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Gravity in dynamically generated dimensions

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A theory of gravity in $d+1$ dimensions is dynamically generated from a theory in d dimensions. As an application we show how N dynamically coupled gravity theories can reduce the effective Planck mass.

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I. INTRODUCTION

The idea that a gauge theory in $(1,d)$ dimensions appears as the low energy limit of a $(1,d-1)$ theory with many fields has been recently put forward [1,2]. One starts with $(1,d-1)$ gauge fields $A_\mu(x,i)$, $\mu=0,1,\ldots,d-1$ with the range of the discrete index i being either infinite of finite and periodic $[A_\mu(x,1)=A_\mu(x,N)]$. Interactions are chosen as to ensure that a field $A_{i,i+1}(x)$ is generated dynamically and whose interactions, in the low energy limit, mimic A_d in a $(1,d)$ dimensional space with i turning into the discrete extra dimension. A feature of this approach is that the $(1,d-1)$ theory has the desired properties of renormalizability and asymptotic freedom.

In this work, we extend this approach to gravity, namely we generate a $(1,d)$ dimensional gravity from a theory in $(1,d-1)$ dimensions. Gravity will be described in a moving frame formalism as an $SO(1,d-1)$ gauge theory on a $(1,d-1)$ manifold. Unlike the situations discussed in Refs. [1,2], where one started with a gauge theory in some dimension and generated the same gauge theory in a space with one higher dimension, in order to generate gravity in the higher dimension we have to start with an extended gravity in the lower dimensional space. Namely, we start with an $SO(1,d)$ gauge theory on a $(1,d-1)$ manifold and dynamically generate the $SO(1,d)$ theory on a $(1,d)$ manifold [3]. Of course, the lower dimensional theory includes gravity in the $SO(1,d-1)$ subgroup of $SO(1,d)$.

In this case we cannot appeal to renormalizability or to asymptotic freedom to justify this approach as the lower dimensional gravity or extended gravity is unlikely to be renormalizable or asymptotically free. What our construction ensures is that the dynamically generated higher dimensional theory is no more singular than the lower dimensional one and that coordinate invariance and local $SO(1,d)$ invariance are maintained at each step. Details, as well as a discussion of $SO(1,d)$ invariant interactions in $(1,d-1)$ dimensional spaces, are presented in Sec. II. In Sec. III we apply this approach to a scenario where in four dimensions the existence of N dynamically coupled gravity theories decreases the effective gravitational coupling by a factor of N.

II. DYNAMICAL GENERATION OF GRAVITY

A. Gravity in $d+1$ dimensions

We shall first discuss gravity theory on a $(1,d)$ dimensional manifold with coordinates $x_\mu;\mu=0,1,\ldots,d$, which we wish to obtain through the dynamical generation of a dimension in some theory on a $(1,d-1)$ dimensional manifold. Our goal is the $(1,d)$ Einstein-Hilbert Lagrangian, which we express in the moving frame or d-ad formalism:

$$L_{\rm EH}=M_{(d+1)}^{d-1}\epsilon^{\mu_1\mu_2\cdots\mu_{d+1}}\epsilon_{a_1a_2\cdots a_{d+1}}[R^{a_1a_2}_{\mu_1\mu_2}(x)+M^2_{(d+1)}\lambda_{(d+1)}e^{a_1}_{\mu_1}(x)e^{a_2}_{\mu_2}(x)]\times e^{a_3}_{\mu_3}(x)e^{a_4}_{\mu_4}(x)\cdots e^{a_{d+1}}_{\mu_{d+1}}(x); \tag{1}$$

in the above the $e^a_\mu(x)$'s, with flat space, Minkowski, indexes $a=0,1,\ldots,d$, are the $(d+1)$-ad's and the $\omega^{ab}_\mu(x)$'s are the spin connections. M_d is the d dimensional Planck mass, $M_d^2\lambda_d$ is a cosmological constant, and the curvature tensor $R^{a_1a_2}_{\mu_1\mu_2}(x)$ is related to the spin connections by

$$R^{ab}_{\mu\nu}(x)=\partial_\mu\omega^{ab}_\nu(x)-\omega^a_{\mu,n}(x)\omega^{nb}_\nu(x)-(\mu\leftrightarrow\nu). \tag{2}$$

