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Authors
Moretto, L.G.
Kataria, S.K.

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L. G. Moretto and S. K. Kataria

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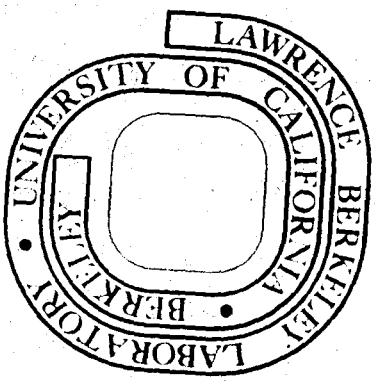

DISCLAIMER

SUPERFLUID PROPERTIES OF EXCITED NUCLEI ARISING FROM A δ-FORCE RESIDUAL INTERACTION†

L. G. Moretto and S. K. Kataria††

Department of Chemistry and
Lawrence Berkeley Laboratory
University of California
Berkeley, California 94720

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A δ-force interaction is used to evaluate the statistical functions for excited shell-model nuclei within the pairing approximation. The dependence of the gap parameters upon temperature is determined and the critical temperature is obtained.

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The short range residual interaction in nuclei is usually accounted for by means of the pairing Hamiltonian.

$$H = \sum_{\pm k} \epsilon_k a_k^\dagger a_k - \sum_{k,k'} G_{kk'} a_k^\dagger, a_{\bar{k}}^\dagger, a_{\bar{k}} a_k \qquad (1)$$

where ϵ_k are the single particle energies, $a_k^\dagger$ and a_k are the creation and annihilation operators and $G_{kk'}$ are the residual interaction matrix elements. A commonly used approximation consists in assuming that all the matrix elements $G_{kk'}$ are equal to a constant G called the pairing strength. In order to avoid divergences and to obtain results in agreement with experiment, this approximation requires a suitably truncated shell model space and a renormalization of the pairing strength G. The pairing strength G is usually varied smoothly as a function

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††On leave from Bhabha Atomic Research Centre, Bombay, India.

of the particle number in an empirical way [1]. This procedure does not account for the fluctuations in the gap parameters Δ_k arising from the matrix elements $G_{kk'}$. Such matrix elements, when calculated for a δ-force residual interaction, can vary by as much as one order of magnitude depending upon the single particle states k,k'. In this work the statistical functions describing excited nuclei have been calculated on the basis of the Nilsson shell model and of a δ-force matrix elements [2]. A comparison is made with calculations performed on the basis of a constant pairing strength G [3].

The logarithm Ω of the grand partition function can be derived from the Hamiltonian [4]:

$$\Omega = -\beta \sum_k (\epsilon_k - \lambda - E_k) + 2 \sum_k \ell n[1 + \exp(-\beta E_k)] \qquad (2)$$

$$-\beta \sum_{kk'} G_{kk'} \chi_k \chi_{k'}$$

where β is the inverse of the temperature, λ is the chemical potential, $E_k = [(\epsilon_k - \lambda)^2 + \Delta_k^2]^{1/2}$ and Δ_k is related to χ_k's by the relation:

$$\Delta_k = \sum_{k'} G_{kk'} \chi_{k'}$$

All the other thermodynamical equations can be obtained by a suitable differentiation of Ω.

The gap equation:

$$\frac{\partial\Omega}{\partial\Delta_k} = 0 \text{ or } \Delta_k = \frac{1}{2}\sum_{k'} G_{kk'}\,\Delta_{k'}\,\frac{\tanh\frac{1}{2}\beta E_{k'}}{E_{k'}} \tag{3}$$

The particle equation:

$$\frac{\partial\Omega}{\partial\alpha} = N = \sum_k \left(1 - \frac{\epsilon_k - \lambda}{E_k}\tanh\frac{1}{2}\beta E_k\right) \tag{4}$$

The energy equation:

$$-\frac{\partial\Omega}{\partial\beta} = E = \sum_k \epsilon_k \left(1 - \frac{\epsilon_k - \lambda}{E_k}\tanh\frac{1}{2}\beta E_k\right) - \sum_{kk'} G_{kk'}\,\chi_k\chi_{k'} \tag{5}$$

where $\alpha = \beta\lambda$. The above equations define the values of Δ_k, λ for a fixed value of N and E. The entropy is given by:

$$S = 2\sum_k \ell n[1 + \exp(-\beta E_k)] + 2\beta\sum_k \frac{E_k}{1 + \exp\beta E_k} \tag{6}$$

The level density is

$$\rho(E,N) = \frac{\exp S}{(2\pi)^{3/2} D^{1/2}} \tag{7}$$

where $D = \det\left[\frac{\partial^2\Omega}{\partial\alpha_i\,\partial\alpha_j}\right]$, α_i being the Langrange multipliers used to fix the constants of motion of the system.

