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### Authors

Dong, Yi  
McCartney, John S  
Lu, Ning

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**Critical Review of Thermal Conductivity Models for Unsaturated Soils**

Yi Dong <sup>1</sup>, John S. McCartney, P.E., M.ACSE <sup>2</sup>, Ning Lu, F.ASCE <sup>3</sup>

January 9, 2014

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<sup>1</sup>Post-doctoral fellow, Department of Civil & Environmental Engineering, Colorado School of Mines, 1012 14<sup>th</sup> St., Golden, CO 80401. Email: ydong@mines.edu  
<sup>2</sup>Associate Professor, Department of Civil, Environmental, and Architectural Engineering, University of Colorado at Boulder, UCB 428, Boulder, CO 80309. Email: john.mccartney@colorado.edu

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25 <sup>3</sup>Professor and corresponding author, Department of Civil & Environmental Engineering,  
26 Colorado School of Mines, 1012 14<sup>th</sup> St., Golden, CO 80401. Email: ninglu@mines.edu

27 **Abstract** Although it is well established that heat conduction in unsaturated soil depends  
28 on liquid saturation, there are several models available to consider the changes in thermal  
29 conductivity during drying and wetting. The key factors affecting thermal conductivity of  
30 unsaturated soil are evaluated through a critical examination of these different models  
31 and their development. Depending on the principles and assumptions employed, these  
32 models are categorized into three groups: mixing models involving series/parallel  
33 elements; empirical models where thermal conductivity values at dry and saturated states  
34 are used; and mathematical models based on phase volume fractions. Experimental data  
35 for different soils are used to assess the quality of prediction for these models. It is found  
36 that all the existing models do not realistically account for pore structure or interface  
37 properties, and thus are not capable of predicting thermal conductivity as a function of  
38 liquid saturation. A conceptual model based on soil-water retention mechanisms, is  
39 proposed to overcome the pitfalls of the existing models and can be used to establish  
40 quantitative thermal conductivity models for variably saturated soils in the future.

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45 **Keywords:** Thermal conductivity, heat transfer, unsaturated soil, soil-water retention.

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4 **46 Introduction**

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7 47 Heat transfer in soil and rock presents a field of fertile research in which the thermal  
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9 48 properties of soils are used in a large number of geotechnical, geophysical and geo-  
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11 49 environmental applications, including geothermal energy resources (White, 1973),  
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14 50 radioactive waste disposal (Li et al., 2012), ground-source heat pump systems (Preene  
15  
16 51 and Powrie, 2009), energy piles (Brandl, 2006), geological CO<sub>2</sub> sequestration (Ebigbo,  
17  
18  
19 52 2005) and recovery of natural methane gas hydrates (Cortes et al., 2009).  
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22 53 Soil is a multi-phase material consisting of solid particles, gas and/or liquid. Thermal  
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24 54 properties of soils are not only determined by the intrinsic physical properties of each  
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27 55 phase, but also affected by environmental variation of each phase. The thermal  
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30 56 conductivity  $k$  of different soil phases varies over two orders of magnitude (e.g. thermal  
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32 57 conductivity of mineral particles  $k_{\text{mineral}} > 3 \text{ W/m}\cdot\text{K}$ , thermal conductivity of water  
33  
34 58  $k_{\text{water}} = 0.56 \text{ W/m}\cdot\text{K}$  (at 0 °C), and thermal conductivity of air  $k_{\text{air}} = 0.026 \text{ W/m}\cdot\text{K}$ )  
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37 59 (Mitchell and Soga, 2005; Yun and Santamarina, 2008). However, the thermal  
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40 60 conductivity of dry soil  $k_{\text{dry\_soil}}$  is almost one order of magnitude lower than that of the  
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42 61 pure mineral solids, in most cases  $k_{\text{dry\_soil}} < 0.5 \text{ W/m}\cdot\text{K}$ , depending on mineral  
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45 62 composition and packing density (Farouki, 1981). This implies that the air obstructs heat  
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48 63 conduction and heat conduction primarily occurs through the particle contacts in dry soil.  
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50 64 At the other end, the thermal conductivity of water-saturated soil is between that of the  
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52 65 pure mineral and that of water, implying that replacement of air with water provides a  
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55 66 significant improvement in the heat conduction through the soil mixture. The ordered  
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57 67 sequence of typical thermal conductivity values are

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4 68  $k_{\text{air}} < k_{\text{dry\_soil}} < k_{\text{water}} < k_{\text{saturated\_soil}} < k_{\text{mineral}}$ . These observations, and many experimental  
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7 69 studies (Brandon and Mitchell, 1989; Farouki, 1981; Smits et al., 2010), confirm that the  
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9 70 varying water saturation in the soil mixture system plays an important role in determining  
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11 71 the bulk thermal properties.  
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15 72 The purposes of this paper are: (1) to identify the controlling physical mechanisms for  
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17 73 the thermal behavior of the soil at various unsaturated conditions, and (2) to assess the  
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19 74 existing models for their predictability of thermal conductivity under varying saturation  
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21 75 conditions.  
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## 25 76 **Heat Transfer Mechanisms and Governing Factors**

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28 77 There are three heat transfer mechanisms in a material medium: conduction through  
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30 78 solids and liquids, convection in fluids, and radiation (which does not require a material  
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32 79 medium). The most effective means of transferring heat in dry particulate materials is  
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34 80 through the solid contacts, while conduction through the gas phase and radiation have  
35  
36 81 less relevant effects (Carslaw and Jaeger, 1959; Murashov and White, 2000). Heat  
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38 82 transfer by convection plays an important role if the particle size  $D_{50}$  is larger than ~  
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40 83 6 mm permitting fluid flow through the porous network (Yun, 2005). In this paper, the  
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42 84 scope of our assessment of thermal conductivity models considers only heat transfer by  
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44 85 conduction. It is assumed that soils are under room temperature and under a small  
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46 86 temperature gradient. Thus, the liquid phase change or water vapor enhancement induced  
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48 87 by high temperature and the heat flux of fluid convection due to high temperature  
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50 88 gradient are ignored. Under these assumptions, the apparent or effective thermal  
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4 89 conductivity of soil is often used to describe the heat transfer capability of multi-phase  
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6 90 materials.  
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10 91 The heat flux  $q$  [W/m<sup>2</sup>] at steady state is proportional to the thermal gradient by the  
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12 92 coefficient of thermal conductivity  $k$  [W/mK], according to the constitutive equation of  
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15 93 Fourier's law in one dimension:  
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19 94 
$$q = k \cdot \frac{dT}{dx} \quad (1)$$
  
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23 95 The rate of heat transfer in transient conditions is equal to the heat stored within the  
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25 96 medium and the rate of internal heat generation, given by the continuity equation:  
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30 97 
$$\frac{dq}{dx} = q_{\text{generated}} - q_{\text{stored}} \quad (2)$$
  
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34 98 where the heat stored in the material is:  $q_{\text{stored}} = \rho \cdot c \cdot (\partial T / \partial t)$ ,  $\rho$  [kg/m<sup>3</sup>] is the material mass  
35  
36 99 density, and  $c$  [J/kgK] is the heat capacity. If there is no heat generation within the  
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39 100 material, the  $q_{\text{generated}}$  term vanishes. Combining the above two equations leads to Fick's  
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41 101 second law:  
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45 102 
$$\frac{\partial^2 T}{\partial x^2} = -D \frac{\partial T}{\partial t} \quad (3)$$
  
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49 103 where  $D = k / (\rho \cdot c)$  [m<sup>2</sup>/s] is the thermal diffusivity which reflects how fast heat will be  
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52 104 transferred through a material. The negative sign indicates that heat transfer occurs in the  
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55 105 direction opposite to the temperature gradient. Common values of thermal conductivity,  
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4 106 diffusivity, and specific heat of different soils and their phase components are  
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6 107 summarized in Table 1.  
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10 108 The thermal properties of soils under isothermal conditions are governed by several  
11  
12 109 particle-level and macro-scale factors: mineralogy, particle size, particle shape, packing  
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14  
15 110 geometry, stress level, water content, porosity, gradation, and cementation.  
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18 111 *Mineralogy.* Solid minerals are the most conducive constituents in the air-water-solid  
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20 112 soil system; thus they define the upper limit of the thermal conductivity. Soil comprised  
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23 113 of different mineral substances has different thermal conductivity (e.g., quartz > mica)  
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25 114 (Gangadhara Rao and Singh, 1999; Tarnawski et al., 2002). Soil with higher quartz  
26  
27  
28 115 content has larger thermal conductivity values (Tarnawski et al., 2009).  
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30

