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CALCULATION OF GEOMETRICAL EFFICIENCY OF COUNTERS

Louis R. Henrich

August 8, 1952

Berkeley, California

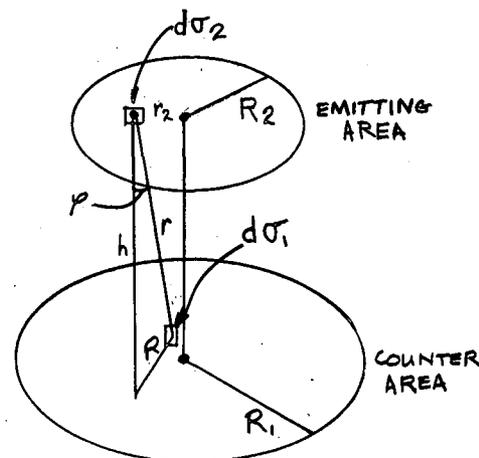
CALCULATION OF GEOMETRICAL EFFICIENCY OF COUNTERS

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August 8, 1952

Let F be the flux of radiation going from a sample of radius R_2 to a counter of radius R_1 then Φ the emitted intensity of radiation per unit solid angle. Then the flux between two elements of area $d\sigma_1$ and $d\sigma_2$ will be given by



$$dF = \Phi \frac{\cos \phi}{r^2} d\sigma_1 d\sigma_2 \quad (1)$$

Since

$$\cos \phi = \frac{h}{r} = \frac{h}{\sqrt{h^2 + R_1^2}} \quad (2)$$

and

$$dF = \Phi \frac{h}{r^3} d\sigma_1 d\sigma_2 \quad (3)$$

a. Point Source ($R_1 \geq R_2$)

If there is only one element of area emitting ($d\sigma_2$) then the total emitted flux is $4\pi\Phi d\sigma_2$ and the geometrical counting efficiency will be given by

$$dG = \frac{dF}{4\pi\Phi d\sigma_2} = \frac{h}{r^3} d\sigma_1 \quad (4)$$

To integrate over σ_1 we set

$$d\sigma_1 = R dR d\alpha \quad (5)$$

Then by symmetry we can limit $0 \leq \alpha \leq \pi$ and put in a factor 2 so that in this case

$$dG = \frac{h}{2\pi r^3} R dR d\alpha \quad (6)$$

Then integrate first over α .

For $R_2 + R \leq R_1$, $0 \leq \alpha \leq \pi$;

and for $R_2 + R \geq R_1$

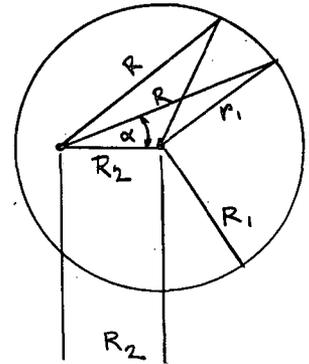
the upper limit on α is determined by

$$R_1^2 = R^2 + R_2^2 - 2 R_2 R \cos \alpha ,$$

(7)

or

$$\alpha = \cos^{-1} \left[\frac{R^2 + R_2^2 - R_1^2}{2 R_2 R} \right] , \quad (8)$$



so

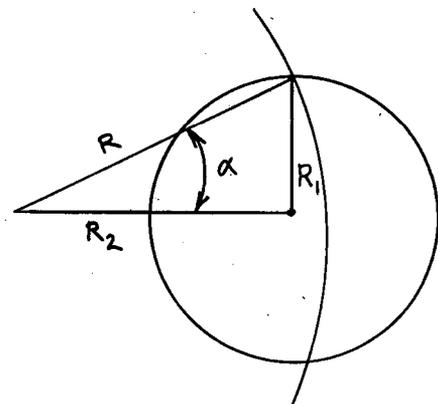
$$G = \frac{h}{2} \int_0^{R_1-R_2} \frac{R \, dR}{r^3} + \frac{h}{2\pi} \int_{R_1-R_2}^{R_1+R_2} \cos^{-1} \left\{ \frac{R^2 + R_2^2 - R_1^2}{2 R_2 R} \right\} \frac{R \, dR}{r^3} \quad (9)$$

or

$$G = \frac{1}{2} \left[1 - \frac{h}{\sqrt{(R_1 - R_2)^2 + h^2}} \right] + \frac{h}{2\pi} \int_{R_1-R_2}^{R_1+R_2} \cos^{-1} \left\{ \frac{R^2 + R_2^2 - R_1^2}{2 R_2 R} \right\} \frac{R \, dR}{r^3} \quad (10)$$

b. Point Source ($R_2 \geq R_1$)

