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Weakened Topological Protection of the Quantum Hall Effect in a Cavity

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We study the quantum Hall effect in a two-dimensional homogeneous electron gas coupled to a quantum cavity field. As initially pointed out by Kohn, Galilean invariance for a homogeneous quantum Hall system implies that the electronic center of mass (c.m.) decouples from the electron-electron interaction, and the energy of the c.m. mode, also known as Kohn mode, is equal to the single particle cyclotron transition. In this work, we point out that strong light-matter hybridization between the Kohn mode and the cavity photons gives rise to collective hybrid modes between the Landau levels and the photons. We provide the exact solution for the collective Landau polaritons and we demonstrate the weakening of topological protection at zero temperature due to the existence of the lower polariton mode which is softer than the Kohn mode. This provides an intrinsic mechanism for the recently observed topological breakdown of the quantum Hall effect in a cavity [F. Appugliese *et al.*, Breakdown of topological protection by cavity vacuum fields in the integer quantum Hall effect, Science **375**, 1030 (2022).]. Importantly, our theory predicts the cavity suppression of the thermal activation gap in the quantum Hall transport. Our work paves the way for future developments in cavity control of quantum materials.

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Interaction and topology give rise to rich exotic phases of matter, among which the integer quantum Hall (IQH) effect and the fractional quantum Hall (FQH) effect stand out [1–4]. On the other side, great progress has been achieved in the manipulation of quantum materials with the use of cavity vacuum fields [5–15]. Specifically, for two-dimensional (2D) materials in magnetic fields, ultrastrong coupling of the Landau levels to the cavity field and the observation of Landau polariton quasiparticles have been achieved [16–20]. Recently, modifications of the magneto-transport properties inside a cavity due to Landau polaritons were reported [21,22] and most significantly cavity modifications of the IQH transport was demonstrated [23,24]. The experimental phenomena was argued to originate from a disorder-assisted cavity-mediated long-range hopping [25].

In this work, given that in experiments the GaAs samples have low disorder and that the cavity field is homogeneous in the bulk of the cavity [23], we study the quantum Hall system in the homogeneous limit with vanishing disorder and we propose an alternative theory for the observed cavity modified IQH transport [23]. Our theory highlights the importance of the hybridization between cavity photons and the collective Kohn mode in the quantum Hall system, and provides the exact solution for the polariton modes. In connection to the experimental findings [23], our theory draws the picture that the transport in the hybrid system is strongly influenced by the polariton states, in contrast to the standard quantum Hall transport which is purely electronic. Crucially, the low energy physics is dictated by the lower polariton mode which is softer than the cyclotron mode. The softening of the cyclotron mode signals the weakened topological protection and provides an intrinsic mechanism for the recently observed topological breakdown [23]. Importantly, our theory predicts that the cavity suppresses the thermal activation gap which can be studied experimentally in the temperature dependence of the quantum Hall transport in the cavity.

Model Hamiltonian.—Our model considers a two-dimensional electron gas coupled to a strong magnetic field and a single-mode homogeneous cavity field, as schematically depicted in Fig. 1(a). The system is described by the Pauli-Fierz Hamiltonian [26–28]

$$H=\sum_{i=1}^{N}\frac{(\boldsymbol{\pi}_i+e\mathbf{A})^2}{2m}+\hbar\omega\left(a^{\dagger}a+\frac{1}{2}\right)+\sum_{i<j}W(\mathbf{r}_i-\mathbf{r}_j), \quad (1)$$

