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The Pricing Risk of Energy Use Intensity for Office and Multifamily Mortgages*

Paulo Issler† Paul Mathew‡ Nancy Wallace§

May 27, 2020

Abstract

Prior studies have shown that energy use is related to commercial mortgage defaults. This report presents the results of a new and tractable methodology to estimate the association between energy use and mortgage pricing sensitivity - specifically the origination contract rate and points - of office and multifamily mortgages. *Source Energy Use Intensity* (Source EUI) is our key energy efficiency measure of interest due to its increasing availability through local energy efficiency benchmarking programs and the potential ease of its construction from the utility bills of commercial buildings. Based upon an empirical model of mortgage default transitions for a sample of 610 securitized office and multifamily mortgages from Trepp, we simulate the loan-by-loan mortgage prices using a four factor dynamic model with: i) a measure of Source EUI that is scaled to the the net operating income of the property, called Scaled Source EUI, ii) the Electricity Price Gap, the cumulative difference between expected and actual realized electricity prices, iii) loan-to-value ratio, and iv) the 10-year LIBOR rate. We find a statistically significant positive association between mortgage default and Scaled Source EUI and a statistically significant and negative association between the Scaled Source EUI of buildings and the simulated market prices of the mortgages written on those buildings. We then derive two sensitivity measures with respect to changes in Scaled Source EUI: i) the sensitivity of mortgage points to a 1% change in Scaled Source EUI; ii) the sensitivity of the mortgage coupon to a 1% change in Scaled Source EUI. We find that sensitivity of mortgage points to 1% shocks to Scaled Source EUI is 7.71 and 4.0 basis points respectively for office and multifamily loans. We find that the sensitivities of mortgage coupons to 1% shocks to Scaled Source EUI is 2.10 and 0.84 basis points respectively for office and multifamily loans.

Key words: Mortgages, Energy risk.

JEL codes: G21

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1 Introduction

Prior studies find mortgage defaults to be related to energy use and volatility (see, for example, Issler, Mathew, Sun, and Wallace, 2017; Jaffee, Issler, Stanton, and Wallace, 2017) and also show that energy ratings and use affect commercial building values (see, for example, Eichholtz, Kok, and Quigley, 2010; Jaffee, Stanton, and Wallace, 2018). The motivation for the study described in this report is to assess the impact of energy use and volatility on mortgage pricing and valuation. Specifically, we show how increases in energy use and volatility translate into higher interest rates and mortgage origination points. Building on the prior studies, we develop a commercial mortgage valuation strategy that accounts for building-level energy risk and mortgage default performance and using a sample of office mortgages originated between between 1999 to 2012 and a sample of multifamily mortgages originated between 1999 to 2014. We call our primary energy efficiency measure the Scaled Source Energy Use Intensity (EUI).[1] For the office properties we obtain the Source EUI from building specific benchmarking data which we then scale by the net operating income of the building which we obtain from the data company, Trepp.[2] For the multifamily properties, we develop a proxy for Source EUI constructed from annual metering data obtained from an energy benchmarking firm in Massachusetts, Wegowise,[3] and then scale it by the net operating income of the building obtained from Trepp.

There are three stages to our valuation methodology. In the first stage, we estimate a model of commercial mortgage default controlling for loan and building characteristics, in which we establish two measures for the energy related costs of operating the building: i) Scaled Source EUI; ii) the electricity price gap (calculated as the cumulative difference between the expected electricity prices at the time of loan origination and the realized electricity prices for the loan performance). The model also includes other standard proxies for exogenous factors that affect default terminations, such as the difference between the coupon on the mortgage and the realized 10 year LIBOR rate, the time to the balloon payment date on the mortgage, and the loan-to-value ratio on the mortgage.

In the second stage of the valuation analysis, we forecast the key determinants of the termination model: electricity forwards for the electricity price gap; the Scaled Source EUI; the LIBOR 10-year forward rates as a proxy for the forward mortgage coupon rates; the office and multifamily loan-to-value ratios computed by simulating the realized price changes for

[1]Source EUI measures the total amount of raw fuel that is required to operate the building per square foot including all transmission, delivery and production losses See, https://www.energystar.gov/buildings/facility-owners-and-managers/existing-buildings/use-portfolio-manager/understand-metrics/difference.

[2]See, https://www.trepp.com/.

[3]See, https://www.wegowise.com/.

the collateral on the mortgage and the amortized loan balance. Our strategy thus accounts for default risk associated with four forecasted exogenous market channels: 1) the dynamics of local-level electricity prices; 2) the level and volatility of national mortgage interest rates; 3) the prices of the properties based on forecasted price indices; 4) the dynamics of Scaled Source EUI.

In the third stage of the simulation process, we derive the loan-level elasticity of the price of each mortgage to a 1% change in its Scaled Source EUI. From the loan-level estimates of the price and elasticity with respect to a 1% shock to Scaled Source EUI, we derive measures of the sensitivity of the mortgage coupon and the mortgage points to 1% shocks to loan-level Scaled Source EUI.[4] These sensitivities give a direct market measure of the energy price risk of the loan and can be used in the risk management decisions of mortgage originators and portfolio lenders.

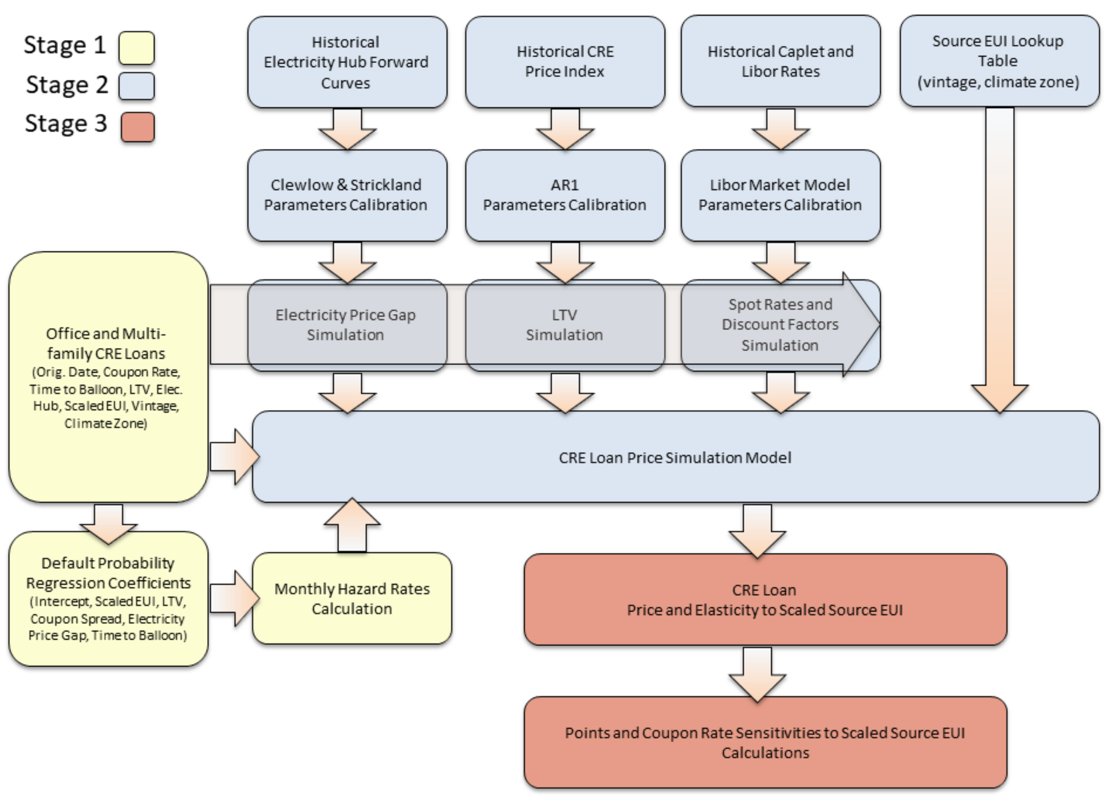

Figure 1: Schematic for the Valuation Model

Figure 1 summarizes the three stages of the valuation model. The first stage, shown in

[4]Our estimate of mortgage points is the interest charge, paid in cash on the loan origination date as a percentage of the initial principal of the loan, to achieve a market value of the loan equal to the amount of principal that is distributed to the borrower (i.e. for the loan to be valued at par on the origination date).

the lower left-hand corner of the schematic, is the estimation of the probability of default using loan-level data from TREPP in a reduced form model.

