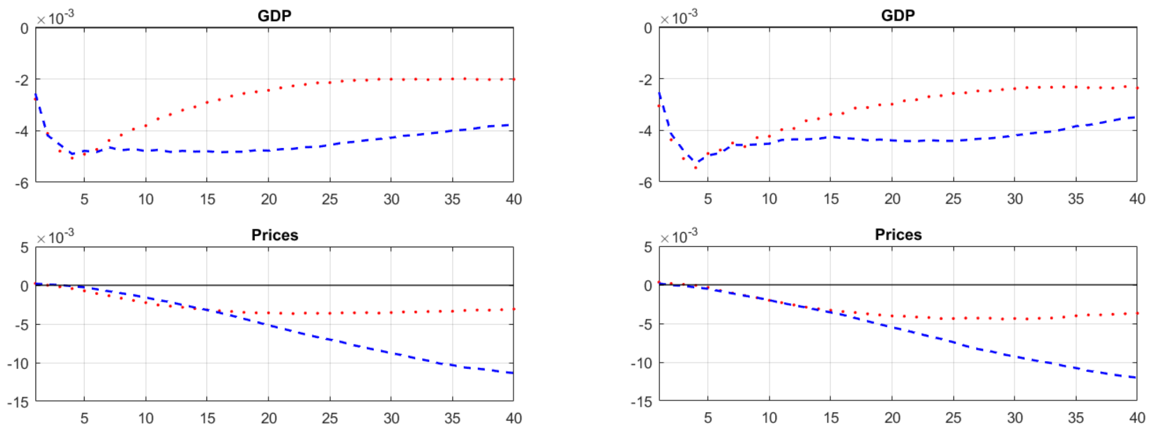
For three lags, the impulse responses are nearly identical to those with two lags. The only noticeable differences come from the FEVD for inflation. The corporate credit shock becomes marginally less important by about two percentage points at each horizon. Also, demand shocks explain only 32% of price-level variation instead of 40% after four years, while supply shocks explain 31% instead of 24% after one year. Overall, changes are minimal for the two credit shocks with one additional lag.

The impulse responses for GDP and inflation with four and five lags are displayed by Figure B.10. The responses are still similar to the baseline, however, GDP does experience a more delayed recovery following the corporate credit shock. Following the shock, GDP remains below -0.4% for up to 15 quarters in the baseline while it takes close to 35 quarters with the extended lag lengths. Both shocks also produce a marginally stronger negative impact on inflation. As for output's variance decomposition, with five lags household credit contributes an additional 3-5% at each horizon while corporate credit explains 1-2% less. Also, the role for monetary policy shocks and output slightly decreases with lag length. The FEVD results for inflation and credit shocks display little variation away from the baseline. The only change is that both shocks explain 1-2% less in the short-run with household shocks explaining 3-4% more after four years and five lags. To summarize, increasing lag length assigns more responsibility to household credit for GDP fluctuations and a slower recovery for corporate credit. However, these changes are marginal and elicit a negligible impact on the paper's primary conclusions.

B.2.4 Prior Selection

The baseline and extended models are estimated using the dummy observation prior with 2,000 iterations and a burn-in of 1,000. Remaining consistent with the literature, hyperparameters are set such that each variable's own first lag coefficient is 0.8, the lag decay is 2, and the overall tightness around the random walk process is 0.1. A tightness value of zero sets the posterior equal to the prior with no information coming from the data while larger values set the posterior closer to OLS estimates. According to Banbura et al. (2010),

Figure B.10: Alternative Lag Length Impulse Responses



(a) 4 lags

(b) 5 lags

Notes: Impulse responses to a one-standard deviation adverse household (dotted line) and corporate (dashed line) credit supply shock with extended lag lengths. Estimates based on median response.

the estimation and empirical accuracy of VARs with as few as six variables can be improved upon by implementing Bayesian shrinkage techniques and tighter priors. It follows that prior tightness should increase with the number of variables included and is especially critical for macroeconomics time series where collinearity is often present.

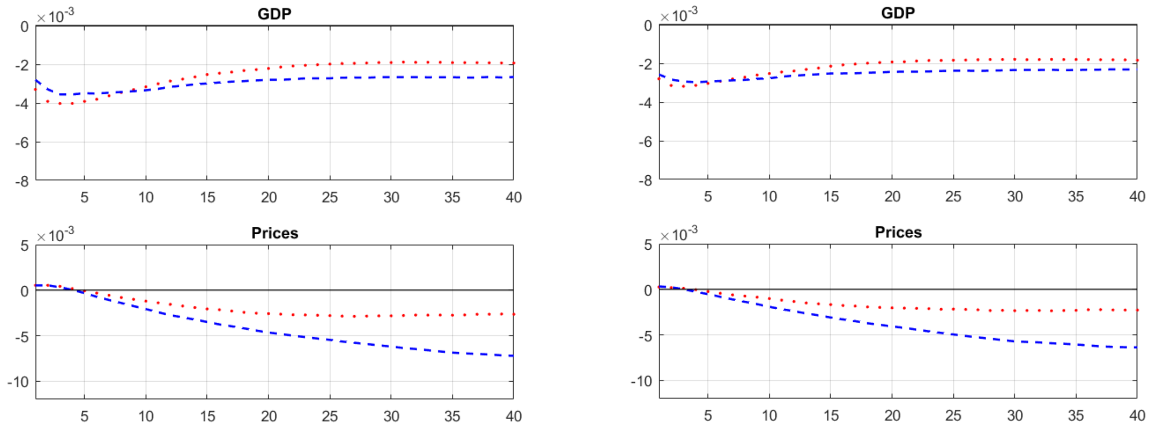
Beginning with diffuse priors on β and Σ, the prior introduces dummy, or pseudo, observations for each regression coefficient to match the moments of the popular Minnesota prior. It's a method of indirectly specifying the same distribution via dummies instead of directly imposing restrictions on the variance-covariance matrix Σ. The Minnesota prior asserts the belief that an independent random-walk process is a reasonable center for the time series dynamics of each variable. However, with respect to the baseline model and five shocks, the Minnesota prior requires the inversion of a (72 x 72) matrix while implementing dummy observations reduces this to only a (12 x 12) matrix. This reduces computation time and makes the estimation considerably more tractable for numerical softwares. As opposed to the Minnesota, dummy observations allow for prior covariance between the VAR coefficients in each equation which becomes critical when using interest rates and spreads of similar maturity, as well as when variables enter the model by themselves and inside mix variables. The prior can then be extended by implementing the "initial dummy observation" or "sum-of-coefficients" extensions to better handle unit root and co-integration processes which become problematic for larger VARs and variables that enter in levels. However, since no unit roots are present for the baseline or extended model estimations, I do not augment the dummy prior with either extension.

