Angle, Distance, Shape, and their Relationship to Projective Relations

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Abstract

The semantics of spatial relations have been intensively studied in linguistics, psychology, and cognitive neuroscience. Angle, distance, and shape are widely considered to be the key factors when establishing spatial relations. In this work an empirical study shows that previous theories overemphasize variation and we clarify the interdependencies between angle, distance, and shape with respect to the acceptability of projective relations. It turned out that the angular deviation plays the key role for relations of this class. The degree of deviation was dependent upon the extension of the reference object perpendicular to the canonical direction of the relation. There was no major effect due to the distance. However, distance interacted with the angular deviation if the located object was very close to the reference object. The experimental results can now be used as a theoretical framework for validating existing computational models of projective relations for their cognitive plausibility.

Introduction

Many research disciplines are concerned with problems related to the domain of *space*. One major point of interest are spatial relationships. In the last couple of decades the semantics of spatial relations have been intensively studied in many areas of cognitive science. The fundamentals can be found in linguistic and psycholinguistic literature (e.g., Clark, 1973; Herskovits, 1986; Lakoff, 1987; Miller & Johnson-Laird, 1976; Talmy, 1983) and in cognitive neuroscience (e.g., Kosslyn, 1980, 1994).

Establishing a spatial relation requires a located object (LO), one or two reference objects (RO), and a certain frame of reference which determines the use of the relation depending on the prevailing context: intrinsic, extrinsic, or deictic (cf. Garnham, 1989; Levelt, 1984; Rock, 1973; Retz-Schmidt, 1988). In a deictic or viewer-centered frame, objects are represented in a retinocentric, head-centric, or body-centric coordinate system based on the viewer's perspective of the world (Carlson-Radvansky & Irwin, 1993). In an intrinsic or object-centered frame, objects are coded with respect to their intrinsic axes. In an extrinsic or environment-centered frame, objects are represented with respect to salient features of the environment. In order to talk about space, horizontal and vertical coordinate axes must be oriented with respect to one of these reference frames so that linguistic spatial terms such as *right*[1] or *above* can be assigned (Miller & Johnson-Laird, 1976).

[1] In the following text only the English expressions for the German prepositions are used. Slight differences between the German and the English may appear.

Spatial relations are mainly divided into two classes: topological (*at*, *near*, etc.) and projective, or directional, (*in front of*, *to the right of*, etc.) relations. Where it is necessary to localize objects more precisely, some languages,such as German, usually use more than one spatial relation. Normally, no more than two relations are combined, e.g. *the LO is in front of and to the right of the RO*. The use of such combinations is very common in German, because they involve direct combinations of the simple terms. For instance, *rechts vor* means the same as *to the right and in front of* in English. Thus, to get a combined localization expression using two prepositions, one has only to combine the elementary expressions appropriately (cf. Gapp, 1994b).

Following Landau and Jackendoff (1993), people do not take into consideration every detail of the objects involved when applying spatial relations. They propose that these kinds of relations depend mainly on the boundedness, surface, or volumetric nature of an object, and on its axial structure (see also Herskovits, 1986; Talmy, 1983). Hence, the process of establishing spatial relations considers only the essential shape properties of objects.

Spatial relations are fuzzy (cf. Freeman, 1975), i.e. the region where a relation is applicable cannot be defined by sharp boundaries (see also (Herskovits, 1985, 1986)). Attempts by speakers to reduce this vagueness can be seen in the use of linguistic hedges like *exactly in front of* or *just behind*, and has therefore to be accounted for when defining a spatial relation's semantics (cf. Kochen, 1974; Kay, 1979, 1983; Lakoff, 1972, 1987).

We restrict our investigations in this work to how people evaluate the applicability of projective relations and their compositions. In existing theoretical and practical approaches three essential factors were considered in establishing projective relations: the angular deviation of the located object from the canonical direction implied by the relation, the distance between the located object and the reference object, and the reference object's shape (cf. Gapp (1994a) for more details).