Figure 1 presents the the second stage across the top of the schematic. Our Monte Carlo simulation of the mortgage default transitions requires forecasts for the following factors: 1) electricity price levels and volatilities; 2) office and multifamily real estate price levels and volatilities; 3) interest rate levels and volatilities; and 4) estimated mean and standard deviations of Source EUI's by building vintage and climate zone in the U.S. The electricity forward and spot markets provide financial contracts that can be traded over-the-counter for individual regional markets [5]. We estimate the monthly loan-to-value ratios using property price processes estimated from Real Capital Analytics (RCA) data for office and multifamily properties and the computed end-of-month loan balance using the contract terms from TREPP for each loan. We fit the interest rate data using a LIBOR Market Model (see, Brace, Gątarek, and Musiela, 1997) to obtain the 10-year LIBOR forward rates (our proxy for the expected 10-year mortgage coupon rates) and for the LIBOR spot rates that are used for discounting. The fourth factor, the Scaled Source EUI means and standard deviations, are deterministic and were developed in a lookup table by location, building type, and vintage. These expected values are then introduced into the first stage hazard estimates to generate the expected mortgage cash flows (CFs) given the default risk exposure and recoveries of 40% of the loan balances. These cash flows are then discounted to obtain the expected market prices of the mortgages at origination.

In the third stage of the model shown at the bottom of the schematic, we derive measures of the sensitivity of the mortgage coupon and the mortgage points to 1% shocks to loan-level Scaled Source EUI.

2 Default Estimation

In this stage, we follow standard methodologies to estimate a reduced-form mortgage termination model.[6] Again, following Figure 1 and Issler et al. (2017), our measure of default is a loan that is ninety days or more delinquent, is in foreclosure, REO, or bankruptcy. The right hand side of the empirical model includes information on the spread between the mortgage coupon rate and the 10-year LIBOR rate, the loan-to-value ratio, and two constructed measures to account for energy risk; the Scaled Source EUI and the electricity price gap.

The Scaled Source EUI is constructed as the building's reported Source EUI scaled by

[5] We obtain the pricing for these regional markets from Platts historical electricity forward prices and the posted locational marginal prices (LMP) recorded by the independent system operators (ISO).

[6] See, for example, Deng, Quigley, and Van Order (2000); Schwartz and Torous (1989); Stanton and Wallace (2018); Titman and Torous (1989).

the net operating income of the building.[7] The Source EUI measures the total amount of raw fuel that is required to operate the building per square foot including all transmission, delivery and production losses. Due to data limitations, we implement a slightly different methodology to construct the Scaled Source EUI for the multifamily mortgage default estimation.[8]

The electricity price gap is a measure of building-specific electricity price risk. It is constructed as the difference between the forecasted and actual electricity costs of a building over the mortgage holding period. The electricity price gap is computed by summing the deviations of the realized monthly energy expenditures from the "expected" monthly expenditures that we assume could have been anticipated by the borrower, and/or lender, at the time of mortgage origination. Our measure of the anticipated monthly energy costs, are those reported to the lender in the building's *pro forma* as required by commercial loan underwriting. The measure accounts for seasonality using an indexing approach such that for each future realized month/year spot we are subtracting the anticipated *pro forma* price for the same month of the year, and thus comparing the appropriate seasonal prices to each other (i.e. comparing *pro forma* March prices to future realized March prices and so on through the seasons).

The estimation results are reported in Table 1 for the office and multifamily mortgages. As shown in the table, the results are quite similar for the multifamily and office mortgages. As is usual in mortgage default model estimates, the loan-to-value ratio is positively and statistically significantly associated with mortgage default for both the office and multifamily loans. Scaled Source EUI is shown to be a positive and statistically significant determinant of default meaning that as the energy expenditures per square foot increase the default probability rises. The electricity price gap is also shown to be a positive and statistically significant determinant of default meaning that the greater the cumulative gap between the loan's pro forma electricity cost estimates and the realized marginal price of electricity the higher the chance of default. Since most of these loans amortize over one horizon and are

[7]See, https://www.energystar.gov/buildings/facility-owners-and-managers/existing-buildings/use-portfolio-manager/understand-metrics/difference. Source EUI is considered the "gold standard" measure of energy efficiency because it provides the most equitable assessment of building-level energy efficiency. Billed site energy use is the primary component of the Site EUI and energy billing structures reflect a combination of primary energy (the raw fuel that is burned to create heat and electricity) and secondary energy (the purchased energy product created from a raw fuel).

[8]For multifamily, we construct a proxy for Source EUI by applying the site-to-source conversion factors from the EPA's Energy Star Portfolio Manager (See, https://portfoliomanager.zendesk.com/hc/en-us/articles/216670148-What-are-the-Site-to-Source-Conversion-Factors) to each of the fuel consumption types reported by Wegowise for each property. We then sum the converted fuel consumption levels to obtain an aggregate proxy for the Source EUI of each property. We multiply the measured proxy Source EUI by the gross square footage of the property and then divide this value by the secured net operating income (NOI) at origination producing a measure we term the Scaled Source EUI.

Table 1: **Linear regression estimates for the probability of default for office and multifamily mortgages.** This table presents the coefficient estimates for a linear probability model of default for office and multifamily mortgages. The table reproduces results first reported in Issler et al. (2017).

	Office	
	Coefficient Estimate	Standard Error
Intercept	0.00538	0.11067
Scaled Source EUI	0.00183***	0.999369
Loan-to-value Ratio	0.00263***	0.00117
Coupon Spread to 10 Year Treasury	0.00751	0.04000
Electricity Price Gap	0.00003**	0.00001
Time to Maturity on Balloon	-0.00203**	0.00068
Origination Year Fixed Effects	Yes	
	N = 339, r^2 = .18	
	Multifamily	
	Coefficient Estimate	Standard Error
Intercept	0.09654	0.09216
Scaled Source EUI	0.00088***	0.00024
Loan-to-value Ratio	0.00109***	0.00009
Coupon Spread to 10 Year Treasury	0.02314	0.01488
Electricity Price Gap	0.000013**	0.00000
Time to Maturity on Balloon	-0.00126**	0.00057
State Fixed Effects	Yes	
Post-Crisis Year Fixed Effects	Yes	
Origination Year Fixed Effects	Yes	
	N = 271, r^2 = .38	

$^{*}P < 0.1$; $^{**}P < 0.05$; $^{***}P < 0.01$

due in another, as shown, the number of months remaining before the full balance of the loan is due is also a statistically significant and negative factor in default, meaning that the closer the loan is to it's balance due date (the balloon payment date) the higher the default probability.

The regression reported in Table 1 is a linear probability model, however the simulation model requires estimates of the conditional probabilities, or hazards of loan default, at the end of every month. More formally, the unconditional probability of default is:

$$g(X(T-t)|\Theta) = \Theta' X(T-t), \tag{1}$$

where Θ is a vector of coefficients, $(\Theta_{EUI}, \Theta_{LTV}, \Theta_{coup\ sprd}, \Theta_{Elec\ gap}, T-t)$, as reported in Table 1 and X is a vector of the corresponding simulated factors as a function of time to balloon patment: $X := X(T-t)$. For the simulations we need an estimate of the hazard of default at the end of each monthly draw also expressed as function of time to balloon payment $\tilde{\lambda} := \tilde{\lambda}(T-t)$. More formally the unconditional probability is:

$$(1 - F(t)) = exp^{-\tilde{\lambda}(T-t)\times t} = g(X(T-t)|\Theta) = \Theta' X(T-t), \tag{2}$$

implying that desired hazard as a function of $(T-t)$ for each month is:

$$\tilde{\lambda}(T-t) = \frac{-ln[\Theta' X(T-t)]}{t}. \tag{3}$$

These hazard estimates are thus computed for each loan-month of the loan's history.

3 Mortgage Valuation Simulations

The objective of the mortgage valuation simulations is to determine the sensitivities of office and multifamily mortgages to changes in Scaled Source EUI which is our primary measure of the energy efficiency of commercial real estate. To calculate this sensitivity, we first simulate the market price of each mortgage in our office and multifamily sample by valuing the embedded default options in these mortgages. As discussed above in Section 2, our linear probability models of multifamily and office mortgage default indicated that the levels of default for these mortgages was importantly determined by the electricity price gap, the loan-to-value ratio of the mortgage, the Scaled Source EUI of the building, and the time to the balloon payment on the mortgage. We thus simulate the probability of the receipt of the principal and interest payments of each mortgage as a function of the estimated time series dynamics of each factor on the monthly hazard of default from the origination date on the loan to its balloon payment date, the loan default date, or the end of our performance data period (end of 2016).

Following the logic of Figure 1, there are four factors that must be simulated to determine the probability of default for each loan and the subsequent loss of principal and interest payments given an assumed 40% recovery rate. The dynamics used to simulate these factors are presented below for 1) the electricity price gap; 2) the office and multifamily real estate prices; 3) Scaled Source EUI; and 4) Interest rates.

3.1 Electricity price gap simulation

In our pricing model, we simulate the electricity price gap using the loan's regional hub electricity forward curve at the time of origination as the proxy for the forecasted monthly energy expenditures, and a simulated spot price as the proxy for the realized energy expenditures.

We model the dynamics of electricity prices following Schwartz (1997) and Clewlow and Strickland (1999), which assumes that an electricity forward price traded at time t and maturing at time T follow the risk-neutral processes given by

$$\frac{dF(t,T)}{F(t,T)} = \sigma e^{-\alpha(T-t)}\, dW(t), \tag{4}$$

where σ is the level of spot-price volatility and α is the rate of decay of the term structure of volatilities.

