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COHERENT PION PROCESSES IN NUCLEAR COLLISIONS

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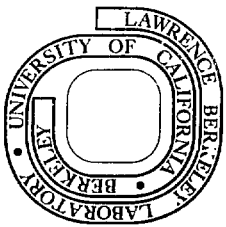

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[illegible] that pionic instabilities could occur [illegible] are reviewed and specific dynamical effects of [illegible]. Then experimental signatures of pionic [illegible] are [illegible]. The single nucleon inclusive cross section [illegible] distribution are shown not to be sensitive to [illegible]. In particular, no copious pion production [illegible]. Finally, [illegible] the $\pi^-\pi^-$ inclusive cross section [illegible] tool to measure the degree of coherence of the [illegible] collisions.

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Introduction

One of the main reasons for studying high energy nuclear collisions is the hope that under the extreme conditions ($\rho \gtrsim 2\rho_0$, $E^* \sim 100$ MeV/nucleon) generated in such collisions new phenomena will be observed associated with the coherent interactions of a large number of nucleons. In this talk I will discuss the possibility that pionic instabilities could occur under those conditions. Other exotic possibilities at high nuclear densities such as density isomers and quark matter are discussed elsewhere in these proceedings (see talks of W. Greiner and A. Kerman). Here, I shall concentrate on coherent pion processes.

The discussion is organized into the following sections.

1. Pion condensation[1] and pionic instabilities[2,3] in excited nuclear systems.
   a) Thermal equilibrated, T ~ 50-100 MeV
   b) Non-equilibrium, E_{kin} ~ GeV/nucleon
2. Dynamical effects of instabilities[2]
   a) Critical scattering phenomena
   b) Unstable pion fields
3. Pion Production
   a) Multiplicity distributions[4]
   b) Pion lasers and the $2\pi^-$ inclusive[5]

1. Pionic Instabilities

The first model calculations of pion condensation were carried out for nuclear matter in its ground state (i.e. T = 0). Since the pioneering work of A. B. Migdal[6], the literature on this problem has been growing steadily (see talk of W. Weise elsewhere in these proceedings as well as Ref. 7 for further references). The present status of such calculations is that if pion condensation in non-excited nuclear matter occurs at all it would be for densities $\rho_c \gtrsim 2\rho_o$. In fact, the most recent calculations of Weise[8] and Meyer-ter-Vehn[9] cast considerable pessimism about the likelihood of pion condensation in finite nuclei.

The point made in Ref. (8) is that the density dependence of the effective nucleon mass $m^*(\rho)$ tends to increase ρ_c very much. The point made in Ref. (9) is that the correlation parameter g, as determined from low lying excited states of finite nuclei, is significantly larger ($g = 0.7 \pm 0.1$) then previously estimated[6,7], $g = 0.5 \pm 0.1$. These two points decrease very much the possibility that pion condensation could occur in non-excited nuclear matter.

However, nuclear matter formed during heavy ion collisions is very far from its ground state and can easily reach densities well over $2\rho_o$. The following points should then also be kept in mind. (1) While $m^*(\rho)$ is a decreasing function of ρ, $m^*(\rho,T)$ may be an increasing function of T. Thus the effect of $m^*(\rho,T)$ on pion condensation could well be less for $T > 0$ systems. (2) The value of g determined from finite nuclei takes into account the strong two body correlations that keep nucleons apart. However, in excited nuclear systems, especially during the non-equilibrium phase of the collisions, the effective

value of $g(E^*)$ may be less than $g(E^* \approx 0)$ since the nucleons can approach each other much closer due to their high relative momenta. Of course nothing is really known about $g(E^*)$, and we can only treat g as an unknown parameter for excited systems. The sensitivity of the results to variations in g must then always be emphasized.

Another important point to note is that collective instabilities in non-equilibrium systems can be fundamentally different from those in equilibrated systems. The instabilities that occur in colliding plasmas are particularly good examples of this (see Ref. (2)). As we will also emphasize below, the most dramatic effects of pionic instabilities are in fact expected during the non-equilibrium phase of the collision.

It is clear that no definitive statements can be made about the existence or non-existence of pionic instabilities in excited nuclear systems. Therefore, the calculations reported here are only presented in the spirit of exploration of what exotic phenonema could at least in principle occur in nuclear collisions. Eventually, we must turn to experiments for definitive statements. For now we simply explore what could happen and what observables could be sensitive to novel phenomena.

The central quantity in these investigations is the pion propagator[6,7]

$$\Delta(\omega,\vec{k}) = (\omega^2-k^2-m_\pi^2 - \Pi(\omega,\vec{k}))^{-1} \quad , \qquad (1)$$

where Π is the pion proper self energy or the polarization operator that results from the πN and NN interactions in the medium. The form

of Π that includes the strong P-wave πNN and $\pi N\Delta$ interactions as well as hard core NN and $N\Delta$ interactions is[2,3,6,7]

$$\Pi = \frac{\Pi_{NN} + \Pi_{N\Delta}}{1-g/k^2(\Pi_{NN} + \Pi_{N\Delta})} \quad . \tag{2}$$

See Refs. (2,3,6,7) and the talk of W. Weise in these proceedings for further details.

The main reasons for studying $\Delta(\omega,\tilde{k})$ are as follows:

(1) The pion spectrum $\omega(\tilde{k})$ in nuclear systems (including collective excitations (phonons) carrying the pion *quantum numbers*) can be obtained from $\Delta^{-1}(\omega,\tilde{k}) = 0$.

(2) The linear response of the medium to a pion depends on the residue of $\Delta(\omega,\tilde{k})$ at its singularities.

