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On the Concept of an Optimal Portfolio of Uncertain Flexible Loads

Julie Rousseau^{*,†}, Philipp Heer[†], Kristina Orehounig[‡], Gabriela Hug^{*}

Abstract—Flexible loads can enhance power system stability by providing reserves, but their limited energy capacity and uncertain availability distinguish them from conventional generators. To accommodate these characteristics, the Danish Transmission System Operator (TSO) recently introduced new reserve market rules that incorporate energy constraints and relax reliability requirements. In this context, the optimal reserve quantification becomes a joint chance-constrained reserve quantification problem, which is difficult to solve. In this paper, we derive two analytical reformulations of this problem: an exact one when a reserve direction dominates and an approximate one, otherwise. Furthermore, when flexible loads must collectively satisfy a reliability requirement, we introduce the concept of an optimal portfolio of flexible loads: adding loads with similar expected values but different stochastic behaviors to an existing portfolio may change the total portfolio's reserves. To support this idea, we theoretically study the marginal increase in reserves resulting from adding a load. Numerical results show that our analytical reformulations closely match the exact formulation, with a mean absolute error of 2.5%. Case studies further demonstrate the existence of optimal load groupings and our ability to predict the portfolio in which a load's marginal value is highest, leveraging our theoretical analysis.

Index Terms—Demand-side flexibility, joint chance-constrained optimization, aggregation, optimal portfolio.

I. INTRODUCTION

In an attempt to reduce global carbon emissions, electric power grids undergo profound transformations: a large number of flexible carbon-intensive power plants are decommissioned and replaced by intermittent renewable energy sources [1]. While conventional power plants used to constantly adapt their production to the electric power demand, renewable power units can only partly do so. Hence, policymakers encourage power consumers to become more flexible, i.e., partly adapt their consumption by optimally planning it in advance or providing ancillary services in real time [2].

In the residential sector, Heating, Ventilation, and Air Conditioning (HVAC) systems and Electric Vehicles (EVs) are among the most promising sources of flexibility [1]. The power consumption of an HVAC can be shifted over time, as long as the rooms it serves remain within predefined temperature bounds. Similarly, the power consumed by an EV can be adjusted, as long as the user's requested state-of-charge at departure is guaranteed. Additionally, the power consumption of both devices is also limited by their power and energy technical ratings. Previous research has proven the technical viability of ancillary service provision by such flexible loads [3], [4].

Compared to conventional generators, flexible loads are limited both in the amount of power and energy they can consume over time [5]. Yet, most ancillary service products are specified in terms of ramping rates, power, and expected

duration [6], but do not specify the amount of energy to be reserved. A crude approach consists in considering the worst-case energy activation, i.e., the delivery of the full power over the duration [7]. While robust, this approach also appears overly conservative, especially for durations of multiple hours or days. Instead, an active energy management is preferable. It consists of continuously adjusting the loads' energy contents through intra-day trades to offset previous energy activations [4], [7]–[9].

Furthermore, the flexibility of HVACs and EVs is also partly uncertain and, therefore, complex to plan. Indeed, an EV's flexibility strongly depends on when the vehicle leaves, which, in most cases, is uncertain [10]. The flexibility of an HVAC is also hard to predict, as it depends on uncertain future weather conditions, the presence and actions of inhabitants, and thermal models with limited accuracy [7]. However, ancillary service markets require the offered flexibility to be almost certain, placing the burden on aggregators to manage risk, e.g., by trading off revenues for flexibility with penalties for failing to deliver the promised reserves [7], [11].

Acknowledging the current lack of specifications for energy and reliability in ancillary service markets, new market rules have been developed in Denmark [12] to attract more flexible loads. Specifically, it defines an amount of energy to be reserved by energy-constrained loads, as well as an amount of power to be reserved in the opposite direction [13]. Both specifications give formal requirements for the implementation of an energy management system in practice. Additionally, the new market rules specify a reduced reliability level, requiring loads to be available with a 90% probability, down from the previous 99.9% [13]. This reliability requirement shifts part of the burden of managing risk from aggregators to TSOs.

Given these new energy and reliability specifications, some studies explore how flexible loads should participate in the Danish reserve markets. The problem is formulated as a joint-chance constrained optimization, which is generally complex to solve. Existing studies solve the optimization problem using various methods, e.g., the iterative ALSO-X method [14], the conservative CVaR method [14], or a distributionally robust reformulation [12]. Alternatively, the authors of [15] suggest separating the joint chance constraints into individual chance constraints using the Bonferroni approximation. While all methods are valid, the authors of [16] observe that an analytical reformulation of the problem exists, offering a computationally efficient, non-conservative approach. However, this approach assumes that loads can only reduce their consumption and no comparison with existing methods is provided. Following a similar approach to [16], we derive an analytical formulation to determine the reserves of a portfolio of loads in both directions and compare it to existing methods.

Besides, when grouping loads, previous studies observe that larger portfolios can provide greater reserves relative to their number of loads [11], [14]. Another phenomenon likely affects a portfolio's reserves: for a given number of loads, grouping different stochastic profiles can yield different reserve amounts. In this paper, we explore the concept of an optimal portfolio, i.e., how to group loads based on their stochastic behavior to maximize the portfolio's reserve amount. The contribution is, therefore, three-fold:

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- We propose an analytical method to determine the maximum amount of reserves a portfolio can offer under the new Danish rules. Technically, we propose an analytical reformulation of a joint chance-constrained optimization.
- We demonstrate a counter-intuitive result showing how wisely pairing loads into different portfolios may increase the total amount of reserves for a given reliability level.
- We develop theoretical foundations to understand the marginal increase in reserves, resulting from adding a load to a portfolio. Specifically, we study the impact of certain load and portfolio characteristics.

The remainder of this paper is organized as follows. Section II presents an analytical formulation to determine the optimal participation of a portfolio, i.e., a set of flexible loads, in the Danish reserve markets. Section III further investigates the concept of an optimal portfolio, i.e., an optimal grouping of loads according to their stochastic properties. Then, Section IV describes the case study, Section V analyzes the results, and Section VI concludes the work.

In the remainder of this paper, the sets $\mathcal{T}$ and $\mathcal{I}$ describe the discrete set of timesteps and loads, respectively. Tildes denote stochastic variables, and Δt designates the time resolution between two discrete timesteps k and $k + 1$.

II. A PORTFOLIO OF FLEXIBLE LOADS PARTICIPATING IN THE DANISH RESERVE MARKETS

This section first describes the flexibility potential of two types of loads, EVs and HVACs, before explaining how they can sell their flexibility in the Danish reserve markets.

A. Flexible Loads

In this paper, we describe load flexibility through the concept of power and energy envelopes [17], and limit ourselves to EVs and HVACs. The methodology extends naturally to any asset with limited power and energy consumption.

1) *EVs*: Once plugged in, an electric vehicle is flexible in the sense that its power consumption can be shifted over time, as long as its final state-of-charge requirement is guaranteed. The EV i arrives at a charging station at $t_{ev,i}^a$ with an initial energy content $e_{ev,i}^0$ and leaves at $t_{ev,i}^d$. Its power and energy consumption are limited by its technical ratings:

$$\begin{aligned} p_{ev,i}^{\min,k} &\leq p_{ev,i}^k \leq p_{ev,i}^{\max,k}, & \forall k \in \mathcal{T}, \\ e_{ev,i}^{\min,k} &\leq e_{ev,i}^0 + \Delta t \sum_{t=t_{ev,i}^a}^k p_{ev,i}^t \leq e_{ev,i}^{\max,k}, & \forall k \in \mathcal{T}, \end{aligned} \quad (1)$$

where $p_{ev,i}^{\min,k}$, $p_{ev,i}^{\max,k}$, $e_{ev,i}^{\min,k}$, and $e_{ev,i}^{\max,k}$ describe its time-varying minimum and maximum power and energy ratings, respectively. The power ratings are equal to the respective EV rating when plugged-in, e.g., $p_{ev,i}^{\min}$ for $p_{ev,i}^{\min,k}$, and 0 otherwise. The time-varying energy bounds correspond to the minimum and maximum energy consumption paths, as described in [18]. Finally, upon departure, at $t_{ev,i}^d$, EV i should be charged to at least the level of $e_{ev,i}^f$, i.e.:

$$e_{ev,i}^0 + \Delta t \sum_{t=t_{ev,i}^a}^k p_{ev,i}^t \geq e_{ev,i}^f, \quad \forall k \geq t_{ev,i}^d. \quad (2)$$

Constraint (2) can be integrated into the time-varying lower energy bound $e_{ev,i}^{\min,k}$ such that the flexibility of EV i is simply delimited by time-varying power and energy bounds.

