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# Contract, Renegotiation, and Hold Up: Results on the Technology of Trade and Investment

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## Abstract

This paper examines a class of contractual relationships with specific investment, a non-durable trading opportunity, and renegotiation. Trade actions are modeled as individual and trade-action-based option contracts (“non-forcing contracts”) are explored. The paper identifies an important distinction, between *divided* and *unified* investment and trade actions, that plays a key role in determining whether efficient investment and trade can be achieved. By using non-forcing contracts, the party without the trade action can be made residual claimant with regard to the investment action, which implies that an efficient outcome can be achieved in the divided case but not typically in the unified case. More generally, the paper shows that, with ex post renegotiation, constraining parties to use “forcing contracts” implies a strict reduction in the set of implementable value functions. Tools are developed for calculating the “punishment values” that determine the sets of implementable post-investment value functions.

The hold-up problem arises in situations in which contracting parties can renegotiate their contract between the time they make unverifiable relation-specific investments and the time at which they can trade.<sup>1</sup> The severity of the hold-up problem depends critically on the productive technology and on the timing of renegotiation opportunities. This paper contributes to the literature by examining how the nature of the “trade action” in a contractual relationship influences the prospects for achieving an efficient outcome. We introduce a new distinction—whether the party who invests also is the one who consummates trade—that plays an important role in determining the outcome of the contractual relationship.

So that we can describe our modeling exercise more precisely, consider an example in which contracting parties “Al” and “Zoe” interact as follows. First Al and Zoe meet and write a contract that has an externally enforced element. Then one of them makes a private investment choice, which influences the *state* of the relationship. The state is commonly observed by the contracting parties but is not verifiable to the external enforcer. Al and

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\*UC San Diego; <http://econ.ucsd.edu/~jwatson/>. The authors thank colleagues at UCSD and Yale, especially Nageeb Ali, Joel Sobel, and Ben Polak, for comments. Part of the analysis reported here was completed while Watson was a visitor at the Cowles Foundation, Yale.

<sup>1</sup>Che and Sákovics (2008) provide a short overview of the hold-up problem, which was first described by Klein, Crawford and Alchian (1978), and Williamson (1975,1977). Analysis was provided by Grout (1984), Grossman and Hart (1986), and Hart and Moore (1988).

Zoe then send individual public messages to the external enforcer. After this, they have an opportunity to renegotiate their contract; this is called “ex-post” renegotiation because it occurs after messages. Finally, the parties have a one-shot opportunity to trade and they also obtain external enforcement. Trade is verifiable to the external enforcer.

Because the investment is unverifiable, the investor cannot be directly rewarded for choosing the efficient investment level. Instead, investment incentives hinge on how the terms of trade can be made sensitive to the investment choice. Typically a conflict arises between the parties’ joint interests prior to investment and their joint interests following investment and messages. In particular, it may be useful for investment incentives to specify an inefficient trade action ex post in some off-equilibrium-path contingencies. But parties then would have the joint incentive to renegotiate and divide the surplus according to their bargaining power (hold up). Because parties rationally anticipate the renegotiation, the incentives to invest are distorted.

The description above obviously leaves the mechanics of trade and enforcement ambiguous. In reality, the parties have individual actions that determine whether and how trade is consummated. Let us suppose that Al selects the individual trade action, which we call  $a$ . This could be a choice of whether to deliver or to install an intermediate good, for example. We then have an *individual-action model*, whereby Al chooses  $a$  and the external enforcer compels a transfer  $t$  as a function of  $a$  and the messages that the parties sent earlier. In contrast, a *public-action model* (or *external-action model*) combines the trade action and the monetary transfer into a single public action  $(a, t)$  that is assumed to be taken by the external enforcer. With this modeling approach, the contract specifies how the public action is conditioned on the parties’ messages.

Although the public-action model may typically be a bit unrealistic, it is simple and lends itself to elegant mechanism-design analysis (for example, as in Maskin and Moore 1999 and Segal and Whinston 2002). On the other hand, Watson (2007) demonstrates that analysis of the individual-action model can be straightforward as well. He also shows that the public-action model is equivalent to examining individual trade actions but constraining attention to “forcing contracts” in which the external enforcer induces a particular trade action as a function of messages sent by the parties (so the trade action is constant in the state). Watson (2007) provides an example in which the restriction to forcing contracts has strict (negative) efficiency consequences.

We provide a more thorough analysis for a large class of contractual relationships. We show that the properties of Watson’s (2007) example are robust. Furthermore, we prove the existence of non-forcing contracts that make Al’s payoff constant in the state, gross of any investment costs. In fact, we show that a straightforward “dual option” contract (in which only Zoe sends a message) suffices. This implies that Zoe can be made the “residual claimant” in the relationship.

We thus have strong conclusions about how the technologies of trade and investment interact to determine whether the efficient outcome can be achieved. If Zoe is the party who makes the investment choice — we call this the *divided case*, because here the investment and trade actions are chosen by different parties — then there is a contract that induces

efficient investment and trade. On the other hand, in the *unified case* in which AI makes the investment choice and also has the trade action, the efficient outcome is generally not attainable because there typically do not exist contracts that make AI the residual claimant.

Our results underscore the usefulness of modeling trade actions as individual. This is particularly salient for the setting of *cross/cooperative investment* (Che and Hausch 1999), where the investment by one party increases the benefit to the *other* party of subsequent trade. The literature has regarded cross investment settings as especially prone to the hold-up problem (and inefficient outcomes as a result). By introducing the distinction between unified and divided investment and trade actions, we thus give a basis for deeper analysis. We find that the hold-up problem can be solved in the case of cross investment and divided actions, whereas hold-up is more problematic in the case of cross investment and unified actions.

Our analysis utilizes mechanism-design techniques. With both the individual-action and public-action modeling approaches, analysis of the contractual problem centers on calculating the set of implementable value functions from just after the state is realized (before messages are sent). Formally, an implementable value function is the state-contingent continuation value that results in equilibrium for a given contract. Let  $V^{\text{EPF}}$  be the set of implementable value functions under ex-post renegotiation when one constrains attention to forcing contracts (the public-action model), and let  $V^{\text{EP}}$  be the corresponding set without the constraint to forcing contracts (the individual-action model). We also examine the case of interim renegotiation, where the parties can renegotiate only before sending messages, and let  $V^{\text{I}}$  be the set of implementable value functions for this case. By their definitions, these three sets satisfy  $V^{\text{EPF}} \subseteq V^{\text{EP}} \subseteq V^{\text{I}}$ . In Watson's example, the inclusion relations are strict so that  $V^{\text{EPF}} \neq V^{\text{EP}} \neq V^{\text{I}}$ .

We provide simple tools to calculate the “punishment values” that determine the implementable sets for the class of relationships we analyze here. Our first theorem establishes that  $V^{\text{EP}}$  contains functions that hold fixed the value of the player with the trade action, and thus they give the other player the full value of the relationship minus this constant. This result is the basis for our insights on the relation between the investment and trade technologies. The result relies on an assumption that investment does not confer a direct gain for some minimal trade action; this assumption is satisfied by the most prominent models in the hold-up literature.

Our second theorem establishes that, for a wide class of contractual relationships, the inequalities  $V^{\text{EPF}} \neq V^{\text{EP}} \neq V^{\text{I}}$  always hold. In particular, in the important setting of ex post renegotiation described above, limiting attention to forcing contracts (studying  $V^{\text{EPF}}$  rather than  $V^{\text{EP}}$ ) reduces the range of state-contingent continuation values, making it more difficult to give the investing party the incentive to invest at the beginning of the relationship.<sup>2</sup>

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<sup>2</sup>This does not mean that a more efficient outcome can always be achieved when actions are modeled as individual, because efficiency depends on what region of the implementable-value set is relevant for giving appropriate investment incentives. That is, in some examples we have  $V^{\text{EP}} \neq V^{\text{EPF}}$  but these sets coincide where it matters to induce optimal investment.

In the class of trade technologies that we study here, a single player (player 1, Al above) has the trade action. The key economic assumption behind our second result is that player 1’s utility is supermodular as a function of the state and trade action. That is, this player’s marginal value of the trade action is monotone in the state. Our results generalize to settings in which both players have trade actions. Our other assumptions are weak technical conditions that guarantee well-defined maxima, non-trivial settings, and the like. We argue that these conditions are likely to hold in a wide range of applications and that they are consistent with what is typically assumed in the literature.