where $\boldsymbol{\pi}_i = i\hbar\nabla_i + e\mathbf{A}_{\rm ext}(\mathbf{r}_i)$ are the dynamical momenta of the electrons, and $\mathbf{A}_{\rm ext}(\mathbf{r}) = -\mathbf{e}_x By$ describes the applied magnetic field $\mathbf{B} = \nabla\times\mathbf{A}_{\rm ext}(\mathbf{r}) = B\mathbf{e}_z$. The cavity field $\mathbf{A} = \sqrt{(\hbar/2\epsilon_0\mathcal{V}\omega)}\mathbf{e}_x(a+a^\dagger)$ is characterized by the in-plane polarization vector $\mathbf{e}_x$ and the photon's bare frequency ω. The $\mathcal{V}$ and ϵ_0 are the effective mode volume and the dielectric constant, respectively. The operators a and $a^\dagger$ represent photonic annihilation and creation operators which satisfy bosonic commutation relations $[a,a^\dagger]=1$. Further, $W(\mathbf{r}_i-\mathbf{r}_j) = 1/4\pi\epsilon_0|\mathbf{r}_i-\mathbf{r}_j|$ is the Coulomb interaction between the electrons. We have parametrized the bare electron dispersion by an effective mass m and assumed Galilean invariance. With Galilean invariance in a homogeneous system, the c.m. is decoupled from the relative motion of the electrons, regardless of the interaction strength [29]. The kinetics of the c.m. and its coupling to light is best described in terms of the c.m. coordinate $\mathbf{R} = (X,Y) = \sum_{i=1}^N \mathbf{r}_i/\sqrt{N}$ where N is the total particle number. Following the derivation presented in the Supplemental Material [30] we obtain the Hamiltonian describing the coupling of the c.m. to light:

$$H_{\rm c.m.} = \frac{1}{2m}(\mathbf{\Pi} + e\sqrt{N}\mathbf{A})^2 + \hbar\omega\left(a^\dagger a + \frac{1}{2}\right), \quad (2)$$

where $\mathbf{\Pi} = i\hbar\nabla_{\mathbf{R}} + e\mathbf{A}_{\rm ext}(\mathbf{R})$ is the dynamical momentum of the c.m. It is important to mention that if we break Galilean invariance or consider a spatially inhomogeneous cavity field, the relative degrees of freedom will couple to quantum light. The c.m. Hamiltonian has the form of two coupled harmonic oscillators, one for the Landau level transition and one for the photons. Such a Hamiltonian is known as the Hopfield Hamiltonian which can be solved by the Hopfield transformation [36]. The Hopfield model has been employed in previous works for the description of single-particle Landau level transitions coupled to cavity photons [19,22]. Here, it shows up for the collective coupling of the electrons which emerges naturally through the c.m. After the Hopfield transformation we find

$$H_{\rm c.m.} = \hbar\Omega_+\left(b_+^\dagger b_+ + \frac{1}{2}\right) + \hbar\Omega_-\left(b_-^\dagger b_- + \frac{1}{2}\right), \quad (3)$$

where $\{b_\pm^\dagger, b_\pm\}$ are the creation and annihilation operators of the Landau polariton quasiparticles satisfying bosonic commutation relations $[b_l, b_{l'}^\dagger] = \delta_{ll'}$ with $l,l' = \pm$. The details about the diagonalization of $H_{\rm c.m.}$ are given in the Supplemental Material [30]. The $\Omega_\pm$ are the upper and lower Landau polariton modes, respectively,

$$\Omega_\pm^2 = \frac{\omega^2+\omega_d^2+\omega_c^2}{2} \pm \sqrt{\omega_d^2\omega_c^2 + \left(\frac{\omega^2+\omega_d^2-\omega_c^2}{2}\right)^2}, \quad (4)$$

where $\omega_d = \sqrt{e^2N/m\epsilon_0\mathcal{V}}$ is the diamagnetic frequency originating from the $\mathbf{A}^2$ term which depends on the number of electrons N and the effective mode volume $\mathcal{V}$, and $\omega_c = eB/m$ is the cyclotron frequency [37]. To define the polariton operators we represent $\{a, a^\dagger\}$ in terms of a displacement coordinate q and its conjugate momentum ∂_q as $a = (q+\partial_q)/\sqrt{2}$ with $a^\dagger$ obtained via conjugation [26,27]. The polariton operators then are written in terms of mixed coordinates as $S_\pm = \sqrt{\hbar/2\Omega_\pm}(b_\pm + b_\pm^\dagger)$ with

