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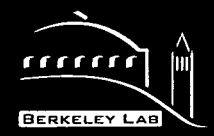

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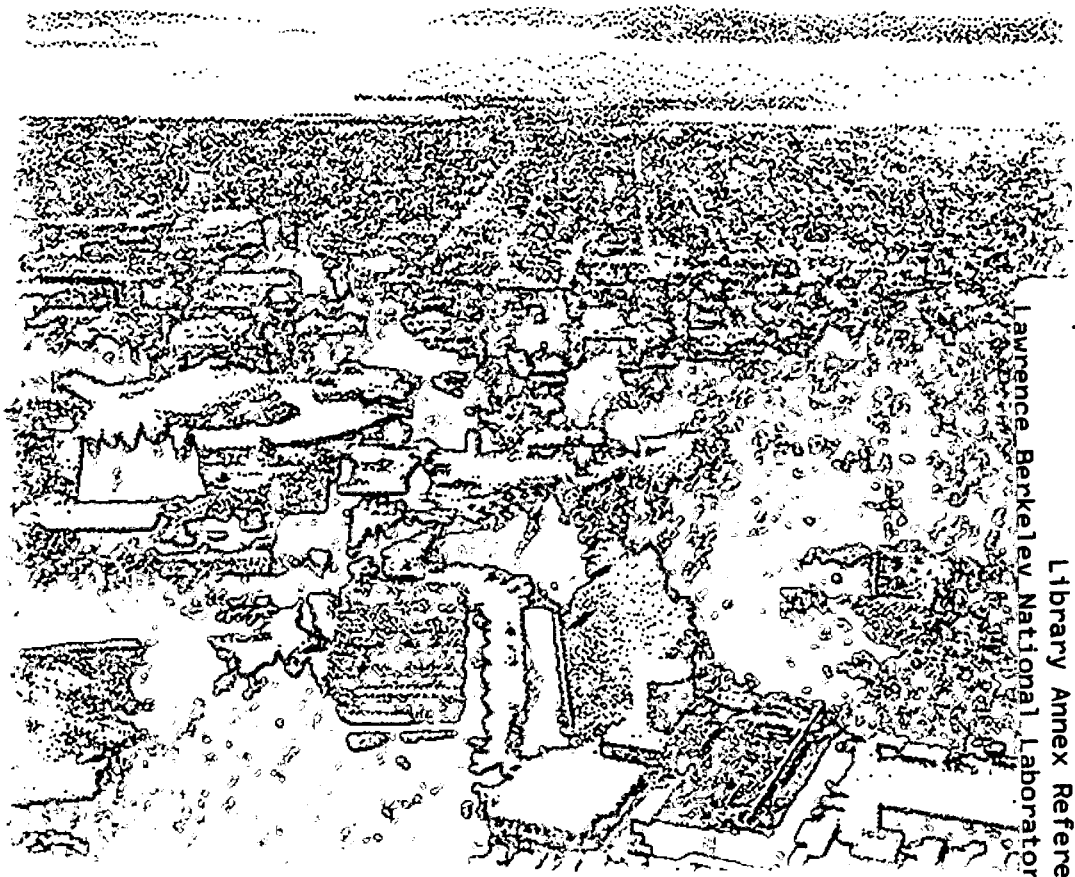

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Not Even Decoupling Can Save Minimal Supersymmetric SU(5)

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(August 18, 2001)

We make explicit the statement that Minimal Supersymmetric SU(5) has been excluded by the Super-Kamiokande search for the process $p \to K^+\overline{\nu}$. This exclusion is made by first placing limits on the colored Higgs triplet mass, by forcing the gauge couplings to unify. We also show that taking the superpartners of the first two generations to be very heavy in order to avoid flavor changing neutral currents, the so-called "decoupling" idea, is insufficient to resurrect the Minimal SUSY SU(5). We comment on various mechanisms to further suppress proton decay in SUSY SU(5). Finally, we address the contributions to proton decay from gauge boson exchange in the Minimal SUSY SU(5) and flipped SU(5) models.

I. INTRODUCTION

Proton decay would be a smoking gun signature for Grand Unified Theories (GUTs). Unfortunately, no such signal has been seen. In fact, very strong experimental limits have been set for this process, placing the minimal GUTs in a very precarious position. Super Kamiokande has set a lower limit on the proton lifetime in the channel $p \to K^+\overline{\nu}$ of 6.7×10^{32} years at the 90% confidence level [1]. This has already placed stringent constraints on SU(5). We explicitly review the situation for proton decay in minimal supersymmetric SU(5) and show that the theory is easily excluded.

Because the minimal case is so easily excluded, one might attempt to tweak the parameters of the theory in some way to push the proton lifetime upwards. One such proposed adjustment can be motivated by the supersymmetric (SUSY) flavor problem. The numerous parameters of the soft SUSY-breaking sector are *a priori* arbitrary, and generically the SUSY-breaking sector will give rise to phenomenologically dangerous flavor-changing neutral current effects. One proposal for avoiding such neutral current difficulties is to decouple the first two generations of superpartners by making them very heavy [2–4]. The lore has been that such a decoupling would also push predictions for proton decay to an acceptable level. We show that this is not the case, and such a modification of the parameters of supersymmetric SU(5) is not enough to save it. After painting this bleak picture for the minimal SU(5) theory, we review variations on the theory that are not yet excluded. Finally, we study the issue of the contributions to proton decay from X and Y gauge boson exchange.