The rest of the paper proceeds as follows. In the next section we provide the details of the model. Section 2 previews the main results by means of an example. Section 3 contains our first result, on making player 2 the residual claimant. Section 4 provides an overview of the basic tools for general analysis, which mostly restates material in Watson (2007). Section 5 contains our result on the difference in implementable sets based on variations regarding when renegotiation can occur and whether one restricts attention to forcing contracts. The Conclusion contains more discussion about the hold-up problem and cross investment, including notes on the case of durable trading opportunities. Some of the technical material and proofs are contained in the appendices.

## 1 The Theoretical Framework

We look at the same class of contracting problems and use the same notation as in Watson (2007), except that we add a bit of structure on the trade technology to focus our analysis. In particular, we examine the case in which a single player has a trade action. Throughout the paper, we use the convention of labeling the player with the trade action as “player 1” and we call the other “player 2.” These two players are the parties engaged in a contractual relationship with a non-durable trading opportunity and external enforcement. Their relationship has the following payoff-relevant components, occurring in the order shown:

**The state of the relationship  $\theta$ .** The state represents unverifiable events that are assumed to happen early in the relationship. The state may be determined by individual investment decisions and/or by random occurrences, depending on the setting. When the state is realized, it becomes commonly known by the players; however, it cannot be verified to the external enforcer. Let  $\Theta$  denote the set of possible states.

**The trade action  $a$ .** This is an individual action chosen by player 1 that determines whether and how the relationship is consummated. The trade action is commonly observed by the players and is verifiable to the external enforcer. Let  $A$  be the set of feasible trade actions.

**The monetary transfers  $t = (t_1, t_2)$ .** Here  $t_i$  denotes the amount given to player  $i$ , for  $i = 1, 2$ , where a negative value represents an amount taken from this player. Transfers

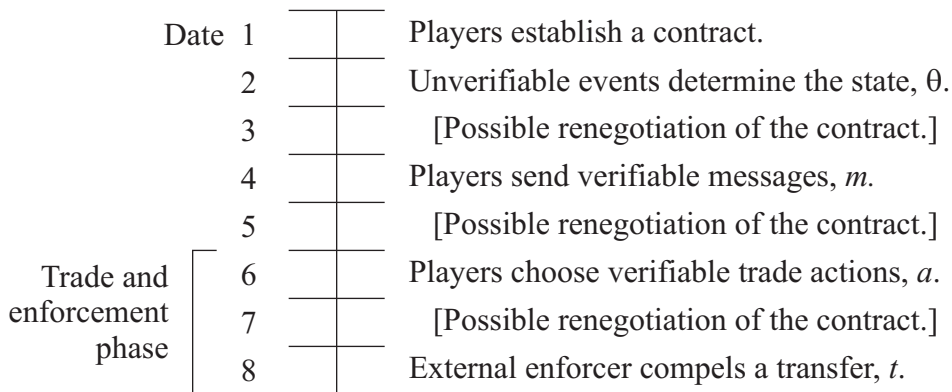


Figure 1: Time line of the contractual relationship.

are compelled by the external enforcer, who is not a strategic player but, rather, who behaves as directed by the contract of players 1 and 2.<sup>3</sup> Assume  $t_1 + t_2 \leq 0$ .

We assume that the players' payoffs are additive in money and are thus defined by a function  $u: A \times \Theta \rightarrow \mathbb{R}^2$ . In state  $\theta$ , with trade action  $a$  and transfer  $t$ , the payoff vector is  $u(a, \theta) + t$ . Define  $U(a, \theta) \equiv u_1(a, \theta) + u_2(a, \theta)$ , which is the joint value of the contractual relationship in state  $\theta$  if trade action  $a$  is selected. We assume that, in each state  $\theta$ , the joint value has a unique maximizer  $a^*(\theta)$ . We let  $\gamma(\theta)$  denote the maximal joint payoff in state  $\theta$ , so we have

$$\gamma(\theta) \equiv U(a^*(\theta), \theta) = \max_{a \in A} U(a, \theta). \quad (1)$$

In addition to the payoff-relevant components of their relationship, we assume that the players can communicate with the external enforcer using public, verifiable messages. Let  $m = (m_1, m_2)$  denote the profile of messages that the players send and let  $M_1$  and  $M_2$  be the sets of feasible messages. The sets  $M_1$  and  $M_2$  will be endogenous in the sense that they are specified by the players in their contract.

Figure 1 shows the time line of the contractual relationship. At even-numbered dates through Date 6, the players make joint observations and they make individual decisions—jointly observing the state at Date 2, sending verifiable messages at Date 4, and selecting the trade actions at Date 6. At Date 8, the external enforcer compels transfers.

At odd-numbered dates, the players make joint contracting decisions—establishing a contract at Date 1 and possibly renegotiating it later. The contract has an externally-enforced component consisting of (i) feasible message spaces  $M_1$  and  $M_2$  and (ii) a *transfer function*  $y: M \times A \rightarrow \mathbb{R}^2$  specifying the transfer  $t$  as a function of the verifiable items  $m$  and  $a$ . That is, having seen  $m$  and  $a$ , the external enforcer compels transfer  $t = y(m, a)$ . The contract also has a self-enforced component, which specifies how the players coordinate their behavior for the times at which they take individual actions. Renegotiation of the

<sup>3</sup>That the external enforcer's role is limited to compelling transfers is consistent with what courts do in practice.

contract amounts to replacing the original transfer function  $y$  with some new function  $y'$ , in which case  $y'$  is the one submitted to the external enforcer at Date 8.

The players' individual actions at Dates 2, 4 and 6 are assumed to be consistent with sequential rationality; that is, each player maximizes his expected payoff, conditional on what occurred earlier and on what the other player does, and anticipating rational behavior in the future.

The joint decisions (initial contracting and renegotiation at odd-numbered periods) are assumed to be consistent with a cooperative bargaining solution in which the players divide surplus according to fixed bargaining weights  $\pi_1$  and  $\pi_2$  for players 1 and 2, respectively. The bargaining weights are nonnegative, sum to one, and are written  $\pi = (\pi_1, \pi_2)$ . The negotiation surplus is the difference between  $\gamma(\theta)$  and the joint value that would result if the players fail to reach an agreement, where the disagreement point is given by an equilibrium in the continuation in which the externally enforced component of the contract has not been altered.<sup>4</sup>

The effect of the renegotiation opportunity at Date 7 is to constrain transfers to be “balanced” — that is, satisfying

$$t \in \mathbb{R}_0^2 \equiv \{t' \in \mathbb{R}^2 \mid t'_1 + t'_2 = 0\}.$$

Thus, we will simply assume that transfers are balanced and then otherwise ignore Date 7.

So far, we have not explicitly included any specific investment technology in the model. That is, we have not described the individual investment actions that determine the state. We shall add this structure in the following sections, where we investigate the interaction between the technology of trade and the technology of investment. Some of our technical results concern only how the trade technology is modeled.

## Public-Action Modeling and Forcing Contracts

Because the trade action  $a$  is assumed to be taken by player 1, we have specified here an individual-action model. A public-action model, in contrast, would abstract by treating the trade action  $a$  as something that the external enforcer directly selects. Watson (2007) shows that specifying a public-action model is equivalent to examining the individual-action model but limiting attention to a particular class of contracts called *forcing contracts* which, for any given message profile, prescribe that player 1 select a particular trade action.

More precisely, a forcing contract specifies a large transfer from player 1 to player 2 in the event that player 1 does not take his contractually-prescribed action. This transfer is sufficiently large to give player 1 the incentive to select the prescribed action in every state. Thus, the induced trade action is constant in the state, conditional on the messages sent earlier.

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<sup>4</sup>The generalized Nash bargaining solution has this representation. The rationality conditions identify a *contractual equilibrium*; see Watson (2004) for notes on the relation between “cooperative” and “noncooperative” approaches to modeling negotiation.