$$S_+ = \frac{\sqrt{m}\bar{Y} + q\Lambda\sqrt{\hbar/\omega}}{\sqrt{1+\Lambda^2}} \quad \text{and} \quad S_- = \frac{-q\sqrt{\hbar/\omega} + \sqrt{m}\Lambda\bar{Y}}{\sqrt{1+\Lambda^2}},$$

where $\bar{Y} = Y + (\hbar K_x/eB)$ is the guiding center and K_x is the electronic wave number in the x direction. Also we introduced the parameter $\Lambda = \alpha - \sqrt{1+\alpha^2}$ with $\alpha = (\omega_c^2 - \omega^2 - \omega_d^2)/2\omega_d\omega_c$, which quantifies the mixing between electrons and photons.

Behavior of polaritons.—The polariton modes $\Omega_\pm$ depend on the cavity frequency ω, the cyclotron frequency ω_c, and the number of electrons through ω_d. The behavior of the polariton modes as a function of the magnetic field strength can be understood from their exact expressions Eq. (4) also shown in Fig. 1(b). Before the avoided crossing Ω_+ follows the cavity frequency ω while Ω_- follows the cyclotron frequency ω_c. After the avoided crossing the situation is inverted. On resonance $\omega = \omega_c$ the two modes are separated by the Rabi splitting $\Omega_R = \Omega_+ - \Omega_-$ which is approximately proportional to ω_d. For the geometry considered in Fig. 1, ω_d can be estimated through the electron

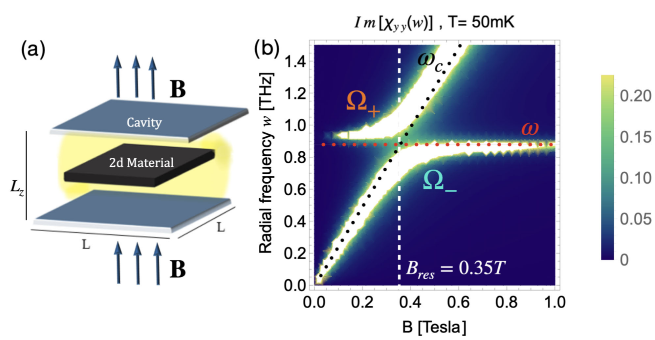

FIG. 1. (a) Two-dimensional material confined in a cavity. The distance between the cavity mirrors is L_z. The system is placed perpendicular to a homogeneous magnetic field $\mathbf{B}$. (b) Imaginary part of the response function $\chi_{yy}(w)$ for IQH system in the cavity. The radial cavity frequency $\omega = 2\pi\times 0.14$ THz, the 2D electron density $n_{2D} = 2\times 10^{11}$ cm^{-2}, the effective electron mass $m = 0.07m_e$ and the temperature $T = 50$ mK are chosen according to the experiment in Ref. [23]. We observe the upper Ω_+ and lower Ω_- polariton, with normalized Rabi splitting $\Omega_R/\omega = 0.33$. The lower polariton is softer than the cyclotron mode $\omega_c = eB/m$. This signals the weakened topological protection of the hybrid system.

density n_{2D} as $\omega_d = \sqrt{e^2 N/m\epsilon_0 \mathcal{V}} = \sqrt{e^2 n_{2D}\omega/\pi c m \epsilon_0}$, where we used the expression for the fundamental cavity frequency $\omega = \pi c/L_z$ [20]. Given the experimental parameters in Ref. [23] for the cavity frequency $\omega = 2\pi \times 0.14$ THz, the 2D electron density $n_{2D} = 2 \times 10^{11}$ cm^{-2}, and the effective electron mass $m = 0.07 m_e$ in GaAs, we find the normalized Rabi splitting $\Omega_R/\omega = 0.33$, which is in good agreement with the experimentally observed value $\Omega_R/\omega|_{\exp} = 0.3$ [23]. We note that $\omega = \omega_c$ for magnetic field strength $B_{\mathrm{res}} = 0.35T$ as it is also observed experimentally [23]. The normalized Rabi splitting is above 10% signaling ultrastrong light-matter coupling [9,10]. The lower polariton is decisive for the low energy physics of the system and we will show that its behavior controls the IQH transport. Approaching the limit $\omega \to 0$, the lower polariton becomes gapless reproducing the result in Refs. [15,20]. In addition, Ω_- decreases as a function of the light-matter coupling strength, controlled via ω_d, i.e., $\Omega_- < \omega_c$ when $\omega_d > 0$. In what follows, we discuss the implications of the polariton states for the quantum Hall transport at zero and finite temperature.

