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Author

Costello, Fintan J.

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A Unified Account of Conjunction and Disjunction Fallacies in People's Judgments of Likelihood

Fintan J. Costello (fintan.costello@ucd.ie)
Department of Computer Science, University College Dublin
Belfield, Dublin 6, Ireland

Abstract

This paper describes a simple continuous-valued logic which aims to explain the occurrence of both conjunction fallacies (where a conjunction $AandB$ is judged more likely than a constituent A) and disjunction fallacies (where a disjunction $AorB$ is judged *less* likely than a constituent A) in people's judgments of likelihood for simple and complex events. In this model both these fallacies are the result of minor rescaling of constituent likelihood judgments. Two experiments tested this model by asking people to judge the likelihood of different everyday weather events and the likelihood of conjunctions and disjunctions of those events. In both experiments the model was able to accurately predict the occurrence of these fallacies in both conjunctions and disjunctions.

Keywords: logical reasoning; probabilistic reasoning; conjunction fallacy; disjunction fallacy.

Introduction

To what extent are human thought processes rational? How closely do people's mental operations correspond to consistent and logical rules? In a influential paper Tversky and Kahnemann (1983) addressed this question by looking at how people carried out the operation of conjunction (the AND operation) for judgments of likelihood. A fundamental law of probability is that a conjunction cannot be more probable than any of its constituents. In most cases people follow this law when assessing conjunctive probability. Tversky and Kahnemann showed, however, that for some conjunctions people reliably deviate from this rule, judging a conjunction to be more likely than one or other of its constituents (committing a 'conjunction fallacy'). This conjunction fallacy has been confirmed in a number of studies (e.g. Stolarz-Fantino, Fantino, and Zizzo, 2003). A similar fallacy can also occur in peoples' judgments of *disjunction* likelihood: this 'disjunction fallacy' arises whenever a disjunction is judged *less* probable than any of its constituents.

How do we explain robust errors such as the conjunction fallacy? Traditional approaches to conjunction assume that people judge the likelihood of a conjunction $A\&B$ by first judging the likelihood of A and the likelihood of B, and then combining those two constituent likelihoods using some conjunction operator or function. To account for the conjunction fallacy we need to provide a function which will respond differently to different conjunctions, giving some conjunctions a lower likelihood than their constituents, but giving other conjunctions a higher likelihood than their constituents. A number of researchers, particularly in the area of fuzzy logic, have proposed various different conjunctive functions, such as Product, Average, Sum, or Minimum (see Osherson and Smith, 1982; Hajek, 1988). However, none of these can respond in different ways to different conjunctions, as required by the conjunction fallacy.

In the light of this difficulty, a variety of different accounts of the conjunction fallacy have been proposed, implicating prototype similarity or representativeness, the use of implicit mental models, the role of conversational implicature, or the influence of causal attribution (Ahn and Bailenson, 1996; Betsch and Fiedler, 1999; Giverenzer and Hertwig, 1996; Tversky and Kahneman 1983). In this paper I return to something close to the traditional view of conjunction and I describe a simple continuous-valued logic for conjunction which can produce both conjunction fallacy responses and standard conjunction responses. This logic is an extension of the standard probability theory account of the operations AND, OR, and NOT. I first describe this logic and its account for the conjunction fallacy. I then describe two experiments testing this model by comparing the model's predictions against people's judgments of likelihood in an everyday domain; that of estimating the likeihood of different types of weather. In testing this model I examine its ability to mirror people's production of three different conjunctive responses (one in which people judge a conjunction to be more likely than both constituents; one in which they judge the conjunction to have an intermediate likelihood between those of its two constituents; and one in which they judge the conjunction to have a lower likelihood than either constituent). I also test this model's ability to account for the relative responses seen in people's judgments of disjunctions in these experiments. In both cases the model gives a good match to people's responses to items in the experiment.

AND, OR, and NOT in probability theory

Perhaps the simplest way to produce a continuous-valued logic for people's judgments of likelihood is to use standard probability theory equations for ANDing and ORing independent probabilities: equations 1, 2, and 3.

$$P(NOTA) = 1 - P(A), \quad (1)$$

$$P(AandB) = P(A) \times P(B), \quad (2)$$

$$P(AorB) = 1 - P(NOTA) \times P(NOTB). \quad (3)$$

These equations assume that the probability of a given event A occurring falls in a range from 0 (certain not to happen) up to 1 (certain to happen). The probability of A not occurring is then simply 1 minus the probability of A occurring. The probability of both A and B occurring is simply the product of the probabilities of A and B. Finally, the probability of A or B occurring is 1 minus the probability of not A occurring and not B occurring.