However, in practice, EVs' future flexibility is often uncertain, e.g., due to uncertain departure times. As a consequence, the time-varying power and energy bounds become stochastic variables denoted as $\tilde{p}_{ev,i}^{\min,k}$, $\tilde{p}_{ev,i}^{\max,k}$, $\tilde{e}_{ev,i}^{\min,k}$, and $\tilde{e}_{ev,i}^{\max,k}$.

2) *HVACs*: Electric heating and cooling systems can adjust their power consumption while maintaining thermal comfort for inhabitants, often defined as an acceptable indoor temperature range. To relate the HVAC i 's power consumption $p_{hvac,i}^k$ to indoor temperatures, we rely on a discretized first-order resistance-capacitance lumped model:

$$C_i \frac{T_i^{k+1} - T_i^k}{\Delta t} = -\frac{1}{R_i} (T_i^k - T_{a,i}^k) + p_{hvac,i}^k + s_i^k, \quad (3)$$

where T_i^k describes the representative indoor temperature of the building heated/cooled by HVAC i , which initially equals T_i^0 . Parameters R_i and C_i are the thermal resistance and capacitance of this building, respectively, whereas $T_{a,i}$ and s_i describe the ambient air temperature and the solar heat gains, respectively. The power consumption of HVAC i is limited by its technical power ratings, $p_{hvac,i}^{\min}$ and $p_{hvac,i}^{\max}$ as:

$$p_{hvac,i}^{\min} \leq p_{hvac,i}^k \leq p_{hvac,i}^{\max}, \quad \forall k \in \mathcal{T}. \quad (4)$$

Additionally, to guarantee the inhabitants' thermal comfort, the indoor temperature must remain within an acceptable range:

$$T_i^{\min} \leq T_i^k \leq T_i^{\max}, \quad \forall k \in \mathcal{T}. \quad (5)$$

Given these temperature constraints and following the methodology of [17], we can determine the time-varying minimum and maximum energy HVAC i can consume over a given horizon, denoted as $e_{hvac,i}^{\min,k}$ and $e_{hvac,i}^{\max,k}$, respectively. These time-varying energy bounds, together with the power bounds defined in (4), delimit the flexibility of HVAC i .

However, the time-varying energy bounds that define the future flexibility may be uncertain, e.g., due to uncertain future weather [7]. Hence, they become stochastic variables, denoted as $\tilde{e}_{hvac,i}^{\min,k}$ and $\tilde{e}_{hvac,i}^{\max,k}$. Additionally, to harmonize notations among loads, we also consider the power bounds to be time-varying stochastic variables, denoted as $\tilde{p}_{hvac,i}^{\min,k}$ and $\tilde{p}_{hvac,i}^{\max,k}$.

B. The Danish FCR-D Market

The Danish TSO recently introduced new requirements to enable uncertain energy-constrained flexibility providers to participate in ancillary service reserve markets [13]. While these new rules apply to all Danish ancillary services, FCR-D has gained particular attention, since it is an hourly asymmetric product. The Frequency Containment Reserves for Disturbances (FCR-D) is an emergency reserve designed to support the regular FCR in case the frequency deviates from the nominal frequency by more than 0.1 Hz. Our methodology complies with the requirements of the FCR-D market, similarly to [12], [14], [16], but could also be adapted to other Danish ancillary services using the requirements detailed in [13].

1) *Energy Requirement*: The Danish TSO proactively defines energy and power requirements for FCR-D reserves, which allow energy-constrained loads to participate in ancillary service markets by managing their energy content [8]. More specifically, the Danish TSO specifies that, for a given amount of reserves in one direction at an hour, the flexibility provider should be able to deliver the full power during one third of the hour, and reserve 20% of the power in the opposite direction. From an aggregator's point of view, this new requirement can be integrated into an optimization

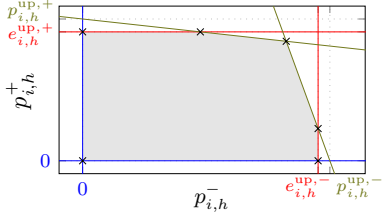


Fig. 1. Feasible space delimited by constraints (7b)-(7e) for load i .

problem, whose objective is to maximize, at each hour h , revenues as [14]:

$$\max_{p_{i,h}^+, p_{i,h}^-} c_h^+ \sum_{i \in \mathcal{I}} p_{i,h}^+ + c_h^- \sum_{i \in \mathcal{I}} p_{i,h}^- \quad (6a)$$

$$\text{s.t. } b_i^k + p_{i,h}^+ + 0.2p_{i,h}^- \leq p_i^{\max,k}, \quad \forall k \in \mathcal{T}_h, \forall i \in \mathcal{I}, \quad (6b)$$

$$b_i^k - p_{i,h}^- - 0.2p_{i,h}^+ \geq p_i^{\min,k}, \quad \forall k \in \mathcal{T}_h, \forall i \in \mathcal{I}, \quad (6c)$$

$$e_{b,i}^k + \frac{1}{3}p_{i,h}^+ \leq e_i^{\max,k}, \quad \forall k \in \mathcal{T}_h, \forall i \in \mathcal{I}, \quad (6d)$$

$$e_{b,i}^k - \frac{1}{3}p_{i,h}^- \geq e_i^{\min,k}, \quad \forall k \in \mathcal{T}_h, \forall i \in \mathcal{I}, \quad (6e)$$

$$p_{i,h}^+, p_{i,h}^- \geq 0, \quad \forall i \in \mathcal{I}, \quad (6f)$$

where $\mathcal{T}_h$ designates the subset of $\mathcal{T}$ included in hour h , and $(p_i^{\min,k}, p_i^{\max,k}, e_i^{\min,k}, e_i^{\max,k})$ are the time-varying bounds defined in Section II-A. Parameters b_i^k and $e_{b,i}^k$ are the baseline power and energy consumption of the load i , respectively. $e_{b,i}^k$ corresponds to the accumulated baseline power consumption, i.e., $e_{b,i}^k = \Delta t \sum_{l=0}^k b_i^l$. The total upward and downward¹ reserves, given by $\sum_{i \in \mathcal{I}} p_{i,h}^+$ and $\sum_{i \in \mathcal{I}} p_{i,h}^-$ respectively, are compensated with the prices c_h^+ and c_h^- . The problem can be expressed in more compact form as:

$$\max_{p_{i,h}^+, p_{i,h}^-} c_h^+ \sum_{i \in \mathcal{I}} p_{i,h}^+ + c_h^- \sum_{i \in \mathcal{I}} p_{i,h}^- \quad (7a)$$

$$\text{s.t. } p_{i,h}^+ + 0.2p_{i,h}^- \leq p_{i,h}^{\text{up},+}, \quad \forall i \in \mathcal{I}, \quad (7b)$$

$$p_{i,h}^- + 0.2p_{i,h}^+ \leq p_{i,h}^{\text{up},-}, \quad \forall i \in \mathcal{I}, \quad (7c)$$

$$0 \leq p_{i,h}^+ \leq e_{i,h}^{\text{up},+}, \quad \forall i \in \mathcal{I}, \quad (7d)$$

$$0 \leq p_{i,h}^- \leq e_{i,h}^{\text{up},-}, \quad \forall i \in \mathcal{I}, \quad (7e)$$

where the parameters $(p_{i,h}^{\text{up},+}, p_{i,h}^{\text{up},-}, e_{i,h}^{\text{up},+}, e_{i,h}^{\text{up},-})$ are derived from the parameters in (6)². This problem can be separated into sub-problems for each flexible load of the portfolio. Each sub-problem can then be efficiently solved using the simplex method. As an example, Fig. 1 illustrates the feasible space (gray area) defined by constraints (7b)-(7e) for load i . This forms a convex space, composed of at most 6 corners. The optimal solution can be found by evaluating the objective function in all 6 corners.

2) *Reliability Requirement*: Additionally, the Danish TSO acknowledges the uncertainty associated with some flexible loads and allows them to provide reserves with a reduced reliability level of 90%. In other words, whenever such flexible loads participate in ancillary service reserves, regardless if they are activated, their promised reserves should be available 90%

¹Throughout the paper, upward reserves indicate an increase in power consumption, equivalent to a decrease in production, while the downward reserves designate a decrease in consumption, equivalent to an increase in production.

