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MULTIPERIPHERAL DYNAMICS

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MULTIPERIPHERAL DYNAMICS

Irvine Review, December 6, 1969

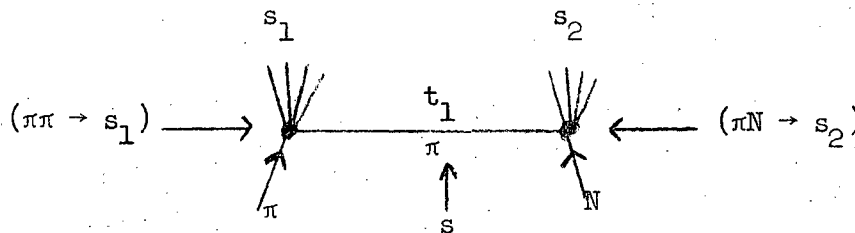
Geoffrey F. Chew

Lawrence Radiation Laboratory and Department of Physics
University of California, Berkeley, California

A. INTRODUCTION

The multiperipheral idea has been reached through many different paths by many different physicists. It is constantly being rediscovered, cast into new forms, and being called by new names. I interpret the 10 year persistence of the idea as indicating its inevitability. To my mind the question is no longer whether the qualitative idea is correct but how we should incorporate it into bootstrap dynamics. In the time available today I cannot review all the physical arguments that have been used to motivate multiperipheralism. Let me remind you of just one.

It is experimentally established that the singly peripheral description of a reaction such as

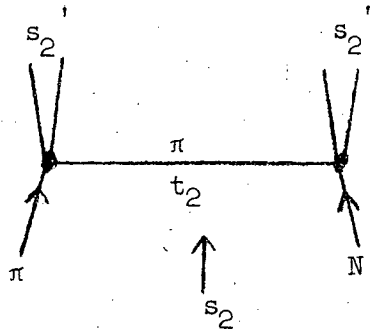


is meaningful when

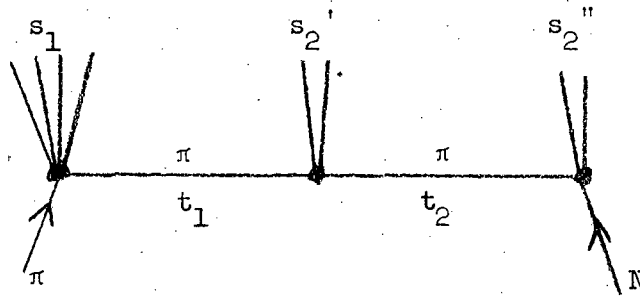
$$t_1^{\min} \approx \frac{s_1 s_2}{s}$$

is sufficiently small. "Singly-peripheral" means that the full amplitude can be approximated as the product of a factor proportional to the $\pi\pi \rightarrow s_1$ amplitude and a factor proportional to the $\pi N \rightarrow s_2$ amplitude. Now if s

is large enough, the singly peripheral description will allow a sufficiently large s_2 that the latter amplitude itself admits a singly peripheral representation:



The criterion now is that $t_2^{\min} \approx s_2' s_2'' / s_2$ be small. The full amplitude then can be represented as



with the requirement that

$$\frac{s_1 s_2' s_2''}{s \tau^2} \lesssim 1$$

where τ is some "interblob" mean momentum transfer squared. Evidently the decomposition can be continued to any number of "blobs" so long as s is large enough to satisfy the appropriate product inequality. I do not see how one can deny this line of reasoning, once the singly peripheral notion is accepted. An important question remains, of course, as to what proportion of all reactions at any given energy can be described as singly peripheral. Another question is the minimum blob size.

Several general features of the multiperipheral mechanism were emphasized by ABFST.¹ One almost immediate consequence is a logarithmic increase of multiplicity with energy. If we agree on a blob of mean mass squared \bar{s} , we can add one of these blobs to the chain each time s is increased by a factor

$$e^w \gtrsim \bar{s}/\tau.$$

To make N blobs, we require

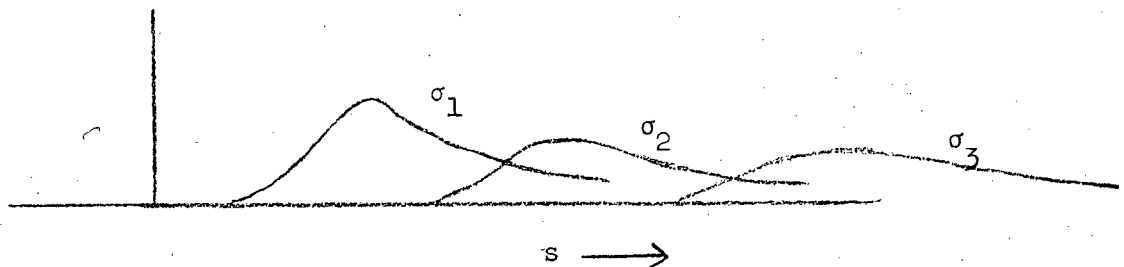
$$s \sim e^{Nw} s_0, \quad \text{or} \quad \ln(s/s_0) \sim Nw,$$

where s_0 is the energy required for a single blob. Thus the mean total multiplicity, with \bar{n} particles per blob is $\sim \frac{\bar{n}}{w} \ln(s/s_0)$.