In the present calculations the single particle eigenvalues ε_k have been obtained using the Nilsson model. The matrix elements $G_{kk'}$ have been computed on the basis of a δ-force residual interaction. The strength of the δ-interaction has been chosen so that the gap parameters Δ_k at the Fermi surface reproduce the overall experimental even-odd mass differences. The particle eq. (4) and the set of gap equations (eq. (3)) have been solved simultaneously by means of an iteration procedure. The convergence in the values of Δ_k can be achieved when the number of levels used in the computation is above 100. This is not commonly done in the calculation since the same results can be obtained with a smaller spectroscopical space and with a renormalized δ-force strength.

Figure 1 shows the temperature dependence of the gap parameters Δ_k associated with each level k for the proton component of the nucleus $^{220}_{86}Rn$. It is seen that in ground state the gap parameters for different levels are different. Furthermore, it is interesting to notice that all the gap parameters Δ_k vanish at a common critical temperature. In the constant G approximation all the gap parameters are identically equal. At the critical temperature the system undergoes a second-order phase transition which appears in the specific heat as a discontinuity. Figure 2a shows the temperature dependence of the denominator of the level density expression (eq. (7)) for the δ-force matrix elements. Figure 2b and 2c show the variation of the nuclear entropy and nuclear level density with temperature.

The validity of the constant-G approximation has been studied by calculating the various statistical thermodynamic functions. The pairing strength G was chosen so as to match the gap parameter Δ to the gap parameter Δ_k associated with the level closest to the Fermi surface, calculated on the basis of the δ-interaction. The temperature dependence of the matched Δ and Δ_k for a single

nucleon component is shown in fig. 3. One notices that the critical temperature of the system with δ-force matrix elements is different from the critical temperature obtained from the constant G calculations. A lower critical temperature for the proton component of the system with δ-force G-matrix elements as compared to the case with constant G matrix elements is observed for most of the cases analyzed. Another possible way of comparing the two formalisms is that of matching the condensation energies:

$$CE = \sum_{k=1}^{\infty} \left\{ \epsilon_k \left(1 - \frac{\epsilon_k - \lambda}{E_k}\right) - \frac{1}{2} \Delta_k^2 / E_k \right\} - \sum_{k=1}^{N/2} \epsilon_k \quad .$$

When this is done, the gap parameter Δ does not agree with the Δ_k at the Fermi surface. However, the critical temperatures for these two systems do match for the few cases analyzed. Figure 2a shows the denominators of the level density expression (eq. (7)) for the two formalisms where the constant-G gap parameter Δ is matched to the Δ_k closest to the Fermi surface. Figures 2b,2c show the behavior of the entropy and of the logarithm of nuclear level density on the temperature for the two formalisms.

The calculations have been performed for the neutron components too. A similar behavior is observed for those cases. It is concluded that the effect of the approximation of constant matrix element G on the nuclear entropy or nuclear level density is not a serious one, provided one has the correct prescription to estimate an effective pairing strength G.

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Figure Captions

Fig. 1. The temperature dependence of the gap parameters Δ_k for the proton component of ^{220}Rn is shown for few selected levels. ε_k denotes the single particle energy eigenvalue. All the gap parameters Δ_k become zero at the same temperature.

Fig. 2. The temperature dependence of the thermodynamical quantities for the proton component of the nucleus $^{220}_{86}$Rn. a) The denominator in the level density expression. b) The nuclear entropy. c) The nuclear level density. The continuous lines refer to the calculation with δ-force pairing matrix elements and the dashed lines are associated with the use of constant G-matrix elements. The magnitude of the pairing strength G was adjusted so as to match the Δ with the gap parameter Δ_F obtained with δ-force.