For example, holding the message profile fixed, the transfer function  $\hat{y}$  defined as follows will force player 1 to select action  $\hat{a}$  and impose the transfer  $\hat{t}$  (as though the external enforcer chose these in a public-action model):

Let  $L$  be such that  $L > \sup_{a,\theta} u_1(a, \theta) - \inf_{a,\theta} u_1(a, \theta)$ . Then define  $\hat{y}(\hat{a}) \equiv \hat{t}$  and, for every  $a \neq \hat{a}$ , set  $\hat{y}(a) \equiv \hat{t} + (-L, L)$ .

We use the term *forcing* for any transfer function that, given the message profile, induces player 1 to select the same trade action over all of the states.<sup>5</sup> We use the term *non-forcing* for transfer functions that induce player 1 to select different actions in at least two different states.

When the players renegotiate at Date 3 or Date 5, they know the state  $\theta$  and so they can select a contract that forces the action  $a^*(\theta)$  to obtain the joint value  $\gamma(\theta)$ .

## Continuation Value Functions

A (*state-contingent*) *value function* is a function from  $\Theta$  to  $\mathbb{R}^2$  that gives the players' expected payoff vector from the start of a given date, as a function of the state. Such a value function represents the continuation values for a given outstanding contract and equilibrium behavior. We adopt the convention of not including any sunk investment costs from Date 2 in the values from later dates.

The continuation values from the start of Date 3 are important to calculate, because these determine the players' incentives to invest at Date 2. Thus, our chief objective is to characterize the set of *implementable value functions* from the start of Date 3.<sup>6</sup> A value function  $v$  for Date 3 is implementable if there is a contract that, if formed at Date 1, would lead to continuation value  $v(\theta)$  in state  $\theta$  from the start of Date 3, for every  $\theta \in \Theta$ .

We examine variations of the model in terms of whether renegotiation is possible at Date 5 (ex post) and/or Date 3 (interim), and whether one restricts attention to forcing contracts. We let  $V^{\text{EPF}}$  be the set of implementable value functions from Date 3 for the case of ex-post renegotiation and with the restriction to forcing contracts. We let  $V^{\text{EP}}$  be the corresponding set for the case of ex-post renegotiation and no contractual restrictions. Further, we let  $V^{\text{I}}$  be the set of implementable value functions for the case in which renegotiation can occur only at Date 3 (the interim stage).<sup>7</sup>

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<sup>5</sup>One could add a public randomization device to the model for the purpose of achieving randomization over trade actions using forcing contracts. Allowing such randomization does not expand the set of implementable value functions here.

<sup>6</sup>Section 4 reviews how to calculate value functions from the various dates in the general model, by backward induction.

<sup>7</sup>In the case of only interim renegotiation, a restriction to forcing contracts does not affect the implementable set. Also, with ex post renegotiation, allowing interim renegotiation has no additional effect.



## Related Literature

Much of the recent contract-theory literature focuses on public-action mechanism-design models. For instance, Che and Hausch (1999), Hart and Moore (1999), Maskin and Moore (1999), Segal (1999), and Segal and Whinston (2002) have basically the same set-up as we do except that their models treat trade actions as public (collapsing together the trade action and enforcement phase), so they focus on forcing contracts.<sup>8</sup> In some related papers, the verbal description of the contracting environment identifies individuals who take the trade actions, but the actions are effectively modeled as public due to an implicit restriction to forcing contracts. In some cases, such as with the contribution of Edlin and Reichelstein (1996), simple forcing contracts (or breach remedies) are sufficient to achieve an efficient outcome and so the restriction does not have efficiency consequences.<sup>9</sup>

Examples of individual-action models in the literature, among others, are the articles of Hart and Moore (1988), MacLeod and Malcomson (1993), and Nöldeke and Schmidt (1995). Also relevant is the work of Myerson (1982, 1991), whose mechanism-design analysis nicely distinguishes between inalienable individual and public actions (he uses the term “collective choice problem” to describe public-action models).

Most closely related to our work is that of Evans (2006, 2008), who emphasizes how efficient outcomes can be achieved by conditioning external enforcement on costly individual actions. Evans (2006) examines general mechanism-design problems; Evans (2008), which we discuss more in the Conclusion, examines contracting problems with specific investment and durable trading opportunities. Related as well is the work of Lyon and Rasmussen (2004), which shares the theme of Watson (2007), and the recent work of Boeckem and Schiller (2008) and Ellman (2006).<sup>10</sup>

In classifying the related literature, another major distinction to make is between models with cross investment and models with “own investment.” In the latter case, investment enhances the investing party’s benefit of trade. We discuss this distinction in more detail in the next two sections. Since the hold-up problem is more problematic in the case of cross investment, and there the distinction between forcing and non-forcing contracts (public-versus individual-action modeling) is critical, we concentrate on settings with significant cross investment.

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<sup>8</sup>Aghion, Dewatripont, and Rey (1994) is another example. The more recent entries by Roider (2004) and Guriev (2003) have the same basic public-action structure. Demski and Sappington (1991), Nöldeke and Schmidt (1998), and Edlin and Hermalin (2000) examine models with sequential investments in a tradeable asset; in these models, as in Maskin and Tirole (1999), transferring the asset is essentially a public action.

<sup>9</sup>Stremitzer (2009) elaborates on Edlin and Reichelstein (1996) by examining the informational requirements of standard breach remedies (specifically, partially verifiable investments).

<sup>10</sup>Also related are some studies of delegation in principal-agent settings with asymmetric information, where implementable outcomes depend on whether it is the principal or agent who has the productive action. As Beaudry and Poitevin (1995) show, ex post renegotiation imposes less of a constraint in the case of “indirect revelation” (where the agent has the productive action). Thus, if it is possible to transfer “ownership” of the productive action to the agent, the threat of ex post renegotiation provides one reason for doing this.

## 2 Cross-Investment Example

In this section, we provide a simple example of specific investment and hold-up to illustrate our results. Consider a contractual setting with what Che and Hausch (1999) call “cooperative investments” (others use the term *cross investments*). One of the players is the *investor* and the other player is the *beneficiary*. The investor chooses a level of investment at some personal cost. The investment generates a benefit if the parties later trade, but it is the beneficiary who stands to obtain much of it. Suppose that there is ex post renegotiation and that the players have equal bargaining weights. Then there are contingencies (typically out of equilibrium) in which the beneficiary can extract surplus from the investor by threatening to hold up trade. This can distort the investor’s incentives and lead to inefficient investment.

We will look at two versions of the example, one in which player 1 (whom we continue to call Al) is the investor and one in which player 2 (Zoe) is the investor. In both cases, Al has the trade action, as we have specified in the general model. Thus, if Al is the investor then we have a setting of *unified* investment and trade actions, whereas if Zoe is the investor then we have the *divided* case.

Here are the numerical details for our example. At Date 2, the investor selects  $\theta \in [1, 3]$  at immediate cost  $c(\theta) = \frac{7}{2}(\theta - 1)$ . That is, the investor takes an action that determines the state. At Date 6 Al selects a trade action  $a \in [0, 8]$ , which we interpret as a quantity of an intermediate good that he trades with Zoe. The beneficiary’s gain from trade is

$$B(a, \theta) = 8a - \frac{6a^2}{\theta},$$

whereas the investor’s gain from trade (gross of investment cost) is

$$C(a, \theta) = \frac{2a^2}{\theta}.$$

Thus, in the unified case where player 1 (Al) has both the investment and trade actions, the payoffs (not including investment cost) are given by

$$u_1(a, \theta) = \frac{2a^2}{\theta} \quad \text{and} \quad u_2(a, \theta) = 8a - \frac{6a^2}{\theta},$$

whereas in the divided case the payoffs are

$$u_1(a, \theta) = 8a - \frac{6a^2}{\theta} \quad \text{and} \quad u_2(a, \theta) = \frac{2a^2}{\theta}$$

gross of investment cost. As we assume throughout, the sunk investment cost is not included in these functions and in the value functions computed below.

The joint value of the relationship in state  $\theta$  is  $U(a, \theta) = 8a - \frac{4a^2}{\theta}$ , which is maximized at  $a^*(\theta) = \theta$ . Therefore the maximal joint value in state  $\theta$  is  $\gamma(\theta) = U(a^*(\theta), \theta) = 4\theta$ . Regardless of who makes the investment, we see that the efficient level of investment  $\theta^*$  solves:

$$\max_{\theta \in [1, 3]} 4\theta - \frac{7}{2}(\theta - 1).$$

Thus the optimal investment level is  $\theta^* = 3$ .