II. DIMENSION FIVE DECAY MECHANISM

The $p \to K^+\overline{\nu}$ channel is predicted to be dominant for supersymmetric SU(5) theories [5–9]. We concentrate on this channel here. This channel is enough to exclude the minimal SUSY SU(5).

The $p \to K^+\overline{\nu}$ decay results from dimension 5 operators, and the associated dressing diagram [10], shown in Fig. 1. The dimension five operators come from colored Higgs triplet exchange, and arise from the following terms in the superpotential:

$$W_Y = h^i Q_i u_i^c H_f + V_{ij}^* f^j Q_i d_j^c \overline{H}_f + f_i e_i^c L_i \overline{H}_f + \frac{1}{2} h^i e^{i\phi_i} Q_i Q_i H_C + V_{ij}^* f^j Q_i L_j \overline{H}_C + h^i V_{ij} u_i^c e_j^c H_C + e^{-i\phi_i} V_{ij}^* f^j u_i^c d_j^c \overline{H}_C. \quad (1)$$

Here, the H and the $\overline{H}$ represent the two different Higgs multiplets that give the up and down type quarks their masses. The H_f is the doublet, while the H_C is the colored Higgs triplet. All fields are superfields. h^i and f^j are Yukawa couplings, V_{ij} is a CKM matrix element, and ϕ_i is a phase, which is subject to the constraint $\phi_1 + \phi_2 + \phi_3 = 0$. We will address the decays that result from Higgs triplet exchange in some detail in the following sections.

III. RGE ARGUMENTS

In a grand unified theory, we expect that the gauge couplings should precisely unify. Particles near the GUT scale provide corrections to the renormalization group trajectories of the coupling constants. These corrections

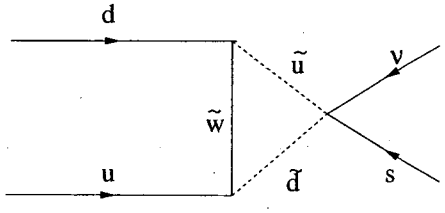

FIG. 1. The dimension five operator results from the exchange of the colored Higgs triplet. The super-particles are then removed from the initial state by chargino exchange. Wino exchange is shown here, but there is an analogous diagram which involves higgsino exchange.

are calculable in terms of the quantum numbers and the masses of the GUT scale particles. Therefore, by imposing the constraint that the gauge couplings exactly unify, we can make statements about the high-energy structure of the theory. This technique has already appeared in the literature [11,9,12]. However, these papers were written when the knowledge of the strong coupling, α_s, was less precise. Measurements at LEP and SLD have allowed a substantially more precise determination of $\alpha_s(m_Z)$. Utilizing this knowledge, we can dramatically improve the constraint on the mass of the colored Higgs triplet, M_{H_C}. Constraining the Higgs triplet mass is of particular importance since it mediates the dominant decay of the proton.

The colored Higgs triplets are not the only new particles at the GUT scale. We expect to have a Σ_{24} Higgs, new vector bosons (denoted collectively by V), in addition to the colored Higgs triplet, H_C, near the GUT scale. One might think that it would be impossible to determine M_C without knowledge of M_Σ, and M_V. However, by examining the RGEs for the gauge couplings at one loop (neglecting the Yukawa couplings):

$$\begin{aligned}
\alpha_3^{-1}(m_Z) &= \alpha_5^{-1}(\Lambda) + \frac{1}{2\pi}\Big[(-2-\frac{2}{3}N_g)\log\frac{m_{SUSY}}{m_Z} \\
&+ (-9+2N_g)\log\frac{\Lambda}{m_Z} - 4\log\frac{\Lambda}{M_V} \\
&+ 3\log\frac{\Lambda}{M_\Sigma} + \log\frac{\Lambda}{M_{H_C}}\Big], \\
\alpha_2^{-1}(m_Z) &= \alpha_5^{-1}(\Lambda) + \frac{1}{2\pi}\Big[(-\frac{13}{6}-\frac{2}{3}N_g)\log\frac{m_{SUSY}}{m_Z} \\
&+ (-5+2N_g)\log\frac{\Lambda}{m_Z} - 6\log\frac{\Lambda}{M_V} + 2\log\frac{\Lambda}{M_\Sigma}\Big], \\
\alpha_1^{-1}(m_Z) &= \alpha_5^{-1}(\Lambda) + \frac{1}{2\pi}\Big[(-\frac{2}{3}N_g-\frac{1}{2})\log\frac{m_{SUSY}}{m_Z} \\
&+ (\frac{3}{5}+2N_g)\log\frac{\Lambda}{m_Z} - 10\log\frac{\Lambda}{M_V} + \frac{2}{5}\log\frac{\Lambda}{M_{H_C}}\Big], \quad (2)
\end{aligned}$$

we find that we can eliminate M_Σ and M_V by taking a judicious combination of the couplings [11]. In the case of the above RGEs, neglecting the Yukawa couplings, we find:

$$\begin{aligned}
&3\alpha_2^{-1}(m_Z) - 2\alpha_3^{-1}(m_Z) - \alpha_1^{-1}(m_Z) = \\
&\quad \frac{1}{2\pi}\Big(\frac{12}{5}\log\frac{M_{H_C}}{m_Z} - 2\log\frac{m_{SUSY}}{m_Z}\Big). \quad (3)
\end{aligned}$$

We can invert the above equation to determine the colored Higgs mass independently of the other masses at the GUT scale.