Fragility of topological protection against polariton lifetimes and ultrastrong light-matter coupling.—A clean or weakly disordered quantum Hall system at zero temperature, as long as it is gapped, is expected to be topologically protected [38]. However, the softening of the cyclotron mode, due to the lower polariton, indicates that the topological protection of the system is weakened. Because of the gap reduction, the transport of the system can be more easily affected by disorder, which leads to a finite lifetime for the polariton quasiparticles. The polariton lifetimes will be included phenomenologically and we will see that their effect combined with ultrastrong coupling enables the breakdown of topological protection [23,24].

The gauge-invariant current operator for homogeneous fields solely depends on the c.m. dynamical momentum and the cavity field [15,37] $\mathbf{J} = -(e\sqrt{N}/m)(\mathbf{\Pi} + e\sqrt{N}\mathbf{A})$. Because of this property and the separability of $H_{\mathrm{c.m.}}$ from the electronic correlations we can compute the transport of the system by focusing only on the states of $H_{\mathrm{c.m.}}$. At $T = 0$ the system is in the polariton vacuum $|\Psi_{\mathrm{gs}}\rangle = |0_+\rangle|0_-\rangle$, which is annihilated by both polariton operators $b_\pm$. Given this state, we employ the standard Kubo formalism [39] for the computation of the current correlators $\chi_{ab}(t) = -\mathrm{i}\Theta(t)\langle\Psi_{\mathrm{gs}}|[J_a(t), J_b]|\Psi_{\mathrm{gs}}\rangle/\hbar$ in the time domain which we transform to the frequency domain in order to obtain the optical conductivities [39] $\sigma_{ab}(w) = [\mathrm{i}/(w + \mathrm{i}\delta)]\{(e^2 n_{2D}/m)\delta_{ab} + [\chi_{ab}(w)/A]\}$, where A and $n_{2D} = N/A$ are the area and the electron density of the 2D material, respectively, δ is the broadening parameter, and δ_{ab} the Kronecker delta with $a, b \in \{x, y\}$. The optical conductivities $\sigma_{ab}(w)$ are given in the frequency domain in terms of the frequency w. The full details for the transport computations are provided in the Supplemental Material [30]. The poles of the response functions $\chi_{ab}(w)$ identify the optical responses of the system and its excitations. As we show in Fig. 1(b) the optical excitations correspond to Landau polariton modes, which have been observed in a multitude of experiments [17,18,23,40]. Note that in Fig. 1(b) we use the parameters reported in Ref. [23] which we described previously. In addition, using the Kubo formula we find the Hall and longitudinal dc ($w = 0$) conductivities

$$\sigma_{xy} = \frac{e^2\nu}{h(1+\Lambda^2)}\left[\frac{\Lambda(\Lambda+\eta)}{\Omega_-^2/\omega_c^2 + \delta^2/\omega_c^2} + \frac{1-\eta\Lambda}{\Omega_+^2/\omega_c^2 + \delta^2/\omega_c^2}\right],$$
$$\sigma_{yy} = \sigma_D\left[1 - \frac{1}{1+\Lambda^2}\left(\frac{\Omega_+^2}{\Omega_+^2+\delta^2} + \frac{\Lambda^2\Omega_-^2}{\Omega_-^2+\delta^2}\right)\right], \qquad (5)$$