Note that, at the ex post optimal trade action for each state, the investor's gain is  $C(\theta, \theta) = 2\theta$  and the beneficiary's gain is  $B(\theta, \theta) = 2\theta$ . The example thus exhibits elements of both cross-investment (since the investment increases the beneficiary's gain from trade) and *own-investment* (since the investment also boosts the investor's gain from trade). The cross-investment element is particularly problematic, as the literature as shown.

This example is a member of the class studied by Che and Hausch (1999). These authors formulate a public-action model, which restricts attention to forcing contracts and thus does not distinguish between the unified and divided cases. Che and Hausch find that in their model the hold-up problem severely restricts implementability and leads to inefficiently low investment. In fact, for the parameters described here, their results establish that the “null contract”—forcing no trade, regardless of the messages—is best.

Unfortunately, the null contract leads to an inefficient level of investment. To see this, note that the players always renegotiate to take the ex post efficient trade action in each state. In our example this implies that in state  $\theta$  the renegotiation surplus equals the joint value  $4\theta$ . Since the investor receives half of the surplus (recall that the bargaining weights are  $1/2$ ), the investor's value from Date 3 is  $2\theta$ . At Date 2, the investor therefore chooses  $\theta$  to solve

$$\max_{\theta \in [1,3]} 2\theta - \frac{7}{2}(\theta - 1).$$

As a result, the inefficiently low level  $\theta = 1$  is chosen.

We next demonstrate that by using non-forcing contracts a more efficient outcome can be achieved (Watson's 2007 point) and that the unified and divided cases behave quite differently. Let us start with the unified case, where payoffs are given by

$$u_1(a, \theta) = \frac{2a^2}{\theta} \quad \text{and} \quad u_2(a, \theta) = 8a - \frac{6a^2}{\theta}.$$

Consider the following contract that gives player 1 a trade-action-based option (messages are not used).<sup>11</sup> If Al chooses  $a = 0$  then no transfer is made. If he chooses  $a = 1$ , a transfer of  $4/3$  is made from Al to Zoe. Finally, if Al chooses any other value of  $a$ , a transfer of 150 is made to Zoe so that Al will never find it optimal to choose  $a \notin \{0, 1\}$ .

Note that Al strictly prefers  $a = 1$  if and only if

$$u_1(1, \theta) - \frac{4}{3} = \frac{2}{\theta} - \frac{4}{3} > 0,$$

which simplifies to  $\theta < 3/2$ . In this case, the surplus to be gained from renegotiation is

$$U(a^*(\theta), \theta) - U(1, \theta) = 4\theta - \left(8 - \frac{4}{\theta}\right),$$

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<sup>11</sup>The literature has emphasized the importance of option contracts for aligning incentives (as in Demski and Sappington 1991 and most of the more recent papers cited in the previous sections). By laying out the details of the trade technology, we are able to differentiate between message-based and trade-action-based options.

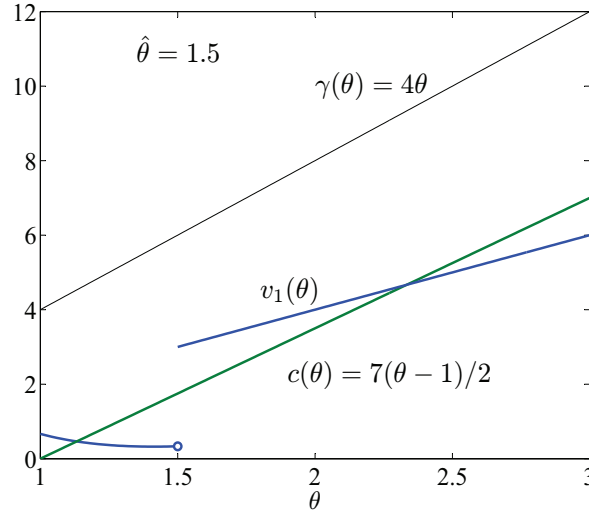


Figure 2: Al's Value Function Under Option Contract.

implying that Al's state-contingent value from Date 3 is

$$\begin{aligned}
 v_1(\theta) &= u_1(1, \theta) - \frac{4}{3} + \frac{1}{2} [U(a^*(\theta), \theta) - U(1, \theta)] \\
 &= \frac{2}{\theta} - \frac{4}{3} + \frac{1}{2} [4\theta - (8 - \frac{4}{\theta})] \\
 &= \frac{4}{\theta} + 2\theta - \frac{16}{3}
 \end{aligned}$$

For all states  $\theta \geq 3/2$ , Al prefers  $a = 0$  and so  $v_1(\theta) = 2\theta$  as under the null contract.

As can be seen from Figure 2, at Date 2 Al maximizes the difference  $v_1(\theta) - c(\theta)$  by choosing  $\theta = 3/2$ , which yields a higher joint value than is achieved with  $\theta = 1$ . Thus, by decreasing Al's implemented value at low states (those below  $3/2$ ), this trade-action-based option contract improves upon the investment incentives of the best forcing contract. However, we can use the results of Appendix C to show that no contract induces the efficient investment level  $\theta = 3$ . So the bottom line is that, in the unified case, non-forcing contracts strictly improve on the best forcing contract but efficient investment is still not achievable.

The results are more dramatic in the divided case, which we now consider. Suppose that Zoe makes the investment so that the payoffs are given by

$$u_1(a, \theta) = 8a - \frac{6a^2}{\theta} \quad \text{and} \quad u_2(a, \theta) = \frac{2a^2}{\theta}.$$

We show that a particular "dual option" contract can be used to make Zoe the residual claimant with respect to the post-investment joint value. The contract requires Zoe (player 2) to send a message at Date 4; her message names a trade action  $\hat{a} \in [0, 8]$ . Then if Al subsequently selects  $a = \hat{a}$ , the external enforcer compels a transfer of  $2\hat{a}$  from Al to Zoe. If Al selects  $a = 0$  then the transfer is zero. Finally, if Al chooses any other value of  $a$ , a transfer of 150 is made to Zoe so that Al will never find it optimal to choose  $a \notin \{0, \hat{a}\}$ .

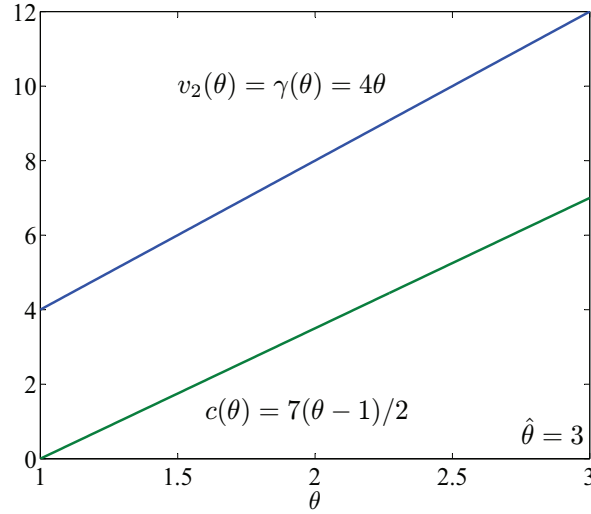


Figure 3: Zoe's Value Function Under Option Contract.

Let us calculate the value function that this contract implements. In any given state  $\theta$ , if Zoe names  $\hat{a} = \theta$  at Date 4 then it makes Al indifferent between selecting  $a = \theta$  and  $a = 0$  at Date 6, so we can prescribe that Al will select  $a = \theta$ . Because this is the ex post efficient trade action in state  $\theta$ , there is no renegotiation at Date 5. This gives Zoe a value of  $4\theta$ , which is exactly  $\gamma(\theta)$ , and it holds Al's payoff at zero. It is easy to see that Zoe can do no better by choosing any other  $\hat{a}$  at Date 4. Thus, for each state  $\theta$ , the contract implements the continuation value  $v(\theta) = (0, 4\theta)$  from Date 3, and thus it makes Zoe the residual claimant.

As Figure 3 illustrates, at Date 2 Zoe maximizes  $v_2(\theta) - c(\theta)$  by choosing  $\theta^* = 3$ . Thus the dual option contract not only improves upon the investment incentives of the best forcing contract but it induces Zoe to choose the efficient investment level.