Note that this result is independent of blob size as long as $\bar{n} \propto \ln \bar{s} \propto w$.

The asymptotic distribution of multiplicities at a given energy, not surprisingly, turns out to be of the Poisson form.

It also can be shown that the partial cross sections for making different numbers of blobs add together so as to produce a total cross section that varies as a power of energy. Since the individual partial cross sections have a much more complicated energy dependence



this prediction of Regge behavior is remarkable. The connection of multiperipheralism with Regge poles is of course the reason for my talk at this meeting.

Another immediate consequence of multiperipheralism is what Yang and collaborators have recently called the "limiting distribution."² The effect can also be described as "short range order" in longitudinal momentum. Particles emitted from a localized portion of the multiperipheral chain evidently have a behavior that in an appropriate local frame of reference is independent of what is occurring at distant portions of the chain. As one moves along the chain, furthermore, one is populating successive regions of increasing (or decreasing) longitudinal momentum. Thus the distribution within a finite interval of longitudinal momentum approaches a limit independent of the length of the chain--that is, of the total energy. Intervals of longitudinal momentum corresponding to regions near the ends of the chain will have different distributions from "central" regions, although a larger and larger fraction of the chain becomes "central" as the energy increases. At the highest currently accessible energies, "end-effects" are still of major importance. A well-known special application of the limiting distribution phenomenon is to transverse momentum, but the argument also applies to distributions in particle type.

A parenthetical remark: When the multiperipheral chain has at most a few links, as at currently accessible energies, statistical fluctuations allow many of its properties to be described in "fireball" terminology. Evidently, however, the number of fireballs is not well-defined.

The above qualitative aspects of multiperipheralism all are supported by experiment. You may have noted that the specific mechanism of pion exchange was not needed. Any mechanism leading to some kind of "factorization" of "short-range order" along the multiperipheral chain

is adequate. [A precise general characterization of what is meant by the "chain" and by "short-range order" is neatly given through Toller variables, that is, by using successive Lorentz transformations rather than particle momenta to describe the system.] A different mechanism from that of ABFST has been intensively studied during the last two years--one in which the chain links are associated with Regge poles. This is the multi-Regge-pole model.³

B. THE MULTI-REGGE-POLE MODEL

One version of the multi-Regge model, developed by Chan, Loskiewicz, and Allison,⁴ has been extensively used in fitting individual reactions. A cruder but similar phenomenological model, proposed by Chew and Pignotti⁵ and further developed by Caneschi and Pignotti,⁶ has been applied to the fitting of total cross sections and corresponding collective distributions. To justify such models one must rely heavily on the duality idea--that Regge pole asymptotic representations have some average validity even in the low energy resonance region. The CLA and CP models reduce the blob size to a single stable particle, usually a pion. Now it is known experimentally that the mean mass of adjacent pion pairs in the multiperipheral chain is less than 1 GeV. Thus, in the CLA and CP models one employs a Regge representation of the $\pi\pi$ amplitude in the region of the rho resonance!

The parameters of the CLA and CP models are adjustable, nevertheless, and the empirical success of the models is understandable. It has been shown, in fact, by Ball and Marchesini⁷ and by Chew, Rogers, and Snider⁸ that the original ABFST model, with a correct treatment of the $\pi\pi$

resonances, is consistent with the multi-Regge model if trajectories and coupling constants are appropriately adjusted. The correspondence of the model chain-links with actual Regge poles and residues, however, becomes so blurred that only the crudest of bootstrap applications are possible. The bootstrap idea here, of course, is that Regge links in the chain should generate a corresponding set of output Regge poles. A similar thought lies beneath any multiperipheral bootstrap model, and I shall postpone further discussion until we take up a model more reliable than the multi-Regge.

One of the most interesting aspects of multi-Reggeism, as distinct from other ways of discussing the multiperipheral idea, is the role of the Pomeranchuk trajectory as a link in the chain. It was shown by Finkelstein and Kajantie⁹ that repetition of the Pomeron link would violate the Froissart limit if $\alpha_P(0) = 1$. This conclusion can be reached independent of duality and in fact applies also to the pion exchange mechanism if one merely says that $\sigma_{\pi\pi}^{\text{tot}} \rightarrow \text{constant}$. When actual numbers based on experiment are inserted, however, the Pomeron coupling turns out to be so weak that with $1 - \alpha_P(0) \gtrsim 0.02$, there is no difficulty in accommodating Froissart. These estimates have been made by Pignotti,⁵ by Ball and Marchesini,⁷ and by Snider.⁸ Two years ago I stated in a review of the status of the Pomeranchuk trajectory that, "Most physicist...feel it would be ugly for total cross sections to almost, but not quite, approach constants at high energy. It would seem frivolous of Mother Nature to tease scientists in such a fashion." On further reflection, however, I conclude that such "teasing" has occurred repeatedly in the history of physics. Whenever a small