Hypotheses

We propose that the cognitive system accounts only for the angular deviation between the located object and the relation's direction, and *not* for the distance, when establishing projective relations. However the proximity of a located object to a certain referent plays an important role in object localization tasks, e.g., answering a question like "*Where is object A?*" (cf. Gapp, 1994c). This corresponds to the findings of Logan and Sadler (in press). They propose that people decide whether a relation applies by fitting a *spatial template* to the objects that represents *regions of acceptability* for the relation in question. Logan and Sadler define three main regions of acceptability: one reflecting good examples, one reflecting less-than-good-but-nevertheless-acceptable examples, and one reflecting unacceptable examples (Figure 1a).

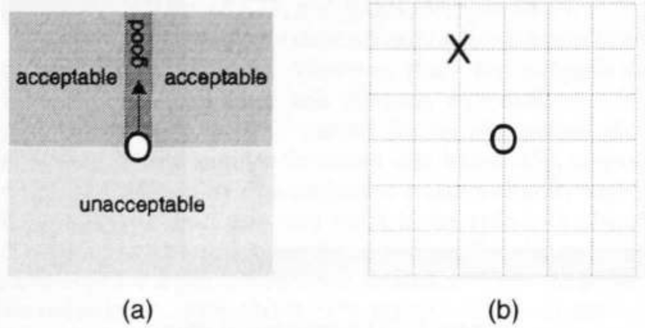

Figure 1: Spatial template layout for *above* (a) and square grid setup (b).

Good and acceptable regions are not distinct, they blend into one another gradually. However Logan and Sadler do not specify how this blending is achieved. This may be due to the nature of their empirical data. Subjects were presented with a spatial relation[2] and a diagram containing an O (the RO) in the center of an invisible 7×7 grid, and an X (the LO) in one of the surrounding positions (Figure 1b). The task was to evaluate the "goodness of fit" of the relation to the object configuration presented. However, using this square grid layout makes it difficult to investigate angular dependency. Alternatively a radial ordered test setup for the located object would be better at measuring the proposed angular effect.

We propose furthermore that the extension of the RO in each dimension influences the scale of the angular deviation and consequently a relation's degree of applicability (*DA*). In Figure 2, for example, even though object L is in the same absolute position compared to R for all configurations (a,b,c), the applicability of the relationship *<right L R>* increases from (a) to (c). This effect becomes increasingly relevant, the more that the extensions of two ROs differ.

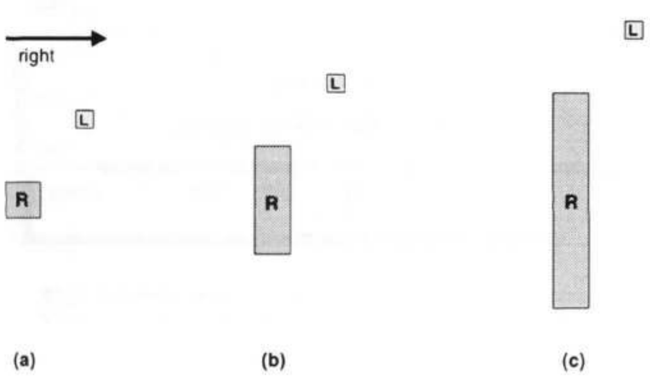

Figure 2: RO's extension influences the applicability of a projective relation.

[2]The relations used were: *above, below, left of, right of, over, under, next to, away from, near to, and far from.*

Experiments

We restrict our account in this paper to the subset of projective relations and their compositions. Experiments were conducted to clarify the following open questions:

1. How does the angular deviation influence the applicability of a projective relation?
2. How is the distance between the located object and the reference object related to a relation's applicability?
3. How does the reference object's shape influence the applicability?
4. Are there distinctions between the applicability regions of *in front of/behind*, *right/left*, and *above/below*?
5. How is the region of applicability for a composition of two projective relations structured? Is there a weighting towards one relation?

Two experiments were designed using different reference frames: a horizontal 2D frame (bird's-eye view) and a vertical 2D frame. Subjects were presented a spatial relation (preposition) and two objects on a computer screen. The task was then to evaluate how well the preposition described the relationship between the located object and the reference object.