Robustness to prior choice is assessed by estimating the baseline with two alternative priors: the normal-Wishart and normal-diffuse. All hyperparameters remain specified according to the baseline's dummy observation prior, except the normal-diffuse introduces cross-variable variance taking a common value of 0.5. The normal-Wishart prior is a natural conjugate that takes the variance-covariance matrix Σ as unknown in contrast to the Minnesota. It still assumes a multivariate normal distribution for the VAR coefficients, but an inverse-Wishart distribution for Σ. It restricts Σ to be diagonal with its elements parameterized according to the residual variance of individual AR models estimated on each variable. Since the prior imposes a Kronecker structure on the prior for the coefficients, β, it creates dependence between the residual variance and coefficients in each equation.

Alternatively, when the researcher wants to remain even more agnostic with respect to their beliefs on Σ, the normal-diffuse can be used. It combines the multivariate normal prior on the parameters of the Minnesota with a diffuse prior on Σ known as Jeffrey's prior. This allows Σ to be non-diagonal imposing less prior information than the normal-Wishart. The Jeffrey's prior is improper as it integrates to infinity instead of one leading to an improper or even bimodal posterior. In contrast to the Minnesota and normal-Wishart, draws cannot be made directly from the posterior and numerical methods such as Gibbs sampling must be employed instead. Also, the two alternative priors both rely on the Minnesota structure which does not allow for prior covariance among coefficients.

Figure B.11 displays the impulse responses to household and corporate credit shocks under

Figure B.11: Alternative Prior Specification Impulse Responses



(a) normal-Wishart (b) normal-diffuse

Notes: Impulse responses to a one-standard deviation adverse household (dotted line) and corporate (dashed line) credit supply shock with alternative prior specifications. Estimates based on median response.

each alternative prior. Compared to the baseline, GDP is less sensitive to credit shocks for both, while being the least sensitive under the normal-diffuse. In the baseline, GDP drops to around -0.5%, while here it doesn't dip below -0.4%. In addition, under both alternatives household shocks elicit a larger initial drop in GDP before recovering marginally quicker. The general recovery paths are similar across both alternatives. The inflation response is also close to the baseline, except the price level remains less than 0.75% below steady-state under both alternatives compared to 1% below steady-state originally.

Transitioning to the FEVD, household credit explains 3% more of the short-run variation in GDP under the normal-Wishart, but explains 5% less under the normal-diffuse. At horizons beyond four years, estimates are mostly unchanged with 1-2% more and 2-3% less for the normal-Wishart and diffuse priors, respectively. For corporate credit, the baseline FEVD estimated a contribution of 31% after four years while here it contributes no more than 24% for the normal-Wishart and 21% under the diffuse at any horizon. Estimates within the first quarter remain unchanged. Under the normal-Wishart, credit shocks are still the most important shocks for GDP fluctuations overall. However, supply shocks become dominant for the normal-diffuse prior explaining up to 23% of GDP fluctuations after four years compared to 19% and 21% for the two credit shocks. As for inflation, results are nearly unchanged. Corporate credit explains a couple percentage points less under both alternatives while household credit explains marginally more under the normal-Wishart.

While the GDP contributions for corporate credit appear to be more sensitive to prior choice than for household credit, they are still up to six percentage points higher than what is estimated for the extended model. It's also worth commenting on the FEVD for monetary policy shocks. The results under both alternative priors assign a larger role to monetary shocks that are closer in magnitude to the extended model's estimates. Considering this, it's possible the monetary shock's contribution may not be as sensitive to the inclusion of bank credit, as it's already being picked up by the five shock specification under alternative priors. If this were the case, including the bank credit shock under the alternative priors may not significantly alter the FEVD estimates for credit supply contributions and maintain conclusions closer to those made for the extended model.

More generally, the Minnesota, normal-Wishart, and normal-diffuse do not allow prior covariance among the coefficients. Within the context of large BVARs, I argue in favor of the dummy observation prior for not only introducing covariance, as is common for macro-finance analyses, but also for reducing dimensionality and computation time. It's important to consider that the strength of Bayesian estimation comes from the ability to supplement the data with private beliefs for improving estimation and accuracy. The more diffuse priors are, the closer estimates are to OLS and MLE which can become bias in large VARs. As argued in Ni and Sun (2003) and Banbura et al. (2010), using Bayesian estimators with shrinkage priors can dominate MLE and Bayesian estimators with diffuse priors. Therefore, dummy observations are a convenient prior choice for overcoming the Minnesota's limitations while maintaining its advantages in a full Bayesian context.