31 116 *Particle size and gradation.* Heat flux between particles is proportional to the radius  
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33 117 of the particles. Larger particles and fewer contacts in a given volume result in higher  
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36 118 thermal conductivity (Aduda, 1996; Gangadhara Rao and Singh, 1999). Well-graded soil  
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39 119 exhibits higher heat transfer as small particles fill the interstitial pore space and increases  
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41  
42 120 the inter-particle coordination (Esch, 2004).  
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45 121 *Packing geometry.* Higher inter-particle coordination increases the thermal  
46  
47 122 conductivity for a given particle size (Lambert and Fletcher, 1997b; Tarnawski et al.,  
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49  
50 123 2002). The contact conductance is more important than the radiational conductance  
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52  
53 124 (Lambert and Fletcher, 1997a). The thermal conduction at contacts results in percolation-  
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55 125 type conduction process (Sahimi and Tsotsis, 1997).  
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126        *Stress level.* Higher stress results in higher contact radii leading to an increase in  
127 thermal conductivity therefore thermal conductivity. Granular chains under higher stress  
128 also gives rise to more developed heat transfer paths (Vargas and McCarthy, 2001).

129        *Water content.* Fluid volume fraction is a dominant factor to the thermal conductivity  
130 in partially saturated soils. At residual water content region, adding small amount of  
131 water dramatically improves the thermal conduction. The increase of thermal  
132 conductivity with increase of water content in unsaturated soils suggests the important  
133 role of the pore fluid conduction (Singh and Devid, 2000).

134        *Porosity.* The lower the void ratio, the higher the thermal conductivity (Brandon and  
135 Mitchell, 1989; Yun and Santamarina, 2008).

136        *Cementation.* Cement and colloidal precipitation at particle contacts increases the  
137 contact area and thus increase the thermal conductivity (Tarnawski et al., 2002).

138        Through the analysis of these factors affecting the effective thermal conductivity of  
139 soils found in the literature, they can be generalized into few essential elements: thermal  
140 conductivity of each constituent (i.e., minerals, liquid, and air), water content, soil type  
141 (e.g., particle size/shape), and particle contacts (e.g., coordination numbers), which can  
142 be affected in a number of macro-level manifestation such as porosity, stress level, and  
143 gradation. The key governing factors are summarized in Table 2.



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4 **144 Thermal Conductivity Models**

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7 145 The accurate prediction of the thermal conductivity of composite materials comprises  
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10 146 a significant portion of the literature about heat transfer in porous media, and a large  
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12 147 number of effective thermal conductivity models have been proposed. New models for  
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14 148 thermal conductivity of soils are emerging, suggesting that, to-date, a unified model or  
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17 149 prediction procedure has not been found with universal applicability. In this section,  
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19 150 several existing models have been reviewed and categorized into the following three  
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22 151 groups based upon their principles:

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25 152 *Mixing models.* This type of model conceptualizes the multi-phase soil system as a  
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27 153 certain combination of series and parallel solid, air and/or water blocks in the cubic cell  
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30 154 or representative elementary volume (REV); and the effective thermal conductivity of the  
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33 155 bulk medium is calculated by mixing those blocks.

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36 156 *Empirical models.* This group of models builds the relationship between relative  
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38 157 thermal conductivity and degree of saturation or water content, by normalizing the  
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41 158 effective thermal conductivity or Kersten number over the difference between saturated  
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44 159 state thermal conductivity  $k_{\text{sat}}$  and dry state thermal conductivity  $k_{\text{dry}}$ .

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46  
47 160 *Mathematic models.* These models were adopted from predictive models of other  
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50 161 physical properties, such as dielectric permittivity, magnetic permeability, electrical  
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52 162 conductivity, and hydraulic conductivity; which are calculated by certain mathematical  
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55 163 algorithm given the thermal conductivity of each component and their volume fractions.

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58 164 ***Mixing Models***

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4 165 The most common models are based on classical mixing laws (arithmetic and  
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6 166 harmonic) of series model and parallel model. The series model imposes a constant heat  
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9 167 flux through each serially connected component (Figure 1a) so that each component  
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11 168 develops different temperature gradients depending on its own thermal conductivity. In  
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13  
14 169 this case, the resistivity (inversion of conductivity) of the bulk material is the arithmetic  
15  
16 170 average of the resistivity of each component weighted by their volume fractions  
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19 171 (harmonic average for conductivity). The parallel model, on the other hand, imposes the  
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21 172 identical temperature gradient to the individual phases or elements (Figure 1b) so that  
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24 173 each phase has the same temperature difference, but conducts different heat flow  
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26 174 depending on the thermal conductivity of each component. In this case, the thermal  
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29 175 conductivity is the arithmetic mean of the thermal conductivity of each component. These  
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31 176 two models are also referred to as upper and lower bounds (or Wiener bounds). They are  
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33 177 the widest bounds with least constraint since no soil structure or fabric is considered. The  
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36 178 larger the difference between the thermal conductivity values of each component, the  
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39 179 wider the bandwidth of the bounds. Some other mixing laws also generate “averaged”  
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41 180 effective thermal conductivity, such as the geometric mean law (hereafter Geo\_Mean)  
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43  
44 181 and the quadratic parallel law (see Table 3).

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47 182 Another group of mixing models can be derived based upon a combination of series  
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49 183 and parallel models. *Mickley's (1951) model* (Mickley, 1951) involves a unit cubic cell  
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51 184 (see Figure 2a) that consists of heat conduction through a  $(1-a)^2$  solid block, heat  
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54 185 conduction through a  $c^2$  air block, and heat conduction through a series of solid, water  
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57 186 film, and air, assuming a water film having a thickness of  $b = a-c$ . This model considers

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4 187 unsaturated soil conditions by introducing the parameters  $a$ ,  $b$ ,  $c$ , which represent the  
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6 188 proportions of solid, air, and water phases that can be determined for given porosity  $n$  and  
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9 189 degree of saturation  $S$ . However, this method does not hold for very porous or dry soil,  
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11 190 due to the poor grain-to-grain contacts when the soil is loose or few water bridges.  
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14  
15 191 *Woodside and Messmer's (1961) model* (Woodside and Messmer, 1961) employed a  
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17 192 three-component model that was originally used for determining electrical conductivity.  
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19 193 The model visualizes three parallel heat flow paths in an idealized unit cube of soil as  
20  
21 194 shown in Figure 2b. It consists of a path of width  $a$  through fluid and solid in series, a  
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23 195 path of width  $b$  through continuous solid material, and a path of width  $c$  through  
24  
25 196 continuous pore fluid. Given the thermal conductivity and volume fractions of each  
26  
27 197 component, the key parameters  $a$ ,  $b$ ,  $c$  can be determined by porosity, and formation  
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29 198 factor borrowed from Archie's law. This model only works for two-phase systems such  
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31 199 as saturated soils, and determination of those parameters are difficult and by empirical  
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33 200 fitting.  
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40 201 *McGaw's (1969) model* (McGaw, 1969) neglected the heat conduction through  
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42 202 particle contacts and considered the heat flow mainly across solids with intervening fluid  
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44 203 and passing entirely within the fluid (see Figure 2.c). This cubic cell can be expanded by  
45  
46 204 adding one block in parallel for heat conducted in air. McGaw defined an interfacial  
47  
48 205 efficiency factor  $\varepsilon$  (at solid/fluid interfaces), that turns to be close to 1- by assuming there  
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50 206 is little temperature gradient within the intervening fluid, and the volume of this  
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52 207 interfacial fluid  $n_c$  is about 0.03 for high saturation sand. To use this model, some  
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4 208 uncertain assumptions have to be made regarding the values of  $\varepsilon$  and  $n_c$ , when the soil is  
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6 209 under low degrees of saturation.  
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10 210 *Gori and Corasaniti's (2002) model* (Gori and Corasaniti, 2002) used the cubic  
11  
12 211 mixing model for thermal conductivity with considerations of different regimes of water  
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14 212 content for three-phase soil in unsaturated conditions (see Figure 2d). The entire range of  
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16 213 water content for a given soil is divided by the water content at the field capacity of the  
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18 214 soil  $\theta_f$  (when water has drained out of the larger pores but the small pores remain filled  
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20 215 with water), water content at the permanent wilting point  $\theta_p$  (water content at  $\sim 1.5$  kPa of  
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22 216 suction pressure), and water content of adsorbed water films  $\theta_c$  (water film around the  
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24 217 solid particles without connecting others, a fraction of  $\theta_p$ ). The model involves  
25  
26 218 arrangement of the solid phase in the center of the gas cubic, with water films/bridges  
27  
28 219 growing around the particle depending on the water content regime. The effective  
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30 220 thermal conductivities of soils in each regime are defined separately by the mixing of  
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32 221 air/water/solid phase. However, the dividing points were determined empirically for  
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34 222 different types of soil.  
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### 43 223 ***Empirical Models***