(3) The decay rate of excited nuclear systems due to pionic instabilities can be estimated via[2,3]

$$\Gamma/V = \mathrm{Re}\int \frac{3d^4k}{(2\pi)^4} \log \varepsilon(\omega,\tilde{k}) \tag{3}$$

where the pion "dielectric" function

$$\varepsilon(\omega,\tilde{k}) = \frac{\Delta_o(\omega,\tilde{k})}{\Delta(\omega,\tilde{k})} \left(1+\frac{g}{k^2}\Pi(\omega,\tilde{k})\right)^{-1} \quad . \tag{4}$$

If there exist <u>complex</u> roots of $\Delta^{-1}(\omega \pm i\gamma,\tilde{k}) = 0$, i.e. complex zeros of $\varepsilon(\omega,\tilde{k})$, then linear response theory shows[2] that the system is unstable. In particular, *small spin-isospin perturbations* in the system are <u>amplified</u> (i.e. grow exponentially in time) at a rate γ. Of course, external perturbations are not necessary to bring on this instability. Quantum fluctuations are sufficient to induce the decay of the system when $\gamma \neq 0$ via Eq. (3).

To study whether pionic instabilities could occur in excited nuclear systems, two extreme models of the nucleon momentum distribution were considered in Refs. (1-3):

(a) Thermal equilibrated

$$n(\tilde{p}) = \left\{ \exp\left[(p^2/2m - \mu)/T \right] + 1 \right\}^{-1} \quad , \tag{5}$$

(b) Non-equilibrium

$$n(\tilde{p}) = \theta\left(p_F - |\tilde{p} + \tilde{p}_{cm}|\right) + \theta\left(p_F - |\tilde{p} - \tilde{p}_{cm}|\right) \quad . \tag{6}$$

With Eq. (5), we calculated the pion analogue of the Curie temperature in ferromagnets. With Eq. (6), we studied the analogue of the two stream instability in colliding plasmas.

The results for the thermal system are shown in Fig. I. The essential feature to note is the rather steep slope of $T_{crit}(\rho)$, indicating that for $\rho > (2\text{-}3)\ \rho_o$ quite a high temperature is required to destroy this <u>second order</u> phase transition. In fact, the solution of the Rankine shock equation (see Ref. 6 in Ref. 1) indicates that for the densities expected during nuclear collisions, the temperature is likely to be below T_{crit} for $\rho > 2\rho_o$. Of course, these results are sensitive to g, but if the appropriate value of g is less than 0.6, then the above conclusion remains valid.

Encouraged by those results, we turn next to the non-equilibrium phase of the collisions where Eq. (6) is appropriate. Again complex roots of $\Delta^{-1} = 0$ can be found.[2,3] The contour plot of the growth rates $\gamma(k_\pi, \theta_\pi)$ (in units of 0.1 m_π) are shown in Fig. II. The angle θ_π is

that between $\tilde{k}_\pi$ and the beam axis $\tilde{p}_{cm}$. In Fig. II the plot for $p_{cm} = 4m_\pi$ (kinetic energy ~ 670 MeV/nucleon) is shown as a typical example. The essential points we focus on here are (1) unstable modes are again found for $g \leq 0.6$, (2) the average growth rates per mode are $\langle\gamma\rangle \sim 0.1\ m_\pi \approx 2 \times 10^{22}\ \text{sec}^{-1}$, and (3) the momenta of the unstable modes is $k_\pi \approx (2\text{-}3)\ m_\pi$. Clearly the exact boundaries of phase space where unstable pion modes occur depend sensitively on g (compare Figs. IIa, IIb). Nevertheless, the features mentioned above are not so sensitive to g. The importance of (2) is that it indicates that during the non-equilibrium phase (lasting $\sim 5 \times 10^{-23}$sec) at least a few phonons can be created in each unstable mode, i.e. where $\gamma(\tilde{k}) \neq 0$. Note in particular that only a few phonons can be created in a given mode $\tilde{k}$, but there are many unstable modes. Typically, the number of such modes is found to be[2,3] $\sim V/\lambda_\pi^3$. Therefore, the term pion condensation is not appropriate for nuclear collisions, since a condensate implies that there are many phonons in a few modes. On the other hand, the term pionic instabilities is quite suitable.

The importance of feature (3) is that the wavelengths of those unstable modes are then small ~(3-5) fm. Therefore, such phonons can exist in the finite dimensions of the nuclear systems considered.

We conclude from these calculations that provided the effective strength of g is less than 0.6, pionic instabilities can occur during nuclear collisions in spite of (1) the high excitation energies, (2) the short interaction times, and (3) the small nuclear dimensions. Having thus shown the possibility of pionic instabilities, we turn now to what effects they could have on the dynamics of nuclear collisions.

2. Dynamical Effects[2]

There are two major effects of collective instabilities familiar from solid state and plasma physics:

a) the modification of two body scattering rates via critical scattering phenomena,

b) the growth of strong collective fields via spontaneous phonon pair production.

The first effect can be easily pictured as follows: In free space, two particles scatter via the exchange of bare mesons so that in the Born approximation their scattering amplitude is proportional to the free meson propagator $\Delta_o(\omega,\tilde{k})$. In a many body medium, the propagation of a meson is modified by its interactions with the medium as illustrated in Fig. IIIa. This interaction "dresses" the meson thereby converting it to a phonon, i.e. a coherent many particle-many hole state. The scattering amplitude is then proportional to the dressed propagator $\Delta(\omega,\tilde{k})$. Therefore, the effective cross section of two particles which are distinguishable from the constituents of the medium is related to the free space cross section via

$$d\sigma_{eff} = \frac{1}{|\varepsilon(\omega,\tilde{k})|^2}\, d\sigma_o \qquad . \qquad (7)$$

Equation (7) is the familiar plasma kinetic theory result relating $d\sigma_{eff}$ and $d\sigma_o$ via the dielectric function.

Critical scattering occurs when there exists a $\omega = 0$ phonon mode $\tilde{k}_c$, i.e. such that $\varepsilon(0,\tilde{k}_c) = 0$. In that case $d\sigma_{eff} \to \infty$!