²The parameter values for hour h are defined as the the lowest parameter values among timesteps $k \in \mathcal{T}_h$.

of the time. This additional requirement transforms problem (7) into the following joint chance-constrained optimization:

$$\max_{p_h^+, p_h^-} c_h^+ p_h^+ + c_h^- p_h^- \quad (8a)$$

$$\text{s.t. } \mathbb{P} \left[\begin{array}{l} p_{i,h}^+ + 0.2p_{i,h}^- \leq \tilde{p}_{i,h}^{\text{up},+}, \quad \forall i \in \mathcal{I}, \\ p_{i,h}^- + 0.2p_{i,h}^+ \leq \tilde{p}_{i,h}^{\text{up},-}, \quad \forall i \in \mathcal{I}, \\ 0 \leq p_{i,h}^+ \leq \tilde{e}_{i,h}^{\text{up},+}, \quad \forall i \in \mathcal{I}, \\ 0 \leq p_{i,h}^- \leq \tilde{e}_{i,h}^{\text{up},-}, \quad \forall i \in \mathcal{I}, \\ p_h^+ \leq \sum_{i \in \mathcal{I}} p_{i,h}^+, \\ p_h^- \leq \sum_{i \in \mathcal{I}} p_{i,h}^-, \end{array} \right] \geq R, \quad (8b)$$

where $R = 90\%$ is the required reliability, and tilde variables are stochastic counterparts of the variables introduced in (7). By introducing p_h^+ and p_h^- , as sums of reserves from individual resources, we aim to provide a robust total amount of reserves rather than robust individual amounts. When a load fails to provide reserves, the flexibility of other loads can compensate for it, maintaining a high level of robust total reserves while the load's robust individual reserve level decreases. Hence, this reduces conservativeness.

To approach this problem, we first consider that one reserve direction is clearly more valuable³ than the other, e.g., the upward direction. In this context, we first set the reserves in the opposite direction, i.e., p_h^- in this case, to 0 such that constraint (8b) can be formulated as:

$$\mathbb{P} \left[\begin{array}{l} p_{i,h}^+ \leq \tilde{p}_{i,h}^{\text{up},+}, \quad \forall i \in \mathcal{I}, \\ 0.2p_{i,h}^+ \leq \tilde{p}_{i,h}^{\text{up},-}, \quad \forall i \in \mathcal{I}, \\ 0 \leq p_{i,h}^+ \leq \tilde{e}_{i,h}^{\text{up},+}, \quad \forall i \in \mathcal{I}, \\ p_h^+ \leq \sum_{i \in \mathcal{I}} p_{i,h}^+, \end{array} \right] \geq R. \quad (9)$$

The maximum of p_h^+ is obtained if the reserves provided by each load are maximized. In other words, each load reserves $p_{i,h}^+$ is equal to its upper bound, such that:

$$\mathbb{P} \left(p_h^+ \leq \underbrace{\sum_{i \in \mathcal{I}} \min \left(\tilde{p}_{i,h}^{\text{up},+}, \frac{1}{0.2} \tilde{p}_{i,h}^{\text{up},-}, \tilde{e}_{i,h}^{\text{up},+} \right)}_{X_h^+} \right) \geq R. \quad (10)$$

Hence, the optimal upward reserve amount p_h^{+*} can be obtained analytically as the $(1 - R)$ quantile of the stochastic variable⁴ X_h^+ , i.e.:

$$p_h^{+*} = q_{X_h^+}(1 - R). \quad (11)$$

Such a quantile value can be evaluated using scenarios. In this paper, we rely on a number of scenarios similar to [14]. Then, fixing the value of upward reserves to p_h^{+*} , we can determine the value of downward reserves using a similar technique⁵.

When reserve prices are significantly different, reserves can be optimized sequentially. However, when prices are comparable, reserve values become coupled through the joint chance constraint, and the optimal reserve values no longer correspond

³In this context, a direction is said to be clearly more valuable if the ratio of the smallest to the highest prices is lower than 0.2.

⁴This approach is similar to the one used in [16], although the authors do not explicitly define the random variable of interest, nor establish the validity of the approach.

⁵This approach is similar to the Naive approach developed in [14]. However, the authors of [14] considered the minimum of the three variables' quantile, instead of the quantile of the minimum of the three variables.

to the quantiles of the marginal distributions. Therefore, applying the previous technique may yield suboptimal results. In such a case, we suggest a scenario-based approach. For each scenario, we determine both total optimal reserves, using the vertex method to solve (7). Then, the optimization aims to find the values of p_h^+ and p_h^- that maximize the objective function, while violating only $(1 - R)$ scenarios. Therefore, we define the reserve values as the $(1 - R)$ quantile of the joint distribution of the optimal values. Nevertheless, it is important to note that this method is sub-optimal. Indeed, we first consider optimal pairs of values for each scenario before evaluating the joint quantile. Yet, in some scenarios, considering a sub-optimal couple may not change the overall quantile in one direction, but increase it in the opposite direction, leading to a higher final objective value.

III. ON THE CONCEPT OF AN OPTIMAL PORTFOLIO

Given a portfolio of flexible loads, Section II provides information about the optimal amount of reserves a portfolio can offer in the Danish reserve markets. However, this amount depends on its constituent loads. In this section, we investigate the design of a portfolio of flexible loads to maximize its reserve capabilities.

Ignoring the reliability requirement introduced in Section II-B2, i.e., considering a deterministic case, a portfolio's reserves are equal to the sum of individual loads' reserves. Hence, for a fixed number of loads, the total amount of reserves across all portfolios remains identical regardless of how the loads are grouped into different portfolios. However, when considering a reliability requirement, this statement no longer holds true. Instead, some groupings may yield a higher total reserve amount, leading to the concept of optimal portfolios, i.e., an optimal grouping of a set of flexible loads to maximize the total reserves. An illustrative example of this concept is provided in Appendix A. In the following, we investigate the additional reserves a load brings to a portfolio.

A. Portfolio's Value Increase After Adding New Loads

According to Section II-B2, the amount of reserves that a portfolio can offer relates to a specific quantile. Without loss of generality, for the rest of this section, we consider that one reserve direction is significantly more valuable, e.g., the upward direction. Therefore, the level of reserves that a portfolio can offer is:

$$p_h^{+*} = q_{X_h^+}(1 - R) = q_X(1 - R), \quad (12)$$

at hour h . For simplicity, here, we simply denote X_h^+ as X .

The concept of an optimal portfolio can be viewed from different perspectives. From a system's point of view, flexible loads should be allocated among portfolios to maximize the total reserves, given a set of loads. However, an aggregator seeks flexible loads that increase its reserves the most. Finally, a flexible load's owner is interested in joining the portfolio in which its own contribution increases the amount of reserves of this portfolio the most. The load's view differs from the aggregator's perspective, as all aggregators want to attract reliable loads with high flexibility, but the load wishes to join only one portfolio. Here, we focus on an individual portfolio and examine how adding a flexible load affects its value.

We consider an initial portfolio, described by the stochastic variable X , to which we add a flexible load. When considered in isolation, the amount of reserves that the new load can provide by itself corresponds to $q_{\varepsilon Y}(1 - R)$, where εY is the

stochastic variable describing the upward reserves of this load. In particular, ε indicates the small amount of reserves this new load brings in comparison to the existing portfolio. The new portfolio's value equals $q_{X+\varepsilon Y}(1 - R)$. This problem can be interpreted as the change in quantile values induced by the addition of a relatively small load, which naturally aligns with the concept of a Taylor expansion. However, the small input perturbation lies in the space of probability distribution.

The Taylor expansion in the space of distributions is called the Taylor-von Mises expansion. Its exact definition is introduced in Appendix C2. Provided a few assumptions, e.g., the existence and derivability of a density, this expansion evaluates the evolution of statistical metrics, such as the quantile value, around an initial distribution. Based on the Taylor-Von Mises expansion as well as a few additional steps described in Appendix C3, we can derive the evolution of the quantile function, in the case of a small disturbance εY in the space of distributions, as:

$$q_{X+\varepsilon Y}(1 - R) = q_X(1 - R) + \varepsilon \mathbb{E}(Y) - \frac{\varepsilon^2}{2} \underbrace{(\ln(f_X))'(q_X(1 - R))}_{\alpha_X} \text{Var}(Y) + o(\varepsilon^2). \quad (13)$$

In the first order, the amount of reserves of the new portfolio increases by the expected flexibility of the new load. In the second order, it depends on the variance and derivative of the log-density of the initial portfolio at its quantile. In the following, we denote this term as α_X . In practice, computing α is challenging, especially based on empirical distributions. This aspect will be discussed in more detail in the next section.

According to (13), a portfolio owner should seek new flexible loads characterized by a large expected flexibility and a low variance in cases where α_X is positive, e.g., if the initial distribution is Gaussian. From a flexible load's point of view, its first-order contribution is identical across portfolios, but its second-order contribution is larger in the portfolio with the smallest α factor. Interestingly, this criterion relies on the derivative of the initial log-density at the quantile, and is, a priori, independent of the initial portfolio's size.