In summary, the example shows that by using non-forcing contracts, the parties can achieve a more efficient outcome than is possible with forcing contracts. Furthermore, the efficiency gain depends on the relation between the technology of trade and the technology of investment. In the divided case, a simple contract induces efficient investment and trade actions. In the unified case, the efficient outcome cannot be attained but a non-forcing contract still is preferred.

### 3 Residual Claimancy and Efficient Investment

In this section we provide a general version of the result on the divided case from the example. Remember that player 1 takes the trade action at Date 6. We shall show that a dual-option contract can be used to make player 1's value from Date 3 constant in the state. Player 2 then becomes the residual claimant with respect to the investment decision. Thus, in the divided case (where player 2 is the investor), player 2 can be given the incentive to

invest efficiently regardless of the distribution of the investment gains.

Our result is facilitated by making the following assumption.

**Assumption 1:** There exists a trade action  $\underline{a} \in A$  such that  $u_1(\underline{a}, \theta) = u_2(\underline{a}, \theta) = 0$  for every  $\theta$ .

Think of  $\underline{a}$  as the “no trade” choice. Assumption 1 means that the investment conveys no direct benefits; that is, investment produces a gain only conditional on the players trading at Date 6. The no-trade payoffs could be normalized to any level; we set them to zero here for simplicity.

**Theorem 1:** Consider any contractual relationship that satisfies Assumption 1. Also assume that there is ex post renegotiation. Let  $k$  be any real number and define value function  $v$  by  $v_1(\theta) = k$  and  $v_2(\theta) = \gamma(\theta) - k$  for all  $\theta \in \Theta$ . Then  $v \in V^{\text{EP}}$ .

Note that Assumption 1 has to do solely with the technology of trade; it puts no constraints on the technology of investment. The proof of Theorem 1 (which may be found in Appendix A) is constructive and runs along the lines of the demonstration for the example. We show how to implement these value functions using a straightforward dual-option contract in which player 2 is required to declare a state  $\hat{\theta}$  at Date 4. The contract then gives player 1 the incentive to tender trade action  $a^*(\hat{\theta})$  or  $\underline{a}$ . Thus, a message is required, but only from player 2.

To formalize the implications of Theorem 1, let us specify the investment technology. As in the example, suppose that one of the players makes an investment choice at Date 2. We have two cases:

- **Unified case** – Player 1 has both the Date 2 investment action and the Date 6 trade action.
- **Divided case** – Player 2 has the Date 2 investment action, whereas player 1 has the Date 6 trade action.

Let the investment choice be denoted  $x \geq 0$ . We assume that the investment imposes an immediate cost of  $x$  on the investor. The state  $\theta$  is then drawn from a distribution  $G(x)$  that depends on the investment choice.<sup>12</sup> Recalling that  $\gamma(\theta) = U(a^*(\theta), \theta)$  is the maximum joint value in state  $\theta$ , we see that the efficient level of investment  $x^*$  solves:

$$\max_{x \geq 0} \int \gamma(\theta) dG(x) - x.$$

Typically  $x^* > 0$ . Thus, letting  $i$  denote the investing party, we will want to implement a value function  $v$  so that  $v_i(\theta)$  is increasing in  $\theta$  to some particular extent. In this way, player  $i$  will be rewarded for investing.

<sup>12</sup>It is natural to assume that  $G$  is increasing in  $x$  in the sense of first-order stochastic dominance and that  $U(a^*(\theta), \theta)$  is increasing in  $\theta$ , so that higher investments increase the expected gains from trade, but these assumptions are not needed for the Corollary presented here.

Consider a value function that satisfies  $v_1(\theta) = k$  and  $v_2(\theta) = \gamma(\theta) - k$  for all  $\theta \in \Theta$  and suppose that the players contract at Date 1 to implement this value function. Let us observe what this implies for investment in the divided case, where player 2 is the investor. Clearly player 2 selects  $x$  at Date 2 to maximize

$$\int v_2(\theta)dG(x) - x = \int \gamma(\theta)dG(x) - x - k.$$

Since  $k$  is a constant, player 2 seeks to maximize the joint value of the relationship and thus player 2's optimal investment level is  $x^*$ . Efficient investment and trade are obtained. At Date 1, the players will select such a value function to maximize the joint value of their relationship, and they will use  $k$  to divide the value between them. We formalize this conclusion by stating:

**Corollary:** *Under Assumption 1 and in the divided case in which player 2 is the investor and player 1 has the trade action, optimal contracting induces efficient investment and trade (the first best outcome).*

Note that this result makes no restrictions on which party stands to gain from the investment. That is, the result holds for settings of cross-investment, own-investment, and any combination of the two. The key is simply that the investment action and trade action are taken by different parties.

The impact of the Corollary relative to the literature is most pronounced when applied to settings of cross-investment.<sup>13</sup> In such a setting, as in our example, one player is the investor and the other is the beneficiary. The beneficiary's gain from trade is  $B(a, \theta)$ , whereas the investor's gain is  $C(a, \theta)$ . We take these to be gross of investment cost (that is, they do not include  $-x$  for the investor). In the divided case, we thus have  $u_1 \equiv B$  and  $u_2 \equiv C$ . In the unified case, we have  $u_1 \equiv C$  and  $u_2 \equiv B$ .

The *null contract* is that which forces the no-trade action  $\underline{a}$  regardless of the message profile. Che and Hausch (1999) have shown that when the investor receives a sufficiently small share of the benefits of the investment, the null contract is optimal among forcing contracts. Under the null contract, the investor shares the gains from trade in proportion to his/her bargaining weight (because the players renegotiate from the no-trade action in order to trade). This means that at Date 2 the investor chooses  $x$  to solve

$$\max_{x \geq 0} \int \pi_i \gamma(\theta) dG(x) - x,$$

where the investor here is denoted “ $i$ .” Since  $\pi_i$  is typically below 1, unfortunately the investor will not select the efficient  $x^*$ .

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<sup>13</sup>The literature has demonstrated that forcing contracts can usually prevent the hold-up problem in the “own-investment” case, where the investing party obtains a large share of the benefit created by the investment. See, for example, Chung (1991), Rogerson (1992), Aghion, Dewatripont, and Rey (1994), Nöldeke and Schmidt (1995), and Edlin and Reichelstein (1996). An exception is the “complexity/ambivalence” setting studied by Segal (1999), Hart and Moore (1999), and Reiche (2006).

We therefore see that in the case of cross-investment, the above Corollary presents a great improvement in comparison to the result from analyses that restrict attention to forcing contracts. However, the picture is not so rosy in the unified case because it is generally not possible to implement a value function that makes player 2's continuation value from Date 3 constant in the state. This is demonstrated in Appendix C, where we provide the technical conditions. However, we can still show that non-forcing contracts outperform forcing contracts in a wide range of contractual settings, which is the focus of the rest of this paper.

Before launching into the general analysis on the comparison of forcing and non-forcing contracts, some intuition from the example may be helpful. Consider the unified case, where player 1 makes the investment and selects the trade action. In the example, his gain from trade is  $C(a, \theta) = 2a^2/\theta$ . Note that this function is supermodular in  $\theta$  and  $-a$ . Thus, for any transfers specified as a function of the trade action, player 1's preferences satisfy the single-crossing property and he weakly prefers higher actions in lower states. Since ex post efficiency requires lower actions in lower states, we can find a non-forcing contract that, relative to the null contract, induces a lower renegotiation surplus in lower states, reducing player 1's incentive to choose the lowest levels of investment.

Similar intuition is at work in settings where we want to give player 2 investment incentives but where Assumption 1 does not hold (so that we cannot rely on Theorem 1 and its Corollary). If player 1's gain from trade is supermodular in  $\theta$  and  $a$ , then a non-forcing contract can be used to induce player 1 to select higher trade actions in higher states. Furthermore, if ex post efficiency calls for higher actions in higher states then player 1 can be induced to select the ex post efficient trade action (so there is no surplus to renegotiate over) and transfer some of his gain to the other party.