where $\eta = \omega_d/\omega_c$. Note that $\sigma_D = e^2 n_{2D}/m\delta$ is the Drude dc conductivity, and that in σ_{xy} we introduced the Landau level filling factor $\nu = n_{2D}h/eB$ [41,42]. For $\delta \to 0$ we find the Hall conductance perfectly quantized $\sigma_{xy} = e^2\nu/h$, consistent with the Thouless flux insertion argument [38]. In the last step we used two properties of the mixing parameter $1 - \eta\Lambda = \Omega_+^2/\omega_c^2$ and $\Lambda(\Omega_-^2/\omega_c^2 - 1) = \eta$ which are deduced from the definition of Λ.

The polariton lifetimes are responsible for the broadening in the transmission spectra observed experimentally [17,18,23,40]. The total lifetime is a result of several mechanisms: scattering by impurities, radiative decay [27], coupling to phonons, as well as to the substrate. Here, we phenomenologically model the polariton lifetime as $\tau = 1/\delta$ by keeping a finite broadening δ which enables to model the experimental optical spectra as for example in Fig. 1(b).

Motivated by the experiments in Refs. [18,21,23] we choose $\delta = 2\pi \times 5 \times 10^{-3}$ THz and in Fig. 2 we plot σ_{xy} and σ_{yy} under ultrastrong light-matter coupling for different values of the magnetic field strength, corresponding to different filling factors. In Fig. 2 we see that $\sigma_{xy}/(\nu e^2/h)$ deviates from unity and σ_{yy} deviates from zero. Both phenomena signal the breakdown of topological protection. For $B = 2T$ we observe that the cavity effects are suppressed in comparison to $B = 1T$. This is physically expected as for larger magnetic fields the vacuum field fluctuations become a small perturbation to the system. The deviations from the expected values occur off-resonance, for a small cavity frequency, because in this regime the lower polariton gap Ω_- is significantly reduced [see Fig. 1(b)]. This relates to the fact that for fixed electron density and small ω the normalized Rabi splitting Ω_R/ω is enhanced. Thus, it is the interplay between the ultrastrong light-matter coupling and the finite polariton lifetime that causes the effects on transport. This intuitive physical picture is in agreement with the observed breakdown of topological protection in Ref. [23], and the disorder-assisted cavity-mediated hopping mechanism [25].

The above analysis is consistent with the result in the long-wavelength limit $\omega \to 0$ and $\delta = 0$ [43]. The Hall

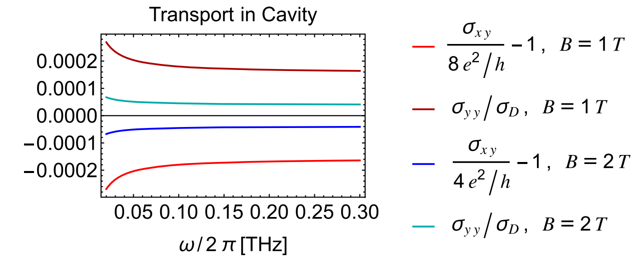

FIG. 2. Quantum Hall transport in a cavity at $T = 0$ with a finite broadening $\delta = 2\pi \times 5 \times 10^{-3}$ THz for two different values of magnetic field strength $B = 1T$ and $B = 2T$ which correspond to the filling factors $\nu = 8$ and $\nu = 4$, respectively. The 2D electron density is $n_{2D} = 2 \times 10^{11}$ cm^{-2} as in Ref. [23]. The Hall and the longitudinal conductivities deviate from the topologically expected values. For the smaller value of the magnetic field (higher filling factor) the deviations of the IQH transport due to the cavity are enhanced.

