Title
POLYNOMIAL CHROMODYNAMICS IN 1 + 1 DIMENSIONS I. CONFINEMENT OF QUARKS AND THE STRUCTURE OF COMPOSITE PARTICLES

Author
Ruck, Herbert M.

Publication Date
1981-02-01

LBL-12316 c.2
Preprint

Lawrence Berkeley Laboratory
UNIVERSITY OF CALIFORNIA

Submitted to Nuclear Physics

POLYNOMIAL CHROMODYNAMICS IN 1 + 1 DIMENSIONS I. CONFINEMENT OF QUARKS AND THE STRUCTURE OF COMPOSITE PARTICLES

Herbert M. Ruck

February 1981

Prepared for the U.S. Department of Energy under Contract W-7405-ENG-48

DISCLAIMER

POLYNOMIAL CHROMODYNAMICS IN 1 + 1 DIMENSIONS I. CONFINEMENT OF QUARKS AND THE STRUCTURE OF COMPOSITE PARTICLES*

Herbert M. Ruck

Nuclear Science Division
Lawrence Berkeley Laboratory
University of California,
Berkeley, California 94720

ABSTRACT

We derive bound states of quarks and gluons in a two-dimensional model with Z(3) symmetry. The discrete symmetry imposes an inseparability theorem for the quark fields that satisfy the field equations. We calculate the masses of the ground state, resonances, colored states and glueballs. The various properties as size, formfactor and mass distributions are analyzed. The phenomenological bag mass formula is reproduced in terms of the mean square extension.

*This work was supported by the Director, Office of Energy Research, Division of Nuclear Physics of the Office of High Energy and Nuclear Physics of the U.S. Department of Energy under Contract W-7405-ENG-48.

1. INTRODUCTION

The forces that mold the strongly interacting elementary particles (hadrons) out of gluons and quarks determine the interaction between these particles themselves.

Any theory of gluons and quarks that explains hadrons provides a natural foundation for the theory of nuclear physics.

Baryons are made of colored [1,2] gluons [3] and quarks [4]. The basic interaction between them is the color exchange force quantitatively described by Yang-Mills type field theories [5,6].

Nuclear forces conventionally are described by the exchange of mesons [7]. The number of mesons available is sufficiently large to provide enough degrees of freedom to describe the complexity of nuclear forces. But this is only a different aspect of the dynamics between nucleons and there should be a direct link between the quark and gluon forces on one hand and nuclear forces on the other hand [8-10].

The mesonic picture and the quark-gluon picture might be equivalent, although the latter one is the more natural. There is also more confidence in the meson picture as mesons one observed directly, whereas the quarks are observed indirectly. The economy of the mesonic description becomes more complicated as the nucleons get closer, whereas the complexity of the quark-gluon descriptions should be independent of the relative distance of the nucleons. Therefore from a fundamental point of view all nuclear physics should derive from the quark-gluon and gluon-gluon interaction.

I will present a model of composite particles made out of "gluons" and "quarks," that features quark confinement [11] and derives manybody forces between these particles, that will cluster to large ensembles of particles.

The results are presented in three papers according to the logical subdivision of the subject:

I. The structure of a single particle

II. Two-particle interactions

III. Formation of nuclei, many-particle interactions.

The assumptions of the model are the following:

1. Quarks and gluons are described by classical fields.
2. The internal symmetry in color space is Z(3) the group of cyclic discrete rotations.
3. The gluons are scalar fields. The "free" gluons have a polynomial self-interaction.

Then exact solutions of the model are found under the following simplifying conditions:

4. Use of a two dimensional configuration space R^2: one time dimension t and one space dimension x with the metric $g_{\mu\nu} = (g_{00}, g_{11}) = (1,-1)$, $\mu,\nu = 0,1$.
5. Partial decoupling of the field equations, such that the gluons determine the motion of the quarks but there is no feedback from the quarks on the gluons.

Some of the results that will be presented in the series of three papers are the following:

1. Confinement of the quarks to bound states.
2. Mass spectrum of color neutral states, resonances, colored states and glueballs.
3. Phenomenological bag type formula for the mass of the composite system.
4. Two body potential between two composite particles with a shallow attraction near the touching point of the particles and a hard core repulsion at short distances. The potential between colored states or glueballs is purely attractive.
5. The many body forces become more attractive with increasing number of particles, and move the particles closer than two body forces alone would predict.
6. Saturation of nuclear force.
7. Constant density in the center of nuclei.
8. Equation of state of nuclear matter.

In Chap. 2, part I, we introduce the classical fields as a dynamical degrees of freedom and define the model by giving its Langrangian in three different representations, discuss the symmetry and field equations.

In Chap. 3 we modify the problem by decoupling the gluons from the quarks but having the quarks still move in the field of the gluon. In this case we may use a previously found solution of the gluon equations [12].

In Chap. 4 then the motion of the quarks in the gluon field is solved and the solution interpreted as permanent confinement, color singlet, vanishing outside, linear rising potential.

In the following chapters the properties of composite particles are analyzed. The energy density Chap. 5, the average extension (Chap. 6), the form factor (Chap. 7) and the mass formula (Chap. 8).

In Chap. 9 the question of the mass spectrum is addressed. Two excitation levels are calculated by an approximation of the nonlinear gluon equations. The energy distribution (time dependence) of the resonance is given.

In Chap. 10 the properties of colored states (states deficient in one quark) and glueballs (states deficient in both quarks) are discussed.

In the last Chap. 11, the results are summarized and conclusions drawn.

2. A Z(3) SYMMETRIC FIELD THEORY MODEL

The Langrangian of the model contains two scalar fields $\phi_1(t,x)$, $\phi_2(t,x)$ that we call gluon fields and two spinor fields $\psi_1(t,x)$, $\psi_2(t,x)$ that we call quark fields.

$$\begin{aligned}\mathcal{L}\,(t,x) = {} & \frac{1}{2}\,(\partial_\mu\phi_1)^2 + \frac{1}{2}\,(\partial_\mu\phi_2)^2 - V(\phi_1,\phi_2) \\ & + i\bar{\psi}_1\gamma_\mu\overleftrightarrow{\partial}_\mu\psi_1 - m\,\bar{\psi}_1\psi_1 + i\bar{\psi}_2\gamma_\mu\overleftrightarrow{\partial}_\mu\psi_2 - m\bar{\psi}_2\psi_2 \\ & - g(j_1\phi_1 + j_2\phi_2)\end{aligned} \tag{2-1}$$

where V is the self-interaction of the free gluon field (Fig. 1):

$$V(\phi_1,\phi_2) = \lambda(\phi_1^2 + \phi_2^2)^2 - \nu(\phi_1^3 - 3\,\phi_1\phi_2^2) - \mu(\phi_1^2 + \phi_2^2) - \gamma \tag{2-2}$$

$$j_1 = \bar{\psi}_1\psi_1 - \bar{\psi}_2\psi_2 \quad ; \quad j_2 = -\bar{\psi}_1\psi_2 - \bar{\psi}_2\psi_1 \quad . \tag{2-3}$$

The self interaction of the gluon is determined by three coupling constants λ, ν, μ. The constant γ is determined such that V is positive definite $V \geq 0$. g is an independent coupling constant in front of the Yukawa coupling term between quarks and gluons.

$$\partial_\mu = \frac{\partial}{\partial x_\mu} \quad ; \quad \overleftrightarrow{\partial}_\mu = \frac{1}{2}\,(\overrightarrow{\partial}_\mu - \overleftarrow{\partial}_\mu) \quad , \quad \left\{\gamma_\mu,\gamma_\nu\right\} = 2\,g_{\mu\nu} \quad , \quad \mu,\nu=0,1\ . \tag{2-4}$$

So far the configuration space can have any dimension.

The two components of quark fields ψ_1, ψ_2 have the significance of two different colors. In a model with quarks having two colors red and blue, e.g., the least number of gluon fields needed to change the colors is two. One component of the gluon field should not change any color whereas the second component should change the color either from red to blue or from blue to red. In the SU(2) nonablian gauge theory three gluon fields are used to change two colors because the gluon fields making the red-blue and blue-red transition are complex conjugates of each other.

An alternative form of the Lagrangian (2-1) is obtained by introducing matrix fields for fermions

$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} \; ; \quad \bar{\psi} = (\bar{\psi}_1, \bar{\psi}_2) \tag{2-5}$$

and for the gluons

$$\phi = \begin{pmatrix} \phi_1 & -\phi_2 \\ -\phi_2 & -\phi_1 \end{pmatrix} = \sigma_3\phi_1 - \sigma_1\phi_2 \tag{2-6}$$

where σ_1, σ_2, σ_3 are the Pauli matrices.

Then the currents (2-3) become:

$$j_1 = \bar{\psi}\,\sigma_3\psi \quad , \quad j_2 = -\bar{\psi}\,\sigma_1\psi \quad . \tag{2-7}$$

The Langranian (2-1) is transformed into

$$\mathcal{L} = \frac{1}{4}\ \mathrm{Tr}(\partial_\mu \phi)^2 - \frac{1}{2}\ \mathrm{Tr}\ [\lambda\phi^4 - \nu(\sigma_3\phi)^3 - \mu\phi^2 - \gamma]$$
$$+\ i\ \bar{\psi}\ \gamma_\mu\ \overleftrightarrow{\partial}_\mu \psi - m\bar{\psi}\psi\ - g\bar{\psi}\phi\psi\ , \tag{2-8}$$

a form that pretends to generalizations to a larger number of colors.

The model is invariant under the transformations of the Poincaré group and has the internal symmetries $\psi_1 \to -\psi_1$, $\psi_2 \to -\psi_2$ and most distinctive the cyclic symmetry Z(3):

$$\phi_1 \to \phi_1\ \cos(\theta n) - \phi_2\ \sin(\theta n)$$
$$\phi_2 \to \phi_1\ \sin(\theta n) + \phi_2\ \cos(\theta n) \tag{2-9}$$

$$\psi_1 \to \psi_1\ \cos(\theta n) - \psi_2\ \sin(\theta n)$$
$$\psi_2 \to \psi_1\ \sin(\theta n) + \psi_2\ \cos(\theta n) \tag{2-10}$$

$$\bar{\psi}_1 \to \bar{\psi}_1\ \cos(\theta n) - \bar{\psi}_2\ \sin(\theta n)$$
$$\bar{\psi}_2 \to \bar{\psi}_1\ \sin(\theta n) + \bar{\psi}_2\ \cos(\theta n)\ . \tag{2-11}$$

In matrix form the same transformations are

$$\phi \to R^{+}\phi R \tag{2-12}$$

$$\psi \to \eta\ R^2\psi \tag{2-13}$$

$$\bar{\psi} \to \eta\ \bar{\psi}\ R^{+2} \tag{2-14}$$

with $\quad R = \exp\left(-i\,\frac{\theta}{2}\,\sigma_2\right) \quad , \quad \eta = \pm 1 \quad ,$ (2-15)

and $\quad \theta = \frac{2\pi}{3}\,n \quad , \quad n = 0,1,2 \quad .$

R is a representation of a cyclic group of order three, and R^3 = Identity guarantees the invariance of $\bar{\psi}\phi\psi$.