Method

Subjects. The subjects were 20 graduate and undergraduate students from 10 different academic areas (7 female, 13 male, aged from 20 to 33) who were each paid DM 13.- for a 1 hr session.

Apparatus and stimuli. The stimuli were displayed on a Hewlett Packard HP98754A 19" color monitor controlled by a HP 9000/720 workstation.

In the experiment with the horizontal reference frame we used the relations *in front of*, *behind*, *right*, *left*, and their compositions, and in the experiment with the vertical frame of reference the relations *above* and *below* were used instead of *in front of* and *behind*.

A design with four independent factors — RO, relation, angle, and distance — varied within subjects was carried out. To investigate the shape's influence five different reference objects were used: Three squares (of 10×10, 30×30, and 50×50 pixels) and two rectangles (of 30×50 and 50×30 pixels). The LOs were represented by a 10×10 pixels square. The color of the LO (red) and the RO (green) made them distinguishable. The LOs occupied fixed positions arranged radially around the RO (Figure 3). A scene consisted of a relation, a located object and a reference object. The initial scene definitions were made in the first quadrant (Figure 4) using the relations *right*, *behind* (*above*), and *right and behind* (*right and above*). All of these relations were valid to a certain degree. To avoid sequence effects in the experiment, the single scenes were randomly mapped to one of the four quadrants, i.e. the coordinates of the LO and the relation were constantly transformed. But relations changed only in their dimension, i.e. *right* to *left*, *in front of* to *behind*, *above* to *below*, and vice versa.[3] The result was a grid of 64 different test positions for the LO, with four different angles ($0^o, 22.5^o, 45^o, 67.5^o$)

[3]This is valid because of the symmetry of the relations in one dimension. In the following text only the first relation will be used as a simplification.

and relative distances (130, 240, 350, and 460 pixels) in each quadrant (Figure 3). The grid layout was slightly different for each of the five ROs because an additional offset to the distance had to be added for the larger ROs so as to keep the distance between the LO and the RO constant.

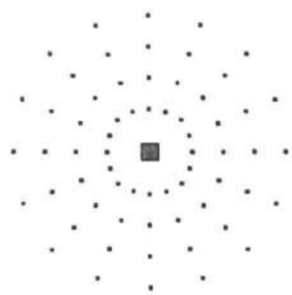

Figure 3: Radial grid layout.

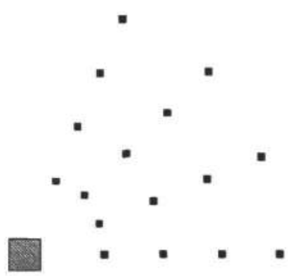

Figure 4: Definition grid.

The reference frame was indicated by a description of the four main directions at the borders of the display (cf. (1) in Figure 5). It was kept fixed throughout the whole experiment. Depending upon the chosen quadrant for the located object, the reference object was placed at fixed positions in the corner of the diagonally opposite quadrant (cf. (2) in Figure 5).

Procedure. There were 244 experimental trials (204 horiz. and 40 vert.). The experiments started with the horizontal and ended with the vertical experiment. The trials were randomly ordered for each subject. The program paused every 70 trials to allow the subjects a brief rest. Separate instructions for each experiment were displayed on the screen. Subjects were told they would see two objects, a red object and a green reference object. Additionally a spatial preposition (or composition) was given (cf. (3) in Figure 5). The task was to evaluate with a sliding scale (cf. (4) in Figure 5), how well the preposition describes the relationship between the two objects, taking the given reference frame into account. The vertical sliding scale was used to indicate no applicability at its lower end and full applicability at the upper end. The scale was set to 0 at the beginning of each trial. Three lines indicated the 0.25, 0.5 and 0.75 positions. There was also an additional digital display with the degrees of applicability ranging from 0.00 to 1.00 (cf. (5) in Figure 5). In the introduction to each experiment all spatial configurations were displayed in a quick run through and, in addition, the subjects were given ten test trials to familiarize themselves with the environment.