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46 224 A number of empirical relationships between effective thermal conductivity and  
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48 225 degree of saturation, porosity and soil types have been established by researchers.  
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50 226 Kersten (Kersten, 1949) performed an extensive series of tests on various types of clays,  
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52 227 silts and sands, and proposed empirical equations of thermal conductivity based on water  
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54 228 content and dry bulk density for silt-clay and sandy soil with separate equations (see  
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4 229 Table 3). This empirical model leads to a discrepancy of ~25 % or higher for high silt-  
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6 230 clay content soils and is only valid for the range of water content over which the  
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9 231 experimental data is available.

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11  
12 232 Johansen introduced a normalized thermal conductivity called Kersten number  $K_e$ ,  
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15 233 given by  $K_e = (k - k_{dry}) / (k_{sat} - k_{dry})$  (Johansen, 1975). A model was developed by the  
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17 234 relationships between the Kersten number and degree of saturation  $S$ . Simple first-order  
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19 235 logarithmic functions of  $S$  for  $K_e$  were used to describe such relationships, where  
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22 236 parameters in these equations were obtained by fitting the experimental results of  
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24  
25 237 different types of soils. To project the Kersten number to the effective thermal  
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27 238 conductivity, the bounds at dry and saturated condition were calculated by thermal  
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30 239 conductivity of single phase (solid, water, air) and other soil properties, such as dry bulk  
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32 240 density, porosity, and quartz content. Johansen's model provided a method to estimate  
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35 241 the effective thermal conductivity by interpolating between the dry and the saturated  
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37 242 values of the thermal conductivity. Yet, issues come up when it applies to low quartz-  
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40 243 content soils.

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43 244 Inspired by the Johansen model, some other empirical models were derived based on  
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45 245 the relative thermal conductivity described using the Kersten number  $K_e$ . Cote and  
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48 246 Konrad modified the logarithmic equation into a hyperbolic equation for  $K_e \sim S$  with a  
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50  
51 247 fitted material parameter  $\kappa$ , and redefined a new equation of determining dry thermal  
52  
53 248 conductivity with two empirical parameters  $\chi$  and  $\eta$  (Côté and Konrad, 2005). The *Cote*  
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55 249 *and Konrad 2002 model* is applicable to more soil types and generates more accurate  
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250 estimation of thermal conductivity for the effective thermal conductivity than the  
251 *Johansen 1975 model*.

252 Other derivative models modified based upon the *Johansen's (1975) model* are *Lu et*  
253 *al.'s (2007) model* which used a modified exponential equation of saturation  $S$  with a  
254 fitted soil-type parameter  $\alpha$ , and *Chen 2008 model* which adapted a modified power  
255 equation with two empirical parameters  $b$  and  $c$  to describe the effective thermal  
256 conductivity behavior with saturation  $S$  (Chen, 2008; Lu et al., 2007). These models  
257 better consider the tail behavior of thermal conductivity for fine-textured soil at low  
258 water content; however, lose applicability to other soil types.

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4 260 ***Mathematical Models***  
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7 261 This group of models usually is analog of other physical properties of mixtures, such  
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9 262 as electrical conductivity, magnetic permittivity, elastic modulus (see Table 3). The  
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11 263 *Hashin-Shtrikman's model*, hereinafter HS (Hashin and Shtrikman, 1962; Hashin and  
12  
13 264 Shtrikman, 1963), employed variational principles and established the effective  
14  
15 265 conductivity bounds that were first used for magnetic permeability and were found to be  
16  
17 266 the best or narrowest possible bounds for composite materials, whose bulk properties  
18  
19 267 could be derived from the property of the constituents and their volume fractions (Carson  
20  
21 268 et al., 2005). The assumptions of HS model are that the composite materials are  
22  
23 269 macroscopically homogeneous, isotropic, multi-phase, and it considers that the spheroidal  
24  
25 270 intrusions are dispersed in a different matrix. It is referred to as “external porosity”  
26  
27 271 material that the low conductivity spherical phase (e.g., air) dispersed in the high  
28  
29 272 conductivity phase matrix (e.g., grains), restrained by the upper bound such as foams or  
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31 273 sponges. The opposite case of high conductivity spherical phase (e.g., grains) dispersed in  
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33 274 low conductivity matrix (e.g., air or water) is the “internal porosity” material such as soils,  
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35 275 restrained by the lower bound (see schematic illustration in Figure 3.a). The HS upper  
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37 276 and lower (HS\_U, HS\_L) bounds always lie within the parallel/series bounds discussed  
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39 277 above, regardless of the component volume fractions or thermal conductivities.  
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50 278 The *Effective Medium Theory* (EMT), as one kind of the self-consistent method  
51  
52 279 (SCM), has been proposed to distinguish the thermal conductivity region bounded by HS  
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54 280 bounds into internal porosity and external porosity (Carson et al., 2005). The EMT model  
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56 281 assumes that the bulk property of a composite material as a result of the interaction of  
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4 282 each phase is accounted for by imaging each phase to be an inclusion embedded in a  
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6 283 homogeneous medium that has the overall property of the composite. It is equivalent to  
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9 284 the composite has no continuous or dispersed phases with different conductivity; instead,  
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11 285 the different phases are randomly distributed with the medium (see schematic illustration  
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14 286 in Figure 3.b).

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17 287 In summary, the *mixing models* are trying to simplify the soil microstructure and the  
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19 288 arrangement of each phase into a simple combination of series and parallel elements in a  
20  
21 289 representative elementary volume to substitute the apparent effective property of the bulk  
22  
23 290 material. Although some more sophisticated models derived base on those mixing  
24  
25 291 principle are trying to describe the effect of pore-water distribution (e.g., *Gori and*  
26  
27 292 *Corasaniti's (2002) model*) or simulate the soil microstructure by fractal model (e.g.,  
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29 293 *Lehmann 2003 model*,(Lehmann et al., 2003)), they are basically lump the property of  
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32 294 small elements into the effective bulk property without a realistic physical mechanism.  
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35 295 The *empirical models* interpolate the effective thermal conductivity between the  
36  
37 296 minimum value (dry thermal conductivity) and the maximum value (saturated thermal  
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39 297 conductivity) by using different types of functions with empirical fitted parameter to  
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41 298 apply different types of soil, whereas each model is only valid for a small group of soil  
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44 299 type, and those parameters lack clear physical meanings, plus the determination of dry  
45  
46 300 and saturated thermal conductivity were also not definite. The *mathematical models*  
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49 301 approximate the effective thermal conductivity only by given property of each  
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51 302 component and their volume fractions, by homogenization assumptions and analog of  
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54 303 phase intrusion of a matrix in an electrical field or elastic energy conservation of  
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4 304 composite material under a field constrain, however, all of them have no consideration of  
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6 305 the soil microstructure, solid particle geometry, water-phase distribution, and pore-size  
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9 306 gradation.

