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April 1990

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Buckley-Leverett Flow in Composite Porous Media

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Abstract

This paper presents a Buckley-Leverett-type analytical solution for one-dimensional immiscible displacement in a linear composite porous medium. The classical Buckley-Leverett theory, applicable only to flow in a homogeneous porous medium, has been extended to flow in an inhomogeneous porous medium, in which the formation system is treated as consisting of a number of flow domains with different rock properties. The analytical solution, obtained under the conditions for the Buckley-Leverett solution for each flow domain, can be used to determine the complete saturation profile in the composite system at all times. The analytical results indicate that noncapillary immiscible displacement of two fluids in a composite system is characterized by discontinuities in saturation profiles across the interfaces between adjacent flow domains.

Introduction

Immiscible flow and displacement of multiple phase fluids in porous media are of fundamental importance to many problems relating to underground natural resource recovery and to storage projects, and waste disposal and contamination transport evaluation. Immiscible and miscible flow of multiple phase fluids through porous media, as compared with single phase flow, is much more complicated and is not well understood in many areas due to the complex interactions of the different fluid phases. Many contributions to this subject have been made since the 1940's. In the petroleum industry, the simultaneous flow of oil, gas and water in reservoirs is important in connection with the production of oil and gas. The flow of moisture in unsaturated soils (i.e., the simultaneous flow of water and air) is often encountered in soil science. Multiple phase flow of water, hydrocarbons, air and chemicals is also involved in evaluating problems of underground contamination.

A fundamental understanding of immiscible displacement of Newtonian fluids in porous media was contributed by Buckley and Leverett (1942) in their classical study of the fractional flow theory. The Buckley-Leverett solution gives a saturation profile with a sharp front along the flow direction, but ignores capillary pressure and gravity effects. As time progresses, the saturation becomes a multiple-valued function of the distance coordinate, x, which can be overcome by material balance considerations. Where the initial saturation is uniform, a simple graphic approach developed by Welge (1952) can be used to determine the sharp saturation front without difficulty. Effects of gravity and capillary pressure on linear waterflood was included by Fayers and Sheldon (1959), Hovanessian and Fayers (1961), by numerical models. Some special analytical solutions of immiscible displacement including the effects of capillary pressure were obtained in Russian and Chinese literature in the 1960's (Chen,1988), and more recently by Yortsos and Fokas (1983) and McWhorter and Sunada (1990).

The state of the art in numerical modeling of immiscible fluid displacement processes has advanced considerably since the 1950's (Douglas et al., 1959; Coats, 1987; Aziz and Settari, 1979; and Thomas, 1982). A large number of numerical models have been developed to simulate the process of waterflooding under quite general operating and reservoir conditions in oil production. However, numerical techniques cannot replace analytical methods completely, since i) numerical methods need checking against analytical solutions because they are subject to discretization errors that may be especially troublesome for problems that involve propagation of sharp fronts, and ii) analytical solutions if available often provide a better insight into the physics of the transport phenomena occurring within the porous media.

In this paper, we extend the Buckley-Leverett theory to the flow problem in a composite porous medium, which is used to approximate more complicated formations. Our formulation considers a one-dimensional linear flow system, consisting of two flow domains with different rock properties, but an extension to an arbitrary number of domains is straightforward. A new analytical solution to describe displacement of two immiscible fluids in this composite system is developed and examined using a numerical simulator. Immiscible displacement in composite systems is found to give rise to complex saturation profiles, which consist of different-shape segments with discontinuities at the interfaces of adjacent flow domains.

2. Mathematical Formulation and Analytical Solution

Two-phase flow of immiscible fluids is considered in an incompressible composite system, consisting of two flow domains (j=1, 2) with each domain having different rock properties. The mathematical formulation of immiscible multiple phase flow in porous media has been discussed extensively in the literature (Willhite, 1986). For the derivation of the analytical solution, the same assumptions as used for the Buckley-Leverett solution are made for each flow domain, namely,

1. the two fluids and the porous medium are incompressible,
2. the capillary pressure gradient is negligible,
3. the flow is one-dimensional linear,
4. the fluid and rock properties are constant within each domain,

and additionally,

5. the formation properties change in discontinuous fashion at the contact between domains 1 and 2.

The flow system under consideration (See Figure 1) is a linear one-dimensional composite flow tube with a constant cross-sectional area A. The system is initially saturated uniformly with a mixture of a non-wetting phase fluid (such as oil) and a wetting phase fluid (such as water), and at time t = 0 injection of the same wetting fluid is started at the inlet (x = 0). It is further assumed that gravity segregation is negligible and that stable displacement exists near the displacement front (no viscous fingering). The fractional flow function for the wetting phase in domain j (j=1, 2) may be written in the following form (Willhite, 1986):

$$f_j = \frac{1}{1+\left[\dfrac{k_{n,j}(S)}{k_{w,j}(S)}\right]\left[\dfrac{\mu_w}{\mu_n}\right]} + \frac{\dfrac{AK_j k_{n,j}(S)}{\mu_n q(t)}(\rho_n-\rho_w)\, g \sin(\alpha)}{1+\left[\dfrac{k_{n,j}(S)}{k_{w,j}(S)}\right]\left[\dfrac{\mu_w}{\mu_n}\right]} \qquad (1)$$

where $k_{n,j}(S)$ and $k_{w,j}(S)$ are relative permeabilities of domain j (j = 1, 2) to non-wetting and wetting phases, respectively, as functions of wetting phase saturation, S; μ_n and μ_w are viscosities of non-wetting and wetting fluids, respectively; K_j is absolute permeabilities of domain j; q(t) is the volumetric injection rate of wetting fluid at the inlet; ρ_n and ρ_w are densities of non-wetting and wetting fluids; and α is the angle of the flow direction with the horizontal plane.

The partial differential flow equation for the wetting phase in each domain can be expressed in terms of saturation and fractional flow as

$$\frac{\partial f_j(S)}{\partial x} + \frac{\phi_j A}{q(t)} \frac{\partial S}{\partial t} = 0 \qquad \text{for } j = 1, 2 \tag{2}$$

where ϕ_j is the formation porosity in domain j. As shown by Buckley and Leverett, this equation describes propagation of different saturations at different characteristic speeds, given by (Willhite, 1986)

$$\left[\frac{dx}{dt}\right]_S = \frac{q(t)}{\phi_j A} \left[\frac{df_j}{dS}\right]_S \qquad \text{for } j = 1, 2 \tag{3}$$

The interface at x = D between domain 1 and 2 is a discontinuity surface for porosity and absolute and relative permeability. This surface is fixed in space, so that the volumetric flow rates for both phases must be continuous at x = D at all times. Thus

$$f_1(S_1^-) = f_2(S_2^+) \qquad \text{at } x = D \tag{4}$$

where S_1^- and S_2^+ are saturations of the wetting phase on the interface $D^- = D - \varepsilon, D^+ = D + \varepsilon$ in domains 1 and 2, respectively (ε is an infinitesimally small increment).

The complete saturation solution for immiscible displacement in a composite medium is obtained in this paper by solving the frontal advance equation (3) in both domains, subject to the continuity condition Equation (4).

Since the downstream conditions of the system have no effects on the upstream flow, the saturation distribution in domain 1 at all times is given by the Buckley-Leverett theory. Suppose that at $t = t^*$, the displacement shock front with saturation $S = S_{f,1}$ in domain 1 reaches the interface. For $t > t^*$, the injected wetting fluid has entered domain 2. The total volume of the injected fluid remaining in domain 1 at time t can be calculated as

$$Q_1(t) = \phi_1 A \int_0^D \left[S(x, t) - S_i^{\,1}\right] dx \tag{5}$$

where $S_i^{\,1}$ is the initial saturation in domain 1.

The volume of injected fluid that has crossed the interface x = D into domain 2 is then given from mass balance considerations as

$$Q_2(t) \;=\; Q(t) - Q_1(t) \tag{6}$$

where $Q(t) = \int_0^t q(\tau)\, d\tau$, the total injected fluid volume. Equation (6) will be used to find the moving shock saturation front in domain 2.