The discrete angle of rotation $\theta = 2\pi/3\ n$ is imposed by the cubic term in the Lagrangian. All other terms and rotationally invariant. In polar coordinates $\phi_1 = \rho\cos\theta$, $\phi_2 = \rho\sin\theta$ the potential

$$V = \lambda\,\rho^4 - \nu\,\rho^3\cos 3\theta - \mu\rho^2 - \gamma \tag{2-16}$$

is a periodic function of θ with period $2\pi/3$.

The Euler-Lagrange field equations are

$$\Box\,\phi_1 = -4\lambda(\phi_1^2+\phi_2^2)\phi_1 + 3\nu(\phi_1^2-\phi_2^2) + 2\mu\phi_1 + g(\bar{\psi}_1\psi_1-\bar{\psi}_2\psi_2) \tag{2-17a}$$

$$\Box\,\phi_2 = -4\lambda(\phi_1^2+\phi_2^2)\phi_2 - 6\nu\,\phi_1\phi_2 + 2\mu\phi_2 + g(-\bar{\psi}_1\psi_2-\bar{\psi}_2\psi_1) \tag{2-17b}$$

$$i\gamma_\mu\partial_\mu\psi_1 - m\psi_1 = g(\psi_1\phi_1 - \psi_2\phi_2) \tag{2-17c}$$

$$i\gamma_\mu\partial_\mu\psi_2 - m\psi_2 = g(-\psi_2\phi_1 - \psi_1\phi_2) \tag{2-17d}$$

In Ref. [12] we found a solution of the gluon equations in the absence of quarks. To make use of these solutions we consider a motion of the gluons independent of the quarks, but the quarks moving in the potential created by the gluon field. We summarize this result in the next section.

3. THE MODIFIED Z(3) PROBLEM. SOLUTION OF THE GLUONIC FIELD EQUATIONS

Consider the following system of equations written in 1+1 dimensions:

$$(\partial_0^2-\partial_1^2)\phi_1 = -4\lambda(\phi_1^2+\phi_2^2)\phi_1 + 3\nu(\phi_1^3-\phi_2^2) + 2\mu\phi_1 \tag{3-1a}$$

$$(\partial_0^2-\partial_1^2)\phi_2 = -4\lambda(\phi_1^2+\phi_2^2)\phi_2 - 6\nu\phi_1\phi_2 + 2\mu\phi_2 \tag{3-1b}$$

$$i(\gamma_0\partial_0-\gamma_1\partial_1)\psi_1 - m\psi_1 = g(\psi_1\phi_1-\psi_2\phi_2) \tag{3-1c}$$

$$i(\gamma_0\partial_0-\gamma_1\partial_1)\psi_2 - m\psi_2 = g(-\psi_2\phi_1-\psi_1\phi_2) \tag{3-1d}$$

where

$$\partial_0 = \frac{\partial}{\partial t} \quad , \quad \partial_1 = \frac{\partial}{\partial x} \quad ; \quad \phi_k = \phi_k(t,x) \quad ; \quad \psi_k = \psi_k(t,x) \qquad k = 1,2 \quad .$$

In two dimensions the kinetic part of the fermion equations is given by the Thirring-model [13] with the representation of the γ matrices:

$$\gamma_0 = -\sigma_2 \ , \ \gamma_1 = i\sigma_1 \ ; \left\{\gamma_\mu,\gamma_\nu\right\} = 2g_{\mu\nu} \quad \mu,\nu = 0,1 \ . \tag{3-2}$$

and the conjugate fields

$$\bar{\psi}_k = \psi_k^+\sigma_2 \qquad\qquad k=1,2 \quad . \tag{3-3}$$

The system of equations (3-1) is not even an approximation of the original set of equations (2-17) in two dimensions because a first order effect is neglected. This is rather a new problem. The feedback of the quarks upon the gluons is neglected. But this new problem has the advantage to render itself to an elegant solution in closed form for the ϕ and ψ fields.

Time independent solutions of the gluon equations [12] are obtained when the coupling constants are related to each other

$$\mu = \lambda\phi_v^2 \quad , \nu = \frac{2}{3}\lambda\phi_v \quad , \gamma = -\frac{2}{3}\lambda\phi_v^4 = -\nu\phi_v^3 \quad . \tag{3-4}$$

The soliton solutions with asymptotic values $\phi_1 = \phi_v$, $\phi_2 = 0$ for $x = +\infty$ and $\phi_1 = -(1/2)\,\phi_v$, $\phi_2 = (\sqrt{3}/2)\,\phi_v$ for $x = -\infty$ are Fig. (3-1):

$$\phi_1 = \frac{1}{4}\,\phi_v(1 + 3X) \quad ; \quad \phi_2 = \frac{\sqrt{3}}{4}\,\phi_v(1-X) \quad , \tag{3-5}$$

ϕ_v is the vacuum field and

$$X\,(x) = \tanh\,(\sqrt{\frac{3\lambda}{2}}\,\phi_v(x-x_0)) \quad . \tag{3-6}$$

The fields (3-5) satisfy the algebraic relation (straight line in field space)

$$\phi_1 + \sqrt{3}\,\phi_2 = \phi_v \quad . \tag{3-7}$$

For the particular values of the coupling constants (3-4) the trajectory (3-7) is a geodesic line in the space of the potential $V(\phi_1,\phi_2)$ that connects the two minima of the potential in which the asymptotic values of the fields reside. There are two more pairs of solutions obtained by rotations by $2\pi n/3$ ($n = 1,2$) in ϕ-space

$$\phi_1 = \frac{1}{2}\,\phi_v \quad , \quad \phi_2 = \frac{\sqrt{3}}{2}\,\phi_v\,\chi \tag{3-8}$$

and

$$\phi_1 = \frac{1}{4}\,\phi_v(1-3\chi) \quad , \; \phi_2 = -\,\frac{\sqrt{3}}{4}\,\phi_v(1+\chi) \; . \tag{3-9}$$

Next we are going to solve the quark equations with ϕ_1 and ϕ_2 from Eq. (3-5). ϕ_1, ϕ_2 are stable soliton solutions. The action for this gluon configuration is finite and less than the action for the vacuum state.

$$S \equiv S(t_0) = \int_{-\infty}^{\infty} dt\;\delta(t - t_0)\int_{-\infty}^{\infty} dx\,\mathcal{L}(t,x) \tag{3-10}$$

$$S_{soliton} < S_{vacuum} = 0 \quad . \tag{3-11}$$

It is important that the action is finite. The sign is not important because the action enters the functional integrals as imaginary exponent in the measure exp(iS).

There is of course a zero-field $\phi_1 = \phi_2 = 0$ solution of the Eqs. (3-1), but then the gluonic part of the Lagrangian is γ and the action is infinite.

4. SOLUTION OF THE QUARK EQUATIONS OF MOTION AND QUARK CONFINEMENT

The field equations for the quarks are:

$$i\gamma_\mu \partial_\mu \psi_1 - m\psi_1 = g(\psi_1\phi_1 - \psi_2\phi_2) \tag{4-1}$$

$$i\gamma_\mu \partial_\mu \psi_2 - m\psi_2 = g(-\psi_2\phi_1 - \psi_1\phi_2) \tag{4-2}$$

with ϕ_1 and ϕ_2 given by Eq. (3-5).

As a consequence of the linearity of the system of differential equations (4-1) and (4-2) in ψ_1 and ψ_2 and the structure of the RHS induced by the Z(3) symmetry the following theorem holds:

Theorem: For massless quarks $m = 0$ any pair of solutions ψ_1 and ψ_2 of the system of equations (4-1), (4-2) are related algebraically:

$$\psi_2(t,x) = i\eta\sigma_3 \psi_1(t,x) \tag{4-3}$$

where $\eta \equiv \text{sign } \psi_2 = \pm 1$ is a phase factor, σ_3 is the third Pauli matrix.

Proof: Substitute ψ_2 from (4-3) into both equations (4-1) and (4-2). Set $m = 0$. We obtain:

$$i\gamma_\mu \partial_\mu \psi_1 = g(\psi_1\phi_1 - i\eta\sigma_3\psi_1\phi_2) \tag{4-4}$$

$$i\gamma_\mu \partial_\mu \, i\eta\sigma_3\psi_1 = g(- i\eta\sigma_3 \, \psi_1\phi_1 - \psi_1\phi_2) \quad . \tag{4-5}$$

Equation (4-5) multiplied from the left by $i\eta\sigma_3$ is identical to Eq. (4-4), because σ_3 anticommutes with γ_0 and γ_1:

$$\eta i\sigma_3\gamma_\mu i\eta\sigma_3 = -\sigma_3\gamma_\mu\sigma_3 = \gamma_\mu \quad . \tag{4-6}$$

and

$$(i\eta\sigma_3)^2 = -1 \qquad \text{QED.}$$

The system of field equations (4-1), (4-2) reduces to a single field equation for ψ_1

$$i\gamma_\mu\partial_\mu\psi_1 = g(\psi_1\phi_1 - i\eta\sigma_3\ \psi_1\ \phi_2) \tag{4-7}$$

and once this is solved ψ_2 is calculated from Eq. (4-3).

The algebraic connection (4-3) for the quarks actually means that the time and space dependence of the two quark fields are the same. The spinors differ by a multiplication by $i\eta\sigma_3$.

As a quick test we can set $\psi_2 = 0$ and it immediately follows from (4-2), independently of (4-3) that ψ_1 must also vanish. The theorem is true for arbitrary functions of ϕ_1 and ϕ_2.

The relation (4-3) between the fields is the basis for confinement of quarks and the formation of color singlets.

The currents (2-3) become functions of ψ_1 alone:

$$j_1 = 2\ \bar{\psi}_1\psi_1 \quad ; \quad j_2 = -2i\eta\ \bar{\psi}_1\sigma_3\psi_1 \quad . \tag{4-8}$$

We turn now to solve Eq. (4-7) [14]. After substituting ϕ_1 and ϕ_2 we observe that the quark equation separates into two parts. The time dependent x-independent part of the equation

$$i\gamma_0\partial_0\psi_1(t,x) = g\,\frac{1}{4}\,\phi_v(1 - i\eta\sigma_3\sqrt{3})\;\psi_1(t,x) \tag{4-9}$$

and the x-dependent part of the equation

$$-i\gamma_1\partial_1\psi_1(t,x) = g\;\frac{3}{4}\,\phi_v\;(x)\;(\sqrt{3} + i\eta\sigma_3)\;\psi_1(t,x) \quad . \tag{4-10}$$

We assume a stationary time dependence for the quark field of the form:

$$\psi_1(t,x) = N\,U\,\exp\,(iEt - H(x)) \quad , \quad H(x) \geq 0 \quad . \tag{4-11}$$

The first equation (4-9) will determine the energy E and the components u_1, u_2 of the spinor U. The second equation (4-10) gives the H(x) function. N is a normalization coefficient that will be defined later.

The algebraic matrix equation obtained from (4-9) is

$$\begin{pmatrix} -g\,\frac{1}{4}\,\phi_v(1-i\eta\sqrt{3}) & -iE \\ iE & -g\,\frac{1}{4}\,\phi_v(1 + i\eta\sqrt{3}) \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = 0 \quad . \tag{4-12}$$

The determinant must vanish to give nontrivial solutions for the spinor. From this condition follows the energy:

$$E = \pm \frac{1}{2}\ |g| \phi_V = (\text{sign } E)(\text{sign } g)\ \frac{1}{2}\ g\ \phi_V \quad . \qquad (4\text{-}13)$$

The negative value of the energy will be used as the binding energy of the quarks. g will always be the chosen to be positive $g > 0$. The negative value of the energy E means the quarks want to get into the bag created by the gluons.