Following Clewlow and Strickland (1999), the exponential functional form of the term structure of volatilities and the non-arbitrage condition that equates the spot price at time t to a forward contract that is maturing also at time t,

$$S(t) = F(t,t),$$

results in a mean-reverting process for the spot price with mean-reversion rate equal to the rate of decay of volatility:

$$\frac{dS(t)}{S(t)} = [\mu(t) - \alpha \ln S(t)]dt + \sigma dW(t). \tag{5}$$

To match the initial forward curve for electricity, we set the drift component $\mu(t)$ in Equation 5 to

$$\mu(t) = \frac{\partial \ln F(0,t)}{\partial t} + \alpha \ln F(0,t) + \frac{\sigma^2}{4}\left(1 - e^{-2\alpha t}\right). \tag{6}$$

In particular, Clewlow and Strickland (1999) show that for a single factor, exponentially decayed term structure of volatilities specification defined in Equation 4, the dynamics the forward (futures) curve, at any future time is simply a function of the spot price at that time, the initial forward (futures) curve, and the volatility function parameters for electricity

$$F(t,T) \quad = \quad F(0,T)\left(\frac{S(t)}{F(0,t)}\right)^{\exp(-\alpha(T-t))} \exp\left[-\frac{\sigma^2}{4\alpha}e^{-\alpha T}\left(e^{2\alpha t}-1\right)\left(e^{-\alpha T}-e^{-\alpha t}\right)\right]. \quad (7)$$

In summary, once we calibrate the model parameters σ and α, we are able to simulate, under no-arbitrage conditions, both the evolution of spot prices and the full dynamics of the forward (futures) curve.

To calibrate the model parameters, we take the same approach detailed in Jaffee et al. (2017) and use historical market data for each major U.S. electricity forward trading region. We use the Clewlow & Strickland model's feature that for a fixed trade date t, as the time to maturity $(T-t)$ increases the instantaneous volatility of forward prices decays exponentially at a rate α. Our calibration approach assesses the instantaneous volatility of forward prices at trade dates taken as the 15^{th} day of each trade month (or the closest date to the 15^{th} when this date is a weekend or holiday). More precisely, we uses a small 22-day sample of historical price data around the 15^{th} day of each trade month to calculate its corresponding annualized historical time series of volatility.[9] Each of these small sample volatilities, and their corresponding number of months to maturity, constitute a data point for fitting an exponential decayed expression of the form $f(t) = \sigma^{-\alpha t}$, where α is the estimated rate of decay and σ is the intercept of the functional form to the t axis.

In Table 2, we report the estimation of the rate of decay of the term structure of volatilities, α, and the level of the spot price volatility, σ. Results show that there is considerable heterogeneity across the electricity hubs in the fitted values of the speed of mean reversion, α, and in the electricity spot price volatility, σ.

First, we mapped each loan in our study to its appropriate electricity region k. We then simulate the electricity spot price for region k using a discrete time version of Equation 5.

$$S_{k,t+1} = S_{k,t}([\mu_t - \alpha_k \ln S_{k,t}]\Delta t + \sigma_k \epsilon_t(0,1)\sqrt{\Delta t}). \quad (8)$$

[9]It is important to note that our historical forward prices reflect quotes from different seasonal packages. Because of averaging effects, it is expected that the larger the seasonal package, the lower the volatility. As a result, when calculating the historical time series of volatility for each trade month, we also include the size of the seasonal package as an explanatory variable.

Table 2: Estimates for the historical values of α and σ in the Clewlow and Strickland Process for the Electricity Hubs (Average 2004–2010)

Region	α	σ
East New York Zone J	0.352	0.313
ERCOT	0.417	0.525
Into Cinergy	0.231	0.384
Into TVA	0.303	0.424
Mass Hub	0.279	0.353
Mid-Columbia	0.175	0.489
Northern Illinois Hub	0.190	0.437
North Path 15	0.236	0.457
Palo Verde	0.206	0.473
PJM Western	0.272	0.347
South Path 15	0.212	0.446

Finally, we compute the electricity price gap for an individual month t as

$$pgap_{k,t} = \sum_{s=1}^{s=t} S_{k,s} - F(0,s), \tag{9}$$

and construct the simulated path by iterating this equation from $t = 0$ to $t = T$.

3.2 Office and Multifamily Real Estate Price Simulations

For our study, we use the Apartment and Office sub-categories for simulating price indexes for the multifamily and office loans respectively. For each loan in the sample, our estimates of the market dynamics are used to obtain the expected future changes in the initial market price of each property starting from the date that the mortgage was originated. These expected future market prices are then divided into the amortized balance of the loan to obtain a dynamic estimate of the end-of-month loan-to-value ratio of each loan.

As shown in Figure 2, we use price index data from Real Capital Analytics (RCA) for office and multifamily properties in the United States. To capture the persistence in the data, we estimate two single-lag, auto-regressive processes (AR(1)) as predictive pricing models for the office and multifamily series.

Our pricing models follows the common form:

$$y_{t_{office,multi}} = \beta y_{t-1_{office,multi}} + \sigma \epsilon_t \sqrt{\Delta t}, \tag{10}$$

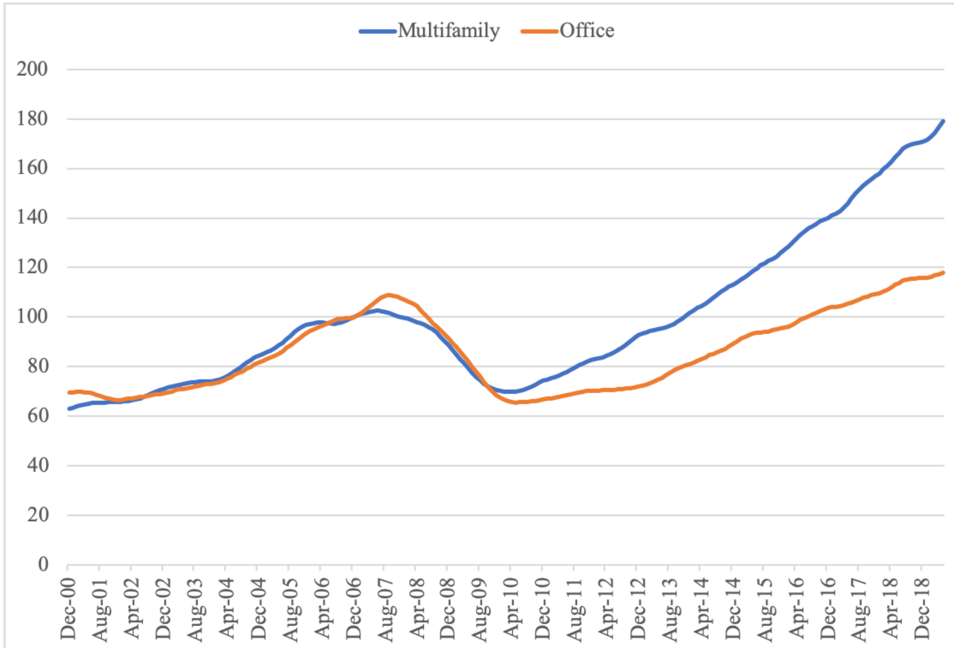

Figure 2: **Apartment and Office Price Indices.** This figure presents the Real Capital Analytics price indices for apartment and office properties in the United States. Source: Real Capital Analytics CPPI TM, where December 2006 = 100.

where $y_{t_{office,multi}}$ is the change in the log-return of the index at time t for either office or multifamily properties. The parameter, β, is the auto-correlation estimated from the regression, σ is estimated from the sample residual volatility, and $\epsilon_t \sim \mathcal{N}(0, 1)$.

Table 3 lists the results for the calibrated AR(1) process for the National price index and other sub-categories.

3.3 Scaled Source EUI Simulations

Table B.1 of the Appendix describes the simulated monthly profile of the average Source EUI, expressed in KBtu/sf., for a representative commercial buildings located within each of the fifteen U.S. climate zones[10] designated by the American Society of Heating, Refrigerating and Air-Conditioning Engineers (ASHRAE) - International Energy Conservation Code (IECC) and broken down by building vintages as classified under the following 3 tiers: **New** (buildings constructed after 2010), **Post-1980** (used for buildings constructed 1980-2010), and **Pre-1980** (used for buildings constructed prior to 1980). Table B.2 in the same Appendix lists the standard deviations, expressed in KBtu/sq., of the monthly Source EUI for the

[10]For more details, refer to https://www.energy.gov/sites/prod/files/2015/10/f27/ba_climate_region_guide_7.3.pdf for a full description of how climate zones are classified.

Table 3: Estimated AR-1 process for the RCA CPPI™ - US National Commercial Real Estate price Indices.