### 10 11 12 307 **Model Assessment by Experimental Data**

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15 308 In this section, the thermal conductivities of collected experimental data of different  
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18 309 types of soil are compared with the prediction of the abovementioned models. Cross  
19  
20 310 validation was used to evaluate each model. The properties of three types of soil used in  
21  
22 311 the model predictions and assessments are listed in Table 5. The normalized root-mean-  
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24 312 square-error (*NRMSE*) and the coefficient of variation of the root mean square error  
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26 313 *CV(RMSE)* defined for the three sets of experimental data and the selected model  
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28 314 predictions are summarized in Table 6. These values are defined as follows:  
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34 315 
$$RMSE = \sqrt{\frac{\sum (k_{\text{exp}} - k_{\text{mod}})^2}{n}} \quad (4)$$

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39 316 
$$NRMSE = \frac{RMSE}{k_{\text{exp}}^{\text{max}} - k_{\text{exp}}^{\text{min}}} \quad (5)$$

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44 317 
$$CV(RMSE) = \frac{RMSE}{\bar{k}_{\text{exp}}} \quad (6)$$

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49 318 The smaller the NRMSE number, the less the model prediction deviates from the  
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51 319 measured data points.

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55 320 A comparison of a set of measured thermal conductivity values for quartz sand and  
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57 321 the calculated thermal conductivity by different models is shown in Figure 4 for the

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322 conditions listed in Table 5. In Figure 4a, it can be clearly seen that the parallel/series  
323 models set the upper and lower limits of the possible thermal conductivity predictions.  
324 These two limits have the largest bandwidth, due to the large difference in the thermal  
325 conductivity values between the solid mineral and the air. The Geometric Mean model  
326 has an upward concave trend as the saturation increases, across the convex experimental  
327 data points. The HS upper and lower bounds lie within the parallel and series bounds, but  
328 still have large width and fail to capture the thermal conductivity variation pattern. All of  
329 the experimental data points fall into the external porosity region (lower than the SCM  
330 model, short dash-dot line). The SCM model is close to the experiment results only at  
331 high degrees of saturation. The De Vries' (1963) model underestimates the thermal  
332 conductivity at most of the range of saturation (De Vries, 1963). All the empirical models  
333 have similar calculation results of the thermal conductivity at different saturations. The  
334 relative thermal conductivity relations defined by the models of Johansen (1975), Cote's  
335 (2005) and Lu et al. (2007) use logarithmic, exponential and power functions,  
336 respectively, and have a similar trend with the measured data points. The functions  
337 describing those relations generate smooth curves and therefore cannot reflect the  
338 characteristic inflection points, which represent the critical saturation values dividing the  
339 pore-water distribution regimes, on the thermal conductivity curve for sand material.  
340 Figure 4b shows the 1:1 line for comparison of the measured and calculated results. None  
341 of the models have data points falling around the diagonal line throughout the entire  
342 range of saturation. The NRMSEs of all models are large (0.253 ~ 0.819), indicating that  
343 very large discrepancies exist between the experimental data and model predictions.

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4 344 Although, the empirical models have lower NRMSE values than the other models, they  
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6 345 still largely underestimate the thermal conductivity.  
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10 346 Figure 5 delineates the comparison of experimental measurements of sandy silt and  
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12 347 model calculations. Evidently, the parallel/series and HS upper/lower bounds set the  
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15 348 upper and lower limits of the thermal conductivity values. Geometric Mean model has the  
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17 349 right trend but concaves to an opposite direction to the experimental data. SCM model  
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20 350 has “quasi-linear” projections and higher predictions over all the saturation range.  
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22 351 De Vries’ (1963) model has close results at mediate saturation range 0.2 ~ 0.6. Empirical  
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25 352 models are quite fit with the experimental data points. Nevertheless, only Lu et al.’s  
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27 353 (2007) model captures the characteristic “flat-tail” behavior of thermal conductivity of  
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30 354 silt at low degrees of saturation with a slightly shift. Also Johansen’s and Lu et al.’s  
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32 355 models underestimate the thermal conductivity at lower saturation and overestimate at  
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35 356 higher saturation. Cote’s model has the best fit with experimental data but fails to  
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37 357 simulate the stasis behavior of thermal conductivity of silt at low degrees of saturation.  
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39 358 Figure 5b shows the 1:1 diagonal comparison, and Cote’s (2005) model has the lowest  
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42 359 NRMSE value of 0.145.  
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45 360 Figure 6 shows the comparison of experimental data of clay soil and the model  
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48 361 calculations. Parallel/Series model and the HS model again set the upper and lower  
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50 362 bounds of the thermal conductivity for the multi-phase mixture. Geometric mean always  
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52 363 cross the experimental curve and has the different shape of the curve. SCM model put all  
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55 364 experimental data points into the “external porosity” zone between the SCM and HS\_L  
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58 365 line. Other mathematical models have higher calculation result than the experimental  
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4 366 results. All empirical models have monotonically increases of thermal conductivity  
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6 367 simulation as the saturation increases, which are all far away from the true sigmoidal  
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9 368 behavior for clay. Figure 6b shows the 1:1 diagonal comparison, with the NRMSE values  
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11 369 ranging from 0.175 ~ 0.839.  
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15 370 By the above comparisons, the parallel/series bounds are much higher/lower than the  
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17 371 real thermal conductivity of the sand measured throughout the saturation range. However,  
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19 372 they set theoretical limits for all model predictions. The reason for that lies in the fact that  
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21 373 the parallel/series models assume each of the components are stacked in layers to form  
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23 374 the multiphase mixture. Those two represent the extreme cases of the heterogeneity of  
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25 375 material and geometry. HS bounds have an additional constraint by assuming the second  
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27 376 or other phases are uniformly dispersed in a continuous matrix. This homogenization  
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29 377 technique generates more realistic upper and lower limits and thus narrows the bandwidth  
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31 378 between two limits for the thermal conductivity of multiphase composites. However,  
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33 379 water distribution in multiphase soil is generally non-uniform. SCM further lowers the  
34  
35 380 upper bound by dividing the region bounded by HS model into “internal” and “external”  
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37 381 porosity parts, making SCM model better in upper bound predictions. The thermal  
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39 382 conductivities of soil at partially saturated conditions are always lower than SCM line  
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41 383 where the “external porosity” region is resulted from the with high-thermal-conductivity  
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43 384 soil particles “dispersed” in relatively low-thermal-conductivity air or water. This is not  
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45 385 physically realistic in unsaturated soil. The Geometric Mean model averages the thermal  
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47 386 conductivity of each phase in the unsaturated soil mixture but overestimates the  
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49 387 contribution of pore-water at low degree of saturation and underestimate it at high degree  
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388 of saturation. The actual thermal conductivity for unsaturated soils evolves from the HS  
389 lower bound at dry and approaches to the SCM line at full saturation. As mentioned, the  
390 averaging and homogenization techniques do not consider the air-water interface and  
391 pore-water distribution in the soil at different stages of saturation. They do incorporate  
392 the governing factors of constituent, and water content (given the thermal conductivity of  
393 each phase and their volume fraction), but miss the factors of soil type and particle  
394 contacts (i.e., the influences of particle geometry, mineral type, and particle connectivity),  
395 and different water retention mechanisms for different saturations.

396 The empirical models use certain mathematical functions to fit the experimental  
397 results and then determine the parameters in these functions to match with certain type of  
398 soil. The difficulty of this group to apply all soil types is the uncertainty in calculating  
399 thermal conductivity of the solid minerals, which makes it harder to determine two end  
400 members, the thermal conductivity of dry and saturated states. Additionally, the smooth  
401 curves generated by logarithmic, exponential or power equations also cannot represent  
402 the different water retention regimes observed in different types of soil. The empirical  
403 models work better for silt and clay than sand, but are still far from accurate when soil  
404 has rich clay-content at low degrees of saturation. The empirical models integrate the key  
405 factors of constituent in terms of the thermal conductivities at dry state  $k_{dry}$  and at full  
406 saturated state  $k_{sat}$ , and consider the factor of water content, but still omit the factors of  
407 soil type and particle contact, which depends on the micro-structure of the material.