IV. CASE STUDY

The considered uncertain flexible loads include EVs and HVACs. To represent EVs, we use the ACN dataset [19], which comprises charging events recorded at three office parking-lot charging stations in California. The dataset indicates the arrival and departure times, the active charging duration, and the total energy delivered for each charging event. For some charging events, it further specifies vehicle-specific power and energy ratings, as well as the initial state-of-charge; for the others, we infer these parameters based on standard EV power and energy ratings and a plausible, randomly drawn state-of-charge. In the base case, we assume that EVs are charged at a constant power to reach their final state of charge at a user-defined departure time. While we know when EVs arrived, their exact departure times are uncertain and can be modeled as a function of arrival times using a Gaussian Mixture Model (GMM) [19]. Finally, at a specific time, we only consider EVs already connected for the determination of the flexibility at future time steps and ignore those that could connect later.

We further model the flexible electric heating systems using the large-scale dataset introduced in [20]. This dataset provides the thermal parameters and ambient weather conditions of 300 single-family houses, for which the HVAC baseline consumption keeps the indoor temperature to 20°C. Uncertain

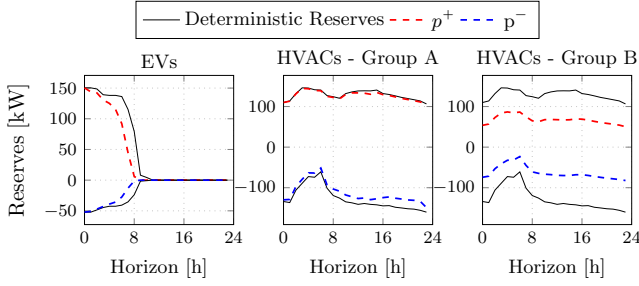


Fig. 2. Reserves offered by different portfolios of loads over one day: a portfolio of EVs (left), a portfolio of HVACs in buildings with a wide indoor temperature comfort range (A, middle), and a portfolio of HVACs in buildings with a tight indoor temperature comfort range (B, right). Downward reserves are shown as negative for clarity in the figure.

future ambient weather conditions are represented by an autoregressive Gaussian process, following the methodology of [7].

V. RESULTS

This section first illustrates the reserves that a portfolio of flexible loads can offer. Then, based on real load data, we assess the validity of our analytical reformulation of (8), described in Section II-B2, and of the theoretical quantile expansion (13), introduced in Section III-A. Finally, we investigate the existence and properties of an optimal portfolio.

A. Participation of a Portfolio in the Danish Reserve Markets

In Denmark, the new reserve market rules introduce energy and reliability requirements designed to open the market to uncertain, flexible loads. Fig. 2 illustrates the reserves that these loads can provide, following these new requirements. More specifically, it depicts the reserve capability of multiple EVs and HVACs over a winter day, starting at 08:00. Deterministic reserves, i.e., in the absence of uncertainty, are shown by solid lines, while reserves in the presence of uncertainty and computed using (11) are shown by dashed lines. Over the first 4 hours, it is likely that EVs remain connected, as they are connected at a workplace. Hence, the dashed and solid lines superpose. Afterwards, the probability that EVs disconnect increases, hence, the level of reserves they can offer with a reliability of 90% decreases. Eventually, EVs are disconnected, and the reserves fall to zero. In contrast, portfolios composed of HVACs provide a stable level of reserves throughout the day. Fig. 2 depicts the reserves of two groups of HVACs. In Group A, HVACs serve buildings with a large acceptable range of indoor temperatures, defined as $[18.5, 21.5]^{\circ}\text{C}$, while in Group B, these buildings are characterized by a tighter range of $[19, 21]^{\circ}\text{C}$. Additionally, we assume that Group B is exposed to more uncertain future weather conditions than Group A. As a consequence, in Group B, the uncertain weather conditions have a greater impact, reducing the available flexibility.

B. Validation of the Analytical Reformulation

In Section II-B2, we reformulated the joint chance-constrained optimization (8) into the analytical problem (11). To validate our approach, we benchmark our analytical method against the ALSO-X method⁶, an established approach for solving joint chance-constrained problems. Fig. 3 compares the amount of reserves that a portfolio of EVs can offer, according to both methods. When one reserve price dominates, Section II-B2 introduces an exact analytical reformulation of the problem, as Fig. 3 illustrates. At the worst hour, we

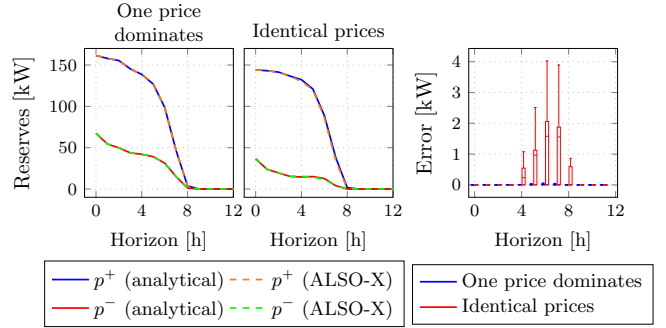


Fig. 3. Comparison between the proposed analytical method and the ALSO-X method, for an example day, when one reserve price dominates (left) and when both are equal (middle), and the distribution of absolute errors over 20 days (right).

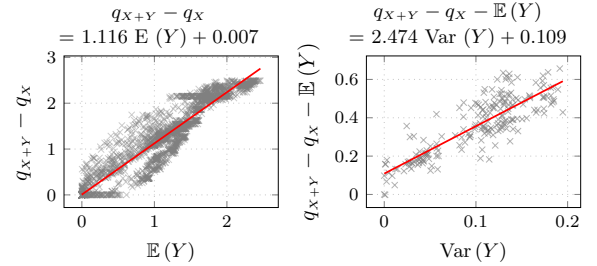


Fig. 4. Validation of (13) for the first order (left) and the second order terms (right).

observe a mean absolute error of 0.0195 kW, a 75%-quantile of 0.035 kW, and a maximum absolute error of 0.063 kW between the two methods. This small difference stems from the sample-based quantile estimation, which introduces small numerical errors. Additionally, the computation time for reserve scheduling over a day drops from around 9 minutes with the ALSO-X method to around 1.2 seconds with our analytical reformulation, on average. When both prices are equal, the analytical reformulation is not exact, as Fig. 3 shows. Yet, the mean absolute error between the two methods remains small relative to the reserve magnitude, with a mean error of 1.32 kW, a 75%-quantile of 2.06 kW, and a maximum absolute error of 4.03 kW at the worst hour, which should be acceptable in practice.

C. Validation of the Quantile Expansion

To study the notion of an optimal portfolio, we derived the quantile expansion (13), approximating the marginal change in reserve capacity when a new flexible load is added to an existing portfolio. To empirically validate this formula, we evaluate the increase in reserves resulting from adding various new loads to an existing EV portfolio. More specifically, we generate 200 loads whose flexibilities are governed by a randomly assigned probability distribution, selected from a set of standard probability distributions, with randomly selected parameters. According to (13), the marginal change in reserves can be approximated by the new load's expected flexibility, in the first order term, which is confirmed in the results shown in Fig. 4. In the second order term, it linearly depends on the variance of the new load, which is also validated in Fig. 4.

The second-order term depends not only on the new load's variance but also on the derivative of the log-density of the initial portfolio evaluated at its quantile. However, this quantity is very challenging to compute using sampled data. It also varies across time and must be estimated for every hour. To overcome this issue, we leverage the correlation observed between the marginal change in reserves and the new load's

⁶A brief description of the ALSO-X formulation is provided in Appendix B.

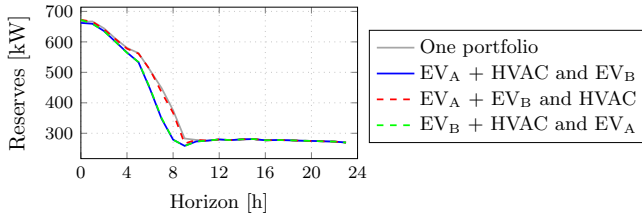


Fig. 5. Total amount of reserves obtained with two groups of EVs, and one group of 200 buildings, for different pairings into portfolios.

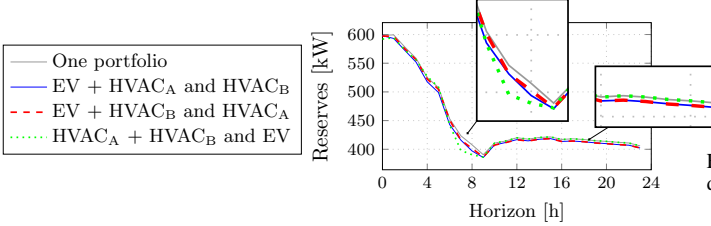


Fig. 6. Total amount of reserves obtained with a group of EVs, and two different groups of 200 buildings each, for different pairings into portfolios.

variance in the second order term. For each hour, we evaluate the correlation and use it to infer the log-density derivative. This approach is used in the rest of the paper.