dimensionless parameter generates corresponding small effects that are difficult to measure, it is likely that models which set the parameter equal to zero appear so beautiful to the eye of the physicist that he resists the seeming complication of a nonzero value. It has always turned out, of course, that some alternative way of looking at the physics eventually has restored the beauty. Given that the Pomeron coupling is small, I no longer feel oppressed by the failure of $1 - \alpha_P(0)$ to be exactly zero. On the contrary, I am pleased to find a connection between two different small parameters.

C. GENERATION OF REGGE POLES AND CUTS

I have referred to the Pomeron's role as a link in the chain, but the Pomeron also appears as the leading asymptotic power in the total cross section that results from summing the multiperipheral series. The ABFST demonstration that Regge poles emerge from such a sum has recently been extended to multi-Regge models through a technique which also could be applied to more general multiperipheral models. Contributing to the development of this technique have been Goldberger and Low,¹⁰ Halliday and Saunders,¹¹ DeTar,¹² Mueller and Muzinich,¹³ and Ciafaloni and Misheloff.¹⁴ This technique makes it plausible that any mechanism based on "short-range order" will lead to Regge poles. It furthermore is almost certain that the leading pole will have the quantum numbers of the vacuum. Acceptance of multiperipheralism, therefore, probably means acceptance of Regge-pole status of the Pomeron.

What about the Pomeron slope? All multiperipheral models so far studied, either of the ABFST or multi-Regge type, have generated "normal" slopes, that is, of the order 1 GeV^{-2} . These models accord the Pomeron

and the rho a similar status, and although the multiperipheral model does not predict whether physical particles will appear on a trajectory, the general similarity of the Pomeron to the rho suggests physical Pomeron particles, in particular a 2 plus unitary singlet meson at a mass of about 1 GeV. The width of this meson might be as narrow as ~ 20 MeV, corresponding to the small Pomeron coupling that I have already emphasized. Experimental developments in this region are being eagerly watched.

What about cuts? Multiperipheral models, whether of the pion-exchange or multi-Regge type, tend to generate Regge branch points as well as Regge poles when the series is summed. The strength and sign of the cut discontinuity depends on the details of the models. In particular, Caneschi¹⁵ has shown how the absorptive type of discontinuity may be generated. *As you heard yesterday from Pignotti, this argument has been further refined.* An interesting contribution from multiperipheral models is the light they have shed on the cut-pole relationship. I mention two aspects: (a) Just as in the energy complex plane, angular momentum cuts tend to be most important when nearby poles lie beneath them. In such an event the effect of the cut is approximately reproduced by moving the pole to the physical sheet and ignoring the cut. (b) Although Regge poles do not mathematically collide with branch points in multiperipheral models, in physical effect they may move smoothly across a branch point. The mechanism works in this way: As a Regge pole on the real axis of the physical sheet approaches a branch point it loses strength; at the same time a second sheet complex pole is approaching. The first sheet pole never crosses the branch point, but its residue becomes negligible while the second sheet complex pole

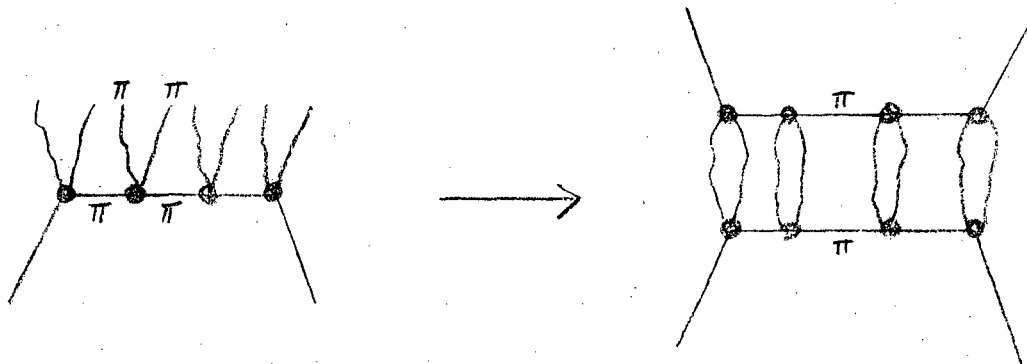
moves parallel to the real axis past the branch point in such a way that a resulting peak in the cut discontinuity reproduces the effect of an unimpeded moving real pole.