Consider a particular saturation S in domain 2, which begins to propagate from the interface x = D at $t = t_S$. Multiplying Equation (3) with dt and integrating from t_S to t, we have

$$x_S = D + \frac{1}{\phi_2 A} \int_{t_S}^{t} \left[\frac{df_2}{dS}\right]_S q(\tau) d\tau \tag{7}$$

where x_S is the travelling distance of saturation S at time t from the inlet.

As normally done for evaluation of the Buckley-Leverett solution, we pick a value of saturation in domain 2, and then use (7) to calculate its location at the given time. The starting time t_S for each saturation $S = S_2^+$ at the interface can be determined by using the continuity condition (4). Indeed, for each value S_2^+ of saturation at $x = D^+$ in domain 2, there exists a unique corresponding saturation S_1^- at $x = D^-$ in domain 1 (see Figure 2), $S_1^- = S_1^-(S_2^+)$, implicitly defined by (4), and there are two possibilities:

i) for $S_1^- \geq S_{f,1}$, i.e., for a value of S_1^- larger than that of the sharp front saturation in domain 1, the time t_S for $S = S_2^+(S_1^-)$ to start travelling into domain 2 is equal to the time at which the corresponding saturation S_1^- reaches the interface of domain 1, given by

$$\int_0^{t_S} q(\tau)\, d\tau = \frac{\phi_1 A D}{\left.\dfrac{df_1}{dS}\right|_{S_1^-}} \tag{8}$$

ii) for $S_1^- < S_{f,1}$, i.e., for values of the corresponding saturation in domain 1 smaller than or equal to that of the sharp front saturation, the actual starting time t_S is the time when the sharp front arrives at the interface,

$$t_S = t^* \tag{9}$$

As in the Buckley-Leverett solution, a direct use of (7) to calculate saturation profiles in domain 2 will result in a multi-valued solution at the displacement front. Physically,this corresponds to the development of a moving saturation shock front in domain 2. The location $x_{f,2}$ and saturation $S_{f,2}$ of the shock front can be obtained from the mass balance constraint,

$$Q_2(t) = \phi_2 A \int_D^{x_{f,2}} \left[S(x, t) - S_{i,2}\right] dx \tag{10}$$

where $S_{i,2}$ is the initial saturation in domain 2. Then, the saturation profile in domain 2 at any injection time t ($t \geq t^*$) is determined by Equations (7) and (10), with the starting time given by Equation (8), or (9).

Discussion of Immiscible Displacement

The fundamental displacement behavior of two immiscible fluids in a composite system can be discussed using the analytical solution obtained above. For simplicity, let us consider a linear horizontal composite system with a constant cross-sectional area A. Initially, the system is saturated with only a non-wetting phase, and a wetting fluid is injected at a constant volumetric rate, q , at the inlet x = 0 from t = 0. Then, the solution (7) for the saturation distribution in domain 2 (x > D) is simplified as

$$x_S = D + \frac{q}{\phi_2 A}\left[\frac{df_2}{dS}\right]_S \left[t - t_S\right] \tag{11}$$

where the starting time t_S for this saturation at x = D in domain 2 is, from Equation (8),

$$t_S = \frac{\phi_1 AD}{q\left[\frac{df_1}{dS}\right]_{S_1}} \qquad \text{for } S_1 > S_{f,1} \tag{12}$$

Here $S_1 = S_1(S)$ is the interface saturation in domain 1, corresponding to S in domain 2 according to Equation (4). When $S_1 \le S_{f,1}$, we have

$$t_S = t^* \tag{13}$$

From Equations (5), (6) and (10), the mass balance for determining the sharp displacement front in domain 2 becomes

$$Q_2(t) = \phi_2 A \int_D^{x_{f,2}} \left[S(x, t) - S_{i,2}\right] dx = q\,t - A\phi_1 D\left[\overline{S}_1 - S_{i,1}\right] \tag{14}$$

where $\overline{S}_1$ is the average saturation in domain 1, which can be determined by the graphic method (Willhite, 1986). The detailed procedure for calculating saturation profiles is given in Appendix A.

Note that the saturation profile in domain 2, described by Equations (11)-(13), is determined from formation porosity and fractional flow curves which, for horizontal flow, depend only on relative permeabilities and viscosities of fluids. Thus we have the important result that, under the approximations of Buckley-Leverett flow, saturation profiles in a composite medium are dependent only on formation porosities and relative permeabilities, and are completely independent of absolute permeabilities. In heterogeneous geological systems, such as layered formations, the relative permeabilities may be quite different in different flow domains, resulting in a diverse variety of possible saturation profiles.

The fluid and rock properties for illustrative examples are given in Table 1, in which the relative permeability functions were chosen typical for oil and water flow in different media (Honarpour et al, 1986). The fractional flow functions for the domains are shown in Figure 2, and the predicted saturation profile after an injection time of $t = 8{,}143.3$ seconds is given in Figure 3. The distinguishing features of immiscible displacement in a composite porous medium, as shown in Figure 3, are that there exists a saturation discontinuity at the interface of the domains, and that the derivative $\partial S/\partial x$ has a discontinuity at a point (x^*, S^*) in domain 2, at which the value of S^* corresponds to the shock front saturation $S_{f,1}$ of domain 1, $f_2(S^*) = f_1(S_{f,1})$.

The wave-traveling behavior of saturation profiles in a two-domain composite medium can be represented by characteristics in the (x, t) space, as shown in Figure 4. Each straight line represents a constant saturation, and travels at different velocity, which is described by the slope of the straight lines. Each value of saturations (S_1, S_2, or S_3) in domain 1 corresponds to a unique saturation wave (S^*_1, S^*_2, or S^*_3) across the interface $x = D$ if $S > S^*$ in domain 2. For saturations in the range $S_{f,2} \leq S \leq S^*$ in domain 2, the starting times for a saturation to travel from the interface are the same, corresponding to the time when the sharp moving front in domain 1 reaches the interface. For a given time $t = T$ ($T > t^*$), the intersections of characteristic straight lines with the vertical line ($t = T$) on Figure 4 give the complete saturation profile, such as given by Figure 3 in S-x space.

A numerical simulator, MULKOM-GWF (Pruess and Wu, 1988), has been used to examine the analytical solution. A comparison of the analytical and numerical results for the data of Table 1 is provided in Figure 5. The numerical simulation shows the familiar numerical dispersion effects (Aziz and Settari, 1979), but generally agrees very well with the analytical solution.

If we switch the fractional flow curves for the two domains, the saturation profile after time t = 29,927 seconds of injection is shown in Figure 6. In this case, the mass balance (14) is satisfied before the moving front reaches the point (x^*, S^*), and there is no discontinuity in $\partial S/\partial x$ versus x in domain 2. The numerical results match the analytical solution very well.

The values of saturation on the interface for both domains are always increasing with time. Equation (11) indicates that the travelling distance of a particular saturation S from the interface in domain 2 is proportional to derivatives of the fractional flow function of domain 2 with respect to saturation. In the above two examples, saturation variations happen to be in a range over which (df_2/dS) decreases as S_2^+ increases, i.e., a higher saturation, later departing from the interface, has a lower velocity (see Figure 7). The physical range for saturations in domain 1 is the range with $f_1 \geq f_1(S_{f,1})$, or $S \geq S_{f,1}$, and for domain 2, as shown in Figure 7, the physical range at a given time when $S = S_2^+$ at the interface is given by $f_2(S_{f,2}) \leq f_2 \leq f_2(S_2^+)$, or $S_{f,2} \leq S \leq S_2^+$.

Since the relative permeabilities in different regions of a composite medium are generally independent, we may have a situation in which the travelling velocity increases in domain 2 as the saturation increases. An example of fractional flow curves with this behavior is given in Figure 8, and the corresponding correlation of the fractional flow and its derivatives is shown in Figure 9. The derivatives of fractional flow with respect to saturation increase as saturation and fractional flow increases in domain 2. The resulting saturation distribution is shown in Figure 10. In this case, $S_{f,2} \geq S^*$, so that there is no discontinuity for $\partial S/\partial x$ in domain 2. The saturation profile in domain 2 has a negative curvature since the derivative df_2/dS decreases as S decreases in domain 2.