D. Optimal Portfolio of Flexible Loads

The existence of an optimal portfolio suggests that appropriately combining flexible loads can increase the total reserve capacity. To illustrate this effect, we consider an example in which three groups of loads are combined into two portfolios. Specifically, we analyze two groups of EVs, each composed of approximately 50 vehicles and one group of HVACs, comprising 200 HVACs with a wide temperature comfort range, similar to Group A depicted in Fig. 2. For the three possible configurations, Fig. 5 depicts the total reserves that both portfolios can offer, as well as reserves that loads can offer when all grouped into one portfolio. One configuration clearly dominates the others, namely when the two EV groups are paired together. The HVAC group is highly reliable, as shown in Fig. 2. Hence, when paired with an EV group, its reliability balances the uncertainty of one of the EV groups, but the other group of EVs is not reliable enough to provide substantial reserves. In contrast, pairing the two EV groups improves their total reliability while allowing the HVAC group to independently deliver a high reserve capacity. At the worst hour, pairing each EV group with the HVAC group reduces total reserve capacity by 25.6% relative to the single-portfolio benchmark, while pairing the two EV groups together narrows this gap to 5.8%. This example indicates that optimal portfolio formation can be interpreted as an optimal allocation of a reliability budget.

Figure 6 presents the results of a similar experiment with one EV group and two HVAC groups A and B, introduced in Fig. 2. In contrast to the previous example, the optimal configuration varies over time. Between hours 4 and 8, when the EV uncertainty is high, pairing EVs with an HVAC group is beneficial, with a slight preference for the more uncertain HVAC group. At the worst hour, the optimal single-portfolio configuration offers 6% more reserves than the worst alternative pairing, with the remaining combinations falling 1.8% and 2.6% short. After hour 12, EVs are no longer available, and pairing them yields no additional reliability or value. In this case, the two HVAC groups are optimally paired. This example highlights that the optimal portfolio composition is not static, but evolves as the uncertainty profiles of the available loads change throughout the day.

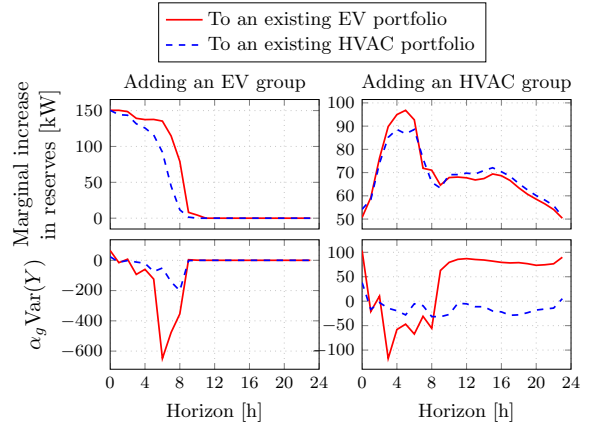


Fig. 7. Marginal increase in reserves resulting from adding different loads to different portfolios.

Flexible loads seeking to join an existing portfolio can leverage the proposed methodology to identify the portfolio that would benefit most from their participation. Indeed, Fig. 7 shows the marginal increase in reserves resulting from adding a group of EVs or a group of HVACs to different portfolios. For the EV group, joining an EV-only portfolio yields a higher marginal increase in reserves, whereas the HVAC group may prefer joining different portfolios at different times of the day. According to the quantile expansion (13), flexible loads should choose to join portfolios for which the product of the log-density derivative, α , and their variance is the smallest. Using the data-based identification of the log-density derivative described earlier, Fig. 7 shows that, by following this criterion, flexible loads can identify which portfolio to join.

Figures 6 and 5 suggest that grouping all loads into a single portfolio tends to yield better results than splitting them across multiple portfolios. We do not prove this holds in general, and counterexamples may exist, for instance when loads are strongly correlated. In practice, multiple portfolios often emerge naturally due to contractual or operational constraints, rather than by design. Our results nonetheless quantify the optimality gap between such fragmented configurations and the single-portfolio benchmark, which may serve as a useful bound for practitioners.

VI. CONCLUSION

This paper proposes an analytical framework to quantify the reserve capacity of portfolios composed of energy-constrained and uncertain flexible loads. The method uses a quantile-based formulation of a portfolio's flexibility, which is exact in some specific cases and results in small approximation errors in most practical settings. Yet, this method is not only a computationally efficient way to compute reliable reserves, but it can also be used to analyze the factors that affect a portfolio's reserves. In particular, it allows us to understand how the marginal contribution of an additional load varies with the stochastic properties of the loads. Based on a few selected examples, we demonstrate the existence of an optimal load grouping, given a fixed number of loads and portfolios. We additionally show that our framework helps to identify portfolios in which individual loads deliver the highest marginal value.

Future work should extend the initial results presented in this paper to larger-scale case studies and larger portfolio sizes. Furthermore, the current theoretical framework assumes that loads are affected by independent sources of uncertainty, which may not hold in practice. Accounting for correlated uncertainties is necessary to extend our methodology to more realistic case studies.

TABLE I
EXAMPLE OF 4 FLEXIBLE LOADS WITH 2 LEVELS OF FLEXIBILITY (LEFT)
AND POSSIBLE SPLITS INTO 2 PORTFOLIOS OF 2 LOADS EACH (MIDDLE
AND RIGHT).

$\mathbb{P}(\text{Reserve})=x$			$\mathbb{P}(\text{Reserve})=x$			$\mathbb{P}(\text{Reserve} \leq x) \geq 70\%$	
x	0.5 kW	1 kW	x	2 kW	1.5 kW	1 kW	Reserves at 70% (x)
A	70%	30%	A & B	9%	42%	49%	A & B
B	70%	30%	C & D	25%	50%	25%	C & D
C	50%	50%	A & C	15%	50%	35%	A & C
D	50%	50%	B & D	15%	50%	35%	B & D

VII. AI USAGE DISCLOSURE

We acknowledge the use of generative AI for spell- and grammar-checking, and style improvements.

APPENDIX

A. Optimal Portfolio: An Illustrative Example

To illustrate the concept of optimal portfolio formations, we introduce a simple example. As indicated in Table I, we consider 4 loads, labeled A to D. We assume that each load can provide a discretized amount of upward flexibility, with each discrete level associated with a reliability. For instance, Load A can increase its consumption by 0.5 kW with a confidence of 70%, and by 1 kW with a confidence of 30%.

Assuming that we want to form two portfolios, each containing two loads, there exist two possible groupings: we form either homogeneous or heterogeneous groups. Table I indicates the reliability associated with different total upward reserve values for each group. For instance, if Loads A and B are paired, their total consumption can be increased by 2 kW if each load increases its individual consumption by 1 kW, with a reliability of $30\% \times 30\% = 9\%$. Similarly, 1 kW of total reserves corresponds to both delivering 0.5 kW of upward reserves, which is associated with a probability of 49%. Finally, Loads A and B can increase their total consumption by 1.5 kW with a reliability of 42%.

Loads A and B can provide at least 1.5 kW of reserves with confidence $9\% + 42\% = 51\%$ and at least 1 kW with full certainty. Hence, given a reliability level of 70%, they can reliably provide 1 kW of upwards reserves. Similarly, analyzing the cumulative distribution of reserves of Loads C and D, we observe that they can deliver an amount of reserves larger than 1.5 kW with a 70% confidence. Indeed, for the second portfolio, the probability that the reserves are larger than 1.5 kW is 75%, exceeding the 70%-reliability requirement. The value of 1.5 kW also corresponds to the 30%-quantile of the discrete stochastic variable describing the reserves of Loads C and D, as introduced in Section II-B2. When grouping loads heterogeneously, each group can only provide 1 kW of upward reserves, under the 70%-requirement, as indicated in Table I. Hence, under these conditions, the total reserve level is higher with homogeneous grouping, indicating that, for a fixed set of resources and number of portfolios, an optimal portfolio allocation exists.