These Regge branch point phenomena have been studied by Frazer,¹⁶ by Pignotti,¹⁷ and by Ball and Marchesini.⁷

Quite apart from the issue of Regge cuts, multiperipheral models also suggest complex Regge poles that could for example, produce damped oscillations in total cross sections. The period of these oscillations corresponds to the interval $\Delta \ln s = w$ already discussed in connection with the logarithmic growth of multiplicity. It is conceivable that the upturn in K^-p total cross section observed near 40 GeV in recent Serpukhov experiments might be a manifestation of such an oscillation. Snider and I, assuming 40 GeV to be a minimum, have attempted to estimate on the basis of the observed rate of growth of multiplicity where the next maximum in the total cross section would occur, and we have arrived at a guess of ~ 150 GeV.¹⁸

D. THE ABFST MODEL

The only multiperipheral model whose kernel can be calculated in a direct way from experiment continues to be that of ABFST:¹



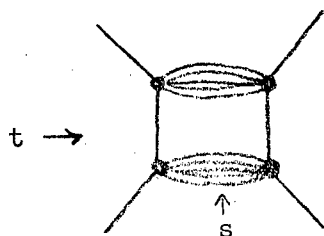
The kernel is proportional to the elastic $\pi\pi$ cross section, and the low energy resonance region dominates. Qualitatively, using available knowledge about $\pi\pi$ scattering, the model does well, predicting $\alpha_P > \alpha_0$ and no high ranking $I = 2$ trajectory.^{7,8} It also predicts a reasonable transverse momentum distribution and trajectory slope. The kernel strength, however, is too weak by a factor 3 - 5. Perhaps more important, the mean interblob momentum transfer, when the blobs are broken down to the two-pion level, turns out to be of the order of 1 GeV^2 . One is therefore not operating sufficiently close to the pion pole to justify the model. The off-mass shell ambiguity, that is to say, is overwhelming. The most recent numerical studies of the ABFST model have been by Ball and Marchesini,⁷ by Tow,²¹ and by Rogers, Snider and myself.⁸ Because of quantitative limitations, and the lack of crossing, neither the multi-Regge or the ABFST models have been as well suited to bootstrap applications as some of us had hoped.

E. THE STRIP MODEL

A more ambitious form of multiperipheral model, which respects the principle of crossing and stays entirely on shell is the strip model.¹⁹ Here the bootstrap possibilities are more promising. The strip model was vigorously discussed following its introduction in 1961, its history being intimately interwoven with the initial hypothesis of Regge asymptotic behavior. The Regge pole idea developed a life of its own, however, as indicated by this Conference, and for some years one has not heard much talk about the strip model. The reason is not that the model failed to reproduce known experimental facts but rather that no one

succeeded in solving the nonlinear equations of the model.

I shall not write down the equations here. Suffice it to say that although these equations involve only elastic scattering amplitudes they do not neglect inelastic processes. Far from it! The multiple production process constitutes a central component of the bootstrap mechanism in the strip model. The key approximation making the model tractable is the representation of inelastic effects by a multiperipheral mechanism. The mechanism is implemented through the Mandelstam double discontinuity, which can be represented by a sum of four-vertex graphs. A well-defined subset of these graphs is



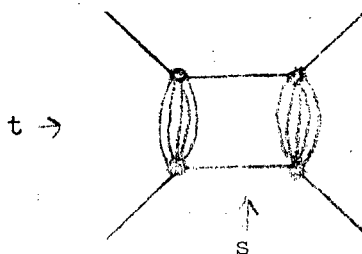
which we call $\rho_{el\ s}(s,t)$, since only two-particle "intermediate states" in the s reaction are included. All other graphs contribute to $\rho_{in\ s}(s,t)$, i.e.,

$$\rho(s,t) = \rho_{el\ s}(s,t) + \rho_{in\ s}(s,t).$$

Now the basic approximation of the strip model is to set

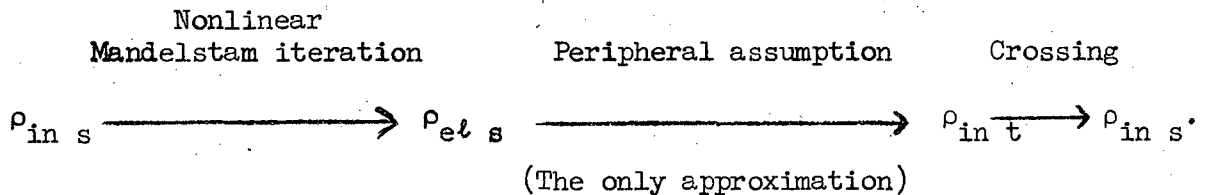
$$\rho_{in\ s}(s,t) \approx \rho_{el\ t}(s,t)$$

which means that in $\rho_{in\ s}$ we keep the following subset of graphs:



This is a singly-peripheral approximation and if one looks at the Mandelstam iterative formula for evaluating this graph through the simple s discontinuity, one recognizes the essential elements of multiperipheralism. It is not easy here, however, to identify a well-defined blob size.