We fix u_1 to be one, then the spinor components are:

$$u_1 = 1 \quad ; \quad u_2 = (\text{sign } E)\ (\text{sign } g)\ (\text{sign } \psi_2)\ \exp\left[i(\text{sign } \psi_2)\ \frac{\pi}{6}\right] ,$$

$$u_1^* u_1 = u_2^* u_2 = 1 \quad . \qquad (4\text{-}14)$$

The x-dependent equation (4-10) reduces to

$$-\frac{d}{dx} H(x) \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ u_2 \end{pmatrix} = g\ \frac{\sqrt{3}}{4}\ \phi_V\ X(x) \begin{pmatrix} \sqrt{3} + i\eta & 0 \\ 0 & \sqrt{3} - i\eta \end{pmatrix} \begin{pmatrix} 1 \\ u_2 \end{pmatrix} . \qquad (4\text{-}15)$$

Both equations are equivalent and reduce to the ordinary differential equation for H(x):

$$-\frac{d}{dx} H(x) = g(\text{sign } E)(\text{sign}\psi_2)\ \frac{\sqrt{3}}{2}\ \phi_V\ X(x) \quad . \qquad (4\text{-}16)$$

The integration is straightforward and gives:

$$H(x) = -|g|\ (\text{sign } E)\ (\text{sign } \psi_2)\ \frac{1}{\sqrt{2\lambda}}\ \ln \cosh\left(\sqrt{\frac{3\lambda}{2}}\ \phi_V x\right) \quad . \tag{4-17}$$

For positive definite H(x), that is a necessity for the normalizability of the quark currents, the product of sign functions must be negative:

$$\text{sign } E \text{ sign } \psi_2 = -1 \quad . \tag{4-18}$$

The sign of the quark coupling constant g does not influence the sign of H(x).

The solutions of the quark fields are in summary:

$$\psi_1 = N \begin{pmatrix} 1 \\ u_2 \end{pmatrix} \exp\ (i\ Et - H(x)) \tag{4-19}$$

$$\psi_2 = (\text{sign } \psi_2) i \sigma_3 \psi_1 \tag{4-20}$$

$$u_2 = -(\text{sign } g) \exp\left[-\ i(\text{sign } E)\ \frac{\pi}{6}\ \right] \tag{4-21}$$

with the energy of the quark fields:

$$E = (\text{sign } E)\ \frac{|g|\phi_V}{2} \tag{4-22}$$

and

$$H(x) = \frac{1}{\sqrt{2\lambda}} \; |g| \;\; \ln \cosh \left(\sqrt{\frac{3\lambda}{2}}\, \phi_V (x-x_0)\right) \quad . \tag{4-23}$$

Instead of (4-3) we could have used $\psi_1 = -i\eta\sigma_3\psi_2$ to express everything in terms of ψ_2.

The two currents are time independent and proportional to each other:

$$j_1(x) = 4N^2 \;(\text{sign } E)(\text{sign } g)\sin\left(\frac{\pi}{6}\right) \exp\left(-2H(x)\right) \tag{4-24}$$

$$j_2(x) = 4N^2 \;(\text{sign } E)(\text{sign } g)\cos\left(\frac{\pi}{6}\right) \exp\left(-2H(x)\right) \tag{4-25}$$

the ratio is a constant:

$$j_1(x)/j_2(x) = \text{tg}\,\frac{\pi}{6} \quad . \tag{4-26}$$

We determine N the normalization constant Eq. (4-11) from the requirement:

$$\int_{-\infty}^{\infty} |j_1| \;\; dx \;=\; 1 \tag{4-27}$$

from where we obtain the square of N [15, Eq. 3.512]

$$N^2 = \left(\frac{3\lambda}{8}\right)^{1/2} \phi_V \, \frac{\Gamma\left(\frac{1}{2} + (2\lambda)^{-1/2} g\right)}{\Gamma\left(\frac{1}{2}\right)\Gamma\left((2\lambda)^{-1/2} g\right)} = \left(\frac{3\lambda}{8}\right)^{1/2} \phi_V B^{-1}\left(\frac{1}{2}\,,(2\lambda)^{-1/2} g\right) \quad . \tag{4-28}$$

where B is the beta-function.

N^2 is adjusted in such a way that the contribution of the quarks to the field energy is twice the binding energy E (4-22).

Consider a composite particle made of gluons and quarks described by the fields (3-5) and (4-19), (4-20).

The confinement of quarks is described by Eq. (4-3) and (4-23):

1.) The algebraic relation (4-3) between two quark fields of different color makes them unseparable. One field goes with the other. Quarks will form a permanent color singlet.

2.) The currents of the quark fields vanish exponentially with the distance from the center of the particle. We therefore have a bound state of quarks. The inverse power of a hyperbolic cosine function is similar to a gaussian function for large arguments.

Bound and unseparable quarks are confined, and cannot be removed by scattering or any other mechanism.

5. THE COMPOSITE PARTICLE: ENERGY DENSITY

The energy density T_{00} calculated from the Lagrangian contains the kinetic and potential gluon energy G_{kin}, G_{pot} and the kinetic and interaction energy of the quarks Q_{kin} and Q_{int}:

$$T_{00}(x) = \sum_{k=1}^{2} \left[\frac{\partial \mathcal{L}}{\partial \phi_{k,0}} \phi_{k,0} + \bar{\psi}_{k,0} \frac{\partial \mathcal{L}}{\partial \bar{\psi}_{k,0}} + \frac{\partial \mathcal{L}}{\partial \psi_{k,0}} \psi_{k,0} \right] - g_{00} \mathcal{L}$$

$$= G_{kin} + G_{pot} + Q_{kin} + Q_{int} \quad , \tag{5-1}$$

where

$$G_{kin}(x) = \frac{1}{2} (\partial_o \phi_1)^2 + \frac{1}{2} (\partial_o \phi_2)^2 + \frac{1}{2} (\partial_1 \phi_1)^2 + \frac{1}{2} (\partial_1 \phi_2)^2 \tag{5-2}$$

$$G_{pot}(x) = V(\phi_1, \phi_2) \tag{5-3}$$

$$Q_{kin}(x) = i \, \bar{\psi}_1 \gamma_1 \partial_1 \psi_1 + i \, \bar{\psi}_2 \gamma_1 \partial_1 \psi_2 \tag{5-4}$$

$$Q_{int}(x) = g(j_1 \phi_1 + j_2 \phi_2) \quad . \tag{5-5}$$

For the fields we obtained previously in Eqs. (3-9), (4-19), (4-20) the kinetic and potential energy of the gluons are equal:

$$G_{kin}(x) = G_{pot}(x) = \frac{9}{16} \lambda \, \phi_v^4 \left[\cosh \left(\sqrt{\frac{3\lambda}{2}} \, \phi_v (x - x_0)\right)\right]^{-4} \quad , \tag{5-6}$$

the kinetic energy of the quarks vanishes

$$Q_{kin} = 2\, i\bar{\psi}_1\, \gamma_1\, \frac{\partial}{\partial x}\, \psi_1 = 0 \tag{5-7}$$

as a consequence of $\bar{U}\, \gamma_1 U = (1-u_2^* u_2) = 0$.

The interaction energy of the quarks is

$$Q_{int} = (\text{sign } E)\; 2\; g\; N^2\, \phi_v\, \exp\,(-2H(x)\,)$$

$$= (\text{sign } E)\; 2|g|N^2\, \phi_v\, [\cosh\,(\sqrt{\tfrac{3\lambda}{2}}\, \phi_v(x-x_0))]^{-|g|\sqrt{2/\lambda}}\,. \tag{5-8}$$

The energy density of the composite system of gluons and quarks is

$$T_{00}(x) = \frac{9}{8}\,\lambda\phi_v^4\left[\cosh\left(\sqrt{\frac{3\lambda}{2}}\phi_v(x-x_0)\right)\right]^{-4}$$

$$+ (\text{sign } E) 2\; g\; N^2\phi_v\left[\cosh\left(\sqrt{\frac{3\lambda}{2}}\,\phi_v(x-x_0)\right)\right]^{-|g|\sqrt{2/\lambda}}\;, \tag{5-9}$$

with sign $E = -1$. Both the gluon and the quark energy are largest when $x-x_0 = 0$, and vanish exponentially with increasing distance from the center x_0 of the composite system. The energy distribution (5-9) is symmetrical around x_0. A graphical representation of Eq. (5-9) is shown in Fig. (5-1) and (5-2).

None of the gluons or quarks has restmass. The entire energy of the particle defined by (5-9) consists of field energies.

The mass M of the particle is the integrated field energy:

$$M = \int_{-\infty}^{\infty} dx\; T_{00}(x) = \sqrt{\frac{3\lambda}{2}}\;\; \phi_v^3 - \;g\;\; \phi_v \tag{5-10}$$

From this mass formula, the first measurable quantity derived so far, ϕ_v--the vacuum field and g--the quark-gluon coupling constant emerge as useful parameters for the description of a particle. The coupling constant λ may be chosen to be one. For constant mass there is only one free parameter ϕ_v or g.

In Fig. (5-2) the change of energy distribution for a sequence of values of g is shown for a particle with constant mass.

The energy distribution is a competitive effect of the positive gluon and negative quark energy, both have approximately the same bell-shape. It is possible to obtain a variety of shapes as seen in Fig. (5-2), from peaked ($g < 4.4$) to flat ($g = 4.4$) and distributions with a dip in the middle ($g > 4.4$). For too large g's the energy may become negative, these values of g are discarded.

In the two dimensional base space R^2 ϕ_v has no dimensionality $[\phi_v] = L^0$. The fermionic field is $[\psi] = L^{-1}$. The square root of λ has the dimension of an energy. We choose $\lambda^{1/2}$ to be the unit of energy, and set therefore the scale for all numerical values $[\lambda] = L^{-2}$, $[\nu] = [\mu] = [\gamma] = [\lambda]$, $[g] = L^{-1}$ [e.g., $\lambda = \text{fermi}^{-2}$ or $\lambda = \text{cm}^{-2}$ etc.].

On the basis of the quark energies we can again show that quarks will be confined to bound states. To prove this we solve the quark field equations (4-1), (4-2) in the gluon vacuum $\phi_1 = \phi_v$, $\phi_2 = 0$ for plane wave states.

The equations are:

$$i\gamma_\mu \partial_\mu \psi_1 = g\phi_V \psi_1 \tag{5-11}$$

$$i\gamma_\mu \partial_\mu \psi_2 = -g\phi_V \psi_2 \tag{5-12}$$

The quarks in the vacuum pick up an effective mass $m_{eff} = g\phi_V$.

The plane wave solution of (5-11) is:

$$\psi_1 = N \begin{pmatrix} 1 \\ \\ i(E-p)/g\phi_V \end{pmatrix} \exp[\, i(Et-px)], \tag{5-13}$$

with E the energy and p the momentum of the quarks that satisfy the relativistic energy-momentum relation:

$$E^2-p^2 = (g\phi_V)^2 \; . \tag{5-14}$$

Because (5-11), (5-12) are just a special case of the general field equations the relation (4-3) between ψ_2 and ψ_1 holds.