Appendix C

C.1 Supplementary Tables

Table C.1: Chapter 3 Data and Sources

Variable	Description	Source
GDP	Log of real GDP	Federal Reserve Bank of St. Louis
GDP Deflator	Log of GDP price index	Federal Reserve Bank of St. Louis
Policy Rate	Federal funds rate	Federal Reserve Bank of St. Louis
Credit Card Loans	Log of real household & nonprofit organizations revolving consumer credit	Federal Reserve Bank of St. Louis
Small Business Loans	Log of real nonfinancial noncorporate business depository institution loans	Federal Reserve Bank of St. Louis

C.2 Proofs and Derivations

C.2.1 Proof of Lemma 3.1

Consider when households are unconstrained. The solution to (3.24) within the context of consumer lending specifies a loan contract with $q = q^*$ and $\phi_h = \theta_h[u(q^*) - q^* - \Delta(z_h)]$.

When constrained, $\bar{b}_h < b_h^* = (1-\theta_h)q^* + \theta_h u(q^*)$ and $z_h < z_h^*$, where z_h^* solves (3.27) when $\chi = 0$. The solution to (3.24) is given by

$$(1-\theta_h)q + \theta_h u(q) = z_h + \bar{b}_h + \theta_h \Delta(z_h), \tag{C.1}$$

where z_h and $\bar{b}_h$ are given by (3.34) and (3.37), respectively. To show uniqueness in a constrained equilibrium, begin by setting $q = 0$. As $q \to q^*$, the left side of (C.1) increases from 0 to b_h^*. For a given $0 \leq \bar{b}_h < b_h^*$ and $0 \leq z_h < z_h^*$, the right side of (C.1) is less than $\bar{b}_h$. This insures the left side will eventually surpass the right side before $q = q^*$, thus, there exists a unique $q > 0$ solving (3.24).

I now show uniqueness of the firm's bargaining solution to (3.24). After rearranging (3.29), the constrained solution to the firm's bargaining problem is given by

$$(1-\theta_f)k + \theta_f f(k) = \chi f(k) + z_f + \bar{b}_f + \theta_f \Delta(z_f), \tag{C.2}$$

where z_f and $\bar{b}_f$ are also given by (3.34) and (3.37), respectively. Without any loss in generality, k can be restricted to the compact interval $[\underline{k}, k^*]$, where $\underline{k}$ solves $\chi f'(\underline{k}) = 1$. Regardless of z_f and $\bar{b}_f$, for any $k < \underline{k}$ a Pareto improvement can be made by increasing k to $\underline{k}$ as the surpluses of the firm and bank are both strictly increasing over that range.

Assume $z_f + \bar{b}_f > 0$. At $k = 0$, the left side of (C.2) is zero while the right side is strictly positive. As $k \to k^*$, the left side of (C.2) increases at rate $(1-\theta_f) + \theta_f f'(k)$ while the right side increases at rate $\chi f'(k)$. Considering firms are constrained by $\chi < \chi^*$, $\bar{b}_f < b_f^*$, and $z_f < z_f^*$, the left side of (C.2) will eventually surpass the right side as long as $(1-\theta_f) > (\chi - \theta_f)f'(k)$. This condition holds for all $k \in [0, k^*]$. □

C.2.2 Proof of Lemma 3.2

Households receive $\Delta(z_h) = u(q^m) - q$ with $q^m = \min\{z_h, q^*\}$ in the event they receive a consumption shock, but are unbanked with probability $\sigma(1-\alpha\mu_h)$. With probability $\sigma\alpha\mu_h$, the household receives a consumption shock and meets a bank where they capture the share $(1-\theta_h)$ of the total match surplus $u(q)-q-\Delta(q^m)$ from a credit match, where q is a function of z_h and $\bar{b}_h$. I check that for $i > 0$, borrowers never carry enough cash to purchase q^* directly from producers which implies $q^m = z_h < q^*$. The household chooses their real balances to maximize their expected surplus from bringing cash into Stage 1 of the following period, net of the opportunity cost of holding money, $i \equiv (1+\pi)(1+\rho)-1$. This optimization problem is written as

$$\underset{z_h \geq 0}{max} \quad \Big\{ -iz_h + \sigma\alpha\mu_h(1-\theta_h)\Delta(q) + \sigma\big[1-\alpha\mu_h(1-\theta_h)\big]\Delta(z_h)\Big\}.$$

Deriving the firm's optimal portfolio choice follows the same methodology as that for households, except with $(\lambda, \mu_f, \theta_f, k, z_f)$ replacing $(\sigma, \mu_h, \theta_h, q, z_h)$. □

C.2.3 Proof of Lemma 3.3

For $i > \bar{\imath}_h$ and $\bar{b}_h > 0$, households have a positive debt limit, but are still constrained by their liquidity holdings such that $\bar{b}_h < b_h^*$ and $z_h < z_h^*$. It follows that $q^* > q > \tilde{q}$, where $\tilde{q} = \tilde{z}_h$ represents the defaulting household's consumption that satisfies $\Delta'(\tilde{z}_h) = i/\sigma$. To determine how z_h responds to an increase in $\bar{b}_h$, consider the constrained solution to (3.32) given by

$$i = \sigma\alpha\mu_h(1-\theta_h)\Delta'(q)\big[1+\theta_h\Delta'(z_h)\big] + \sigma\big[1-\alpha\mu_h(1-\theta_h)\big]\Delta'(z_h),$$