As an aside we note that it is easy to see why the existence of $\omega = 0$ modes is intimately related to collective instabilities. A phase transition occurs when the state ϕ of the system becomes degenerate with some other states ϕ'. When $\omega = 0$ modes are available, ϕ becomes degenerate with the set of states ϕ' differing from ϕ by any number of these soft phonon excitations. Consequently, the system decays by building these collective excitations into itself. In the case of pion condensation the new phase is characterized by a spin-isospin lattice.[6,7]

Returning to the scattering mechanism, we note that Eq. (7) is not generally applicable. Self-consistency and finite size effects modify Eq. (7) as[2,3]

$$d\sigma_{eff} = P(\omega,\tilde{k})d\sigma_o \tag{8}$$

where the polarization from factor $P(\omega,\tilde{k})$ is much less singular[2,3] than $|\varepsilon(\omega,\tilde{k})|^{-2}$. For detailed discussion of Eq. (8) refer to Refs. (2,3). The only point we stress here is that the scattering rates are modified due to pionic instabilities as in Eq. (8).

The second major effect of instabilities, i.e. the growth of collective fields, can be understood in terms of spontaneous phonon pair creation as illustrated in Fig. IIIb for $\pi^+\pi^-$ phonons. This is associated with those quantum fluctuations in the system involving virtual phonon pair creation. For example, a virtual π^+ meson is created at some time together with a neutron particle-proton hole ($n\bar{p}$) excitation. As the π^+ propagates through the medium its interactions (further particle-hole excitations) converts it into a π^+ phonon. Similarly, as the $n\bar{p}$ excitation propagates through the medium, it

is converted into a π^- phonon. Note that we use the term phonon here as a generic name for the quanta associated with the singularities of $\Delta(\omega,\tilde{k})$. As mentioned before, they are coherent many particle-many hole states of the system.

If there exists an $\omega \approx 0$ mode for some $\tilde{k}_c$, then the virtual fluctuation described above can "materialize" since both energy and momentum can be conserved. As more and more phonons are created in this way, a structural change (i.e. phase transition) of the system occurs. In addition, after sufficiently many phonons are created the pion field will acquire a finite expectation value; $\langle\phi(x)\rangle \neq 0$. This is quite analogous to the growth of collective electric fields as a result of plasma instabilities in colliding plasmas.

Both effects of collective instabilities described above are closely related.[2] In fact the only difference between (a) and (b) is that in (a) the energy momentum $(\omega,\tilde{k})$ of the phonons is on the particle-hole branch of the phonon spectrum. These phonons simply decay to 1 particle - 1 hole excitations. The decay of a pair of such phonons is thus completely equivalent to a two body scattering as illustrated in Fig. IIIa.

Having discussed these effects, we turn to an estimate of their importance in nuclear collisions. To that end we note the results of Refs. (2,3), where it was shown that the decay rate of the system can be estimated from its complex correlation energy (sum of ring diagrams in RPA) giving

$$\Gamma = \Gamma_{col} + \Gamma_{scat}$$

$$= \sum_{\tilde{k}} 3\gamma(\tilde{k}) + \frac{1}{2} \sum_{\tilde{p}_1,\tilde{p}_2} n(\tilde{p}_1) n(\tilde{p}_2) \sigma^{eff}(\tilde{p}_1\tilde{p}_2) \frac{|\tilde{p}_1 - \tilde{p}_2|}{mV} \quad , \qquad (9)$$

In Eq. (9), Γ_{col} measures the rate of spontaneous phonon pair creation for phonons that do not decay to 1p-1h excitation, and Γ_{scat} is the Boltzmann collision integral measuring the rate of two body scattering in the medium. The effective cross section in Γ_{scat} includes both elastic ($NN \to NN$) and inelastic ($NN \to N\Delta$ and $NN \to \Delta\Delta$) contributions.

The polarization form factor in Eq. (8) was computed in Ref. (3) (Eq.(26)). It was found that $P(\omega,\tilde{k})$ had logarithmic singularities that led to typical enhancements of $\sigma_{eff}/\sigma_o \sim (2\text{-}4)$ for <u>both</u> the elastic and inelastic cross sections. In addition, we found[3] that $\Gamma_{col} < \Gamma_{scat}$ in these models calculations indicating that effect (a) should dominate effect (b). In other words, two body scattering is expected to dominate mean field effects.

The first important conclusion we draw from these calculations is that we do <u>not</u> expect any drastic effects of pionic instabilities! What we can expect is a <u>moderate</u> increase of the scattering rates that would lead to more rapid thermalization in the system. Taken together with $\Gamma_{col} < \Gamma_{scat}$, these results suggest then that a hydrodynamic or cascade approach to nuclear dynamics is most appropriate.

We can now see the following dilemma with regard to observing effects of such instabilities. If pionic instabilities occur, then they will lead to more rapid thermalization than if $d\sigma_{eff}$ were

identical to $d\sigma_o$ in Eq. (8). However, once the nucleons reach thermal equilibrium, the memory of the interesting dynamical path is lost! Therefore, the bulk of the single nucleon inclusive cross section, $d\sigma_N$, would not show any effects of such instabilities.

It may seem at first sight that the "knock-on" contribution[10] to $d\sigma_N$ would be sensitive to the form of $d\sigma_{eff}$ in Eq. (8). This knock-on contribution is in the region of phase space beyond the quasi-elastic peak, i.e. the high momentum end of the nucleon spectrum. Those nucleons are thought to be projectile nucleons that have undergone only one scattering in the target. Therefore, it is tempting to assume that this knock-on region of $d\sigma_N$ could be used to determine $P(\omega,\tilde{k})$. However, most of the nucleons observed at high momenta come from the peripheral region of the interaction region. That is simply because the number of mean free paths a projectile nucleon must traverse is much smaller in the perpheral region than in the central region. (Experimentally, the perpheral nature of the knock-on contribution could be tested by looking for an $A^{1/3}$ dependence.) In a peripheral collision, however, only a small amount of nuclear matter interacts, and, consequently, collective phenomena would be greatly supressed, i.e. $P(\omega,\tilde{k}) \to 1$ in peripheral collisions. Therefore, we would only measure the free space cross sections in the knock-on region.