The quark-gluon energy density (5-5) becomes a positive constant:

$$Q_{int}^{\text{plane-wave}}(x) = g j_1^{\text{plane wave}} \phi_V = 4N^2(E-p)>0 \; . \tag{5-14}$$

Due to the Z(3) invariance of (5-5) the interaction energy density is the same as (5-14) in any of the three vacuum states of the gluon field.

Therefore a plane wave packet of quarks that is made to spread out in vacuum over a domain (o, ℓ) requires an energy proportional to the length of the domain:

$$\int_0^{\ell} Q_{int}^{\text{plane wave}}(x)\, dx = 4N^2(E-p)\ell > 0. \qquad (5\text{-}15)$$

This is equivalent to a linearly increasing potential with distance for plane wave quarks.

The gluon fields are smaller at the center of the particle $x=x_0$:

$$\phi_1^2(0) + \phi_2^2(0) = \frac{1}{4}\,\phi_v \qquad (5\text{-}16)$$

than outside of the particle: if $x \to \pm\infty$ then:

$$\phi_1^2 + \phi_2^2 = \phi_v^2 \quad . \qquad (5\text{-}17)$$

This indicates the quarks occupy a region where there is a minimum of color exchange and avoid the regions of vacuum with potentially intensive color interaction (Fig. (3-1).

6. THE AVERAGE EXTENSION OF A PARTICLE

One space-dimension is the most important in physics. In one space dimension the concept of extension and distance is implemented. One can therefore distinguish extended particles from pointlike particles and consider the interaction between particles as a function of their relative distance. Three space-dimensions add volume to the extended particles but nothing new to the relative distance between two particles.

We define the half-length of the particle (analog of the radius) by the second moment of the energy distribution ($T_{00}(x) = T_{00}(-x)$):

$$R = \left(3\int_0^\infty dx\ x^2\ T_{00}(x) \Big/ \int_0^\infty dx\ T_{00}(x)\right)^{1/2} \quad . \tag{6-1}$$

Formula (6-1) is chosen in such a way as to reproduce for a box-like energy distribution of height h ranging in the interval $x \varepsilon (-s,s)$ the correct length $2R = 2s$.

Since the gluons are decoupled from the quarks we consider the length R_b of the gluon distribution (gluon bag) separately

$$R_b = \left(3\int_0^\infty dx\ x^2\ G_{kin}(x)\ dx \Big/ \int_0^\infty dx\ G_{kin}(x)\right)^{1/2} \quad , \tag{6-2}$$

that has an explicit dependence on λ and ϕ_v:

$$R_b = (0.80305\ldots)\ \lambda^{-1/2}\ \phi_v^{-1} \quad . \tag{6-3}$$

The length of the entire particle (6-1) must be computed numerically.

A comparison of the gluon half-length (analog of radius) and the particle half-length is shown in Fig. (6-1) for constant mass M=10 and different values of quark-gluon coupling constant. A small admixture of quarks makes the system shrink whereas for larger g the quarks push the gluon bag apart.

7. THE FORM FACTOR

A measure of the shape of a particle is the form factor, that can be obtained from scattering experiments. We consider the formfactor to be the Fourier transform of the matter distribution. In electron scattering the formfactor is the Fourier-transform of the electrical charge distribution. Because the energy or matter distribution is symmetric about the origin the form factor is a real function of momentum transfer q. The Fourier transform reduces to a cosinus transform:

$$F(q) = \int_{-\infty}^{\infty} dx\; T_{00}(x) e^{iqx} = \int_{-\infty}^{\infty} dx\; T_{00}(x) \cos qx \quad . \tag{7-1}$$

This function can be calculated in a closed form. Introducing the function [15, Eq. (3.985(1)], [16, Eq. 6.1.25]

$$\begin{aligned} \mathrm{coco}(a;\delta;b) &= \int_{0}^{\infty} dx \cos(ax) \cosh^{-\delta}(bx) \\ &= \frac{2^{\delta-2}}{b\Gamma(\delta)} \Gamma^2\left(\frac{\delta}{2}\right) \prod_{n=0}^{\infty} \left[1 + \frac{\left(\frac{a}{2b}\right)^2}{\left(\frac{\delta}{2}+n\right)^2}\right]^{-1} \end{aligned} \tag{7-2}$$

we obtain the form factor:

$$\begin{aligned} F(q) &= \frac{g}{4} \lambda \phi_V^4 \,\mathrm{coco}(q;-4;\sqrt{3\lambda/2}\;\phi_V) \\ &\quad + |g| \,\mathrm{sign}\, E\, \phi_V\, 2N^2 \,\mathrm{coco}(q;-\sqrt{2/\lambda}\;|g|;\sqrt{3\lambda/2}\;\phi_V) \end{aligned} \tag{7-3}$$

The square of the form factor for the configuration in Fig. (5-1) is presented on logarithmic scale in Fig. (7-1) as a function of Rq.

8. DERIVATION OF A PHENOMENOLOGICAL BAG MASS FORMULA

In the MIT bag model [17] the mass of a particle is given by the volume energy and correction terms due to quarks :

$$M = \frac{4\pi}{3} B R^3 + \frac{\alpha_c}{R} - \frac{Z_0}{R} + \text{fine structure} \tag{8-1}$$

where R is the radius of the spherical bag, B the volume energy density. α_c and Z_0 are related to the motion of fermions inside the bag [18].

This formula and the theory behind it explains well the mass-spectrum of single hadrons. There is a natural difficulty associated with this geometrical model when it comes to describe the interaction between two hadrons.

On the other hand every field theory should derive a mass formula similar to the above (8-1), in terms of mean values of extension of the particle.

The problem is to express the mass of the particle:

$$M = \sqrt{\frac{3\lambda}{2}} \phi_v^3 - g \phi_v \tag{8-2}$$

in terms of the size of the system and a bag constant. The constant γ, that is substracted from the potential in the Lagrangian (2-1), is a natural candidate for the bag constant.

We define for our model a bag-constant:

$$B = -\gamma = \frac{2}{3}\lambda\,\phi_V^4 \qquad (8\text{-}3)$$

such that B is positive. Only the gluon distribution length is explicitly useable as a radius. This is very much in the spirit of the bag models where the bag is determined by the gluons.

From Eqs. (6-3) and (8-3) we obtain :

$$\phi_V = 0.80305\ \lambda^{-1/2}\ R_b^{-1} \quad ; \quad \text{and} \quad BR_b = \frac{2}{3}\,0.80305\ \lambda^{1/2}\ \phi_V^3 \ . \qquad (8\text{-}4)$$

Therefore the mass formula (8-2) is cast into the form:

$$M = 1.1438\ B(2R_b) - 0.80305\ \ g\ \lambda^{-1/2}\ \frac{1}{R_b} \ . \qquad (8\text{-}5)$$

This formula of the energy in terms of the radius and the constant $B = -\gamma$ is the analog in the one dimensional case of the phenomenological bag formula.

In one space dimension there is no distinction between volume and linear size or radius.

There is however an important difference between the formula (8-1) and (8-5). The sign of the energy of the energy of the quarks is negative in our case but is positive in the MIT-model Eq. (8-1). This would make it impossible to find the physical value for R_b from the minimum of M: $\partial M/\partial R_b = 0$, because there is no real solution in R_b

to this equation. However, this proceedure is not necessary because R_b in this field theoretical model is calculated as a expectation value, whereas in the MIT-model R_b is a geometrical parameter that in the end is calculated from $\partial M/\partial R_b = 0$.

9. RESONANCES AND THE STABILITY OF THE GROUND STATE

Excited states of the gluon field are given by fluctuations in time about the classical fields ϕ_1 and ϕ_2.

Because of the nonlinear selfinteraction of the gluon field one can make progress only by choosing a convenient expansion of the fields and calculate the field correction perturbatively.

The expansion of the fields in our case is obtained by adding a perturbation P(t,x) [19], to the classical fields ϕ_1, ϕ_2:

$$\tilde{\phi}_1(t,x) = \phi_1(x) + P(t,x) \tag{9-1}$$

and

$$\tilde{\phi}_2(t,x) = \phi_2(x) - \frac{1}{\sqrt{3}} P(t,x) \tag{9-2}$$

such that the new fields $\tilde{\phi}_1$ and $\tilde{\phi}_2$ satisfy the same algebraic relation (3-7) as the classical fields ϕ_1, ϕ_2:

$$\tilde{\phi}_1(t,x) + \sqrt{3}\,\tilde{\phi}_2(t,x) = \phi_V \tag{9-3}$$

that has the property to decouple the gluon field equations. The expansion of the fields (9-1) and (9-2) may be viewed as an expansion of the χ function (3-6) alone by writing the fields in the form (3-5)

$$\tilde{\phi}_1(t,x) = \frac{1}{4}\,\phi_V(1 + 3\tilde{\chi}(t,x)) \tag{9-4}$$

$$\tilde{\phi}_2(t,x) = \frac{\sqrt{3}}{4}\,\phi_V(1 - \tilde{\chi}(t,x)) \tag{9-5}$$

where

$$\tilde{\chi}(t,x) = \chi(x) + \frac{4}{3}\,\phi_V^{-1}P(t,x) = \chi(x) + \Sigma(t,x)\ . \tag{9-6}$$

With the ansatz (9-4) and (9-5) the coupled gluon equations (3-1a,b) reduce to a single equation for $\tilde{\chi}$ [12]:

$$(\partial_o^2 - \partial_t^2)\ \tilde{\chi} = -2\alpha^2(\tilde{\chi}^3 - \tilde{\chi}) \tag{9-7}$$

where $2\alpha^2 = 3\lambda\phi_V^2$. The equation for $\Sigma(t,x)$ that follows from (9-7) is:

$$(\partial_o^2 - \partial_1^2)\Sigma = -2\alpha^2[(3\chi^2 - 1)\Sigma + 3\chi\Sigma^2 + \Sigma^3]\qquad . \tag{9-8}$$

We solve for Σ in a linear approximation by neglecting Σ^2 and Σ^3 in (9-8). The linearized equation might be considered as well to be the equation for P(t,x):

$$(\partial_o^2 - \partial_1^2)\ P(t,x) = -2\alpha^2(3\ \tanh^2\ \alpha\ (x - x_o) - 1)\ P(t,x) \tag{9-9}$$

(the proportionality factor between Σ and P drops out). The ansatz (A is a constant):

$$P(t,x) = A\cos(Et)\,\phi(x) \tag{9-10}$$

leaves the gluon fields real and reduces (9-9) to a second order differential equation in the space coordinate:

$$\frac{d^2}{dx^2}\,\phi(x) + [(E^2 + 2\alpha^2) - 6\alpha^2 \tanh^2 \alpha\,(x - x_0)]\,\phi(x) = 0\ . \tag{9-11}$$

The general method of integration of this equation is given in Ref. [20].

There are two sets of solutions that are finite at infinity:

1. $E^{(1)} = \sqrt{3}\ \alpha\ ;\ P^{(1)}(t,x) = A\cos(Et)\,\mathrm{sech}(\alpha x)\,\tanh(\alpha x)$ (9-12)

and

2. $E^{(2)} = 2\alpha\ ;\ P^{(2)}(t,x) = A\cos(Et)\left[1 - \frac{3}{2}\,\mathrm{sech}^2(\alpha x)\right]$. (9-13)

Both solutions are consistent with the approximation that leads to the linearized equation if the maximal values of (9-12) $P^{(1)}_{max} = A/2$; $\Sigma^{(1)}_{max} = 2A\phi_V^{-1}/3$ and of (9-13) $P^{(2)}_{max} = A$, $\Sigma^{(2)}_{max} = 4A\phi_V^{-1}/3$ satisfy the inequality $\Sigma \gg \Sigma^2 \gg \Sigma^3$. This works best when $\phi_V \gg (4/3)A$.