This one loop example gives the basic procedure. In the numerical calculation that follows, we use the two loop RGEs for the gauge and Yukawa couplings between the SUSY scale and the GUT scale, which can be found, for example, in [13]. Here, the SUSY scale is defined as the mass scale above which all superpartners contribute to the RGEs. We include only the Yukawa couplings of the third generation, all others are neglected. We use one loop RGEs for all running between m_Z and the SUSY scale. We also include the one loop finite effects at the wino and gluino threshold, using the results of [14]. There is no simple analytic solution for the colored Higgs mass, so we must do a numerical analysis.

It is further necessary to take into account the splitting of the supersymmetric particle spectrum. We make the approximation that all the supersymmetric particles, aside from the gauginos, are degenerate at a TeV. As long as the splitting between the sparticles within each SU(5) multiplet is not too large, this is a reasonable approximation. Because the proton decay constraint ends up requiring scalars to be somewhat heavy, the expected splittings within each SU(5) multiplet due to the gaugino contribution in the RGE is small.

From the ratio between the couplings near the SUSY scale, we expect $\frac{M_3}{M_2}$ to be 3.5. With this approximation, we are left with M_2 and $\tan\beta$ as free parameters. In the limits quoted below, we set M_2=200 GeV. We scan over $\tan\beta$ between 1.8 and 4. Large values of $\tan\beta$ are very bad for proton decay, and the top Yukawa becomes non-perturbative below 1.8. In fact, recent results from Higgs searches at LEP [15] suggest that $\tan\beta > 2.4$. However, these bounds can probably be avoided by modifying the Higgs sector.* Therefore, we conservatively scan the interval between 1.8 and 4, a scan between 2.4 and 4 would only make things worse for SU(5).

We use the following precision measurements as inputs [16]:

$$\alpha_{s_{\overline{MS}}}(m_Z) = .1185 \pm .002 \quad (4)$$

$$\sin^2\theta_{w_{\overline{MS}}}(m_Z) = .23117 \pm .00016 \quad (5)$$

$$\alpha_{em_{\overline{MS}}}(m_Z) = \frac{1}{127.943 \pm .027}. \quad (6)$$

All these quantities are given in the $\overline{MS}$ scheme. However, the step function approximation at particle thresholds is good only in the $\overline{DR}$ scheme [17]. Yukawa couplings and gauge couplings must therefore be converted from $\overline{MS}$; the dictionary for this conversion may by found in reference [18].

Operationally, we use a given colored Higgs mass along with the renormalization group equations to predict the data of Eqns. (4,5,6). We find that SU(5) prediction of exact unification agrees with the data (using a χ^2 fit for

*For example, by including a singlet field as in the NMSSM, one can weaken these bounds using the larger Higgs self-coupling and/or the invisible decay of Higgs into singlet scalars. The tadpole problem in the NMSSM can be avoided even with GUT if the supersymmetry breaking originates in gauge mediation at low energies.

the one degree of freedom: M_{H_C}) only for colored Higgs masses of:

$$3.5 \times 10^{14} \leq M_{H_C} \leq 3.6 \times 10^{15}\text{GeV} \quad \text{(90\% confidence level)}. \tag{7}$$

We find that varying M_2 within a reasonable range (100-400 GeV) causes a change in the M_{H_C} bounds on the order of 10%. The previous upper limit of reference [12], was $M_{H_C} < 2.4 \times 10^{16}$ GeV. The improvement is largely due to the improvement in the precision on α_s.

Note that the above limit will not be drastically affected in the case where we take the scalars of the first and second generations to have masses on the order of 10 TeV. This is because changing the energy scale of an entire SU(5) multiplet does not change the unification condition, and hence the RGE bound, at one-loop. A small sparticle splitting within a multiplet relative of the sparticles masses is especially well motivated if the first and second generation scalars are pushed up to 10 TeV, otherwise a problematic Fayet–Illiopoulos D-term [2] is induced. This fact will be of use when we move on to discuss the decoupling scenario in section IV.

We also note that it is possible to place a constraint on the combination $(M_\Sigma M_V^2)^{1/3}$. This is done by looking at the combination $5\alpha_1^{-1} - 3\alpha_2^{-1} - 2\alpha_3^{-1}$ [11]. We find that this scale is very tightly constrained:

$$1.7 \times 10^{16} \leq (M_\Sigma M_V^2)^{1/3} \leq 2.0 \times 10^{16}\text{ GeV} \quad \text{(90\% confidence level)}. \tag{8}$$

In what follows, we refer to the scale $(M_\Sigma M_V^2)^{1/3}$ as M_{GUT}. Incidentally, the above bounds of Eqns. (7,8), are not uncorrelated. We show the allowed region in the $M_{H_C} - M_{GUT}$ plane in Fig. 2. The bounds that result from projecting the ellipse in the figure on to one of the axes are weaker than those in Eqns. (7,8). This is because the ellipse is found by performing a fit using a χ^2-distribution with two degrees of freedom, whereas the bounds in the equations are found using a χ^2-distribution with one degree of freedom.

