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Large Displacement Analysis of Axisymmetic Shells

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LARGE DISPLACEMENT ANALYSIS

OF

AXISYMMETRIC SHELLS

by

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NOTATION

r - radial coordinate

z - axial coordinate

L_0, L_1 - initial and deformed element lengths

ϕ - angle defining slope of element

s - element local coordinate

ζ - element local dimensionless coordinate

u - meridional displacement

w - normal displacement

$\bar{u}$ - axial displacement

$\bar{w}$ - radial displacement

$\varepsilon_s, \varepsilon_\theta, \chi_s, \chi_\theta$ - strain components

$N_s, N_\theta, M_s, M_\theta$ - stress resultants

U, U_0, U_1, U_2 - strain energy quantities

E - Young's modulus

υ - Poisson's ratio

$[D]$ - elasticity matrix

$\Delta D_{11}, \Delta D_{22}, \Delta D_{33}, \Delta D_{44}$ - increments of diagonal elements of elasticity matrix due to orthotropic stiffeners

$[K^0]$ - element elastic stiffness matrix

$[K^1]$ - element geometric stiffness matrix

$[K]$ - structural stiffness matrix

$\{\alpha\}$ - generalized coordinates

$[L], [L^{-1}]$ - generalized coordinate transformation matrix

$[\lambda]$ - coordinate transformation matrix

[W] - strain interpolation function in terms of generalized coordinates

[T] - strain interpolation function in terms of element local coordinates

[S],[G] - matrices associated with strain energy leads to geometric stiffness

$\{P^e\}$,{P*} - element nodal force vectors

{P} - structural nodal load vector

$\{\delta\}$ - displacement vector in local coordinates

$\{\bar{\delta}\}$ - displacement vector in globe coordinates

{r*}- "equivalent infinitesimal" displacement vector

INTRODUCTION

Axisymmetric shells subjected to axisymmetric loads are common engineering structures. The failure mode for a structure of this type is very often due to buckling or large deformation behavior. In this report a method of analysis and a computer program are presented for the determination of the displacements and stresses in an axisymmetric shell loaded into the large displacement range.

The finite element idealization has been used extensively in recent years for the analysis of axisymmetric shells. Details of its application are presented in references [1] to [9] and only a brief outline will be presented in this report.

In this investigation the finite element method for the analysis of axisymmetric shells is extended to include large displacement. The approach seeks an equilibrium position in the deformed configuration of the structure by a step-by-step iterative solution technique. Previously, this approach has been successfully applied to the large deflection analysis of plates [10].

The particular finite element which is ultilized is the truncated cone which may be reinforced with stiffeners arranged orthotropically. The computer program developed is for axisymmetric shells of arbitrary geometry. Examples are presented which illustrate the accuracy of the method. The analysis of a toroid shell subjected to deep ocean enviroment is presented to illustrate the practicability of the program.

FINITE ELEMENT ANALYSIS OF SHELLS OF REVOLUTION

A. Strain Energy

In this investigation, both the shell and the loading are axisymmetric. In a typical conical frustrum element as shown in figure 1, the four strain components are given by.

$$\{\varepsilon\}^T = \langle \varepsilon_s, \varepsilon_\theta, \chi_s, \chi_\theta \rangle$$

The four associated internal stress resultants are

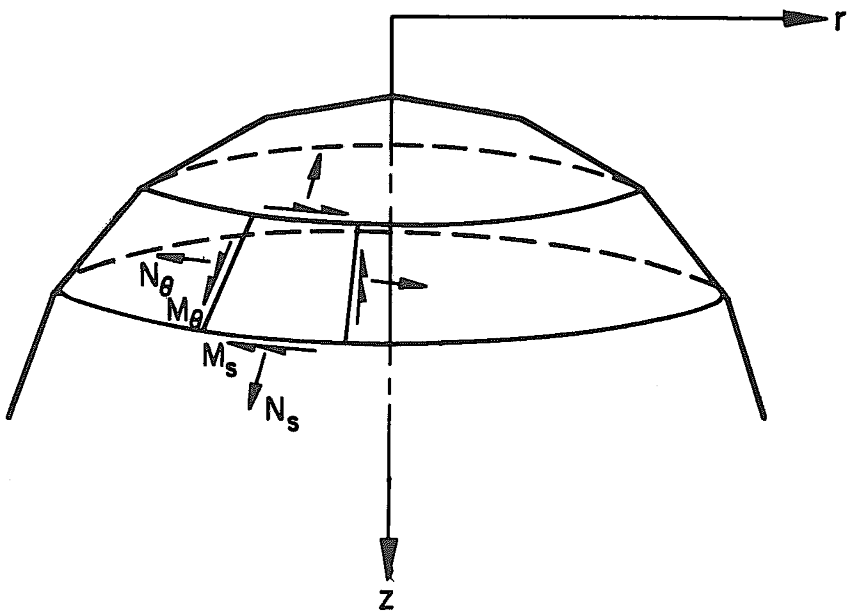
$$\{\sigma\}^T = \langle N_s, N_\theta, M_s, M_\theta \rangle$$

When the shell is subjected to a system of conservative axisymmetric loads, and it is in a state of stable equilibrium, let $\{\varepsilon^\circ\}$ and $\{\sigma^\circ\}$ be the strains and stress resultants respectively. For additional loads, additional strains $\{\varepsilon^a\}$ and stress resultants $\{\sigma^a\}$ are developed. The total strains are

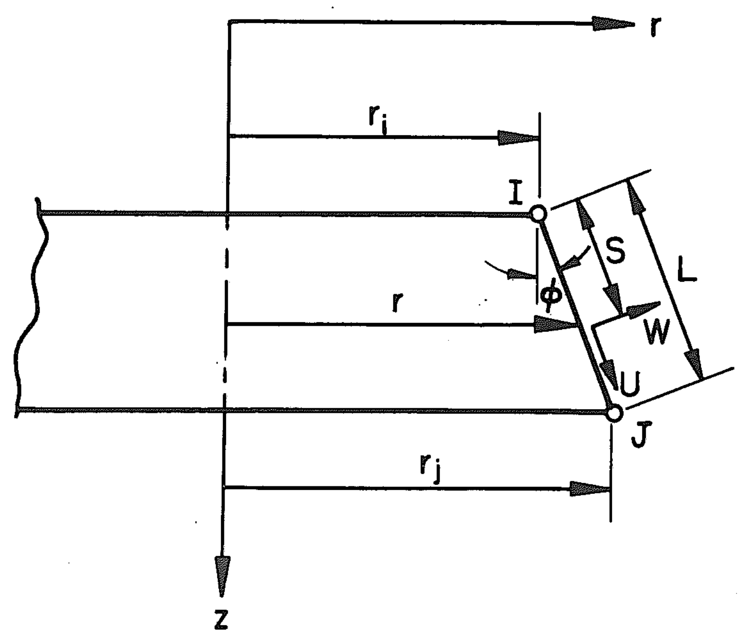
$$\{\varepsilon\} = \{\varepsilon^\circ\} + \{\varepsilon^a\}$$

The total stress resultants are

$$\{\sigma\} = \{\sigma^\circ\} + \{\sigma^a\}$$



STRESS RESULTANTS IN IDEALIZED SHELL OF REVOLUTION



ELEMENT LOCAL COORDINATES

FIG. I TYPICAL CONICAL FRUSTRUM ELEMENT

The total strain energy U may be written as

$$U = \frac{1}{2} \iint_A \{\sigma\}^T \{\varepsilon\} \, dA$$

Let [D] be the stress-strain matrix, such that

$$\{\sigma\} = [D] \{\varepsilon\}$$

It follows that,

$$U = \frac{1}{2} \iint_A \{\varepsilon\}^T [D] \{\varepsilon\} \, dA$$

$$= \frac{1}{2} \iint_A \{\underline{\varepsilon}^\circ + \underline{\varepsilon}^a\}^T [D] \{\underline{\varepsilon}^\circ + \underline{\varepsilon}^a\} \, dA$$

$$= U_0 + U_1 + U_2$$

where

$$U_0 = \frac{1}{2} \iint_A \{\varepsilon^\circ\}^T [D] \{\varepsilon^\circ\} \, dA$$

$$U_1 = \frac{1}{2} \iint_A \{\varepsilon^a\}^T [D] \{\varepsilon^a\} \, dA$$

$$U_2 = \iint_A \{\varepsilon^\circ\}^T [D] \{\varepsilon^a\} \, dA$$

U_0 is simply the strain energy present prior to the imposition of the additional loads, it can be treated as a constant, with respect to the new load increment, therefore, it has no contribution to the stiffness of the element at this stage.

U_1 depends on the additional strains, and as it is assumed to be the same as the conventional, small deflection, elastic case, it must yield elastic stiffness $[k^o]$.

U_2 depends on the initial stress resultants, hence, it must yield the geometric stiffness $[k^1]$.

B. Constitutive Law

For isotropic linear elastic material, the constitutive law in elasticity matrix form is

$$[D] = \frac{Et}{1 - \nu^2} \begin{bmatrix} 1 & \nu & 0 & 0 \\ \nu & 1 & 0 & 0 \\ 0 & 0 & \frac{t^2}{12} & \frac{\nu t^2}{12} \\ 0 & 0 & \frac{\nu t^2}{12} & \frac{t^2}{12} \end{bmatrix}$$

When axisymmetric reinforcement is used, the stiffeners are arranged in meridional and tangential directions. In an axisymmetric loading condition, there is no torsional deformation meridional stiffeners. Torsional deformation in the tangential stiffeners is small and is neglected. Along each stiffener, most places can be freely expanded in lateral direction. Therefore, the stiffeners can be described as one dimensional structural members in axial and bending deformations. These axial and bending rigidities are assumed to be uniformly distributed along the meridional and tangential directions. although the shell is made with isotropic linear elastic material. Due to geometric arrangement of stiffeners, the structure has orthotropic behavior. Because the coupling forces between meridional stiffeners and tangential stiffeners are small, these effects are negelected, therefore, it is only necessary to modify the diagonal elements in the elasticity matrix.

The modified elasticity matrix is

$$[D] = \begin{bmatrix} \frac{Et}{1-\nu^2} + \Delta D_{11} & \frac{\nu Et}{1-\nu^2} & 0 & 0 \\ \frac{\nu Et}{1-\nu^2} & \frac{Et}{1-\nu^2} + \Delta D_{22} & 0 & 0 \\ 0 & 0 & \frac{Et^3}{12(1-\nu^2)} + \Delta D_{33} & \frac{\nu Et^3}{12(1-\nu^2)} \\ 0 & 0 & \frac{\nu Et^3}{12(1-\nu^2)} & \frac{Et^3}{12(1-\nu^2)} + \Delta D_{44} \end{bmatrix}$$

The meridional stiffeners are uniformly spaced along the element circle, the number n and size $t_m x h_m$ will change for different elements.

The total length of the midpoint circle is

$$l_m = \pi(r_i + r_j)$$

let

$$A_{sm} = l_m t$$

$$A_m = n t_m h_m$$

$$I_m = n t_m h_m^3/12.$$

$$d_m = \frac{A_{sm}(t+hm)}{2(A_{sm}+A_m)}$$

Then, ΔD_{11}, the increment of area per unit length times Young's modulus, is given as

$$\Delta D_{11} = EAm/lm$$

The increment of moment of inertia per unit length times Young's modulus, ΔD_{33} , is given as

$$\Delta D_{33} = \frac{E}{lm}[A_{sm}d_m^2 + I_m + \frac{Am}{4}(t + h_m - 2d_m)^2]$$

The tangential stiffeners are also rectangular bars, the space C_t and size $t_t x h_t$ are the same within an element.

Let

$$A_{st} = C_t t$$

$$A_t = t_t h_t$$

$$I_t = t_t h_t^3/12$$

$$d_t = \frac{A_{st}(t+h_t)}{2(A_{st}+A_t)}$$

Therefore

$$\Delta D_{22} = EA_t/C_t$$

$$\Delta D_{44} = \frac{E}{C_t}[A_{st}d_t^2 + I_t + \frac{A_t}{4}(t + h_t - 2d_t)^2]$$

C. Kinematic Assumptions and Strain-Displacement Relations

In a conical frustrum element between Joint I (r_i,z_i) and Joint J (r_j,z_j), the displacement of any point in the middle surface of the shell element, (defined by coordinate S), is defined by the displacement u and w, meridional and normal to the shell surface respectively. (Because of axial symmetry, there is not dispacement in the circumferential direction). Therefore, the displacement field is function of meridional distance S only, furthermore, this displacement pattern is assumed as

$$u = \alpha_1 + \alpha_2 S$$

$$w = \alpha_3 + \alpha_4 S + \alpha_5 S^2 + \alpha_6 S^3$$

The continuous system is represented with six degrees of freedom, the displacement functions can be determined in terms of the nodal point displacements and rotations.

$$\{\delta\} = [L]\{\alpha\}$$

or

$$\begin{Bmatrix} u_i \\ w_i \\ (\frac{\partial w}{\partial s})_i \\ u_j \\ w_j \\ (\frac{\partial w}{\partial s})_j \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & L & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & L & L^2 & L^3 \\ 0 & 0 & 0 & 1 & 2L & 3L^2 \end{bmatrix} \begin{Bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \\ \alpha_5 \\ \alpha_6 \end{Bmatrix}$$

and

$$\{\alpha\} = [L^{-1}]\{\delta\}$$

or

$$\begin{Bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \\ \alpha_5 \\ \alpha_6 \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ -\frac{1}{L} & 0 & 0 & \frac{1}{L} & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & -\frac{3}{L^2} & -\frac{2}{L} & 0 & \frac{3}{L^2} & -\frac{1}{L} \\ 0 & \frac{2}{L^3} & \frac{1}{L^2} & 0 & -\frac{2}{L^3} & \frac{1}{L^2} \end{bmatrix} \begin{Bmatrix} u_i \\ w_i \\ (\frac{\partial w}{\partial s})_i \\ u_j \\ w_j \\ (\frac{\partial w}{\partial s})_j \end{Bmatrix}$$

For a thin conical shell the strain-displacement equations are

$$\{\varepsilon\} = \begin{Bmatrix} \varepsilon_s \\ \varepsilon_\theta \\ \chi_s \\ \chi_\theta \end{Bmatrix} = \begin{Bmatrix} \frac{du}{ds} + \frac{1}{2}(\frac{dw}{ds})^2 \\ (w\cos\phi + u\sin\phi)/r \\ \frac{d^2w}{ds^2} \\ -\frac{\sin\phi}{r}\frac{dw}{ds} \end{Bmatrix}$$

or

$$\varepsilon_s = \alpha_2 + \frac{1}{2}(\alpha_4 + 2\alpha_5 s + 3\alpha_6 s^2)^2$$

$$\varepsilon_\theta = \frac{1}{r}[(\alpha_1 + \alpha_2 s)\sin\phi + (\alpha_3 + \alpha_4 s + \alpha_5 s^2 + \alpha_6 s^3)\cos\phi]$$

$$\chi_s = 2\alpha_5 + 6\,\alpha_6 s$$

$$\chi_\theta = -\frac{\sin\phi}{r}(\alpha_4 + 2\alpha_5 s + 3\alpha_6 s^2)$$

D. <u>Element Elastic Stiffness</u>

The part of strain energy which yields element elastic stiffness is rewritten as

$$U_2 = \frac{1}{2} \int_A\int \{\varepsilon^a\}^T [D] \{\varepsilon^a\} \, dA$$

Before substituting the strain vector in the above equation. It is worthwhile to investigate the strain components first. ε_θ, χ_s , χ_θ , have linear terms of the nodal displacements and rotations, if they were substituted in U_2, the quaqratic terms in the nodal displacements and rotations are generated. But, in the meridional strain expression:

$$\varepsilon_s = \frac{du}{ds} + \frac{1}{2}\left(\frac{dw}{ds}\right)^2$$

The first term is linear, and the second term is quadratic, if substituting in U_2, the latter will only generate cubic and higher order terms of the nodal displacements and rotations, it is obvious that the mathematical model is a nonlinear one. In a usual case, $\frac{1}{2}\left(\frac{dw}{ds}\right)^2$ is one order smaller than $\frac{du}{ds}$, it is dropped, as in the case in the classical nonlinear theory. After linearization, the strain vector is given as

$$\{\varepsilon\} = [W] \{\alpha\}$$

or

$$\begin{Bmatrix} \varepsilon_s \\ \varepsilon_\theta \\ \chi_s \\ \chi_\theta \end{Bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ \frac{\sin\phi}{r} & \frac{S\sin\phi}{r} & \frac{\cos\phi}{r} & \frac{S\cos\phi}{r} & \frac{S^2\cos\phi}{r} & \frac{S^3\cos\phi}{r} \\ 0 & 0 & 0 & 0 & 2 & 6S \\ 0 & 0 & 0 & -\frac{\sin\phi}{r} & -\frac{2S\sin\phi}{r} & -\frac{3S^2\sin\phi}{r} \end{bmatrix} \begin{Bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \\ \alpha_5 \\ \alpha_6 \end{Bmatrix}$$

It follows that

$$\begin{aligned} \{\varepsilon\} &= [W]\,\{\alpha\} \\ &= [W]\,[L^{-1}]\,\{\delta\} \\ &= [T]\,\{\delta\} \\ &= [\underline{T_i} \,\vdots\, \underline{T_j}]\,\left\{\frac{\delta_i}{\underline{\delta_j}}\right\} \end{aligned}$$

Let

$$\zeta = \frac{s}{L}$$

Carry out the multiplication, the strain interpolation function is given as

$$[T_i] = \begin{bmatrix} -\frac{1}{L} & 0 & 0 \\ (1-\zeta)\frac{\sin\phi}{r} & (1-3\zeta^2+2\zeta^3)\frac{\cos\phi}{r} & (\zeta-2\zeta^2+\zeta^3)\frac{L\cos\phi}{r} \\ 0 & (-6+12\zeta)\,\frac{1}{L^2} & (-4+6\zeta)\,\frac{1}{L} \\ 0 & (6\zeta-6\zeta^2)\frac{\sin\phi}{rL} & (-1+4\zeta-3\zeta^2)\frac{\sin\phi}{r} \end{bmatrix}$$

and

$$[T_j] = \begin{bmatrix} \frac{1}{L} & 0 & 0 \\ \frac{\zeta\sin\alpha}{r} & (3\zeta^2-2\zeta^3)\frac{\cos\phi}{r} & (-\zeta^2+\zeta^3)\ \frac{L\cos\phi}{r} \\ 0 & (6-12\zeta)\frac{1}{L^2} & (-2+6\zeta)\ \frac{1}{L} \\ 0 & (-6\zeta+6\zeta^2)\frac{\sin\phi}{rL} & (2\zeta-3\zeta^2)\ \frac{\sin\phi}{r} \end{bmatrix}$$

Then, the strain energy is given as

$$U_1 = \frac{1}{2} \iint_A \{\delta\}^T[T]^T[D][T]\{\delta\}\ dA$$

$$= \frac{1}{2}\{\delta\}^T\ [\iint_A [T]^T[D][T]\ dA]\ \{\delta\}$$

Apply Castigliano's first theorem. The element elastic stiffness is

$$[K] = [\iint_A [T]^T[D][T]\ dA]$$

since

$$dA = 2\pi rLd\zeta$$

it follows that

$$[K] = 2\pi L\ [\ \int_0^1 [T]^T[D][T]rd\zeta]$$

This integral is evaluated numerically within the computer program using a 5-point gauss quadrature integration procedure.