where $\Delta'(q) = \big[u'(q) - 1\big]/\big[1 - \theta_h + \theta_h u'(q)\big] > 0$. Taking $\bar{b}_h$ as exogenous, the household's liquidity premium can be differentiated to obtain

$$\frac{\partial z_h}{\partial \bar{b}_h} = -\left\{1 + \theta_h \Delta'(z_h) + \frac{\Big(1 - \alpha\mu_h(1-\theta_h)\big[1 - \theta_h\Delta'(q)\big]\Big)\Delta''(z_h)}{\alpha\mu_h(1-\theta_h)\big[1+\theta_h\Delta'(z_h)\big]\Delta''(q)}\right\}^{-1} \tag{C.3}$$

$$\in \Big(-1/\big[1+\theta_h\Delta'(z_h)\big], 0\Big),$$

where $\Delta''(z_h) = u''(z_h) < 0$ and

$$\Delta''(q) = \frac{u''(q)}{\big[1 - \theta_h + \theta_h u'(q)\big]^3} < 0.$$

Let $\Psi_h = z_h + \theta_h \Delta(z_h) + \bar{b}_h$ denote the household's total financing capacity comprised of their down payment, captured share of $\Delta(z_h)$ via interest fees, and debt limit. This corresponds to the right side of (3.29) rewritten within the context of consumer lending. Differentiating with respect to $\bar{b}_h$, it follows that $d\Psi_h/d\bar{b}_h = \big[1 + \theta_h\Delta'(z_h)\big]dz_h/d\bar{b}_h + 1$. Considering (C.3), households respond to a higher debt limit with a reduction in real balances that is just strong enough to ensure $d\Psi_h/d\bar{b}_h > 0$ and therefore $dq/d\bar{b}_h > 0$.

For firms, the proof for showing $d\Psi_f/d\bar{b}_f > 0$ is analogous to that for households above. The only difference is that $\Delta'(k)$ and $\Delta''(k)$ depend on pledgeability. To see this, differentiate $\Delta(k) = f(k) - k$ and $\Delta'(k)$ to obtain

$$\Delta'(k) = \frac{f'(k) - 1}{1 - \theta_f + (\theta_f - \chi_b)f'(k)} > 0;$$

$$\Delta''(k) = \frac{(1-\chi)f''(k)}{\big[1 - \theta_f + (\theta_f - \chi_b)f'(k)\big]^3} < 0.$$

□

C.2.4 Proof of Lemma 3.4

A necessary condition for $b_f^{max} > 0$ to be feasible is given by (3.38). To show that there exists a $\hat{k} < \overline{k}$ such that (3.38) holds, $\Delta'(k)$ can be used to rewrite it as

$$\frac{\lambda\alpha\mu_f(1-\theta_f)\big[f'(\hat{k})-1\big]}{1-\theta_f+(\theta_f-\chi_b)f'(\hat{k})} > \rho.$$

Solving for $f'(\hat{k})$, one can obtain

$$f'(\hat{k}) > \frac{(1-\theta_f)[\rho+\lambda\alpha\mu_f]}{\rho(\chi-\theta_f)+\lambda\alpha\mu_f(1-\theta_f)} \equiv f'(\overline{k}),$$

where $\rho(\chi-\theta_f)+\lambda\alpha\mu_f(1-\theta_f) > 0$ for $\chi \in (0,1)$ and $\lambda\alpha\mu_f(1-\theta_f) > \rho\theta_f$. Using $f'(k^*) = 1$ and $f'(\underline{k}) = 1/\chi$, one can then show that $\underline{k} < \overline{k} < k^*$. Given the concavity of $f'(k)$, it follows that there exists a $\hat{k} < \overline{k} \in (\underline{k}, k^*)$ such that (3.38) holds and $b_f^{max} > 0$. $\square$

C.2.5 Proof of Lemma 3.5

Following the steps outlined in Section 3.4.4 to derive $\bar{b}_f$, the consumer debt limit $\bar{b}_h$ solves

$$\rho\bar{b}_h \leq \max_{z_h\geq 0}\Big\{-iz_h+\sigma\alpha\mu_h(1-\theta_h)\Delta(q)+\sigma\big[1-\alpha\mu_h(1-\theta_h)\big]\Delta(z_h)\Big\}$$
$$-\max_{\tilde{z}_h\geq 0}\Big\{-i\tilde{z}_h+\sigma\Delta(\tilde{z}_h)\Big\}. \tag{C.4}$$

To perform comparative statics for $\bar{b}_j$, debt limits must be "not-too-tight" such that $\rho\bar{b}_j = \Omega(\bar{b}_j)$ with $\bar{b}_j = b_j^{max}$. I begin by showing $\bar{b}_j$ is positively related to the credit market matching and loan approval rates, α and μ_j, respectively. Starting with households, differentiate (C.4) at equality with respect to α to obtain

$$\rho d\bar{b}_h = \sigma\mu_h(1-\theta_h)[\alpha\Delta'(q)d\bar{b}_h + \Delta(q)d\alpha - \Delta(z_h)d\alpha].$$

After dividing through by $d\alpha$, one can obtain

$$\frac{d\bar{b}_h}{d\alpha} = \frac{\sigma\mu_h(1-\theta_h)[\Delta(q)-\Delta(z_h)]}{\rho - \sigma\alpha\mu_h(1-\theta_h)\Delta'(q)} > 0,$$

where $\Delta(q) > \Delta(z_h)$ and $\rho > \sigma\alpha\mu_h(1-\theta_h)\Delta'(q) = \Omega'(\bar{b}_h)$ which holds as $\rho\bar{b}_h$ intersects $\Omega(\bar{b}_h)$ at $\bar{b}_h = b_h^{max}$. The household's debt limit is an increasing function of α.