	β	σ	Adj. R^2
National All-Property	0.9928	0.0011	0.9820
Apartment	0.9865	0.0015	0.9687
Core Commercial	0.9915	0.0011	0.9817
Retail	0.9911	0.0010	0.9834
Industrial	0.9691	0.0022	0.9306
Office - CBD	0.9760	0.0020	0.9524
Office - Suburban	0.9828	0.0017	0.9653
Office	0.9872	0.0014	0.9743
Major Markets (All-Property)	0.9903	0.0013	0.9728
Non-Major Markets (All-Property)	0.9942	0.0009	0.9878

corresponding climate zones and vintages described in table B.1. Figure 3 presents the ASHRAE-IECC climate zone map for the continental U.S.

We construct the Scaled Source EUI variables listed in the linear regression coefficients in Table 1 by linearly scaling the building's Source EUI by the reported loan's net operating income (NOI) per square feet at the time of the loan origination[11]. We implement the simulation for Scale Source EUI by first simulating all Monte Carlo paths for a loan's monthly Source EUI in Table B.1, and then scaling the resulting paths by the factor

$$scale\_factor = \frac{Loan\_Scale\_Source\_EUI}{Source\_EUI_{month(t0)}}. \tag{11}$$

More specifically, we identify the loan's monthly Source EUI in Table B.1 from the the building's climate zone, building vintage, and the loan origination month. We then construct each Monte Carlo path for the monthly Source EUI by iterating Equation 12 from $t = 1$ to the time of balloon payment $t = T_{Balloon}$.

$$Source\_EUI_t = Source\_EUI_{t-1} + \mu_{month(t)}\Delta t + \sigma_{month(t)} \times \epsilon_t(0,1)\sqrt{\Delta t}, \tag{12}$$

where $Source\_EUI_0$ is set to the loan's origination monthly Source EUI at the lookup Table B.1, $Source\_EUI_t$ is the simulated monthly Source EUI for period t, $\mu_{month(t)}$ is the monthly Source EUI drift for the Brownian motion (defined as the difference between the average monthly Source EUI in Table B.1 for the months of the year of periods t and $t-1$),

[11] In our prior study we found that Scaled Source EUI is a better predictor than Source EUI because Source EUI in and of itself is not as important as how high EUI is relative to NOI.

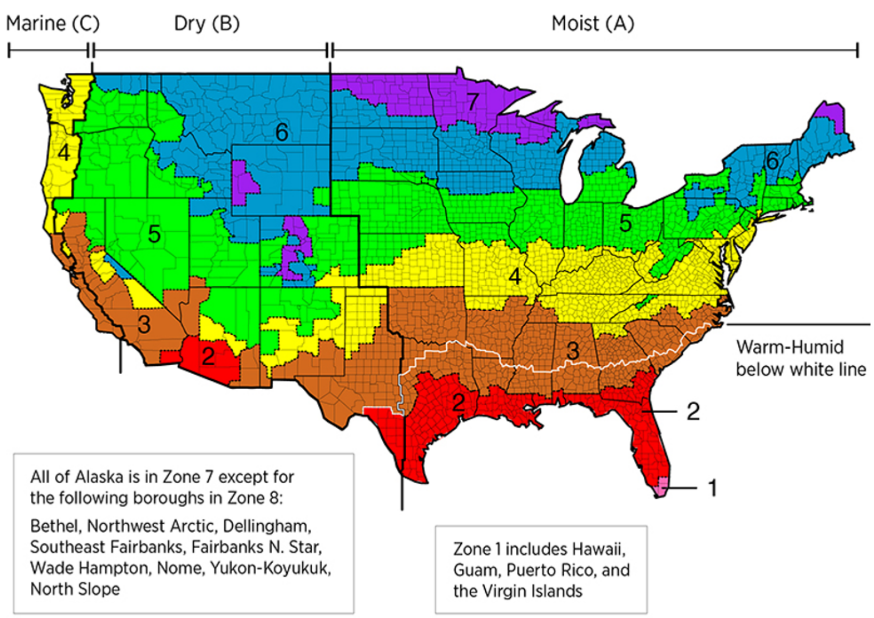

Figure 3: **Climate Zones in the United States.** This figure presents the six climate zones in the continental United States. Image source: https://basc.pnnl.gov/images/iecc-climate-zone-map 2020 IECC - International Energy Conservation code. https://codes.iccsafe.org/content/IECC2012P5

$\sigma_{month(t)}$ is the monthly Source EUI standard deviation for the month of the year and listed at the lookup table B.2, $\epsilon_t(0,1)$ is a draw from a standard normal distribution, and $\Delta t = 1/12$ is the monthly time step measured in years.

Finally, we scale all generated Monte Carlo paths by $scale\_factor$ defined in Equation 11.

3.4 Interest Rate Simulations

As shown in Figure 1, we fit the interest rate processes using a LIBOR market model (LMM). The LMM model introduced by Brace et al. (1997) is a common choice made by practitioners since the model can be nearly exactly calibrated to the quoted market spot and volatility term structures. The LMM model specifies the dynamics of forward rates. For each of the mortgage loans that are priced, we simulate the quarterly compounded forward rates from the loan's origination date ($t = 0$) to 10 years beyond the loan's maturity date ($t = t_T + 10$). The extra 10 years allow the 10 year spot rate to be simulated up to the maturity of the loan since the 10-year rate is needed to compute the coupon spread in the linear probability model.

LMM assumes that the forward rate $f(t, Ti, T_{i+1})$ follow a zero-drift log-normal diffusion process under the T_{i+1}-forward risk neutral measure:

$$\frac{df(t,T_i,T_{i+1})}{f(t,T_i,T_{i+1})} = \sigma_f^{i+1}(t)dW_t, \tag{13}$$

where dW_i is the standard Weiner process, T_i is the forward rate reset time, T_{i+1} is the maturity time of the forward contract, $\sigma_f^{i+1}(t)$ is the forward rate volatility, and $\Delta T = T_{i+1} - T_i = 0.25$. The dynamics of the other forward rates, under the same T_{i+1}-forward risk neutral measure, are given by:[12]

$$\frac{df(t,T_i,T_{i+1})}{f(t,T_i,T_{i+1})} = \begin{cases} -(\Sigma_{k=j}^{i-1} \frac{\Delta T f(t,T_k,Tk+1\sigma_f^{j+1}(t)}{1+\Delta T f(t,T_k,T_{k+1})})dt + \sigma_f^{j+1}(t)dW_t, for\ j < i \\ -(\Sigma_{k=i}^{j} \frac{\Delta T f(t,T_k,Tk+1\sigma_f^{j+1}(t)}{1+\Delta T f(t,T_k,T_{k+1})})dt + \sigma_f^{j+1}(t)dW_t, for\ j > i. \end{cases} \tag{14}$$

Apart from the initial forward rates $f(0, T_j, T_{j+1})$, the forward rate volatilities also need to be specified. Following the usual practice with LMM models, we assume that σ_f^{j+1} is

[12]This formulation is taken from the one-factor LMM formulation presented by Veronesi (2010). Within the one-facotor formulation, the inter-tenor correlations among the forward rates are not modeled explicitly, as only one independent Wiener process is present in a one-factor LMM This maintains the simulation time for the entire sample of mortgages within a practical limit.

constant and equal to the corresponding caplet forward volatility of the same maturity:

$$\sigma_f^{j+1}(t) = \sigma_{f,\ caplet}^{j+1}. \tag{15}$$

Once the model is specified, it needs to be calibrated to the LIBOR market data as of the pricing dates. Daily quotes are obtained from the Bloomberg terminal. Bootstrapping is then applied to a piece-wise constant, instantaneous forward rate (IFR) curve from the spot and swap rate data. For the intervals between two quoted tenors, the bootstrapping involves root-solving and linear interpolation on the time integral of IFR. Once the IFR curve is built, the quarterly compounded forward rates at $t = 0$ can be extracted as:

$$f(0, T, T + 0.25) = \frac{exp \int_T^{T+0.25} f(t)dt - 1}{0.25}, \tag{16}$$

where $f(t)$ is the IFR at time t.

Once the model is calibrated for each pricing date, the future evolution of each quarterly forward rate can be simulated by Monte Carlo simulation. Given the log-normal diffusion dynamics as specified in Equation 11, forward rates can be simulated by monthly increments:

$$f(t+, T_j, T_{j+1}) \approx f(t, T_j, T_{j+1}) exp \left\{ \left[\mu_{j+1}(t) - \frac{(\sigma_f^{j+1})^2}{} \right] \Delta t + \sigma_f^{j+1} \sqrt{\Delta t}\ \epsilon \right\}, \tag{17}$$

for all j, where $\mu_{j+1}(t)$ is one of the drifts specified in Equation 11, ϵ is an independent random draw from the standard normal distribution, and $\Delta t = t/12$. Antithetic random draws are used for variance reduction. The LMM simulations produce monthly 10-year spot rates that are our proxies for the future 10-year mortgage coupon rate and discount factors for all tenors.