E. Element Geometric Stiffness

The part of strain energy which leads to the geometric stiffness is

$$U_2 = \int_A\int \{\varepsilon^{\circ}\}^T[D]\ \{\varepsilon^{a}\}\ dA$$

using the constitutive equations

$$\{\sigma^{\circ}\}^T = \{\varepsilon^{\circ}\}^T[D]$$

$\{\sigma^{\circ}\}$ is simply an initial stress resultants vector. It can be treated as a constant vector with respect to the new load increment.

The components of the additional strain vector contain both linear and quadratic terms of the nodal displacements and rotations, since the linear terms do not contribute to the geometric stiffness. They are dropped in the following derivation. Retaining the quadratic terms only, the strain energy is given as

$$\begin{aligned} U_2 &= \int_A\int \{\sigma^{\circ}\}^T\{\varepsilon^{a}\}\ dA \\ &= \int_A\int N_s\ \frac{1}{2}\left(\frac{dw}{ds}\right)^2\ dA \\ &= \frac{1}{2}\int_A\int N_s(\alpha_4 + 2\alpha_5 s + 3\alpha_6 s^2)^2\ dA \end{aligned}$$

Express U_2 in matrix form

$$U_2 = \frac{1}{2}\{\alpha\}^T\ [\int_A\int [s]\ dA]\ \{\alpha\}$$

where

$$[S] = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & N_s & 2N_sS & 3N_sS^2 \\ 0 & 0 & 0 & 2N_sS & 4N_sS^2 & 6N_sS^3 \\ 0 & 0 & 0 & 3N_sS^2 & 6N_sS^3 & 9N_sS^4 \end{bmatrix}$$

From experience, the variation of N_s in an element is very small, using the midpoint (s = 0.5L) value. Also using ($2r = r_i + r_j$), the integral can be evaluated directly. It gives:

$$U_2 = \frac{1}{2}\{\alpha\}^T[G]\{\alpha\}$$

where

$$[G] = \pi(r_i + r_j)N_s \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & L & L^2 & L^3 \\ 0 & 0 & 0 & L^2 & \frac{4L^3}{3} & \frac{3L^4}{2} \\ 0 & 0 & 0 & L^3 & \frac{3L^4}{2} & \frac{9L^5}{5} \end{bmatrix}$$

It follows that

$$U_2 = \frac{1}{2}\{\delta\}^T[L^{-1}]^T[G]\,[L^{-1}]\{\delta\}$$

$$= \frac{1}{2}\{\delta\}^T\,[K^1]\,\{\delta\}$$

Apply the Castigliano's first theorem, the element geometric stiffness is

$$[K^1] = [L^{-1}]^T[G][L^{-1}]$$

or

$$[K^1] = \pi(r_i+r_j)N_s\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{6}{5L} & \frac{1}{10} & 0 & -\frac{6}{5L} & \frac{1}{10} \\ 0 & \frac{1}{10} & \frac{2L}{15} & 0 & -\frac{1}{10} & -\frac{L}{30} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{6}{5L} & -\frac{1}{10} & 0 & \frac{6}{5L} & -\frac{L}{10} \\ 0 & \frac{1}{10} & -\frac{L}{30} & 0 & -\frac{1}{10} & \frac{2L}{15} \end{bmatrix}$$

F. Direct Stiffness Formulation

The element elastic stiffness and element geometric stiffness in the local coordinate system (u,w) are transformed to the global coordinate system (z,r), as follows:

$$[\bar{K}] = [\lambda]^T[K][\lambda]$$

$$[\bar{K}^1] = [\lambda]^T[K^1][\lambda]$$

where the transformation matrix is given as

$$[\lambda] = \begin{vmatrix} \cos\phi & \sin\phi & 0 & 0 & 0 & 0 \\ -\sin\phi & \cos\phi & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos\phi & \sin\phi & 0 \\ 0 & 0 & 0 & -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \end{vmatrix}$$

The element stiffness is written as

$$[K^e] = [\bar{K}] + [\bar{K}^1]$$

Modifications to the element stiffness matrix to allow for specified displacement boundary conditions are done by making the terms of the appropriate row and column zero, and the diagonal term unity. The load vector is also appropriately adjusted.

The structural stiffness matrix [K] is formed by adding the stiffness coefficients of the submatrices $[K_i^e]$ into the proper location in [K]. Symbolically, this process is shown as

$$[K] = \sum_{i=1}^{m} [k_i^e]$$

with the summation including all elements.

[K] is stored in symmetric banded matrix form within the computer storage.

G. Nodal Loading Vector

In the computer program, the input is in the form of externally applied load which may be either in the form of axisymmetric line force around the nodal ring, or specified nodal displacement. For water pressure distributed over the element, the tributary area concept is used to lump the distributed load for finding the equivalent nodal loading.

For water pressure P lb./sq.in. the total pressure resultant over the element is

$$P^T = \pi PL(r_i + r_j)$$
$$= 2\pi PL(r_i + \frac{L}{2}\sin\phi)$$

In the global coordinates, the statical equivalent line load vector around the nodal rings is

$$\{P^e\} = 2\pi LP \begin{Bmatrix} (\frac{r_i}{2} + \frac{L\sin\phi}{6})\sin\phi \\ -(\frac{r_i}{2} + \frac{L\cos\phi}{6})\cos\phi \\ 0 \\ (\frac{r_i}{2} + \frac{L\sin\phi}{3})\sin\phi \\ -(\frac{r_i}{2} - \frac{L\cos\phi}{3})\cos\phi \\ 0 \end{Bmatrix}$$

The structural load vector is formed by similar technique previous used to yield structural stiffness.

$$\{P\} = \sum_{i=1}^{m} \{P_i^e\}$$

LARGE DEFLECTION ANALYSIS

The development given in the previous section is for an incremental displacement about a known deformed position. The basic assumption made is that the engineering strains are small and that the incremental rotations are large.

In the past, the most common approach to the large displacement problem has been the Incremental Method, coupled with the use of an accurate geometric stiffness. In this method, the large displacement problem is treated as a series of linear small displacement problems, with the load being applied in increments and the corresponding incremental displacements found. The complete stiffness is recomputed at each step. This method has been used successfully where an accurate formulation of both the elastic and the geometric stiffness terms is possible (11). However, the method presented in this paper (referred to here as the Equilibrium approach) does not require the formulation of an accurate geometric stiffness.

In the Equilibrium approach a structural configuration is sought in which the total applied loads are equilibrated by the internal structure resisting forces at the nodes. The final configuration is assumed to be unique, and it is therefore irrelevant how it is obtained, provided an equilibrium balance is achieved. A discussion on the uniqueness of the solution is given by Murray (10).

The general approach is therefore to estimate a deformed configuration, from which a set of out of balance forces between the applied loading (total) and the structure is obtained. This out of balance is used to compute an improved estimate of the displacements, and the process is continued until the out of balance vanishes sufficiently.

A. Algorithm to Obtain an Equilibrium Balance

For each movement of load the following can be considered a general interactive algorithm to obtain a configuration in which the total external loads are equilibrated by the internal forces:

1. Assume the displaced nodal locations (in global co-ordinate system) by an incremental small deformation analysis; thereby establishing a displaced local co-ordinate system for each element.
2. Determine the true element deformations in its displaced local co-ordinate system.
3. Using the deformations established in the displaced geometry, compute the element resisting forces.
4. The element stiffness and resisting forces are added to the global system. Steps (2), (3), and (4) are repeated for each element.
5. A set of out of balance nodal forces on the structure (in displaced configuration) is now obtained, from the difference between the summed element resisting forces and the total applied loads.
6. This set of out of balance loads is applied to the structure in this configuration, and a set of incremental displacements is obtained. (The stiffness matrix used in this step has been determined in Step 3.) Hence a new and improved estimate

of the displaced shape of the structure is obtained.

Steps (2) through (6) are repeated until an equilibrium balance is achieved with the externally applied loads.

Some comments on the above method can now be made:

1. Each incremental displacement analysis is a small displacements analysis and hence the analysis procedure as formulated for small displacements is still valid for this step. Because the structural stiffness is only required to obtain a new estimate of the displaced position, it need not be exact; therefore, an exact Geometric stiffness is not needed.
2. The main problem of the method is, given an estimate of the nodal displacements, to compute accurately the vector of element resisting forces $\{\bar{P}^*\}$. The basic idea is to find a set of nodal displacements $\{\bar{r}^*\}$, which, when applied to the structure in its deformed geometry, will produce the correct element deformations (these deformations are by definition small). This vector of displacements $\{\bar{r}^*\}$ can be considered as equivalent small displacements, and the required element resisting forces are obtained using the element stiffness $[\bar{K}]$, i.e.

$$\{\bar{P}^*\} = [\bar{K}]\,\{\bar{r}^*\}$$

The structural stiffness is always formed in the deformed geometry, hence the effect of the change of geometry on the equilibrium is included.

B. Element Resisting Forces

It is necessary to compute a set of nodal forces which holds the structure in a known deformed configuration. The resisting forces of each element in the deformed shape are computed and summed at each node point of the structure. This set of forces is compared with the total applied loads to determine whether an equilibrium balance has been achieved.

The conical element is allowed to have a finite deflection $\{\delta^e\}$, as shown in figure (2). It is convenient to find the a "equivalent infinitesimal" nodal displacement vector $\{\bar{r}^*\}$, as shown in figure (3), such that the element resisting forces are defined by the conventional, small deflection, elastic theory.

$$\{P^*\} = [\bar{K}]\{\bar{r}^*\}$$

where $[\bar{K}]$ is the elastic stiffness in the deformed geometry.

To establish the relationship between $\{\delta^e\}$ and $\{\bar{r}^*\}$. The assumption is made that the extensions of the element in both positions are the same.

The nodal displacements $\{\delta^e\}$ yield the element extension is given as

$$\Delta L = L_1 - L_0$$

where
$$L_0^2 = (r_{0_j} - r_{0_i})^2 + (z_{0_j} - z_{0_i})^2$$

$$L_1^2 = (r_{1_j} - r_{1_j})^2 + (z_{1_j} - z_{1_i})^2$$

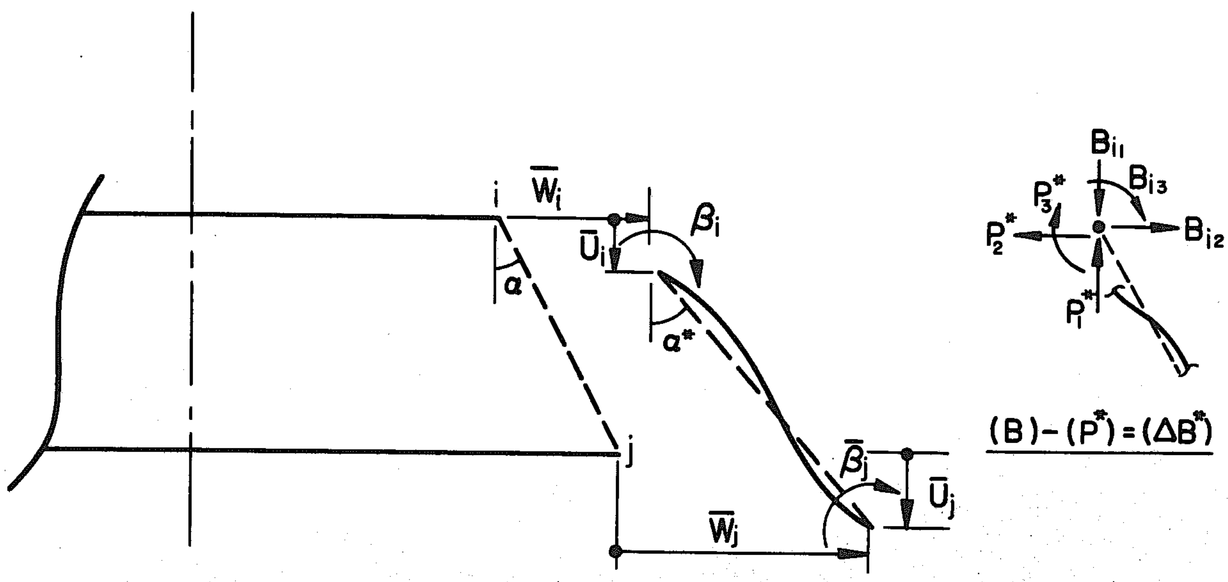


FIG. 2 ASSUMED DISPLACEMENTS - PRODUCE RESISTING FORCES (P^*)

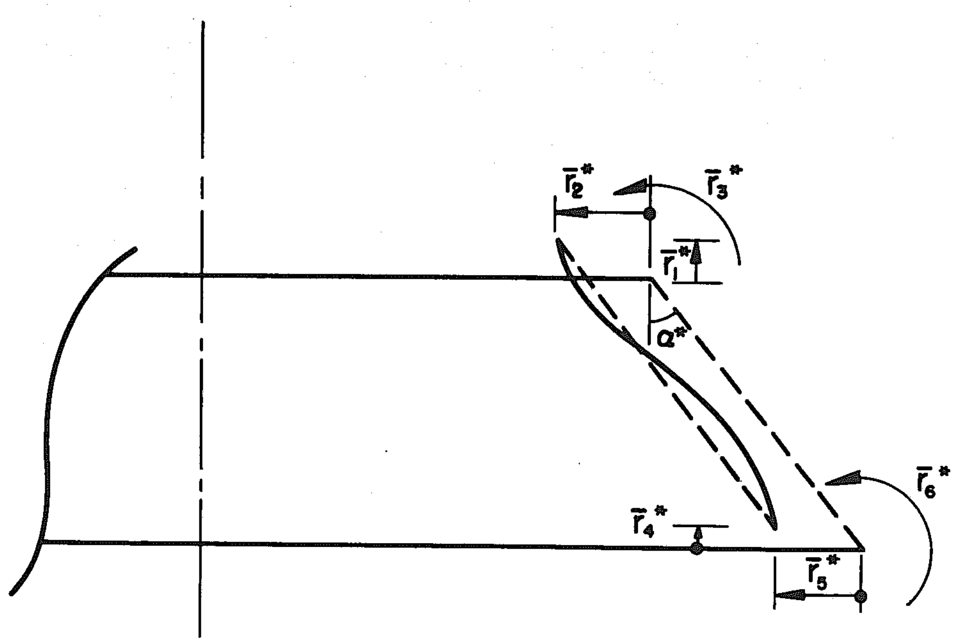


FIG. 3 NODAL DISPLACEMENTS IN DISPLACED GEOMETRY ($\bar{r}^*$) - PRODUCED BY (P^*)

The extension associated with $\{\bar{r}^*\}$ is defined by

$$\Delta L = (r_4^* - r_1^*)\cos\phi^* + (r_5^* - r_2^*)\sin\phi^*$$

Selections are made for $\bar{r}_1^*$, $\bar{r}_2^*$ and $\bar{r}_5^*$ as

$$\bar{r}_1^* = \bar{u}_i$$
$$\bar{r}_2^* = \bar{w}_i$$
$$\bar{r}_5^* = \bar{w}_j$$

it follows that

$$\bar{r}_4^* = \bar{r}_1^* + \frac{\Delta L}{\cos\phi^*} - (\bar{r}_5^* - \bar{r}_2^*)\tan\phi^*$$

This will introduce additional relative rotation at both nodes of the element. It is given as

$$\psi^* = \frac{\bar{u}_j \sin\phi}{L} - \frac{\bar{r}_4^* \sin\phi^*}{L}$$

To compensate for ψ^*, $\bar{r}_3^*$ and $\bar{r}_6^*$ are selected as

$$\bar{r}_3^* = \bar{\beta}_i - \psi^*$$

$$\bar{r}_6^* = \bar{\beta}_j - \psi^*$$

C. Stress Computation

When the solution for nodal displacements has been obtained, the stress components $\{\sigma\}$ in the shell are computed using the following relationships

$$\{\varepsilon\} = [T]\{\delta\}$$

$$\therefore \{\sigma\} = [D]\{\varepsilon\} = [D]\,[T]\,\{\delta\}$$

To ensure the most accurate evaluation of shell stresses, they are computed at the midpoint of the element, where the true slope of the shell is most nearly represented by the straight line approximation of the conical element. In this manner, one of the main disadvantages of the conical element is avoided, i.e., the discontinuity of slope at the nodal point in the finite element approximation. No stresses are actually computed at the nodes. Hence, in this investigation, stresses were computed using the equations:

$$\{\varepsilon\}_{(\zeta=\frac{1}{2})} = [T]_{(\zeta=\frac{1}{2})}\,[\lambda]\,\{\bar{\delta}\}$$

$$\{\sigma\} = [D]\,\{\varepsilon\}$$

It should be noted that an alternative approach exists to compute the meridional stress components N_s and M_s, that is, by using the element stiffness matrix and the nodal displacements. (This yields nodal point stresses.) However, this has been found to yield inaccurate results, especially for distributed loading. Again, this is most likely due to the poor representation of the true geometry of the shell at the nodal points. In particular, for a shell, under pressure, where classical analyses predict predominantly membrane action, this method produced a large area of almost constant, but substantial, moment. This effect was noted by Popov (11) and Navaratna (12).