Summary

A Buckley-Leverett type analytical solution for one-dimensional two-phase immiscible displacement in a composite porous medium has been developed. Our treatment has considered a composite medium consisting of two domains with uniform initial conditions; an extension to an arbitrary number of domains, to non-uniform initial saturation distribution, and to one-dimensional horizontal flow in a composite system with non-constant cross-sectional areas is straightforward. The analytical solution has been examined using a numerical simulator and excellent agreement has been obtained between analytical and numerical calculations.

Immiscible displacement in composite porous media is found to be characterized by discontinuities in saturation profiles across the interfaces of adjacent flow domains, and by discontinuous saturation derivatives. Saturation profiles for horizontal displacement depend only on relative permeability curves and ratio of fluid viscosities, and are independent of absolute permeability.

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Nomenclature

Symbol	Description	Units
A	Cross-sectional area	m^2
D	Length of Domain 1	m
f_j	Fractional flow of wetting phase in domain j (j=1, 2)	
g	Magnitude of the gravitational acceleration	m/s^2
K_j	Absolute permeability in domain j	m^2
$k_{n,j}$	Relative permeability to non-wetting phase	
$k_{w,j}$	Relative permeability to wetting phase	
q, $q(t)$	Volumetric injection rate	m^3/s
$Q(t)$	Cumulative injected fluid volume	m^3
$Q_j(t)$	Fluid volume injected into domain j	m^3
S	Saturation of wetting phase	
$S_{f,j}$	Sharp front saturation of domain j	
$S_{i,j}$	Initial wetting phase saturation in domain j (j=1,2)	
S^k	Distributed saturation in domain 2	
$\overline{S}_1$	Average wetting phase saturation in domain 1	
S_1^-	Wetting phase saturation at interface in domain 1	
S_2^+	Wetting phase saturation at interface in domain 2	
t	Time	s
t_S	Time for saturation S to begin to propagate into domain 2	s
V	Injected fluid volume in domain 2	m^3
x	Distance from inlet, coordinate	m
$x_{f,j}$	Distance to shock saturation front in domain j	m
x_S	Distance to saturation S	m
x_{S^k}	Distance to saturation S^k in domain 2	m

Greek Symbols

α	Angle with horizontal plane	
μ_n	Non-wetting phase viscosity	Pa·s
μ_w	Wetting phase viscosity	Pa·s
ρ_n	Density of non-wetting fluid	kg/m^3
ρ_w	Density of wetting fluid	kg/m^3
ϕ_j	Porosity of domain j	

Subscripts

f,j	Shock front in domain j
j	Domain index, j=1, 2
n	Non-wetting phase
w	Wetting phase
rn,j	Relative to non-wetting phase in domain j
rw,j	Relative to wetting phase in domain j
t	Time
t	Total

REFERENCES

Aziz, K. and Settari, A. (1979) : *Petroleum Reservoir Simulation, Applied Science, London.*

Buckley, S. E. and Leverett, M. C. (1942) : " Mechanism of Fluid Displacement in Sands, " *Trans.* AIME 146, 107-116.

Chen, Z. X. (1988) : "Some Invariant Solutions to Two-Phase Fluid Displacement Problems Including Capillary Effect," *SPE Reservoir Engineering,* 691-700.

Coats, K. H. (1987) : "Reservoir Simulation," *Petroleum Engineering Handbook,* SPE., Richardson, TX, 48.1-48.13.

Douglas, J. Jr., Peaceman, D. W. and Rachford, H. H. Jr. (1959) : "A Method for Calculating Mutidimensional Immiscible Displacement," *Trans.,* AIME 216, 147-155.

Honarpour, M., Koederitz, L. and Harvey, A. L. (1986) : *Relative Permeability of Petroleum Reservoirs,* CRC Press, Inc., Boca Raton, Florida.

McWhorter, D. B. and Sunada, D. K. (1990) : "Exact Integral Solutions for Two-Phase Flow," *Water Resour. Res., 23(3), 399-413.*

Pruess, K. and Wu, Y. -S. : "On PVT-Data, Well Treatment, and Preparation of Input Data for an Isothermal Gas-Water-Foam Version of MULKOM," Report LBL-25783, UC-403, Earth Sciences Division, Lawrence Berkeley Laboratory, Berkeley, CA.

Sheldon, J. W., Zondek, B. and Cardwell, W. T. Jr. (1959) : "One-Dimensional,Incompressible, Noncapillary, Two-Phase Fluid Flow in a Porous Medium," *trans,* AIME. 216, 290-296.

Thomas, G. W. (1982) : *Principles of Hydrocarbon Reservoir Simulation,* International Human Resources Development Co., Boston.

Welge, H. J. (1952) : "A Simplified Method for Computing Oil Recovery by Gas or Water Drive," *Trans.*, AIME 3, 108.

Willhite, G. P. : *Waterflooding* , SPE Textbook Series, Society of Petroleum Engineers, Richardson, TX. 1986.

Yortsos, Y. C. and Fokas, A. S. (1983) : "An Analytical Solution for Linear Waterflood Including the Effects of Capillary Pressure," *Sec. Pet. Eng. J.,* 115-124.

Appendix A.

Evaluation of Saturation Profiles in Domain 2

1. Determine the sharp front saturation $S_{f,1}$ in domain 1 by the Welge method or the mass balance calculation. Calculate the time t^* at which $S_{f,1}$ reaches the interface by

$$t^* = \frac{\phi_1 AD}{q\left[\frac{df_1}{dS}\right]_{S_{f,1}}} \tag{A.1}$$

2. For a given time t ($t > t^*$), calculate S_1^- at the interface in domain 1 by

$$q\,t = \frac{\phi_1 AD}{\left[\frac{df_1}{dS}\right]_{S_1^-}} \tag{A.2}$$

Then solve for S_2^+ in domain 2 by Equation (4). Calculate the average saturation $\bar{S}_1$ in domain 1 by

$$\bar{S}_1 = S_1^- + \frac{\left[1 - f_1(S_1^-)\right]}{\left[\frac{df_1}{dS}\right]_{S_1^-}} \tag{A.3}$$

3. Calculate the saturation profile in domain 2 as follows:

   i) choose a saturation S^k ($k = 0, 1, 2, \ldots N$), in which $S_2^+ = S^0 > S^1 > S^2 > S^3 >, \cdots$;

   ii) calculate the travelling distance x_{S^k} of a saturation S^k from $x = D$ by (11);

   iii) calculate the injected fluid volume V contained from $x = D$ to $x = x_{S^k}$ in the portion of domain 2 with $S > S^k$ by

$$V = A\phi_2 \int_{x=D}^{x_{S^*}} \left[S_w - S_{wir,2}\right] dx$$

$$\approx A\phi_2 \sum_{j=1}^{j=k} \left[S^j - S_{i,2}\right] \Delta x_j \tag{A.4}$$

where $\Delta x_j = x_j - x_{j-1}$, and $x_0 = D$.

vi) compare V with $Q_2(t)$ as given by Equation (14). If $V < Q_2(t)$, the saturation shock front has not been reached yet, and the process is continued with the next saturation value S^{k+1}. If $V \geq Q_2(t)$, stop the calculation, and set

$$S_{f,2} = S^k \tag{A.5}$$

and

$$x_{f,2} = x_k \tag{A.6}$$

If $S^k - S^{k+1}$ is taken to be sufficiently small, the calculation of the sharp front will be accurate. In this work, we have used $\Delta S^k = .001$. The above procedure has been programmed and carried out by computer.

The above procedure can be easily to extended to composite media with an arbitrary number of domains, because of saturation profiles are solely determined from upstream conditions. Given the time-dependence of saturations at the interface between domains N and N + 1, our method will yield the saturation distribution in domain N + 1, which in turn defines the time dependence of saturations at the interface to domain N + 2, etc.