4 Mortgage Valuation Results

As shown in Table 4, energy driven default does affect the simulated market prices of both office and multifamily mortgages. We find that the office loans are priced at a slight discount of \$98.90 per \$100 of the initial loan principal for office loans and \$95.21 per \$100 of the initial loan principal for multifamily loans. The key sensitivity of interest, however, is the estimated price elasticity of the mortgages with respect to a 1% change in Scaled Source EUI. As shown in the Table, this price elasticity is 8.97% for the office mortgages with a standard deviation of 1.66% and is .46% for the multifamily mortgage with a standard deviation of .11%.

Table 4: **Mortgage Valuation Results** This table presents the simulation results for the loan-level estimates of the market price of the mortgages and estimates for the elasticity of the market price with respect to 1% change in Scaled Source EUI.

	Simulated Market Price per $100 of Loan Principal		Price Elasticity with respect to 1% Change in Scaled Source EUI	
	Mean $	Standard Deviation $	Mean %	Standard Deviation %
Office Loans	98.80	11.31	8.97	1.66
Multifamily Loans	95.21	9.40	0.46	0.11

To our knowledge, this is the first paper in either the mortgage pricing or energy efficiency literature that explicitly computes the mortgage price elasticity associated with defaults induced by a 1% shock to Scaled Source EUI. Thus, a natural question to ask is whether the levels of these estimates are reasonable and what might explain the important differences between the commercial office and multifamily elasticities. With respect to the first point, the effect of default on commercial and multifamily mortgage pricing through the shocks to energy appear more muted than the effects of shocks to interest rates and property prices on these mortgages (Stanton and Wallace, 2018).

One possible reason for the important empirical difference in our estimated elasticities for office and multifamily loans, concerns the contractual differences between the allocation of energy expenditures in multifamily and commercial leases generally and especially in our data sets. The energy utilitization data for our multifamily loans is from Wegowise, and all of the multifamily properties in this sample have metered and priced each tenant's individual electricity use. Thus, the only electricity consumption that is attributable to the borrower/property owner is for the common areas, reception, grounds and offices of the property and Wegowise reports the exact square footage that is assignable to the landlord and the electricity usage that is assignable to that square footage. For most of these properties the landlord controlled space is a small proportion of the overall square footage of the property and energy consumption whereas the unobserved electricity consumption of the tenants is unobserved and thus is not part of our calculations.

Unfortunately, we have less complete data for the lease contracts for the buildings with commercial loans (i.e. we do not know whether the leases are full service or triple net) and we only know the overall electricity energy through the Scaled Source EUI at the property level. Thus, the elasticity calculations for the commercial loans is implicitly assuming the the leases are full service so that the building owner is exposed to the full impact of shocks to electricity prices. As a result, this elasticity estimate should be considered as an upper bound.

Given the simulation results for the market prices, we then computed the amount of interest that would have to be charged as a cash payment on the origination date of the loan (the number of points to be charged at origination) for the market value of the loan to equal the amount of principal distributed to the borrower (i.e. for the loan to be valued at 100% of its balance, or par). Figure 4 presents the estimated relationship between the number of points that would have to be charged at origination for the loans to be priced at par for properties with various average levels of Scaled Source EUI over the holding period.

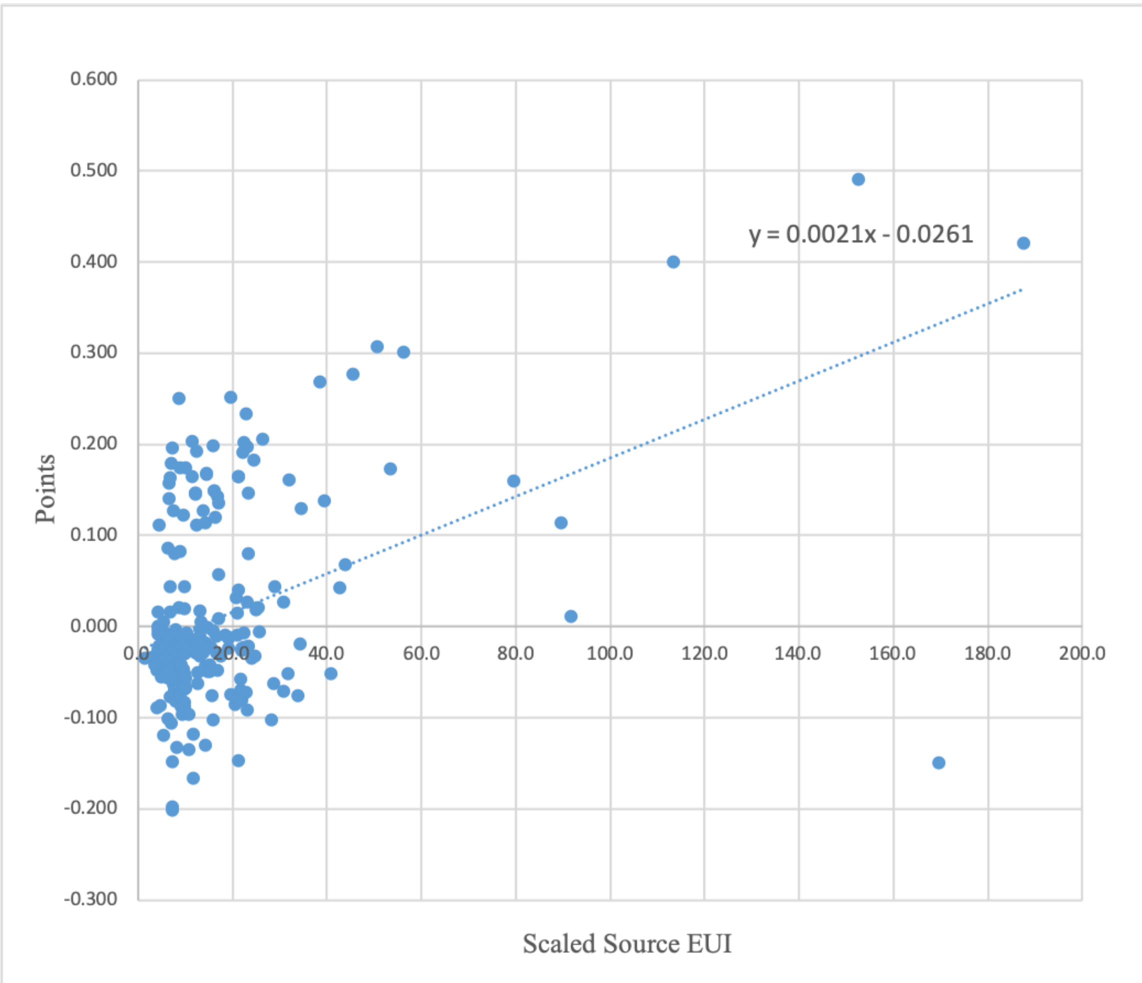

Figure 4: **Estimated points required to price each office loan to par given the Scaled Source EUI of the property.** This figure presents the estimated points that would have to be charged at origination so that each office loan would price to par given the Scaled Source EUI of the property.

As shown in Figure 4, office properties that exhibited higher Scaled Source EUI required