EXAMPLES

A. Spherical Shell-Edge Moment Loading

A uniform spherical shell, loaded with a bending moment of 10 in. Kips/in. at the edge, was selected to illustrate the application of the method to the elastic small deformation analysis of axisymmetric shells. The shell has the following properties:

Radius = a = 100"

Thickness = t = 1.0"

Young's Modulus = E = 30×10^6 psi

Poisson's Ratio $\nu = 0.3$

The analysis was carried out using 20 elements @ 3°.

The results are plotted on figure 4. The exact solution was obtained from Timoshenko (14). Results compared in this case are this displacements, i.e., the horizontal displacement and the rotation at any point. Note that, as with most thin shells, the effect of an edge action is damped out very rapidly across the shell.

B. Spherical Shell-Uniform Pressure Loading

A spherical shell with fixed edges was analysed and the results were again compared with a known exact solution (Timoshenko (14). The purpose of this example was to check that good results could also be obtained with distributed loading. The loading considered was uniform internal pressure. The results are presented in figure 5 for the distribution of meridional moment (M_S) and hoopstress (N_θ).

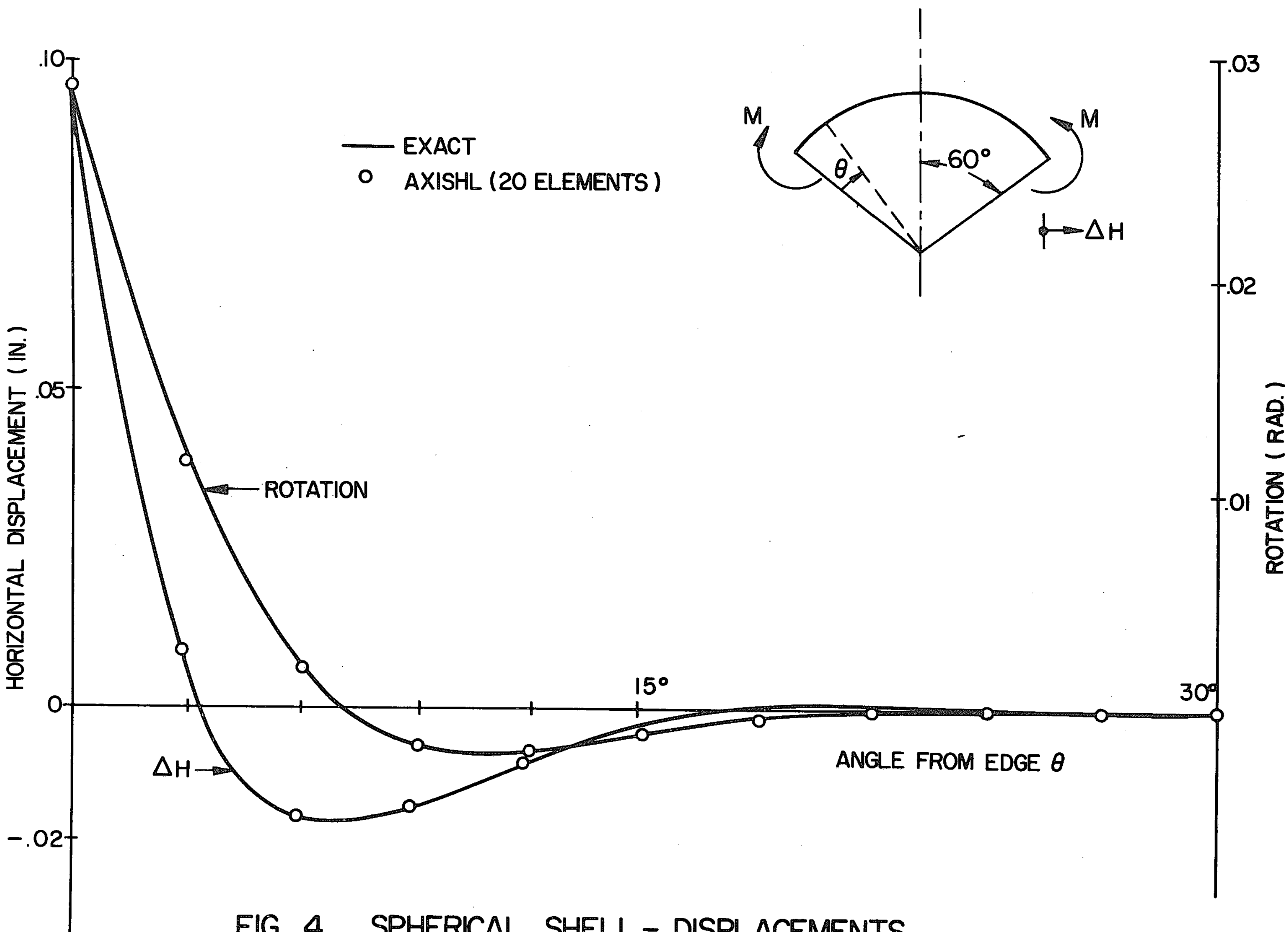


FIG. 4 SPHERICAL SHELL - DISPLACEMENTS

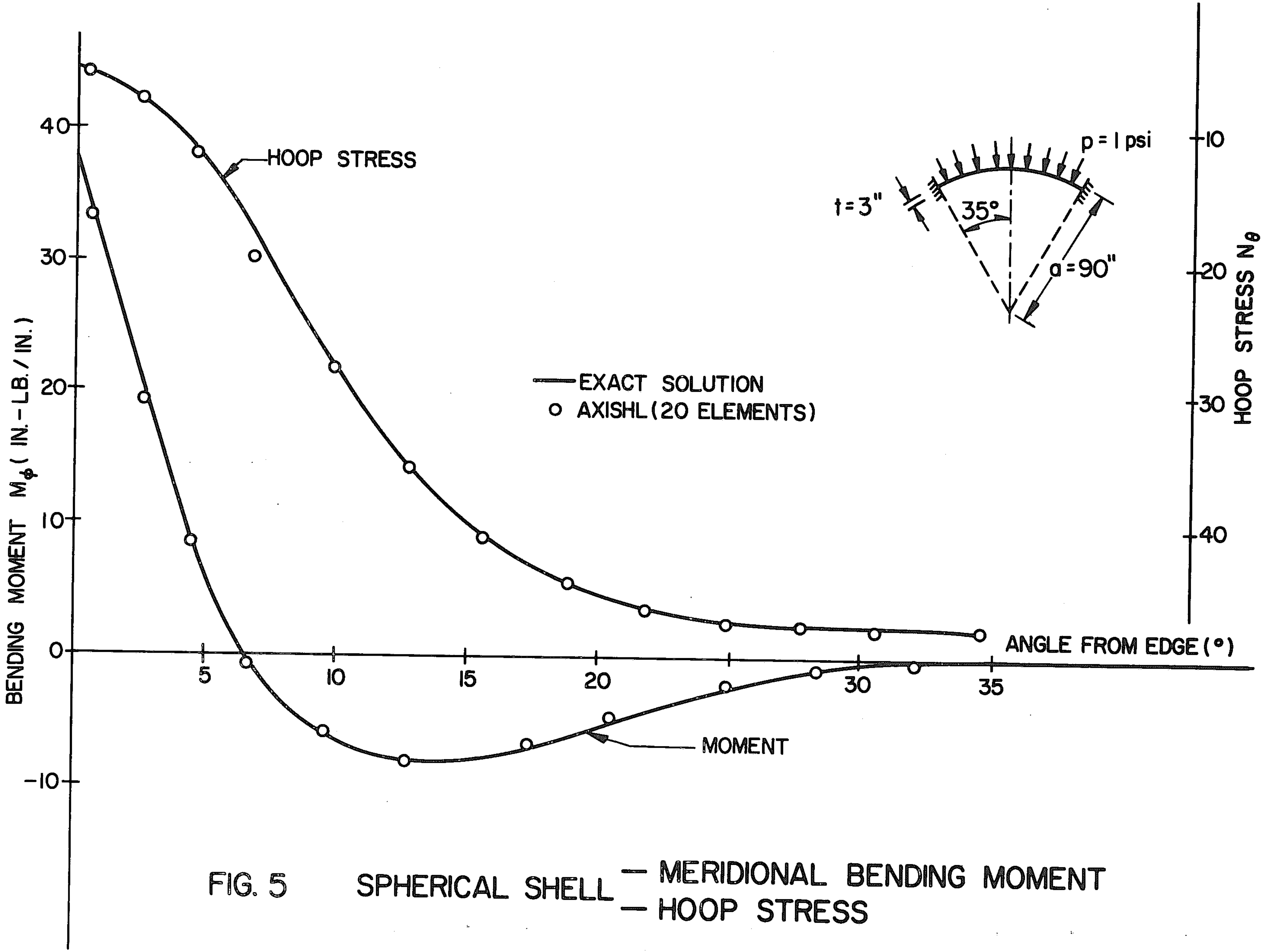


FIG. 5 SPHERICAL SHELL — MERIDIONAL BENDING MOMENT — HOOP STRESS

A concrete shell was considered with the following properties:

$a = 90''$

$\alpha = 35°$ (semi-angle)

$t = 3''$

$\nu = 1/6$

$E = 1. \times 10^6$ psi

Internal pressure $p = 1$ psi

20 elements were used (5 @ 1°, 15 @ 2°).

Once again, the results obtained using AXISHL show very good agreement with the correct ones, for both the moment M_S and the direct stress N_θ. Thus it appears that the approach to stress computation presented here produces results as good as possible using the simple conical element.

C. Large Displacement Analysis of A Flat Plate

The example chosen was that of a flat plate, infinite in one direction and simply supported along its edges on unyielding supports. An "exact" solution for this has been presented (Timoshenko (14)) and this example was also evaluated by Murray (10).

The plate was considered as an axisymmetric shell, by considering it as a flat "donut" shape, with a very large inside radius. Thus, the plate is effectively infinite in the transverse direction and the curvature small. The results are shown in Figures 6 and 7.

The properties of the plate were:

span $L = 20$ ins

$t = 0.5$ ins

$E = 30 \times 10^6$ psi

$\nu = 0.3$

The load was applied in 8 increments of 625 psi up to 5000 psi. This system showed extreme non-linearities in both displacement and stress, and was considered a good test of the method of analysis, e.g., under the 5000 psi load, using simple beam theory = 30.5 ins., however, as can be seen from Figure 6, the true deflection is little more than 1 inch.

It is seen that the agreement with the correct solution for both deflection and stresses is excellent.

D. Large Displacement Analysis of Torus

A toroid shell of 1¼" thick, subjected to hydrostatic pressure, is shown in figure 6. One half of the steel torus is idealized by a finite element model consisting of 18 elements and 19 nodal points. The numbering sequences for the nodes and elements is shown in figure 9. Various displacement components are shown in figure 10, 11 and 12. The deformed shape of the torus is shown in figure 13 for a depth of 2200 ft. The stress resultants are plotted in figures 14, 15, 16 and 17.

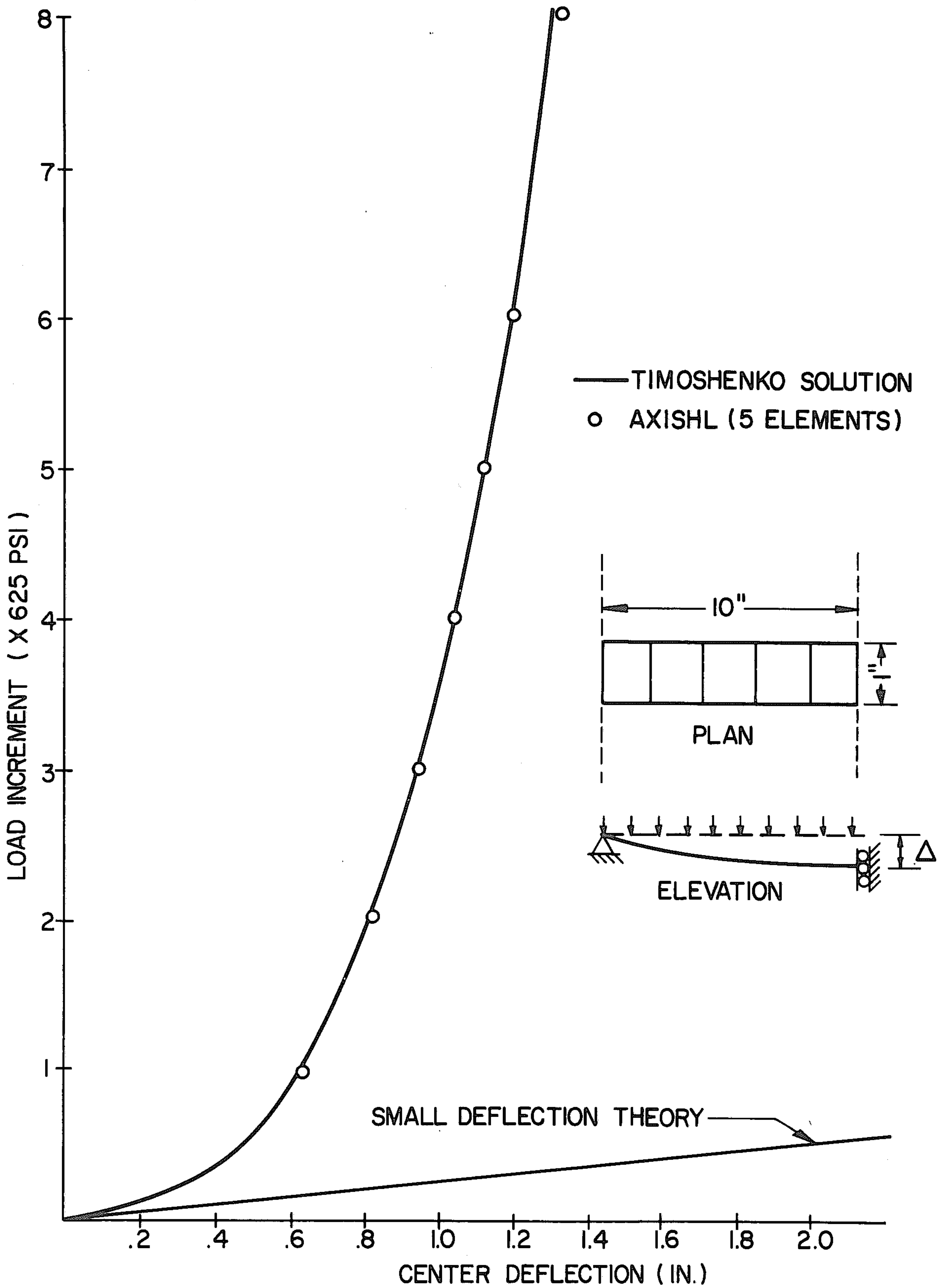
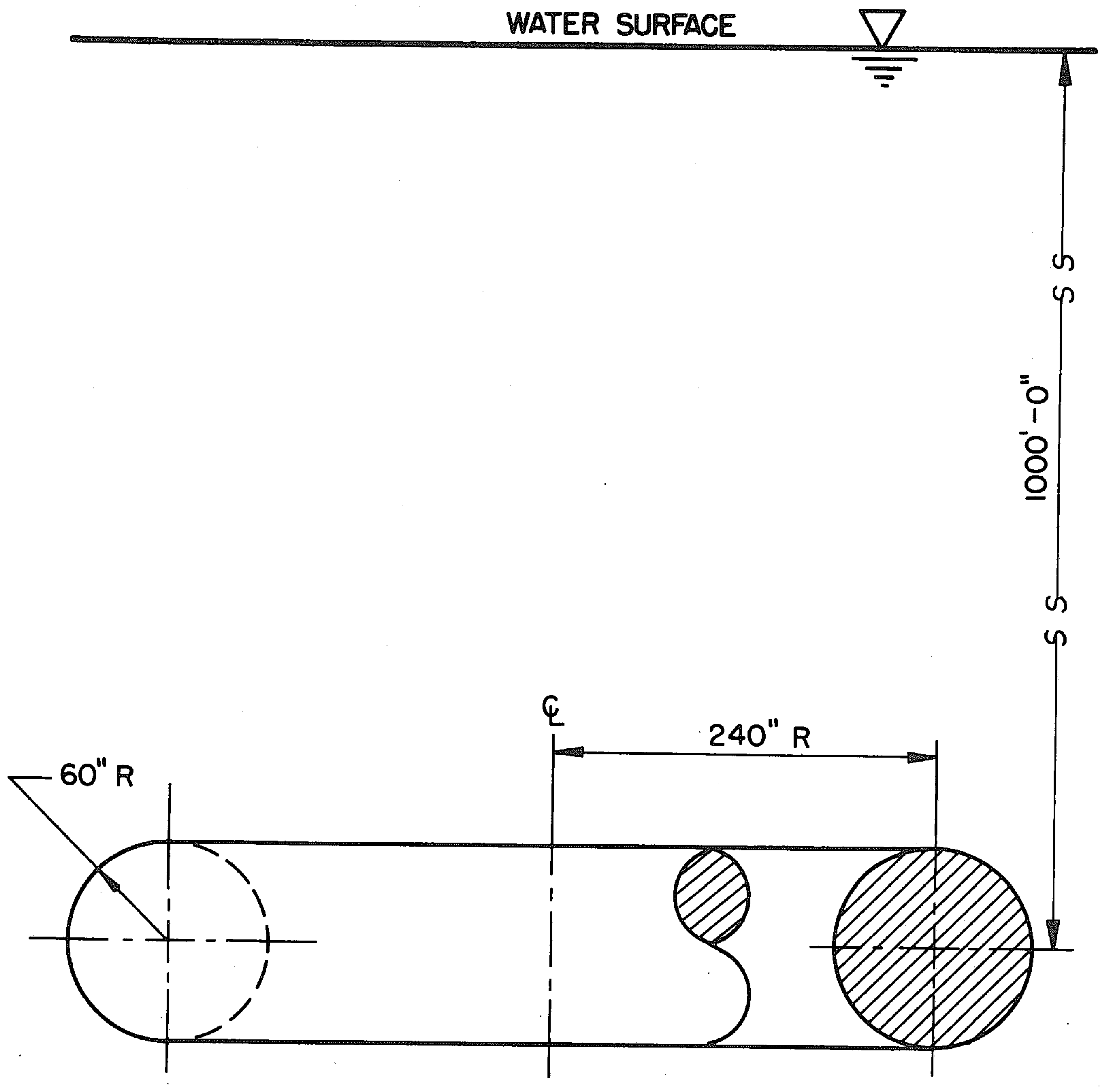