The same methodology can be applied for firms and $\bar{b}_f$. After differentiating (3.37) at equality with respect to α and rearranging, one can obtain

$$\frac{d\bar{b}_f}{d\alpha} = \frac{\lambda\mu_f(1-\theta_f)[\Delta(k)-\Delta(z_f)-\Delta(\hat{k})+\Delta(\tilde{z}_f)]}{\rho - \lambda\alpha\mu_f(1-\theta_f)\Delta'(k)} > 0,$$

where $\Delta(k) - \Delta(z_f) > \Delta(\hat{k}) - \Delta(\hat{z}_f)$ for all $k > \hat{k}$. The firm's debt limit is an increasing function of α. Due to symmetry, it follows that $d\bar{b}_h/d\mu_h > 0$ and $d\bar{b}_f/d\mu_f > 0$ can be obtained following the same methodology for α.

To show the debt limit is increasing with σ and λ, $\bar{b}_j$ can be differentiated with respect to each to obtain

$$\frac{d\bar{b}_h}{d\sigma} = \frac{\alpha\mu_h(1-\theta_h)\Delta(q) + \left[1-\alpha\mu_h(1-\theta_h)\right]\Delta(z_h) - \Delta(\tilde{z}_h)}{\rho - \sigma\alpha\mu_h(1-\theta_h)\Delta'(q)}; \tag{C.5}$$

$$\frac{d\bar{b}_f}{d\lambda} = \frac{\alpha\mu_f(1-\theta_f)\left[\Delta(k)-\Delta(\hat{k})\right] + \left[1-\alpha\mu_f(1-\theta_f)\right]\left[\Delta(z_f)-\Delta(\hat{z}_f)\right]}{\rho - \lambda\alpha\mu_f(1-\theta_f)\Delta'(k)}, \tag{C.6}$$

respectively. If $\alpha\mu_j(1-\theta_j) = 0$, then $\bar{b}_j = 0$, $\Delta(q) = \Delta(\tilde{q})$, and $\Delta(k) = \Delta(\hat{k})$. Thus, the numerators of (C.5) and (C.6) converge to zero as do $d\bar{b}_h/d\sigma$ and $d\bar{b}_f/d\lambda$. As $\alpha\mu_j(1-\theta_j)$ increases towards unity, the numerators in (C.5) and (C.6) become $\Delta(q) - \Delta(\tilde{z}_h)$ and $\Delta(k) -$

$\Delta(\hat{k})$. Given $q > \tilde{z}_h$ and $k > \hat{k}$ when $\bar{b}_j = b_j^{max}$, the debt limit is increasing in σ and λ.

Turning to bargaining power, one can differentiate $\bar{b}_h$ with respect to θ_h to obtain

$$\frac{d\bar{b}_h}{d\theta_h} = \frac{\sigma\alpha\mu_h\Big[\Delta(z_h) - \Delta(q) + (1-\theta_h)\Delta'(q)\frac{\partial q}{\partial \theta_h}\Big]}{\rho - \sigma\alpha\mu_h(1-\theta_h)\Delta'(q)} < 0,$$

where $\partial q/\partial\theta_h = \big[\Delta(z_h) - \Delta(q)\big]/\big[(1-\theta_h) + \theta_h u'(q)\big] < 0$. The effect of θ_f on $\bar{b}_f$ is less clear as θ_f has intensive margin effects for business defaulters. Differentiating $\bar{b}_f$ gives

$$\frac{d\bar{b}_f}{d\theta_f} = \frac{\lambda\alpha\mu_f\Big[\Delta(\hat{k}) - \Delta(k) + \Delta(z_f) - \Delta(\hat{z}_f)\Big] + \lambda\alpha\mu_f(1-\theta_f)\Big[\Delta'(k)\frac{\partial k}{\partial\theta_f} - \Delta'(\hat{k})\frac{\partial\hat{k}}{\partial\theta_f}\Big]}{\rho - \lambda\alpha\mu_f(1-\theta_f)\Delta'(k)}; \tag{C.7}$$

where $\partial k/\partial\theta_f = \big[\Delta(z_f) - \Delta(k)\big]/\big[(1-\theta_f) + (\theta_f - \chi_b)f'(k)\big] < 0$. Due to the concavity of $\Delta'(k)$, it follows that $\Delta'(k) < \Delta'(\hat{k})$ while $|\partial k/\partial\theta_f| > |\partial\hat{k}/\partial\theta_f|$. There can exist a $\bar{b}_f$ such $\Delta'(k)\partial k/\partial\theta_f - \Delta'(\hat{k})\partial\hat{k}/\partial\theta_f > 0$ and if large enough, can turn the numerator on the right side of (C.7) positive to yield $d\bar{b}_f/d\theta_f > 0$. However, this is less likely to hold for k in the neighborhood of k^* given $\Delta'(k^*) = 0$. Therefore, $\bar{b}_h$ decreases with θ_h while $\bar{b}_f$ may decrease or increase with θ_f.