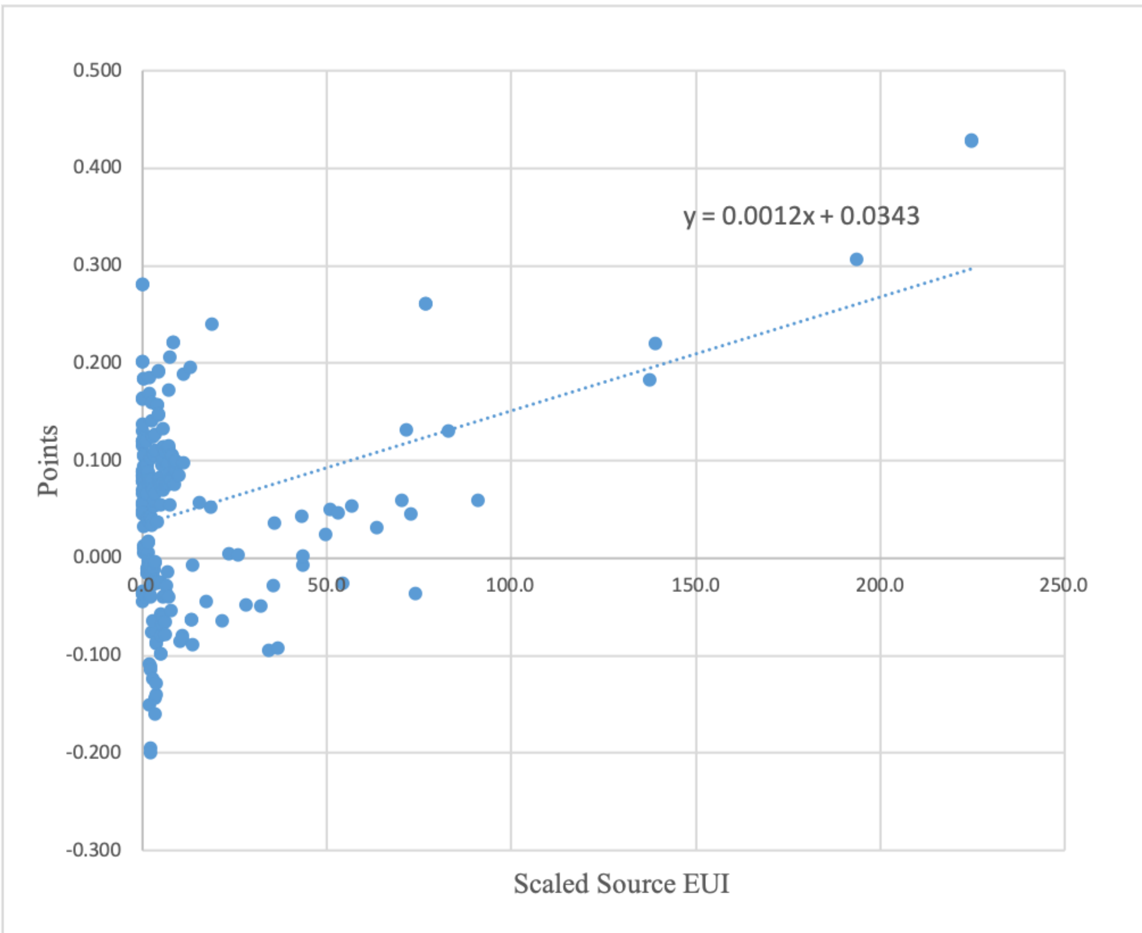

Figure 5: **Estimated points required to price each multifamily loan to par given the Scaled Source EUI for the property.** This figure presents the estimated points that would have to be charged at origination so that each office loan would price to par given the Scaled Source EUI of the property.

more points to be charged at origination for the market price (the sum of the forecast price plus the cash payment of points as a percentage of initial balance) of these loans to equal par on origination. In other words, buildings that are more energy efficient should be charged fewer points to compensate for the lower expected default risk of these mortgages. Similar results are shown in Figure 5 for the multifamily loans. Here again, we find that multifamily buildings that are more energy efficient should be charged fewer points to compensate for the lower expected default risk of these mortgages. In addition, although not shown, the plots for the relationship between the mortgage coupon and the Scaled Source EUI show exactly the same positive relationship for both office and multifamily mortgages.

4.1 Sensitivities

Our empirical estimation and simulation results indicate that more energy inefficient buildings are associated with riskier mortgages due to their elevated levels of default. Our goal, however, is to determine the relative sensitivities of the mortgage coupon and mortgage points in response to shocks to Scaled Source EUI. Following the derivations presented in Appendix A, the sensitivity of points to shocks to Scaled Source EUI can be written as:

$$\begin{aligned}\frac{\Delta points}{\frac{\Delta EUI}{EUI}} &= \frac{\frac{\Delta price(c_m)}{price(c_m)}}{\frac{\Delta EUI}{EUI}} \times \frac{\Delta points}{price(c_m)} \times price(c_m) \\ &= price\ elasticity_{EUI} \times \frac{-price(c_m)}{Bal_0},\end{aligned} \tag{18}$$

Where the Bal_0 is the balance of the loan at origination, $price(c_m)$ is the simulated market price at origination of a loan with a given monthly coupon, c_m, points are defined as $(1 - \frac{price(c_m)_0}{Bal_0})$, EUI is the Scaled Source EUI, and the price elasticity with respect to Scaled Source EUI, $price\ elasticity_{EUI}$, is obtained from the simulations.

Similarly, as derived in Appendix A, the sensitivity of the annual coupon, c_a, on the mortgage with respect to Scaled Source EUI is:

$$\frac{\Delta c_a}{\frac{\Delta EUI}{EUI}} = -price\ elasticity_{EUI} \times price(c_m) \times \frac{12}{\frac{\Delta price(c_m)}{\Delta c_m}}, \tag{19}$$

Where c_a is the annual coupon on the loan (12 $\times$ the monthly coupon).

The sensitivity results are reported in Table 5. As shown in the Table, the points sensitivity to a 1% change in the Scaled Source EUI is 7.71 basis points for office loans and is 4 basis points for multifamily loans. The coupon sensitivity to a 1% change in Scaled

Source EUI is 2.1 basis points for office loans and .84 basis points for multifamily loans. The computed standard errors for the measures are all quite small.

Table 5: **Points and Coupon Sensitivities to Source EUI Shocks (Δ1.0%).** This table presents the simulation results for the points and coupon sensitivities to 1% shocks to Source EUI.

	Points Sensitivity to 1% Change in Scaled Source EUI		Coupon Sensitivity to 1% Change in Scaled Source EUI	
	Mean Basis Points	Standard Error Basis Points	Mean Basis Points	Standard Error Basis Points
Office Loans	7.71	0.79	2.10	0.24
Multifamily Loans	4.0	0.56	0.84	0.14

Although these sensitivity values may seem small, realized shocks to Scaled Source EUI can be very large. For example, a prior case study analysis of five buildings found that reasonably common changes in building operational practices and occupancy characteristics could change Source EUI from -62% to +183% in office buildings (See, Mathew, Sun, Ravache, Issler, and Wallace (2011)). In addition, Mills (2011) finds that building commissioning alone (which only involves changes to how the building is operated) can result in 10-20% reduction in energy use.

5 Conclusions

This report presents the results of a new and tractable methodology to obtain statistically significant estimates of the association between an important energy efficiency measure, the Scaled Source EUI, and the default risk and mortgage pricing sensitivity of office and multifamily mortgages. Building upon prior studies (see, for example, Issler et al., 2017; Jaffee et al., 2017), we expand on standard commercial mortgage pricing techniques by first estimating a linear probability model of mortgage default, defined as 90 days or more delinquent, that includes factors for Scaled Source EUI, the Electricity Pricing Gap, mortgage contract terms, fixed effects, and the time to the balloon payment on the mortgage. This analysis was carried out using loan-level performance and origination data from Trepp for 610 mortgages. We then introduce regional forecasts of two energy related metrics, Scaled Source EUI and the Electricity Pricing Gap, into the loan-level pricing simulations. The Electricity Pricing Gap is forecast using standard econometric methods strategies to fit regional electricity forward prices (see, for example, Clewlow and Strickland, 1999; Jaffee et al., 2017). The Scaled Source EUI is simulated using deterministic models of EUI by geographic region, building type and building age. Our pricing methodology also includes the more typical mortgage pricing factors of the market prices of the buildings to forecast dynamic loan-to-value ratios, and the term structure of LIBOR rates to forecast the spread between the mortgage coupon and realized expected future mortgage rates.

Our key energy efficiency measure of interest, due to the increasing ease of access for this measure from local benchmarking efforts and the potential ease of its construction from the utility bills of commercial buildings, is the *Scaled Source EUI* of the building that is the collateral for each loan. We simulated the market prices of our sample of mortages, using a four factor dynamic model: i) Scaled Source EUI, ii) the Electricity Pricing Gap, iii) loan-to-value ratio, and iv) the 10-year coupon rate proxied by the 10-year LIBOR rate. We find a statistically significant negative association between the Scaled Source EUI of buildings and the simulated market prices of the mortgages written on those building. We then derive two sensitivity measures with respect to changes in Scaled Source EUI: i) the sensitivity of mortgage points to 1% changes to Scaled Source EUI; ii)the sensitivity of the mortgage coupon to 1% changes Scaled Source EUI. We find that the points sensitivities to 1% shocks to Scaled Source EUI are 7.71 and 4.0 basis points respectively for office and multifamily loans. We find that the coupon sensitivities to 1% shocks to Scaled Source EUI are 2.10 and 0.84 basis points respectively for office and multifamily loans. These sensitivities give a direct market measure of the energy price risk of the loan and could be used in the risk management decisions of mortgage originators and portfolio lenders.

A Derivation of the Sensitivity Formulae

As discussed in Section 4, the simulations provide information on the simulated market price of each mortgage and a measure of the elasticity of each mortgage price with respect to a 1% shock to Scaled Source EUI. Based on these estimates we derive two sensitivity measures: 1) the sensitivity of points with respect to a 1% change in Scaled Source EUI; 2) the sensitivity of coupon with respect to a 1% change in Scaled Source EUI. These derivations require employing the loan balance equations and its derivatives, and expressing them in terms of the simulated market prices of the loans, the underlying coupons, and/or the points charged at origination for the loans.

The simulated market price of a loan can be expressed as:

$$Price(c_m) = \sum_{t=1}^{T_B} \frac{pmt(c_m)}{(1+y)^t} + \frac{Bal(c_m)_{T_B}}{(1+y)^{T_B}}, \tag{20}$$

where pmt is the monthly loan payment, $Bal(c_m)_{T_B}$ is the balance due on the loan on the balloon payment date, T_B given its monthly coupon, c_m, (these mortgages amortize over one maturity period and are due at an earlier period called the balloon payment date), and y is the market yield to maturity. Notice that from Equation 20, for $Price(c_m)$ to be well defined, we need to solve for the implied yield to maturity y for the loan given the realized monthly coupon payments and the final balloon payment. Thus, for each loan we numerically solve for the yield to maturity.