FIG. 6 LOAD-DEFLECTION, PLATE BENDING EXAMPLE



ELEVATION OF THE TORUS

FIG. 8 TORUS SUBJECTED TO HYDROSTATIC PRESSURE

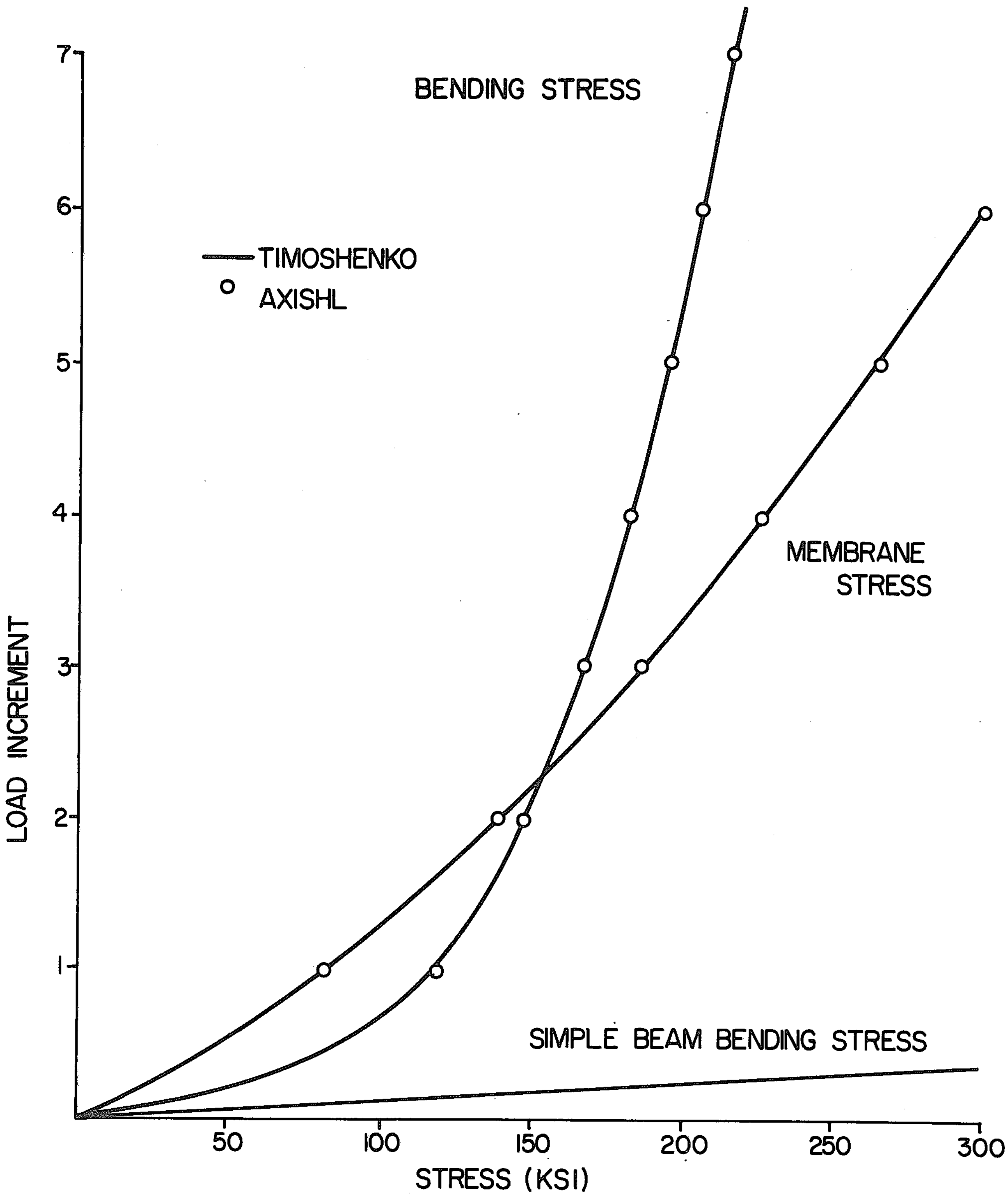


FIG. 7 LOAD - STRESS , PLATE BENDING EXAMPLE

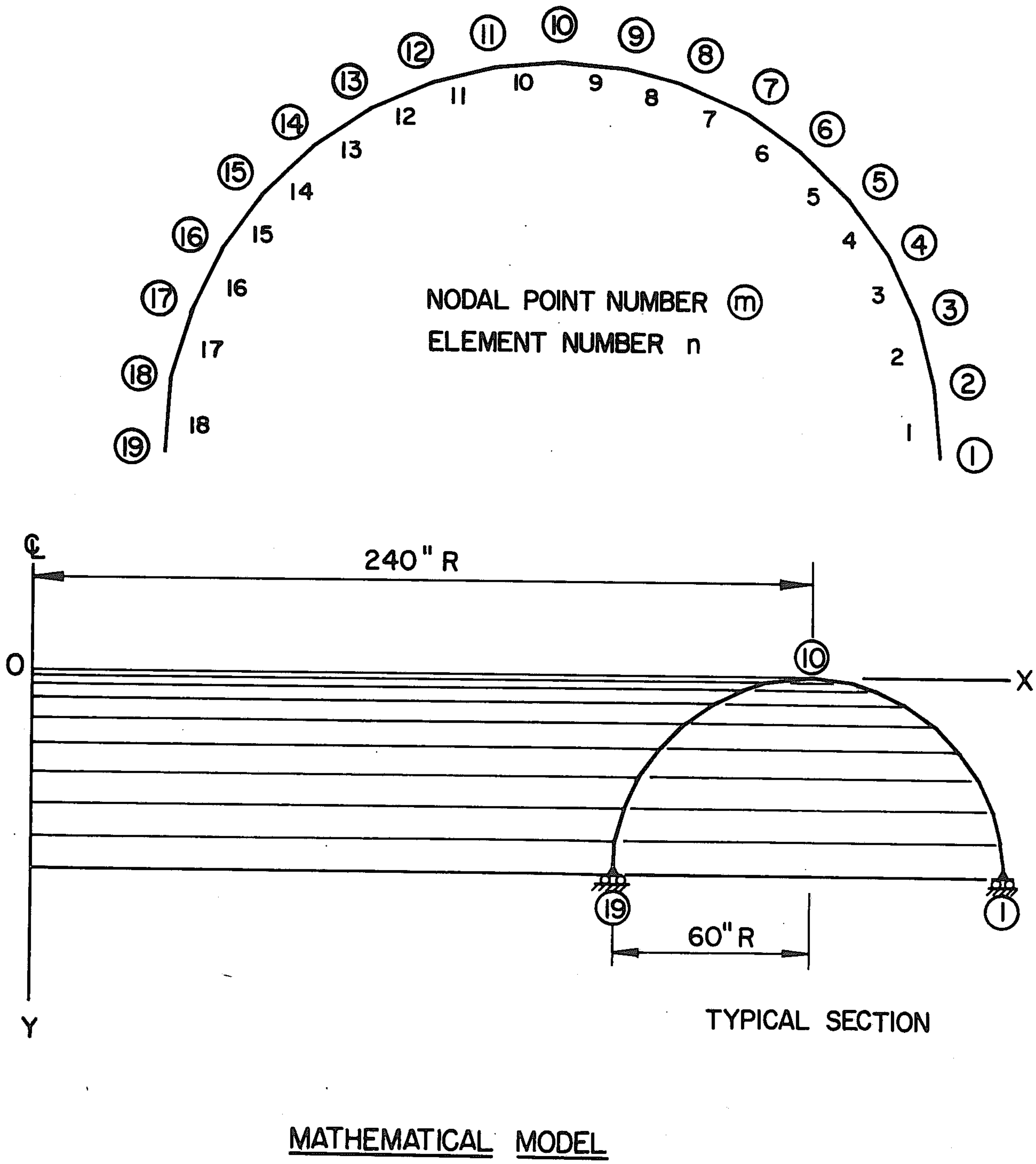


FIG. 9 FINITE ELEMENT REPRESENTATION OF TORUS

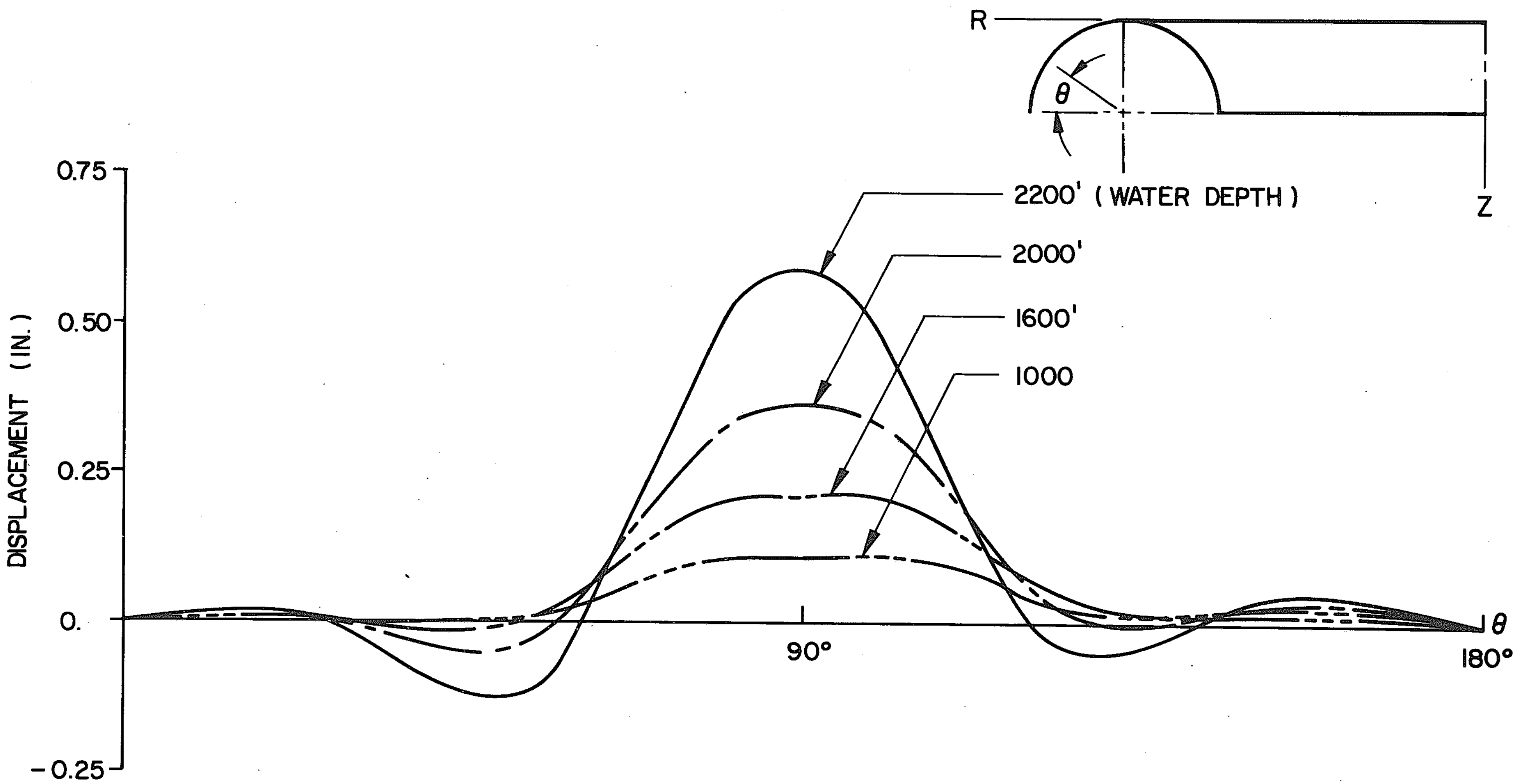


FIG. 10 AXIAL DISPLACEMENTS

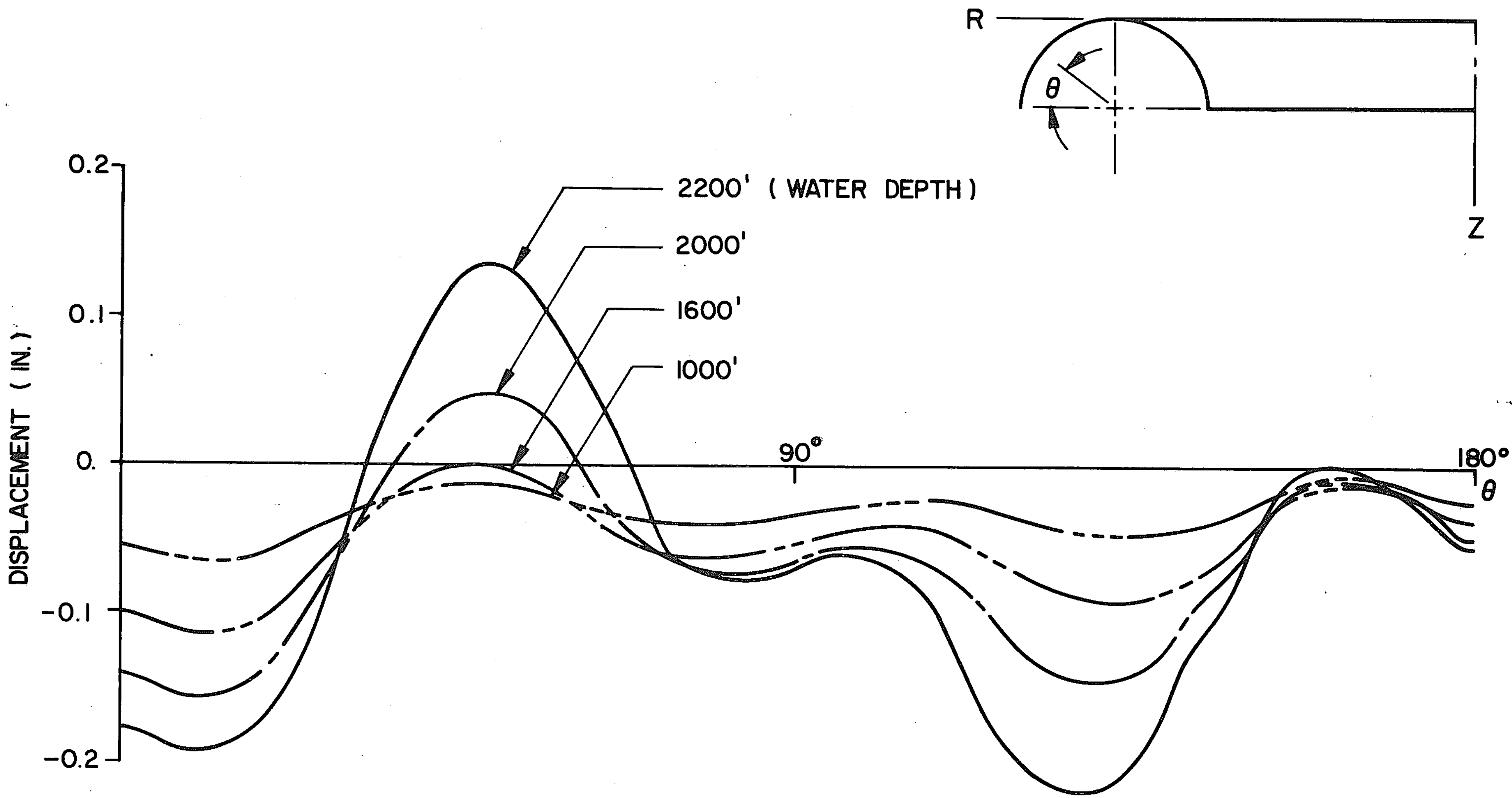


FIG. 11 RADIAL DISPLACEMENTS

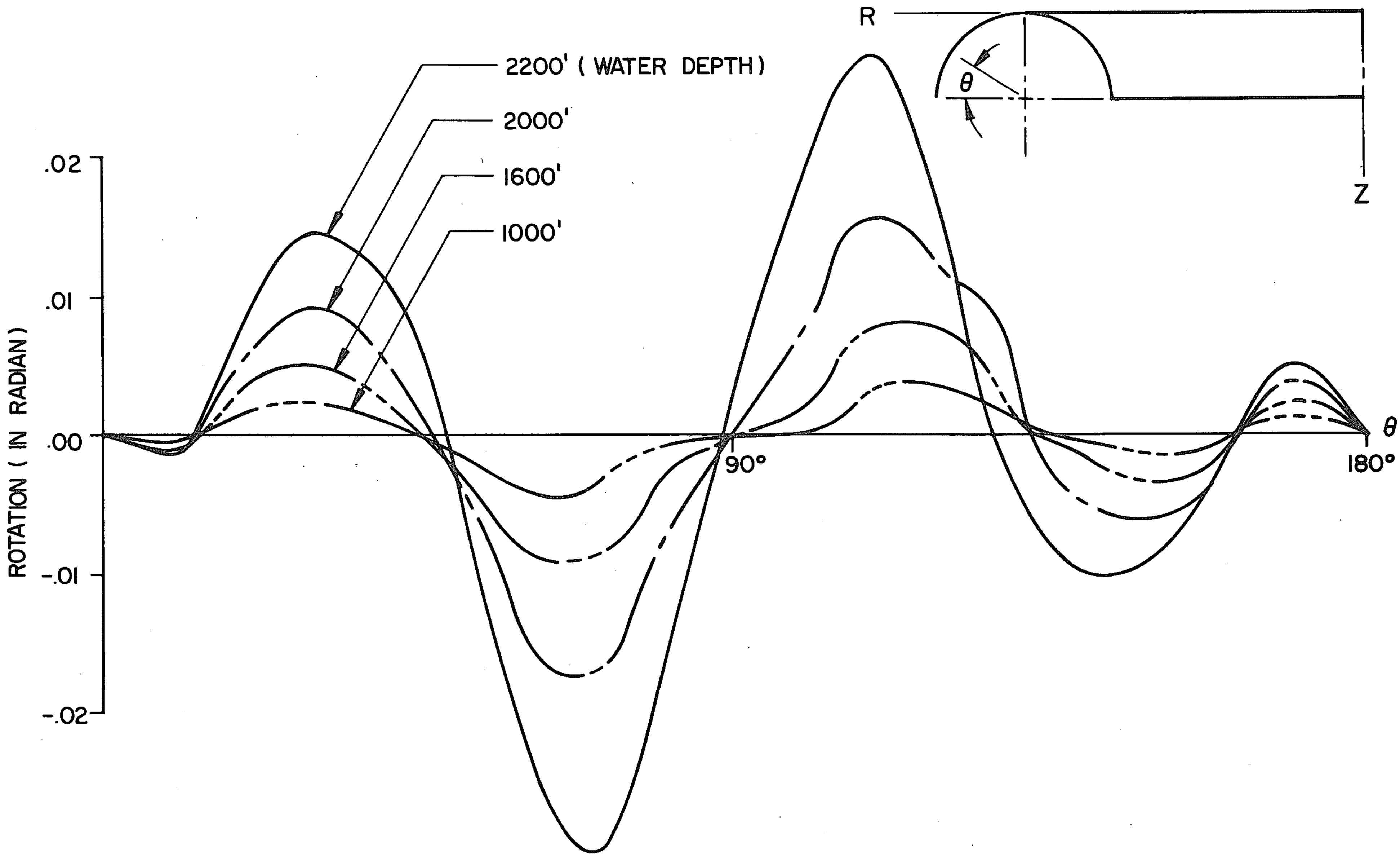


FIG. 12 ROTATION

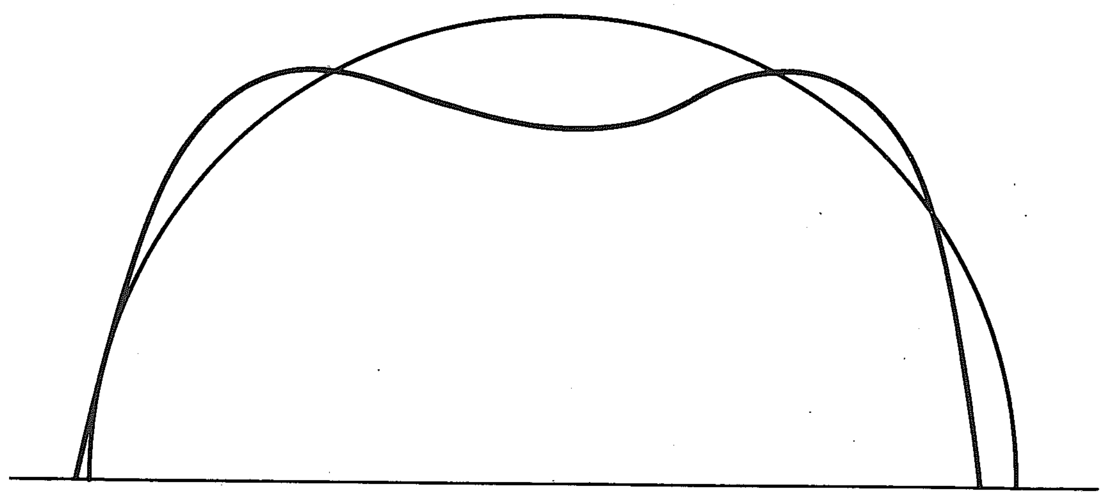


FIG. 13 DEFORMATION SHAPE AT WATER DEPTH OF 2200 FT.