With the assumption of second degree analyticity, the double discontinuity determines the single discontinuities, and the Mandelstam formula provides a nonlinear relation between $\rho_{in s}$ and $\rho_{el s}$. The bootstrap cycle, schematically, is then



Although the strip model equations are easily written down, the possibility must be considered that no solutions exist. The reason is the nonlinearity of the equations, which has defied all attempts at systematic analysis. It can be made plausible that if Regge behavior is assumed as $s \rightarrow \infty$ then the model equations imply Regge behavior as $t \rightarrow \infty$, with all output parameters determined by input parameters. Even granted that such is true, however, it has been impossible to prove that there exists a choice of input which will lead exactly to the bootstrap requirement:

$$\text{output} = \text{input}.$$

For a number of years computational difficulties were a further obstacle. That is, given an input we were not able to calculate the

output with sufficient accuracy. A host of inadequate approximations have confused the literature.

There was continuous activity along strip model lines in Berkeley until 1966, involving Steve Frautschi, Ed Jones, Vic Teplitz, Naren Bali, and Shu-Yuan Chu. Each of these theorists, after making what is now recognizable as important progress, eventually fell in weariness by the wayside and turned to other tasks. Peter Collins had more stamina; after two years in Berkeley he went to Durham and continued his efforts with a research student, R. C. Johnson, for three more years.

Collins identified and remedied a crucial defect in the previous method of computation and then settled down, with Johnson, to a patient search for an input to the $\pi\pi$ amplitude which would generate a matching output. Let me tell you what they found.

F. THE COLLINS-JOHNSON RESULTS²⁰

To begin, I must say something about cutoffs and cuts. The strip model predicts Regge cuts as well as Regge poles, which is a virtue not a vice, but which adds to the computational difficulties. I have spoken of how the effect of the cut can be approximately reproduced by moving the underlying pole to the physical sheet. Now it turns out that in multiperipheral models one accomplishes this replacement of cuts by underlying poles through the introduction of cutoffs in certain integrations that in principle should go over an infinite energy range. Dale Snider and I have been able to show that the results are insensitive to such cutoffs if they are chosen above the region of prominent low energy resonances.

The strip model is a variety of multiperipheral model, so we expect it to admit a cut-off prescription that eliminates Regge cuts in favor of poles. The Collins-Johnson model in fact employs such a device and the authors have studied the numerical dependence of their results on the particular cut-off choice. They find negligible dependence if the cut-off energy is chosen above 4 GeV. We therefore should not consider their cutoff as an arbitrary parameter; it is merely a device of convenience.

Although the Collins-Johnson model inputs the physical value of the pion mass, it appears that the results would not appreciably be changed if m_π were set equal to zero. The model consequently may be described as containing no parameters, although we must not forget that without the small pion mass the strip model would lack motivation. In any event the model sets its own energy scale, an unavoidable property of a bootstrap mechanism. Bootstraps, that is to say, only generate ratios of energies.

It turns out that the Collins-Johnson model predicts the existence of a meson with the quantum numbers of the rho, and they choose the mass of this meson to be 760 MeV to set their scale. They succeed in finding a solution with approximate input-output consistency in the region $|s|, |t| \lesssim m_\rho^2$, and only one solution, the chief properties of which are as follows:

1. There is a single high-ranking $I = 1$ trajectory, which they of course identify with the rho. Near $t = 0$ the trajectory is quite linear, with an intercept

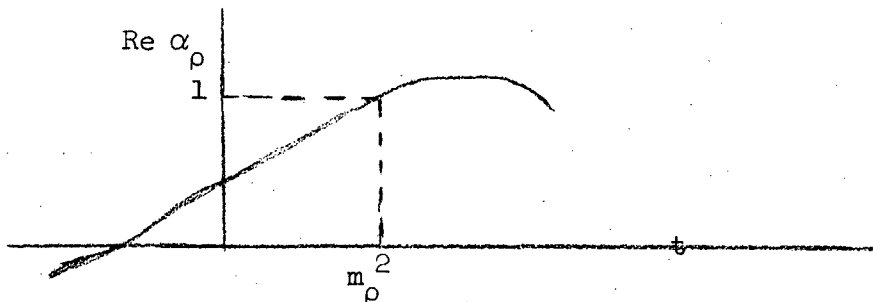
$$\alpha_\rho(0) = 0.5 \pm 0.2,$$

the limits corresponding to their estimate of how bad a mismatch of input with output is tolerable in view of the known limitations of the model. The $I = 1$ residue determines the rho width; Collins and Johnson find

$$\Gamma_{\rho} = 140 \pm 40 \text{ MeV,}$$

a result that will be fully appreciated only by bootstrappers who have tried to calculate it from less accurate models. The usual theoretical prediction is 3 to 4 times larger.

The Collins-Johnson rho trajectory turns over soon after passing $J = 1$ at positive t and does not generate higher spin resonances of small width. At large negative t the trajectory flattens out, approaching an asymptotic limit of $J = -0.35$.