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4 **408 A Unified Conceptual Model**  
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7 409 The change in thermal conductivity of soils with increasing degree of saturation  
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9 410 differs remarkably for different soil types. For sandy soils, the thermal conductivity  
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11 411 increases immediately once the soil experiences a small increase in degree of saturation,  
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13 412 with a sharp jump to almost 70 % of the heat transfer capability at full saturation during  
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15 413 wetting from dry conditions to approximately 5 % ~ 10 % degree of saturation. The  
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17 414 increase in thermal conductivity becomes inconspicuous as the soil approaches fully  
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19 415 saturated conditions. Inflection points are clearly observed for those separate regions at  
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21 416 different critical degrees of saturation, indicating transition from one heat transfer  
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23 417 mechanism to another.  
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30 418 For silt and clay soils, it can be observed that the thermal conductivity evolution with  
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32 419 changing degree of saturation is much smoother than sand, without sharp inflection  
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34 420 points. It can also be observed that the thermal conductivity values of silt or clay do not  
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36 421 increase until the degree of saturation increases up to a certain threshold. The higher fines  
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38 422 content of a soil, the longer the flat-tail development of thermal conductivity occurs at  
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40 423 low degrees of saturation (e.g., clay has longer stasis development of thermal  
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42 424 conductivity at low degrees of saturation, and therefore larger value of such threshold of  
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44 425 saturation than silt). As the saturation continues increasing, the thermal conductivity  
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46 426 gradually increases to gain most of the heat transfer capability in a short range of  
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48 427 saturation (e.g. 5 % ~ 20 % for silt, and 35% ~ 55 % for clay in this case). Next, the  
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50 428 thermal conductivity slowly increases to the maximum value and then keeps constant, as  
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52 429 the saturation increases. Generally the thermal conductivity of silt and clay is lower than  
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4 430 that of the sand over the entire range of saturation, due to the lower thermal property of  
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6 431 mineral constituents consist of fine soil than sands.  
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10 432 Based on the afore-mentioned analysis and observation, a conceptual model unifying  
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12 433 thermal conductivity behavior and soil water retention mechanisms is proposed and  
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15 434 shown in Figure 7. The full range of thermal conductivity variation with saturation can be  
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17 435 distinguished and conceptualized in four regimes: hydration, pendular, funicular, and  
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19 436 capillary. The range and boundary of each regime depends on the soil and liquid type and  
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21 437 micro-structures (i.e., particle geometry, particle/pore size distribution, pore-water  
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23 438 arrangement, and interfacial properties), which can be identified as the key governing  
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27 439 factors shown in Table 2. These factors have been largely ignored in all the existing  
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29 440 models. More importantly, these factors can also be characterized by the soil-water  
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31 441 retention curve (SWRC). At thermodynamic or multiphase equilibrium for a given  
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34 442 saturation, matric suction value indicates the information of certain pore size that holds  
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37 443 certain amount of pore water of that soil. The characteristic changing of slopes and  
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39 444 turning features on the SWRC curves reflect the effects of soil and liquid type and pore  
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41 445 size distribution. Therefore, SWRC is a promising tool to unify all significant governing  
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44 446 factors of the thermal conductivity of unsaturated soils for the proposed conceptual model.  
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48 447 As illustrated in Figure 7, from dry state to the first critical degree of saturation  
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50 448 defines the hydration state (i.e., regime *I*), where only hydration water formed with the  
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52 449 interaction of clay minerals. In this stage, water molecules are absorbed into mineral  
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55 450 complex, which do not change the soil particle network or connectivity, thus the thermal  
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57 451 properties of each constituent in soil. This is the physical reason why in rich fine-content  
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4 452 soil like silt and clay at the low degree of saturation, the thermal conductivity of bulk soil  
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6 453 barely changes (e.g.,  $S$  from 0 to 0.05 for silt or 0.35 for clay). The higher the fines  
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9 454 content, the wider the range of this regime, and the higher value of this first critical  
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11 455 saturation value. In the hydration regime, although pore water has been involved, the soil  
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14 456 system behaves still like a two-phase mixture (solid/air), where the thermal conductivity  
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16 457 follows the path close to the lower bounds (series/HS lower).  
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20 458 As the saturation increases, pore water starts to form thin film around the soil particle  
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22 459 and build individual water bridges at the particle contacts, which is defined as pendular  
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24 460 regime 2. The water film/bridge overcomes the huge contact resistance in the dry soil-air-  
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27 461 solid contact conduction and significantly improves the connectivity of thermal transfer  
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30 462 paths, which contributes the rapid increase of the thermal conductivity. Sandy soil has  
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32 463 very limited hydration stage; therefore the pendular regime dominates the low saturation  
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34 464 region (e.g.,  $S = 0.05 \sim 0.1$ ). However, silty and clayey soil can have considerable amount  
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37 465 of hydration water, which prevails the low saturation region and gradually transits to the  
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39 466 pendular regime. The hydration water makes the thermal conductivity almost constant at  
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41 467 the early stage of wetting, and the overlap between hydration and pendular regime makes  
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44 468 the rapid smooth increase of thermal conductivity of silt or clay as the water content  
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47 469 increase but not as steep as that of sand. The significant increase in water meniscus at the  
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49 470 particle contacts is the physical reason why thermal conductivity in all types of soil  
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51 471 increases rapidly within the pendular regime. The pendular regime can be  $S = 0.05 \sim 0.15$   
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53 472 for silt or  $0.35 \sim 0.55$  for clay. Of all the previous models, Only Cote's equation predicts  
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56 473 closely to the measured data in this regime for silt. All empirical models fail to capture  
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4 474 the rapid change of the thermal conductivity of sand in this regime, and these models also  
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6 475 cannot accurately predict thermal conductivity variations in clayey soil.  
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10 476 As the saturation continues to increase, the liquid bridges grow and begin to merge  
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12 477 with adjacent ones and start to form a thick connected water-film around the soil particles,  
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14 478 fill up the void space of pores. This regime carries on the buildup of the pore-water  
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16 479 network, and this is the physical reason why the connectivity of heat transfer paths is  
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18 480 further enhanced, leading to a gradual increase of thermal conductivity in funicular  
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20 481 regime 3 (Figure 7). Because of the high thermal conductivity of solid minerals and the  
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22 482 pore-water network, the heat flow goes in preference of the solid skeleton connected by  
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24 483 pore-water at particle contacts. Hence, further increase of water content does not  
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26 484 remarkably improve the thermal conductivity of the system as much as that in pendular  
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28 485 regime. Here, the thermal conductivity of soil gradually increases to approach the  
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30 486 maximum value (e.g.,  $S = 0.1 \sim 0.8$  for soil,  $0.15 \sim 0.85$  for silt and  $0.55 \sim 0.85$  for clay).  
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32 487 The three empirical models have good fit in this section with silt, but underestimate the  
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34 488 values of sand and overestimate the values of clay.  
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43 489 At even higher saturation, the pore-water occupies most of the voids instead of air,  
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45 490 which does not further increase the thermal conductivity of the soil mixture in capillary  
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47 491 regime 4 (Figure 7). In this regime, the increase of the pore water either does not further  
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49 492 alter the preference of heat flow through the soil skeleton or improve the connectivity of  
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51 493 heat transfer pathway. The range of capillary regime for sand is around  $0.8 \sim 1$ , and  
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53 494  $0.85 \sim 1$  for silt and clay. From mediate to high degree of saturation, all models are  
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55 495 converging to the upper bound (SCM line). Nevertheless for the empirical models, which  
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4 496 are better than others, the performance is limited by the accurate calculation the saturated  
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6 497 thermal conductivity, which is estimated by a geometric mean of solid and liquid phase.  
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10 498 As shown, by linking the water retention mechanisms and thermal conductivity  
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12 499 variation behavior, and by comparing the previous models with the experimental data for  
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14 500 different types of soil, the large discrepancies predicted by the previous models lay in the  
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16 501 overlook of the roles of different water retention regimes in thermal conductivity  
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18 502 variation. None of the models explicitly used the governing factors of soil and liquid  
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20 503 types and pore size distribution that characterize the soil water retention. The existing  
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22 504 models are all pore-structure independent. In two-phase systems, the previous theoretical  
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24 505 models are close to experiment results for certain type of soil. However, when air and  
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26 506 water are both involved, the complexity of pore-water network and connections is  
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28 507 increased due to the air-water interfaces. How water phase distributed in the pore space at  
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30 508 different degrees of saturation defines the connectivity of the pore water in the soil  
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32 509 skeleton and therefore determines the thermal behavior of unsaturated soils.  
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## 40 510 **Conclusions**

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43 511 This paper reviews the existing models of thermal conductivity for unsaturated soil,  
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45 512 assess these models' predictability, and thereby identify the controlling factors for  
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47 513 thermal conduction in porous media. The existing thermal conductivity models are  
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49 514 assessed and categorized into three groups based on their assumptions and principles.  
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51 515 *Mixing models* simulate the thermal conductivity of mixtures by a combination of simple  
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53 516 parallel or series elements consisting of the composite. *Mathematical models* adopt the  
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4 517 homogenization approaches to calculate the thermal conductivity of a uniform composite.  
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6 518 *Empirical models* use different functions to fit the experimental measurements and their  
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9 519 parameters to apply to certain soil types. All model predictions are compared with  
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11 520 experimental data of three types of unsaturated soil: sand, silt and clay. It is found that  
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14 521 these models are either work only at portions of saturation range or apply to certain soil  
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16 522 type. Most importantly, these models lack the consideration of some key governing  
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19 523 factors such as soil and liquid type and pore size distribution. It is found that the pore-  
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21 524 scale water distribution based on thermodynamic equilibrium conditions among air-  
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24 525 water-solid interfaces during different saturation states have not been reflected in all the  
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26 526 existing models.