In this case α is determined as in Eq. (8). The limits on integration are different so that here we have



$$G = \frac{h}{2\pi} \int_{R_2-R_1}^{R_2+R_1} \cos^{-1} \left\{ \frac{R^2 + R_2^2 - R_1^2}{2 R_2 R} \right\} \frac{R \, dR}{r^3} \quad (11)$$

This agrees with the stated expression in Calvin, "Isotopic Carbon", App. IV, for $R_2 > R_1$.

b. Circular Source $R_1 \geq R_2$

In this case the total flux is $4\pi\Phi(\pi R_2^2)$ so that

$$dG = \frac{dF}{4\pi^2 R_2^2 \Phi} = \frac{h}{4\pi^2 R_2^2} \frac{d\sigma_1 d\sigma_2}{r^3} \quad (12)$$

Let

$$d\sigma_2 = r_2 dr_2 d\theta \quad (13)$$

$$d\sigma_1 = R dR d\alpha \quad (14)$$

We can integrate readily over θ (and restrict $0 \leq \alpha \leq \pi$ so that we get

$$dG(r_2, R, \alpha) = \frac{h}{\pi R_2^2} \frac{r_2 dr_2 R dR d\alpha}{r^3} \quad (15)$$

Now integrate over α

$$dG(r_2, R) = \left(\frac{h}{\pi R_2^2} \frac{R dR}{r^3} \right) (\alpha r_2 dr_2) \quad (16)$$

Now either we shall have

$$\alpha = \pi$$

or

$$\alpha = \cos^{-1} \left\{ \frac{R^2 + r_2^2 - R_1^2}{2 r_2 R} \right\}, \quad (17)$$

depending on the circumstances.

We now want to find out over what ranges the integration of R and r_2 must be carried for different values of φ . If we take a fixed r_2 we can draw as follows the ranges of integration for R .

Let $r_2 \leq R_1$ then

for $r_2 + R \leq R_1$; $\varphi = \pi$; and $0 \leq R \leq R_1 - r_2$

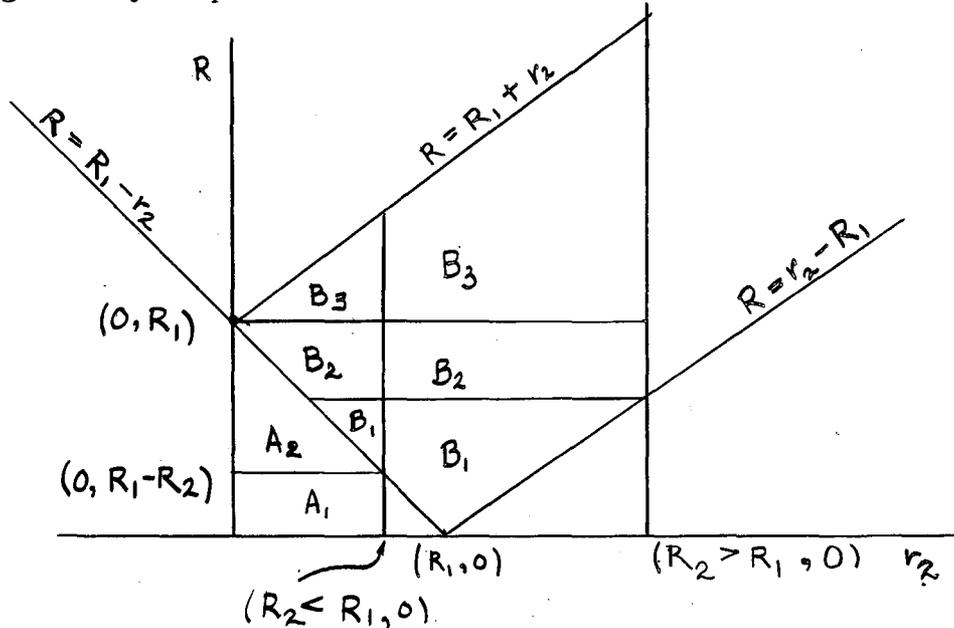
$$r_2 + R \geq R_1; \quad \varphi = \cos^{-1} \left\{ \frac{R^2 + r_2^2 - R_1^2}{2 r_2 R} \right\}; \quad \text{and}$$

$$R_1 - r_2 \leq R \leq R_1 + r_2.$$

While for $r_2 \geq R_1$; $\varphi = \cos^{-1} \left\{ \frac{R^2 + r_2^2 - R_1^2}{2 r_2 R} \right\}$; and

$$r_2 - R_1 \leq R \leq r_2 + R_1.$$

These regions may be plotted in two dimensions as follows in the R, r_2 plane



If we change the order of integration to an integration over r_2 first, we have various regions to consider.

b1 (R₁ > R₂)