What are the consequences of such a strong limit on the colored Higgs mass for minimal SUSY SU(5)? They are not good. Our calculation of the proton lifetime follows the methods of reference [6].† Although values of μ on the order of 800 GeV are favored by the electroweak symmetry breaking condition, we take μ as a free parameter in our phenomenological analysis. We keep the M_2 as a free parameter, and determine the other

†Our calculation shows an approximate factor of two discrepancy with that reference. Our predicted lifetime is shorter, but in any case, it will not affect the qualitative nature of our results in any way.

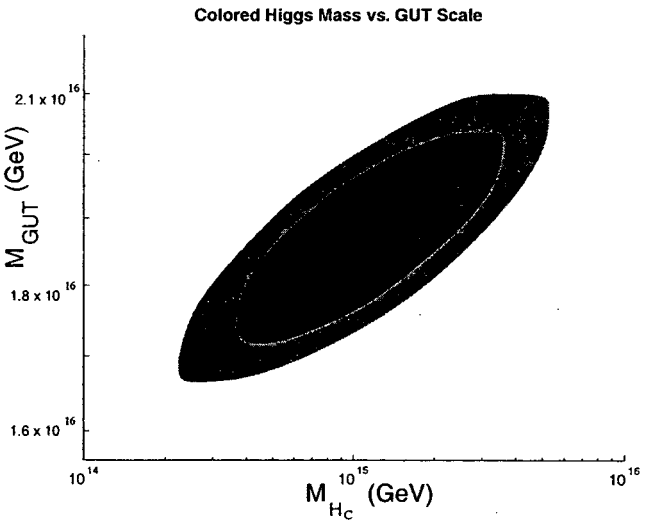

FIG. 2. Plot showing 68% and 90% contours allowed by the renormalization group analysis for the color Higgs triplet mass, M_{H_C}, and the GUT scale, $M_{GUT} \equiv (M_\Sigma M_V^2)^{1/3}$.

gaugino masses through the unification condition. For the scalars, we take the stop soft masses to be 400 and 800 GeV at the weak scale, and set the masses of all other SUSY particles to have masses of 1 TeV. We neglect squark and slepton mixing, except for the stops. With these assumptions in place, we scan over the parameters μ, M_2, $\tan\beta$, and the independent phases ϕ_1 and ϕ_2, to maximize the lifetime as a function of M_{H_C}. We allow $\tan\beta$ to vary in the interval $\tan\beta \in (1.8, 4)$; M_2 to vary in the interval $M_2 \in (100, 400)$, and $\mu \in (100, 1000)$. We eliminate those points which have a too-light chargino mass, using the constraint from LEP II [19], $m_{\chi^+} > 103.5$ GeV. The Yukawa couplings are extracted from the central values of the quark masses listed in reference [16].

In our calculation, we take into account both short and long range renormalization effects. Yukawa couplings must be run up to the GUT scale. The Wilson coefficients of the effective dimension five operators must be run back down to the SUSY scale. We use the RGEs from the appendix of reference [6], ignoring all Yukawa couplings except for that of the top quark. The one-loop renormalization of the Wilson coefficients of the dimension six operators from the weak scale to 1 GeV can be extracted from reference [5]. The renormalization of the Yukawa couplings (quark masses) from 2 GeV to the weak scale is done to three loops.

Using the newer limit from Super Kamiokande of 6.7×10^{32} years (90 % confidence level), we find that search for proton decay imposes the constraint:

$$M_{H_C} \geq 7.6 \times 10^{16}\text{ GeV}. \tag{9}$$

Comparing this equation with Eqn. (7), we find that the

minimal SUSY SU(5) theory is excluded by a lot.

It should be noted that this is a very conservative value. In particular, this calculation utilizes the traditionally most conservative value of the hadronic parameter $\beta_H = \langle 0|u_L u_L d_L|p\rangle = 0.003$ GeV^3. Recently, however, there has been progress on the evaluation of this parameter by the JLQCD group [20]. They find a value, $\beta_H = .014 \pm .001$ GeV^3. However, this result is to be evaluated at a scale of 2.3 GeV, whereas the value $\beta_H = 0.003$ GeV^3, was to be utilized at a scale of 1 GeV. This difference causes the enhancement of the decay rate to be somewhat less than the naive factor of twenty. Repeating the above analysis, utilizing the central JLQCD value for β_H, we find the even more stringent constraint

$$M_{H_C} \geq 2.0 \times 10^{17} \text{ GeV}. \quad (10)$$

This result is in even sharper conflict with Eqn. (7).