Table 1

Parameters for Immiscible Displacement in a Composite System

Porosity of domain 1	$\phi_1=0.20$
Porosity of domain 2	$\phi_2=0.20$
Cross-Sectional Area	$A = 1\ m^2$
Injection Rate	$q=1\times10^{-5} m^3/s$
Wetting Phase Viscosity	$\mu_w=1cp$
Non-Wetting Phase Viscosity	$\mu_n=5cp$
Permeability of Domain 1	$K_1=100md$
Permeability of Domain 2	$K_2=10md$
Initial Wetting-Phase Saturation	$S_{i,1}=0.00$
Initial Wetting-Phase Saturation	$S_{i,2}=0.00$
Length of Domain 1	$D = 0.25, 0.5, 1$ m
Relative Permeability	$k_{w,1} = 1.831S^4$
Relative Permeability	$k_{n,1} = 0.75(1-1.25S)^2[1-1.652S^2]$
Relative Permeability	$k_{w,2} = 0.4687S^2$
Relative Permeability	$k_{n,2} = 0.5\left[1-1.25S\right]^2$

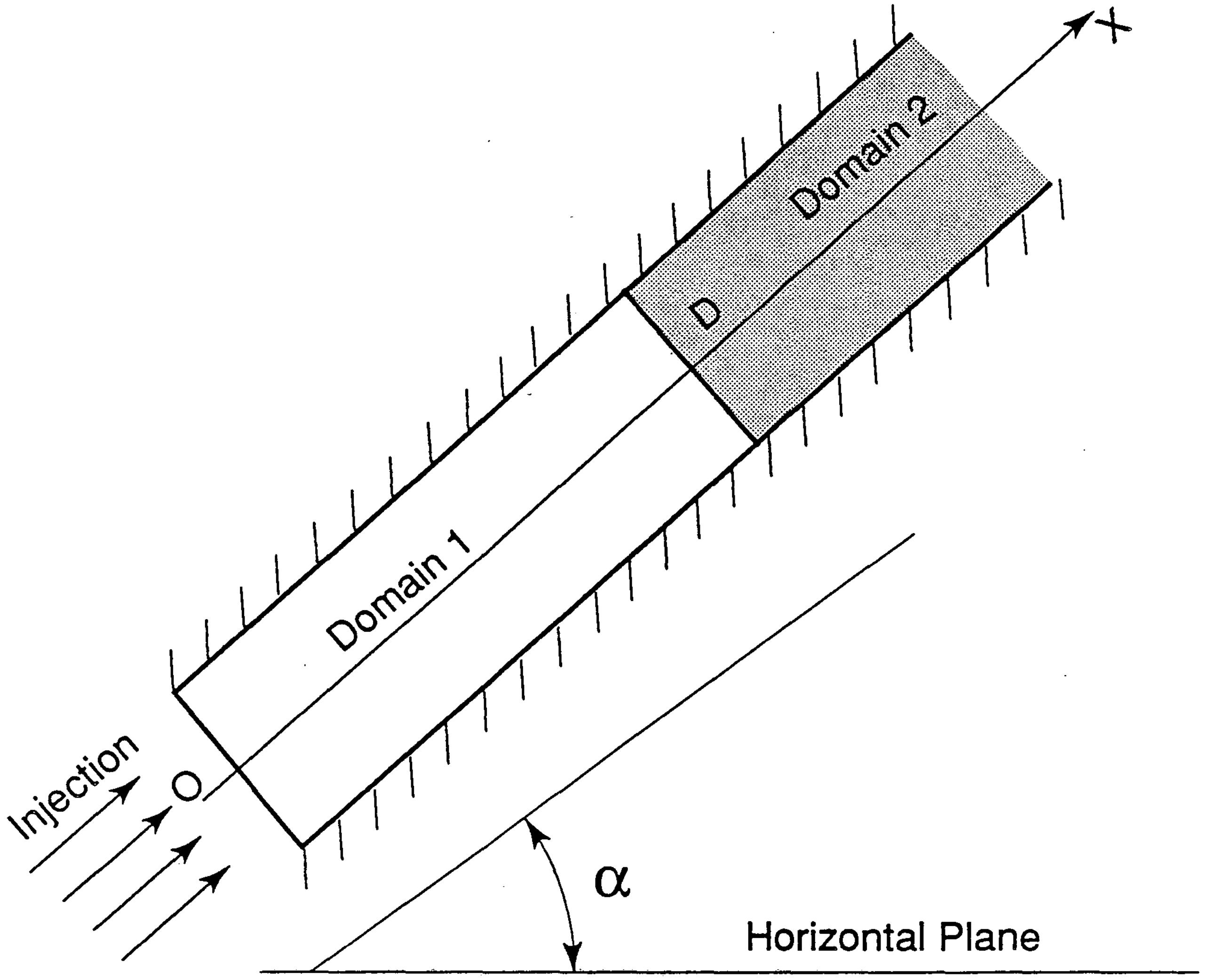

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Figure 1. Schematic of a Two-Domain Composite Porous Medium System.

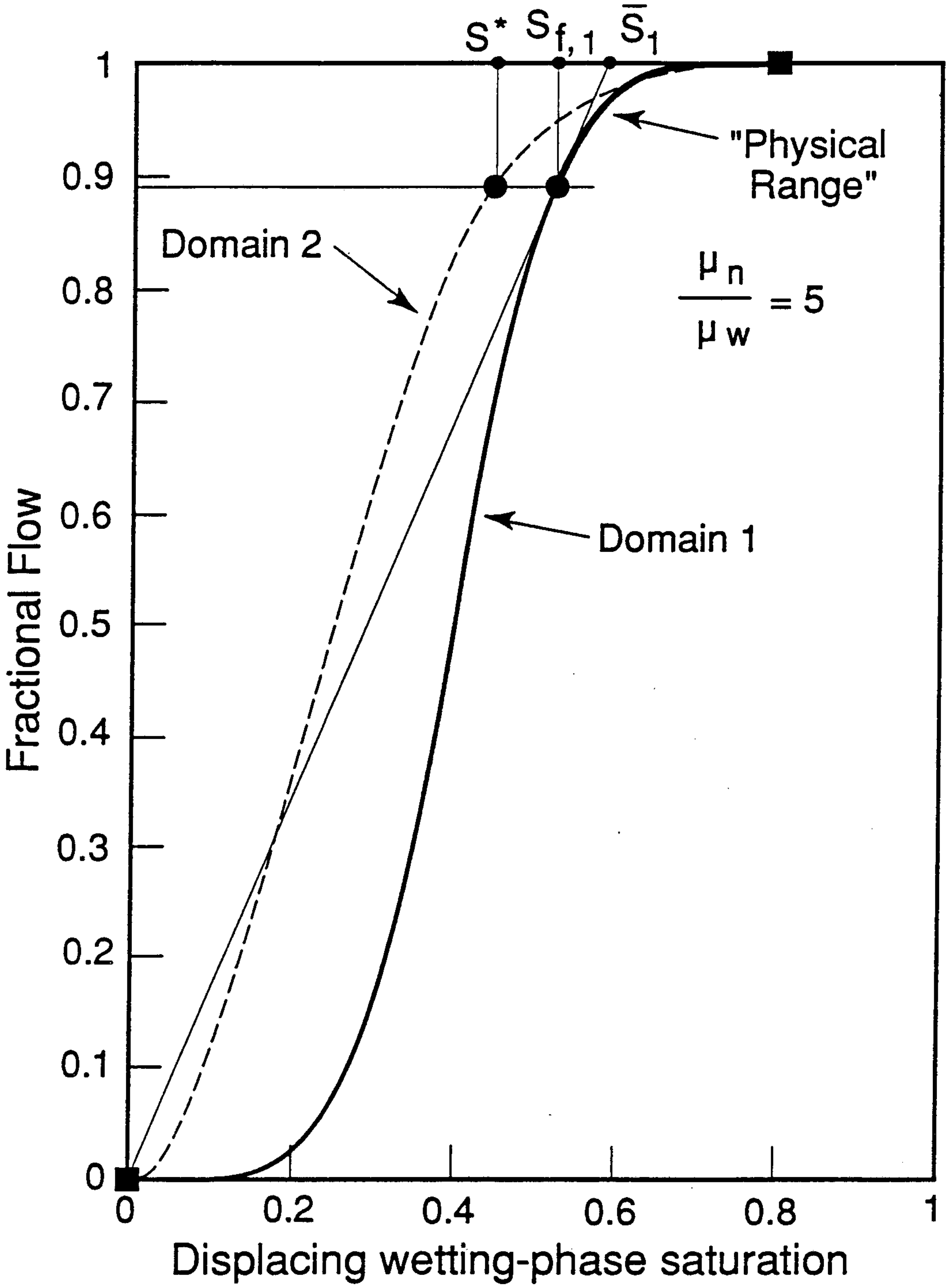

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Figure 2. Fractional Flow Curves for the Two-Domain Composite System; the Range of Physically Possible Saturations Is Indicated by a Bold Line - "Physical Range".