B. ALSO-X Formulation

Using a scenario approach, the joint chance-constrained problem (8) can be reformulated as:

$$\min_{p_h^+, p_h^-} c_h^+ p_h^+ + c_h^- p_h^- \quad (14a)$$

$$\text{s.t.} \quad p_{i,h,s}^+ + 0.2p_{i,h,s}^- \leq p_{i,h,s}^{\text{up},+}, \quad \forall i \in \mathcal{I}, \forall s \in \mathcal{S}, \quad (14b)$$

$$p_{i,h,s}^- + 0.2p_{i,h,s}^+ \leq p_{i,h,s}^{\text{up},-}, \quad \forall i \in \mathcal{I}, \forall s \in \mathcal{S}, \quad (14c)$$

$$0 \leq p_{i,h,s}^+ \leq e_{i,h,s}^{\text{up},+}, \quad \forall i \in \mathcal{I}, \forall s \in \mathcal{S}, \quad (14d)$$

$$0 \leq p_{i,h,s}^- \leq e_{i,h,s}^{\text{up},-}, \quad \forall i \in \mathcal{I}, \forall s \in \mathcal{S}, \quad (14e)$$

$$p_h^+ \leq \sum_{i \in \mathcal{I}} p_{i,h,s}^+ + M y_{h,s}, \quad \forall s \in \mathcal{S}, \quad (14f)$$

$$p_h^- \leq \sum_{i \in \mathcal{I}} p_{i,h,s}^- + M y_{h,s}, \quad \forall s \in \mathcal{S}, \quad (14g)$$

$$y_{h,s} \in \{0, 1\}, \quad \forall s \in \mathcal{S}, \quad (14h)$$

$$\frac{1}{|\mathcal{S}|} \sum_{s \in \mathcal{S}} y_{h,s} \leq 1 - R, \quad (14i)$$

where the binary variables $y_{h,s}$ and (14i) allow the violation of constraints in at most $(1 - R)\%$ of the scenarios contained in the set $\mathcal{S}$. The ALSO-X method relaxes this problem by considering the variables $y_{h,s}$ as continuous. In return, Constraint (14i) is transformed into a budget constraint as:

$$\sum_{s \in \mathcal{S}} y_{h,s} \leq Q, \quad (15)$$

where the violation budget Q is iteratively changed until only $(1 - R)\%$ of the scenarios are violated. A more rigorous definition of the problem can be found in [14].

C. Von Mises-Taylor Expansion of the Quantile Function

1) *Statistical Functions*: Von Mises' theory relies on the observation that statistical functions, such as the quantile function, can be seen as functions of a stochastic cumulative distribution [21]. Mathematically, this means that:

$$q_x(1 - R) = T(F_x), \quad (16)$$

where F_x is the cumulative density function of a stochastic variable X and T is defined for a fixed reliability R .

2) *Von Mises-Taylor Expansion*: Based on the concept of statistical functions, Von Mises "developed a theory for the analysis of the asymptotic distribution of statistical [functions], using a form of Taylor expansion involving the derivatives of the [statistical functions]" [21]. In other words, Von Mises developed an equivalent to the Taylor expansion, but for statistical functions in the space of probability distributions.

A prerequisite to the Von Mises expansion is the definition of the derivative of a statistical function in the space of probability distributions. The Von Mises derivative of a statistical function T is defined if there exists a function ϕ such that:

$$\left. \frac{d}{dt} T(F_x + t(F_z - F_x)) \right|_{t=0} = \int \phi(F_x, u) d(F_z - F_x)(u). \quad (17)$$

From this definition, we can see that the derivative is directional, i.e., it depends on the chosen direction F_z . In the following, we will denote the derivative of T , at F_x , in direction F_z , as $T'(F_x; F_z)$. Additionally, we will use the following notation:

$$F_{x,t} = F_x + t(F_z - F_x) = (1 - t)F_x + tF_z. \quad (18)$$

Under the assumption that T is twice-differentiable in F_x , the Von Mises-Taylor expansion at the second order is:

$$T(F_z) - T(F_x) = T'(F_x; F_z) + \frac{1}{2} T''(F_x; F_z) + \text{Rem}(F_x - F_z), \quad (19)$$

where $\text{Rem}(F_x - F_z)$ designates a remainder term.

3) *Application to the Quantile Function*: Developing the Von Mises-Taylor expansion of the quantile function requires first demonstrating that the quantile function, as a statistical function, is twice differentiable. Starting with the first-order derivative, the authors of [22] prove its differentiability and find the first derivative's value using the following property of the quantile function:

$$F_{X,t}(T(F_{X,t})) = 1 - R, \quad (20)$$

such that:

$$\frac{d}{dt} F_{X,t}(T(F_{X,t})) = 0. \quad (21)$$

Based on this equality, we apply the chain rule and obtain:

$$\begin{aligned} & \left. \frac{d}{dt} ((1-t)F_X(T(F_{X,t})) + tF_Z(T(F_{X,t}))) \right|_{t=0} \\ &= \left[-F_X(T(F_{X,t})) + (1-t)F'_X(T(F_{X,t})) \frac{d}{dt} T(F_{X,t}) \right. \\ & \quad \left. + F_Z(T(F_{X,t})) + tF'_Z(T(F_{X,t})) \frac{d}{dt} T(F_{X,t}) \right] \Big|_{t=0} \\ &= -F_X(T(F_X)) + F'_X(T(F_X)) T'(F_X; F_Z) + F_Z(T(F_X)) \\ &= 0. \end{aligned} \quad (22)$$

Eventually, we can conclude that:

$$T'(F_X; F_Z) = \frac{F_X(T(F_X)) - F_Z(T(F_X))}{F'_X(T(F_X))}. \quad (23)$$

Similarly, we can derive the second-order derivative. Using the same approach as in (22), we compute:

$$\begin{aligned} & \left. \frac{d^2}{dt^2} ((1-t)F_X(T(F_{X,t})) + tF_Z(T(F_{X,t}))) \right|_{t=0} \\ &= \frac{d}{dt} \left[-F_X(T(F_{X,t})) \right. \\ & \quad \left. + (1-t)F'_X(T(F_{X,t})) \frac{d}{dt} T(F_{X,t}) \right. \\ & \quad \left. + F_Z(T(F_{X,t})) + tF'_Z(T(F_{X,t})) \frac{d}{dt} T(F_{X,t}) \right] \Big|_{t=0} \\ &= \left[-F'_X(T(F_{X,t})) \frac{d}{dt} T(F_{X,t}) \right. \\ & \quad \left. - F'_X(T(F_{X,t})) \frac{d}{dt} T(F_{X,t}) \right. \\ & \quad \left. + (1-t)F''_X(T(F_{X,t})) \left(\frac{d}{dt} T(F_{X,t}) \right)^2 \right. \\ & \quad \left. + (1-t)F'_X(T(F_{X,t})) \frac{d^2}{dt^2} T(F_{X,t}) \right. \\ & \quad \left. + F'_Z(T(F_{X,t})) \frac{d}{dt} T(F_{X,t}) \right. \\ & \quad \left. + F'_Z(T(F_{X,t})) \frac{d}{dt} T(F_{X,t}) \right. \\ & \quad \left. + tF''_Z(T(F_{X,t})) \left(\frac{d}{dt} T(F_{X,t}) \right)^2 \right. \\ & \quad \left. + tF'_Z(T(F_{X,t})) \frac{d^2}{dt^2} T(F_{X,t}) \right] \Big|_{t=0} \\ &= 2[F'_Z(T(F_X)) - F'_X(T(F_X))] T'(F_X; F_Z) \\ & \quad + F''_X(T(F_X)) (T'(F_X; F_Z))^2 \\ & \quad + F'_X(T(F_X)) T''(F_X; F_Z) \\ &= 0. \end{aligned} \quad (24)$$

Finally, we obtain:

$$\begin{aligned} T''(F_X; F_Z) &= \frac{2(F'_X(T(F_X)) - F'_Z(T(F_X))) T'(F_X; F_Z)}{F'_X(T(F_X))} \\ & \quad - \frac{F''_X(T(F_X)) (T'(F_X; F_Z))^2}{F'_X(T(F_X))}. \end{aligned} \quad (25)$$

Replacing the first and second-order derivative formulae in (19), we obtain:

$$\begin{aligned} q_Z - q_X &= \frac{F_X(q_X) - F_Z(q_X)}{f_X(q_X)} \\ & \quad + \frac{(f_X(q_X) - f_Z(q_X)) (F_X(q_X) - F_Z(q_X))}{f_X(q_X)^2} \\ & \quad - \frac{f'_X(q_X) (F_X(q_X) - F_Z(q_X))^2}{2f_X(q_X)^3}, \end{aligned} \quad (26)$$

where the simplified notation q_X designates $q_X(1-R)$.