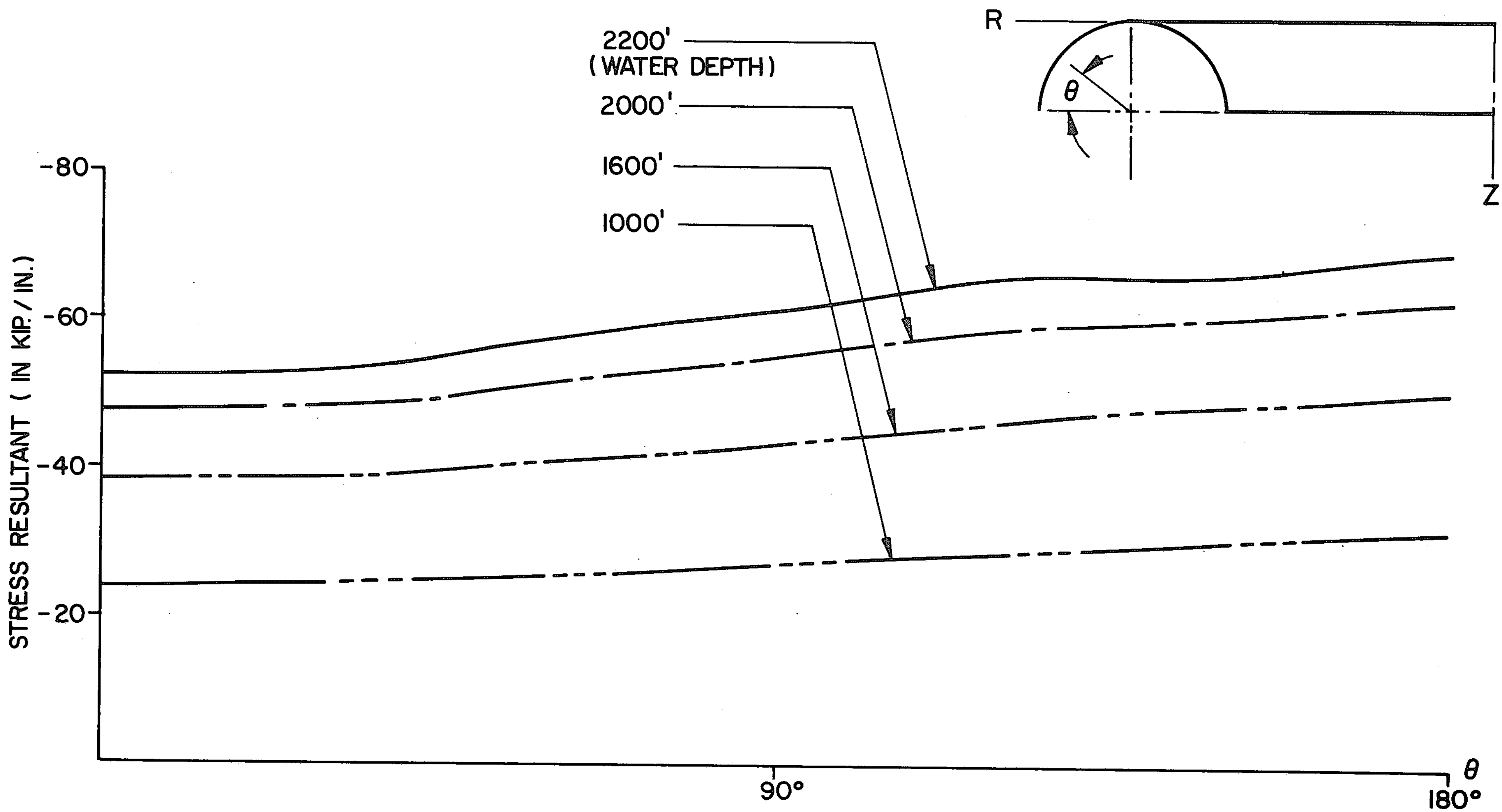


FIG. 14 MERIDIONAL STRESS RESULTANT

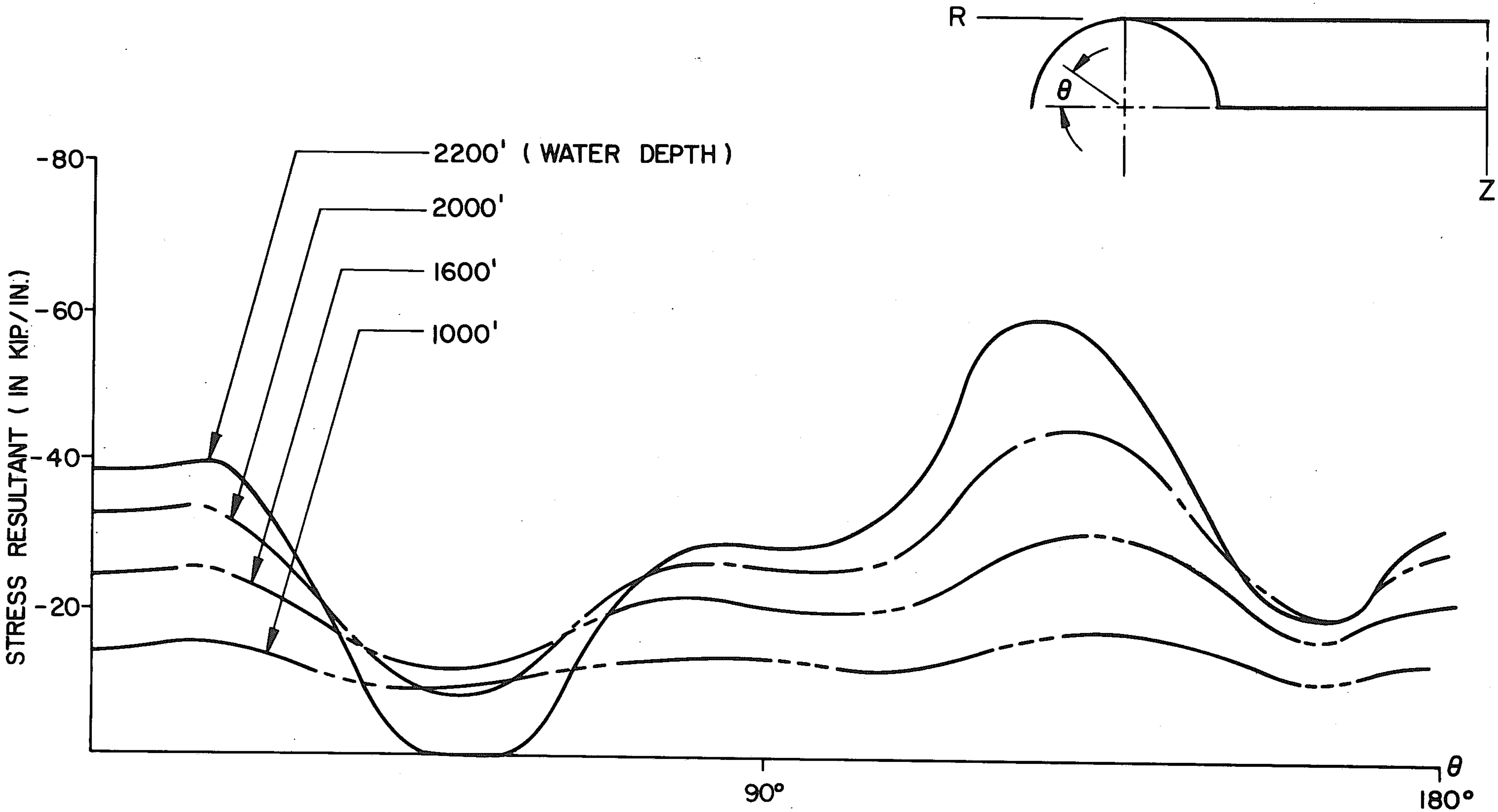


FIG. 15 TANGENTIAL STRESS RESULTANT

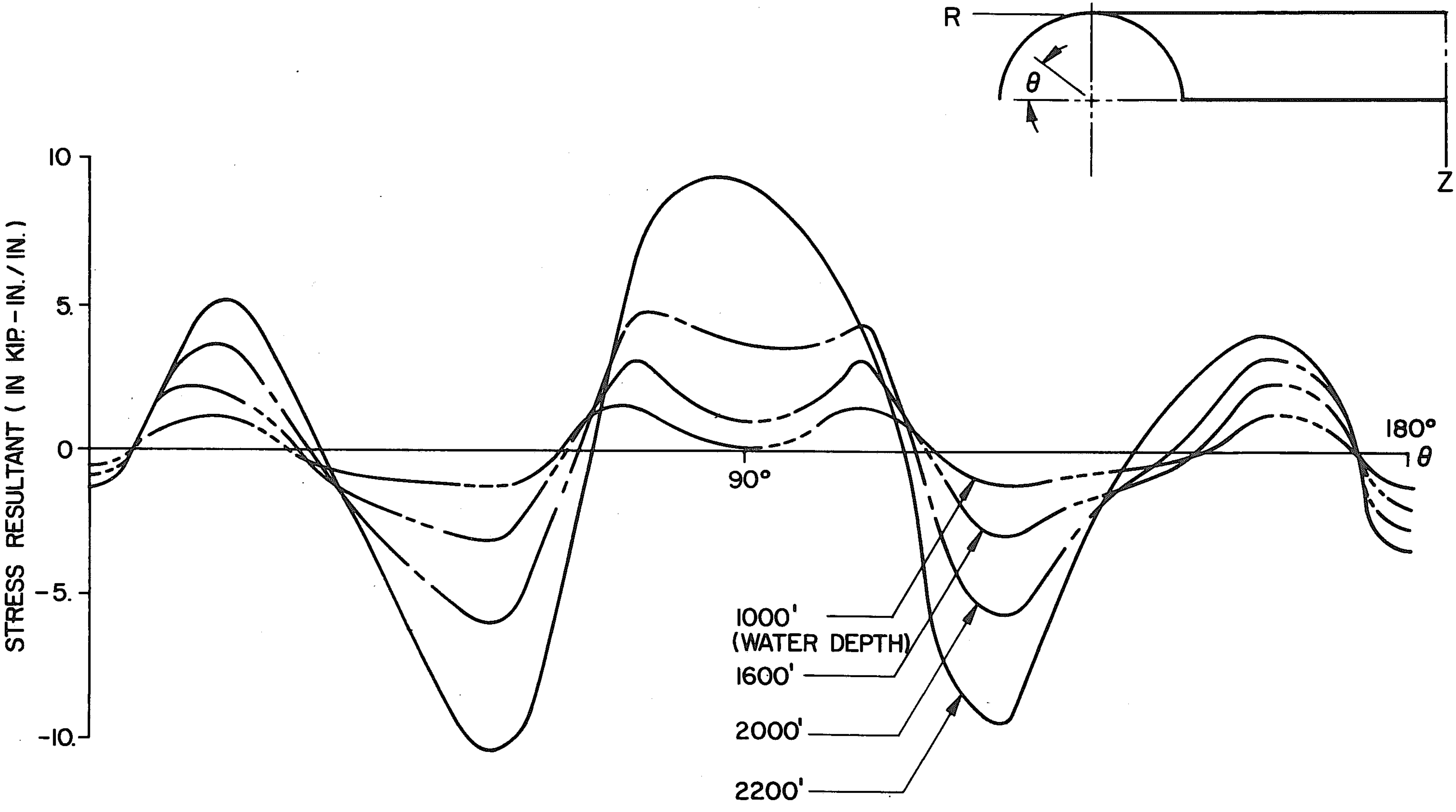


FIG. 16 MERIDIONAL MOMENT

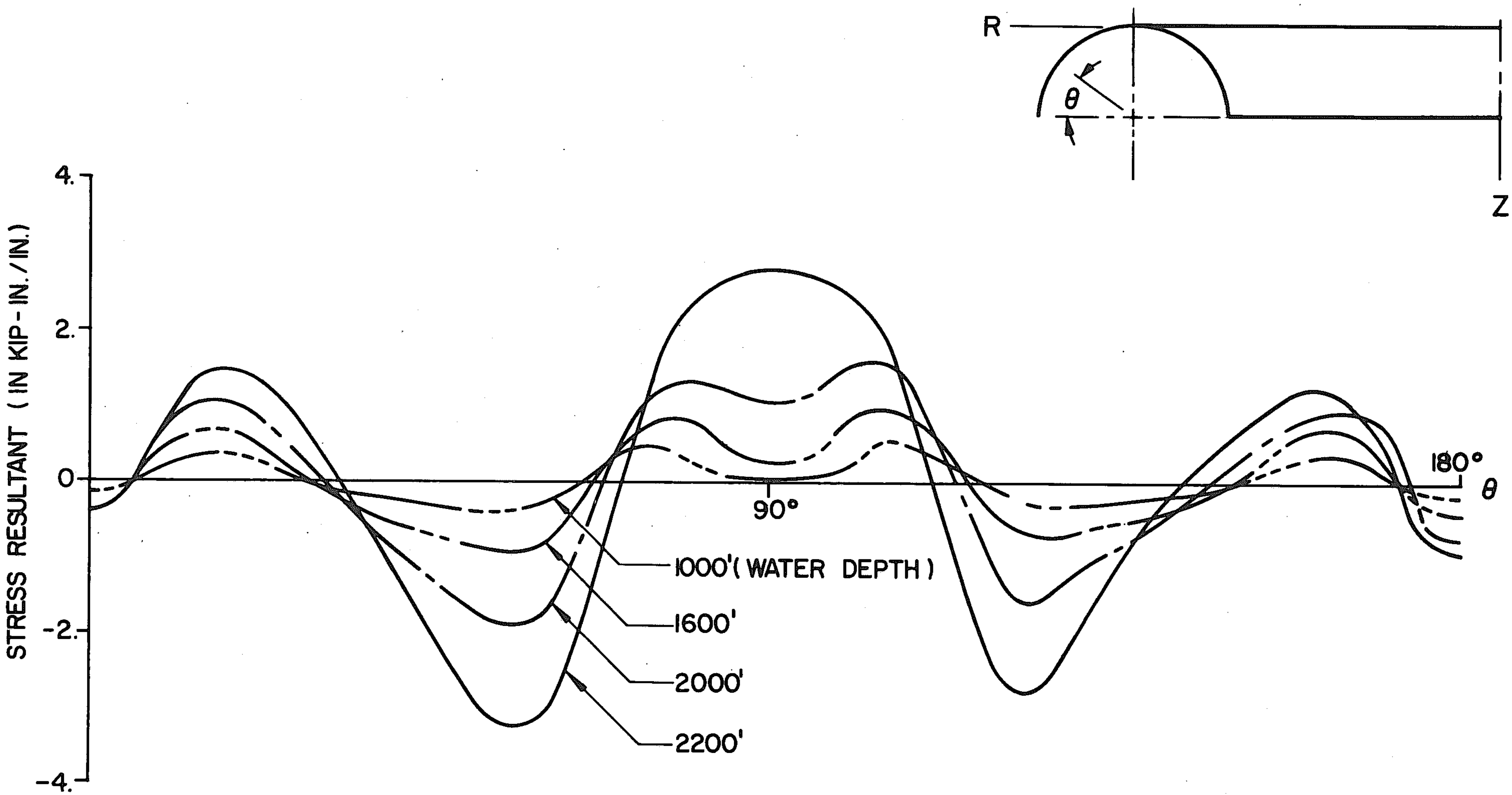


FIG. 17 TANGENTIAL MOMENT

CONCLUSION

A general approach to the large displacement analysis of Axisymmetric Shells has been presented. The method is based on a force equilibrium balance in the deformed geometry. It has been shown also, that the use of the simple conical element yields sufficiently accurate results. Most of the problems associated with its use can be overcome simply by using a smaller mesh size. A digital computer-program based on this approach has been developed. A Fortron IV listing of the program and a description of its use are given in the Appendices of this report.

REFERENCES

1. P. E. Grafton and D. R. Strome, "Analysis of Axisymmetrical Shells by the Direct Stiffness Method", AIAA Jour., Vol. 1, No. 10, Oct. 1963, pp. 2342-2347.

2. J. H. Percy, T. H. H. Pian, S. Klein, and D. R. Havaratna, "Application of Matrix Displacement Method to Linear Elastic Analysis of Shells of Revolution", AIAA Jour., Vol. 3, No. 11, Nov. 1965, pp. 2138-2145.

3. S. Klein, "A Study of the Matrix Displacement Method as Applied to Shells of Revolution", Proc. of the Cont. on "Matrix Methods in Structural Mech." , Dayton, Ohio, Oct. 1965, AFFDL-TR-66-80, Nov. 1966, pp. 275-298.

4. S. Klein and R. J. Sylvester, "The Linear Elastic Dynamic Analysis of Shells of Revolution by the Matrix Displacement Method", see Ref. 5, pp. 299-328.

5. R. E. Jones and D. R. Strome, "A Survey of the Analysis of Shells by the Displacement Method, see Ref. 5, pp. 205-230.

6. S. B. Dong, "Analysis of Laminated Shells of Revolution", Jour. Eng. Mech. Div., Proc. ASCE, Vol. 92, No. 6, Dec. 1966, pp. 135-155.

7. L. R. Herrmann, R. L. Taylor, and D. R. Green, "Finite Element Analysis for Solid Rocket Motor Cases", Report, SESM 67-4, 1967.

8. R. E. Jones and D. R. Strome, "Direct Stiffness Method Analysis of Shells of Revolution Utilizing Curved Element", AIAA Jour., Vol. 4, No. 9, Sept. 1966, pp. 1519-1525.

9. M. Khojasteh - Bakht, "Analysis of Elastic-Plastic Shells of Revolution by the Finite Element Method", SESM Report 67-1.

10. Murray, D. W., "Large Deflection Analysis of Plates," SESM Report 67-44, University of California, Berkeley, 1967.

11. Popov, E. P., J. Penzien, and Z. A. Lu, "Finite Element Solution for Axisymmetric Shells," Proceedings, A.S.C.E., Engineering Mechanics Division, October, 1964.

12. Navaratna, D. R., T. H. H. Pian, and E. A. Witmer, "Stability Analysis of Shells of Revolution by the Finite Element Method," Jnl.AIAA, February 1968.

13. Timoshenko, S. P. and J. M. Gere, THEORY OF ELASTIC STABILITY, McGraw-Hill, 1961.

14. Timoshenko, S. P. and S. Woinowsky-Krieger, THEORY OF PLATES AND SHELLS, McGraw Hill, 1961.

APPENDIX A

Description of Computer Program

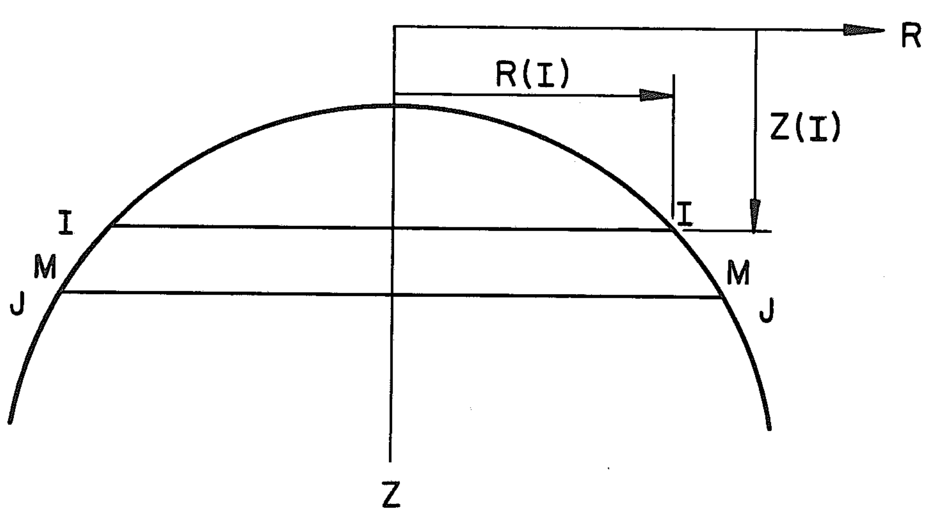
I. IDENTIFICATION

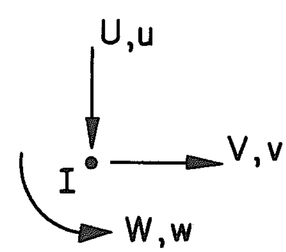
AXISHL - Axisymmetric Shell Large Deflection Analysis.

Programmed by L. Jones and T. Hsueh.

II. PURPOSE

The purpose of this computer program is to determine nodal displacements and element stress resultants of axisymmetric thin shells subjected to axisymmetric loading. The loading is applied in arbitrary increments and a complete solution is given at each increment. The positive definitions for the input and output data are shown below:





III. INPUT DATA

For each shell to be analysed, a group of punch cards is required in this sequence.

A. TITLE CARD (12A6)

Columns 1 - 72 Alphanumeric data for problem identification

B. CONTROL CARD (5I5)

Columns	1 - 5	Number of nodal circles (maximum = 100)
	6 - 10	Number of elements (maximum = 100)