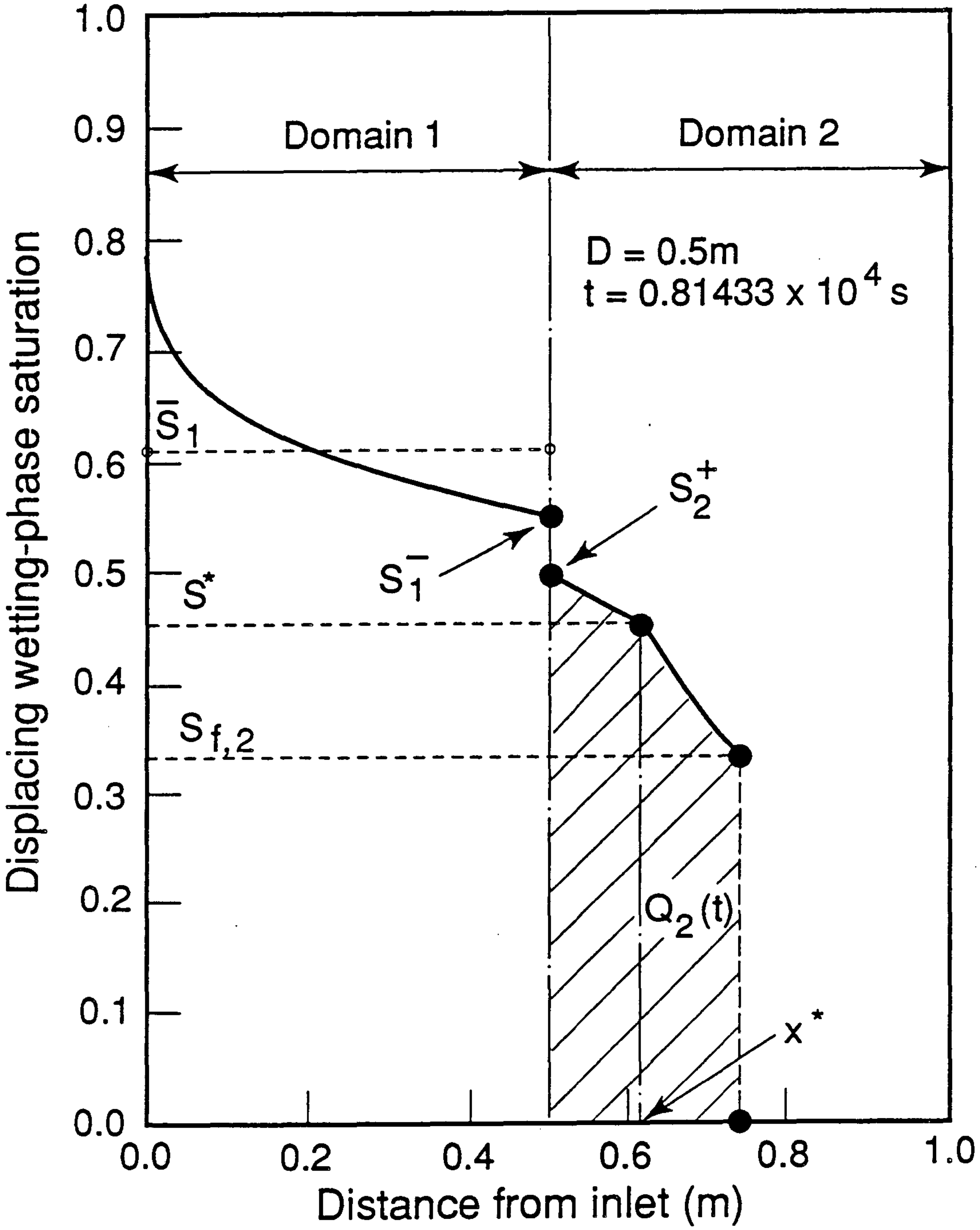

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Figure 3. Saturation Profiles in the Two-Domain System.

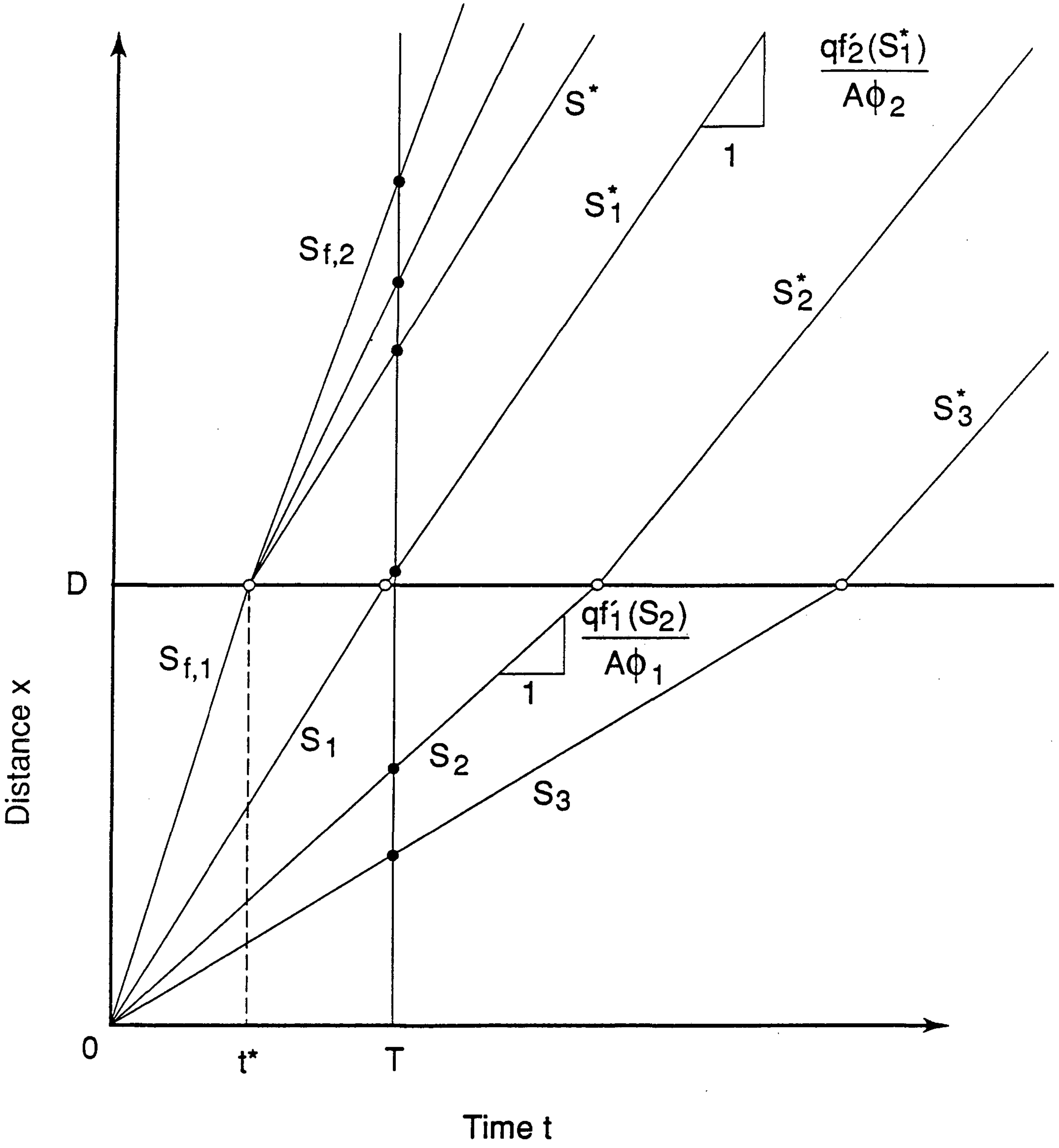

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Figure 4. Characteristics for Buckley-Leverett Flow in a Two-Domain Composite System.