Region	Limits	α
A ₁	$0 \leq r_2 \leq R_2 ; \quad 0 \leq R \leq R_1 - R_2$	π
A ₂	$0 \leq r_2 \leq R_1 - R ; \quad R - R_2 \leq R \leq R_1$	π
B ₁ + B ₂	$R_1 - R \leq r_2 \leq R_2 ; \quad R_1 - R_2 \leq R \leq R_1$	Eq. (17)
B ₃	$R - R_1 \leq r_2 \leq R_2 ; \quad R_1 \leq R \leq R_1 + R_2$	Eq. (17).

b2 (R₁ < R₂)

Region	Limits	α
A	$0 \leq r_2 \leq R_1 - R ; \quad 0 \leq R \leq R_1$	π
B ₁	$R_1 - R \leq r_2 \leq R + R_1 ; \quad 0 \leq R \leq R_2 - R_1$	Eq. (17)
B ₂	$R_1 - R \leq r_2 \leq R_2 ; \quad R_2 - R_1 \leq R \leq R_1$	Eq. (17)
B ₃	$R - R_1 \leq r_2 \leq R_2 ; \quad R_1 \leq R \leq R_1 + R_2$	Eq. (17)

Now to evaluate the expressions for G in the two cases we have to integrate expressions of the type

$$\int \varphi r_2 dr_2$$

where $\varphi = \pi$ or $\varphi = \cos^{-1} \left[\frac{R^2 + r_2^2 - R_1^2}{2 R r_2} \right]$

for $\varphi = \pi$ $\int_a^b \varphi r_2 dr_2 = \frac{\pi}{2} r_2^2 \Big|_a^b$ (18)

For the other case

$$\int_a^b \left\{ \cos^{-1} \left[\frac{R^2 - R_1^2 + r_2^2}{2 R r_2} \right] r_2 \right\} dr_2$$

$$= \left\{ \begin{aligned} & \frac{r_2^2}{2} \cos^{-1} \left[\frac{R^2 - R_1^2 + r_2^2}{2 R r_2} \right] - \frac{R_1^2}{2} \sin^{-1} \frac{R^2 + R_1^2 - r_2^2}{2 R R_1} \\ & - \frac{1}{4} \sqrt{4 r_2^2 R^2 - (R^2 - R_1^2 + r_2^2)^2} \end{aligned} \right\}_a^b$$

(19)

Evaluating this \int at the various limits needed, we find that

$$\begin{aligned}
 & \int^{R_2} \cos^{-1} \left[\frac{R^2 - R_1^2 + r_2^2}{2 R r_2} \right] r_2 dr_2 \\
 &= \frac{R_2^2}{2} \cos^{-1} \left[\frac{R^2 - R_1^2 + R_2^2}{2 R R_2} \right] - \frac{R_1^2}{2} \sin^{-1} \left[\frac{R^2 + R_1^2 - R_2^2}{2 R R_1} \right] \\
 &\quad - \frac{1}{4} \sqrt{4 R_2^2 R - (R^2 - R_1^2 + R_2^2)^2} + C \\
 &= \frac{\pi R_2^2}{4} - \frac{\pi R_1^2}{4} + f(R_2) + C, \tag{20}
 \end{aligned}$$

where

$$\begin{aligned}
 f(R_2) &= \frac{R_1^2}{2} \cos^{-1} \left[\frac{R^2 + R_1^2 - R_2^2}{2 R R_1} \right] - \frac{R_2^2}{2} \sin^{-1} \left[\frac{R^2 + R_2^2 - R_1^2}{2 R R_2} \right] \\
 &\quad - \frac{1}{4} \sqrt{4 R^2 R_1^2 - (R^2 + R_1^2 - R_2^2)^2}. \tag{21}
 \end{aligned}$$

$$\int^{R_1-R} \cos^{-1} \left[\frac{R^2 - R_1^2 + r_2^2}{2 R r_2} \right] r_2 dr_2 = \frac{\pi}{2} (R_1 - R)^2 - \frac{\pi}{4} R_1^2 + C \tag{20a}$$

$$\int_{R-R_1}^R \cos^{-1} \left[\frac{R^2 - R_1^2 + r_2^2}{2 R r_2} \right] r_2 dr_2 = -\frac{\pi}{4} R_1^2 + C \quad (20b)$$

$$\int_{R_1+R}^R \cos^{-1} \left[\frac{R^2 - R_1^2 + r_2^2}{2 R r_2} \right] r_2 dr_2 = \frac{\pi R_1^2}{4} + C \quad (20c)$$

$$\int_{R_2}^R \cos^{-1} \left[\frac{R^2 - R_1^2 + r_2^2}{2 R r_2} \right] r_2 dr_2 = \frac{\pi R_2^2}{4} - \frac{\pi R_1^2}{4} + f(R_2) + C. \quad (20d)$$

We now substitute the limits in the expression for G.

