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Use of projectional phase space data to infer a 4D particle distribution

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We consider beams which are described by a 4D transverse distribution $f(x, y, x', y')$, where $x' \equiv p_x/p_z$ and z is the axial coordinate. A two-slit scanner is commonly employed to measure, over a sequence of shots, a 2D projection of such a beam's phase space, *e.g.*, $f(x, x')$. Another scanner might yield $f(y, y')$ or, using crossed slits, $f(x, y)$. A small set of such 2D scans does not uniquely specify $f(x, y, x', y')$. We have developed "tomographic" techniques to synthesize a "reasonable" set of particles in a 4D phase space having 2D densities consistent with the experimental data. These techniques are described in a separate document [A. Friedman, *et. al.*, submitted to Phys. Rev. ST-AB, 2002]. Here we briefly summarize one method and describe progress in validating it, using simulations of the High Current Experiment at Lawrence Berkeley National Laboratory.

I. INTRODUCTION

In Heavy Ion Fusion research, it is often desirable to simulate beam dynamics over some section of an accelerator or beamline, using measured data at the entrance to that section to specify the initial beam particle distribution. Use of an idealized (*e.g.*, semi-Gaussian) distribution with the measured low-order moments often fails to yield the correct behavior (as shown below). We consider beams which are described by a 4D transverse distribution $f(x, y, x', y')$, where $x' \equiv p_x/p_z$ and z is the axial coordinate. A two-slit scanner is commonly employed to measure, over a sequence of shots, a 2D projection of the beam's phase space, *e.g.*, $f(x, x')$. Another scanner might yield $f(y, y')$ or, using crossed slits, $f(x, y)$. The key challenge is that a small set of such 2D scans does not uniquely specify $f(x, y, x', y')$. Thus, despite the underdetermined nature of the problem, we seek to synthesize a "reasonable" $f(x, y, x', y')$ which is consistent with the measured data. We have developed Monte-Carlo techniques to carry out such syntheses. These differ from classical tomographic techniques used to synthesize 2D distributions from multiple 1D views [1–4], and are described in a separate document [5]. Here we briefly summarize one method (Sec. II), describe progress in validating it using simulations of the High Current Experiment (HCX) at Lawrence Berkeley National Laboratory (Sec. III), and offer concluding comments (Sec. IV).

II. METHODS

The first viable 4D synthesis method was developed by one of us (Staples), extended by discussions with J. Stovall [6]. It has been applied to problems associated with injecting a beam into an RFQ accelerator. This method draws random points from a 4-box and accepts each with a likelihood proportional to $f(x, x') \times f(y, y')$, but then it applies "clipping," *i.e.*, removal of points out-side a prescribed sampling 4-volume. A practical volume consists of the intersection of the interiors of a 4-ellipsoid and four 4-cylinders; the semi-axes of these volumes are scaled to the extent of the data along the principal axes, times user-specified multiplicative factors of order unity. Due to the clipping, the synthesized (x, x') and (y, y') densities differ slightly from the measured ones.

More recently, we have developed algorithms which exactly reproduce, in the limit of many particles and fine data grids, the measured (x, x') and (y, y') data, as well as the measured (x, y) data when it is available. We briefly describe one family of methods here, omitting such details as data thresholding, recentering, etc. The two-plane method with which we have had the greatest success begins by assigning target "counts" $N(x, x')$ and $N(y, y')$ of the numbers of particles to be loaded into each "bin" in (x, x') and (y, y'), proportional to the measured f in that bin. The bin counts must be integers which sum to the desired total number of simulation particles, while f is often quantized to a set of discrete values by the diagnostic. Because of this, scaling and rounding of $f(x, x')$ and $f(y, y')$ to obtain the bin counts often fails— a small change to the scaling factor either has no effect or changes the counts in several bins at once, making it impossible to obtain the correct sum. We address this by adding a very small random number to the input f values. The following steps are repeated until the requisite number of particles have been generated and all bin counts have been decremented to zero or a maximum number of passes have been completed: (1) Generate a random point in the 4-box bounded by the extremes of the measured data; (2) Accept the randomly-placed point only if it falls within the sampling region, and also in a pair of bins both having nonzero counts; (3) If the point is accepted, decrement the counts in the corresponding bins by one. The use of bin counts minimizes statistical noise since the correct number of particles (within ± 1) is loaded into each bin. The three-plane method proceeds identically except that it employs a third set of bin counts $N(x, y)$.