Fig. 3. The temperature dependence of the gap parameters Δ for the proton component of ^{220}Rn for the level closest to the Fermi surface for the three cases. The continuous curve is obtained by using the δ-force pairing matrix elements. The dashed curves are obtained within the constant G approximation. The dashed curve (--) is obtained by matching the condensation energy with constant G and the condensation energy with δ-force matrix elements. The dashed curve (— - —) is obtained by matching the gap parameter Δ in the constant G approximation to the gap parameter Δ_F for the level nearest to the Fermi surface calculated with the δ-force.

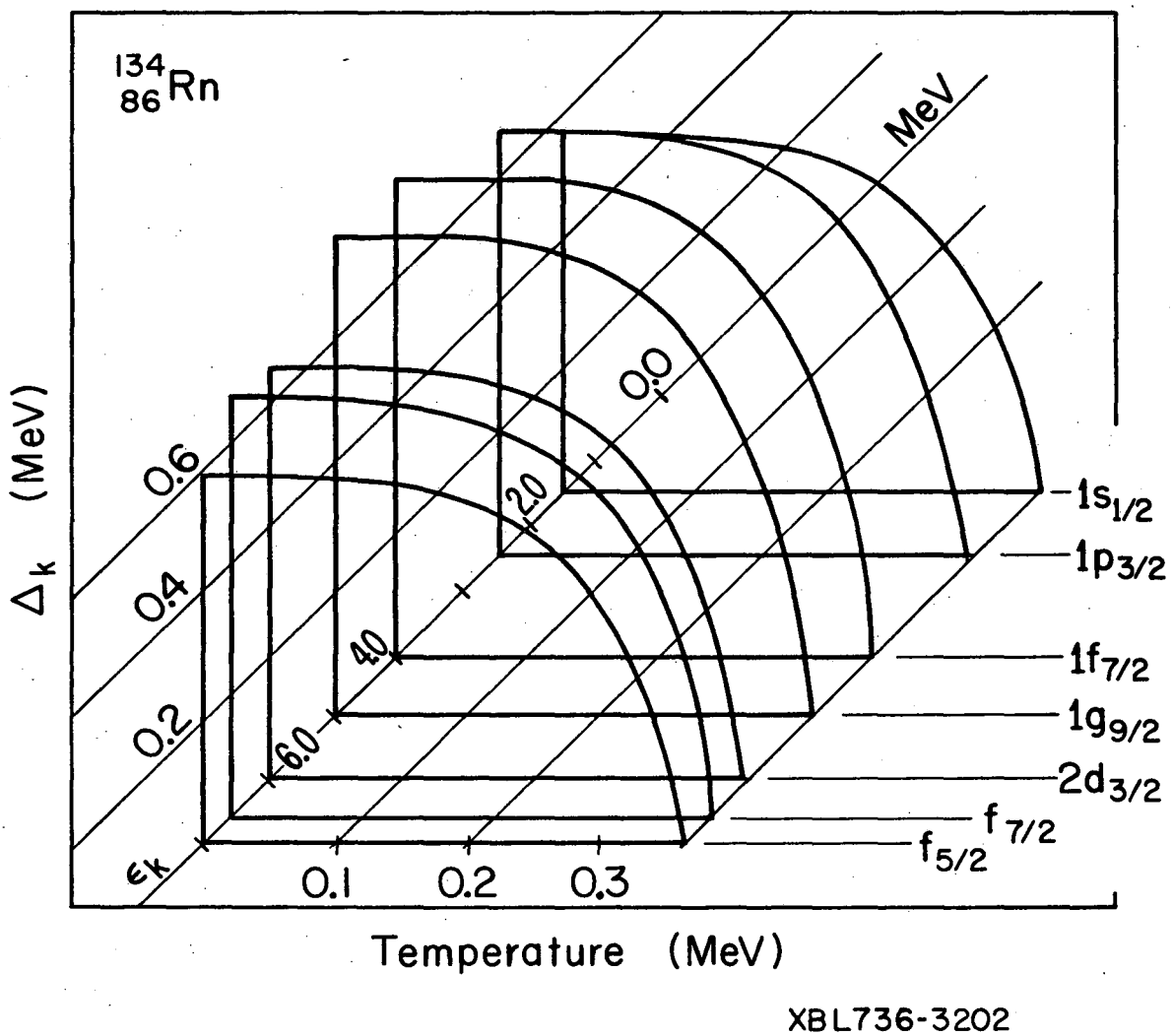

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Fig. 1

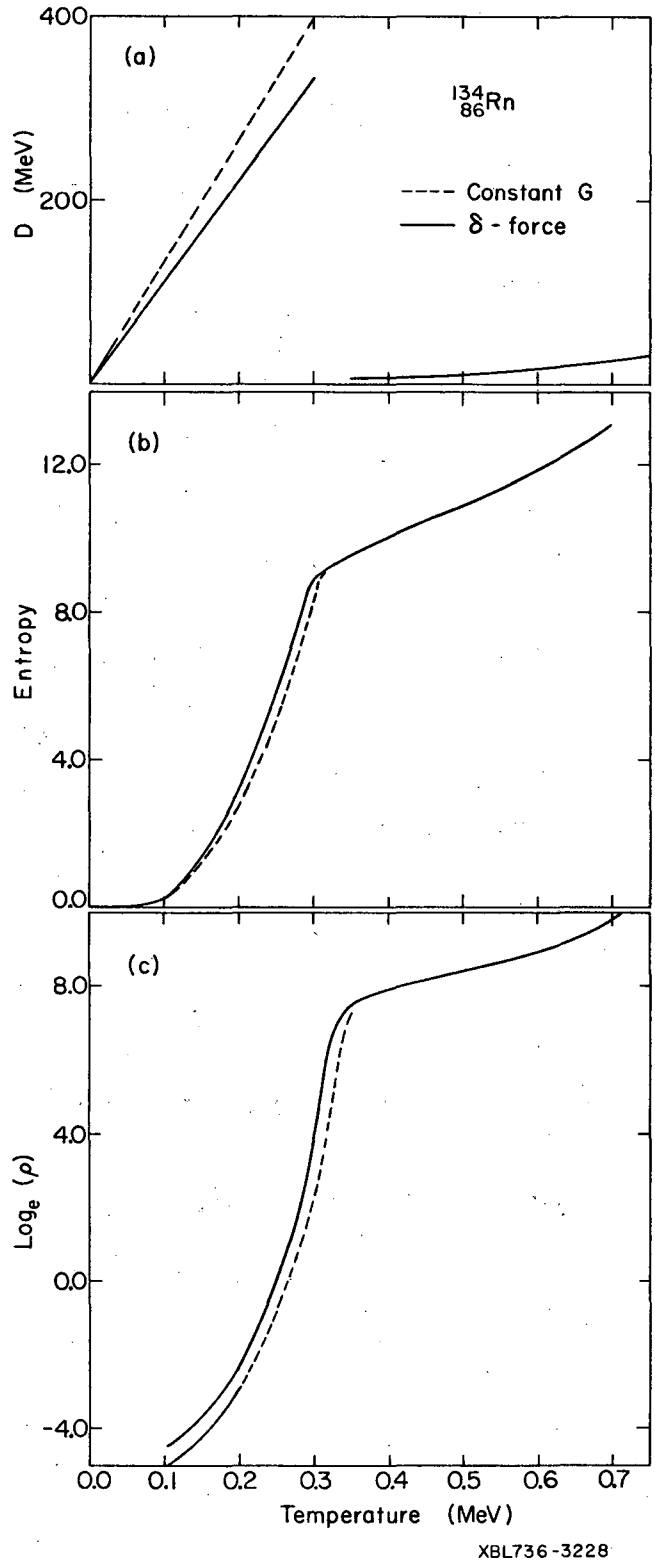
(a)
$^{134}_{86}Rn$
---- Constant G
—— δ - force
D (MeV)
400
200
(b)
Entropy
12.0
8.0
4.0
0.0
(c)
$Log_e(\rho)$
8.0
4.0
0.0
-4.0
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
Temperature (MeV)
XBL736-3228