A similar non-monotonicity can arise with respect to χ due to its simultaneous effects on the intensive margins for defaulting and non-defaulting firms' loan terms. Taking the derivative of $\bar{b}_f$ with respect to χ gives

$$\frac{d\bar{b}_f}{d\chi} = \frac{\lambda\alpha\mu_f\Big[\Delta'(k)\frac{\partial k}{\partial\chi} - \Delta'(\hat{k})\frac{\partial\hat{k}}{\partial\chi}\Big]}{\rho - \lambda\alpha\mu_f(1-\theta_f)\Delta'(k)},$$

where $\partial k/\partial\chi = f(k)/\big[\Delta(z_f) - \Delta(k)\big]/\big[(1-\theta_f) + (\theta_f - \chi_b)f'(k)\big] > 0$. With $\partial k/\partial\chi > \partial\hat{k}/\partial\chi$, there can exist a $\bar{b}_f$ eliciting a combination of k and $\hat{k}$ such that $\partial\bar{b}_f/d\chi > 0$ or < 0, and as a result, the effect of χ on $\bar{b}_f$ is non-monotonic.

The final two parameters to consider are ρ and i. Comparative statics with respect to both are straightforward as neither elicit extensive margin effects. For households, differentiating $\bar{b}_h$ with respect to ρ yields $d\bar{b}_h/d\rho = -\bar{b}_h/\big[\rho - \sigma\alpha\mu_h(1-\theta_h)\Delta'(q)\big] < 0$, while doing so for i yields

$$\frac{d\bar{b}_h}{di} = \frac{\tilde{z}_h - z_h}{\rho - \sigma\alpha\mu_h(1-\theta_h)\Delta'(q)} > 0, \tag{C.8}$$

where $\tilde{z}_h > z_h$. These effects hold for firms with $\hat{z}_f > z_f$ to show both debt limits are increasing in i and decreasing in ρ. □

C.2.6 Proof of Proposition 3.1

Similar to (3.33) and (3.34) for firms, the unconstrained and constrained liquidity premiums for households are given by

$$i = \sigma\big[1 - \alpha\mu_h(1-\theta_h)\big]\Delta'(z_h); \tag{C.9}$$

$$i = \sigma\alpha\mu_h(1-\theta_h)\Delta'(q)\big[1 + \theta_h\Delta'(z_h)\big] + \sigma\big[1 - \alpha\mu_h(1-\theta_h)\big]\Delta'(z_h), \tag{C.10}$$

respectively. Consider Part (i) when lending is unconstrained and $q = q^*$. Differentiating (C.9) with respect to i yields $dz_h/di = 1/\sigma\big[1-\alpha\mu_h(1-\theta_h)\big]u''(z_h) < 0$. With $\ell_f = q^* - z_h$ and q^* independent of i, it follows that $d\ell_f/di = -dz_h/di > 0$. Repeating for $\phi_h = \theta_h\big[\Delta(q^*) - \Delta(z_h)\big]$, one can show $d\phi_h/di = -[u'(z_h) - 1]dz_h/di > 0$.

Moving to Parts (ii) and (iii) for $q < q^*$, (C.10) can be differentiated with respect to i to obtain

$$\frac{dz_h}{di} = \Big\{\sigma\alpha\mu_h(1-\theta_h)\big[1 + \theta_h\Delta'(z_h)\big]^2\Delta''(q) + \sigma\big[1 - \alpha\mu_h(1-\theta_h)(1-\theta_h\Delta'(q)\big]\Delta''(z_h)\Big\}^{-1},$$

where $dz_h/di < 0$. Totally differentiating Ψ_h, q, ℓ_h, and ϕ_h with respect to i, one can obtain

$$\frac{d\Psi_h}{di} = \left(\frac{\partial \Psi_h}{\partial z_h}\frac{dz_h}{di} + \frac{\partial \Psi_h}{\partial \bar{b}_h}\frac{d\bar{b}_h}{di}\right) = \left[1 + \theta_h \Delta'(z_h)\right] \underbrace{\frac{dz_h}{di}}_{-} + \Bigg(\underbrace{1 + \left[1 + \theta_h \Delta'(z_h)\right]\frac{dz_h}{d\bar{b}_h}}_{+}\Bigg) \underbrace{\frac{d\bar{b}_h}{di}}_{+};$$

$$\frac{dq}{di} = \left(\frac{1}{1 - \theta_h + \theta_h u'(q)}\right)\frac{d\Psi_h}{di};$$

$$\frac{d\ell_f}{di} = \theta_h \Bigg(\underbrace{\frac{u'(z_h) - u'(q)}{1 - \theta_h + \theta_h u'(q)}}_{+}\Bigg)\frac{dz_h}{di} + \Bigg(\underbrace{\frac{1 + [1 + \theta_h \Delta'(z_h)]\frac{dz_h}{d\bar{b}_h}}{1 - \theta_h + \theta_h u'(q)}}_{+}\Bigg)\frac{d\bar{b}_h}{di};$$

$$\frac{d\phi_h}{di} = \theta_h \Bigg(\underbrace{\frac{u'(q) - u'(z_h)}{1 - \theta_h + \theta_h u'(q)}}_{-}\Bigg)\frac{dz_h}{di} + \theta_h \Delta'(q)\Bigg(\underbrace{1 + \left[1 + \theta_h \Delta'(z_h)\right]\frac{dz_h}{d\bar{b}_h}}_{+}\Bigg)\frac{d\bar{b}_h}{di} > 0.$$

Part (ii) of Proposition 3.1 assumes $\bar{b}_h < b_h^{max}$ and therefore $d\bar{b}_h/di = 0$. As a result, $d\Psi_h/di < 0$ and q and ℓ_f are decreasing; while ϕ_h is increasing with i when the debt limit subchannel is inactive.