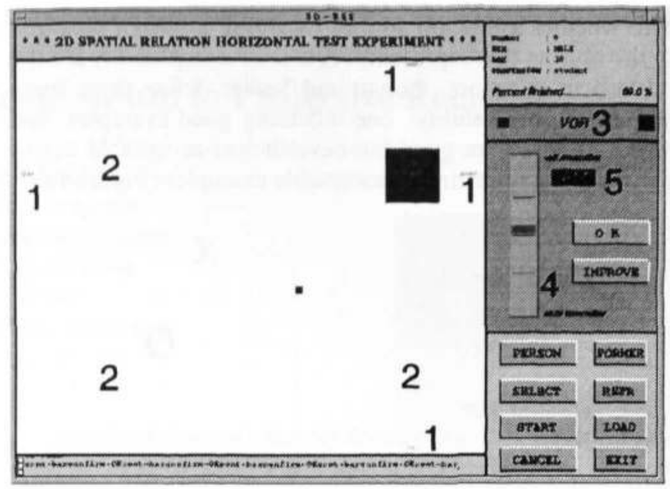

Figure 5: 2D experimental layout.

Results

Angle and Distance. Mean *DAs* across all the subjects were computed for each combination of relation, angle, distance and reference object. The standard error of the mean was 0.17. The means of the *DAs* for the middle sized square reference object across all angles/distances are plotted in Figures 6 and 7. The *DA* decreased linearly with the angular deviation for all tested projective relations. However the distance had no significant effect on the evaluation. A projective relation was fully applicable ($DA_{proj} = 1.0$), if there was no angular deviation. The degree of applicability got closer to 0.0 as the angle approached 90^o.

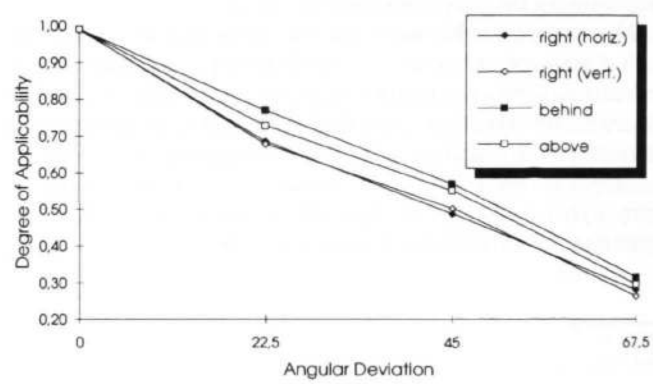

Figure 6: Mean *DAs* (angles).

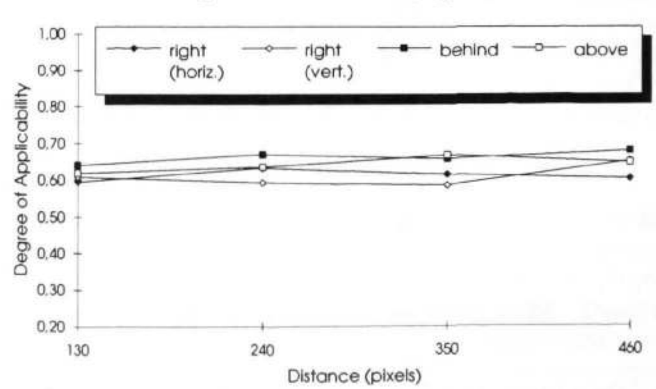

Figure 7: Mean *DAs* (distances).

These conclusions were supported in a 2 (relations: *right*, *behind*) × 4 (angles) × 4 (distances) multivariate analysis of variance (MANOVA), with repeated measurements of the mean *DAs*. The predominant effect of the angle was highly significant, $F(3,608) = 521.82, p < 0.001, MS_e = 13.83$, in the horizontal experiment, as well as in the vertical experiment, $F(3,608) = 487.15, p < 0.001, MS_e = 14.27$.