These equations make sense as a logic for graded probability. In a rational account of human reasoning, we would hope to see people making judgments about conjunction and disjunction in a way that followed these equations. A problem, however, is that the product function representing AND (Equation 2) is unable to account for conjuction fallacy responses. With constituent probabilities limited to the range from 0 to 1, this product function always produces the same relationship between conjunctive probability and constituent probabilities: a 'less than' relationship where the probability of a conjunction is always less than or equal to the probability of both constituents of that conjunction (for example, if $A = 0.9$ and $B = 0.8$, $AandB = 0.72$, which is less than both). People do not follow this rule: for some conjunctions they give conjunctive probabilities that are less than constituent probabilities, but for others they give conjunctive probabilities that are greater.

Generating different relationships between constituent and conjunction likelihoods

The product function for conjunction will always produce conjunction likelihood ratings that are less than constituent ratings, as long as we assume that likelihood ratings (like probabilities) are limited to the range 0 to 1. Interestingly, however, if we allow ratings scores to move above 1, the product function will produce three different relations between constituent likelihood values and conjunctive values. If the values for both constituents A and B are below one, the conjunctive value ($A \times B$) is lower than both. If one value is above 1 and the other is below 1, however, the conjunctive value $A \times B$ will fall between both values (if $A = 0.9$ and $B = 1.1$, $A \times B = 0.99$, higher than A but lower than B). Finally, if both constituent scores are above 1, the conjunctive score $A \times B$ will be higher than both values (if $A = 1.1$ and $B = 1.2$, $A \times B = 1.32$, higher than both).

This observation leads to a new proposal as to why people sometimes produce the conjunction fallacy in judgments of likelihood: they are making use of the product function for forming conjunctions of constituent judgments, but some of those constituent judgments are higher than 1. In any conjunction where one or both constituent have likelihood judgments greater than 1, the conjunction fallacy will occur. The following equations formalise this idea in a 'rescaled' version of the standard probabilistic logic. In this 'rescaled' model we make a distinction between people's probability judgments and their responses, and assume that their responses are rescaled so that some fall above 1 and others below 1.

$$R(A) = P(A) + s, \quad (4)$$

$$R(NOTA) = P(NOTA) + s, \quad (5)$$

$$R(AandB) = R(A) \times R(B), \quad (6)$$

$$R(AorB) = 1 - R(NOTA) \times R(NOTB). \quad (7)$$

In these equations, the likelihood judgment for item A is equal to the probability of that item plus a rescaling factor s. This rescaling factor moves some likelihood judgments over the 1 boundary. Similarly, the likelihood judgment for $NOTA$ is the probability of $NOTA$, rescaled by the factor s. Importantly for the current proposal, the likelihood judgment for a conjunction $AandB$ is simply equal to the product of the rescaled likelihood judgments for A and B. In some cases one of these constituent judgments will will be above 1, producing conjunction fallacy responses). Finally, the likelihood judgment for a disjunction will be as in the standard probability approach, but making use of the rescaled likelihood judgments for $NOTA$ and $NOTB$, rather than the standard probability judgments.

In the next section I describe a preliminary experiment which tests this 'rescaled' model in two ways. This experiment first examines the extent to which the equation for AND (Equation 6) above can account for people's pattern of response for conjunctive judgments, and for the occurrence of conjunction fallacy responses. Next, the experiment examines the extent to which the derived equation for OR (Equation 7) can account for people's pattern of response disjunctive likelihood judgments and for the occurence of any *disjunction* fallacy responses.

Experiment 1

This experiment asked participants to rate the likelihood of various different single weather events ('cold', 'sunny','windy') and the likelihood of various different conjunctions and disjunctions ('cold and windy', 'cold or windy') of those events. Weather events were used because people frequently need to assess the likelihood of such events in their everyday lives (deciding whether they need a coat when going out, whether they should bring sunglasses, and so on), and so the assessment of likelihood in the experiment should be a natural task for participants to carry out.