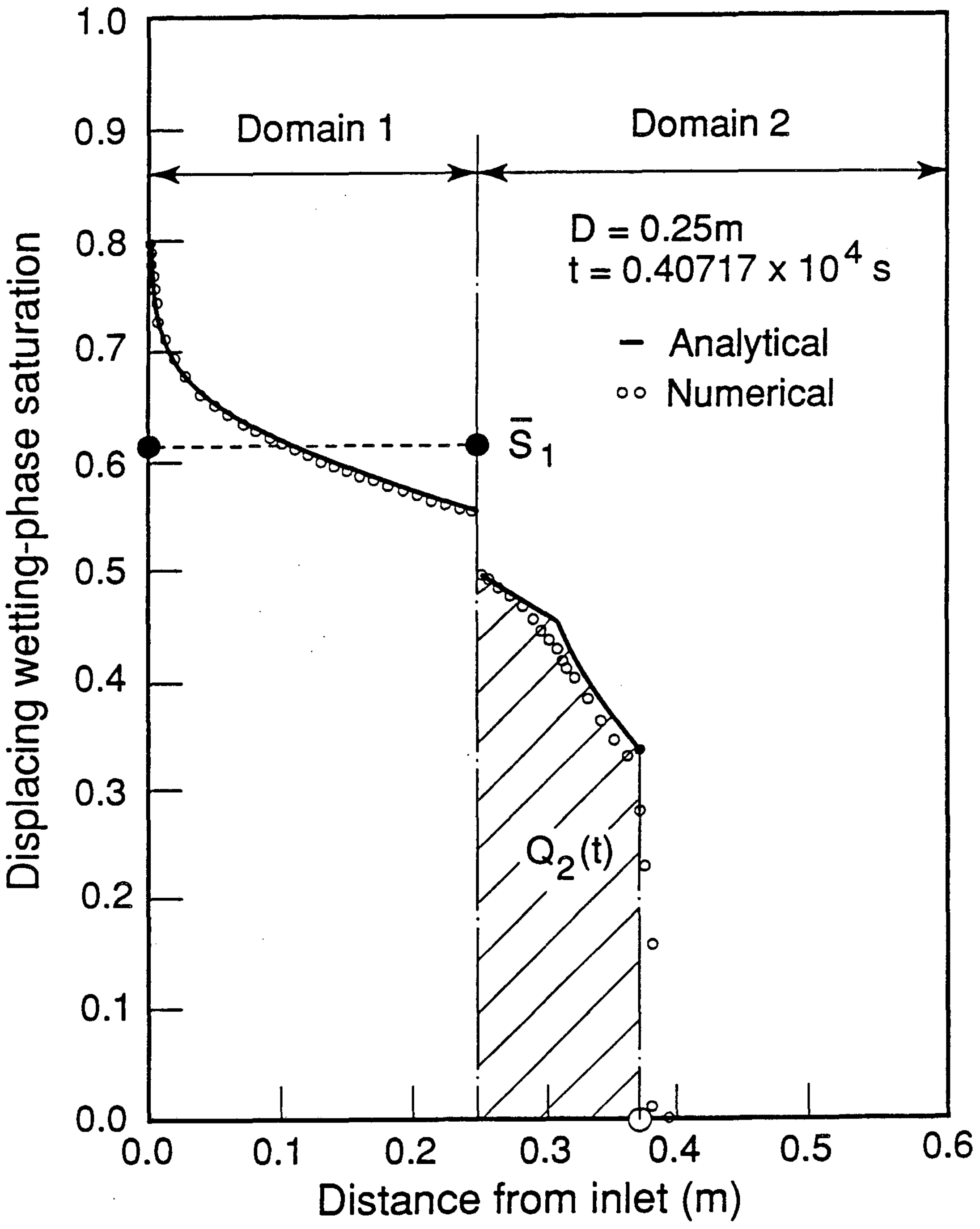

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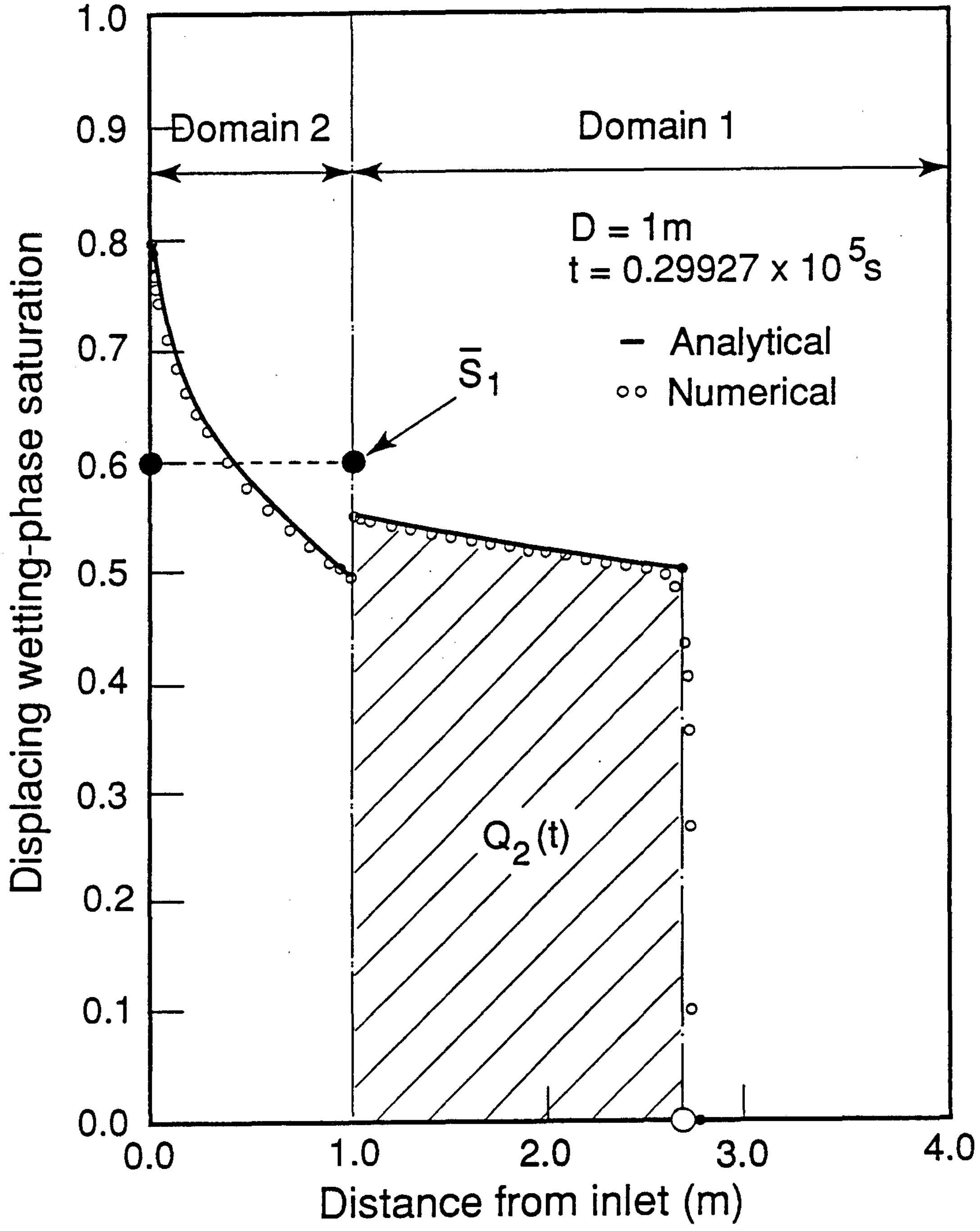
Figure 5. Comparison of Saturation Profiles Calculated from Analytical and Numerical Solutions.