Case b₁. (R₁ > R₂)

$$G = \frac{h}{\pi R_2^2} \left\{ \int_0^{R_1-R_2} \frac{R dR}{r^3} \left[\frac{\pi}{2} R_2^2 \right] \int_{R_1-R_2}^{R_1} \frac{R dR}{r^3} \frac{\pi}{2} (R_1 - R)^2 \right. \\ \left. + \int_{R_1-R_2}^{R_1} \frac{R dR}{r^3} \left[\frac{\pi R_2^2}{4} - \frac{\pi R_1^2}{4} + f(R_2) - \frac{\pi}{2} (R_1 - R)^2 + \frac{\pi}{4} R_1^2 \right] \right. \\ \left. + \int_{R_1}^{R_1+R_2} \frac{R dR}{r^3} \left[\frac{\pi R_2^2}{4} - \frac{\pi R_1^2}{4} + f(R_2) + \frac{\pi}{4} R_1^2 \right] \right\}$$

(22) Cont.

$$\begin{aligned}
 &= \frac{h}{2} \int_0^{R_1-R_2} \frac{R \, dR}{r^3} + \frac{h}{4} \int_{R_1-R_2}^{R_1+R_2} \frac{R \, dR}{r^3} + \frac{h}{R_2^2} \int_{R_1-R_2}^{R_1+R_2} f(R_2) \frac{R \, dR}{r^3} \\
 &= \frac{1}{2} \left[1 - \frac{h}{\sqrt{h^2 + (R_1 - R_2)^2}} \right] + \frac{1}{4} \left[\frac{h}{\sqrt{h^2 + (R_1 - R_2)^2}} - \frac{h}{\sqrt{h^2 + (R_1 + R_2)^2}} \right] \\
 &\quad + \frac{h}{R_2^2} \int_{R_1-R_2}^{R_1+R_2} f(R_2) \frac{R \, dR}{r^3} \\
 &= \frac{1}{2} \left[1 - \frac{1}{2} \frac{h}{\sqrt{h^2 + (R_1 - R_2)^2}} + \frac{h}{\sqrt{h^2 + (R_1 + R_2)^2}} \right] \\
 &\quad + \frac{h}{R_2^2} \int_{R_1-R_2}^{R_1+R_2} f(R_2) \frac{R \, dR}{r^3}
 \end{aligned}$$

(22)

which is formula (4) in Calvin, "Isotopic Carbon", App. IV.

Case b₂ (R₁ < R₂):

$$\begin{aligned}
 G &= \frac{h}{\pi R_2^2} \left\{ \int_0^{R_1} \frac{R \, dR}{r^3} \left[\frac{\pi}{2} (R_1 - R)^2 \right] + \int_0^{R_2 - R_1} \frac{R \, dR}{r^3} \left[\frac{\pi R_1^2}{4} - \frac{\pi (R_1 - R)^2}{2} + \frac{\pi R_1^2}{4} \right] \right. \\
 &\quad \left. + \int_{R_2 - R_1}^{R_1} \frac{R \, dR}{r^3} \left[\frac{\pi R_2^2}{4} - \frac{\pi R_1^2}{4} + f(R_2) - \frac{\pi (R_1 - R)^2}{2} + \frac{\pi R_1^2}{4} \right] \right. \\
 &\quad \left. + \int_{R_1}^{R_1 + R_2} \frac{R \, dR}{r^3} \left[\frac{\pi R_2^2}{4} - \frac{\pi R_1^2}{4} + f(R_2) + \frac{\pi R_1^2}{4} \right] \right\} \\
 &= \frac{h}{\pi R_2^2} \left\{ \frac{\pi R_1^2}{2} \int_0^{R_2 - R_1} \frac{R \, dR}{r^3} + \frac{\pi R_2^2}{4} \int_{R_2 - R_1}^{R_1 + R_2} \frac{R \, dR}{r^3} + \int_{R_2 - R_1}^{R_1 + R_2} f(R_2) \frac{R \, dR}{r^3} \right\} \\
 &= \frac{1}{2} \left\{ \frac{R_1^2}{R_2^2} \left[1 - \frac{h}{\sqrt{h^2 + (R_2 - R_1)^2}} \right] + \frac{1}{2} \left[\frac{h}{\sqrt{h^2 + (R_2 - R_1)^2}} - \frac{h}{\sqrt{h^2 + (R_1 + R_2)^2}} \right] \right\} \\
 &\quad + \frac{h}{\pi R_2^2} \int_{R_2 - R_1}^{R_1 + R_2} f(R_2) \frac{R \, dR}{r^3}, \tag{23}
 \end{aligned}$$

which is formula (5) in Calvin.