III.   APPLICATION TO MODEL OF HCX

To assess various prescriptions for synthesis we employed a self-consistent WARP [7] simulation of HCX. This, rather than early experimental data, was chosen as a first step because we wished to know the true 4D distribution as a benchmark, and such information was not available from the experiment. The reference simulation used a 6D phase space through the injector exit, then a transverse 4D phase space thenceforth. For a depiction of such a simulation, see [8]. Input to the syntheses consisted of projectional phase-space densities (obtained by nearest-grid-point weighting) in the (x, x') and (y, y') planes, and, for the 3-plane synthesis, the (x, y) plane. Projections of the input distribution are shown in Fig. 1.
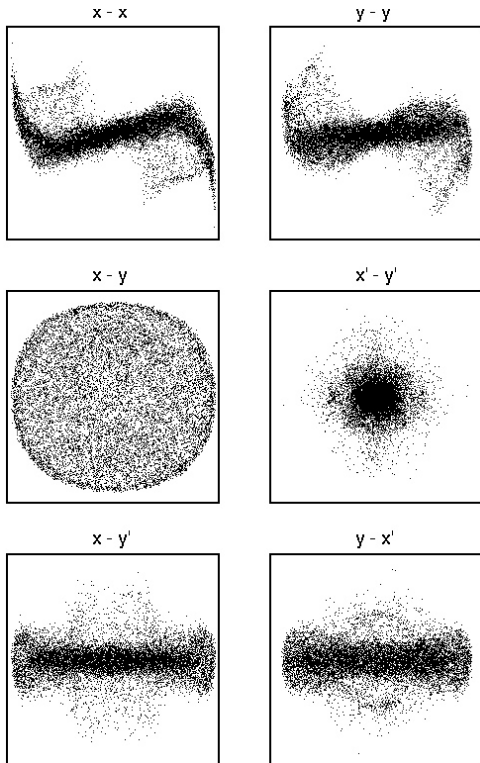

FIG. 1: Projections of sampled particles onto principal planes at exit of injector, for the self-consistent reference simulation.

The results of the syntheses are shown in Fig. 2; here the input planes (x, x') and (y, y') are not shown because they are faithfully reproduced. In the two-plane synthesis, the absence of a "rim" in the (x, y) view is noteworthy; the three-plane synthesis captures this feature, which seems to have an important effect on the downstream dynamics. Note also that neither of the syntheses recreates the structures in the (x, y') and (y, x') planes.

The results of simulations initiated with synthesized particle distributions [9] are compared with the self-consistent WARP run, beginning at the injector exit, in Fig. 3. Even in the "full" HCX with 30 or 40 quadrupoles, no near-steady state is reached. Thus, understanding
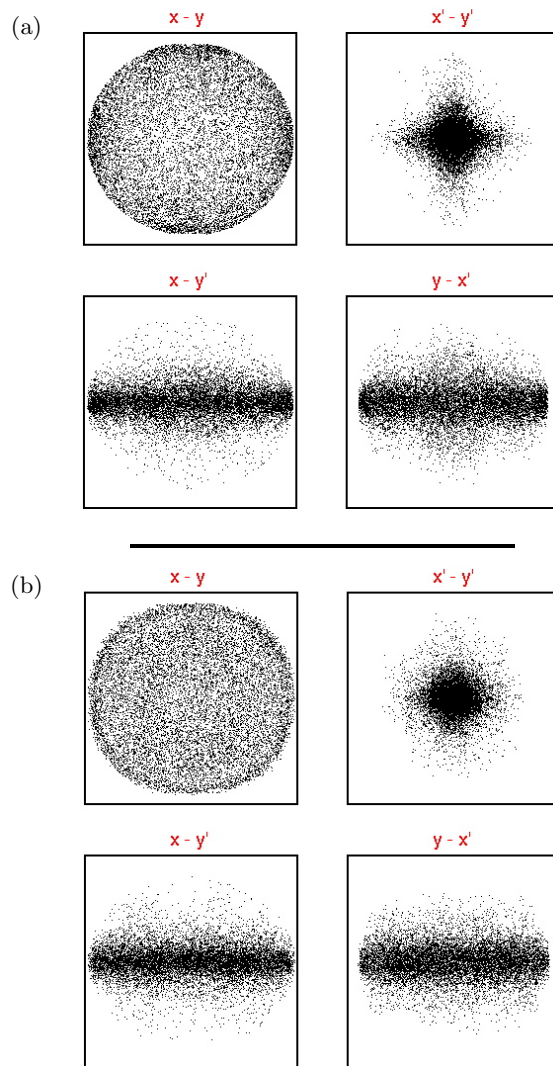

FIG. 2: Projections of sampled particles onto principal planes at exit of injector, for (a) the two-plane reconstruction, and (b) the three-plane reconstruction.

the oscillations of the phase space will be important. The semi-Gaussian utterly fails to capture the dynamics. The "two-plane" synthesized distribution did much better, but good fidelity was not achieved until the spatial density data was incorporated in the "3-plane" synthesis. That spatial information guarantees that the potential energy will be correct. Because the kinetic energy involves the velocity components in quadrature, and all synthesis methods reproduce the x and y velocity distributions well, inclusion of this information guarantees that the total initial transverse energy will be correct.