The excitation energy can be expressed in two ways. First as the positive eigenvalues of the field perturbation $P(t,x)$. Then the masses of the resonances are:

$$M^{*(k)} = M + E^{(k)} \quad , \quad (k = 1,2) \tag{9-14}$$

with $E^{(1)} = \sqrt{3}\,\alpha$ and $E^{(2)} = 2\alpha$ ($\alpha = \sqrt{\frac{3\lambda}{2}}\ \phi_V$).

And second the excitation energy can be expressed as a perturbation of the field energy. In this case the mass of the resonances is:

$$M^{*(k)} = M + \frac{1}{T}\int_0^T dt \int_{-\infty}^{\infty} dx \Delta T_{00}^{(k)}(t,x) \quad (k = 1,2) \tag{9-15}$$

where $T = 2\pi/E^{(k)}$ is the period of time oscillations. The energy density of the perturbation up to second order in $P(t,x)$ is:

$$\Delta T_{00}(t,x) = \frac{2}{3}(\partial_0 P)^2 + \partial_1 \chi \partial_1 P + \frac{2}{3}(\partial_1 P)^2 + 3\lambda\phi_V^3 \chi(\chi^2 - 1)\, P$$
$$+ 2\lambda\phi_V^2(3\chi^2 - 1)\, P^2 + O(\phi_V^{-2} P^2) \quad . \tag{9-16}$$

The field energy (9-16) contains the amplitude A, that cannot be determined from Eq. (9-11) because of the linear approximation taken. Demanding that both sources of information on the energy of the resonance give the same answer we can calculate A. The knowledge of A is of value only in the case when the interaction of the resonance with other particles is investigated.

There is no change in the quark energy due to the perturbation of the gluon field because the quark-gluon interaction energy:

$$j_1\tilde{\phi}_1 + j_2\tilde{\phi}_2 = j_1(\tilde{\phi}_1 + \sqrt{3}\,\tilde{\phi}_2) = j_1\phi_v \tag{9-17}$$

is the same as for the unperturbed fields.

The correction to the energy density (9-15) is an even function in x for the first state of excitation (9-12) but is a mixture of even and odd functions for the second excitation (9-13). The dipole moment of the first resonance therefore vanishes. The second resonance has a time dependent dipole moment equal to:

$$D^{(2)}(t) = \frac{1}{M^*}\int_{-\infty}^{\infty} dx\ x\ \Delta T_{00}^{(2)}(t,x) = A\cos(Et)\ \frac{2(1-\phi_v)}{M^*}\ , \tag{9-18}$$

M* is the mass of the resonance.

The excitation energy of the gluon field is larger than the inverse size of the gluon-bag R_b Eq. (6-3). This is an expression of an universal law of quantum theory [21].

The existence of perturbations of finite energy of the classical soliton solutions is a proof of their stability. An example of an energy spectrum is shown in Fig. 9-1.

10. COLORED STATES AND GLUEBALLS

We can think of removing one quark. The energy of this process may depend on the way the quark is removed. Only if the energy (or the action) becomes infinite for any conceivable process of removing one quark we have perfect confinement of the quarks [22].

The action becomes infinite if quarks are removed as plane waves. Hence the removal of a quark that will continue to exist as a free quark is forbidden Eq. (5-15).

Models that use a single scalar gluon field ϕ with the selfinteraction $(\phi^2 - f^2)^2$ and a single quark ψ coupled in a Yukawa form $\bar{\psi}\psi\phi$ escape the above argument because despite the constant quark current $\bar{\psi}\psi$ = const, the action is zero

$$S \sim \text{const} \int_{-\infty}^{\infty} dx \tanh (x - x_0) = 0.$$

In such models [23] free quarks are predicted to exist.

Returning to our model there is, however, a possibility to remove one quark from a particle and attach the same quark as a bound state to another particle. This process violates the theorem Eq. (4-3) and thus costs energy. However, the energy required in finite and the action associated with it remains finite too.

Colored States

To calculate the energy of a particle from which one quark has been removed we set ψ_1 or ψ_2 zero, while preserving the original wavefunction of the quark that stays behind.

In both cases the second current disappears:

$$j_2(x) = 0 \tag{10-1}$$

The interaction energy in the two possible cases of the quarks is proportional to:

$$1. \quad \psi_1 = 0 \quad T^c_{00} \sim j^c_1\phi_1 = -\overline{\psi}_2\psi_2\phi_1 \tag{10-2}$$

$$2. \quad \psi_2 = 0 \quad T^c_{00} \sim j^c_1\phi_1 = \overline{\psi}_1\psi_1\phi_1 \quad . \tag{10-3}$$

With the original values of the fields

$$\overline{\psi}_1\psi_1 = -\overline{\psi}_2\psi_2 \tag{10-4}$$

therefore

$$j_1^{colored} = \frac{1}{2}\, j_1^{neutral} \quad . \tag{10-5}$$

It makes no difference which quark is removed, the change in mass will be the same. The integrated energy of the quarks is

$$T_{00}^{colored} = g2N^2(\mathrm{sign}E)(\mathrm{sign}g)\sin\frac{\pi}{6}\int_{-\infty}^{\infty} dx e^{-2H(x)}\, \frac{1}{4}\,\phi_v(1 + 3\tanh\alpha x) \quad . \tag{10-6}$$

The hyperbolic tangent does not contribute to the integral being an odd function:

$$T_{00}^{\text{colored}} = g2N^2(\text{sign } E)(\text{sign } g)\sin \frac{\pi}{6} \frac{1}{4} \phi_v \int_{-\infty}^{\infty} dx e^{-2H(x)} \tag{10-7}$$

$$= g \frac{1}{4} \phi_v(\text{sign } E)\ (\text{sign } g) \frac{1}{2} \int_{-\infty}^{\infty} dx\ j_1^{\text{neutral}} \tag{10-8}$$

The integral of j_1 is one, this is the normalization condition (Chapter 6). Thus:

$$T_{00}^{\text{colored}} = g \frac{1}{8} \phi_v(\text{sign } E)(\text{sign } g) \quad . \tag{10-9}$$

The mass of a colored system with either quarks ψ_1 or ψ_2 removed is

$$M^c = \sqrt{\frac{3\lambda}{2}}\, \phi_v^3 - \frac{1}{8} g\phi_v = M + \frac{7}{8}\ g\ \phi_v \tag{10-10}$$

where M is the mass of the complete neutral system Eq. (5-10).

The mass of the colored system is therefore larger than the ground state.

<u>Glueballs</u>

Removing finally two quarks $j_1 = 0$, $j_2 = 0$ leaves a glueball behind with the mass:

$$M^g = M + \ g\ \phi_v = \sqrt{\frac{3\lambda}{2}}\, \phi_v^3 \tag{10-11}$$

The mass spectrum is shown in Fig. 9-1.

The size of the colored state does not differ much from the size R_b of the glueball shown in Fig. 6-1.

The colored state has an asymmetric mass distribution, i.e., a mass dipole.

The process discussed in this chapter becomes possible if two neighbor particles exchange one or two quarks. The exchange has to be made in accordance with the Pauli exclusion principle. Then the energy of a three quark system is the same as that of a single quark system given by Eq. (10-10). Once the quarks are exchanged the two colored particles or the two glueballs attract each other violently at short distances, but cease to interact at large distances. If enough energy is spend to exchange the quarks and afterwards to overcome the attraction of the particles it is possible to separate colored states at artibrary distance from each other with a finite amount of energy. This result has validity only in the framework of this model. Calculations will be presented in the second part of this work.

11. CONCLUSION

In this paper we study a model of gluons and quarks with the internal discrete symmetry in color space Z(3)--the cyclic group of order three. The gluons represented by scalar classical fields have zero spin. The selfinteraction of the gluons is polynomial, with a quartic and cubic term, that suggested the name of polynomial chromodynamics. The quarks are classical spinor fields with spin 1/2.

There are only two colors present in the model, e.g., red and blue. However, this is enough to display the essential features of the color interaction in the same way as the minimal spin-1/2 or isospin-1/2 systems display all characteristics of systems with larger spin or isospin.

The effect of two colors is that mesons and baryons have the same structure. The mesons are made of a quark and an antiquark whereas the baryons are made out of two quarks with the two colors red and blue. Colors can add up to zero either by combining colors with their anticolors or by saturation attained by mixing all colors together. The form of the Lagrangian written in matrix form Eq. (2-8) seems especially suited for generalization to three colors.

Initially there are five parameters in the model λ, ν, μ, γ and g. The four parameters λ, ν, μ and γ in the Lagrangian determine the selfinteraction of the gluous. In order to obtain a solvable system they have to be related to each other, and only two of them can finally assume arbitrary values. Instead of using two independent coupling constants we choose λ and the vacuum field ϕ_V as the free parameters.

The quarks are massless and the strength of their interaction with the gluons is given by the coupling constant g.

The model contains, therefore, three free parameters λ, ϕ_V, and g. All three parameters appear explicitely in different physical quantities like the mass of the composite system in the ground state M Eq. (5-10) the size of the gluon-bag R_b Eq. (6-3) the masses of the resonances M*, M** (Eqs. (9-14)) etc. Therefore, we can substitute for the parameters three physical significant quantities in any reasonable combination like M, M*, R_b etc.

In 1 + 1 dimensions λ is measured in units of mass squared and will set the scale for all measurable quantities.

Prior to any calculation we obtain from the interaction imposed by the discrete internal symmetry group theorem (Eq. (4-3)) that holds in two or four dimensions, and states that the two quark fields ψ_1 and ψ_2 that solve the field equation do no separate. This is the basis for confinement at low energies. Of course in a classical field theory one can violate this theorem and the field equations at the expense of energy.

The field equations are solvable in a closed form in 1 + 1 dimensions when the gluons are subjected only to their selfinteraction and not influenced by the quarks, but the motion of the quarks is determined by the gluons.

Whereas in 1 + 1 dimensions this change (Eqs. (2-17)→(3-1)) in the field equations is useful in order to solve the resulting field equations, because there exist stable soliton solutions, it is well known that in four dimensions it is precisely this trick that is not going to work as there are no spherical symmetric soliton solutions [24] of the modified field equations (Eq. (3-1)). In four dimensions the feedback of the quarks and gluons cannot and should not be neglected.

We take advantage of this situation in lower dimensions where the calculations are easy and transparent to develop and explore the concepts associated with quark-gluon physics.

The ground state is a bound system of quarks and gluons. The gluon field is more intens in the vacuum than inside a composite particle. The color exchange is proportional to the gluon field intensity. This means that the quarks shy away from the vacuum where intense color exchange would take place and prefer energetically to localize in the area of less intense color exchange, i.e., the center of the soliton configuration of the gluon field.

The mass (Eq. (5-10)) of the ground state when expressed in terms of the size (Eq. (6-3)) of the particle (the size is calculated as an expectation value) resembles the mass formula of the phenomenological bag model (Eq. (8-5)) with the two terms of volume energy and quark energy.