There was *no* significant dependence on the distance in either experiment ($F < 1$). However, there was a significant interaction between angle and distance, $F(9,608) = 1.97$, $p < 0.04$, $MS_e = 0.5$, caused by an unexpected phenomenon. If the angular deviation was below 45° subjects rated the applicability of a projective relation slightly higher if the located object was very close to the reference object, than if the located object was further away. The closer the located object was positioned to the reference object the greater the reduction in applicability. The reverse effect was noticed above 45° (cf. Figure 8). The mean *DAs* of the angular deviation 22.5° and 67.5° for the relations *right* and *behind* across the three square reference objects are plotted in Figure 9. Using only these two angles in the MANOVA yielded a highly significant interaction, $F(3,608) = 9.5, p < 0.001$, $MS_e = 0.33$, between angle and distance.

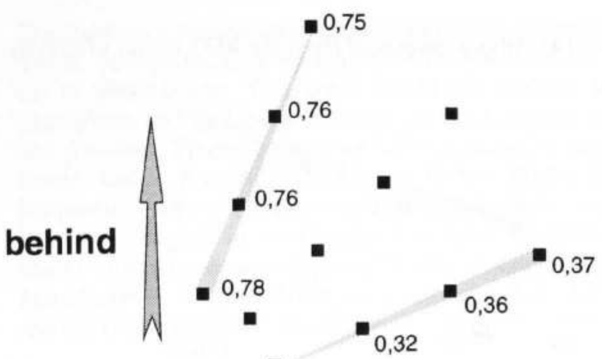

Figure 8: Means at 22.5° and 67.5°.

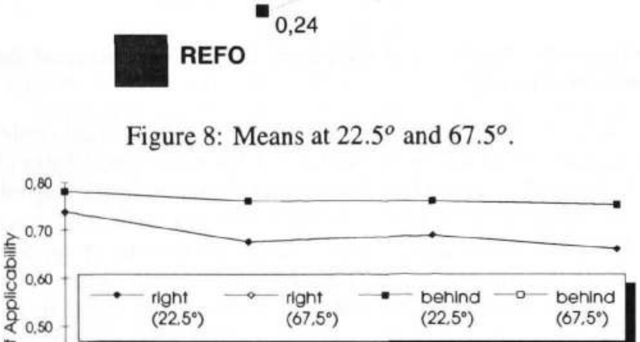

Figure 9: Slight distance effect.

Extension of the reference object. The mean *DAs* of *right* and *behind* across subjects and the three differently shaped reference objects (middle sized square, vertical and horizontal aligned rectangles) are plotted in Figures 10 and 11. Although there was only a very slight difference between the reference objects' extensions, a difference in the ratings consistently appeared, depending upon the extensions of the reference objects. As shown in Figure 2, the extension of the reference object perpendicular to the projective relation's direction, significantly influenced the degree of applicability. The dependency from the reference object was confirmed by a MANOVA using the three differently shaped reference objects. The result was a highly significant interaction between relation and reference object, $F(1,912) = 13.48, p < 0.001$, $MS_e = 0.45$.

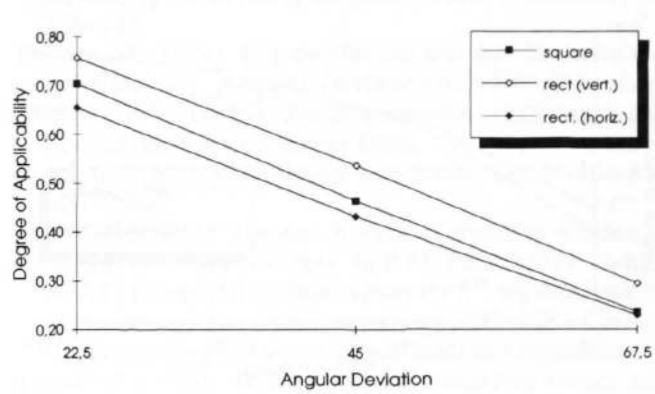

Figure 10: Extension effect: *right*.