The aim in this experiment was to examine the various different relationships that can hold between people's judgments of likelihood for complex weather events (conjunctions and disjunctions) and their judgments of likelihood for the constituents of those complex events. Three different relationships were examined: the conjunctive event being more likely than both constituent events (a 'greater than' response); the conjunctive event being more likely than one constituent event, but less likely than the other (an 'intermediate' response); and the conjunctive event being judged less likely than both constituents (a 'less than' response). Specifically, this experiment was intended to compare the frequency of these different types of response for conjunctions and disjunctions with the occurrence of these responses as predicted by the 'rescaled' model. Given people's rated likelihood for single weather events, to what extent would this model combine those likelihoods to produce 'greater than', 'intermediate' and 'less than' responses for the conjunctions and disjunctions used in the experiment? To what extent would these responses correspond with those produced by participants in the experiment for the same conjunctions and disjunctions?

Materials and procedure

Twenty-four familiar types of weather event ('cold', 'windy', 'sleet', 'sunshine', and so on) were selected and formed into 12 pairs (e.g. 'cold-windy', 'sleet-sunshine'). For each pair, a conjunctive event was generated by ANDing the two constituents and a disjunctive event was generated by ORing the constituents (see Table 1 below for a full list). To provide a realistic range of weather events the 12 pairs used were selected to provide a range of different degrees of relationship between constituent events, ranging from closely related events to unrelated events. The materials used in the experiment consisted of the 12 conjunctive events and the 12 disjunctive events generated in this way, plus the 24 original single weather events.

The 24 single events and 24 complex events (12 conjunctions and 12 disjunctions) were printed two per page in a questionnaire given to participants. Each questionnaire contained all events, events were in a different random order in each questionnaire. Each event was accompanied by a a request to the participant to 'Please rate the following statement on the corresponding scale according to the likelihood with which you believe the weather type may occur many times in Ireland over the year'. This request was followed by a 7-point rating scale going from 'highly unlikely' (-3) to 'highly likely'(+3). The cover page of this questionnaire explained to participants that they would be asked to rate the likelihood of different types of weather occurring, and contained two worked examples using different weather events to demonstrate the task.

A total of 16 participants were given these questionnaires. The questionnaire typically took between 20 and 40 minutes to complete.

Results

To examine the reliablility of participants' likelihood judgments in the experiment, participants were divided into two groups (even-numbered participants in one group, odd-numbered participants in the other). There was a significant Spearman-Brown split-half correlation between likelihood ratings for weather events in the two groups ($r_{sb} = 0.86, p < .01$) across single and complex events, confirming the reliability of these ratings.

To examine the occurrence of 'greater than', 'intermediate' and 'less than' responses for conjunctions in the experiment, each participant's conjunction and constituent likelihood judgments were individually compared. In total there were 192 distinct conjunction likelihood judgments in the experiment (16 participants $\times$ 12 conjunctions). The most frequent pattern across these judgments was a logically correct 'conjunction less than constituents' response, which occured in 113 cases (59% of the total). The two other fallacious response types were rarer: a 'conjunction intermediate' response was given in 38 cases (20%) , and a 'conjunction greater than' response in 34 cases (18%), giving a total of 72 cases (38%) of fallacious responses overall. In the remaining 3% of cases the conjunction likelihood was equal to both constituent likelihoods: all three likelihood rating were the same.

A similar count of 'greater than', 'intermediate' and 'less than' responses for *d*isjunctions in the experiment was also carried out. The logically correct 'disjunction greater than' response occured in 75 cases (39%). The fallacious 'disjunction intermediate' and 'disjunction less than' responses occured in 72 cases (38%) and 35 cases (18%) respectively, giving a total of 107 (56%) cases in which people made fallacious judgments. In the remaining 5% of cases all three likelihood rating were the same.

Both conjunction fallacy and disjunction fallacy responses occured quite often in this experiment, with the disjunction fallacy being particularly frequent. Can the 'rescaling' model account for these results? In the next section I apply the model to the experimental data and examine its ability to predict the occurrence of different responses for different conjunctions and disjunctions.

Comparison with the rescaling model

To apply the rescaling model to the experimental data, the average likelihood ratings produced for participants for single weather events were transformed from the -3 to $+3$ scale used in the experiment onto the range 0 to 1. A value for the rescaling parameter s was selected and added to all these single-event likelihoods, and these rescaled single-event likelihood scores were used as input for the equations for AND (Equation 6) and OR (Equation 7). Using this procedure, values were computed for each of the 12 event conjunctions which participants rated in the experiment, and each of the 12 event disjunctions. A conservative rescaling parameter value of $s = 0.1$ was used; adding this value to all 24 single-event likelihoods resulted the likelihood of only one event moving above 1.