Time dependent oscillations of the gluon field, similar to vibrations of the gluon bag give the resonance states (Eqs. (9-12) and (9-13)).

Next, we disregard theorem (4-3) and remove one quark. This process requires a finite amount of energy. The mass of the state deficient in one quark increases.

Finally, removing the last quark leaves a glueball behind that is even heavier. If the increase in mass due to quark removal is measured in percent then the removal of the first quark contributes 87.5% (7/8 parts) of the total jump in mass from the ground state to glueball, and the removal of the second quark contributes only the remaining 12.5% (1/8 parts) (Eq. (9-14)).

The size of the colored states and glueballs relative to the ground state depends on the value of the coupling constant g between quarks and gluons. This can be read off Fig. (6-1). There is a g_0 such that if $0 < g < g_0$ the withdraw of quarks makes the system increase in size and the geometrical scattering cross section would be larger. If $g > g_0$ the system will shrink in size and the scattering cross section will be smaller for colored states and glueballs than for complete configurations. The value of g_0 depends on mass of the ground state M and λ.

In conclusion this very simple model seems to contain the essentials of quark-gluon physics and gives a variety of single composite particle configuration.

REFERENCES

[1] M. Gell-Mann, Acta Phys. Austriaca, Suppl. IX (1972), 733.

[2] O. W. Greenberg, Phys. Rev. Lett. 13 (1964), 598.

[3] H. Fritzsch, M. Gell-Mann, and H. Leutwyler, Phys. Lett. 47B (1973), 365.

[4] M. Gell-Mann, Phys. Lett. 8 (1964), 118.

[5] C. N. Yang and R. Mills, Phys. Rev. 96 (1954), 191.

[6] R. Utiyama, Phys. Rev. 101, (1956), 1597.

[7] H. Yukawa, Proc. Phys.-Mat. Soc. Japan 17 (1935), 48; Rev. Mod. Phys. 21 (1949), 474; H. Yukawa and C. Kikuchi, Am. J. Phys. 18 (1950), 154.

[8] M. B. Kisslinger, Phys. Lett. 79B (1978), 474.

[9] C. DeTar, Phys. Rev. D 17 (1978) 323, Errata, Phys. Rev. D 19 (1979), 1028; Phys. Rev. D 19 (1979), 1451.

[10] H. J. Pirner, Phys. Lett. 85B (1979), 190.

[11] L. W. Jones, Rev. Mod. Phys. 49 (1977), 717, and "Quark Searches" in Review of particle properties, Rev. Mod. Phys. 52 (1980), S108-S111; G. S. LaRue, J. D. Phillips and W. M. Fairbank, Phys. Rev. Lett. 46 (1981) 967.

[12] H. M. Ruck, Nucl. Phys. B 167 (1980), 320.

[13] W. E. Thirring, Ann. Phys. 3 (1958), 91.

[14] H. M. Ruck, Lecture Notes in Physics, vol. 135 (1980), 307, Springer Verlag, Berlin.

[15] I. S. Gradsteyn and I. M. Ryzhik, Table of Integrals, Series, and Product, Fourth Edition, Academic Press, New York (1965).

[16] M. Abramowitz and I. A. Stegun, Handbook of Mathematical Functions, Dover Publications, Inc., New York (1972).

[17] A. Chodos, R. L. Jaffe, K. Johnson, C. B. Thorn and V. F. Weisskopf, Phys. Rev. D 9 (1974), 3471.

[18] K. Johnson, "A simple model of the ground state of quantum chromodynamics," preprint SLAC-PUB-2436, November (1979).

[19] R. Jackiw, Rev. Mod. Phys. 49 (1977), 681.

[20] P. M. Morse and H. Feshbach, "Methods of Theoretical Physics," McGraw-Hill, New York (1953), p. 1651.

[21] V. F. Weisskopf, Comments Nucl. Part. Phys. 9 (1980), 49.

[22] L. C. P. van Hove, private communication.

[23] W. A. Bardeen, M. S. Chanowitz, S. D. Drell, M. Weinstein and T. M. Yan, Phys. Rev. D 11 (1975), 1094.

[24] G. Derrick, J. Math. Phys. 5 (1964), 1252.

ACKNOWLEDGEMENTS

This work was supported by the Director, Office of Energy Research, Division of Nuclear Physics of the Office of High Energy and Nuclear Physics of the U.S. Department of Energy under Contract W-7405-ENG-48.

FIGURE CAPTIONS

Fig. (2-1) Equipotential lines $V(\phi_1,\phi_2)$ = const. of the potential part of the Lagrangian. The topology displays the symmetry under rotation by multiples of 120° in the field space. There are three minima corresponding to the three vacuum states of the gluon field. The parameters are $\lambda = 1$, $\phi_V = 3$.

Fig. (3-1) The gluon fields ϕ_1 and ϕ_2 (Eq. (3-5)), with the center of the soliton at $x = 0$. The parameters are $\lambda = 1$, $\phi_V =$ 2.6717 . . . At $x = +\infty$ the fields occupy the vacuum state $(\phi_V,0)$ at $x = -\infty$ the vacuum state $(-1/2\ \phi_V, \sqrt{3}/2\ \phi_V)$. The tunneling from one state to the other takes place over the saddle point between the two vacuum states (Fig. (2-1)).

Fig. (5-1) The internal energy distribution of a composite particle made of gluons and quarks (Eq. (5-9)). The upper curve (the gluon field energy) is the first term in Eq. (5-9) and the lower curve (the quark-gluon interaction energy) is the second term in Eq. (5-9). The heavy curve with a dip in the center is the sum of both energies. The quark contribution to the energy is negative and thus lowers the energy of a glueball and explains why quarks prefer to enter into a bound state with gluons. The mass of the particle (integrated field energy) is $M = 10$—obtained with the following combination of parameters $\lambda = 1$, $\phi_V = 2.6717$, $g = 5.0$.

Fig. (5-2) A sequence of energy (matter) distributions (Eq. (5-9)) of composite particles of constant mass $M = 10$, $\lambda = 1$, as a function of the quark-gluon coupling constant g.

Fig. (6-1) The size of the composite particle of constant mass $M = 10$, $\lambda = 1$, as a function of the quark-gluon coupling constant g. The monotonically descending line is the size of the gluon-bag (or glueball) that is obtained if at each point the quarks would be removed from the composite particle. The mass of the gluon-bag is therefore increasing from the left to the right.

Fig. (7-1) The square of the formfactor as a function Rq (R is the size of the particle, q the momentum transfer) of a particle of mass $M = 10$, $\lambda = 1$, $\phi_V = 2.6717$, $g = 5$.

Fig. (9-1) The mass spectrum of a composite particle. The mass of the ground state is chosen to be ten ($M = 10$), to give an easy reference point for the rest of the spectrum. The coupling constants are $\lambda = 1$, $g = 5$ and the value of the vacuum field is $\phi_V = 2.6717$. To the left are the excitations of the particle with intact structural composition, to the right the excitations obtained by removing the quarks. The mass is measured in units of λ.