	11 - 15	Number of material types (maximum = 10)
	16 - 20	Number of load increments (maximum = 20)
	21 - 25	Maximum number of cycles of iteration allowed per load increment (any number)

C. LOAD INCREMENT PROPORTIONAL FACTORS (8F10.0)

In each problem, the load patterns are identical for all load increments, but the relative magnitude can be varied. The load factor for a given increment is directly proportional to its loading level. For example, a shell under hydrostatic pressure is to be analysed. Five load increments are used. The corresponding water depthes are 1000', 1500', 1800', 1900' and 2000' in sequence for the increments. Load condition as input data is corresponding to the first load increment (i.e. water depth of 1000 feet). The load increment proportional factors can be taken as following:

Columns	1 - 10	1000.
	11 - 20	1500.
	21 - 30	1800.
	31 - 40	1900.
	41 - 50	2000.

In case more than 8 load increments are used, more than one card should be needed.

D. MATERIAL PROPERTIES (I5, 2F10.4)

A card must be supplied for each different material.

Columns	1 - 5	Material Identification (any number)
	6 - 15	Young's Modulus
	16 - 25	Poisson's Ratio

E. NODAL CIRCLE DATA (I5, 2F10.4, I5, 3F10.4, I1, F11.0)

One card for each nodal circle.

Columns	1 - 5	Nodal circle number
	6 - 15	R - coordinate
	16 - 25	Z - coordinate
	26 - 30	Nodal boundary code - see * below
	31 - 40	Vertical load U
	41 - 50	Horizontal load V
	51 - 60	Moment load W
	61	ID - See ** below
	62 - 72	Curvature - see ** below

* The boundary code is a 3 digit number (consisting of 0 or 1) which specifies if applied "loads" are forces or displacements.

1 specified displacement

0 specified force

e.g. 101 U(I) is specified displacement

V(I) is specified force

W(I) is specified rotation

** These are optional input data needed if automatic nodal point data is being used. See later section.

F. ELEMENT DATA. (3I5, F10.4, I5, F10.4, 5F7.0)

One card for each element.

Columns	1 - 5	Element number
	6 - 10	Number of node I
	11 - 15	Number of node J
	16 - 25	Element thickness
	26 - 30	Number of material type
	31 - 40	Normal pressure (force per unit area)
	41 - 45	Number of meridional stiffeners
	46 - 52	Depth of meridional stiffeners
	53 - 59	Thickness of meridional stiffeners
	60 - 66	Spacing of tangential stiffeners
	67 - 73	Depth of tangential stiffeners
	74 - 80	Thickness of tangential stiffeners

Automatic Data Generation

A. Nodal Circle Data

This may be used if a series of nodes lie on

a) A straight line; or

b) A circle having its center on the z axis.

Automatic data generation is activated if two successive nodal circle numbers differ by more than one. Nodal circle cards are required for the first and last nodes in the series. The nodes must have either

a) Zero boundary code - set ID = 0 on the last card in the serier; or

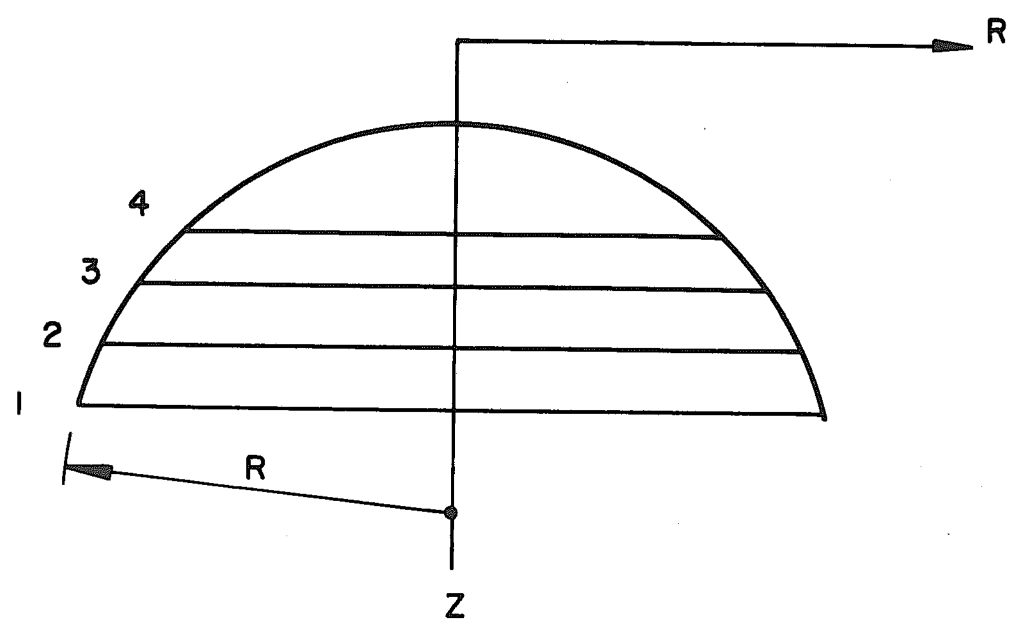
b) Same boundary code as the last - set ID = 1 on the last card in the series.

The last card must also contain the curvature of the line,

a) Straight line - Curvature = 0

b) Circle of radius R - Curvature = 1./R

The sketch below shows positive curvature and appropriate nodal circle numbering sequence:



B. Element data

This may be used if a series of elements has the same

a) Material

b) Thickness

c) Reinforced stiffeners

d) Normal pressure

and the nodal circle numbering is sequential from the first to the last. Element cards are required for the first and last elements in the series.

IV. OUTPUT INFORMATION

The program prints the following output:

A. Input and generated data.

B. The results of analysis for each load increment.

a) Summation of mean square of applied loads

b) Summation of mean square of out of balance forces for every iterative cycle.

c) Nodal circle displacements

d) Stress resultants evaluated at the mid-circle of each element.