We conclude, therefore, that the nucleon inclusive cross section is not expected to be sensitive to dynamical effects of pionic instabilities. In the next section, we therefore turn to pion production with the hope that it is more sensitive to such phenomena.

3. Pion Production

A. Multiplicity Distribution

In Ref. (3), we found that one possible effect of pionic instabilities is to enhance the inelastic rates $\Gamma(NN \to N\Delta)$ by a factor $\sim(2\text{-}4)$. It is then natural to ask whether this should lead to an enhancement of the average number of pions $\langle n_\pi \rangle$ produced. As we shall see, the answer is no!

To see why $\langle n_\pi \rangle$ remains approximately the same, note that pions are produced mainly during the non-equilibrium phase of the collision. After the nucleons reach thermal equilibrium with $T \sim 50$ MeV, the relative momenta are generally below π production threshold. Simple estimates, in fact, reveal that π production rates during equilibrium are smaller by a factor ~ 10 or more than the rates during the non-equilibrium phase, where the relative momenta are well above threshold. Therefore, we can estimate

$$\langle n_\pi \rangle \approx \Gamma(NN \to N\Delta)\ \tau_{th} \quad , \tag{10}$$

where τ_{th} is a characteristic thermalization time

$$\tau_{th} \propto \frac{1}{\sigma_{NN}^{tot}} \propto \frac{1}{\Gamma(NN \to NN) + \Gamma(NN \to N\Delta)} \quad . \tag{11}$$

Now if both the elastic and inelastic rates are increased by a factor X (due to pionic instabilities), then pions are indeed produced X times as fast, but only for a time 1/X as long. This decrease of the thermalization time compensates for the increase of the π production rate!

We therefore do not expect copious pion production as a result of the pionic instabilities discussed here.

To check whether the observed pion multiplicity distribution[11] can in fact be accounted for without copious pion production, we report here on two simple model calculations:

a) Thermodynamic Fireball Model[4]

b) Non-equilibrium Impulse Approximation

In (a) the pions are assumed to come to chemical as well as thermal equilibrium[12] with the nucleons. The average pion multiplicity in a fireball of volume V and temperature T is then[12]

$$\langle n_\pi \rangle = \int \frac{3Vd^3k}{(2\pi)^3} \left(e^{\omega_k/T} - 1\right)^{-1} \quad , \tag{12}$$

The thermodynamic calculation[4] for the multiplicity distribution $P(n_\pi)$ is found to give a convoluted multiple Poisson. However, for the volumes and temperatures expected in nuclear collision at energies $\lesssim$ 2 GeV/nucleon, it was shown in Ref. (4) that $P(n_\pi)$ reduces to a simple Poisson for each impact parameter b, i.e.

$$P(n_\pi;b) = e^{-\langle n_\pi(b)\rangle} \frac{\langle n_\pi(b)\rangle^{n_\pi}}{n_\pi!} \quad (1 \pm 0.05) \quad , \tag{13}$$

with correction $\lesssim$ 5%.

The results of that calculation[4] for the π^- distribution in Ar + Pb_3O_4 at 1.8 GeV/nucleon are shown in Fig. IVa. Curve 2, which incorporates the trigger bias of the data,[11] is seen to correctly account for all features of the data. Figure IVb shows the contributions,

Eq. (13), to $P(n_\pi)$ from different impact parameters. Note, in particular, that Fig. IVa shows that no copious pion production is observed.

Of course, the Poisson aspect of $P(n_\pi,b)$ in Eq. (13) is not severely tested in Fig. IVa. It would be, in fact, very desirable to isolate experimentally the $b \approx 0$ contribution to $P(n_\pi)$ to test for the Poisson form. As shown in Ref. (4), deviations from the Poisson form could serve as strong evidence for unusual correlated pion production mechanisms. The point is that the average pion multiplicity $\langle n_\pi \rangle$ can be correctly accounted for by a large variety of production mechanisms. However, the fluctuations around $\langle n_\pi \rangle$, i.e. the form of $P(n_\pi;b)$, is sensitive to correlated pion production mechanism.

An example of a completely different (uncorrelated) production mechanism that yields identical results to the thermal model is model (b). The assumption here is that pions are produced only during the first collision each projectile nucleon suffers. Then

$$\langle n_\pi(b) \rangle = \langle n_\pi \rangle_{pp} \, A_p(b) \quad , \tag{14}$$

where $A_p(b)$ is the number of projectile nucleons that interact with the target at a fixed b, and $\langle n_\pi \rangle_{pp} \approx 0.7$ is average number of pions produced per nucleon-nucleon collision at these energies. (The average number of π^- is one-third of Eq. (14)). Equation (14) together with the fact that the multiplicity distribution in elementary NN collisions is approximately Poisson then imply that $P(n_\pi,b)$ is a Poisson in Eq. (13) with mean in Eq. (14). Numerically, this model gives the same quality of fit as curve 2 in Fig. IVa.

To conclude this discussion of $P(n_\pi)$, we emphasize again that the only way to gain information on coherent pion processes from $P(n_\pi)$ is by isolating a single impact parameter (preferably $b \approx 0$) and by looking for deviations from Poisson.

B. Pion Lasers and the $\pi^-\pi^-$ inclusive

In searching for more sensitive tools to study pion coherence, we turn next to the $\pi^-\pi^-$ inclusive cross section. As we shall see, this offers perhaps the most exciting possibility to eventually determine the degree of coherence of the pion field.