IV. THE FAILURE OF DECOUPLING

Previous calculations of the proton lifetime have assumed nearly degenerate scalars at the weak scale, or order 1 TeV in mass. We made this same assumption in our calculation in the previous section. It seems that one possible escape for the SUSY SU(5) theory with the minimal field content would be the interesting possibility raised by reference [2]. This scenario allows the first and second generations of scalars to be heavy without severe fine-tuning because they do not affect the Higgs boson self-energy at the one-loop level. Even though there is a naturalness problem at the two-loop level [21], the scenario in [4] achieves it without compromising naturalness (the model in [3] does not seem to allow a large splitting). Since the proton decay amplitude goes like $m_\chi/m_{\tilde{q}^2}$, it seems like we might get a large suppression by making the squarks ultra-heavy. However, we will see that even this will not save us. This point is made clear by looking at the main contributions to proton decay. We can write the contributions to $\Gamma(p \to K^+\overline{\nu})$ as:

$$\begin{aligned} A(p \to K^+\overline{\nu_e}) &\approx [e^{i\phi_2} A_e(\tilde{c}_L) + e^{i\phi_3} A_e(\tilde{t}_L)]_{LLLL} \\ A(p \to K^+\overline{\nu_\mu}) &\approx [e^{\phi_2} A_\mu(\tilde{c}_L) + e^{i\phi_3} A_\mu(\tilde{t}_L)]_{LLLL} \\ A(p \to K^+\overline{\nu_\tau}) &\approx [e^{i\phi_2} A_\tau(\tilde{c}_L) + e^{i\phi_3} A_\tau(\tilde{t}_L)]_{LLLL} \\ &\quad + e^{i\phi_1} A_\tau(\tilde{t}_R)_{RRRR}. \end{aligned} \quad (11)$$

Here, the LLLL subscript refers to the contribution that arises from dressing the dimension five operator with four left-handed particles, while RRRR refers to the contribution that arises from dressing the dimension five operator with for right-handed particles. The RRRR operator will obviously only have a higgsino piece, and not a wino piece. As such, it will only contribute for the ν_τ case, where third generation Yukawa couplings allow it to become big [6]. This contribution was overlooked in earlier analyses, presumably because the large Yukawa coupling of the top quark was unanticipated.

When we write the contributions to proton decay as above, it becomes clear why the decoupling of the first two generations does not save us. Although we are able to eliminate the contribution due to the exchange of the $\tilde{c}$ squark, the contribution due to the stop still persists. In the limit of the very heavy scharm, we can rewrite Eqn. (11) as:

$$\begin{aligned} A(p \to K^+\overline{\nu_e}) &\approx e^{i\phi_3} A_e(\tilde{t}_L)_{LLLL} \\ A(p \to K^+\overline{\nu_\mu}) &\approx e^{i\phi_3} A_\mu(\tilde{t}_L)_{LLLL} \\ A(p \to K^+\overline{\nu_\tau}) &\approx e^{i\phi_3} A_\tau(\tilde{t}_L)_{LLLL} + e^{i\phi_1} A_\tau(\tilde{t}_R)_{RRRR}. \end{aligned} \quad (12)$$

We have not helped matters by making the scharm heavy. In fact, we are in many ways worse off, because we cannot use the $\tilde{c}$ contribution to help cancel off the large RRRR contributions to $p \to K^+\overline{\nu}_\tau$. The basic point is that proton decay has an important contribution from the exchange of third generation sparticles. This causes the decoupling idea to fail. We present our quantitative results below.

We took the third generation sparticles to weigh 1 TeV at the weak scale, except for the top squarks, which, as before, we give soft masses of 800 and 400 GeV at the weak scale. We take the first two generation sparticles at 10 TeV. In the case that the squarks and sleptons are much heavier than the chargino, the triangle loop gives a contribution that goes like $m_\chi/m_{\tilde{q}}^2$. Therefore, placing them at 10 TeV effectively decouples them, by suppressing their contribution to the amplitude by a factor of 10^2.

Again, we scan over the relevant parameter space to determine the maximum proton lifetime. However, there are fewer free parameters than the case where all generations of sparticles contribute. In particular, we can already see that the phase $e^{i\phi_{23}} = e^{i\phi_2}/e^{i\phi_3}$ drops out completely. What is more, if we wish to conservatively maximize the lifetime predicted by such a theory, we find that ϕ_{13} is determined to be π. This effects the largest possible cancellation between the two contributions to $A(p \to K^+\overline{\nu_\tau})$. The remaining free parameters in our calculation are $\tan\beta$, M_2, and μ. Because the RRRR contribution that arises from higgsino exchange is much larger than the contribution from wino exchange, it turns out the the amplitude does not depend strongly on the value of M_2. When the decay rate is higgsino-exchange dominated, nearly the entire branching ratio is to $K^+\overline{\nu}_\tau$. We plot the proton lifetime in the M_2-μ plane in Fig. 3 for a fixed value of $\tan\beta$. There is a relatively strong dependence on $\tan\beta$. It has long been known that the large $\tan\beta$ region is bad for proton decay. This can be seen explicitly in Fig. 4, where we show the region between $\tan\beta$ of 1.8 and 20.