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30 527 A conceptual model is proposed based on the analysis of heat transfer mechanisms  
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32 528 and the discussion about missing factors from existing models in terms of the effect of  
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35 529 pore water distribution at different water retention regimes. These water retention  
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37 530 regimes are governed by the SWRC and can be used to fully reconcile variations of  
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39 531 thermal conductivity with degree of saturation for all types of soil. Quantitative linkages  
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42 532 between the pore size distribution, soil-water interaction mechanisms, and the bulk  
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44 533 thermal conductivity variation behavior can be established in light of the SWRC.

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649 uses solid lines, group II dot-dash lines, and group III dot lines), and (b) 1:1  
650 comparison space for predicted and measured values (group I uses symbols -, +, x;  
651 group II hollow symbols  $\square$ ,  $\diamond$ ,  $\Delta$ ,  $\circ$ ; group III solid-filled symbols  $\square$ ,  $\Delta$ ,  $\circ$ ).

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656 group II hollow symbols  $\square$ ,  $\diamond$ ,  $\Delta$ ,  $\circ$ ; group III solid-filled symbols  $\square$ ,  $\Delta$ ,  $\circ$ ).

657 **Figure 7.** Conceptual model of thermal conductivity: (a) thermal conductivity and soil-  
658 water retention, and (b) regimes of pore-water distribution.

**Table 1**

Material	Density [kg/m <sup>3</sup> ]	Heat capacity [kJ/kg·K]	Thermal conductivity [W/m·K]	Thermal diffusivity [m <sup>2</sup> /s] × 10 <sup>-7</sup>
Air (10 °C)	1.25	1.000	0.0026	0.21
Water (25 °C)	999.87	4.200	0.56	1.43
Water vapor (1 atm, 400 K)	-	1.901	0.016	233.8
Ice (0 °C)	917	2.040	2.25	12
Quartz	2660	0.733	8.40	43.08
Granite	2750	0.890	1.70 ~ 4.00	~ 12
Gypsum	1000	1.090	0.51	4.7
Limestone	2300	0.900	1.26 ~ 1.33	~ 5
Marble	2600	0.810	2.80	13
Mica	2883	0.880	0.75	2.956
Clay	1450	0.880	1.28	10
Sandstone	~ 2270	0.710	1.60 ~ 2.10	10 ~ 13

Source: Bejan, Adrian; Kraus, Allan D. (2003). Heat Transfer Handbook. John Wiley & Sons.

**Table 2**

Key governing factors	Features
Constituent	• Thermal conductivity of solid minerals dominates the bulk property.
Soil type	• Different type of soil has different soil minerals, particle sizes and shapes.
Water content	• The volume fraction of water phase determines the thermal conductivity of partially saturated soils.
Particle contact	• The coordination number affected by stress level, packing density, gradation et al. defines the thermal conduction skeleton.

**Table 3**

Models	Equations
Series	$\frac{1}{k_{\text{eff}}} = \sum_{i=1}^3 \phi_i \cdot \frac{1}{k_i}$
Parallel	$k_{\text{eff}} = \sum_{i=1}^3 \phi_i \cdot k_i$
Geometric mean	$k_{\text{eff}} = \prod_{i=1}^3 k_i^{\phi_i}$
Quadratic parallel	$k_{\text{eff}} = \left( \sum_{i=1}^3 \phi_i \cdot \sqrt{k_i} \right)^2$
Effective medium	$\sum_i^3 \left( \phi_i \frac{k_i - k_{\text{eff}}}{k_i + 2k_{\text{eff}}} \right) = 0$
Self-consistent	$k_{\text{eff}} = \frac{1}{3} \left( \sum_{i=1}^3 \frac{\phi_i}{k_i + 2k_{\text{eff}}} \right)^{-1}$
De Vries (1963)	$k_{\text{eff}} = \frac{\phi_w k_w + F_s \phi_s k_s + F_a \phi_a k_a}{\phi_w + F_s \phi_s + F_a \phi_a},$ <p><math>F_i</math> and <math>g_i</math> are weight and shape factors</p>
Hashin-Strikmann	$k_{\text{HS}}^L = k_1 + 3k_1 \frac{\sum_{i=2}^3 (\phi_i / (1 + c_i^L))}{\phi_1 + \sum_{i=2}^3 (\phi_i c_i^L / (1 + c_i^L))},$ $c_i^L = 3k_1 / (k_i - k_1)$ $k_{\text{HS}}^U = k_3 + 3k_3 \frac{\sum_{i=1}^2 (\phi_i / (1 + c_i^U))}{\phi_3 + \sum_{i=1}^2 (\phi_i c_i^U / (1 + c_i^U))},$ $c_i^U = 3k_3 / (k_i - k_3)$

Key factors:

$\phi_i$  - volume fraction of each phase, i can be a (air) w (water) s (solid), [-]

$k_i$  - thermal conductivity of each phase, ( $k_a = 0.56$ ,  $k_w = 0.026$ ), [W/mK]

**Table 4**

Models	Parameters					Equations
	$k_i$	$S$ ( $\theta$ )	$n$	$\gamma_{dry}$	$k_e, k_{dry}, k_{sat}$	
Kersten (1949)		-	-	-		Silt- clay: $k_{eff} = 0.1442(\log \theta - 0.2) \cdot 10^{0.6243 \cdot \gamma_{dry}}$ Sandy soil: $k_{eff} = 0.1442(\log \theta - 0.2) \cdot 10^{0.6243 \cdot \gamma_{dry}}$
Johansen (1975)	$k_s = k_q^q k_o^{(1-q)}$	-	-	-	$k_{sat} = k_s^{(1-n)} k_w^n$ $k_{dry} = \frac{0.135 \gamma_{dry} + 64.7}{2700 + 0.947 \gamma_{dry}}$	Coarse sand: $k_e = 0.7 \log S + 1.0$ Fine soil: $k_e = \log S + 1.0$
Cote (2005)	$k_s = k_q^q k_o^{(1-q)}$	-	-	-	$k_{sat} = k_s^{(1-n)} k_w^n$ $k_{dry} = \chi \times 10^{-\eta n}$	$k_e = \frac{\kappa S}{1 + (\kappa - 1)S}$ $\chi$
Lu (2007)	$k_s = k_q^q k_o^{(1-q)}$	-	-	-	-	$k_e = \exp[\alpha(1 - S^{\alpha-1.33})]$ $\alpha$
Chen (2008)	-	-	-	-	-	$k_{eff} = k_s^{1-n} k_w^n n \cdot [(1-b)S + b]^{cn}$ $b, c$

**Key parameters:**

$S$  - degree of saturation,  $S = \square_w / (1 - \square_s)$ , [-]

$\theta$  - volumetric water content,  $\theta = (n \cdot S)$ , [-]

$n$  - porosity,  $n = (1 - \square_s)$ , [-]

$\gamma_{dry}$  - dry bulk density,  $\gamma_{dry} = G_s(1 - n)$ , [kg/m<sup>3</sup>]

$k_q$  - thermal conductivity of quartz, [W/mK]

$k_o$  - thermal conductivity of organic materials, [W/mK]

$k_e$  - Kersten number,  $k_e = (k_{eff} - k_{dry}) / (k_{sat} - k_{dry})$ , [-]