XBL904-5439

Figure 6. Saturation Profile and Comparison with Numerical Solution.

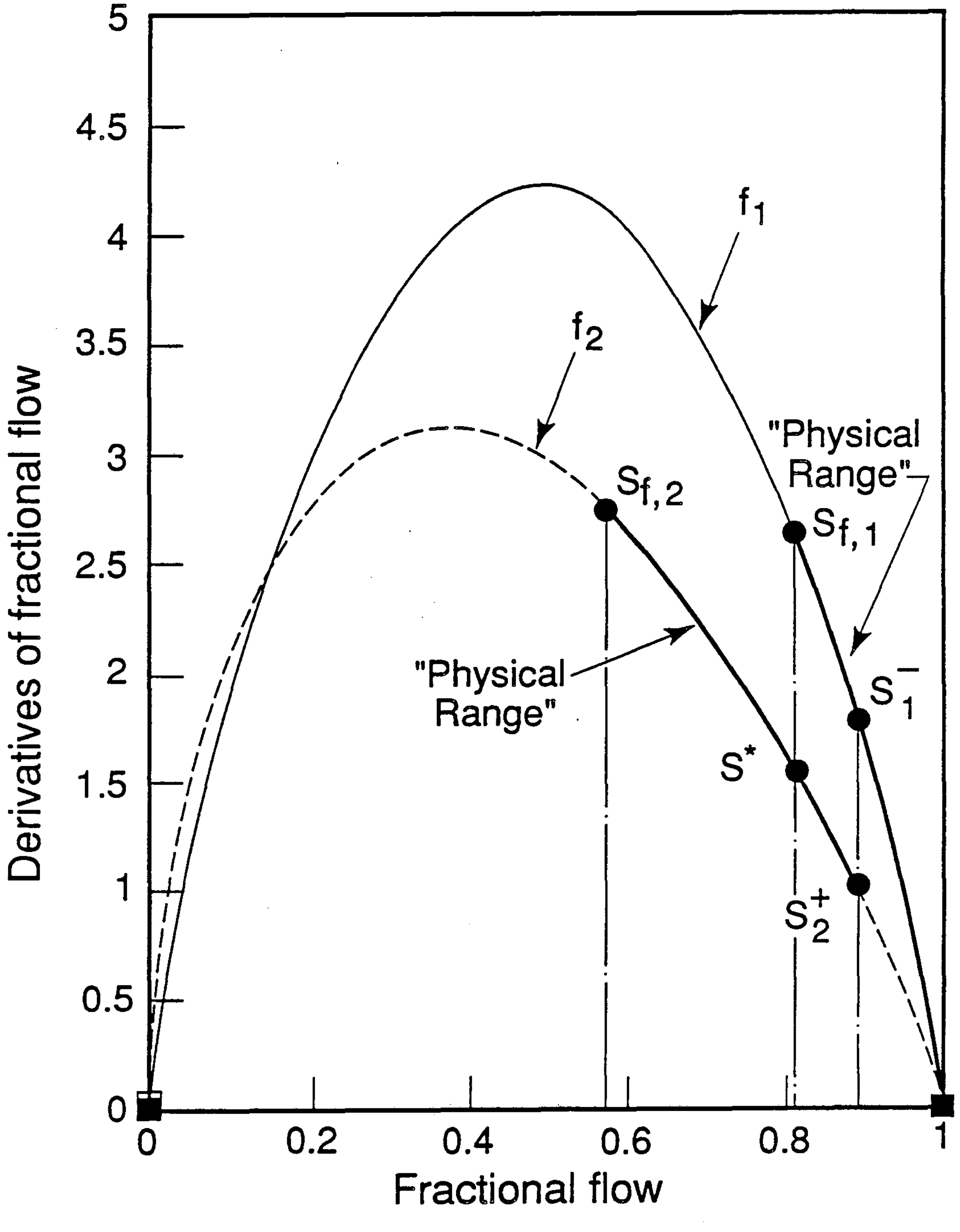


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Figure 7. Relationship of Fractional Flow and Its Derivatives in the Two Flow Domains.

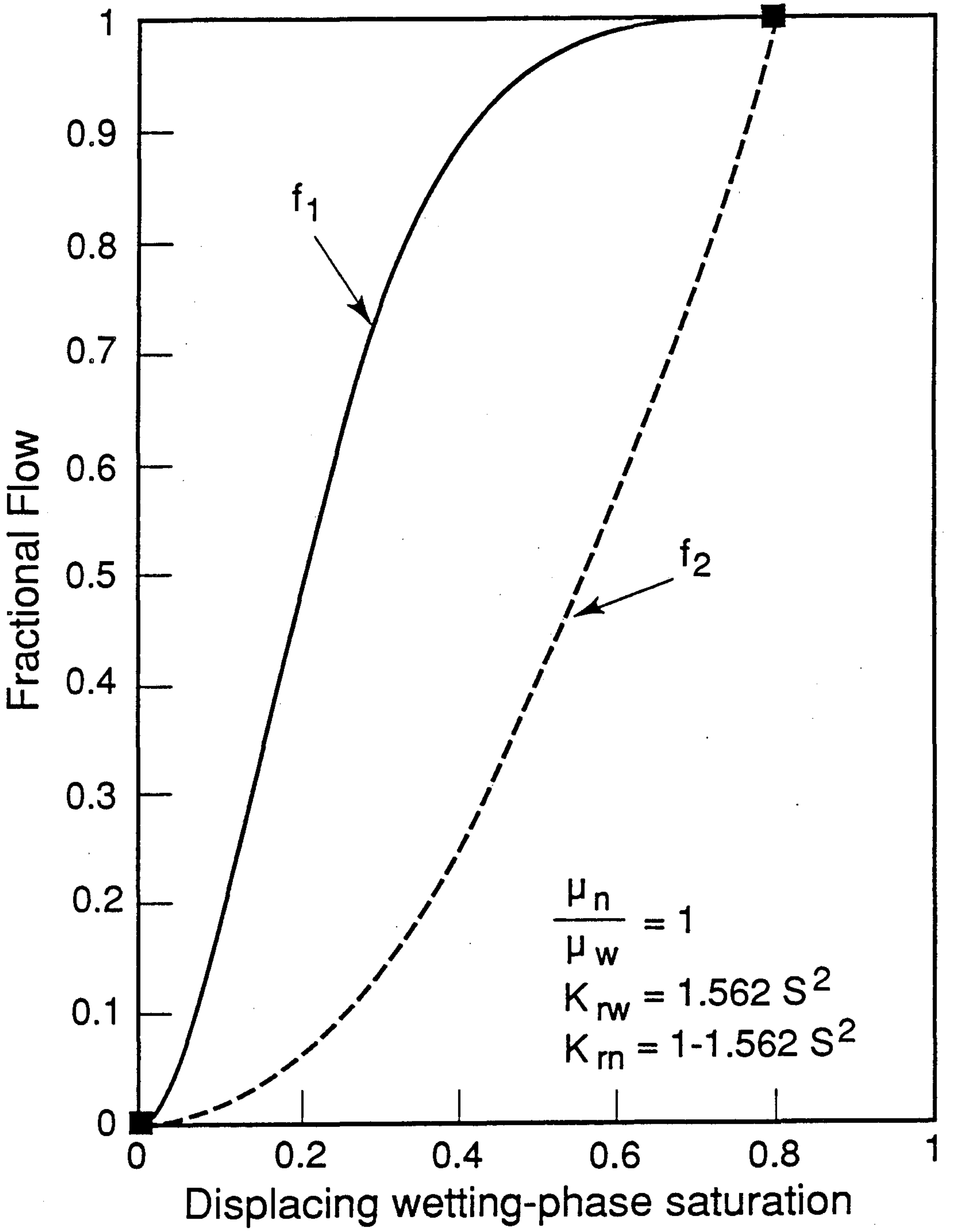

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Figure 8. Fractional Flow Curves, Showing Increases in Derivatives of Fractional Flow with Saturation in Domain 2.