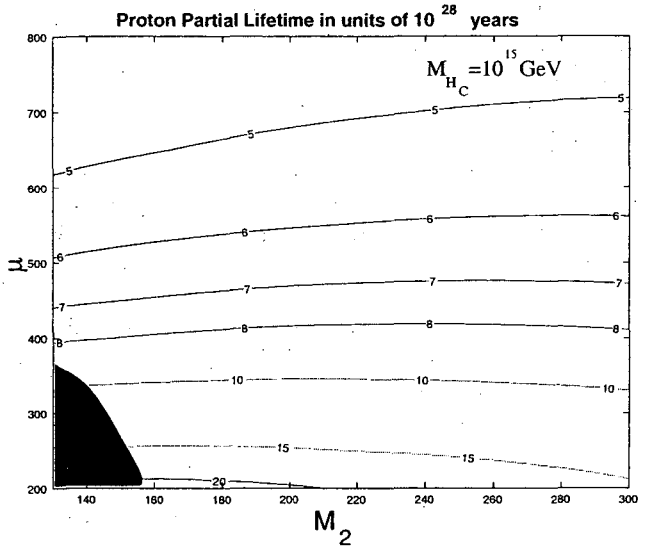

FIG. 3. A contour plot of the proton partial lifetime, $\tau(p \to K^+\bar{\nu})$, in the case where the 1^{st} and 2^{nd} generation scalars are taken to be 10 TeV. The third generation scalars are taken to have masses order 1 TeV, except for the stops, which are given soft masses of 800 GeV and 400 GeV. We fix $\tan\beta$ to be 2.1. Note that the lifetime is approximately proportional to μ, and essentially independent of M_2. The shaded region is excluded by chargino searches at LEP II. Lifetimes for other values of M_{H_C} can be found by noting that the lifetime goes as $M_{H_C}^2$.

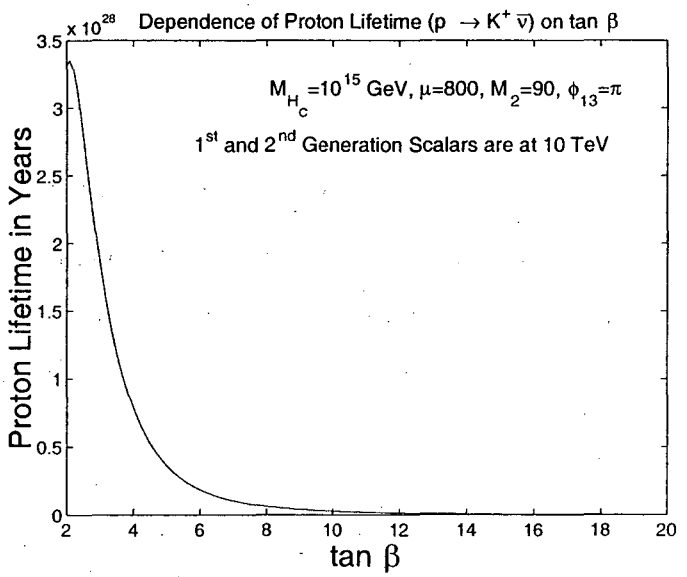

FIG. 4. A plot of proton partial lifetime, $\tau(p \to K^+\bar{\nu})$, vs. $\tan\beta$. Top squark masses are 400 GeV and 800 GeV, while all other 3^{rd} generation sparticles have masses are set to 1 TeV. All other variables are fixed as stated. It is seen that the lifetime peaks for values of $\tan\beta$ slightly greater than 2 in this case. Lifetimes for other values of M_{H_C} can be found by noting that the lifetime goes as $M_{H_C}^2$.

The maximum value of the proton lifetime was found by scanning the parameter space from $\mu \in (80, 400)$, $M_2 \in (100, 400)$, $\tan\beta \in (1.8, 3.0)$. As before, we eliminate those points which have a too-light chargino mass, using the constraint from LEP II [19], $m_{\chi^+} > 103.5$ GeV. Using the maximum value of the colored Higgs mass allowed by our RGE analysis (at 90% confidence), 3.6×10^{15} GeV, we find that the maximum value of the proton partial lifetime is:

$$\tau(p \to K^+\bar{\nu}) \leq 2.9 \times 10^{30} \text{ yrs.} \qquad (13)$$

Therefore, even the situation with very heavy first and second generation scalars is *easily* excluded at the 90% confidence level. We should reiterate that our RGE analysis is largely unaffected by our decoupling the first two generations of particles. First of all, we are only separating the sparticles from the third generation by one decade in energy. Moreover, we have argued that the splitting within the second generation of superpartners is small, and decoupling entire generations of superpartners has no effect on the unification condition at one loop. For the sake of completeness, we also quote the bound on M_{H_C}, independent of the RGE analysis. We find

$$M_{H_C} > 5.7 \times 10^{16} \text{ GeV}. \qquad (14)$$

The statement that this theory is excluded is equivalent to the statement that the above equation is in conflict with 7. Again, upon utilization of the JLQCD central value for $\beta_H = 0.014$ GeV^3, we find that the maximum proton lifetime is even smaller. In particular, we find that:

$$\tau(p \to K^+\bar{\nu}) \leq 2.5 \times 10^{29} \text{ yrs.}, \qquad (15)$$

making the situation even worse.