**Empirical parameters:**

$\alpha, \chi, b, c$  - fitted parameters for different

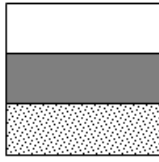
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**Table 5**

Experimental Data		Parameters					
		$k_s$ [W/mK]	$G_s$ [unitless]	$n$ [unitless]	$\gamma_{dry}$ [kg/m <sup>3</sup> ]	$k_{dry}$ [W/mK]	$k_{sat}$ [W/mK]
Sands	Accusand 40/50	4.75	2.65	0.37	1.696	0.07/0.28	2.27
Silt	Bonny silt	2.5	2.53	0.39	1.25	0.06/0.26	1.42
Clay	Tamawaski (2002)	2.5	2.65	0.5	1.325	0.06/0.19	1.18

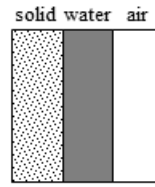
Cross validation		Mixing models			Mathematical models				Empirical models		
		Parallel	Series	Geo_Mean	HS_L	HS_U	SCM	De Vries 1952	Johansen 1975	Cote 2005	Lu 2007
Sand	NRMSE	0.631	0.819	0.395	0.747	0.456	0.333	0.253	0.374	0.282	0.330
	CV(RMSE)	0.745	0.966	0.466	0.881	0.538	0.393	0.299	0.441	0.333	0.389
Silt	NRMSE	1.019	0.788	0.219	0.628	0.769	0.576	0.295	0.224	0.145	0.310
	CV(RMSE)	1.097	0.848	0.236	0.676	0.828	0.620	0.317	0.241	0.156	0.334
Clay	NRMSE	0.839	0.612	0.175	0.462	0.635	0.437	0.311	0.297	0.355	0.255
	CV(RMSE)	0.997	0.728	0.208	0.549	0.755	0.520	0.370	0.353	0.422	0.303

**Table 6**



Series (Wiener lower)

$$k_{\text{eff}} = \sum_{i=1}^3 (\phi_i \cdot k_i)$$



Parallel (Wiener upper)

$$\frac{1}{k_{\text{eff}}} = \sum_{i=1}^3 \left( \frac{\phi_i}{k_i} \right)$$

Figure 1

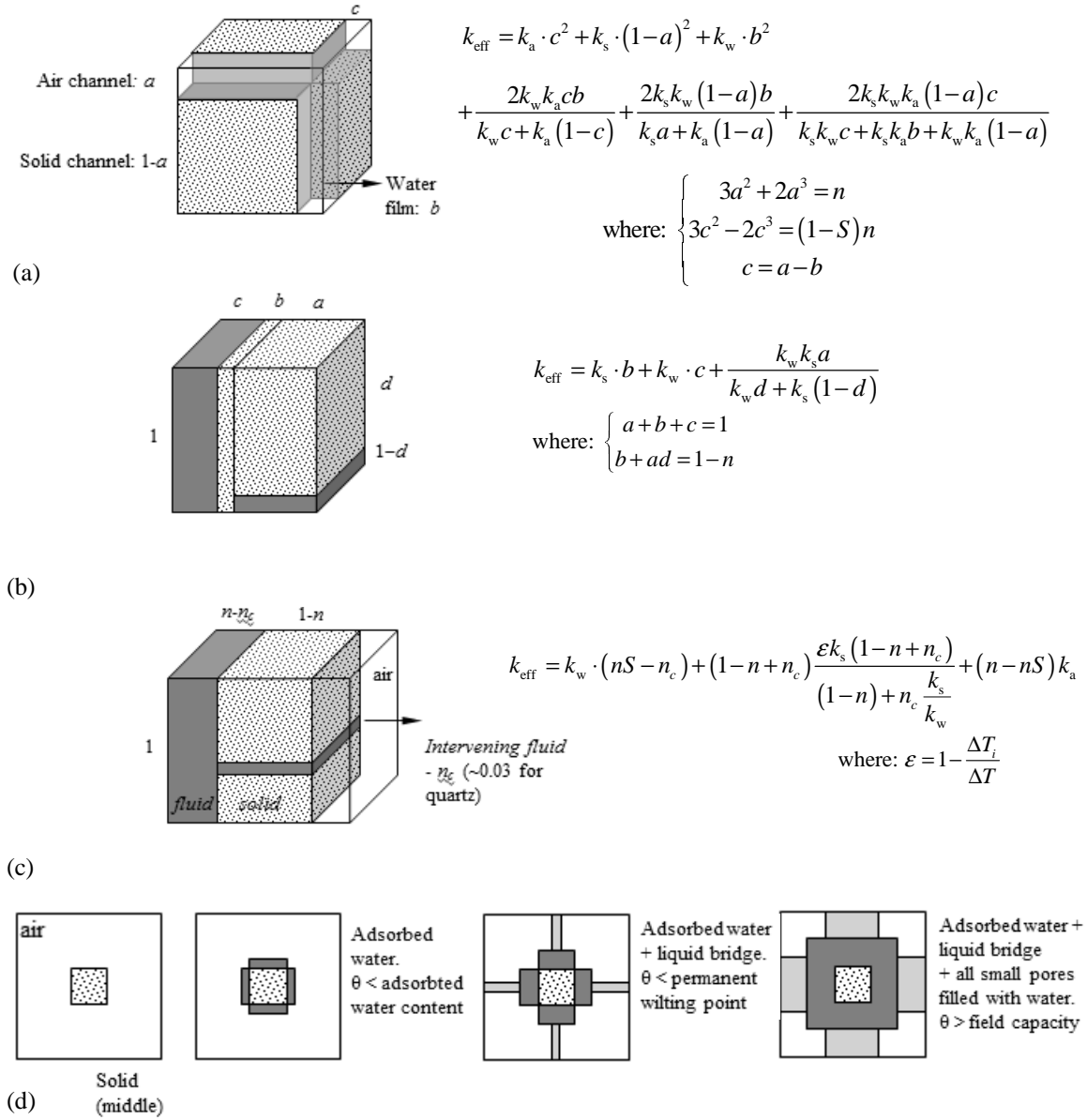
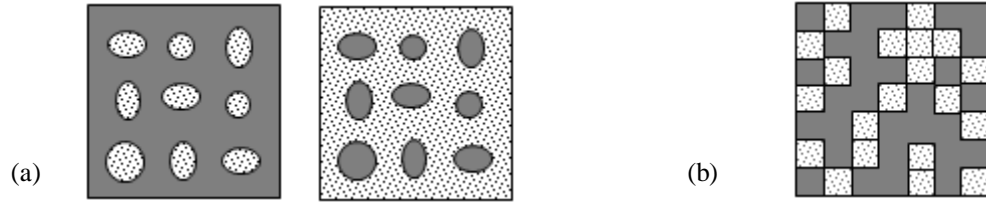
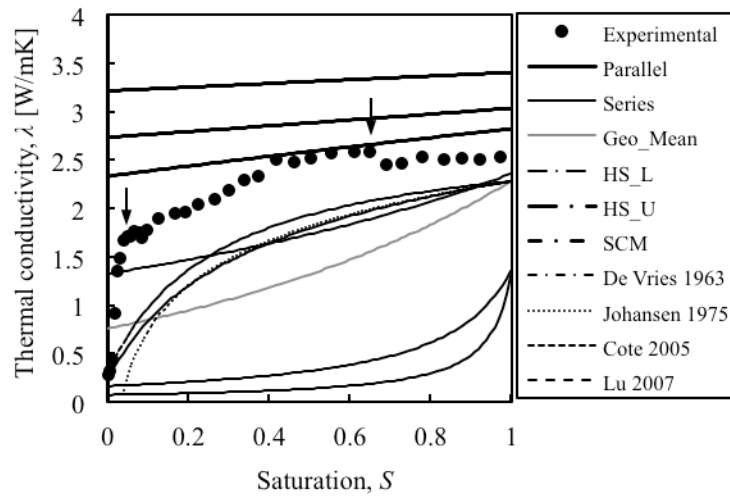