The basic idea here is simply to extend familiar concepts of quantum optics[13] into the realm of pion physics. A detailed discussion of "Pion Quantum Optics" is in preparation.[5]

The starting point is the assumption that the pion source can be treated as a classical (c-number) function, i.e.

$$\mathscr{L}_{int} = \phi(\tilde{x},t)\ J\ (\tilde{x},t) \qquad , \qquad (15)$$

where ϕ is the pion Heisenberg field and $J(\tilde{x},t)$ is a space-time source function. For illustrative purposes ϕ is treated as a scalar field here. Complications due to the isospin structure of ϕ and J will be considered in Ref. (5).

In reality, J is a transition current operator involving the nucleon and possibly Δ_{33} field operators. The physical assumption we make is that as long as the amplitude of the pion field produced is small, we can replace J by its matrix elements between nuclear wave-functions. Thus the recoupling of the emitted pion fields to the nucleon fields is neglected. Since only a relatively small fraction[12]

($\lesssim$ 10%) of hadronic matter in nuclear collisions consists of pions, this may be a rather good approximation. We therefore assume $J(\tilde{x},t)$ is a given function of space-time.

With Eq. (15), the pion field $\phi(\tilde{x},t)$ can be explicitly solved for . More importantly, the S-matrix in the pion sector can be obtained in closed form[14]

$$S = \exp\left\{ i\int d^3k \left[j(k)a^+(k) + j^*(k)a(k) \right] \right\} \quad , \tag{16}$$

where $a^+(k)$, $a(k)$ are the creation and destruction operators of the pions and

$$j(k) = \int d^4x \, \frac{e^{ikx}}{\sqrt{(2\pi)^3 2\omega_k}} \, J(\tilde{x},t) \tag{17}$$

with $k_o = \omega_k = \sqrt{k^2 + m_\pi^2}$ being the (on-shell) space-time Fourier transform of the pion source current.

Given Eq. (16), the asymptotic outgoing pion field is given by

$$|\pi,\text{out}\rangle = S^+|0,\text{in}\rangle$$

$$= e^{-\bar{n}/2} \exp\left\{ -i\int d^3k \; j(k)a^+(k) \right\} \; |0,\text{in}\rangle \quad , \tag{18}$$

where

$$\bar{n} = \int d^3k \; |j(k)|^2 \tag{19}$$

The important point to note about Eq. (18) is that $|\pi,\text{out}\rangle$ is a coherent superposition of states with arbitrary numbers of pions. Such states are known as coherent states in quantum optics[13] and actually characterize laser fields! Thus, a classical source produces a laser field. We return to this point below.

However, first we note that the exclusive and inclusive distributions of pions are simply obtained with Eq. (16). Of particular interest is the inclusive probability $P_m(k_1, \ldots, k_m)$ for observing m pions with momenta $k_1,\ldots,k_m$:

$$P_m(k_1,\ldots,k_m) = \sum_{n=m}^{\infty} \frac{1}{(n-m)!}\int dk_{m+1}\ldots dk_n |\langle k_1\ldots k_n|S^\dagger|0\rangle|^2$$

$$= |j(k_1)|^2 \ldots |j(k_m)|^2 \quad . \tag{20}$$

This incredibly simple result has also been derived in Ref. (15). However, the connection of the j's to the pion source currents was not made there.

It is also easy to show that the multiplicity distribution of the pions is a simple Poisson with a mean $\bar{n}$ given in Eq. (19).

We can now define an m^{th} order correlation function

$$C_m(k_1,\ldots,k_m) = \frac{P_m(k_1,\ldots,k_m)}{P_1(k_1)\ldots P_1(k_m)} - 1 \quad . \tag{21}$$

For a pure classical source, Eq. (20) then shows that $C_m = 0$ for all m. This is in fact the distinguishing property of laser fields from chaotic

fields that we use to characterize what exactly is meant by a pion laser.

To see how chaotic fields are produced, consider a source $j(k)$ that is composed of an incoherent sum of sources j_i as

$$j(k) = \sum_{i=1}^{N} e^{i\phi_i} j_i(k) \quad , \tag{22}$$

where the ϕ_i are random phases. Then, the inclusive distribution in Eq. (20) must be averaged over these phases

$$P_m(k_1,\ldots,k_m) = \int_0^{2\pi} \frac{d\phi_1}{2\pi} \cdots \frac{d\phi_N}{2\pi} \quad |j(k_1)|^2 \ldots |j(k_m)|^2 \quad . \tag{23}$$

For a chaotic source (N>1) in Eq. (22), this averaging leads to $C_m \neq 0$. In fact, the Hanbury Brow-Twiss (HBT) effect, which for pions is called the GGLP effect,[16] can be derived from Eq. (23) in the limit $N \to \infty$. This is accomplished by considering sources in Eq. (22) of the form

$$j_i(k) = \int d^4x \, \frac{e^{ikx}}{\sqrt{(2\pi)^3 2\omega_k}} \quad f(\tilde{x} - \tilde{x}_i) g(t - t_i) \quad , \tag{24}$$

where f and g describe the spacial and temporal development of the individual currents, and the space-time points $(\tilde{x}_i, t_i)$ are randomly distributed with probability density $\rho(\tilde{x},t)$. It then follows[5] in the limit $N \to \infty$ that

$$C_2(\tilde{k}, \tilde{k}+\tilde{q}) = |\rho(\tilde{q}, \sqrt{\tilde{q}^2 + m_\pi^2})|^2 \quad . \tag{25}$$

Therefore C_2 measures directly the space time Fourier transform of the pion source region. Equation (25) is the GGLP effect[16] which this derivation shows holds only for completely chaotic sources ($N \to \infty$). While this limit is appropriate for light emerging from stars as in the original HBT effect, for pions from nucleon collisions, this limit may not be applicable. In particular, Eq. (25) does not hold if there is coherence in the pion field.[17]

An important feature of Eq. (25) is that the $\tilde{q} = 0$ point is

$$C_2(\tilde{k},\tilde{k}) \underset{\text{chaotic}}{=} 1 \quad . \tag{26}$$

for completely chaotic sources, which is in sharp contrast to

$$C_2(\tilde{k},\tilde{k}) \underset{\text{coherent}}{=} 0 \tag{27}$$

for coherent sources. An example of such a coherent source is one where all the j_i in Eq. (22) add with fixed phases.