V. AVOIDING THE CONSTRAINT

We wish to stress that, while things look grim for the minimal SU(5) theory, our result does not mean that no SU(5) theory is viable. There exists a host of ideas that allow one to evade the difficulties outlined in the previous two sections. They fall into two main categories. The first category consists of ideas to evade the constraints from the RGE arguments. The second strategy is to somehow suppress the contribution from the dimension five operators.

In the first strategy, the goal is to push the mass of the colored Higgs triplet very heavy, thereby suppressing the dimension five operators. Then a way must be found to avoid the RGE constraint of section III. To do this, one must include fields that make additional contributions to the GUT-scale threshold corrections. Although there are several ways to accomplish this feat, perhaps the simplest

way to do this is to include a second pair of Higgs bosons in the $5+\overline{5}$ representation without any Yukawa coupling to matter multiplets. However, in this pair one makes the triplet *lighter* than the doublet. As such, the threshold corrections to unification from this pair will work in a way opposite from the correction from the usual Higgs multiplet, and can allow the original Higgs triplet to be heavier.

The second strategy is to suppress the dimension five operators in some way. A number of ideas exist in the literature for accomplishing this goal. Most recently, some interesting ways of eliminating the dimension five operators entirely in an extra-dimensional framework [22] have appeared. Another attempt utilizes a somewhat complicated Higgs sector, but succeeds in suppressing dimension five operators or even removing them entirely [23]. In general, the dimension-five operators are sensitive to the mechanism of doublet-triplet splitting, arguably the least pleasant aspect of GUT. In some models that achieve the doublet-triplet splitting in a natural way, dimension-five operators are eliminated, such as in flipped SU(5) [24]. Yet another method for suppressing the dimension five operators is to somehow suppress the Yukawa couplings between the standard model fermions and the colored Higgs triplet. In the past, this might have been considered the favored mechanism for suppressing proton decay, simply because there were already problems in the minimal SU(5) with GUT relationships like $m_\mu = m_s$. It was assumed that attempts to remedy these fermion mass relationships would somehow also remedy the proton decay problem. However, since it is now recognized that there is a dominant contribution from the RRRR operator, which is proportional to the 3^{rd} generation Yukawas, one would have to modify the flavor structure of the third generation in some way as well, which is less likely.

Finally, methods exist to suppress the dimension five operators where the two strategies mentioned above are combined. For example, one mechanism includes an additional pair of Higgs triplets, H'_C and $\overline{H}'_C$, that exist solely to give the original pair of Higgs triplets a mass. In this case, the operator that arises from integrating out the $M_{H_C}\overline{H}_C H_C$ term can be forbidden by a Peccei–Quinn symmetry [10]. However, the symmetry needs to be eventually broken, and it turns out that the RGE bound constrains the combination relevant for the dimension five operator [25]. So, something must be added to the model to help avoid this bound. Inspiration comes from the missing partner model [26], which utilizes a SU(5)-Higgs in the **75** representation. This generates an additional threshold correction that pushes the RGE limit on the color-triplet Higgs higher [27]. However, the simplest incarnation of the missing partner model model has the problem that the gauge coupling becomes non-perturbative soon above the GUT-scale. The answer comes in combining the two models: adding the the Peccei–Quinn symmetry to the missing partner model can be used to postpone the peturbativity problem. The resulting suppression from the symmetry is sufficient to make the dangerous proton decay of the previous sections benign [9,28].

SO(10) models, having more multiplets at the GUT-scale, allow larger threshold corrections and hence can loosen the bound on the color-triplet Higgs mass if the threshold correction comes with the correct sign. Moreover, there are many color-triplet Higgses which mix with each other. Even though suppressing proton decay and achieving the correct threshold correction often have tension, one can build models to achieve an overall suppression [29].

VI. DIMENSION SIX PROTON DECAY

In general, the dimension six operator arising from X and Y gauge boson exchange provides a less model-dependent decay rate.[‡] With the old evaluation of the hadronic matrix elements, it was thought that the dimension six operators would be completely out of reach for the foreseeable future. However, with the updated value of the hadronic matrix element from the JLQCD collaboration, the prospects of detection are slightly less bleak. Reference [31] has already re-examined this question for the minimal SU(5) model. The decay rate can be written as:

$$\Gamma(p \to \pi^0 e^+) = \alpha_H^2 \frac{m_p}{64\pi f_{\pi^2}}(1+D+F)^2\Big(\frac{g_5^2 A_R}{M_V^2}\Big)^2 \times (1+(1+|V_{ud}|^2)^2) \qquad (16)$$

Here, α_H, is the hadronic matrix element, evaluated to the JLQCD collaboration to be $\alpha_H = 0.015 \pm .001\ \mathrm{GeV}^3$. A_R is a overall renormalization factor that contains both a long and short-distance piece [32]. F and D are chiral Lagrangian parameters. The piece $(1+|V_{ud}|^2)^2$ comes from the operator $\mathbf{10}_i^* \mathbf{10}_i \mathbf{10}_j^* \mathbf{10}_j$, while the piece 1 comes from the operator $\mathbf{10}_i^* \mathbf{10}_i \overline{\mathbf{5}}_1^* \overline{\mathbf{5}}_1$. Our numerical evaluation yields:

$$\frac{1}{\Gamma(p \to \pi^0 e^+)} = 8 \times 10^{34}\ \mathrm{yrs.} \times \Big(\frac{0.015\mathrm{GeV}^3}{\alpha_H}\Big)^2 \Big(\frac{M_V}{10^{16}\mathrm{GeV}}\Big)^4. \qquad (17)$$

[‡] If SU(5) is broken on an orbifold by a boundary condition, and if matter fields live on the fixed point where X, Y bosons vanish, dimension-six operators can be eliminated. This may be viewed as a partial explicit breaking of SU(5) [30].