In the optimal portfolio problem, we wish to approximate the quantile of the portfolio $Z = X + \varepsilon Y$, where X represents the initial portfolio and εY the added load. To this aim, according to (26), we should evaluate the cumulative density function Z , in q_X . However, the existing portfolio would not reasonably share its stochastic profile with the new load and vice versa when prospecting for new loads. In practice, we could evaluate some stochastic properties of X or εY , but not Z . To solve this issue, one approach is to linearize the cumulative distribution function F_Z around F_X using a Taylor expansion. If X and εY are independent, we can prove that:

$$F_Z(u) = F_{X+\varepsilon Y}(u) = \mathbb{E}[F_X(u - \varepsilon Y)]. \quad (27)$$

Therefore, we first linearize F_X around u and, then, consider the expected value of the expression to obtain:

$$\begin{aligned} F_X(u) - F_Z(u) &= \varepsilon \mathbb{E}(Y) f_X(u) \\ & \quad - \frac{\varepsilon^2}{2} \mathbb{E}(Y^2) f'_X(u) + o(\varepsilon^2). \end{aligned} \quad (28)$$

Considering $u = q_X$, we can simplify (26) to obtain the following second-order expression:

$$\begin{aligned} q_Z - q_X &= \varepsilon \mathbb{E}(Y) - \frac{\varepsilon^2}{2} \mathbb{E}(Y^2) \frac{f'_X(q_X)}{f_X(q_X)} \\ & \quad + \varepsilon^2 \mathbb{E}(Y)^2 \frac{f'_X(q_X)}{f_X(q_X)} - \frac{\varepsilon^2}{2} \mathbb{E}(Y)^2 \frac{f'_X(q_X)}{f_X(q_X)} + o(\varepsilon^2) \\ &= \varepsilon \mathbb{E}(Y) - \frac{\varepsilon^2}{2} \frac{f'_X(q_X)}{f_X(q_X)} (\mathbb{E}(Y^2) - \mathbb{E}(Y)^2) + o(\varepsilon^2) \\ &= \varepsilon \mathbb{E}(Y) - \frac{\varepsilon^2}{2} \frac{f'_X(q_X)}{f_X(q_X)} \text{Var}(Y) + o(\varepsilon^2). \end{aligned} \quad (29)$$

Finally, in (19), we only consider a second-order expansion. As a consequence, there is a remainder term which could introduce first and second-order ε terms. Hence, to complete this development, we should prove that the remainder term in (19) is $o(\varepsilon^2)$. To this aim, we should demonstrate that the k^{th} derivative of T evaluated at F_X in direction F_Z exists and is of order ε^k . This can be proven by recurrence, using a similar approach to that for the first- and second-order derivatives. However, for brevity, we omit the detailed proof here.

4) *Verification in the Gaussian Case:* We want to verify (13) in the Gaussian case, for which there exists an explicit formulation of the quantile function. If a stochastic variable X is Gaussian, we can relate its $(1 - R)$ -quantile to the $(1 - R)$ -quantile of the centered standard Gaussian, $q_{0,1}$, as:

$$q_X = \mu_X + \sqrt{\sigma_X^2} q_{0,1}, \quad (30)$$

where μ_X and σ_X^2 denote the mean and variance of X , respectively. Similarly, considering $Z = X + \varepsilon Y$, where X and Y are two independent Gaussian variables, we know that Z is Gaussian and its $(1 - R)$ -quantile verifies:

$$q_Z = \mu_X + \varepsilon \mu_Y + \sqrt{\sigma_X^2 + \varepsilon^2 \sigma_Y^2} q_{0,1}. \quad (31)$$

The change of $(1 - R)$ -quantile resulting from adding εY to X is:

$$q_Z - q_X = \varepsilon \mu_Y + \underbrace{\left(\sqrt{\sigma_X^2 + \varepsilon^2 \sigma_Y^2} - \sqrt{\sigma_X^2} \right)}_{\Delta_\sigma} q_{0,1}. \quad (32)$$

Considering that ε is small, we can linearize Δ_σ around σ_X^2 , such that, at the second order, we obtain:

$$q_Z - q_X = \varepsilon \mu_Y + \frac{1}{2} \frac{\sigma_Y^2}{\sigma_X} q_{0,1} \varepsilon^2 + o(\varepsilon^2). \quad (33)$$

Finally, in the case of a Gaussian variable X , we can demonstrate that α_X appearing in (13) is:

$$\alpha_X = \frac{f'_X(q_X)}{f_X(q_X)} = -\frac{q_X - \mu_X}{\sigma_X^2} = \frac{q_{0,1}}{\sigma_X}, \quad (34)$$

which confirms that, in the Gaussian case,

$$q_Z - q_X = \varepsilon \mu_Y + \frac{\varepsilon^2}{2} \sigma_Y^2 \frac{f'_X(q_X)}{f_X(q_X)} + o(\varepsilon^2). \quad (35)$$

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A. Paper summary

The paper considers portfolios of flexible loads (EVs and HVAC units) for ancillary service products such as grid reserves (e.g., FCR-D). Models are presented for EVs and HVAC units that form an aggregator's portfolio, which under uncertainty is equivalent to a joint chance-constrained problem. An analytical reformulation is presented that leverages quantiles. For a given portfolio, the paper then derives how an individual load added to an existing portfolio impacts (enhances) the quantile, which aids in optimally augmenting portfolios. The different methods are simulated and tested and show small discrepancy for portfolios of 200+ devices.

B. Comments for authors

Strengths

- Well written paper.
- Excellent introduction that clearly motivates an interesting problem.
- The explication of the different roles (System, Aggregator, Load) throughout is excellent and valuable. Highlight this angle more formally.
- The development of the energy bound-based modeling of flexible resources is well done and interesting (albeit not entirely novel).
- Section III is well done and educational. In particular, (13) is insightful and clever as a method for optimally augmenting an existing portfolio with a (small group of) load(s).
- Results in Figs. 5-7 are interesting and aligns with the broader DER literature showing that diversity in device types enhances aggregate flexibility. It is good to see this important lesson confirmed in this context.

Weaknesses

Most weaknesses are not major, unless otherwise noted, but attempts to provide authors with feedback.

1. Breadth of Literature Review The literature review encompasses almost entirely work by the DTU group of Kazempour in [12, 14–15] (with Zwart in [15]), except for [16]. Have others not worked on portfolio optimization of flexible loads? There is an *incredibly* rich literature on generator portfolios that is relevant here. For example, consider Kirschen 2011, McCalley 2013, Wierman 2016, and Bouffard 2020. Consider broadening the literature review to better place your results in context.

2. Framing of Second Contribution

The second bullet point in the contributions does not read as a contribution in its current phrasing. Consider posing this in terms of a counter-intuitive finding: e.g., "We demonstrate a counter-intuitive result showing how wisely pairing loads into different portfolios..." This would better convey the significance.

3. Formulation Clarity: (6) and (7)

- Is formulation (6) standard? If so, a reference should be provided. If this is the first time it has been formulated, that novelty should be noted.
- It seems that (6) is incomplete: it is unclear what e^k is supposed to be (a free variable?). The reviewer assumes $e^k := e_0 + \Delta t \sum_t p^t$.
- In (7), the elimination of b and e baselines is clever, but there is an added subscript h in (7b)–(7e) (e.g., $p_{i,h}^{up,+}$) while neither b_i^k nor $p_i^{\max,k}$ carry subscript h . This should be explained or corrected.
- Are (6) and (7) equivalent problems? What is the reason for introducing (6) when the normalized/baselined models could be developed directly in the modeling of Section II.A?

4. Significance and Novelty of Analytical Reformulation

Major: The paper states matter-of-factly that (11) is an analytical reformulation of (8) and the derivation is presented as rather straightforward. However, the significance is unclear: is this a novel result or a textbook result? Some discussion is needed. Additionally: how many scenarios are needed for the reformulation to be reliable? What is meant by "may yield suboptimal results"? What is the intuition behind identical prices causing the analytical reformulation to be inexact – this was not mentioned during the derivation.

5. Portfolio Optimization Contribution (Section III.A)

Major: While Section III is well written and educational, portfolio optimization is a well-studied field. It is therefore not surprising that "an optimal portfolio allocation exists," which makes the paper's second contribution claim somewhat weak. Consider condensing III.A or moving some material to an appendix to make room for strengthening other parts of the paper.

6. Section III.B: Subsection Title and Scope

The subsection header "Forming the Optimal Portfolio" is misleading: the method optimally *improves* (or augments) an existing portfolio rather than forming one from scratch. To fully deliver on the title, the authors would need to provide a methodology for forming a portfolio or set of portfolios from $M \gg 1$ loads. This would enhance the paper.

Additionally, when presenting (13), the authors should note upfront that α_x is not easy to compute from sampled data—this assumption is only mentioned later in V.C. Furthermore, how is (13) applied when the portfolio groups (EV_A/B and HVAC) are not "epsilon"-small (i.e., single loads)?