It is now obvious that partially coherent fields can also be produced for which $C_2(\tilde{k},\tilde{k})$ is between 0 and 1. In particular for finite N, Eqs. (22,23) yield

$$C_2(\tilde{k},\tilde{k+q}) = \frac{\sum_{i\neq j=1}^{N} \left(j_i(\tilde{k})\, j_i^*(\tilde{k}+\tilde{q})\, j_j^*(\tilde{k})\, j_j(\tilde{k}+\tilde{q}) \right)}{\left(\sum_{i=1}^{N} |j_i(\tilde{k})|^2 \right) \left(\sum_{i=1}^{N} |j_i(\tilde{k}+\tilde{q})|^2 \right)} \quad . \tag{28}$$

Therefore, the $\tilde{q} = 0$ intercept is given by

$$C_2(\tilde{k},\tilde{k}) = 1 - \frac{\sum_{i=1}^{N} |j_i(\tilde{k})|^4}{\left(\sum_{i=1}^{N} |j_i(\tilde{k})|^2\right)^2} \tag{29}$$

$$= 1 - d(\tilde{k}) \tag{30}$$

We now define the degree of coherence of the pion mode $\tilde{k}$ as $d(\tilde{k})$ from Eqs. (29,30). The properties of $d(\tilde{k})$ that make it particularly attractive as the definition of that degree of coherence are

(1) $0 \leq d(\tilde{k}) \leq 1$

(2) $d(\tilde{k})$ could differ for different $\tilde{k}$

(3) $d(\tilde{k}) = 1$ for N = 1, i.e a pure coherent source as in Eq. (22) with fixed phases ϕ_i

(4) $d(\tilde{k}) = \frac{1}{N}$ for N identical chaotic sources with random ϕ_i

(5) $d(\tilde{k}) \to 0$ for completely chatoic source $(N \to \infty)$

In particular, Eqs. (26,27) are included as special cases of Eq. (30).

The important point we want to stress here is that not only the space-time evolution of the pion source, but also the degree of coherence of the pion field can be deduced from the second order correlation function. Experimentally, C_2 can be measured from the $2\pi^-$ and $1\pi^-$ inclusive cross sections.

This double virtue of $C_2(\tilde{k},\tilde{k}+\tilde{q})$ as illustrated in Fig. V has also been emphasized by Fowler and Weiner in Ref. (17). What we have

presented here is a simple field theoretic derivation from which the connection of these effects to specific properties of the pion source can be easily seen.

We are now in the position to discuss how the measurement of C_2 would be useful in looking for coherent pion processes as discussed in sections 1 and 2. As we discussed before, neither the nucleon inclusive cross section nor the average pion multiplicity are expected to be sensitive to pionic instabilities. On the other hand, $d(\tilde{k})$ provides a subtle measure, mode by mode, of coherence. Thus even if no unusual signal is observed in either the nucleon or pion distributions, $d(\tilde{k})$ could still be non-zero for certain critical modes ($k_c \sim (2\text{-}3)\ m_\pi$).

However, it is also clear that $d(\tilde{k})$ is not easy to measure. First, a fixed impact parameter ($b \approx 0$) must be isolated to avoid a b average in Eq. (23). Then high statistics in a small region of phase space are required. Finally, dynamical correlations from final state $\pi^-\pi^-$ interactions must be eliminated by methods such as those used in Ref. (18). Nevertheless, the effort is worth it! In the absence of any more spectacular signals, $d(\tilde{k})$ offers a powerful tool to look for coherent pion processes in nuclear collisions.

References:

1. V. Ruck, M. Gyulassy, W. Greiner, Z. Phys. A277, 391 (1976).
2. M. Gyulassy, W. Greiner, Ann. Phys. 109, 485 (1977).
3. M. Gyulassy, Proceedings of the Topical Conference on Heavy Ion Collisions, Fall Creek Falls State Park, 13 June 1977, (Conf. - 770602, October 1977, p. 457), LBL-6525 preprint.
4. M. Gyulassy, S. K. Kauffmann, Phys. Rev. Lett. 40, 298 (1978); S. K. Kauffmann, M. Gyulassy, Multiplicity Distributions of Created Bosons: The Combinants Tool, LBL-7700 preprint 1978.
5. M. Gyulassy, Y. Karant, S. K. Kauffmann, in preparation.
6. A. B. Migdal, O. A. Markin, I. I. Mishustin, Sov. Phys. JETP 39 212 (1974).
7. G. E. Brown, W. Weise, Phys. Rep. 27C, 1 (1976).
8. See contribution of W. Weise; Univ. of Regensburg preprint 1978.
9. See contribution of J. Meyer-ter-Vehn, Institute für Kernphysik Jülich, preprint 1978.
10. S. E. Koonin, Phys. Rev. Lett. 39, 680 (1977).
11. S. Y. Fung et al., Phys. Rev. Lett. 40, 292 (1978).
12. J. I. Kapusta, Phys. Rev. C16, 1493 (1977) and references therein.
13. J. R. Klauder, E.C.G. Sudarshan, Fundamentals of Quantum Optics, (W. A. Benjamin, Inc., New York, 1968).
14. J. D. Bjorken and S. D. Drell, Relativistic Quantum Fields, (McGraw-Hill, New York, 1965), pp. 202-207.
15. J. C. Botke, D. J. Scalapino, R. L. Sugar, Phys. Rev. D9, 813 (1974); see also F. Cooper, D. H. Sharp, Phys. Rev. D12, 1123 (1975).