From the loan balance equations, the monthly payment, $pmt(c_m)$, is expressed as:

$$pmt(c_m) = Bal_0 \times \left[\frac{c_m(1+c_m)^T}{(1+c_m)^T - 1}\right], \tag{21}$$

where c_m is the monthly coupon rate on the loan, $c_a = c_m \times 12$ is the annual coupon rate on the loan, Bal_0 is the initial balance on the loan, and T is the final maturity date on the loan.

The balance on the loan at the balloon date, T_B, is:

$$Bal(c_m)_{T_B} = Bal_0 \times \frac{\left[(1+c_m)^T - (1+c_m)^{T_B}\right]}{(1+c_m)^T - 1} \tag{22}$$

The derivative of the price with respect to the monthly coupon rate is:

$$\frac{dprice(c_m)}{dc_m} = \frac{dpmt(c_m)}{dc_m} \times \left[\sum_{t=1}^{T_B} \frac{1}{(1+y)^t}\right] + \frac{dBal(c_m)_{T_B}}{dc_m} \frac{1}{(1+y)^{T_B}}, \tag{23}$$

where the derivative of the payment, $pmt(c_m)$, with respect to the monthly coupon, c_m, is:

$$\frac{dpmt(c_m)}{dc_m} = Bal_0 \times \frac{\left[[(1+c_m)^T - 1][c_m T(1+c_m)^{T-1} + (1+c_m)^T] - [c_m(1+c_m)^T][T(1+c_m)^{T-1}]\right]}{\left[(1+c_m)^T - 1\right]^2}, \tag{24}$$

and the derivative of the balance, $Bal(c_m)$, on the mortgage with respect to monthly coupon, c_m, is:

$$\frac{dBal(c_m)}{dc_m} = Bal_0 \times \frac{\left[[(1+c_m)^T - 1][T(1+c_m)^{T-1} - T_B(1+c_m)^{T_B-1}] - [[(1+c_m)^T - (1+c_m)^{T_B}]T(1+c_m)^{T-1}]\right]}{\left[(1+c_m)^T - 1\right]^2} \tag{25}$$

The price elasticity of the mortgage with respect to Scaled Source EUI is:

$$Price\ elasticity_{EUI} = \frac{\frac{\Delta price(c_m)}{price(c_m)}}{\frac{\Delta EUI}{EUI}}. \tag{26}$$

Scaling Equation 28 by price and multiplying through by $\Delta c_a/\Delta c_a$ gives:

$$Price\ elasticity_{EUI} \times price(c_m) = \frac{-\Delta c_a}{\frac{\Delta EUI}{EUI}} \times \frac{\Delta price(cm)}{\Delta c_a} \tag{27}$$

$$\frac{\Delta c_a}{\frac{\Delta EUI}{EUI}} = -price\ elasticity_{EUI} \times price(c_m) \tag{28}$$

$$\frac{\Delta c_a}{\frac{\Delta EUI}{EUI}} = -price\ elasticity_{EUI} \times price(c_m) \times \frac{12}{\frac{\Delta price(c_m)}{\Delta c_m}} \tag{29}$$

where the negative sign on Δc_a arises because every change in the price of the mortgage with respect to a shock to EUI (recalling that shocks to EUI are negatively associated with price due to the effects on the probability of default) must be accompanied by a symmetric and opposite shock to the coupon on the mortgage so as to keep the mortgage equivalently priced.

B Monthly Source EUI Lookup Tables

The data in the Source EUI lookup tables shown below were derived from parametric whole building energy simulations of the DOE Reference Models[13] for large offices, using the EnergyPlus simulation software.[14] Each reference model is specific to one of 15 climate zones and three building vintages (pre-1980, post-1980, new construction). For each reference model we conducted parametric simulations to estimate the variation in source energy use intensity due to changes in over 20 operational practices and occupancy characteristics, including lighting controls, HVAC controls, plug loads, occupant density, occupant schedules. For each operational parameter, we defined three levels of practice: good, average and poor. Good practice represents design intent or optimal performance of the building. For average and poor practice, the analysis assumes the building has the capability to run at the good practice level, but runs less efficiently due to average or poorer facility management or occupant behavior. In each parametric simulation a level of practice for each O&M parameter was selected, assuming 50% probability for average practice and 25% probability for good and poor practice. Each simulation yielded monthly and annual Source EUI. We then calculated the means (Table B.1) and standard deviations (Table B.2) for annual and monthly energy use for each of the reference models.

[13] DOE 2019a. Commercial Reference Buildings. https://www.energy.gov/eere/buildings/commercial-reference-buildings. U.S. Department of Energy. Accessed July 2019

[14] DOE 2019b. EnergyPlus. https://energyplus.net/.U.S.DepartmentofEnergy. Accessed July 2019.

Table B.1: Annual and monthly Source EUI means, measured in KBTU/sqft, for the eight U.S. climate zones and building vintage tiers.

Climate Zone	Vintage	Annual Mean Source EUI (KBTU/sqft)	Monthly Source EUI (KBTU/sqft) Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
1A	New	160	10.6	9.9	11.7	12.4	15.4	16.9	16.5	17.7	13.4	13.0	11.9	10.6
1A	Post	192	12.8	12.0	14.1	14.9	18.5	20.1	19.6	21.1	16.0	15.6	14.4	12.8
1A	Pre	201	13.2	12.4	14.6	15.6	19.4	21.3	20.8	22.4	16.8	16.2	14.9	13.2
2A	New	163	11.2	10.2	11.7	11.9	15.2	17.9	17.8	19.3	13.7	11.8	11.2	11.0
2A	Post	196	13.7	12.3	14.2	14.3	18.2	21.3	21.1	23.0	16.4	14.2	13.5	13.3
2A	Pre	204	13.7	12.5	14.5	15.0	19.1	22.7	22.7	24.6	17.3	14.7	13.9	13.6
2B	New	139	9.4	8.7	10.2	10.4	12.2	15.1	15.5	16.4	11.6	10.7	9.5	9.4
2B	Post	167	11.4	10.6	12.3	12.5	14.7	18.0	18.3	19.5	14.0	12.9	11.5	11.4
2B	Pre	174	11.5	10.8	12.6	13.0	15.4	19.0	19.5	20.7	14.6	13.4	11.8	11.4
3A	New	137	11.4	9.6	10.3	9.7	11.2	13.6	14.4	16.2	11.0	10.1	9.7	10.2
3A	Post	170	14.7	12.2	12.8	11.9	13.7	16.7	17.6	19.8	13.4	12.4	12.1	12.9
3A	Pre	178	15.1	12.5	13.1	12.4	14.4	17.8	18.9	21.2	14.1	12.9	12.4	13.2
3B	New	124	10.0	8.9	10.3	9.7	10.4	10.6	10.9	11.7	11.0	10.6	10.0	9.8
3B	Post	152	12.5	11.1	12.8	12.0	12.8	13.0	13.2	14.3	13.3	12.8	12.2	12.0
3B	Pre	157	12.7	11.2	13.0	12.3	13.1	13.5	13.8	15.0	13.9	13.2	12.5	12.3
3C	New	106	9.5	8.1	9.2	8.4	9.1	9.0	8.6	9.4	8.7	8.8	8.6	9.0
3C	Post	128	11.5	9.8	11.2	10.1	10.9	10.9	10.3	11.4	10.4	10.6	10.3	10.9
3C	Pre	131	11.6	9.9	11.3	10.3	11.2	11.1	10.6	11.7	10.6	10.8	10.5	11.0
4A	New	146	13.7	11.5	11.7	9.9	11.7	13.3	14.9	14.6	11.6	10.5	10.5	12.4
4A	Post	178	17.0	14.1	14.4	12.0	14.1	15.9	17.7	17.6	13.9	12.7	12.8	15.3
4A	Pre	188	18.1	15.0	15.0	12.5	14.9	17.0	19.0	18.8	14.6	13.1	13.4	16.2
4B	New	127	11.1	9.7	10.3	9.3	10.0	11.5	12.1	12.0	10.1	10.0	9.8	11.1
4B	Post	156	13.9	12.1	12.7	11.3	12.1	14.0	14.6	14.6	12.2	12.2	12.1	13.9
4B	Pre	161	14.2	12.4	13.0	11.6	12.5	14.6	15.3	15.3	12.6	12.5	12.3	14.3
4C	New	71	7.6	6.3	6.4	5.5	5.4	5.1	5.0	5.6	5.0	5.5	6.4	7.2
4C	Post	89	9.6	8.0	8.1	6.8	6.6	6.3	6.1	6.9	6.1	6.9	8.0	9.1
4C	Pre	93	10.2	8.4	8.5	7.1	6.8	6.5	6.4	7.2	6.3	7.1	8.4	9.6
5A	New	150	15.1	12.8	12.8	11.2	10.7	12.4	14.2	13.5	10.6	10.3	11.8	14.4
5A	Post	183	18.7	15.9	15.8	13.7	13.0	15.0	17.0	16.3	12.8	12.6	14.6	17.9
5A	Pre	183	18.5	15.8	15.7	13.6	13.0	15.1	17.2	16.5	12.8	12.6	14.5	17.8
5B	New	130	11.8	10.8	11.0	10.0	10.0	11.4	10.7	11.4	9.9	10.1	11.0	11.4
5B	Post	158	14.6	13.4	13.5	12.4	12.1	13.8	12.9	13.8	11.9	12.3	13.5	14.1
5B	Pre	158	14.5	13.3	13.4	12.3	12.1	13.9	13.0	13.9	12.0	12.2	13.4	13.9
6A	New	151	16.9	13.2	12.9	11.1	10.7	11.2	13.0	12.6	10.3	10.6	12.9	15.5
6A	Post	184	20.9	16.3	15.9	13.5	13.0	13.5	15.6	15.2	12.5	12.9	15.9	19.0
6A	Pre	185	20.9	16.3	15.9	13.5	13.1	13.6	15.8	15.3	12.5	12.9	15.9	19.1
6B	New	140	14.1	12.7	12.8	10.7	10.3	10.3	10.8	11.1	10.0	11.2	12.2	14.2
6B	Post	171	17.4	15.6	15.7	13.0	12.5	12.5	12.9	13.5	12.1	13.6	15.0	17.4
6B	Pre	171	17.4	15.6	15.7	13.0	12.5	12.6	13.0	13.5	12.1	13.6	14.9	17.4
7A	New	154	17.9	14.3	14.5	11.4	10.6	10.8	10.6	11.2	9.8	11.5	14.6	16.8
7A	Post	188	22.1	17.7	17.8	13.9	12.9	13.1	12.7	13.5	11.9	14.1	18.0	20.7
7A	Pre	185	21.5	17.3	17.5	13.6	12.7	12.9	12.5	13.3	11.7	13.8	17.6	20.2
8A	New	170	19.7	16.2	17.0	12.0	10.3	10.0	9.9	10.6	10.9	14.4	18.6	20.0
8A	Post	231	27.9	22.8	23.5	15.9	13.1	12.7	12.5	13.6	14.3	19.6	26.3	28.4
8A	Pre	217	25.8	21.2	22.1	15.2	12.7	12.3	12.1	13.2	13.7	18.5	24.4	26.2