APPENDIX B

Fortrain IV Listing of Computer Program

```
      PROGRAM AXISHL (INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT,TAPE1)
C*******************************************************************
C
C     LARGE DEFLECTION STATIC ANALYSIS OF AXISYMMETRIC SHELLS.
C     AXISYMMETRIC LOADING.    LINEAR ELASTIC,ISOTROPIC MATERIAL.
C     INCLUDES MERIDIONAL AND TANGENTIAL STIFFENERS.
C     USES SIMPLE ITERATION TO SEEK AN EQUILIBRIUM CONFIGURATION.
C      USES CONICAL FRUSTRUM ELEMENT.
C ****************************************************
      COMMON NJ,NE,NEQ,MBAND,NLING,NCYC,XNL(20),B(30 ),SK(300,20)
      COMMON/JUNK/HED(13),X(100),Y(100),KODE(100),JI(100),JJ(100),
     1MATERL(100),E(10),PS(10),PRES(100),T(100),RMN(100),RMH(100),
     2RMT(100),RTC(100),RTH(100),RTT(100)
      COMMON/EXTRA/DLO(100),SNO(100),DISPL(300)
      COMMON/ADDNL/NLOAD,NCYCLE,IFLAG,MMM,DL,SN,CS,P(6),D(4,4),S(6,6),
     1Z(6,6),SG(6,6),U(6)
C
C
C     INPUT AND GENERATE MISSING DATA.
C
  500 READ (5,100)     HED
      WRITE (6,99)     HED
C
C     READ AND PRINT CONTROL DATA.
C
      READ (5,101)   NJ,NE,NMAT,NLINC,NCYC
      WRITE (6,201)  NJ,NE,NMAT,NLINC,NCYC
C
      READ (5,111) (XNL(NNN),NNN=1,NLINC)
      WRITE (6,301)
      WRITE (6,302)(I,XNL(I),I=1,NLINC)
C
      NEQ=3*NJ
      NLOAD=1
C
C     READ AND PRINT MATERIAL PROPERTIES.
C
      WRITE (6,210)
      DO 53 I=1,NMAT
      READ (5,110)   N,E(N),PS(N)
   53 WRITE (6,211)  N,E(N),PS(N)
C
C   AUTOMATIC GENERATION OF NODAL POINT DATA.
C   IF A SERIES OF NODAL POINTS LIE EITHER
C      (1) ON A STRAIGHT LINE
C      (2) ON A CIRCLE HAVING ITS CENTER ON THE AXIS OF REVOLUTION.
C     NODAL POINT CARDS NEEDED FOR FIRST AND LAST IN SERIES.
C     BOUNDARY KODE FOR INTERMEDIATE NODES MAY BE EITHER...
C      (1) FREE NODES (KODE=0)   SET ID=0
C      (2) SAME KODE AS LAST IN SERIES    SET ID=1
C         AND HAVE THE SAME LOADS.
C
C     NODAL POINTS GENERATED WHEN TWO SUCCESSIVE INPUT
C     NODE NUMBERS DIFFER BY MORE THAN ONE.
C
```

```
      NPREV=0
      N=0
      WRITE(6,202)
   11 IF(N.EQ.NJ) GO TO 67
      NPREV=N
      READ (5,103) N,X(N),Y(N),KODE(N),B(3*N-2),B(3*N-1),B(3*N), ID,CURV
      IF(N-NPREV-1) 11,11,12
   12 NX=N
      IF(ID) 14,14,15
   15 KOD=KODE(N)
      B1=B(3*N-2)
      B2=B(3*N-1)
      B3=B(3*N)
      GO TO 16
   14 KOD=0
      B1=0.
      B2=0.
      B3=0.
   16 KON=NX-NPREV
      KX=1
      KY=1
      DX=X(NX)-X(NPREV)
      DY=Y(NX)-Y(NPREV)
      DL=SQRT(DX**2+DY**2)
      ANGLE=2.*ASIN (DL*CURV/2.)
      AINC=ANGLE/KON
      BETA=ACOS(X(NPREV)*CURV)+AINC/2.
      DEL=SIN(AINC/2.)/CURV
      N=NPREV
   68 N=N+1
      IF(N.EQ.NX) GO TO 11
      IF(CURV) 17,18,17
   17 DIFY=2.*DEL*COS(BETA)
      DIFX=2.*DEL*SIN(BETA)
      BETA=BETA+AINC
      IF((X(NX)).LT.(X(NPREV)))KX=-1
      IF((Y(NX)).LT.(Y(NPREV)))KY=-1
      GO TO 22
   18 DIFX=DX/KON
      DIFY=DY/KON
   22 X(N)=X(N-1)+KX*DIFX
      Y(N)=Y(N-1)+KY*DIFY
      KODE(N)=KOD
      B(3*N-2)=B1
      B(3*N-1)=B2
      B(3*N )=B3
      GO TO 68
   67 CONTINUE
      DO 69 N=1,NJ
   69 WRITE (6,203)   N,X(N),Y(N),KODE(N),B(3*N-2),B(3*N-1),B(3*N)
C
C     READ  AND PRINT ELEMENT DATA.
C
C     AUTOMATIC GENERATION OF ELEMENT DATA.
C      USE IF ELEMENTS OF A SERIES HAVE..
```

```
C        SEQUENTIAL NUMBERING OF CORRESPONDING NODAL POINTS
C       AND COMMON...
C          (1)MATERIAL PROPERTIES,
C          (2) THICKNESS,
C          (3) DISTRIBUTED LOADING.
C       CARDS REQUIRED FOR FIRST AND LAST ELEMENT IN SERIES.
C
        N=0
        MBAND=0
        WRITE (6,204)
 52     READ (5,105) NX,JINX,JJNX,TN,MATL,ELPRES,NMR,HMR,TMR,CTR,HTR,TTR
C
        IF (NX.GT.(N+1)) GO TO 2
        N=NX
        JIN=JINX
        JJN=JJNX
        GO TO 3
      2 KON=NX-N
        INC=(JINX-JIN)/KON
      4 JIN=JIN+INC
        JJN=JJN+INC
        N=N+1
      3 CONTINUE
        JI(N)=JIN
        JJ(N)=JJN
        T(N)=TN
        MATERL(N)=MATL
        PRES(N)=ELPRES
        RMN(N) = NMR
        RMH(N) = HMR
        RMT(N) = TMR
        RTC(N) = CTR
        RTH(N) = HTR
        RTT(N) = TTR
        WRITE(6,205) N,JIN,JJN,TN,MATL,ELPRES,NMR,HMR,TMR,CTR,HTR,TTR
C
C       COMPUTE BANDWIDTH.
C
        MB=IABS(JIN-JJN)
        IF(MB.GT.MBAND)  MBAND=MB
        IF (N.LT.NX) GO TO 4
        IF(NE-N) 51,51,52
     51 CONTINUE
        MBAND=3*MBAND+3
C
C  ZERO INITIAL DISPLACEMENTS ESTIMATE
C
        DO 752 I=1,NEQ
    752 DISPL(I)=0.
    600 CONTINUE
        REWIND 1
        WRITE (6,206) NLOAD
C*******************************************************************
C   SOLVE FOR NODAL DISPLACEMENTS  (INCLUDING LARGE DEFLECTIONS)
        CALL ANLYSE
```

```
C***********************************************************************
C
C    PRINT OUT DISPLACEMENTS.
C
      WRITE (6,208)
      DO 70  N=1,NJ
   70 WRITE (6,209) N,DISPL(3*N-2),DISPL(3*N-1),DISPL(3*N)
C
C***********************************************************************
C     COMPUTE ELEMENT STRESSES
      CALL ELSTRS
C***********************************************************************
C
C     CHECK IF FINAL LOAD INCREMENT REACHED.
C
      IF (NLOAD.EQ.NLINC)  GO TO 601
C     INCREMENT LOADS.
      NLOAD=NLOAD+1
      DO 5 I=1,NEQ
    5 B(I) =B(I) *XNL(NLOAD) / XNL(NLOAD - 1)
C
C     NODAL LOAD VECTOR FORMED IN CURRENT GEOMETRY.
C
      DO 6 I=1,NE
6     PRES(I)=PRES(I)*XNL(NLOAD)/XNL(NLOAD-1)
      GO TO 600
  601 CONTINUE
      GO TO 500
C
   99 FORMAT (1H1,13A6)
  100 FORMAT (13A6)
101   FORMAT (5I5)
  103 FORMAT (I5,2F10.4,I5,3F10.4,I1,F11.0)
105   FORMAT (3I5,F10.4, I5, F10.4, I5, 5F7.0)
  110 FORMAT (I5,2F10.4)
  111 FORMAT (8F10.0)
  201 FORMAT (1H0,5X,24HNUMBER OF JOINTS          =,I5//
     1       6X,24HNUMBER OF ELEMENTS        =,I5//
     2             6X,24HNUMBER OF MATERIALS       =,I5//
     3             6X,24HNUMBER LOAD INCREMENTS =,I5//
     4             6X,24HNUMBER OF CYCLES          =,I5)
  202 FORMAT (1H1,6X,9HNODE DATA,5X,6HNUMBER,10X,
     1        8HX CO-ORD,10X,8HY CO-ORD,10X,4HKODE,
     2       10X,6HLOAD U,10X,6HLOAD V,10X,6HLOAD W)
  203 FORMAT (1H0,19X,I5,2(9X,F10.4),9X,I5,3(6X,F10.4))
204   FORMAT ( 1H1, 12HELEMENT DATA////
     1 1X, 132H    NUMBER      NODE I      NODE J  THICKNESS    MATERIAL
     2PRESSURE  MER.R.NO.  MER.R. D.  MER.R.TH.  TAN.R.C-C  TAN.R. D.   T
     3AN.R.TH. )
205   FORMAT (1H0,      3( 5X,I5, 1X), F10.4, 1X, 5X, I5, 1X, F10.4, 1X,
     1 5X, I5, 1X, 5(F10.4, 1X))
  206 FORMAT (1H1,*    LOAD INCREMENT NUMBER.......*,I5)
  208 FORMAT(1H1,5X,18HNODE DISPLACEMENTS,6X,4HNODE,5X,
     110HY-DISPL(U),7X,10HX-DISPL(V),7X,11HROTATION(W))
  209 FORMAT(1H0,27X,I5,3(5X,E12.5))
```

```
  210 FORMAT (1H1,6X,19HMATERIAL PROPERTIES,17X,4HTYPE,
     1       6X,14HYOUNGS MODULUS,6X,13HPOISSON RATIO)
  211 FORMAT (1H0,41X,I5,10X,F10.1,10X,F10.4)
301     FORMAT (1H1,10X,25H NUMBER OF LOAD INCREMENT,1 X,
     1 16H INCREMENT RATIO//)
302     FORMAT (1H0,25X,I5,20X,F10.4)
C
        END
```

```
      SUBROUTINE ANLYSE
C
C*******************************************M************************
C   SOLVES FOR NODAL DISPLACEMENTS (INCLUDING LARGE DISPLACEMENTS)
C******************************************************************
      COMMON NJ,NE,NEQ,MBAND,NLING,NCYC,XNL(20),B(30 ),SK(300,20)
      COMMON/JUNK/HED(13),X(100),Y(100),KODE(100),JI(100),JJ(100),
     1MATERL(100),E(10),PS(10),PRES(100),T(100),RMN(100),RMH(100),
     2RMT(100),RTC(100),RTH(100),RTT(100)
      COMMON/EXTRA/DLO(100),SNO(100),DISPL(300)
      COMMON/ADDNL/NLOAD,NCYCLE,IFLAG,MMM,DL,SN,CS,P(6),D(4,4),S(6,6),
     1Z(6,6),SG(6,6),U(6)
      DIMENSION DB(300),CNFV(6),LM(6),XDB(300)
      NCYCLE=0
      IFLAG=0
   50 NCYCLE=NCYCLE+1
C
C     CHECK FOR CONVERGENCE.
C     CONVERGENCE ASSUMED WHEN RMS OF OUT OF BALANCE IS
C     LESS THAN 0.001 OF THE RMS OF THE APPLIED LOADS.
C
      IF(XDBSQ.LE.SQLD)IFLAG=1
C
C     CHECK IF MAX NO OF CYCLES REACHED.
      IF (NCYCLE .EQ. NCYC)  IFLAG = 1
      IF(NCYCLE.EQ.1)IFLAG=0
C     ZERO STRUCT. STIFF.
      DO 1 I=1,NEQ
      DO 1 J=1,MBAND
    1 SK(I,J)=0.
C  SET INITIAL OUT OF BALANCE LOADS.
      DO 51 I=1,NEQ
   51 DB(I)=B(I)
C
      DO 41 MMM=1,NE
C  ****************************************************
C     FORM ELEMENT STIFFNESS,
C     IN GLOBAL COORDS, USING CURRENT GEOMETRY.
      CALL ELSTIF
C  ****************************************************
      JIN=JI(MMM)
      JJN=JJ(MMM)
      XI=X(JIN)
C
C     FORM CONSISTENT FORCE VECTOR DUE TO NORMAL UNIFORM LOADS
C       (USING CURRENT GEOMETRY)
C
      ELPRES=PRES(MMM)
    2 CON=6.283185308*DL*ELPRES
      CI=(XI/2.+DL*SN/6.)
      CJ=(XI/2.+DL*SN/3.)
      CNFV(1)=CON*(-SN)*CI
      CNFV(2)=CON*CS*CI
      CNFV(4)=CON*(-SN)*CJ
      CNFV(5)=CON*CS*CJ
```

```
      CNFV(3)=0.
      CNFV(6)=0.
C***********************************************************************
C    COMPUTE ELEMENT RESISTING FORCES P(6)
      CALL RESIST
C***********************************************************************
C
C***********************************************************************
C FORM MODIFIED STIFFNESS (ADD GEOMETRIC STIFFNESS)
      CALL GEOSTI
C***********************************************************************
C
C
C     MODIFY (S),(B), AND (CNFV) FOR DISPLACEMENT BOUNDARY CONDITIONS.
C
   25 KK=KODE(JIN)
      KOUNT=1
      GO TO 27
   26 KK=KODE(JJN)
      KOUNT=2
   27 XK=KK
      IF(KK) 33,33,28
   28 CONTINUE
      DO 31 K=1,3
      XK=XK/10.
      KK=KK/10
      DKK=KK
      IF(XK-DKK) 31,31,32
   32 CONTINUE
C
C     MODIFY (B)
      IJ=JIN
      IF(KOUNT.EQ.2) IJ=JJN
      IS=3*KOUNT-K+1
      PARTY=DB(3*IJ-K+1)
      DO 30 NI=1,3
30    DB(3*JIN-3+NI) = DB(3*JIN-3+NI) - SG(NI,IS) * PARTY
      DO 49 NI=1,3
49    DB(3*JJN-3+NI) = DB(3*JJN-3+NI) - SG(NI+3,IS) * PARTY
      DB(3*IJ-K+1)=PARTY
C
C     MODIFY (S)
      DO 37 NI=1,6
      SG(NI,IS) = 0.
37    SG(IS,NI) = 0.
      SG(IS,IS) = 1.
C
C    MODIFY (CNFV) AND (P)
      CNFV(IS)=0.
      P(IS)=0.
   31 XK=DKK
   33 CONTINUE
      GO TO (26,35),KOUNT
   35 CONTINUE
C
```

```
C      ADD ELEMENT TO STRUCTURE STIFFNESS.
C
       LM(3)=3*JIN
       LM(2)=LM(3)-1
       LM(1)=LM(2)-1
       LM(6)=3*JJN
       LM(5)=LM(6)-1
       LM(4)=LM(5)-1
       DO 20  I=1,6
       IL = LM(I)
       DO 20  J=1,6
       JL = LM(J) - IL + 1
       IF (JL) 20,20,22
22     SK(IL,JL) = SK(IL,JL) + SG(I,J)
   20  CONTINUE
C
C      FORM STRUCTURE LOAD VECTOR.
C
       DO 38 I=1,3
38     DB(3*JIN-3+I)=DB(3*JIN-3+I)-CNFV(I)
       DO 39 I=4,6
39     DB(3*JJN-6+I)=DB(3*JJN-6+I)-CNFV(I)
C
C      FORM VECTOR OF OUT OF BALANCE NODAL FORCES.
       DO 54 K=1,3
       JOG=3*JI(MMM)-3+K
   54  DB(JOG)=DB(JOG)-P(K)
       DO 55 K=4,6
       JOG=3*JJ(MMM)-6+K
   55  DB(JOG)=DB(JOG)-P(K)
   41  CONTINUE
C
C CALCULATE THE SUM OF THE TOTAL APPLIED LOADS AT THE FIRST CYCLE
C
       IF (NCYCLE .NE. 1) GO TO 224
       IF(NLOAD.NE.1) GO TO 225
       SQLD=0.
       DO 226 I=1,NEQ
226    SQLD=SQLD+DB(I)**2
       SQL=SQRT(SQLD)
       GO TO 227
225    SQL=SQL*XNL(NLOAD)/XNL(NLOAD-1)
227    CONTINUE
       WRITE (6,223) SQL
       WRITE (6,222)
C
       SQLD=0.001*SQL
C
224    CONTINUE
C
      DO906 I=1,NEQ
  906 XDB(I)=DB(I)
C
C*******************************************************************
C      SOLVE FOR INCREMENTAL NODE DISPLACEMENTS.
```

```
      CALL SYMSOL (SK,DB,NEQ,MBAND,1)
      CALL SYMSOL (SK,DB,NEQ,MBAND,2)
C*****************************************************************