In order to see what theory in a $(1,d-1)$ space we should start with, we foliate the $(1,d)$ dimensional manifold into $(1,d-1)$ dimensional ones. Specifically we single out the last coordinate x_d. Coordinates in the $(1,d)$ dimensional space are written as (x_μ,x_d), where now $\mu=0,1,\ldots,d-1$. We leave the first d d-ad's as they were but separate out the "shift" vector [4]

$$e^a_d(x)=N^a(x); \tag{3}$$

the Minkowski index a still ranges over $(d+1)$ values. In terms of this shift vector, Eq. (1) becomes [5]

$$L_{\rm EH}=L_A+L_B, \tag{4}$$

with

$$L_A=M_{(d+1)}^{d-1}\epsilon^{\mu_1\mu_2\cdots\mu_d}\epsilon_{a_1a_2\cdots a_{d+1}}[(d-1)R^{a_1a_2}_{\mu_1\mu_2}(x)+(d+1)M^2_{(d+1)}\lambda_{(d+1)}e^{a_1}_{\mu_1}(x)e^{a_3}_{\mu_3}(x)]\times e^{a_3}_{\mu_3}(x)e^{a_4}_{\mu_4}(x)\cdots e^{a_d}_{\mu_d}(x)N^{a_{d+1}}(x), \tag{5}$$

$$L_B=2M_{(d+1)}^{d-1}\epsilon^{\mu_1\mu_2\cdots\mu_d}\epsilon_{a_1a_2\cdots a_{d+1}}R^{a_1a_2}_{d\mu_1}(x)\times e^{a_3}_{\mu_2}(x)e^{a_4}_{\mu_3}(x)\cdots e^{a_{d+1}}_{\mu_d}(x). \tag{6}$$

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We note that L_A does not contain any derivatives in the x_d direction nor any terms involving $\omega^{ab}_{d\mu}(x)$ while L_B does but, in turn, does not involve the shift vectors $N^a(x)$. L_A will determine the $(1,d-1)$ theory we start out with, and our goal will be to generate dynamically L_B.

We may also note that L_A describes the $(1,d-1)$ theory obtained from a $(1,d)$ gravity by requiring that all fields be independent of the coordinate x_d and choosing the gauge $\omega^{ab}_d=0$. This is the reason we have to start with this Lagrangian in the lower dimension, rather than with pure $(1,d-1)$ dimensional gravity. For gauge theories such a dimensional reduction results in just a lower dimensional gauge theory with a group structure identical to the higher dimensional one, permitting the procedure of Refs. [1,2].

B. $SO(1,d)$ gauge theory in d dimension

The Lagrangian L_A in Eq. (5) describes an $SO(1,d)$ gauge theory in on a $(1,d-1)$ manifold; as $SO(1,d-1)$ is a subgroup of $SO(1,d)$, this model includes gravity, an $SO(1,d-1)$ gauge theory on a $(1,d-1)$ manifold. We shall now obtain some of the properties of this extended model; these properties are needed by other fields and their interactions. Following Eq. (5) we rewrite the Lagrangian as

$$L_{SO(1,d)}=M_d^{d-2}\epsilon^{\mu_1\mu_2\cdots\mu_d}\epsilon_{a_1a_2\cdots a_{d+1}}[R^{a_1a_2}_{\mu_1\mu_2}(x)+M_d^2\lambda_d e^{a_1}_{\mu_1}(x)e^{a_2}_{\mu_2}(x)]\times e^{a_3}_{\mu_3}(x)e^{a_4}_{\mu_4}(x)\cdots e^{a_d}_{\mu_d}(x)N^{a_{d+1}}(x). \tag{7}$$

As $e^a_\mu(x)$ is not a square matrix we may not define an $e^\mu_a(x)$ as its inverse. We can, however, introduce a $d\times d$ metric tensor

$$g_{\mu\nu}(x)=e_{\mu,a}(x)e^a_\nu(x) \tag{8}$$

as well as its inverse $g^{\mu\nu}(x)$, thus allowing us to raise and lower the curved space coordinate indexes. Using the $SO(1,d)$ Clifford algebra

$$\{\gamma_a,\gamma_b\}=2\,\eta_{ab} \tag{9}$$

and its associated spin matrices $\Sigma_{ab}=[\gamma_a,\gamma_b]/2i$ the Dirac Lagrangian for an $SO(1,d)$ spinor field $\psi(x)$ is

$$L_D=\epsilon^{\mu_1\mu_2\cdots\mu_d}\epsilon_{a_1a_2\cdots a_{d+1}}\overline{\psi}(x)\gamma^{a_1}D_{\mu_1}\psi(x)\times e^{a_2}_{\mu_2}(x)e^{a_3}_{\mu_3}(x)\cdots e^{a_d}_{\mu_d}(x)N^{a_{d+1}}(x), \tag{10}$$

with the covariant derivative

$$D_\mu\psi(x)=\left(\partial_\mu-\frac{1}{2}\Sigma_{ab}\omega^{ab}_\mu\right)\psi(x). \tag{11}$$

C. Dynamical generation of gravity in $d+1$ dimensions

In order to generate an extra dimension we study many mutually noninteracting $SO(1,d)$ theories described by d-ad's $e^a_\mu(x,i)$, spin connections $\omega^{ab}_\mu(x,i)$ and fields $N^a(x,i)$; at this point the range of the i's need not be specified. The Lagrangian for this collection of theories is

$$L_0=M_d^{d-2}\sum_i\epsilon^{\mu_1\mu_2\cdots\mu_d}\epsilon_{a_1a_2\cdots a_{d+1}}[R^{a_1a_2}_{\mu_1\mu_2}(x,i)+M_d^2\lambda_d e^{a_1}_{\mu_1}(x,i)e^{a_2}_{\mu_2}(x,i)]\times e^{a_3}_{\mu_3}(x,i)e^{a_4}_{\mu_4}(x,i)\cdots e^{a_d}_{\mu_d}(x,i)N^{a_{d+1}}(x,i). \tag{12}$$

It is invariant under the product group $\cdots\times SO^i(1,d)\times SO^{(i+1)}(1,d)\times\cdots$.