Table B.2: Annual and monthly Source EUI standard deviations, measured in KBTU/sqft, for the eight U.S. climate zones and building vintage tiers.

Climate Zone	Vintage	Annual STD Source EUI (KBTU/sqft)	Monthly Source EUI STD (KBTU/sqft)											
			Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
1A	New	25	1.7	1.6	1.9	1.9	2.3	2.4	2.3	2.5	3.4	2.9	2.3	1.7
1A	Post	28	1.9	1.8	2.2	2.2	2.6	2.7	2.6	2.8	3.9	3.4	2.7	1.9
1A	Pre	30	2.1	1.9	2.4	2.3	2.8	2.9	2.7	3.0	4.2	3.7	2.9	2.1
2A	New	24	1.5	1.4	1.8	1.8	2.3	2.5	2.5	2.7	3.1	1.9	1.8	1.6
2A	Post	27	1.7	1.6	2.1	2.1	2.7	2.9	2.7	3.0	3.5	2.2	2.0	1.8
2A	Pre	29	1.8	1.8	2.3	2.3	2.8	3.0	2.9	3.2	3.8	2.4	2.2	1.9
2B	New	20	1.4	1.3	1.6	1.6	1.8	2.0	2.1	2.2	2.3	1.7	1.5	1.3
2B	Post	23	1.5	1.5	1.8	1.8	2.2	2.3	2.3	2.5	2.7	2.0	1.7	1.5
2B	Pre	24	1.6	1.6	1.9	2.0	2.3	2.5	2.5	2.6	2.8	2.1	1.8	1.6
3A	New	21	1.4	1.3	1.7	1.6	1.9	2.2	2.2	2.4	1.9	1.6	1.5	1.4
3A	Post	23	1.5	1.5	1.9	1.8	2.2	2.5	2.5	2.7	2.2	1.8	1.7	1.5
3A	Pre	24	1.7	1.5	2.0	1.9	2.3	2.6	2.6	2.9	2.3	1.9	1.7	1.6
3B	New	20	1.6	1.5	1.7	1.6	1.7	1.8	1.8	2.0	1.8	1.8	1.6	1.6
3B	Post	23	1.8	1.6	1.9	1.8	2.0	2.1	2.1	2.3	2.0	2.0	1.9	1.7
3B	Pre	24	1.9	1.7	2.1	1.9	2.1	2.2	2.2	2.5	2.2	2.1	2.0	1.8
3C	New	16	1.3	1.2	1.5	1.3	1.5	1.5	1.4	1.5	1.4	1.4	1.3	1.3
3C	Post	19	1.4	1.4	1.7	1.5	1.7	1.7	1.6	1.8	1.7	1.7	1.5	1.4
3C	Pre	20	1.5	1.4	1.7	1.6	1.8	1.8	1.7	1.9	1.7	1.7	1.5	1.5
4A	New	22	1.6	1.5	1.7	1.6	1.9	2.2	2.3	2.3	2.0	1.7	1.5	1.5
4A	Post	25	1.8	1.6	2.0	1.8	2.2	2.6	2.6	2.8	2.3	1.9	1.8	1.7
4A	Pre	25	1.8	1.6	2.0	1.8	2.3	2.6	2.7	2.8	2.4	1.9	1.8	1.7
4B	New	20	1.5	1.4	1.7	1.6	1.7	2.0	2.0	2.1	1.7	1.6	1.5	1.4
4B	Post	23	1.6	1.5	1.9	1.8	2.0	2.3	2.3	2.5	1.9	1.8	1.7	1.6
4B	Pre	23	1.6	1.5	1.9	1.8	2.0	2.3	2.3	2.5	1.9	1.8	1.7	1.6
4C	New	10	0.8	0.7	0.9	0.8	0.8	0.8	0.8	0.9	0.8	0.8	0.8	0.8
4C	Post	11	0.9	0.8	1.0	0.9	1.0	1.0	1.0	1.1	0.9	0.9	0.9	0.9
4C	Pre	11	0.9	0.8	1.0	0.9	1.0	1.0	1.0	1.1	0.9	0.9	0.9	0.9
5A	New	21	1.7	1.5	1.8	1.6	1.8	2.1	2.2	2.2	1.7	1.6	1.6	1.6
5A	Post	24	1.9	1.7	2.0	1.9	2.1	2.5	2.6	2.6	2.0	1.9	1.8	1.8
5A	Pre	24	1.8	1.6	2.0	1.9	2.1	2.5	2.6	2.6	2.0	1.8	1.8	1.8
5B	New	20	1.5	1.4	1.7	1.5	1.7	2.0	1.8	2.0	1.7	1.6	1.5	1.5
5B	Post	23	1.7	1.5	1.9	1.7	2.0	2.3	2.1	2.3	1.9	1.8	1.7	1.7
5B	Pre	22	1.7	1.5	1.9	1.7	2.0	2.3	2.1	2.3	1.9	1.8	1.7	1.6
6A	New	21	1.9	1.5	1.8	1.6	1.8	1.9	2.0	2.0	1.7	1.6	1.6	1.7
6A	Post	24	2.1	1.7	2.0	1.9	2.1	2.2	2.3	2.4	1.9	1.8	1.8	1.9
6A	Pre	23	2.0	1.6	1.9	1.8	2.1	2.2	2.3	2.4	1.9	1.8	1.7	1.8
6B	New	21	1.6	1.6	1.9	1.6	1.7	1.8	1.9	1.9	1.7	1.7	1.6	1.7
6B	Post	23	1.8	1.8	2.1	1.9	2.0	2.1	2.2	2.3	1.9	1.9	1.8	1.9
6B	Pre	23	1.8	1.7	2.0	1.8	2.0	2.1	2.2	2.3	1.9	1.9	1.8	1.8
7A	New	21	2.0	1.6	1.9	1.7	1.8	1.8	1.7	1.9	1.6	1.6	1.7	1.8
7A	Post	23	2.2	1.8	2.1	2.0	2.1	2.1	2.0	2.2	1.8	1.9	1.9	2.0
7A	Pre	22	2.0	1.7	2.0	1.8	2.0	2.1	2.0	2.2	1.8	1.8	1.8	1.8
8A	New	22	2.1	1.8	2.1	1.8	1.7	1.7	1.7	1.8	1.7	1.8	2.0	2.0
8A	Post	25	2.5	2.1	2.4	2.1	2.1	2.1	2.0	2.2	1.9	2.0	2.3	2.4
8A	Pre	24	2.2	1.9	2.2	1.9	2.0	2.1	2.0	2.1	1.8	1.9	2.1	2.2

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