To compare the model's predicted responses with those seen in the experiment, I counted the number of participants who gave the response patterns 'less than', 'intermediate', or 'greater than' for each conjunction and disjunction in the experiment. If a given response pattern was produced by 50% or more of participants for any given conjunction or disjunction, that was taken to be the dominant response pattern for that item. These dominant response patterns from the experimental data were then compared with the predicted response patterns computed by the rescaling model (see Table 1). Note that the blank entries in the response columns in Table 1 represent cases where there was no dominant response for that conjunction or disjunction. The model's

Table 1: Response patterns given by at least 50% of participants to conjunctions and disjunctions in Experiment 1, with response patterns as computed by the rescaling model.

	response patterns for conjunction			response patterns for disjunction	
conjunction	observed	computed	disjunction	observed	computed
cold and windy	less than (correct)	less than (correct)	cold or windy	greater than (correct)	greater than (correct)
frost and drizzle	less than (correct)	less than (correct)	frost or drizzle		
fair and hot	less than (correct)	less than (correct)	fair or hot	intermediate (fallacy)	intermediate (fallacy)
rain and thunder	intermediate (fallacy)	intermediate (fallacy)	rain or thunder	intermediate (fallacy)	intermediate (fallacy)
rough winds and hail			rough winds or hail	greater than (correct)	greater than (correct)
warm and humid	less than (correct)	less than (correct)	warm or humid	greater than (correct)	intermediate (fallacy)
wet and bright	less than (correct)	less than (correct)	wet or bright		
overcast and calm			overcast or calm	intermediate (fallacy)	intermediate (fallacy)
showers and sunny	less than (correct)	less than (correct)	showers or sunny	intermediate (fallacy)	intermediate (fallacy)
sleet and sunshine	less than (correct)	less than (correct)	sleet or sunshine		
gale winds and hazy	less than (correct)	less than (correct)	gale winds or hazy		
icy and cloudy	less than (correct)	less than (correct)	icy or cloudy		

responses are not compared with participants' responses in these cases.

As Table 1 shows, there was a close association between the response patterns predicted by the model and the dominant response patterns in the experiment. For example, there were 10 conjunctions for which the response 'less than' was dominant; the model also produced that response for those conjunctions. There was one conjunction ('thunder and rain') for which the response 'intermediate' was dominant (a conjunction fallacy response, in that the conjunction likelihood is higher than one of the constituent likelihoods. The model also produced that response for that conjunction. In total there were 17 cases where there was a dominant response for a conjunction or disjuction; in 16 out of that 17 the model produced the same response ($p < .01, binomial$). The model thus seemed to be able to mirror people's production of both logically correct responses ('less than' for conjunctions; 'greater than' for disjunctions) and for both conjunction and disjunction fallacies ('intermediate' responses in both cases).

To further investigate the aggeement between the model's computed conjunction and disjunction scores and participants' responses in the experiment, the models scores were compared with the average likelihood scores for conjunction and disjunctions in the experiment. For disjunctions there was a significant correlation between observed average likelihoods and those computed from constituent scores by the rescaling model ($r = 0.72, \%var = 0.52, p < .01$). For conjunctions, however, the correlation was less significant ($r = 0.61, \%var = 0.37, p < .05$). Given the model's good account of the occurence of the different sorts of conjunction and disjunction response patterns in the experiment, these relatively low correlations are surprising. One possible explanation for these relatively low correlations comes from the fact that participant's likelihood judgments for single events and for conjunctions and disjunctions in the experiment fall into quite a narrow range; most single and complex events were judged likely by most participants; few events were judged unlikely. To address this possibility the next experiment repeated the task of the current experiment, but using a set of single events that were distributed more evenly across the range of different likelihood judgments.

Experiment 2

As before, this experiment asked participants to rate the likelihood of various different single weather events and the likelihood of conjunctions and disjunctions of those events. In this experiment, single events were selected to have a range of different likelihoods, from highly unlikely to highly likely.