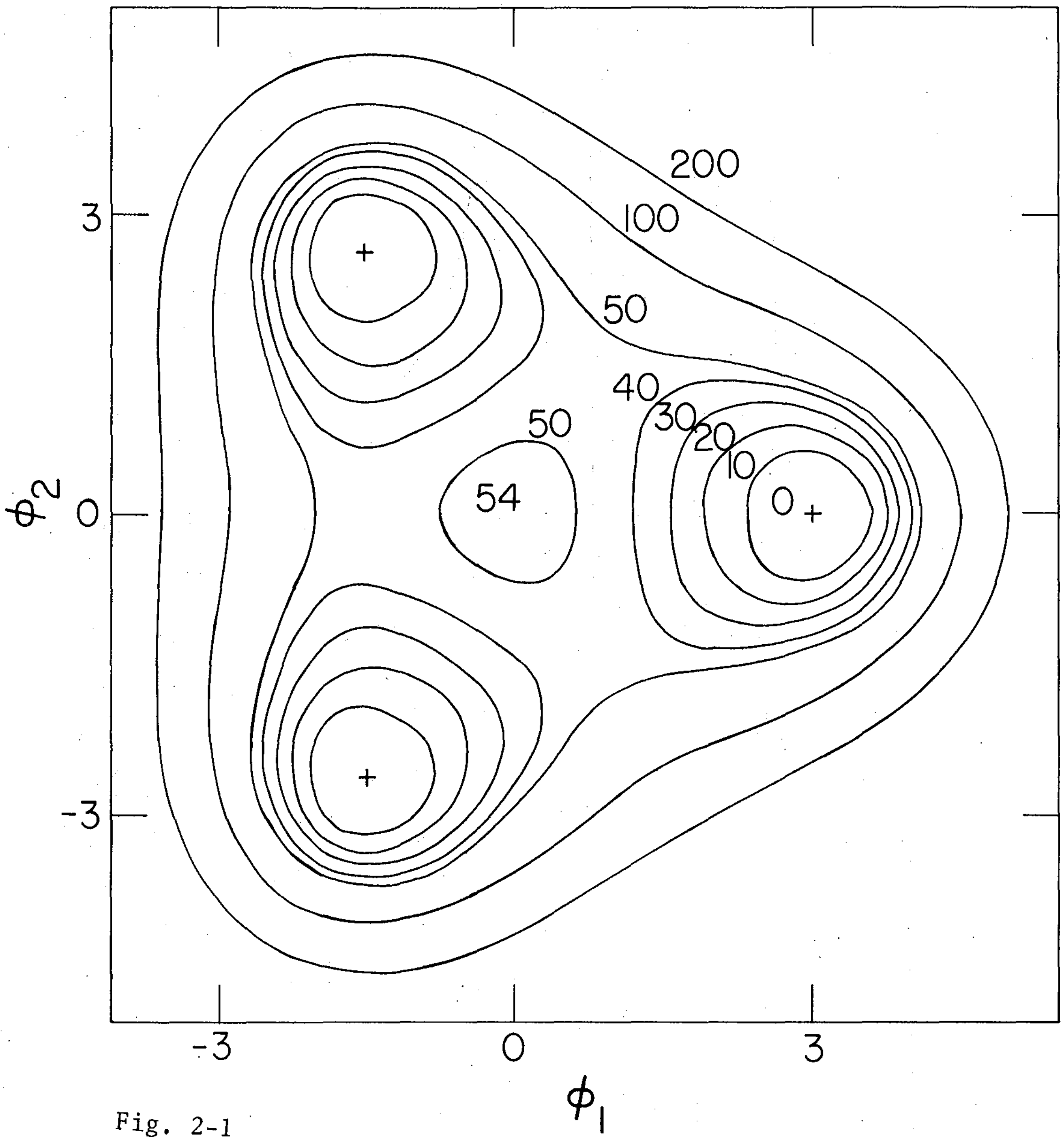

Fig. 2-1

XBL 812-2164

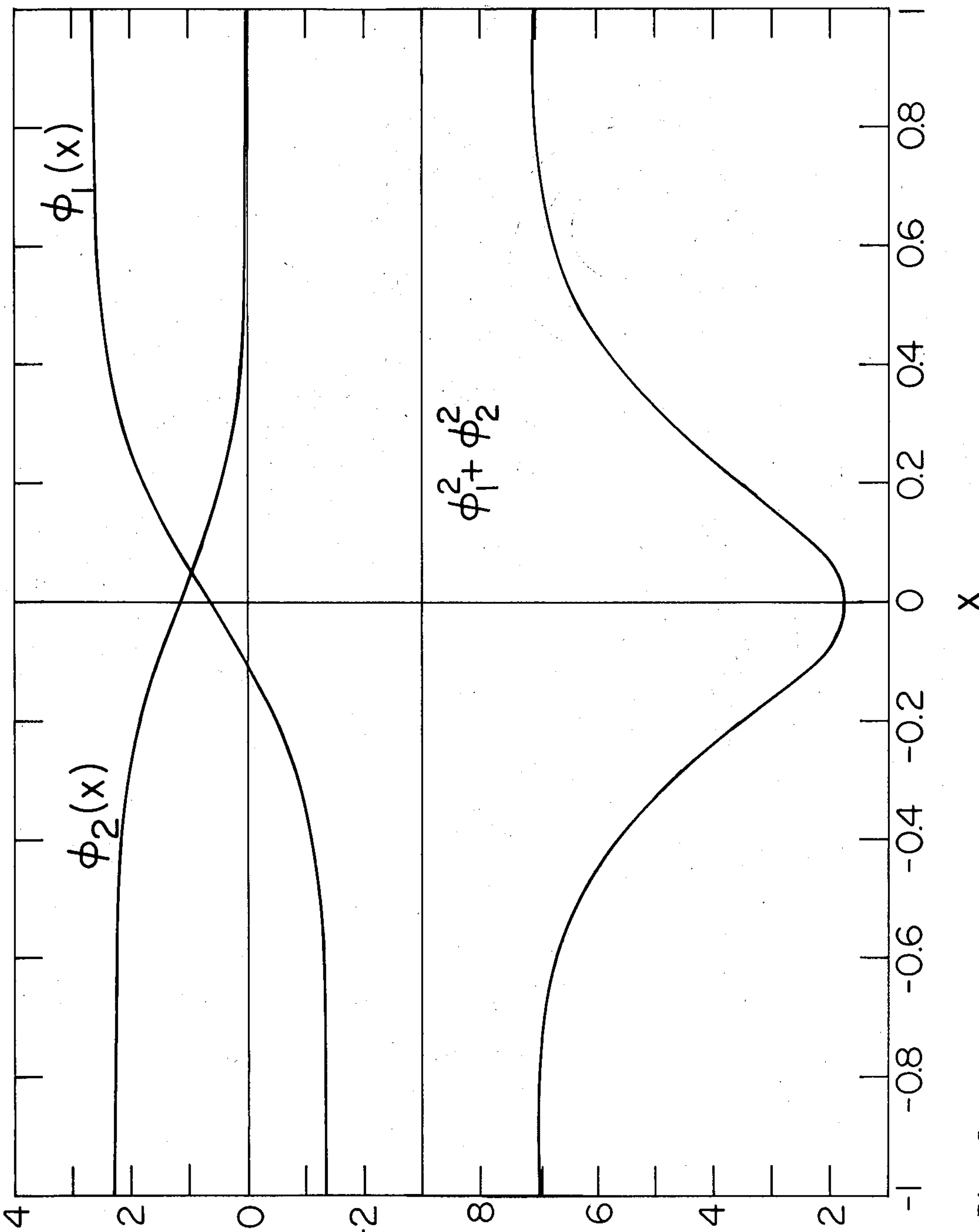

Fig. 3-1

XBL814-2236

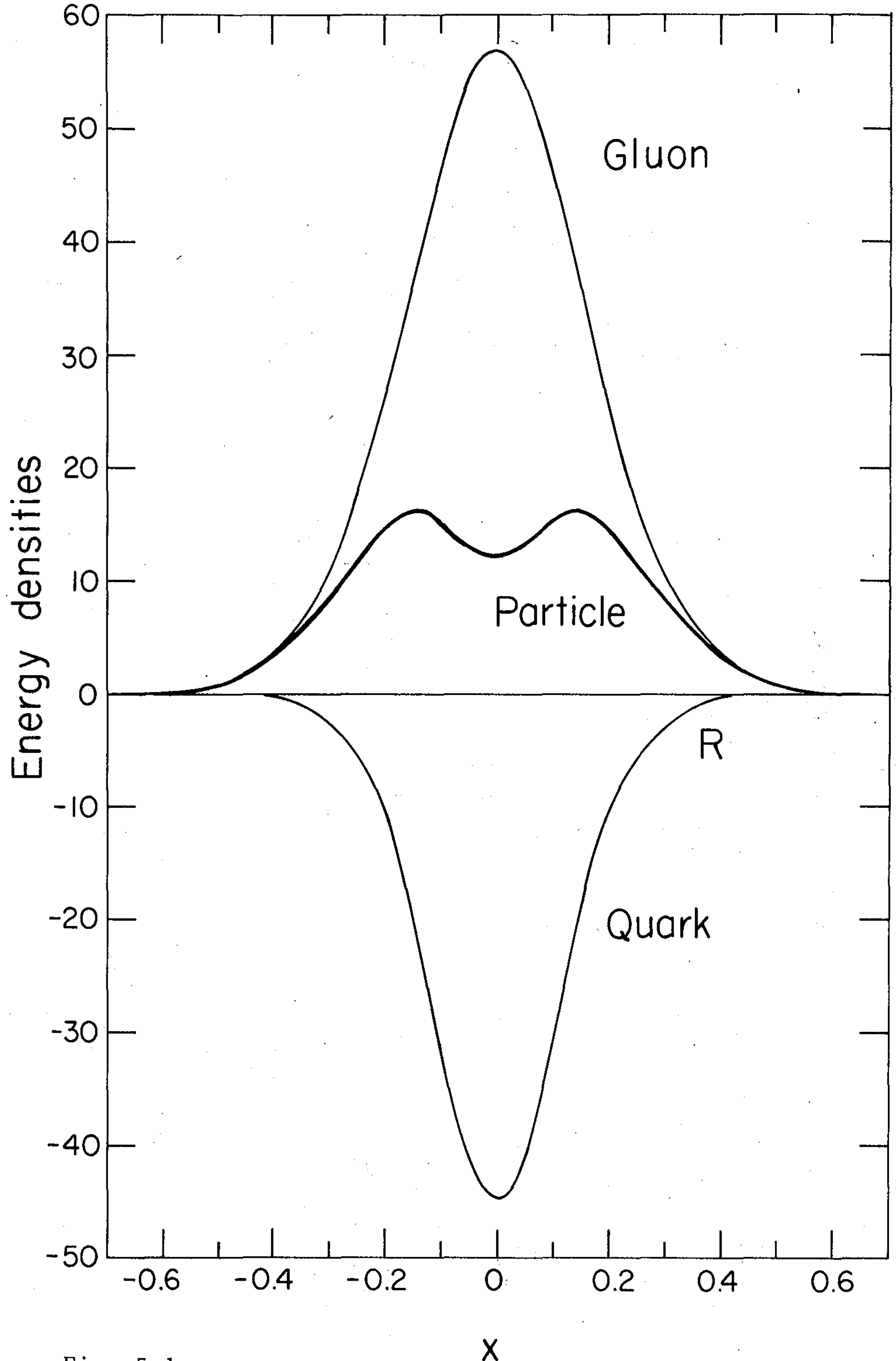

Fig. 5-1

XBL812-2155

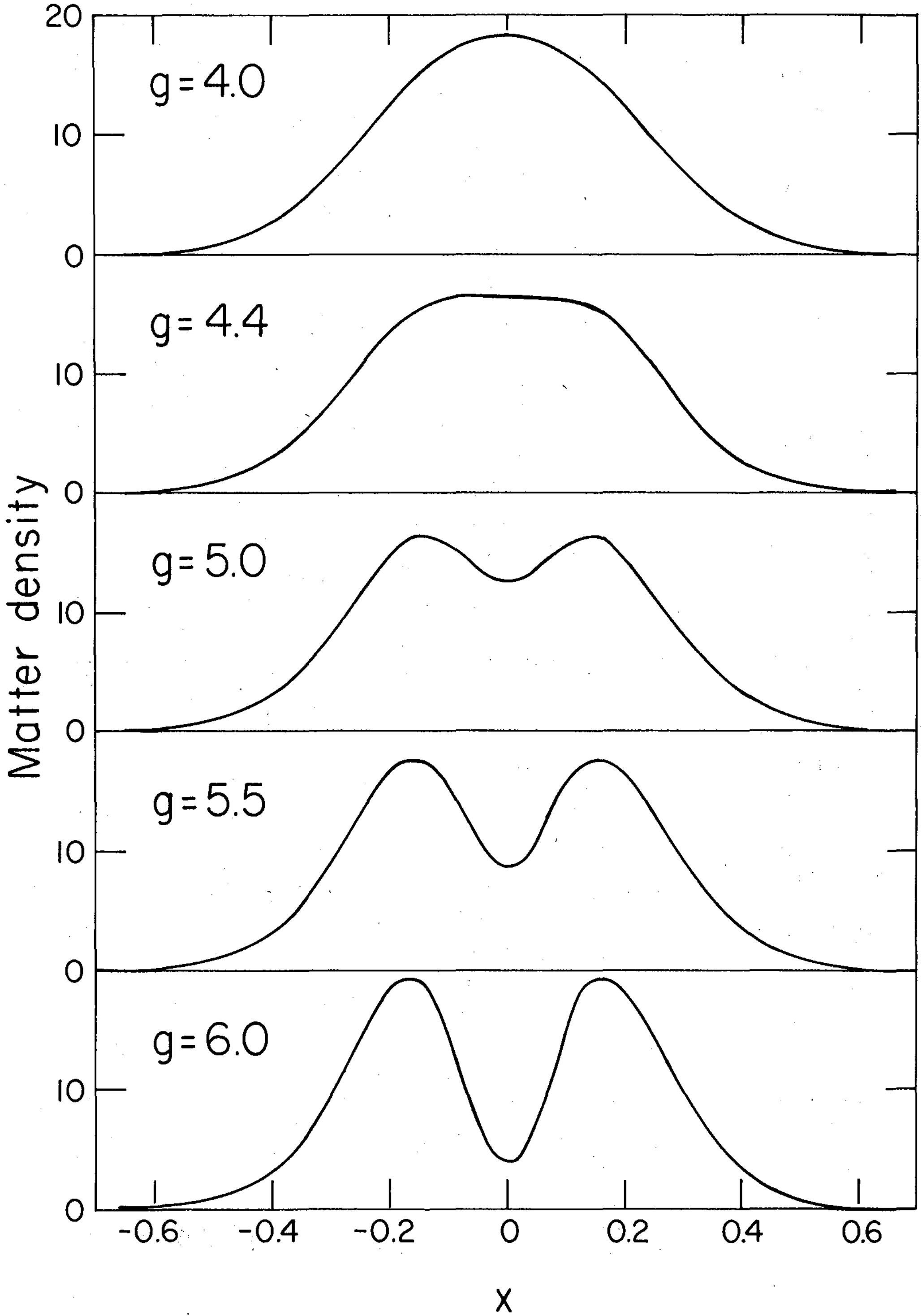