I may interject here that there is no reason for disappointment over the model's failure to generate indefinitely rising trajectories. The higher spin resonances on the experimentally observed leading trajectories are coupled primarily to channels other than $\pi\pi$. Since these resonances have almost no effect on the $\pi\pi$ amplitude they cannot be expected to emerge from a model based on this amplitude alone. Only those poles dominantly coupled to $\pi\pi$ can reasonably be anticipated at the level of approximation of the strip model.

2. There is one high ranking $I = 0$ trajectory, with intercept

$$1.05 \pm 0.15,$$

which Collins and Johnson correspondingly call "the Pomeron." (Had they not employed their cutoff, the mechanism of the Frossart limit would have prevented an intercept above 1.) This P trajectory, whose slope at $t = 0$ is $\approx 1 \text{ GeV}^{-2}$, turns over before reaching $J = 2$, but the residue at $t = 0$ can be converted into

$$\sigma_{\text{tot}}^{\pi\pi}(\omega) = 26 \text{ mb},$$

to be compared to the usual estimate of 15 mb.

3. There is no high ranking $I = 2$ trajectory.

Again an interjection. One may be surprised that a model with crossing, Regge behavior, and no $I = 2$ resonances should fail to exhibit the $\rho - P'$ degeneracy which is such a striking feature of linearized models such as that of Veneziano. A possible explanation involves three observations:

(a) The Collins-Johnson P residue is too large, so a single $I = 0$ trajectory in the model may be combining, in some average sense, the effect of P and P' .

(b) Although the P trajectory lies above the rho, the difference is only a half-unit of J . One therefore does have a very rough $I = 0,1$ degeneracy.

(c) Although no prominent $I = 2$ resonances appear in the model, there is present nevertheless a substantial $I = 2$ cross section, so exact $I = 0,1$ degeneracy cannot in any case be expected.

4. Let us turn finally to the low energy S-wave $\pi\pi$ phase shifts, which to me is the most impressive aspect of the model. There could be no hidden feedback here from experimental knowledge, because the search for input-output consistency was carried out entirely in terms of the rho and Pomeron trajectory and residue. Collins and Johnson had no knowledge of the S-wave content of their solution until the package was all wrapped up. In fact, they published their solution before remembering that there was S-wave content. A supplementary, still unpublished, paper gives the $l = 0$ results.

The first slide shows the $I = 0$ S-wave phase shift. The flags do not represent experimental knowledge; they give the theoretical uncertainty according to the same criteria used for assigning uncertainty to Regge pole parameters. The difference between dashed and solid curves measures the failure of the model to achieve exact crossing symmetry. One of these curves is projected from the t reaction and one from the s reaction. The discrepancy between the two is seen to be small.

The threshold derivative of the phase shift corresponds to an $I = 0$ scattering length of $0.8 m_\rho^{-1}$, or in more familiar units for this quantity, $0.15 m_\pi^{-1}$. This number and the trend of the curve up to 1 GeV are in accord with estimates that have emerged from a combination of experiment with more phenomenological models.

It has been known for a long time that once the width of the rho is correct, crossing allows only one free parameter in the low energy $\pi\pi$ S wave, which is usually taken as the $I = 0$ scattering length. Until the advent of current algebra with PCAC, however, there was great theoretical uncertainty about the value of this scattering length. A

major triumph of current algebra was the prediction of this parameter, a development distressing to bootstrappers because PCAC invokes principles not included in the usual statement of the hadron bootstrap hypothesis.

If the Collins-Johnson result is not accidental it now appears that the hadronic content of current algebra and PCAC is contained within general S-matrix principles, once given that the pion mass is small. This would be a development of tremendous significance.

The next slide shows the $I = 2$ S-wave phase shift. There are no surprises here. Once given the rho width and the $I = 0$ scattering length, crossing leaves no freedom for $I = 2$.

G. CONCLUSION

Explanation of the $\pi\pi$ interaction below 1 GeV, without the aid of arbitrary parameters, is a unique achievement for multiperipheralism. Where do we go from here?