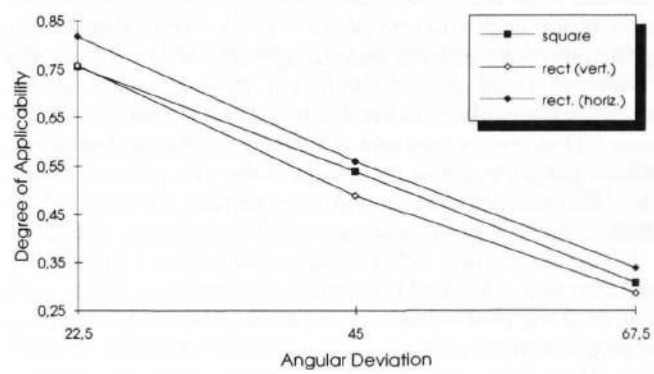

Figure 11: Extension effect: *behind*.

Regions of applicability. There were slight differences between the angular dependence of the three main directions *in front of/behind*, *right/left*, and *above/below*. As plotted in Figure 6, the *in front of/behind* direction was rated highest, followed by the above/below direction.

Compositions. Mean *DAs* of the angular deviation of *right and behind* and *right and above* (RO: middle square) across subjects were computed and plotted in Figure 12. There was a slightly lower weighting for *right* in both compositions. However, the difference between *right* and *above* (0.5) was less than the difference between *right* and *behind* (0.9). There was also a small distance effect (angle: ±22.5°) with the same characteristics measured for elementary projective relations at an angular deviation of 67.5° (cf. Figure 13).

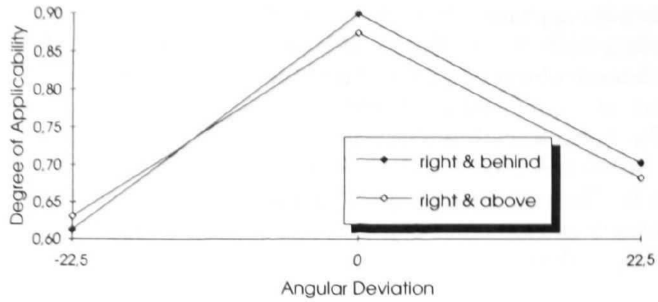

Figure 12: Angular effect.

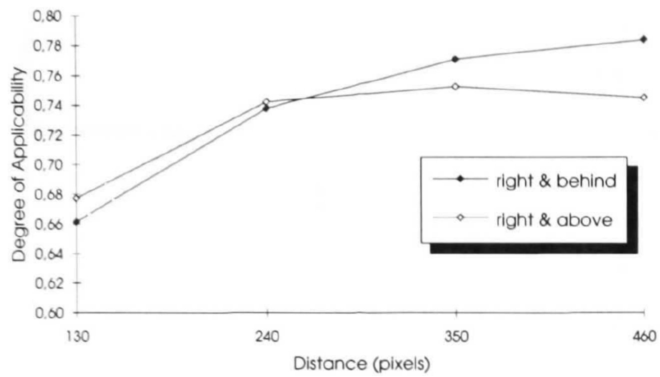

Figure 13: Slight distance effect at $\pm 22.5^o$ (compositions).

Discussion

The data from the experiments showed clearly the characteristics of the evaluation process for projective relations. The applicability of a relation was strongly affected by the angular deviation. There was no significant dependence on the distance. This correlates to the findings of Logan and Sadler (in press). However, we are now able to give a reliable description of how people evaluate the acceptability of a projective relation. This was possible through an experimental design which allowed the independent examination of angle and distance.

In the evaluation of composite projective relations the *right/left* axis was rated lower than the other two. This corresponds to the phenomenon that the subjects rated the *in front of/behind* and the *above/below* axes higher than the *right/left* axis. This slightly higher rating might be due to the fact that the in front of/behind and the above/below axes are easier to perceive (cf. Bryant, Tversky, & Franklin, 1992).

The dependence of the *DA* upon the extension of the reference object was obvious even though the rectangular reference objects were only two-thirds longer/higher than the related square reference object. The larger the extension of the reference object perpendicular to the canonical direction of the relation, the larger the relation's region of applicability in this perpendicular direction. This validates our proposed hypothesis in Figure 2c that the located object L is considered to be more to the right of the reference object R than in Figures 2a and 2b. An exact prediction of how the reference object's extension scales the angular deviation, e.g. linear or logarithmic, requires further investigation with additional, differently shaped reference objects.