7. Intra-Hour Dynamics

Major: What effects do intra-hour dynamics have on this formulation? The one-hour blocks are rather coarse relative to HVAC temperature dynamics (on the order of 20–30 minutes) and Level 3 charging (e.g., high $p_{EV}^{\max}$). This is never mentioned and unclear if validation considers this.

8. Simulation Results

- For Section V.B, consider reporting worst-case absolute errors (or quartiles) rather than mean errors, even if Fig. 3 already show this. In fact, Fig. 3 show errors > 0 for the "one price dominates" case (upper right) when this case is supposed to be exact. Why?
- It seems that the One Portfolio strategy dominates all cases. If so, why pursue multiple portfolios? Some discussion of when multiple portfolios would be advantageous would be helpful.

9. Computational Complexity

Computational complexity, solve times, and scalability are not mentioned except in future work. Some indication of potential scalability challenges would be welcome.

Minor Comments

- The HVAC acronym is incorrect in Section I. Please update.
- Equation (1) states that when not plugged in, EV power/energy ratings are zero. However, this appears to conflict with (1) since $e_0 \geq 0$ and for $t > t^d$ the sum of powers is also non-zero. Please rephrase.
- Please clarify that a first-order lumped HVAC model is used (as opposed to second-order lumped models with mass temperature).
- The conversion to energy bounds is detailed for HVAC via [17], but for EVs, a detailed definition of these bounds is missing. Please provide this, and also consider citing a related EV modeling approach, e.g., : Madjidian, Roozbehani & Dahleh, "Energy Storage From Aggregate Deferrable Demand: Fundamental Trade-Offs and Scheduling Policies," IEEE Trans. Power Systems, vol. 33, no. 4, 2018.
- Is (7) missing a constraint corresponding to (6f)?
- Please cite the TVM expansion in the main text (or at least direct the reader to the appendix).
- The System/Aggregator/Load role comparison is currently buried in text across different locations. Adding a summary table would enhance readability. – When a load fails to provide reserves, how are other loads in the portfolio notified of the emerging slack that needs to be met?

C. Summarize your recommendation in one to two sentences

Well-written paper with a clever portfolio augmentation analysis (Eq. 13) and solid simulations, but the novelty of the analytical reformulation/contribution needs clearer positioning, the literature review should be broadened beyond a single research group, and several formulation details require clarification. A minor-to-moderate revision is recommended.

— Anonymous reviewer

II. REVIEW #128B

A. Paper summary

This papers describes a method of allocating reserves into groups that must provide reserves with a particular reliability level. The approach is developed from Danish TSO rules that enable energy-constrained sources with stochasticity to participate. They show that when flexible loads with different stochastic behavior are combined in different ways, that there is a notion of an optimal portfolio, and they provide an analytical approach to determining how to allocate these. Results are presented that support their algorithmic approaches.

B. Comments for authors

Overall, this is a very strong submission. The contributions are clearly defined, are clearly linked to implications, are contextualized well with respect to existing work, and are described as advancements beyond that existing work. The innovation was clearly explained and the results supported the developed approaches.

My suggestions are minor.

- One thing that would be nice, is when errors are given in raw units (e.g., kW), to quantify them against the nominal scenario. For example, in Section VB, the errors could be quantified as a percent, so there is some context to the magnitude.
- It would be interesting to include or see some commentary about whether this would apply to any set of devices that have quantifiable randomness in power and energy limits.
- Are there other scenarios or settings where this approach could be used? For example, it might be generally applicable in quantifying the amount of load reduction or increase a set of devices could contribute based on their characteristics, as well as what the marginal gain of the next device would be.

C. Summarize your recommendation in one to two sentences

I recommend that this paper be accepted.

— Greg Ledva

III. REVIEW #128C

A. Paper summary

This paper presents two formulations of the optimal reserve quantification problem, using the Danish market with EVs and HVACs as an example. The exact formulation involves joint chance constraints, but they find similar results with their reformulation (according to numeric results).

B. Comments for authors

The authors study the interesting problem of determining the optimal reserve requirements and complete numeric tests using a combination of EV and HVAC loads. The problem is well motivated and the results are interesting, although I found the following weaknesses:

- The paper is presented as though the reformulations are the main contributions. In this case I do not think it appropriate to have them in an appendix, and to spend so much space on examples and results.
- Some of the results are difficult to interpret – e.g. for Fig. 6 rather than visualizing very minor differences in the horizon curve it would be more useful to better understand the differences in reserves procured in each case.
- Overall there are a lot of different cases presented, and it is not totally clear what the authors are trying to illustrate.
- The motivation of the paper was computational complexity of the JCC, but I see very little discussion of the computational cost of each method.

C. Summarize your recommendation in one to two sentences

An interesting paper with important results, although the presentation and overall "message" are a little messy.

— Constance Crozier

IV. BRIEF RESPONSE TO REVIEWERS (OPTIONAL)

We thank all reviewers for their detailed feedback. We have incorporated most of the comments into the revised manuscript, as described in the Summary of Changes. Below, we address the four points we did not integrate and explain our reasoning for each.

Reviewer 1 (#128A)

Comment 1 - Breadth of literature review

The reviewer points out that our literature review draws predominantly from a single research group, and suggests broadening it to better contextualize our results within the wider portfolio optimization literature. We fully agree with this observation. However, expanding the survey would exceed the page limit of this conference paper. We have kept the references most directly relevant to our specific formulation (reserve provision under joint power, energy, and reliability constraints), and we will broaden the review in a future journal extension.

Comment 6 - Generalisation beyond ε -small loads

The reviewer asks how formulation (13) applies when portfolio groups are not ε -small. This is a valid concern. Our augmentation formula relies on a second-order Taylor expansion of the portfolio quantile, which is valid under the assumption that the added loads are small relative to the existing portfolio. When this assumption is violated, the expansion may become a coarse approximation of the true quantile of the augmented portfolio, and the accuracy of (13) can degrade. Assessing how well the approximation holds as the size of the addition increases and establishing practical bounds on when it remains acceptable will be considered in future work.

Minor comment - Summary table for System/Aggregator/Load roles

The reviewer suggests adding a summary table to make the distinction among the System / Aggregator / Load roles more prominent. We agree this would improve readability, but a table is not feasible within the current page budget. We will include it in a future journal version.

Minor comment - Load dispatch and signal disaggregation

The reviewer asks how other loads in the portfolio are notified when one load fails to provide its share of reserves. This is an important and valid question, but it lies beyond the scope of this paper, which focuses on determining reserve capacities offline rather than on real-time dispatch.

V. BRIEF SUMMARY OF CHANGES MADE TO THE FINAL PAPER

We thank all three reviewers for their constructive feedback. We describe the main changes below, grouped by theme. We have integrated most reviewer comments into the revised manuscript; the remaining points are addressed in the Response to Reviewers.

Contribution framing and positioning

We reframed the second contribution bullet to highlight the counterintuitive finding that an optimal pairing of loads into different portfolios exists. We also repositioned the novelty of the analytical reformulation (11) more clearly, with a stronger statement of what distinguishes our approach from existing literature.

Formulation clarity

We added several clarifications around formulations (6), (7), and (11). We added a reference for (6) and defined the accumulated energy variable e_b^k more precisely. A new footnote explains the transition from (6) to (7), and specifically why the superscript k disappears. We updated the title of Section III.B to reflect that the method augments an existing portfolio rather than forming one from scratch, and we added a sentence after (13) noting that α_X is not straightforward to compute from sampled data. For (11), we indicate the number of scenarios through a link to an appropriate reference, and we provide an intuition for why the reformulation is exact under a single reserve direction but may fail when both directions are co-optimized. We also moved Section III-A to the appendix and added a reference to the TVM appendix in the main text.

Results and computational aspects

We now report the 75th-percentile and maximum absolute errors in Section V.B alongside mean errors, and we express results as percentages of the nominal scenario to ease interpretation. We explain the small residual errors in the 'one price dominates' case as numerical artifacts in the quantile computation, and we added a discussion of the observation that a single portfolio consistently outperforms multiple portfolios. We also included computation times.

Scope and applicability

We added a sentence clarifying that the methodology applies to any assets that can be represented by power and energy bounds with a continuous consumption variable.

Minor corrections

We corrected the HVAC acronym in Section I, fixed the claim that EV energy is zero when unplugged (only power is zero), and explicitly described the HVAC model as first-order lumped. We added a reference for the EV energy bound derivation. The reformulation proof remains in the appendix due to page constraints; we plan to promote it to the main text in a journal extension.