Finally a synthesis using a coarse grid with just a few cells across the beam was carried out; the (x, x') projection is shown in Fig. 4. This resolution is typical of that used in the experimental system commissioning. Simulations using such a synthesis showed diminished fidelity relative to those using a fine data grid.
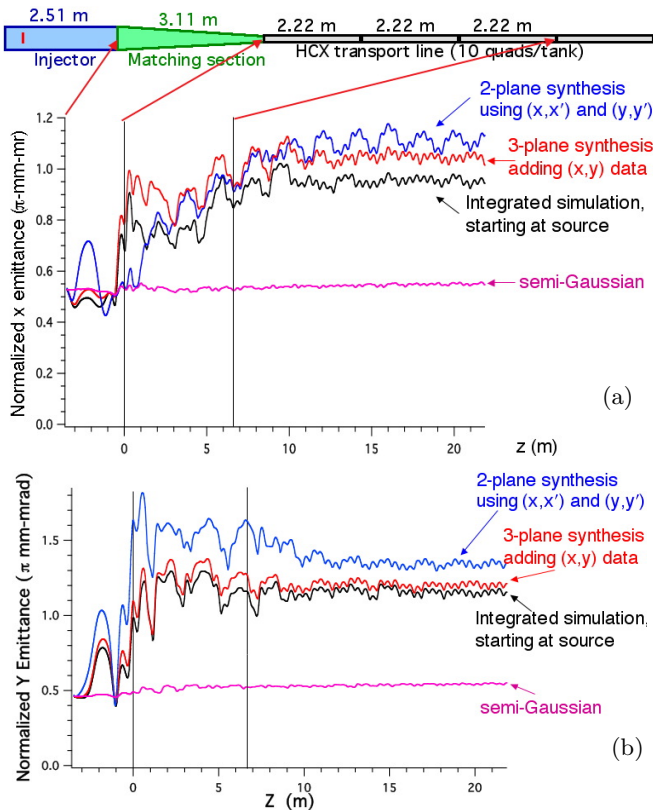
FIG. 3: (Color) WARPxy simulation results: downstream evolution of (a) (x, x') and (b) (y, y') emittances for self-consistent, 2-plane reconstruction, 3-plane reconstruction, and semi-Gaussian beams.

IV. DISCUSSION

The methods described in this paper performed well on the HCX model problem, while an idealized distribution performed poorly. This provides a strong incentive to employ the best available synthesis techniques. The superior performance of the three-plane synthesis motivates the use of crossed-slit scans in HCX experimental practice. These results also indicate that high-resolution scans are needed if key features are to be represented accurately in the synthesized distribution.

It should be possible to improve on the methods. A "parameter-free" method (without a user-specified sampling region) should be sought. Our first attempt at such a method, described in Ref. [5], worked well on a model problem but poorly on the HCX example. It would also be desirable to be able to incorporate any extra information, e.g., localized data at high resolution, into the synthesis. An axisymmetric method specialized to synthesis at the "gun" exit would be valuable. Finally, the application of traditional maximum-entropy methods [2] to 4D syntheses should be explored.
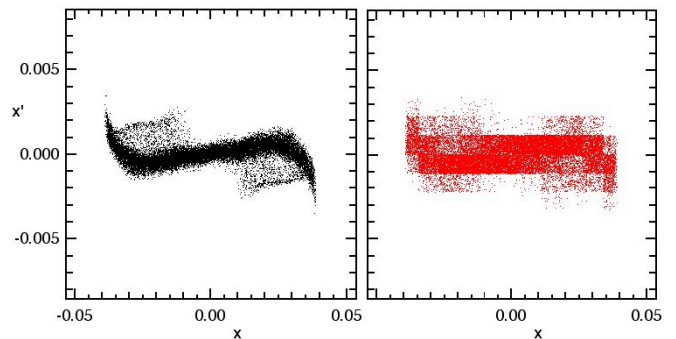

FIG. 4: Input (x, x') projection (left) and synthesis at low resolution (right).

[1] Koehler, H. A., Jacoby, B. A., and Nelson, M., "Time-resolved tomographic images of a relativistic electron beam," Proc. SPIE, ug. 19-24, 1984

[2] Bevensee, R. M., *Maximum Entropy Solutions to Scientific Problems* (Prentice-Hall, NJ, 1993), p. 70.

[3] Hancock, S., Lindroos, M., and Koscielniak, S., "Longitudinal phase space tomography with space charge," Phys. Rev. Special Topics - Accel. and Beams **3**, 124202 (2000);

[4] Hancock, S., Jansson, A., and Lindroos, M., "Tomographic reconstruction of transverse phase space from turn-by-turn profile data," Proc. 1999 Part. Accel. Conf., New York, March 29 - April 2, 1999; CERN/PS99-029(OP).

[5] Friedman, A., Grote, D. P., Celata, C. M., and Staples, J. W., "Synthesizing a four-dimensional beam particle distribution from multiple two-dimensional views," submitted to Phys. Rev. Special Topics - Accel. and Beams, 2002.

[6] Stovall, J., Los Alamos National Laboratory, private communication.

[7] Grote, D. P., Friedman, A., Craig, G. D., Haber, I., and Sharp, W. M., Nucl. Instr. and Meth. in Phys. Res. **A. 464**, 563 (2001).

[8] Davidson, R. C., *et. al.*, these *Proceedings*.

[9] Celata, C. M., *et. al.*, these *Proceedings*.