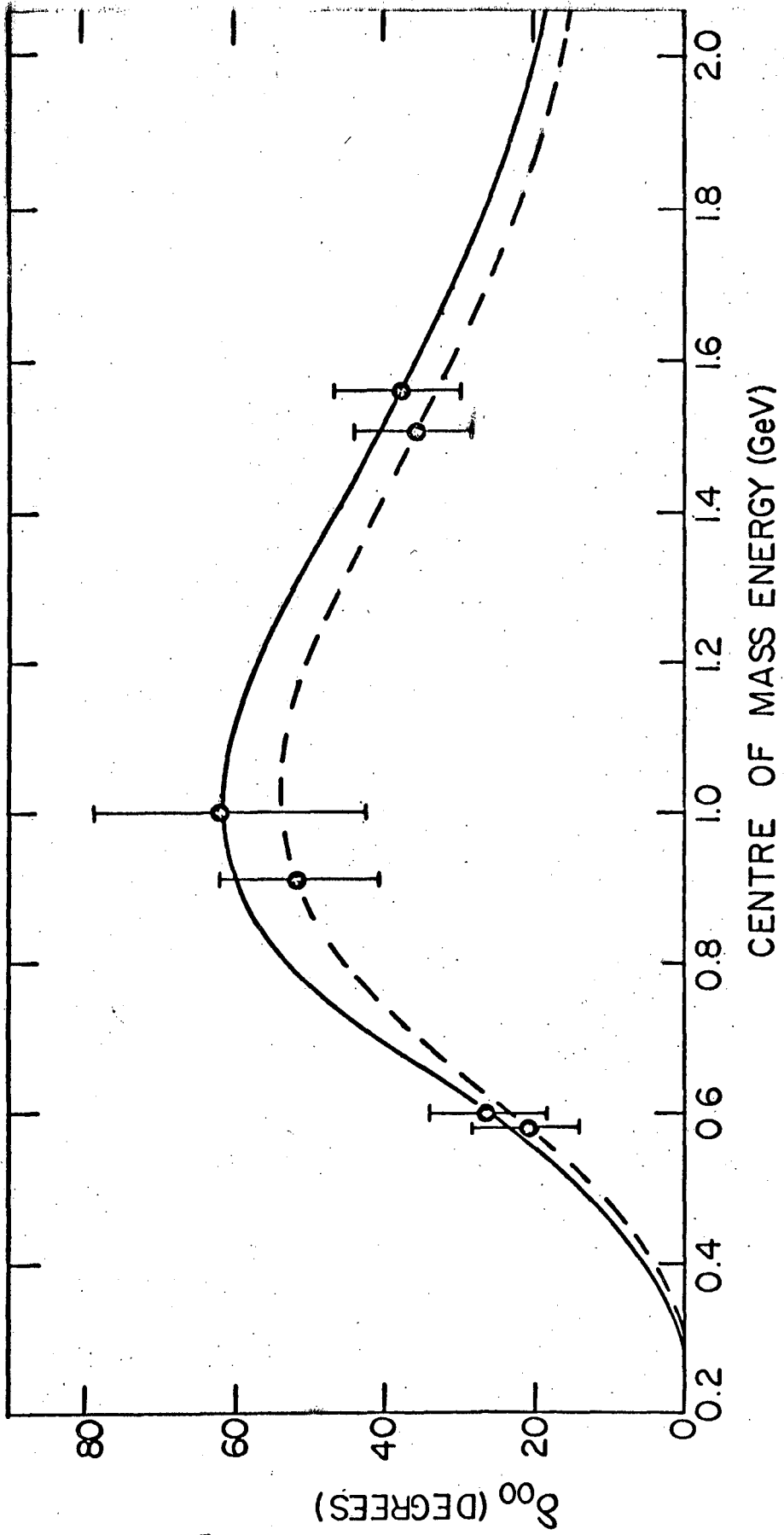
The first task is to check carefully the Collins-Johnson calculation, not so much for computational errors as for the legitimacy of their methods and the uniqueness of their solution. Because of computer limitations they did not employ completely straightforward methods, cutting corners whenever they felt it safe to do so. Questions of judgment arose, and just as with an experiment, independent check is required before the result can comfortably be accepted.

Let us suppose for the sake of argument that the Collins-Johnson conclusion is confirmed. What then? The $\pi\pi$ strip model only covers a tiny corner of the hadron S matrix, but it explains the small rho width and the large spacing between trajectories, thus providing motivation

and specificity for linearized models, such as those of the Veneziano type, which may cover, even if crudely, a much larger chunk of hadronic phenomena. If multiperipheral models can successfully be extended to systems with nonzero hypercharge and baryon number, the constraints on linearized models will be far reaching, and the converse may also be anticipated. One can only guess at the combined potentiality of these two complementary approaches to the hadron bootstrap.