16. R. Hanbury Brown, R. Q Twiss, Nature (London) 178, 1046 (1956);
G. Goldhaber, S. Goldhaber, W. Lee, A. Pais, Phys. Rev. 120, 300 (1960);
G. A. Kopylov, M. I. Podgoretskii, Sov. J. Nucl. Phys. 18, 336 (1974).

17. G. N. Fowler, R. M. Weiner, Phys. Letts. 70B, 201 (1977); and Phys. Rev. C to be published.

18. N. N. Biswas, et al., Phys. Rev. Lett. 37, 175 (1976); C. Enzell, et al., Phys. Rev. Lett. 38, 873 (1977).

Figure Captions:

I. Critical temperature for pion condensation as a function of density. T_{crit} is shown for g = 0.5 (See Ref. 1 for other g). Curves AB and BC illustrate the compression and decompression phases during nuclear collisions.

II a) Contour plot of growth rates $\gamma(k_\pi,\theta_\pi)$ for pion modes in non-equilibrium nuclear matter, Eq. (6). This example is for $p_{cm} = 4m_\pi$ and g = 0.5. Units of contours are multiples of 0.1 m_π.

b) Contour plot of growth rates $\gamma(k_\pi,\theta_\pi)$ for pion modes in non-equilibrium nuclear matter, Eq. (6). This example is for $p_{cm} = 4m_\pi$ and g = 0.6. Units of contours are multiples of 0.1 m_π.

IIIa) Illustrating difference between two body scattering in free space and in a many body medium

b) Illustrating spontaneous $\pi^+\pi^-$ phonon pair production.

IV a) π^- multiplicity distribution for Ar + Pb_3O_4 at 1.8 GeV/nucleon. Data from Ref. (11), curve 2 is the final curve for the thermal model from Ref. (4).

b) $P(n_\pi,b)$ for various impact parameters for Ar + Pb at 1.8 GeV/nucleon.

V. The two π^- correlation function for the general case of partial coherence. ρ is the Fourier transform of the space time region containing the pion source points (x_i,t_i) in Eq. (24). $d(\tilde{k})$ is the degree of coherence of pion mode $\tilde{k}$.

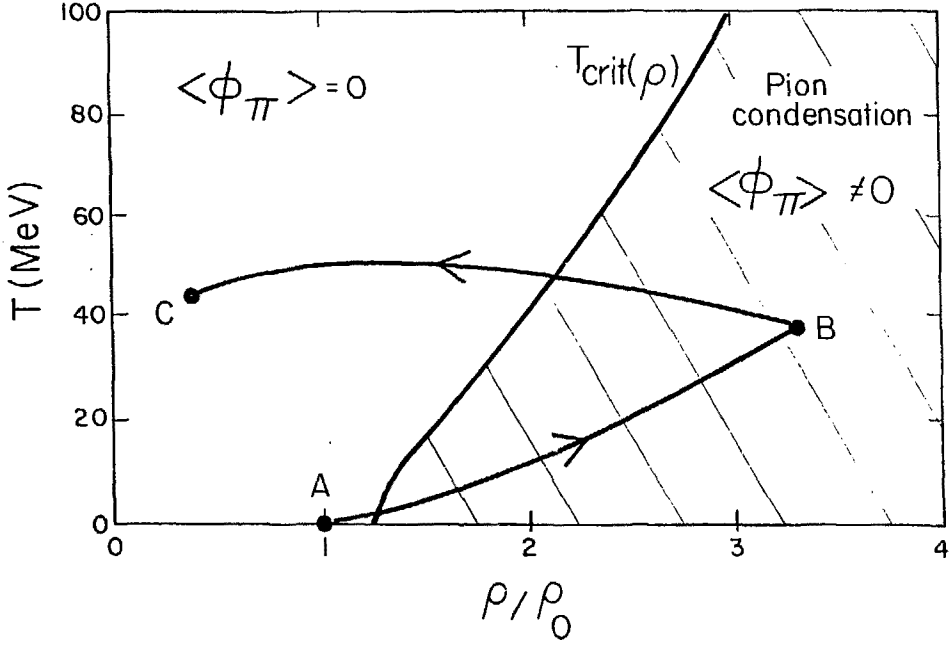

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Fig. I

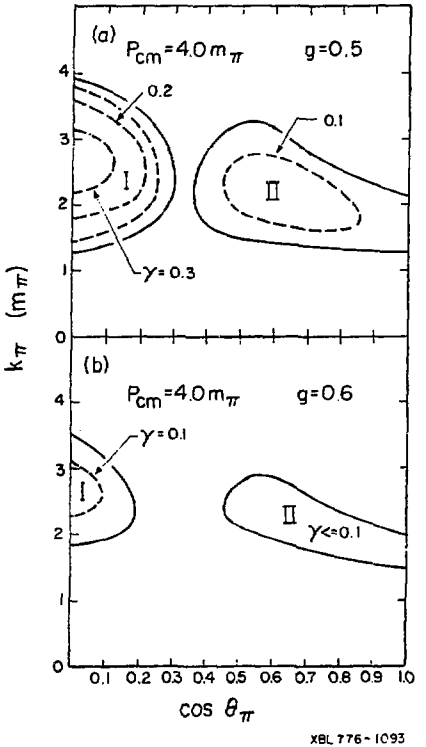

(a)
$P_{cm}=4.0\,m_\pi$
g=0.5
0.2
0.1
I
II
γ=0.3
(b)
$P_{cm}=4.0\,m_\pi$
g=0.6
γ=0.1
I
II
γ<=0.1
k_π (m_π)
cos θ_π
0 1 2 3 4
0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0
XBL 776-1093