Materials and procedure

From the set of 24 single events used in Experiment 1, two sets of 4 single events were selected so that each set of events included events of a range of different likelihoods. By combining each single event from one set with every single event in the other set, a collection of 16 event pairs were constructed. For each pair, a conjunctive event was generated by ANDing the two constituents and a disjunctive event was generated by ORing the constituents. There were thus 8 single events, 16 conjunctive complex events, and 16 disjunctive complex events, used as materials in the experiment. These single and complex events were presented to experimental participants on a standard web browser. Each participants saw events in a different random order. The format and instructions used were as in Experiment 1. A total of 21 participants were given these web-based questionnaires. The questionnaire typically took between 20 and 40 minutes to complete.

Table 2: Response patterns given by at least 50% of participants to conjunctions and disjunctions in Experiment 2, with response patterns as computed by the rescaling model.

	response patterns for conjunction			Response patterns for disjunction	
conjunction	observed	computed	disjunction	observed	computed
rain and cloudy	all equal (correct)	all equal (correct)	rain or cloudy	all equal (correct)	all equal (correct)
windy and rain	all equal (correct)	all equal (correct)	windy or rain	all equal (correct)	all equal (correct)
rain and sunny	less than (correct)	intermediate (fallacy)	rain or sunny		
thunder and rain[a]	intermediate (fallacy)	intermediate (fallacy)	thunder or rain		
cold and cloudy	all equal (correct)	all equal (correct)	cold or cloudy	all equal (correct)	all equal (correct)
windy and cold	all equal (correct)	all equal (correct)	windy or cold	all equal (correct)	all equal (correct)
cold and sunny	less than (correct)	less than (correct)	cold or sunny		
thunder and cold	less than (correct)	less than (correct)	thunder or cold[b]	intermediate (fallacy)	intermediate (fallacy)
frost and cloudy	less than (correct)	intermediate (fallacy)	frost or cloudy		
windy and frost	less than (correct)	intermediate (fallacy)	windy or frost		
frost and sunny	less than (correct)	less than (correct)	frost or sunny	greater than (correct)	greater than (correct)
thunder and frost	less than (correct)	less than (correct)	thunder or frost	greater than (correct)	greater than (correct)
sleet and cloudy			sleet or cloudy		
windy and sleet[a]	intermediate (fallacy)	intermediate (fallacy)	windy or sleet[b]	intermediate (fallacy)	intermediate (fallacy)
sleet and sunny	less than (correct)	less than (correct)	sleet or sunny	greater than (correct)	greater than (correct)
thunder and sleet	less than (correct)	less than (correct)	thunder or sleet	greater than (correct)	greater than (correct)

[a] The two conjunctions for which an 'intermediate' response was produced by the highest proportion of participants (43% in both cases)
[b] The two disjunctions for which an 'intermediate' response was produced by the highest proportion of participants (38% in both cases)

Results

As before, to examine the reliability of participants' likelihood judgments in the experiments, participants were divided into two groups. There was a significant Spearman-Brown split-half correlation between likelihood ratings for weather events in the two groups ($r_{sb} = 0.93, p < .01$) across single and complex events, confirming reliability.

As in Experiment 1, each participant's judgments of likelihood for conjunctions and disjunctions were examined individually. In total there were 336 distinct conjunction likelihood judgments in the experiment (21 participants × 16 conjunctions). The most frequent pattern across these conjunctive judgments was a logically correct ' less than' response, occuring in 173 cases (51%). The two fallacious response types were rarer: in 64 cases (19%) an 'intermediate' response was given, and in 36 cases (11%) a 'greater than' response occured, giving a total of 100 (30%) cases in which people made fallacious judgments. In 63 cases (19%) the conjunction likelihood was equal to both constituent likelihoods: all three likelihood rating were the same. Apart from these 'all equal' responses, the pattern here was similar to that seen in Experiment 1 (where logically correct 'conjunction less than' responses also dominated).

Disjunction responses were also similar to those from Experiment 1 (apart from an increase in 'all equal' responses). The logically correct 'greater than' response occured in 120 cases (36%). The fallacious 'intermediate' and 'less than' responses occured in 66 cases (20%) and 41 cases (12%) respectively, giving a total of 107 (32%) fallacious judgments. Finally, in 109 cases (32%) an 'all equal' response was returned.

Again, both the conjunction fallacy and disjunction fallacy responses occured quite often in this experiment, although somewhat less frequently than in Experiment 1. Can the rescaling model account for the pattern of responses seen for conjunctions and disjunctions? Will the correlation between model and data be improved by the greater spread of constituent likelihood scores in this experiment?