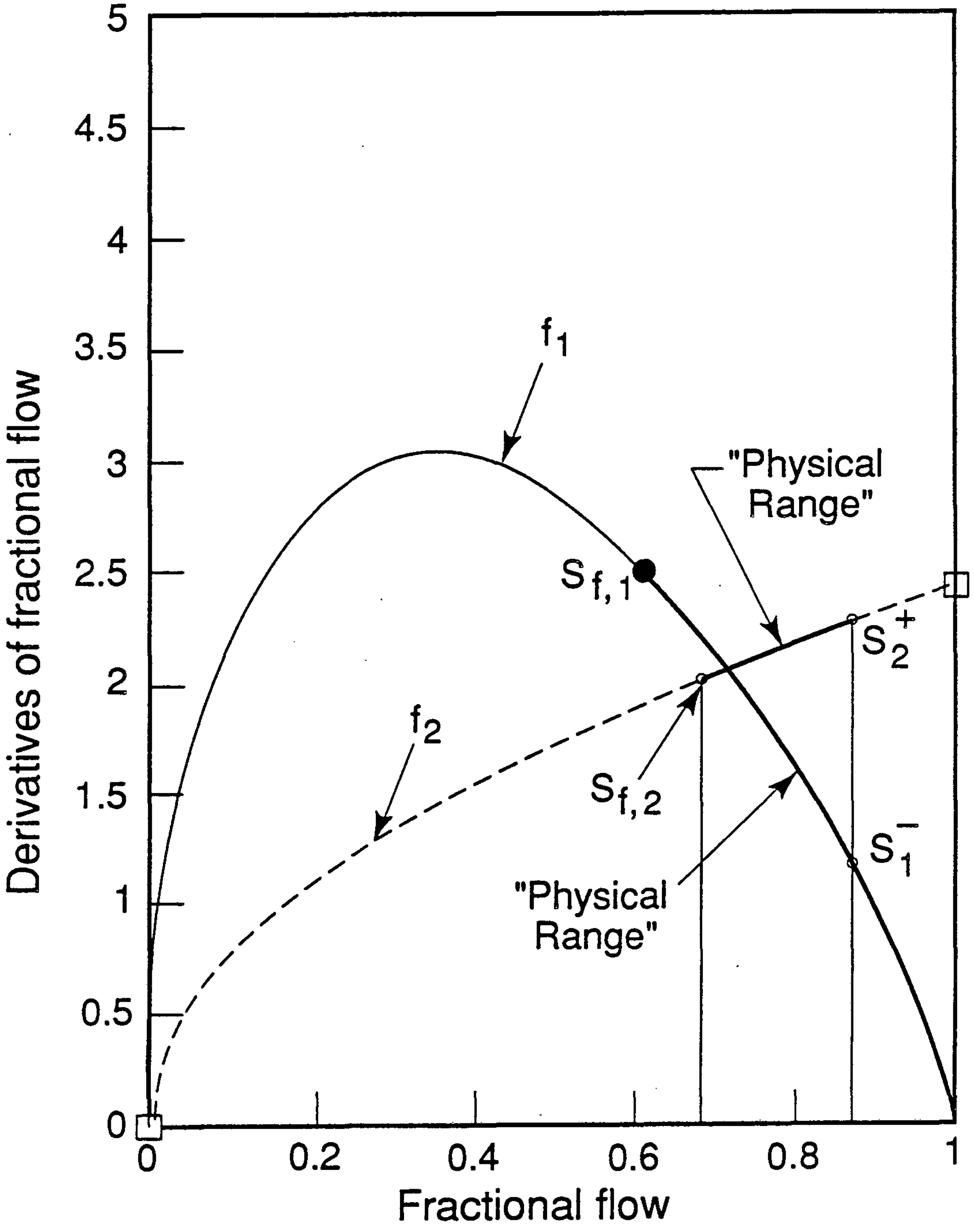

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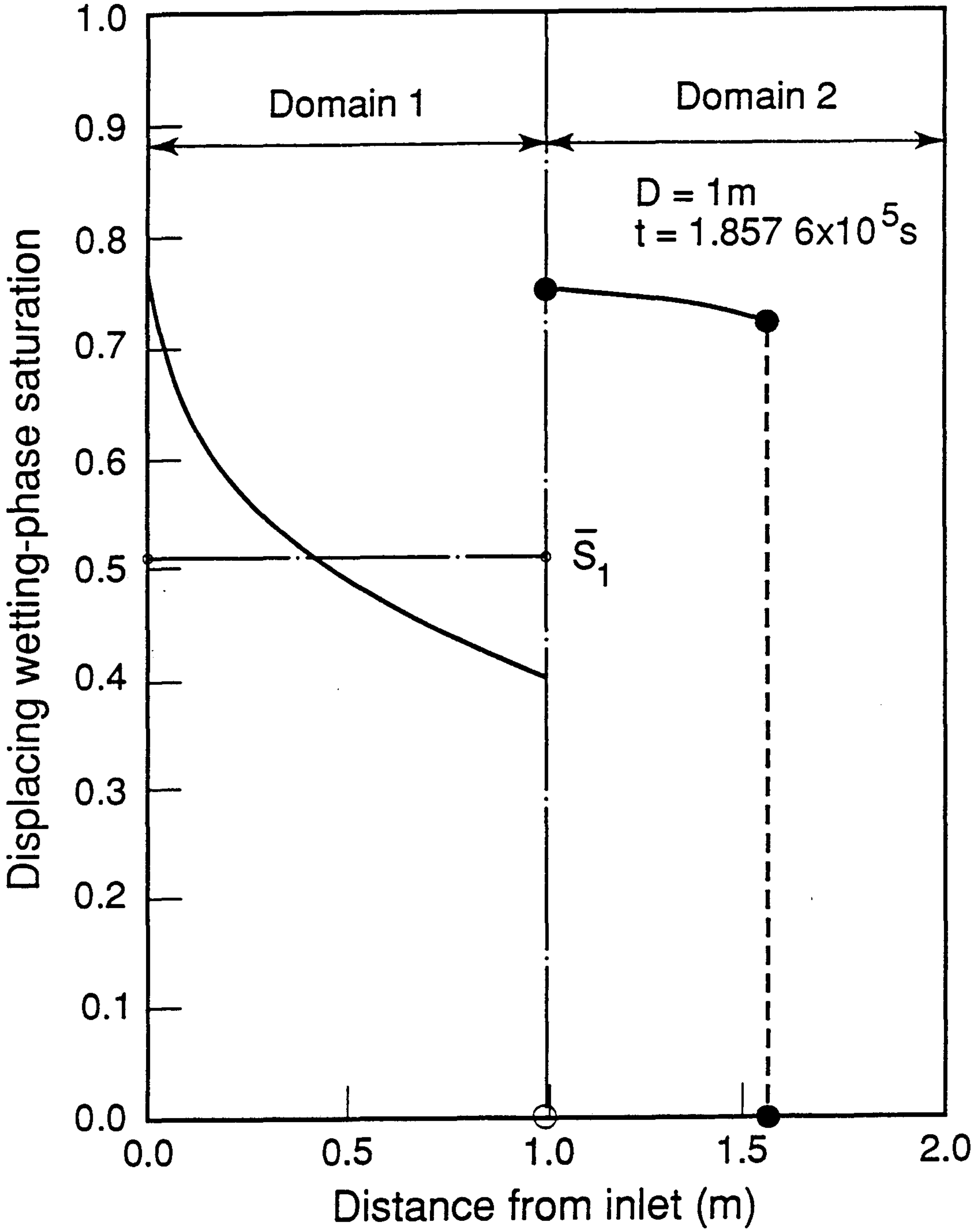
Figure 9. Relationship of Fractional Flow and Its Derivatives for Fractional Flow Curves in Figure 8.



XBL904-5435

Figure 10. Saturation Profiles for Fractional Flow Curves in Figure 8.

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