Now consider Part (iii) when $d\bar{b}_h/di > 0$. To show that $d\Psi_h/di > 0$, recall $\bar{\imath}_h = \rho\sigma/\left[\sigma\alpha\mu_h(1-\theta_h) - \rho\theta_h\right]$ from (3.41). At $i = \bar{\imath}_h$, $\bar{b}_h = 0$ and $q = \tilde{q}$ solves $u'(q) = 1 + \bar{\imath}_h/\sigma$. Once $i > \bar{\imath}_h$, a necessary condition for $\bar{b}_h = b_h^{max} > 0$ is $\rho > \sigma\alpha\mu_h(1-\theta_h)\Delta'(q)$ which comes from $d\bar{b}_h/di$ and the denominator in (C.8). Solving for $u'(q)$, one can obtain

$$u'(q) < \frac{(1-\theta_h)\left[\rho + \sigma\alpha\mu_h\right]}{\sigma\alpha\mu_h(1-\theta_h) - \rho\theta_h} \equiv u'(\bar{q}), \tag{C.11}$$

where $\bar{q}$ solves $1 + \bar{\imath}_h/\sigma$ which is equivalent to $\tilde{q}$ when $i = \bar{\imath}_h$. Similar to Lemma 3.4 and $\bar{k}$ for firms, (C.11) implies $q > \bar{q}$ once $\bar{b}_h > 0$ and $d\bar{b}_h/di > 0$. Hence, $d\Psi_h/di > 0$ and q, ℓ_h, and ϕ_h are increasing with i when the debt limit subchannel is active. Considering $d\bar{b}_h/di > 0$ and taking the limit as $i \to i_h^*$ and $q \to q^*$, $dq/di > 0$ holds for all $i \in (\bar{\imath}_h, i_h^*)$. □

C.2.7 Proof of Proposition 3.2

The proof for Proposition 3.2 is analogous to that for Proposition 3.1 and consumer lending. When unconstrained, an increase in i reduces z_f rendering $d\ell_f/di = -dz_f/di > 0$ and $d\phi_f/di > 0$. When constrained with $\bar{b}_f > 0$ and $d\bar{b}_f/di > 0$, then $k > \bar{k}$ and firms adjust their real balances in the same manner as households leaving their total financing capacity, Ψ_f, larger following an increase in i. Therefore, the results from Parts (i) and (iii) of Proposition 3.1 carry over to firms for Parts (i) and (iii) of Proposition 3.2 with respect to k, ℓ_f and ϕ_f.

The one difference between Propositions 3.1 and 3.2 arises for Part (ii) and the non-monotonic response from ϕ_f to i due to $\chi > 0$. To illustrate analytically, differentiate $\phi_f = \theta_f\big[\Delta(k) - \Delta(z_f)\big]$ with respect to i with $d\bar{b}_f/di = 0$ to obtain

$$\frac{d\phi_f}{di} = \theta_f\left(\underbrace{\frac{(1-\chi)f'(k) - \big(1-\chi f'(k)\big)f'(z_f)}{1-\theta_f+(\theta_f-\chi)f'(k)}}_{+/-}\right)\frac{dz_f}{di}.$$

The sign on $d\phi_f/di$ depends on the sign taken by the numerator in the right side of $d\phi_f/di$. Solving $(1-\chi)f'(k) - \big(1-\chi f'(k)\big)f'(z_f) < 0$ for $f'(z_f)$, the following condition can be derived to ensure the numerator of $d\phi_f/di$ takes a negative sign:

$$f'(z_f) > \frac{(1-\chi)f'(k)}{1-\chi f'(k)}. \tag{C.12}$$

Setting $f'(z_f) = (1-\chi)f'(k)/\big[1-\chi f'(k)\big]$ and plugging into (3.34) to solve for $f'(k)$, (C.12) can be rewritten as $f'(z_f) > 1+i/\lambda$ where $f'(\tilde{z}_f) = 1+i/\lambda$ is the solution to (3.34) when firms lack secured and unsecured credit access. Hence, when $z_f < \tilde{z}_f$, the numerator in $d\phi_f/di$ is negative and $d\phi_f/di > 0$. The converse is true when $f'(z_f) < f'(\tilde{z}_f)$ and $z_f > \tilde{z}_f$. □

C.2.8 Proof of Corollary 3.1

Evaluating (3.27) at $\theta_f = 0$ gives $z_f^* = k^* - \chi f(k^*) - \bar{b}_f$. Beginning with Part (i), when $k = k^*$, z_f and z_f^* solve

$$i = \lambda(1 - \alpha\mu_f)\big[f'(z_f) - 1\big];$$

$$i_f^* = \lambda(1 - \alpha\mu_f)\big[f'(z_f^*) - 1\big],$$

respectively. When $i_f^* - i_f > 0$, but small, $z_f \approx z_f^*$. A first-order Taylor series approximation of $f'(z_f)$ in the neighborhood of $f'(z_f^*)$ yields

$$z_f \approx z_f^* + \frac{i - i_f^*}{\lambda(1 - \alpha\mu_f)f''(z_f^*)};$$

$$L_f \approx \lambda\alpha\mu_f\left[k^* - z_f^* - \frac{i - i_f^*}{\lambda(1 - \alpha\mu_f)f''(z_f^*)}\right],$$

where $i - i_f^* < 0$ and $f''(z_f^*) < 0$. Differentiating z_f and the natural logarithm of L_f with respect to i, one can obtain

$$\frac{dz_f}{di} \approx \frac{1}{\lambda(1 - \alpha\mu_f)f''(z_f^*)} < 0; \tag{C.13}$$

$$\frac{d\ln L_f}{di} \approx \frac{-1}{\lambda(1 - \alpha\mu_f)\big[k^* - z_f^*\big]f''(z_f^*) - (i - i_f^*)} > 0, \tag{C.14}$$