Figure 2

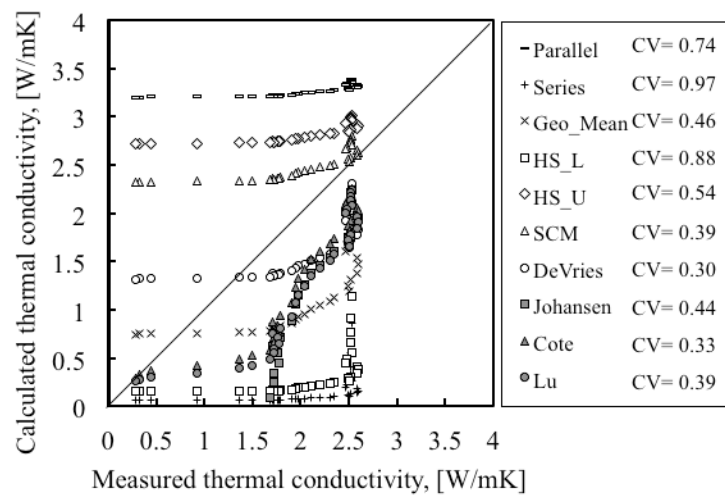




**Figure 3**

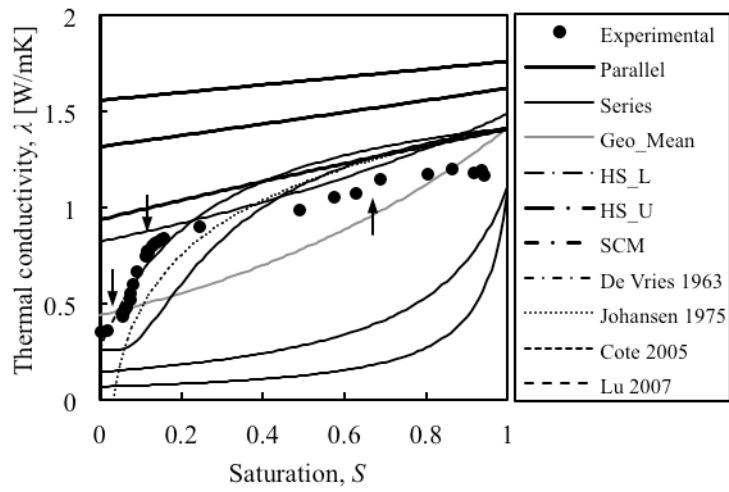


(a)

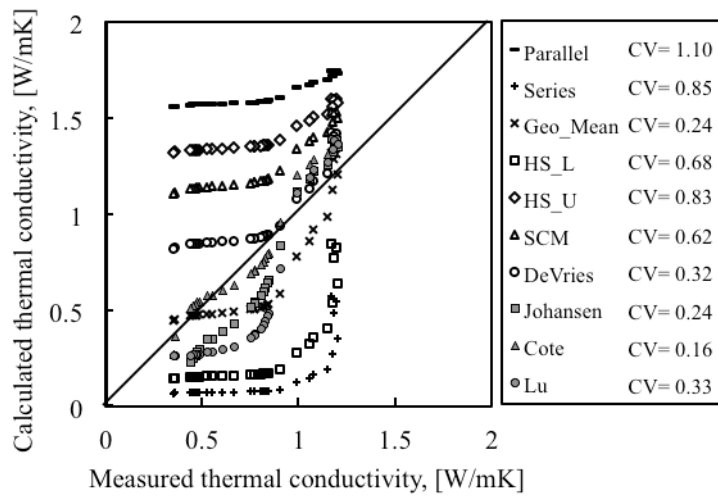


(b)

Figure 4

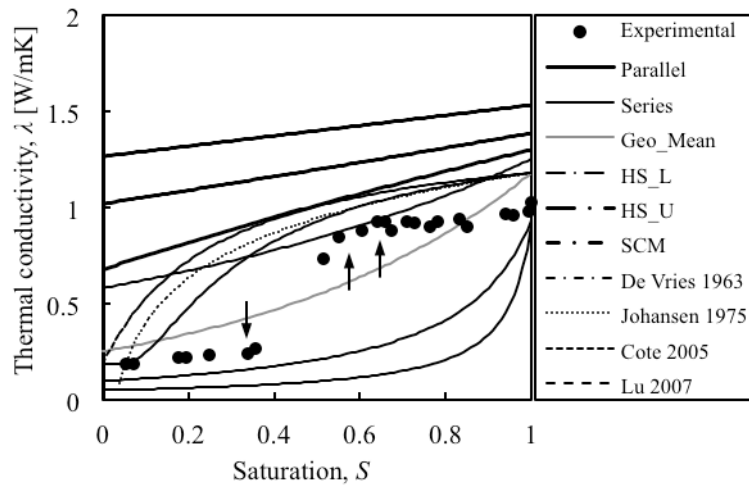


(a)

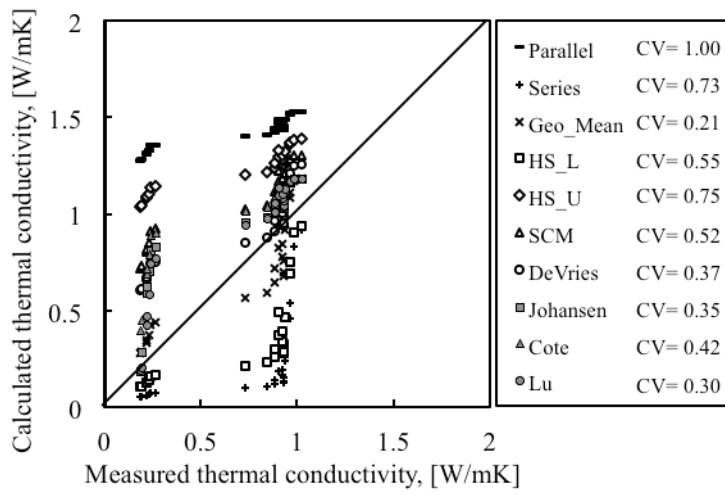


(b)

Figure 5

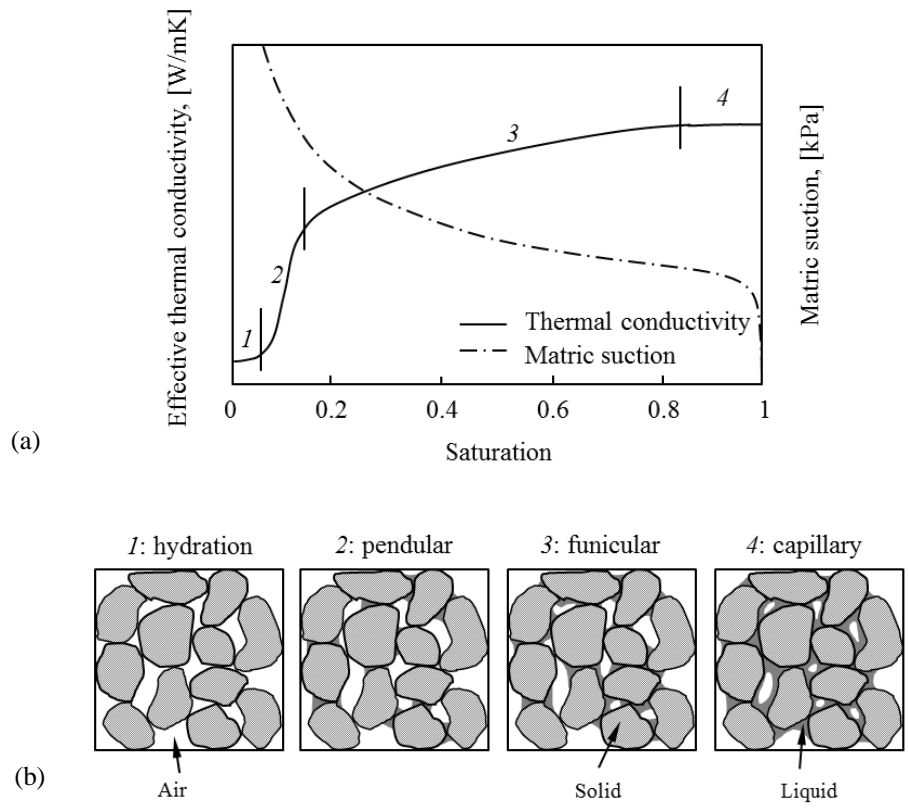


(a)



(b)

Figure 6



**Figure 7**