These are but two rough sketches. Clearly, there are many cases to consider and they are differentiated on the basis of which player is the investor, whether player 1's trade gains are supermodular or submodular (or neither), and whether  $a^*(\theta)$  is increasing or decreasing. We can clearly transform a setting with submodularity into one with supermodularity by relabeling the trade action as  $-a$  rather than  $a$ . For the rest of the paper we take a broad perspective that focuses just on the trade technology. With supermodularity of  $u_1$  and some other mild technical assumptions, we show that, in terms of implementability, non-forcing contracts generally improve on forcing contracts. Appendix C provides additional details for some cases of cross-investment.

## 4 Implementable Value Functions

Our second result requires a complete characterization of the implementable value functions from Date 3. In this section, we analyze equilibrium behavior and provide the characterization. Much of the analysis here repeats material in Watson (2007), so we keep this text brief and ask the reader to see Watson (2007) for more details. The culmination of the basic analysis here are some simple characterization results from Watson (2007), which we



build upon in the subsequent section.

The set of implementable value functions depends on whether renegotiation is possible at Dates 3 or 5.<sup>14</sup> We will examine the cases of *ex post renegotiation*, where the parties can renegotiate at Date 5, and *interim renegotiation*, where the parties can renegotiate at Date 3 but cannot do so at Date 5. We can characterize the implementable value functions by backward induction, starting with Date 6 where player 1 selects the trade action.

## State-Contingent Values from Date 6

To calculate the value functions that are supported from Date 6 (the “trade and enforcement phase” shown in Figure 1), we can ignore the payoff-irrelevant messages sent earlier (or equivalently, fix a message profile from Date 4) and simply write the externally enforced transfer function as  $\hat{y}: A \rightarrow \mathbb{R}^2$ . That is,  $\hat{y}$  gives the monetary transfer as a function of player 1’s trade action.

Given the state  $\theta$ ,  $\hat{y}$  defines a *trading game* in which player 1 selects an action  $a \in A$  and the payoff vector is then  $u(a, \theta) + \hat{y}(a)$ . Focusing on pure strategies, we let  $\hat{a}(\theta)$  denote the action chosen by player 1 in state  $\theta$ . This specification is rational for player 1 if, for every  $\theta \in \Theta$ ,  $\hat{a}$  maximizes  $u_1(a, \theta) + \hat{y}_1(a)$  by choice of  $a$ . The state-contingent payoff vector from Date 6 is then given by the *outcome function*  $w: \Theta \rightarrow \mathbb{R}^2$  defined by

$$w(\theta) \equiv u(\hat{a}(\theta), \theta) + \hat{y}(\hat{a}(\theta)). \quad (2)$$

Let  $W$  denote the set of supportable outcome functions. That is,  $w \in W$  if and only if there are functions  $\hat{y}$  and  $\hat{a}$  such that  $\hat{a}$  is rational for player 1 and, for every  $\theta \in \Theta$ , Equation 2 holds. Furthermore, let  $W^F$  be the subset of outcomes that can be supported using forcing contracts. It is easy to see that  $w \in W^F$  if and only if there is a trade action  $\hat{a}$  and a transfer vector  $\hat{t}$  such that  $w(\theta) = u(\hat{a}, \theta) + \hat{t}$  for all  $\theta \in \Theta$ . We can compare individual-action and public-action models by determining whether the restriction to forcing contracts implies a significant constraint on the set of implementable value functions.

## State-Contingent Values from Date 5

We next step back to Date 5. If there is no opportunity for *ex post* renegotiation, then nothing happens at Date 5 and so  $W$  and  $W^F$  are the supported state-contingent value sets from the start of Date 5 as well. On the other hand, if *ex post* renegotiation is allowed, then at Date 5 the players have an opportunity to discard their originally specified contract  $y$  and replace it with another,  $y'$ . The original contract  $y$  would have led to a particular outcome  $w$  given the message profile from Date 4.

By picking a new contract  $y'$ , the players are effectively choosing a new outcome function  $w'$ , which is freely selected from the set  $W$  or the set  $W^F$  (depending on whether there is a restriction to forcing contracts). If the outcome  $w$  would be inefficient given the realized state and message profile, the players will renegotiate to select an efficient outcome

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<sup>14</sup>As noted earlier, we do not need to explicitly model the Date 2 investment technology in order to calculate the set of implementable value functions from Date 3.

$w'$ . The players divide the renegotiation surplus according to the fixed bargaining weights  $\pi_1$  and  $\pi_2$ . Dividing the surplus in this way is feasible because  $W$  and  $W^F$  are closed under constant transfers.

Clearly, we have  $\gamma(\theta) = \max_{w \in W^F} [w_1(\theta) + w_2(\theta)]$  because the trade action that solves the maximization problem in Equation 1 can be specified in a forcing contract to yield the desired outcome. If the original contract would lead to outcome  $w$  in state  $\theta$ , then the *renegotiation surplus* is

$$r(w, \theta) \equiv \gamma(\theta) - w_1(\theta) - w_2(\theta).$$

The bargaining solution implies that the players settle on a new outcome in which the payoff vector in state  $\theta$  is  $w(\theta) + \pi r(w, \theta)$ .

An *ex post renegotiation outcome* is a state-contingent payoff vector that results when, in every state, the players renegotiate from a fixed outcome in  $W$ . That is, a value function  $z$  is an ex post renegotiation outcome if and only if there is an outcome  $w \in W$  such that  $z(\theta) = w(\theta) + \pi r(w, \theta)$  for every  $\theta \in \Theta$ . Let  $Z$  denote the set of ex post renegotiation outcomes. Note that all elements of  $Z$  are efficient in every state; also,  $Z$  and  $W$  are generally not ranked by inclusion. If trade actions are treated as public (and so attention is limited to forcing contracts) then the set of ex post renegotiation outcomes contains only the value functions of the form  $z = w + \pi r(w, \cdot)$  with the constraint that  $w \in W^F$ . Let  $Z^F$  denote the set of ex post renegotiation outcomes under forcing contracts.

With ex post renegotiation, the set of supportable state-contingent values from the start of Date 5 is  $Z$  in the case of the individual-action model and is  $Z^F$  in the case of the public-action model. We will be a bit loose with terminology and refer to functions in  $Z$  and  $Z^F$ , in addition to functions in  $W$  and  $W^F$ , simply as “outcomes.”

### State-Contingent Values from Dates 4 and 3

Analysis of contract selection and incentives at Date 4 can be viewed as a standard mechanism-design problem. The players’ contract is equivalent to a mechanism that maps messages sent at Date 4 to outcomes induced in the trade and enforcement phase (possibly renegotiated at Date 5). The revelation principle applies in the following sense. We can restrict attention to direct-revelation mechanisms, each of which is defined by (i) a message space  $M \equiv \Theta^2$  and (ii) a function that maps  $\Theta^2$  to the relevant outcome set that gives the state-contingent value functions from the start of Date 5. The outcome set is either  $W$ ,  $W^F$ ,  $Z$ , or  $Z^F$ , depending on whether ex post renegotiation and/or non-forcing contracts are allowed. We can concentrate on Nash equilibria of the mechanism in which the parties report truthfully in each state.<sup>15</sup>

Let us write  $\psi^{\theta_1 \theta_2}$  for the outcome that the mechanism prescribes when player 1 reports the state to be  $\theta_1$  and player 2 reports the state to be  $\theta_2$ . Note that, in any given state  $\theta$  (the actual state that occurred), the mechanism implies a “message game” with strategy space

<sup>15</sup>The revelation principle usually requires a public randomization device to create lotteries over outcomes (or that the outcome set is a mixture space), but it is not needed here.

$\Theta^2$  and payoffs given by  $\psi^{\theta_1\theta_2}(\theta)$  for each strategy profile  $(\theta_1, \theta_2)$ . For truthful reporting to be a Nash equilibrium of this game, it must be that  $\psi_1^{\theta\theta}(\theta) \geq \psi_1^{\theta'\theta}(\theta)$  and  $\psi_2^{\theta\theta}(\theta) \geq \psi_2^{\theta\theta'}(\theta)$  for all  $\theta \in \Theta$ .