In section III, we constrained the product: $(M_V^2 M_\Sigma)^{1/3}$. We now try to disentangle the product. The case $M_V \gg M_\Sigma$ is perfectly allowed, and conceivably, the mass of M_V might be as high as the Planck Scale, so the dimension six decay might be completely out of reach. On the other hand, M_V cannot be arbitrarily small. $\mathcal{W} \ni \frac{f}{3}\mathrm{Tr}\Sigma^3$, and we can write $M_\Sigma = \frac{f M_V}{2\sqrt{2}g_5}$. Imposing the constraint that the Higgs trilinear self-coupling, f, should not blow up before the Planck scale, reference [9] found $M_V > 0.56 M_\Sigma$. Taken with Eqn. (8), we find that $M_V > 1.4 \times 10^{16}$ GeV. If M_V is indeed close to this limit, it is conceivable that dimension six proton decay might be accessible at a next-generation nucleon decay experiment.

The above discussion of dimension six decays can be easily modified to discuss the flipped-SU(5) model [24]. In this model, dimension five operators are absent. However, the dimension six operators arising from the exchange of X bosons are still present. In this model, the scale of the X bosons is determined solely by the unification of the SU(2) and SU(3) couplings (the "exact" unification of the three couplings must be viewed as something of an accident). In this case, the decay rate becomes:

$$\Gamma(p \to \pi^0 e^+) = \alpha_H^2 \frac{m_p}{64\pi f_{\pi^2}}(1+D+F)^2\Big(\frac{g_5^2 A_R}{M_V^2}\Big)^2. \quad (18)$$

This decay rate is is smaller than Eqn. (16) by almost a factor of five, because only the $\mathbf{10}_i^* \mathbf{10}_i \bar{\mathbf{5}}_1^* \bar{\mathbf{5}}_1$ operator contributes to this mode and hence the factor of $(1 + |V_{ud}|^2)^2$ is absent. (This point had not been made in the literature to the best of our knowledge.) However, it turns out that the mass of the gauge bosons, M_V, can be lower than in the minimal SU(5) case, thereby allowing a higher decay rate for flipped-SU(5) theories. Let us now determine how small M_V can actually be. In this case, we cannot use the same method we used for minimal SU(5) to constrain the mass of M_V, as the condition that only two couplings unify is less stringent. On the other had, there is no Σ that gives threshold corrections to the couplings. So, by using the condition that α_2 and α_3 unify, we can determine a bound on the combination $(M_V^2 M_{H_C})^{1/3}$. We find

$$3.3 \times 10^{15} \leq (M_V^2 M_{H_C})^{1/3} \leq 8.2 \times 10^{15} \text{ GeV} \quad (90\% \text{ confidence level}). \quad (19)$$

Now, we expect that M_{H_C} should be near (or below) the GUT scale, as it arises from a coupling times a GUT scale vacuum expectation value. Using this peturbativity argument, reference [9] has shown that $M_{H_C} < 2.0\, M_V$. Applying this result in Eqn. (19), we find that $M_V > 2.6 \times 10^{15}$ GeV. On the other hand, the Super Kamiokande bound [33] on the $p \to \pi^0 e^+$ channel of $\tau_p > 2.6 \times 10^{33}$ years translates into a limit of $M_V > 2.8 \times 10^{15}$ GeV. Therefore, current nucleon decay experiments have just begun to probe the dimension-six operators of the flipped-SU(5) model.

VII. CONCLUSIONS

In conclusion, we find that by forcing the gauge couplings to unify, we can place a rather stringent bound on the colored Higgs mass in the Minimal SUSY SU(5). A more precise determination of $\alpha_s(m_Z)$ has greatly improved this bound. In light of this, LEP has done a great deal to constrain a SUSY SU(5) theory. Using the constraint on the colored Higgs, we find that the minimal SUSY SU(5) grand unified theory has been easily excluded by the Super Kamiokande experiment. Even a scenario allowing for heavy scalars in the first two generations does not allow SU(5) to avoid the experimental bounds.

However, we have also mentioned several theoretical approaches that can substantially suppress the dimension five decay. It is not yet possible to exclude these options. So, while it is is impossible to say that no SU(5) theory is correct, it is correct to say the the minimal SUSY SU(5) theory is excluded, even if the superpartners are taken to be very heavy. It is hoped that future nucleon decay experiments can probe the dimension six operators in the future, providing conclusive evidence for a grand unified theory.

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