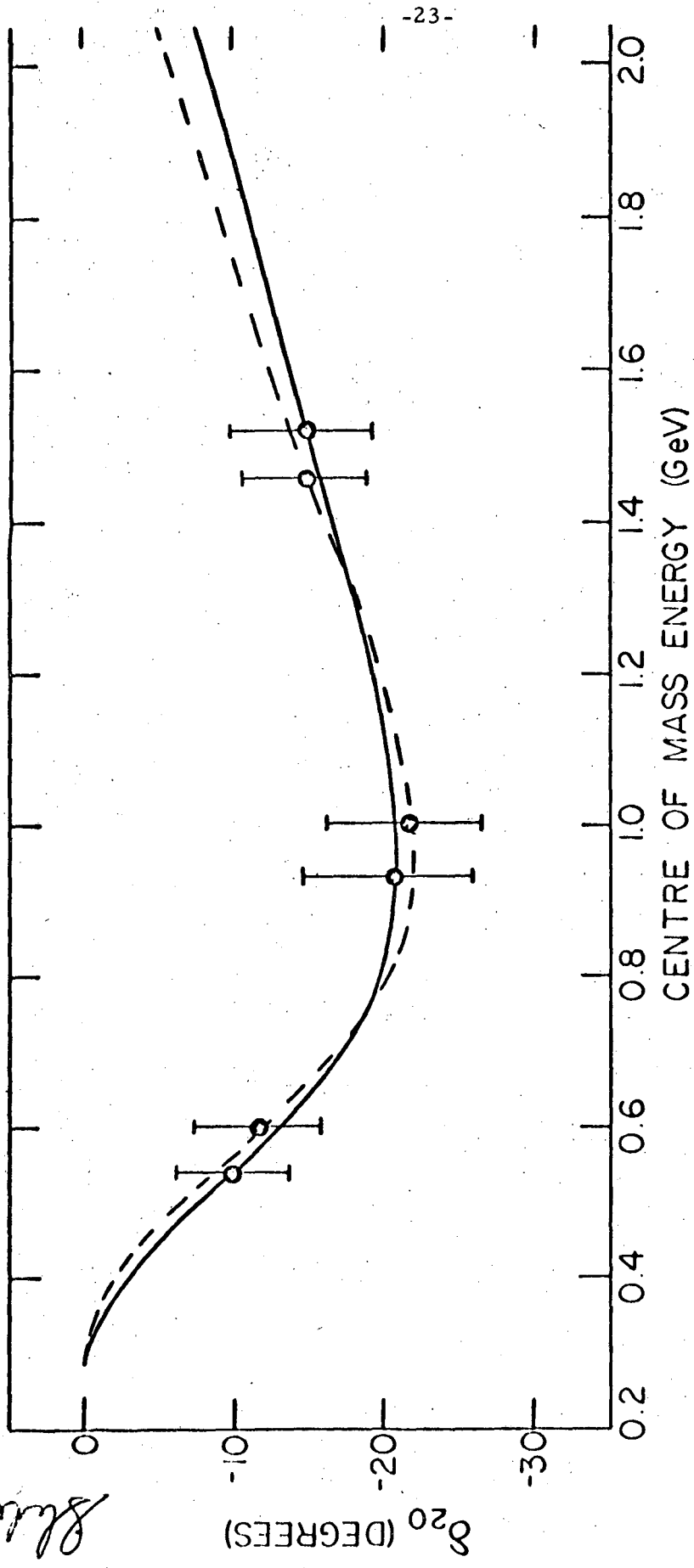
1. L. Bertocchi, S. Fubini, and M. Toni, *Nuovo Cimento* 25, 626 (1962);
D. Amati, A. Stanghellini, and S. Fubini, *Nuovo Cimento* 26, 6 (1962).
2. T. T. Chou, J. Benecke, and C. N. Yang, Stony Brook preprint (1969),
to be published in *Phys. Rev.*
3. A conscientious list of references to multi-Regge-pole publications
may be found in Ref. 5.
4. Chan Hong-Mo, J. Loshiewicz, and W. W. M. Allison, *Nuovo Cimento* 57A,
93 (1968).
5. G. F. Chew and A. Pignotti, *Phys. Rev.* 176, 2112 (1968).
6. L. Caneschi and A. Pignotti, *Phys. Rev. Letters* 22, 1219 (1969);
Phys. Rev. 180, 1525 (1969).
7. J. S. Ball and G. Marchesini, Lawrence Radiation Laboratory report
UCRL-19282 (1969), to be published in *Phys. Rev.*
8. G. F. Chew, T. W. Rogers, and D. R. Snider, private communication.
9. J. Finkelstein and K. Kajantie, *Physics Letters* 26B, 305 (1968).
10. G. F. Chew, M. L. Goldberger, and F. E. Low, *Phys. Rev. Letters* 22,
208 (1969).
11. I. G. Halliday and L. M. Saunders, *Nuovo Cimento* 60, 494 (1969).
12. G. F. Chew and C. DeTar, *Phys. Rev.* 180, 1577 (1969).
13. A. H. Mueller and I. Muzinich, Brookhaven preprints (1969), to be
published in *Phys. Rev.*
14. M. Ciafaloni, C. DeTar, and M. Misheloff, Lawrence Radiation
Laboratory report UCRL-19286 (1969), to be published in *Phys. Rev.*
15. L. Caneschi, *Phys. Rev. Letters* 23, 254 (1969).
16. G. F. Chew and W. R. Frazer, *Phys. Rev.* 181, 1914 (1969).
17. A. Pignotti, University of Washington, private communication (1969).

18. G. F. Chew and D. R. Snider, Lawrence Radiation Laboratory report UCRL-19409 (1969), submitted to Physics Letters.
19. Early strip model developments are described in G. F. Chew, The Analytic S Matrix (Benjamin, New York, 1966), Chaps. 10, 11, and 12.
20. P. D. B. Collins and R. C. Johnson, Phys. Rev. 182, 1755 (1969); Toronto preprint, May 1969.
21. D. Tow, private communication, Berkeley (1969).



Plot # 1

Plot # 2



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