Fig. 2

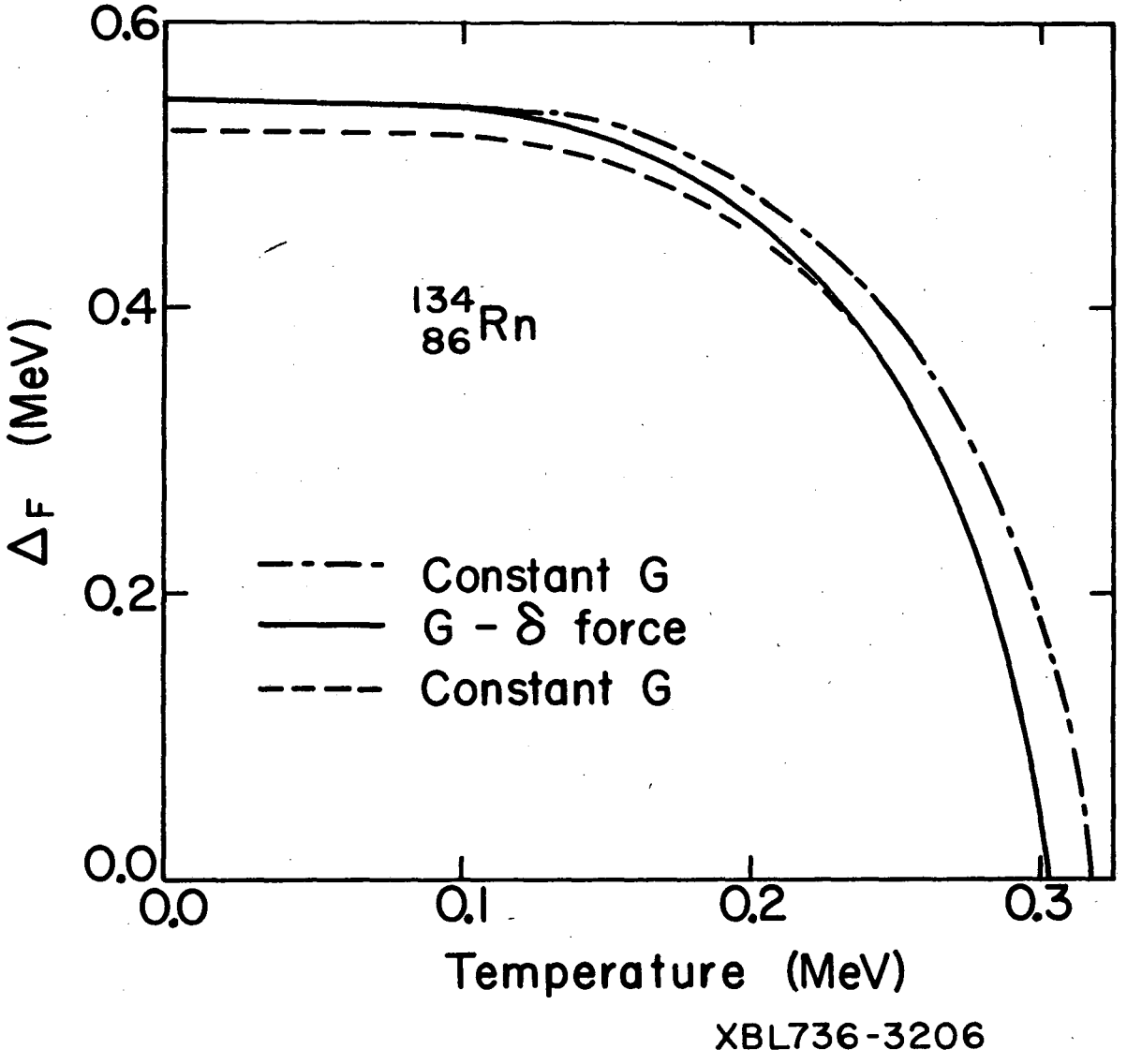
0.6
0.4
0.2
0.0
Δ_F (MeV)
$^{134}_{86}$Rn
Constant G
G - δ force
Constant G
0.0
0.1
0.2
0.3
Temperature (MeV)
XBL736-3206

Fig. 3

TECHNICAL INFORMATION DIVISION
LAWRENCE BERKELEY LABORATORY
UNIVERSITY OF CALIFORNIA
BERKELEY, CALIFORNIA 94720