In order to couple theories at different i's we have to introduce several more fields. For each pair $(i,i+1)$ there is a non-Abelian gauge field $A^{i,i+1}_\mu(x)$; the only requirement on the group G under which these fields transforms and the strength of the gauge coupling is that certain fermion condensates, to be discussed below, are induced. In addition, for each i, we have two Weyl fermion fields. One, $\psi^i(x)$, couples to $A^{i,i+1}_\mu(x)$ as a fundamental under G while the other one, $\chi^i(x)$, couples as an antifundamental under to $A^{i-1,i}_\mu(x)$. We assume that the $SO^i(1,d)\times SO^{(i+1)}(1,d)$ symmetry is broken by a condensate

$$\langle\psi^i(x)\chi^{i+1}(x)\rangle\sim f_G^{d-1}\exp\left[\frac{i}{2}\Sigma_{ab}\mathcal{O}^{ab}_{i,i+1}(x)\right]; \tag{13}$$

$\mathcal{O}^{ab}_{i,i+1}(x)$ is an $SO(1,d)$ Lorentz transformation matrix and f_G parametrizes the strength of the condensate. The low energy effective theory for the fields $\mathcal{O}^{ab}_{i,i+1}(x)$ is governed by the Lagrangian

$$L_1=2f_G^{d-1}\sum_i\epsilon^{\mu_1\mu_2\cdots\mu_d}\epsilon_{a_1a_2\cdots a_{d+1}}\mathcal{O}^{a_1n}_{i,i+1}(x)\times D_{\mu_1}\mathcal{O}^{a_2}_{i,i+1;n}(x)e^{a_3}_{\mu_2}(x,i)e^{a_4}_{\mu_3}(x,i)\cdots e^{a_{d+1}}_{\mu_d}(x,i); \tag{14}$$

the covariant derivative is

$$D_\mu\mathcal{O}^{ab}_{i,i+1}(x)=[\partial_\mu\mathcal{O}^{ab}_{i,i+1}(x)+\omega^a_{\mu;n}(x,i)\mathcal{O}^{nb}_{i,i+1}(x)-\mathcal{O}^{an}_{i,i+1}(x)\omega^b_{\mu;n}(x;i+1)]. \tag{15}$$

In the continuum limit we may expand $\mathcal{O}^{ab}_{i,i+1}(x)$

$$\mathcal{O}^{ab}_{i,i+1}(x)=\eta^{ab}+a\,\omega^{ab}_{i,i+1}(x)+\cdots, \tag{16}$$

where a is the lattice separation. With the following identifications we recover the discrete version of L_B [Eq. (6)]:

$$a = M_d^{d-2}/f_G^{d-1},$$

$$f_G = (d-1)M_{d+1},$$

$$M_d^2\lambda_d = (d+1)M_{d+1}\lambda_{d+1}. \tag{17}$$

III. EXTRA DISCRETE DIMENSIONS

Recently, extensive research has been carried out on the possibility that extra compact but large dimensions may account for the apparently large value of the Planck mass [6,7]. The present work shows how to formulate a discrete version of such schemes. In the continuum case phenomenology demands that we have more than one extra dimension. For simplicity we shall discuss only one extra ''large'' discrete dimension [8]. We envisage a four dimensional manifold with many $SO(1,4)$ theories. The Lagrangian is the sum of Eqs. (12) and (14) with $d=3$, $i=0,1,\ldots,N-1$, and periodic conditions on the discrete index i, $e_\mu^a(x,N) = e_\mu^a(x,0), \ldots$. All other nongravity fields appear only once and couple only to $e_\mu^a(x,0)$; in the continuum language this would indicate that these extra fields do not propagate into the extra dimension. The dynamical mechanism discussed in Sec. II generates a fifth dimension of circumference Na. Using techniques similar to those discussed in Ref. [1] we find that the potential for two masses coupled only to the $i=0$ gravity is

$$V(r) = \frac{m_1 m_2}{N M_4^2}\frac{1}{r}\sum_{m=0}^{N-1}\exp\left(-2\frac{r}{a}\sin\frac{m\pi}{N}\right). \tag{18}$$

For $r \gg Na$ we recover the $1/r$ potential with an effective Planck mass $M_P^2 = N M_4^2$.

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