C
C    ESTABLISH NEW ESTIMATE OF DISPLACEMENTS AND GEOMETRY
      DO 56 K=1,NJ
      Y(K)=Y(K)+DB(3*K-2)
      X(K)=X(K)+DB(3*K-1)
   56 CONTINUE
      DO 57 K=1,NEQ
   57 DISPL(K)=DISPL(K)+DB(K)
C
C CALCULATE THE SUM OF OUT OF BALANCE FORCES
C
      XDBSQ=0.
      DO 107 I=1,NEQ,3
  107 XDBSQ=XDBSQ+XDB(I)**2
      DO 1107 I=2,NEQ,3
 1107 XDBSQ=XDBSQ+XDB(I)**2
      XDBSQ=SQRT(XDBSQ)
      WRITE (6,201) NCYCLE, XDBSQ
      IF (IFLAG.EQ.1) GO TO 59
      GO TO 50
   59 CONTINUE
      RETURN
C
  201 FORMAT (1H0,20H        CYCLE NUMBER,I5, 15X,E16.8)
222   FORMAT (1H0,30X,30H SUM OF OUT OF BALANCE FORCES /)
223   FORMAT (1H0,10X,22H TOTAL APPLIED LOAD = ,E16.8//)
      END
```

```
      SUBROUTINE ELSTIF
C
C*************************************************************************
C   FORM ELEMENT STIFFNESS  (GLOBAL CO-ORDS.)
C*************************************************************************
      COMMON NJ,NE,NEQ,MBAND,NLING,NCYC,XNL(20),B(30 ),SK(300,20)
      COMMON/JUNK/HED(13),X(100),Y(100),KODE(100),JI(100),JJ(100),
     1MATERL(100),E(10),PS(10),PRES(100),T(100),RMN(100),RMH(100),
     2RMT(100),RTC(100),RTH(100),RTT(100)
      COMMON/EXTRA/DLO(100),SNO(100),DISPL(300)
      COMMON/ADDNL/NLOAD,NCYCLE,IFLAG,MMM,DL,SN,CS,P(6),D(4,4),S(6,6),
     1Z(6,6),SG(6,6),U(6)
      DIMENSION TS(6,6),A(4,6),TEMP(4,6),TK(6,6),TZ(6,6)
C
C     STIFFNESS IS FORMED USING CURRENT ESTIMATE OF NODAL CO-ORDINATES
C
      JIN=JI(MMM)
      JJN=JJ(MMM)
      DX=X(JJN)-X(JIN)
      DY=Y(JJN)-Y(JIN)
      DL = SQRT ( DX**2 +DY**2 )
      SN=DX/DL
      MUM=MATERL(MMM)
      EE=E(MUM)
      PPS=PS(MUM)
      TT=T(MMM)
      XI=X(JIN)
      CS=DY/DL
C
C  SET INITIAL LENGTH AND SLOPE OF ELEMENT.
C
      IF((NCYCLE.EQ.1).AND.(NLOAD.EQ.1)) SNO(MMM)=SN
      IF((NCYCLE.EQ.1).AND.(NLOAD.EQ.1)) DLO(MMM)=DL
C
C   FORM 6X6 ELEMENT STIFFNESS MATRIX
C
      DO 15  I=1,6
      DO 15  J=1,6
   15 TS(I,J)=0.0
C
C   FORM ELASTICITY MATRIX (D) 4X4  (LINEAR,ELASTIC,ISOTROPIC)
C
      DO 3  I=1,4
      DO 3  J=1,4
    3 D(I,J)=0.0
C
      C=EE*TT/(1.-PPS**2)
      D(1,1)=C
      D(1,2)=PPS*D(1,1)
      D(2,2)=C
      D(3,3)=C*(TT**2)/12.
      D(3,4)=PPS*D(3,3)
      D(4,4)=D(3,3)
      D(2,1)=D(1,2)
      D(4,3)=D(3,4)
```

```
      IF (RMN(MMM)) 21, 22, 21
21    RRMN = RMN(MMM)
      RRMH = RMH(MMM)
      RRMT =RMT(MMM)
      TLM = 3.141592654 * ( X(JJN) + X(JIN) )
      TAMS = TLM * TT
      TAMR = RRMN * RRMH * RRMT
      TMIMR = RRMN *RRMT *RRMH ** 3/ 12.0
      DCGM = TAMR * (TT+ RRMH ) / ( 2.0 * ( TAMS + TAMR ))
      DTM = EE * TAMR / TLM
      DMIM=EE*(TAMS*DCGM**2+TMINR+TAMR*((TT+RRMH)/2. DCGM)**2)/TLM
      D(1,1) = D(1,1) + DTM
      D(3,3) = D(3,3) + DMIM
22    CONTINUE
      IF ( RTC(MMM)) 23,24,23
23    RRTC = RTC(MMM)
      RRTH = RTH(MMM)
      RRTT = RTT(MMM)
      TAR = RRTH *RRTT
      TMIR = RRTT * RRTH**3/ 12.0
      TAS = RRTC * TT
      DCGT = TAR * ( TT + RRTH ) / (2.*( TAR + TAS))
      DTT = EE * TAR / RRTC
      DMIT=EE*(TAS*DCGT**2+TMIR+TAR*((TT+RRTH)/2.-DCGT)**2) / RRTC
      D(2,2) = D(2,2) + DTT
      D(4,4) = D(4,4) + DMIT
24    CONTINUE
C
C    NUMERICAL INTEGRATION TO EVALUATE ELEMENT STIFFNESS.
C    GAUSS QUADRATURE METHOD (5 POINT INTEGRATION)
C
      DO 4  M=1,5
C
      IF(M.NE.1)  GO TO 5
      XS=0.046910077030668
      XH=0.118463442528094
      GO TO 10
    5 IF(M.NE.2)  GO TO 6
      XS=0.230765344947158
      XH=0.239314335249683
      GO TO 10
    6 IF(M.NE.3)  GO TO 7
      XS=0.5
      XH=0.284444444444444
      GO TO 10
    7 IF(M.NE.4)  GO TO 8
      XS=0.769234655052842
      XH=0.239314335249683
      GO TO 10
    8 IF(M.NE.5)  GO TO 10
      XS=0.953089922969332
      XH=0.118463442528094
   10 CONTINUE
C
C     EVALUATE STRAIN DISPLACEMENT MATRIX.
```

```
      XJ=XI+DL*SN
      R=XI+XS*DL*SN
      A(1,1)=-1./DL
      A(1,2)=0.
      A(1,3)=0.
      A(1,4)=-A(1,1)
      A(1,5)=0.0
      A(1,6)=0.0
      A(2,1)=(1.-XS)*SN/R
      A(2,2)=(1.-(XS**2)*(3.-2.*XS))*CS/R
      A(2,3)=DL*XS*(1.-XS*(2.-1.*XS))*CS/R
      A(2,4)=XS*SN/R
      A(2,5)=(XS**2)*(3.-2.*XS)*CS/R
      A(2,6)=DL*XS**2*(-1.+XS)*CS/R
      A(3,1)=0.0
      A(3,2)=(-6.+12.*XS)/DL**2
      A(3,3)=(-4.+6.*XS)/DL
      A(3,4)=0.0
      A(3,5)=-A(3,2)
      A(3,6)=(-2.+6.*XS)/DL
      A(4,1)=0.0
      A(4,2)=XS*(6.-6.*XS)*SN/(R*DL)
      A(4,3)=(-1.+XS*(4.-3.*XS))*SN/R
      A(4,4)=0.0
      A(4,5)=-A(4,2)
      A(4,6)=XS*(2.-3.*XS)*SN/R
C
      DO 11  I=1,4
      DO 11  J=1,6
      TEMP(I,J)=0.0
      DO 11  MM=1,4
   11 TEMP(I,J)=TEMP(I,J)+D(I,MM)*A(MM,J)
C
      DO 12  I=1,6
      DO 12  J=1,6
      TK(I,J)=0.0
      DO 12  MM=1,4
   12 TK(I,J)=TK(I,J)+A(MM,I)*TEMP(MM,J)
C
      DO 13  I=1,6
      DO 13  J=1,6
   13 TK(I,J)=TK(I,J)*R*XH
C
C    ADD TERM TO SUM IN INTEGRATION
C
      DO 14  I=1,6
      DO 14  J=1,6
   14 TS(I,J)=TS(I,J)+TK(I,J)
C
    4 CONTINUE
C
C     FORM CO ORDINATE TRANSFORMATION MATRIX.
C    (Z)  CHANGES GLOBAL CO-ORDS TO LOCAL.
C
      DO 16  I=1,6
```

```
      DO 16   J=1,6
   16 Z(I,J)=0.0
      Z(1,1)=CS
      Z(1,2)=SN
      Z(2,1)=-SN
      Z(2,2)=CS
      Z(3,3)=-1.0
      Z(4,4)=CS
      Z(4,5)=SN
      Z(5,4)=-SN
      Z(5,5)=CS
      Z(6,6)=-1.0
C
      DO 17   I=1,6
      DO 17   J=1,6
      TZ(I,J)=0.
      DO 17   M=1,6
   17 TZ(I,J)=TZ(I,J)+TS(I,M)*Z(M,J)
C
      DO 18   I=1,6
      DO 18   J=1,6
      S(I,J)=0.0
      DO 18   M=1,6
   18 S(I,J)=S(I,J)+Z(M,I)*TZ(M,J)
C
C   ELEMENT STIFFNESS MATRIX (S)   6X6
C
      COMM=2.*3.141592654*DL
      DO 19   I=1,6
      DO 19   J=1,6
   19 S(I,J)=S(I,J)*COMM
      RETURN
      END
```

```
      SUBROUTINE GEOSTI
C   ***************************************************************
C     FORM GEOMETRIC STIFFNESS AND ADD TO ELASTIC.
C     EFFECT OF INITIAL MERIDIONAL STRESS IS CONSIDERED.
C*****************************************************************************
      COMMON NJ,NE,NEQ,MBAND,NLING,NCYC,XNL(20),B(30 ),SK(300,20)
      COMMON/JUNK/HED(13),X(100),Y(100),KODE(100),JI(100),JJ(100),
     1MATERL(100),E(10),PS(10),PRES(100),T(100),RMN(100),RMH(100),
     2RMT(100),RTC(100),RTH(100),RTT(100)
      COMMON/EXTRA/DLO(100),SNO(100),DISPL(300)
      COMMON/ADDNL/NLOAD,NCYCLE,IFLAG,MMM,DL,SN,CS,P(6),D(4,4),S(6,6),
     1Z(6,6),SG(6,6),U(6)
      DIMENSION GS(6,6),GTS(6,6),GTZ(6,6)
C
C CALCULATE INITIAL MERIDIONAL STRESS
C
      GSP=CS*(P(4)-P(1))/2.+SN*(P(5)-P(2))/2.
C
      DO 210 I = 1,6
      DO 210 J = 1,6
210   SG(I,J) = 0.
      DO 200 I = 1,6
      DO 200 J = 1,6
200   GS(I,J) = 0.0
      GS(2,2) = GSP* 6.0 / (5.0 *DL )
      GS(2,3) = GSP/ 10.0
      GS(2,5) = -GS(2,2)
      GS(2,6) = GS(2,3)
      GS(3,3) = GSP * 2.0 * DL /15.0
      GS(3,5) = -GS(2,3)
      GS(3,6) = -GSP * DL / 30.0
      GS(5,5) = GS(2,2)
      GS(5,6) =-GS(2,3)
      GS(6,6) = GS(3,3)
      GS(3,2) = GS(2,3)
      GS(5,2) = GS(2,5)
      GS(6,2) = GS(2,6)
      GS(5,3) = GS(3,5)
      GS(6,3) = GS(3,6)
      GS(6,5) = GS(5,6)
C
C TRANSFORMATION OF GEOMETRIC STIFFNESS FROM LOCAL COORS TO GLOBAL
C
      DO 270 I = 1,6
      DO 270 J = 1,6
      GTZ(I,J) = 0.
      DO 270 M = 1,6
270   GTZ(I,J) = GTZ(I,J) +  GS(I,M) * Z(M,J)
      DO 280 I = 1,6
      DO 280 J = 1,6
      SG(I,J) = 0.
      DO 280 M = 1,6
280   SG(I,J) = SG(I,J) + Z(M,I) * GTZ(M,J)
C
C ADD GEOMETRIC STIFFNESS TO ELASTIC STIFFNESS
```

```
C
      DO 290 I = 1,6
      DO 290 J = 1,6
290   SG(I,J) = SG(I,J) + S(I,J)
      RETURN
      END
```

```
      SUBROUTINE ELSTRS
C
C*************************************************************************
C   ELEMENT STRESSES AT MIDPOINT OF ELEMENT
C*************************************************************************
      COMMON NJ,NE,NEQ,MBAND,NLING,NCYC,XNL(20),B(30 ),SK(300,20)
      COMMON/JUNK/HED(13),X(100),Y(100),KODE(100),JI(100),JJ(100),
     1MATERL(100),E(10),PS(10),PRES(100),T(100),RMN(100),RMH(100),
     2RMT(100),RTC(100),RTH(100),RTT(100)
      COMMON/EXTRA/DLO(100),SNO(100),DISPL(300)
      COMMON/ADDNL/NLOAD,NCYCLE,IFLAG,MMM,DL,SN,CS,P(6),D(4,4),S(6,6),
     1Z(6,6),SG(6,6),U(6)
      DIMENSION EPS(8),STRESS(8)
      REWIND 1
C
C     DISPLACEMENT VECTOR (R*) USED TO COMPUTE STRESSES.
C
      WRITE(6,487)
      DO 93 MM=1,NE
C
C     READ DISPLS OFF TAPE 1
C
      READ (1)                       (U(I),I=1,6),((D(I,J),I=1,4),J=1,4)
      IN=JI(MM)
      JN=JJ(MM)
      DX=X(JN)-X(IN)
      XI=X(IN)
      DY=Y(JN)-Y(IN)
      DL = SQRT ( DX**2 +DY**2 )
      SN=DX/DL
      CS=DY/DL
      XJ=X(JN)
      IF((XI.EQ.0.).OR.(XJ.EQ.0.)) GO TO 93
      RL1=U(1)*CS+U(2)*SN
      RL2=-SN*U(1)+CS*U(2)
      RL3=-U(3)
      RL4=CS*U(4)+SN*U(5)
      RL5=-SN*U(4)+CS*U(5)
      RL6=-U(6)
C
C     EVALUATE MID-ELEMENT STRAINS.
      R=XI+0.5*DL*SN
      EPS(1)=(1./DL)*(-RL1+RL4)
      EPS(2)=(0.5/R)*(SN*(RL1+RL4)+CS*(RL2+RL5)+(0.25*DL)*CS*(RL3-RL6))
      EPS(3)=(1./DL)*(-RL3+RL6)
      EPS(4)=(SN/R)*((1.5/DL)*(RL2-RL5)+0.25*(RL3+RL6))
C
C     EVALUATE MID-ELEMENT STRESSES.
      DO 98 K=1,4
      STRESS(K)=0.
      DO 98 J=1,4
   98 STRESS(K)=STRESS(K)+D(K,J)*EPS(J)
      WRITE (6,488) MM,(STRESS(K),K=1,4)
  488 FORMAT (1H0,I10,14X,4(8X,F12.5))
   93 CONTINUE
```

```
486 FORMAT (1H0,4X,2I10,4(8X,F12.5)/15X,I10,4(8X,F12.5))
487 FORMAT (1H1,30H                SHELL STRESSES/1H0,104H       ELEME
   1NT       NODE       MERID. STRESS         HOOP STRESS        MERID.
   2MOMENT          HOOP MOMENT))
    RETURN
    END
```

```
      SUBROUTINE RESIST
C
C*************************************************************************
C   COMPUTE ELEMENT RESISTING FORCES IN DISPLACED POSITION
C*************************************************************************
      COMMON NJ,NE,NEQ,MBAND,NLING,NCYC,XNL(20),B(30 ),SK(300,20)
      COMMON/JUNK/HED(13),X(100),Y(100),KODE(100),JI(100),JJ(100),
     1MATERL(100),E(10),PS(10),PRES(100),T(100),RMN(100),RMH(100),
     2RMT(100),RTC(100),RTH(100),RTT(100)
      COMMON/EXTRA/DLO(100),SNO(100),DISPL(300)
      COMMON/ADDNL/NLOAD,NCYCLE,IFLAG,MMM,DL,SN,CS,P(6),D(4,4),S(6,6),
     1Z(6,6),SG(6,6),U(6)
      J=JJ(MMM)
      I=JI(MMM)
C
C   COMPUTE TRUE DEFORMATIONS IN THIS POSITION (DISPL. VECTOR (R*))
C
      U(1)=DISPL(3*I-2)
      U(2)=DISPL(3*I-1)
      U(5)=DISPL(3*J-1)
      IF(CS.EQ.0.) GO TO 4
      U(4)=U(1)+(DL-DLO(MMM))/CS+(U(2)-U(5))*(SN/CS)
      GO TO 6
    4 U(4)=DISPL(3*J-2)
      GO TO 6
    6 ALPHA=(DISPL(3*J-2)*SNO(MMM)-U(4)*SN)/DL
      U(3)=DISPL(3*I)-ALPHA
      U(6)=DISPL(3*J)-ALPHA
      IF (IFLAG.EQ.1)   WRITE (1) (U(I),I=1,6),((D(I,J),I=1,4),J=1,4)
C   COMPUTE (P*) - ELEMENT RESISTING FORCES.
      DO 3 K=1,6
      P(K)=0.
      DO 3 KK=1,6
    3 P(K)=P(K)+S(K,KK)*U(KK)
      RETURN
      END
```

```
      SUBROUTINE SYMSOL (A,B,NN,MM,KKK)
C
C
      DIMENSION A(300,20),B(300)
C
C
      GO TO (1000,2000),KKK
C
C     REDUCE MATRIX
C
 1000 DO 280 N=1,NN
      DO 260 L=2,MM
      C=A(N,L)/A(N,1)
      I = N+L-1
      IF(NN-I) 260,240,240
  240 J=0
      DO 250 K=L,MM
      J=J+1
  250 A(I,J)=A(I,J)-C*A(N,K)
  260 A(N,L)=C
  280 CONTINUE
      GO TO 500
C
C     REDUCE VECTOR
C
 2000 DO 290 N=1,NN
      DO 285 L=2,MM
      I=N+L-1
      IF(NN-I) 290,285,285
  285 B(I)=B(I)-A(N,L)*B(N)
  290 B(N)=B(N)/A(N,1)
C
C     BACK SUBSTITUTION
C
      N=NN
  300 N = N-1
      IF(N) 350,500,350
  350 DO 400 K=2,MM
      L = N+K-1
      IF(NN-L) 400,370,370
  370 B(N) = B(N) - A(N,K) * B(L)
  400 CONTINUE
      GO TO 300
C
  500 RETURN
      END
```