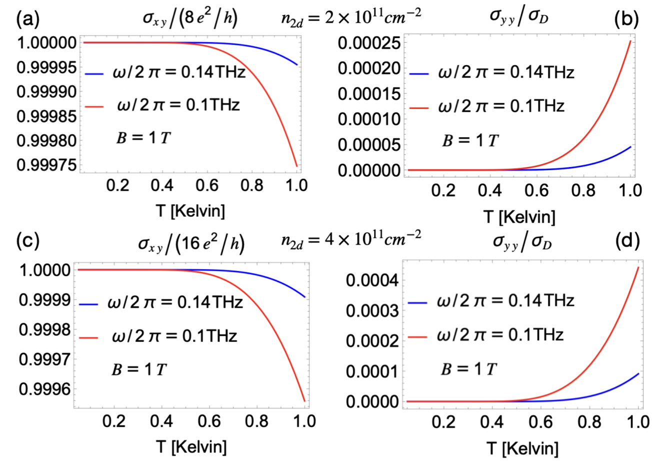

FIG. 3. Low temperature transport at magnetic field strength $B = 1T$. In (a) and (b) $n_{2D} = 2 \times 10^{11}$ cm^{-2} while in (c) and (d) $n_{2D} = 4 \times 10^{11}$ cm^{-2}. The light-matter coupling strongly affects the quantum Hall transport. For the smaller cavity frequency $\omega = 2\pi \times 0.1$ THz and the larger electron density the deviation from the topologically protected values maximizes. This relates to the behavior of Ω_- which controls the thermal activation in the system. The broadening parameter is chosen very small $\delta = 2\pi \times 10^{-4}$ THz to avoid influencing transport, but guarantee numerical convergence.

conductivity for $\delta = 0$ is quantized for all $\omega > 0$ (finite gap) but drops to $e^2\nu/h/(1+\eta^2)$ for $\omega \to 0$ (gapless) [43]. At this point the canonical transformation to the polariton basis becomes singular. In this sense, the broadening δ regularizes the long-wavelength limit result.

Cavity suppression of the thermal activation gap.—Finite temperature transport properties are also strongly influenced by coupling the electrons to the cavity. This can be understood from the formula for the thermal behavior of the longitudinal transport $\sigma_{yy}(T)/\sigma_{yy}(T=0) \approx \exp(-\beta\Delta)$, where Δ is the activation gap of the system and $\beta = 1/kT$. For the hybrid system, $\Delta = \Omega_-$. Thereby for the IQH effect, the coupling to the cavity generally speaking reduces the activation gap from ω_c to Ω_- and makes the Hall transport easier to be modified by temperature.

The quantitative description of the thermal activation gap is obtained from the finite temperature Kubo formula [39], through $\chi_{\alpha\beta}(w)$ which is the retarded current correlation function,

$$\chi_{ab}(w) = \sum_{M,Q} \frac{e^{-\beta E_M} - e^{-\beta E_Q}}{\mathcal{Z}} \frac{\langle \Psi_M | J_a | \Psi_Q \rangle \langle \Psi_Q | J_b | \Psi_M \rangle}{w + (E_M - E_Q)/\hbar + i\delta}, \quad (6)$$

where $|\Psi_M\rangle, |\Psi_Q\rangle$ are the many-body states with eigenenergies E_M, E_Q, respectively, and $\mathcal{Z}$ is the partition function $\mathcal{Z} = \sum_M \exp(-\beta E_M)$. The details of the temperature dependent transport are given in the Supplemental Material [30]. For the temperature dependent computations presented in Fig. 3 we use two set of parameters for the cavity frequency $\omega = 2\pi \times \{0.14, 0.1\}$ THz and the 2D electron density $n_{2D} = \{2, 4\} \times 10^{11}$ cm^{-2}. The first ones correspond to the parameters reported in Ref. [23] and the second are used for comparison. The magnetic field strength is $B = 1T$. Figure 3 demonstrates that the finite temperature transport of the IQH system can be modified by the cavity. From the behavior of both conductivities it is evident that the dependence of transport on temperature is enhanced for the lower cavity frequency $\omega = 2\pi \times 0.1$ THz. This is directly connected to the gap reduction in the system as Ω_- takes a smaller value for a smaller ω. Additionally, we observe that the temperature effect is also enhanced by the electron density by comparing Figs. 3(a) and 3(b) to Figs. 3(c) and 3(d). This is to be expected since the electron density controls the Rabi splitting Ω_R. It is important to mention that in comparison to Fig. 2, the broadening parameter in Fig. 3 is 1 order of magnitude smaller $\delta = 2\pi \times 10^{-4}$ THz such that effect from the polariton lifetime becomes negligible. In the Supplemental Material [30] we provide the finite temperature transport for the parameters used in Fig. 2. The low temperature transport is consistent with the $T = 0$ results in Fig. 2.