Fig. 5-2

XBL 812 - 2161

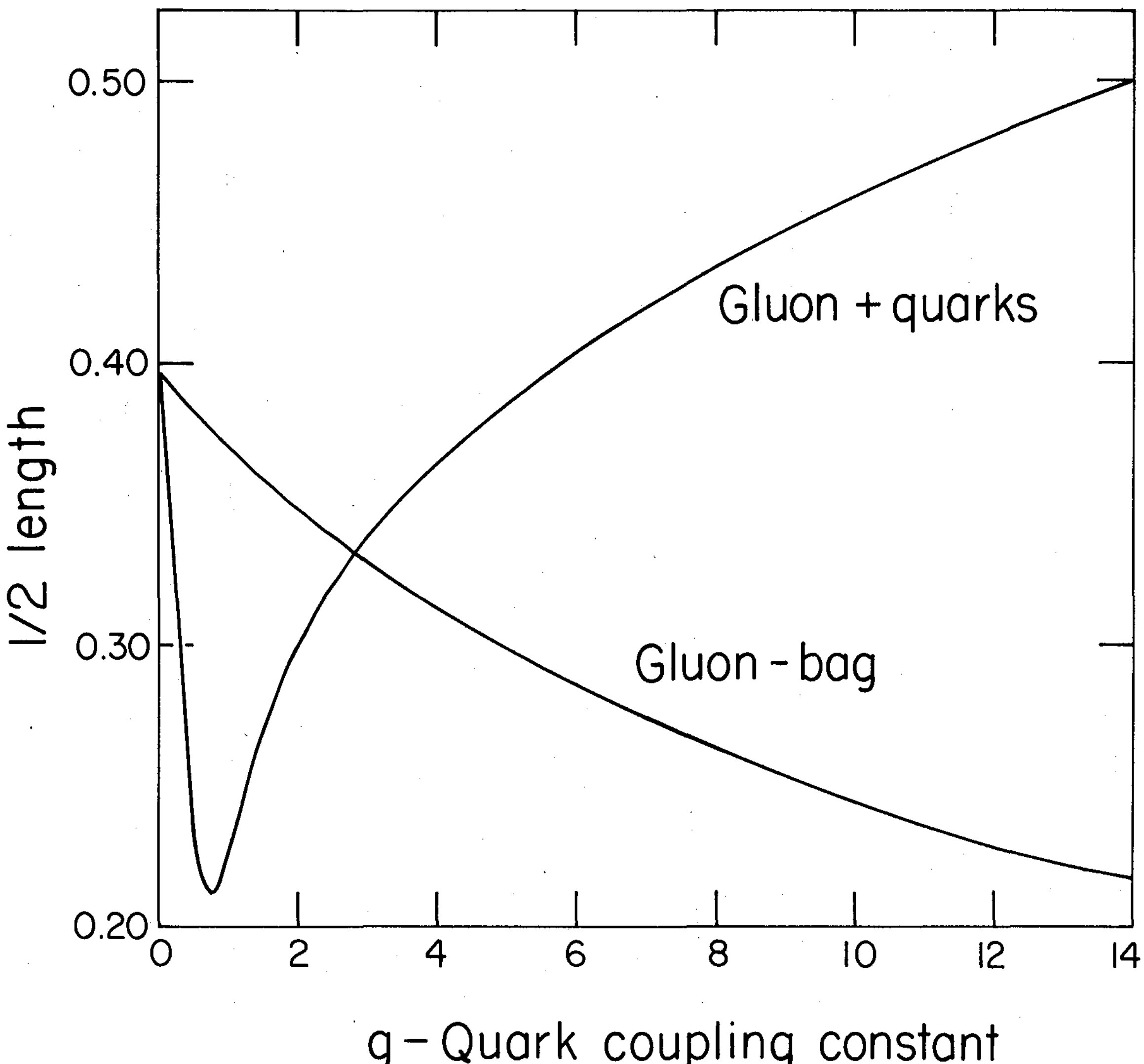

Fig. 6-1

XBL 812-2159

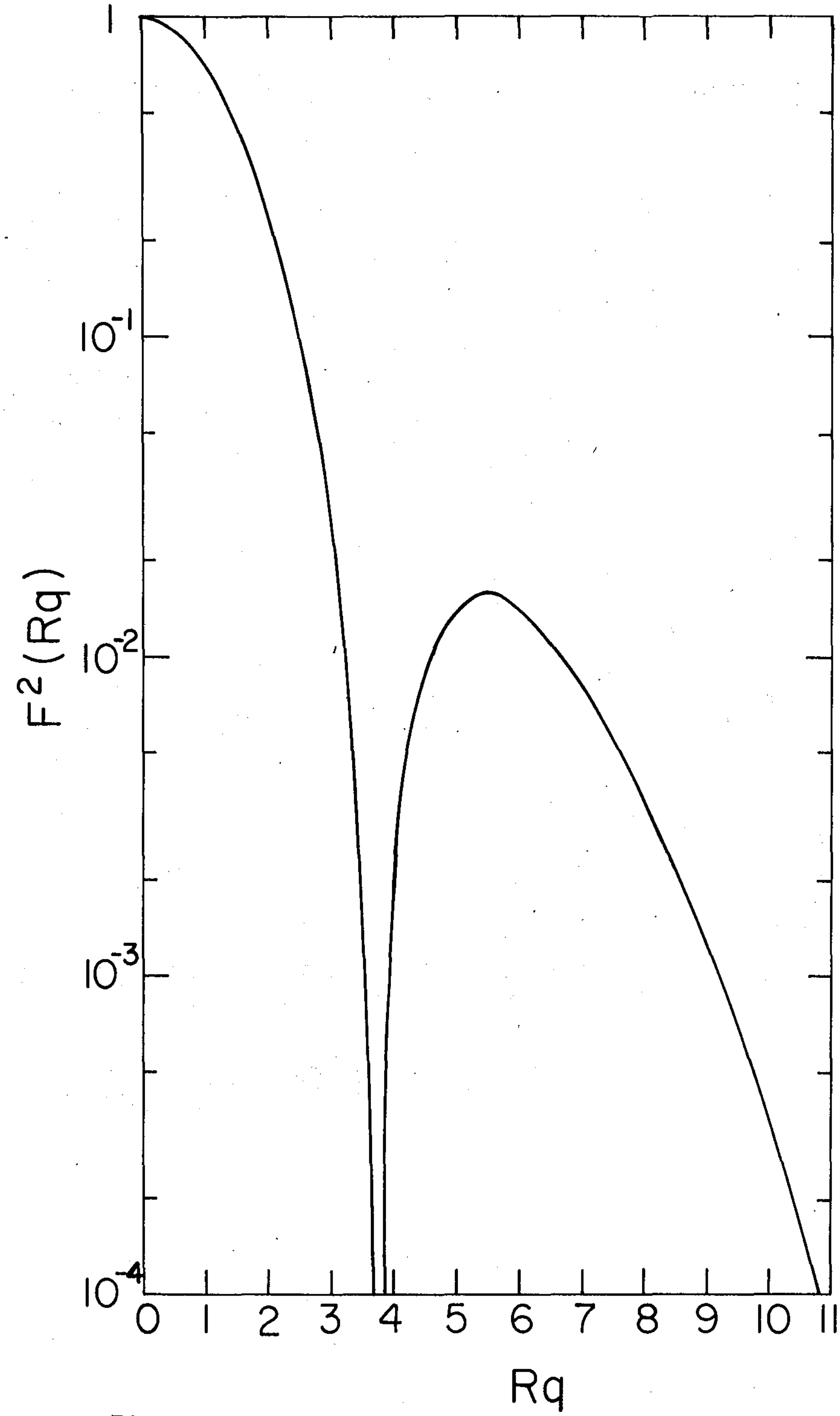

Fig. 7-1

XBL 812-2154

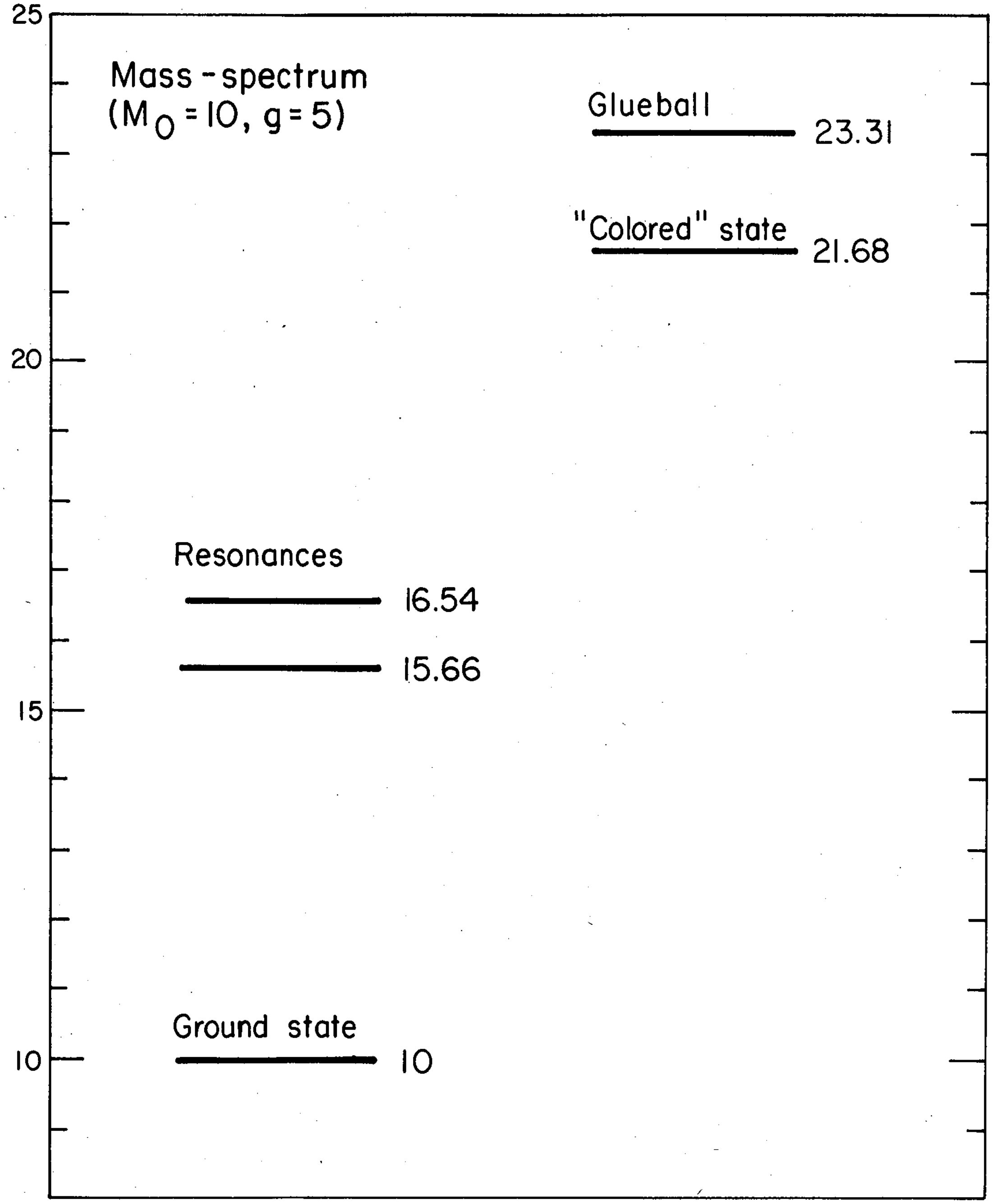

Fig. 9-1

XBL 814-2227