We proceed using standard techniques for mechanism design with transfers, following Watson (2007). The key step is observing that, for any two states  $\theta$  and  $\theta'$ , the outcome specified for the “off-diagonal” message profile  $(\theta', \theta)$  must be sufficient to simultaneously (i) dissuade player 1 from declaring the state to be  $\theta'$  when the state is actually  $\theta$  and (ii) discourage player 2 from declaring “ $\theta$ ” in state  $\theta'$ . Thus, we require

$$\psi_1^{\theta\theta}(\theta) \geq \psi_1^{\theta'\theta}(\theta) \quad \text{and} \quad \psi_2^{\theta'\theta'}(\theta') \geq \psi_2^{\theta'\theta}(\theta').$$

Because the outcome sets are closed under constant transfers, we can choose the outcome to effectively raise or lower  $\psi_1^{\theta'\theta}$  and  $\psi_2^{\theta'\theta}$  while keeping the sum constant. Thus, a sufficient condition for these two inequalities is that the sum of the two holds. Letting  $\psi \equiv \psi^{\theta\theta}$  and  $\psi' \equiv \psi^{\theta'\theta'}$ , we thus have the following necessary condition for implementing outcome  $\psi$  in state  $\theta$  and outcome  $\psi'$  in state  $\theta'$ :

$$\text{(IC)} \quad \text{There exists an outcome } \hat{\psi} \text{ satisfying } \psi_1(\theta) + \psi_2'(\theta') \geq \hat{\psi}_1(\theta) + \hat{\psi}_2(\theta').$$

This condition, applied to all ordered pairs  $(\theta, \theta')$ , is necessary and sufficient for implementation. The sum  $\hat{\psi}_1(\theta) + \hat{\psi}_2(\theta')$  is called the *punishment value* corresponding to the ordered pair  $(\theta, \theta')$ . The punishment value plays a central role in our analysis. Lower punishment values imply a greater set of implementable outcomes.

If interim renegotiation is not allowed, then the analysis above completely determines the implementable set of value functions from Date 3. Allowing interim renegotiation has the effect of requiring each “on-diagonal” outcome to be efficient in the relevant state; that is, for each  $\theta$  we need  $\psi^{\theta\theta}$  to be efficient in this state. In the case of ex post renegotiation, allowing interim renegotiation entails no further constraint because every outcome in  $Z$  is efficient in every state.

It is also the case that without ex post renegotiation,  $W$  and  $W^F$  yield the same set of implementable value functions from Date 3. In other words, a restriction to forcing contracts does not reduce the implementable set in the case of interim renegotiation (Watson 2007, Lemma 3). Therefore, we have three settings to compare: unrestricted contracts with ex post renegotiation, forcing contracts (public-actions) with ex post renegotiation, and forcing contracts with interim (but not ex post) renegotiation. As stated earlier, we denote the implementable value functions for these three settings by, respectively,  $V^{\text{EP}}$ ,  $V^{\text{EPF}}$ , and  $V^{\text{I}}$ .

A value function  $v: \Theta \rightarrow \mathbb{R}^2$  is called *efficient* if  $v_1(\theta) + v_2(\theta) = \gamma(\theta)$  for every  $\theta \in \Theta$ . The following results summarize the characterization of  $V^{\text{EP}}$ ,  $V^{\text{EPF}}$ , and  $V^{\text{I}}$  and provide a general comparison:

**Result 1 [Watson 2007]:** *Consider any value function  $v: \Theta \rightarrow \mathbb{R}^2$ .*

- *Implementation with Interim Renegotiation:  $v$  is an element of  $V^{\text{I}}$  if and only if  $v$  is efficient and, for every pair of states  $\theta$  and  $\theta'$ , there is an outcome  $\hat{w} \in W^F$  such that  $v_1(\theta) + v_2(\theta') \geq \hat{w}_1(\theta) + \hat{w}_2(\theta')$ .*

- **Implementation with Ex Post Renegotiation:**  $v$  is an element of  $V^{\text{EP}}$  if and only if  $v$  is efficient and, for every pair of states  $\theta$  and  $\theta'$ , there is an outcome  $\hat{z} \in Z$  such that  $v_1(\theta) + v_2(\theta') \geq \hat{z}_1(\theta) + \hat{z}_2(\theta')$ .
- **Implementation with Ex Post Renegotiation and Forcing Contracts:**  $v$  is an element of  $V^{\text{EPF}}$  if and only if  $v$  is efficient and, for every pair of states  $\theta$  and  $\theta'$ , there is an outcome  $\hat{z} \in Z^{\text{F}}$  such that  $v_1(\theta) + v_2(\theta') \geq \hat{z}_1(\theta) + \hat{z}_2(\theta')$ .

Furthermore, the sets  $V^{\text{EP}}$ ,  $V^{\text{EPF}}$ , and  $V^1$  are closed under constant transfers.

**Result 2 [Watson 2007]:** The implementable sets are weakly nested in that  $V^{\text{EPF}} \subseteq V^{\text{EP}} \subseteq V^1$ . Furthermore,  $V^{\text{EPF}} = V^{\text{EP}}$  if and only if, for every pair of states  $\theta, \theta' \in \Theta$  and every  $\hat{z} \in Z$ , there is an ex post renegotiation outcome  $\tilde{z} \in Z^{\text{F}}$  such that  $\tilde{z}_1(\theta) + \tilde{z}_2(\theta') \leq \hat{z}_1(\theta) + \hat{z}_2(\theta')$ . Likewise,  $V^{\text{EP}} = V^1$  if and only if, for all  $\theta, \theta' \in \Theta$  and every  $\hat{w} \in W^{\text{F}}$ , there is an ex post renegotiation outcome  $\hat{z} \in Z$  such that  $\hat{z}_1(\theta) + \hat{z}_2(\theta') \leq \hat{w}_1(\theta) + \hat{w}_2(\theta')$ .<sup>16</sup>

To summarize, we have thus far analyzed the players' behavior at the various dates in the contractual relationship, leading to a simple characterization of implementable value functions from Date 3. The characterization is in terms of the minimum punishment values for each pair of states, which yields a way of relating the implementable sets for the cases of interim renegotiation, ex post renegotiation, and ex post renegotiation and forcing contracts. We next turn to investigate the relation more deeply.

## 5 The Value of Non-Forcing Contracts: Robustness for a Class of Trade Technologies

The example from Watson (2007) and ours in Section 2 provide illustrations of  $V^{\text{EPF}} \neq V^{\text{EP}} \neq V^1$ . Our main objective in this section is to examine the robustness of this conclusion. We consider the wide class of contractual relationships that satisfy the following assumptions.

**Assumption 2:** The sets  $A$  and  $\Theta$  are compact subsets of  $\mathbb{R}$  and contain at least two elements, and  $u_1(\cdot, \theta)$  and  $u_2(\cdot, \theta)$  are continuous functions of  $a$  for every  $\theta \in \Theta$ .

Define  $\underline{a} \equiv \min A$ ,  $\bar{a} \equiv \max A$ ,  $\underline{\theta} \equiv \min \Theta$ ,  $\bar{\theta} \equiv \max \Theta$ . Regarding our assumption that  $U(a, \theta)$  has a unique maximizer  $a^*(\theta)$  for every state  $\theta$ , we now make a slightly stronger assumption on  $U(\cdot, \theta)$ :

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<sup>16</sup>Watson's (2006) Lemma 1 provides some of the supporting analysis (which was not explained fully in the relevant proof in Watson 2007). This lemma establishes that, for any given ordered pair of states  $\theta$  and  $\theta'$  and any supportable outcome  $\psi$ , there exists an implementable value function  $v$  for which  $v_1(\theta) + v_2(\theta') = \psi_1(\theta) + \psi_2(\theta')$ . Because the minimum punishment values exists, in each case we can let  $\psi$  equal the outcome that attains the minimum.

**Assumption 3:**  $U(\cdot, \theta)$  is strictly quasiconcave for every  $\theta \in \Theta$ .

**Assumption 4:**  $u_1$  is supermodular, meaning that  $u_1(a, \theta) - u_1(a', \theta) \geq u_1(a, \theta') - u_1(a', \theta')$  whenever  $a \geq a'$  and  $\theta \geq \theta'$ .

**Assumption 5:** There exist states  $\theta^1, \theta^2 \in \Theta$  such that  $\theta^1 > \theta^2$  and either  $U(\underline{a}, \theta^2) < U(\bar{a}, \theta^2)$  or  $U(\underline{a}, \theta^1) > U(\bar{a}, \theta^1)$ .