where $d\ln L_f/di = -1/\lambda(1 - \alpha\mu_f)[k^* - z]f''(z_f^*)$. From (C.13), one can show that $|dz_f/di|$ is increasing with α and μ_f; and decreasing with λ and χ. From (C.14), $d\ln L_f/di$ decreases with i and χ when ℓ_f is high. This follows from differentiating z_f with respect to i and χ to show $dz_f/di < 0$ and $dz_f/d\chi < 0$. The effects from λ, α and μ_f are non-monotonic and depend on their magnitudes relative to ℓ_f. For example, $d\ln L_f/di$ increases with λ if $\lambda dz_f/d\lambda > \ell_f$, or $d\ln L_f/di > 1/i$, which is most likely to hold when χ and ℓ_f are low; while z_f and $|dz_f/di|$ are high. In contrast, $|d\ln L_f/di|$ increases with α and μ_f when $d\ln L_f/di < 1/i$, which is more likely to hold when χ and ℓ_f are high; while z_f and $|dz_f/di|$ are low.

Now consider Part (ii) when $i_f - i_f^* > 0$, but small, with k and z_f determined by

$$k = \chi f(k) + z_f + \bar{b}_f;$$

$$i = \lambda\alpha\mu_f \frac{(1-\chi)f'(k)}{1-\chi f'(k)} + \lambda(1-\alpha\mu_f)\big[f'(z_f) - 1\big],$$

respectively. Linearizing these relationships in the neighborhood of $(k, z_f) = (k^*, z_f^*)$, one can obtain

$$z_f - z_f^* \approx (1-\chi)(k - k^*);$$

$$i - i_f^* \approx \lambda(1-\alpha\mu_f)f''(z_f^*)(z_f - z_f^*) + \lambda\alpha\mu_f \frac{f''(k^*)}{1-\chi}(k - k^*),$$

where $i_f^* = \lambda(1-\alpha\mu_f)\big[f'(z_f^*) - 1\big]$. Substituting $z_f - z_f^*$ into $i - i_f^*$, approximations for z_f, k, and L_f are given by

$$k \approx k^* + \frac{i - i_f^*}{D};$$

$$z_f \approx z_f^* + \frac{(1-\chi)(i - i_f^*)}{D};$$

$$L_f \approx \lambda\alpha\mu_f \left[k^* - z_f^* + \frac{\chi(i - i_f^*)}{D}\right],$$

respectively, where $D = \lambda\alpha\mu_f f''(k^*)/(1-\chi) + \lambda(1-\alpha\mu_f)(1-\chi)f''(z_f^*) < 0$. Differentiating the approximations for z_f and the natural logarithm of L_f with respect to i gives

$$\frac{dz_f}{di} \approx \frac{1-\chi}{D} + \left\{ \frac{(1-\chi)\lambda(1-\alpha\mu_f)}{D^2} \big[f''(z_f^*)D + (1-\chi)(i-i_f^*)f'''(z_f^*) \big] - 1 \right\} \frac{d\bar{b}_f}{di}; \tag{C.15}$$

$$\frac{d\ln L_f}{di} \approx \frac{\chi}{(k^*-z_f^*)D + \chi(i-i_f^*)} + \left\{ \frac{D^2 + \chi\lambda(1-\alpha\mu_f)\big[f''(z_f^*)D + (1-\chi)(i-i_f^*)f'''(z_f^*) \big]}{(k^*-z_f^*)D^2 + \chi(i-i_f^*)D} \right\} \frac{d\bar{b}_f}{di} > 0, \tag{C.16}$$

respectively. For $\bar{b}_f < b_f^{max}$, $d\bar{b}_f/di = 0$ and the second term on the right sides of (C.15) and (C.16) reduce to zero and $d\ln L_f/di < 0$. One can show that $|dz_f/di|$ is decreasing with λ and χ; while $|d\ln L_f/di|$ is decreasing with λ, but increasing with i. To determine the effects from χ, let $i - \bar{\imath}_f > 0$ be sufficiently small such that $\bar{b}_f \approx 0$ and $k^* - z_f^* \approx \chi f(k^*)$. Rewriting (C.16) as $d\ln L_f/di = \chi/\ell_f D$ and differentiating with respect to χ, it follows that $|d\ln L_f/di|$ decreases with χ when $dD/d\chi < -\lambda(1-\alpha\mu_f)f''(z_f^*)$. This condition always holds after differentiating D to obtain

$$\frac{dD}{d\chi} = \frac{\lambda\alpha\mu_f f''(k^*)}{(1-\chi)^2} - \lambda(1-\alpha\mu_f)\big[f''(z_f^*) + (1-\chi)f'''(z_f^*)f(k^*) \big].$$

As for α and μ_f, comparative statics again depend on χ and ℓ_f. After differentiating (C.16) at $\bar{b}_f < b_f^{max}$ with respect to α, $|d\ln L_f/di|$ increases with α when $(k^*-z_f^*)dD/d\alpha > \chi di_f^*/d\alpha$. Since $di_f^*/d\alpha < 0$, a sufficient, but not necessary condition for $|d\ln L_f/di|$ to increase with α is $dD/d\alpha \geq 0$, or

$$\chi \leq 1 - \sqrt{\frac{f''(k^*)}{f''(z_f^*)}} \equiv \overline{\chi},$$

where $f''(k^*)/f''(z_f^*) < 1$ and $\overline{\chi} \in (0,1)$ for a general $f(\cdot)$ exhibiting decreasing returns to scale. The above condition also applies for $d|d\ln L_f/di|/d\mu_f > 0$. □