The slight distance effect for positions of the located object very close to the reference object at angles below 45^o also seems to be dependent upon the extension of the reference object: the larger the reference object the greater the effect. The effect should therefore disappear if one presumes a point-like reference object. This gives rise to the following explanation of the effect: the angles used for the definition of the radial grid setup were measured from the reference object's center of gravity, despite the reference object's extension (cf. Figure 14). Now consider Figure 15. If one uses the nearest point of the reference object from the located object (the corner c in the example) for measuring the angle, then the angles differ in the same way as the degrees of applicability did in the experiments. That is, $\alpha' < \alpha'' < \alpha''' < \alpha'''' < \alpha$ and $|\alpha' - \alpha''| > |\alpha'' - \alpha'''| > |\alpha''' - \alpha''''|$, if $\alpha \leq 45^o$ and vice versa.

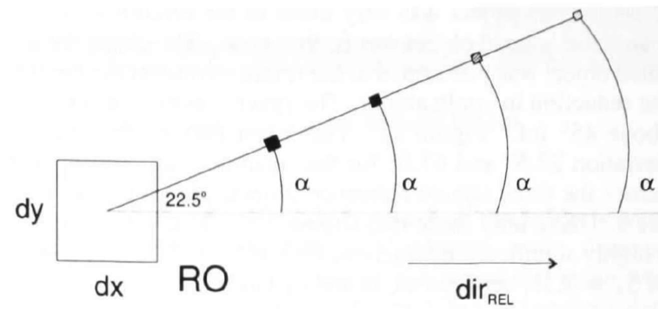

Figure 14: Angles measured from the RO's center of gravity.

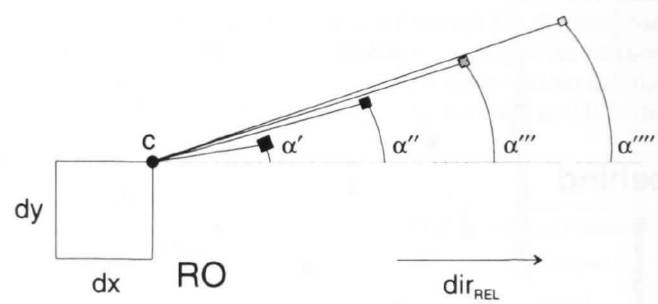

Figure 15: Angles measured from the nearest corner of the reference object.

This leads to the conclusion that in a computational model a certain offset has to be subtracted from the coordinates of the located object before the angular deviation is considered. The offset again depends on the extension of the reference object and additionally on the frame of reference because in deictic use the extension of the reference object perpendicular to the relation's direction is not necessarily equal to the perpendicular extension in intrinsic use (cf. Gapp, 1994a). In the example the offset is exactly half of the reference object's extension in each dimension $(\frac{dx}{2}, \frac{dy}{2})$.

Conclusion and Future Work

Increasingly sophisticated computational models of spatial relations have been developed in the last couple of decades. However, none of these have been empirically validated to prove their correctness. We therefore designed empirical studies to clarify the interdependencies of angle, distance and shape when establishing projective spatial relations.

The experimental findings made clear that the angular deviation is the crucial point when establishing projective relationships. The scale of the angle becomes influenced by the extension of the reference object in each dimension. At angles between $(0^o, 45^o)$ and $(45^o, 90^o)$, and in particular at 22.5^o and 67.5^o, the extension of the reference object also caused a slight distance effect if and only if the located object was very close to the reference object and the corresponding angle was measured from the reference object's center. There exists a slight tendency to rate the *in front of/behind* and the *above/below* regions higher than the *right/left* region. This effect is also reflected in the evaluation of composite projective relations. With the help of these results it is now possible to test existing computational models for their cognitive plausibility.

In future work we intend to try to validate the computational model we presented in Gapp (1994a). This model accounts for all the crucial hypotheses verified in this work. First results were very promising for a positive validation of the models cognitive plausibility. Furthermore, we will also investigate the distance and shape interdependencies for the class of topological relations and extend our model to cope with more complex environments, introducing contextual dependencies and intervening objects.

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