Comparison with the rescaling model

As before, to apply the model the average likelihood ratings produced for participants for single weather events were transformed onto the range 0 to 1. A value for the rescaling parameter s was selected (a conservative value of 0.06 was used) and added to all these single-event likelihoods, and these rescaled single-event likelihood scores were used as input for the equations for AND (Equation 6) and OR (Equation 7).

In this experiment, unlike in Experiment 1, there was a significant proportion of responses where both constituent event likelihoods and conjunctive or disjunctive event likelihood were all equal. Since the rescaling model was computing conjunctive and disjunctive likelihoods on the basis of average likelihood scores for constituent events, the model was extremely unlikely to produce a response in which both rescaled constituent scores and computed conjunction (or disjunction) likelihood were all exactly equal. To attempt to account for the 'all equal' responses in the experiment, a second parameter was added to the model, a distance parameter d such that if any likelihoods computed by the model fell with d of each other, they would be taken to be equal. This distance parameter would give the model a chance to account for the 'all equal' responses seen in the experiment. A value of $d = 0.05$ was chosen for this parameter.

As before, to compare the model's predicted responses with those seen in the Experiment, I counted the number of participants who gave the response patterns 'less than', 'intermediate', or 'greater than' for each conjunction and disjunction in the experiment. If a given response pattern was produced by 50% or more of participants for any given conjunction or disjunction, that was taken to be the dominant response for that item. There were no cases where an 'intermediate' response was produced by 50% or more of participants for either a conjunction or a disjunction. Without such responses there would be no cases of the conjunction fallacy or the disjunction fallacy against which the model's predictions could be compared. To avoid this problem, the conjunctions for which the 'intermediate' response was most frequent, and the disjunctions for which the 'intermediate' response was most frequent, were also included in the set of dominant responses (see Table 2). Again, the blank entries in the response columns in Table 2 represent cases where there was no dominant response for that conjunction or disjunction. The model's and participants' responses are not compared in these cases.

As in Experiment 1, there was a close association between the response patterns predicted by the model and the dominant response patterns in the experiment. In total there were 25 cases where there was a dominant response for a conjunction or disjunction; the model produced the same response for 22 out of that 25 ($p < .01, binomial$).

Finally, the model's computed scores were compared with the average likelihood scores for conjunction and disjunctions in the experiment, giving a significant correlation both for conjunctions ($r = 0.85, \%var = 0.72, p < .01$) and disjunctions ($r = 0.92, \%var = 0.85, p < .01$). In both cases these correlations are higher than those seen in Experiment 1, and confirm the model's account for the experimental data.

Conclusions

This paper started by asking about the correspondance between people's mental operations and logical and consistent rules. Both the conjunction fallacy and similar disjunction errors have been widely used as evidence that people do not think logically; and indeed, both conjunction fallacy and disjunction fallacy responses must be seen as logically incorrect. From some perspectives, this is very worrying: if even our mental mechanisms for conjunction and disjunction (two of the simplest possible operations) are not logically consistent and correct, how can we rely on our more complex thought processes? The rescaling model of continuous-valued logic described here may address these concerns. In this model, the our mechanisms for both conjunction and disjunction are logically justifiable and consistent (the equations for AND and for OR are simply transferred directly from probability theory). Errors in reasoning, such as conjunction and disjunction fallacies, do not arise because our mental operations for conjunction or disjunction are illogical. Rather they arise because of 'rescaling' of the inputs to these operations.

The main contribution of this paper has been to describe how a continuous-valued logic which includes this 'rescaling' can account for the various patterns seen in people's responses when judging the likelihood of conjunctive events. This model can account for cases in which people judge a conjunctive event to be less likely than both constituents of that event, and can also account for conjunction fallacy responses, in which a conjunction is judged more likely than constituent events. This model does not need any extra information or assumptions (no conjunctive prototype or representativeness heuristic is required) to account for both normal conjunction responses and for conjunction fallacy responses. Not only can this logic account for the various patterns seen in conjunctive likelihood judgments, the same approach can also account for the various patterns seen in people's judgments of disjunctive likelihood, and for the occurrence of disjunction fallacy responses for some disjunctions. Evidence from the two experiments described above suggest that this model can give a reasonably good account of people's judgment of likelihood in conjunctions and disjunctions; however, there is a significant amount of work yet to be done in testing this approach.

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