Fig. II

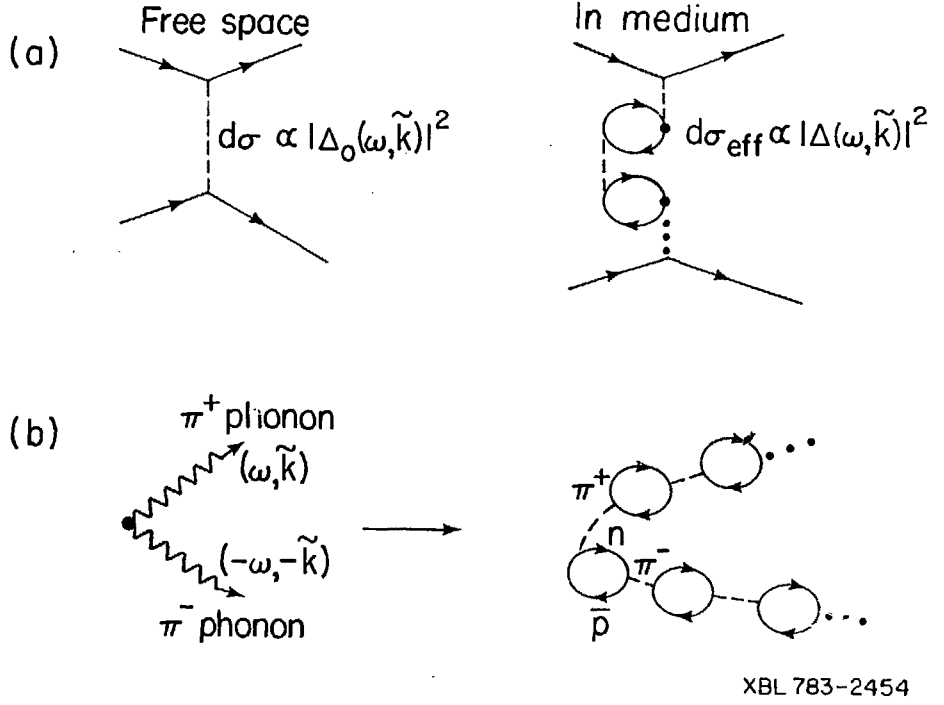

(a)
Free space
$d\sigma \propto |\Delta_0(\omega,\tilde{k})|^2$
In medium
$d\sigma_{eff} \propto |\Delta(\omega,\tilde{k})|^2$
(b)
π^+ phonon
$(\omega,\tilde{k})$
$(-\omega,-\tilde{k})$
π^- phonon
π^+
n
π^-
$\bar{p}$
XBL 783-2454

Fig. III

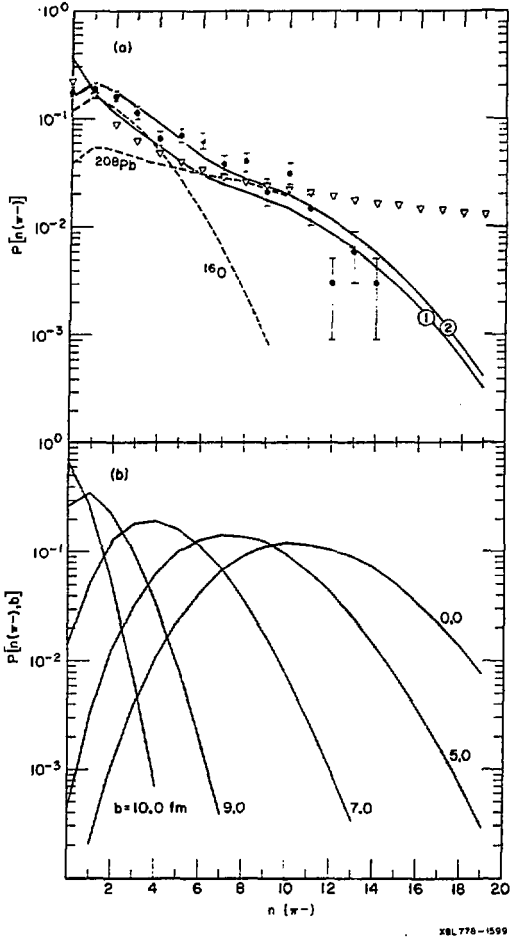
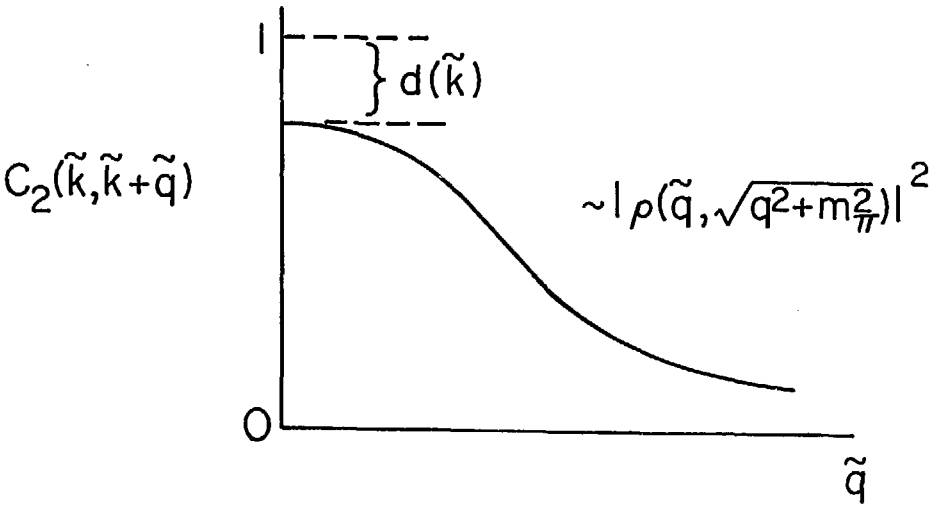


Fig. IV



XBL 783-2455

Fig. V