Connections to experiments and future directions.—The above analysis suggests that the activation gap of the hybrid system is strongly suppressed by coupling to cavity modes. Importantly, our model enables the theoretical estimate of the activation gap and direct comparison to experiment.

Additionally, we discuss the difference reported experimentally between the odd and the even plateaus [23]. The odd plateaus in the IQH effect are due to the Zeeman gap. In the experiment [23] the Zeeman gap is 20% of the cyclotron gap. Thus, we can think of the Zeeman gap effectively as a cyclotron gap with an effective magnetic field strength reduced by the factor 1/5 as compared to the actual magnetic field, i.e., $\Delta_{\text{Zeeman}} = \omega_c/5 = eB_{\text{eff}}/m$, where $B_{\text{eff}} = B/5$. Under this assumption we compute the deviations of IQH transport in the cavity for the odd plateau. The deviation of the longitudinal transport from zero for $\omega = 2\pi \times 0.14$ THz, at $T = 0$ and $B = 1T$ is $\sigma_{yy}/\sigma_D|_B = 1.7 \times 10^{-4}$ (see also Fig. 2), while for the respective odd plateau with $B_{\text{eff}} = 0.2T$ is 1 order of magnitude larger, $\sigma_{yy}/\sigma_D|_{B_{\text{eff}}} = 4.3 \times 10^{-3}$. The Hall conductivities behave similarly as it can be understood from Fig. 2. This analysis shows that odd plateaus are much more vulnerable to the cavity than the even ones. A more rigorous treatment of this effect requires the inclusion of the spin degrees of freedom. This is an interesting problem for future investigation.

Further, we comment on the FQH effect. In samples with low disorder, the activation gap of the FQH effect is given by the many-body gap, closely related to the magnetoroton energy [44], which we assume to be smaller than Ω_- and therefore protected from cavity effects. This picture is consistent with the experimental observations that FQH plateaus are relatively immune to the cavity [23]. From this analysis we anticipate that the FQH effect can be modified at low temperature when Ω_- becomes softer than the many-body gap.

To summarize, using a Galilean invariant quantum Hall model coupled to a homogeneous single-mode cavity field, we provide the exact solution for the polariton states and discuss their experimental implications for quantum Hall transport in cavities. The lower polariton is softer than the cyclotron mode and leads to the weakening of topological protection. This provides an intrinsic mechanism for the recently observed breakdown of the topological protection of the IQH effect due to cavity vacuum fluctuations [23]. Having understood analytically the homogeneous setting, our work paves the way for future investigations going beyond this limit, such that the interaction between the polaritons and the electron correlations comes into play. In this setting the interplay between polaritons and anyons is an interesting future research question, with potential applications to quantum computing [45]. It is important to mention that in Ref. [23] in the edges of the sample the cavity field is not perfectly homogeneous. Despite this fact the bulk polariton modes are observed in the transmission spectrum. This proves the robustness of the polaritons against the field inhomogeneities. Nevertheless, it is an interesting future direction to study the influence of the inhomogeneities of the cavity field to edge modes in disordered samples. The inclusion of impurities will be important for a more precise understanding of transport. Incorporating leakage and the multimode structure of the cavity will enable a more realistic description of transport phenomena in general electromagnetic environments [46]. Finally, we highlight that the quantum Hall system has played a crucial role in redefining units in terms of constants of nature [47]. Thus, the cavity induced phenomena could potentially have implications for metrology as pointed out in Ref. [23].

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