**Assumption 6:** Player 1's bargaining weight is positive:  $\pi_1 > 0$ .

Assumptions 2, 3, 5, and 6 are mild technical assumptions. Assumptions 2 and 3 give us a convenient and familiar technical structure to deal with. Assumption 5 removes a knife-edge case concerning the relative joint values of the extreme trade actions in the various states. For instance, if  $\Theta$  has more than two elements and  $U(\underline{a}, \theta) \neq U(\bar{a}, \theta)$  for some  $\theta$  strictly between  $\underline{\theta}$  and  $\bar{\theta}$ , then Assumption 5 is satisfied. If  $\Theta$  has just two elements ( $\underline{\theta}$  and  $\bar{\theta}$ ), then Assumption 5 requires that either  $\underline{a}$  is the efficient trade action in the high state or  $\bar{a}$  is the efficient trade action in the low state.<sup>17</sup>

Assumption 4 puts some structure on the payoff of player 1, the player with the trade action: Without considering transfers, player 1's marginal value of increasing his trade action rises weakly with the state. In other words, higher trade actions are weakly more attractive to him as the state increases. Note that if in a given application  $u_1$  satisfies submodularity, one can redefine the trade action to be  $-a$  and then Assumption 4 would be satisfied.

Many interesting examples studied in the literature satisfy these assumptions. For instance, consider a buyer/seller relationship in which  $a$  is the number of units of an intermediate good to be transferred from the seller to the buyer. The buyer's benefit of obtaining  $a$  units in state  $\theta$  is  $B(a, \theta)$ . The seller's cost of production and delivery is  $d(a, \theta)$ , and we let  $C(a, \theta) = -d(a, \theta)$ . Suppose, as one would typically do, that  $B$  is increasing and concave in  $a$  and that  $d$  is increasing and convex in  $a$ . If  $a$  is the buyer's action (he selects how many units to install, for example), then the buyer would be player 1 and so we have  $u_1 \equiv B$  and  $u_2 \equiv C$ . If the seller chooses  $a$  (she decides how many units to deliver, say), then the seller is player 1 and so we have  $u_1 \equiv C$  and  $u_2 \equiv B$ . In either case, Assumptions 2 and 3 are satisfied. Assumption 4 adds the weak supermodularity requirement on the payoff of the player who selects  $a$ .

We have the following robustness result:

**Theorem 2:** *Consider any contractual relationship that satisfies Assumptions 2-6. The sets of implementable value functions in the cases of unrestricted contracts with ex post renegotiation, forcing contracts with ex post renegotiation, and interim renegotiation are all distinct. That is,  $V^{\text{EPF}} \neq V^{\text{EP}} \neq V^{\text{I}}$ .*

<sup>17</sup>In Watson's (2007) example, which has two states and two trade actions,  $\bar{a}$  is the efficient trade action in both states.

The analysis underlying Theorem 2 amounts to characterizing and comparing the minimum punishment values that can be supported for each of the settings of interest. Recall that the punishment value for the ordered pair  $(\theta, \theta')$  is the value  $\psi_1(\theta) + \psi_2(\theta')$ , where  $\psi$  is the outcome specified in the message game when player 1 reports the state to be  $\theta'$  and player 2 reports the state to be  $\theta$ . Lower punishment values serve to relax incentive conditions, so to completely characterize the sets of implementable value functions we must find the minimum punishment values. We let  $P^I$ ,  $P^{EP}$ , and  $P^{EPF}$  denote the minimum punishment values for the settings of interim renegotiation, ex post renegotiation, and ex post renegotiation and forcing contracts, respectively:

$$\begin{aligned} P^I(\theta, \theta') &\equiv \min_{w \in W^F} w_1(\theta) + w_2(\theta'), \\ P^{EP}(\theta, \theta') &\equiv \min_{\hat{z} \in Z} \hat{z}_1(\theta) + \hat{z}_2(\theta'), \\ P^{EPF}(\theta, \theta') &\equiv \min_{\hat{z} \in Z^F} \hat{z}_1(\theta) + \hat{z}_2(\theta'). \end{aligned}$$

Our assumptions on the trade technology guarantee that these minima exist.

From Result 2, we know that Theorem 2 is equivalent to saying that there exist states  $\theta, \theta' \in \Theta$  such that  $P^I(\theta, \theta') < P^{EP}(\theta, \theta')$  and there exist (possibly different) states  $\theta, \theta' \in \Theta$  such that  $P^{EP}(\theta, \theta') < P^{EPF}(\theta, \theta')$ . Thus, to prove Theorem 2, we examine the punishment values achieved by various contractual specifications in the different settings. We develop some elements of the proof in the remainder of this section; Appendix B contains the rest of the analysis. We shall focus in this section on the relation between  $V^{EPF}$  and  $V^{EP}$ . The analysis of the relation between  $V^{EP}$  and  $V^I$  is considerably simpler and is wholly contained in Appendix B.

We will establish  $P^{EP} < P^{EPF}$  by comparing the punishment values implied by (i) the outcome in which player 1 would be forced to take a particular trade action (such as one that yields the lowest punishment value in this class), and (ii) a related non-forcing specification in which player 1 would be given the incentive to select some action  $a$  in state  $\theta$  and a different action  $a'$  in state  $\theta'$ . We derive conditions under which  $a$  and  $a'$  can be arranged to strictly lower the punishment value for  $(\theta, \theta')$ , relative to the best forcing case. We then find states  $\theta^1$  and  $\theta^2$  such that the conditions must hold for at least one of the ordered pairs  $(\theta^1, \theta^2)$  and  $(\theta^2, \theta^1)$ .

To explore the possible outcomes in the cases of ex post renegotiation, consider player 1's incentives at Date 6. For any given transfer function  $\hat{y}$ , the following are necessary conditions for player 1 to select trade action  $a$  in state  $\theta$  and action  $a'$  in state  $\theta'$ :

$$\begin{aligned} u_1(a, \theta) + \hat{y}_1(a) &\geq u_1(a', \theta) + \hat{y}_1(a') \quad \text{and} \\ u_1(a', \theta') + \hat{y}_1(a') &\geq u_1(a, \theta') + \hat{y}_1(a) \quad . \end{aligned} \tag{3}$$

Transfer function  $\hat{y}$  can be specified so that player 1 is harshly punished for selecting any trade action other than  $a$  or  $a'$ . Then, in every state, either  $a$  or  $a'$  maximizes player 1's payoff from Date 6. Thus, we have:

**Fact 1:** Consider two states  $\theta, \theta' \in \Theta$  and two trade actions  $a, a' \in A$ . Expression 3 is necessary and sufficient for the existence of a transfer function  $\hat{y} : A \rightarrow \mathbb{R}_0^2$  (defined over all trade actions) such that player 1's optimal trade action in state  $\theta$  is  $a$  and player 1's optimal trade action in state  $\theta'$  is  $a'$ .

Summing the inequalities of Expression 3, we see that there are values  $\hat{y}(a), \hat{y}(a') \in \mathbb{R}_0^2$  that satisfy (3) if and only if

$$u_1(a, \theta) - u_1(a', \theta) \geq u_1(a, \theta') - u_1(a', \theta'). \quad (4)$$

Assumption 4 then implies:

**Fact 2:** If  $\theta > \theta'$  then  $a \geq a'$  implies Inequality 4. If  $\theta < \theta'$  then  $a \leq a'$  implies Inequality 4.

Note that Fact 2 gives sufficient conditions. In the case in which  $u_1(\cdot, \cdot)$  is strictly supermodular (replacing weak inequalities in Assumption 4 with strict inequalities), player 1 can only be given the incentive to choose greater trade actions in higher states.

For any two states  $\theta, \theta' \in \Theta$ , define

$$E(\theta, \theta') \equiv \{(a, a') \in A \times A \mid \text{Inequality 4 is satisfied.}\}.$$

Also, for states  $\theta, \theta' \in \Theta$  and trade actions  $a, a' \in A$  with  $(a, a') \in E(\theta, \theta')$ , define

$$Y(a, a', \theta, \theta') \equiv \{\hat{y} : A \rightarrow \mathbb{R}_0^2 \mid \text{Condition 3 is satisfied.}\}.$$

Condition 3, combined with the identity  $\hat{y}_1 = -\hat{y}_2$ , implies:

**Fact 3:** For any  $\theta, \theta' \in \Theta$  and  $a, a' \in A$ , with  $(a, a') \in E(\theta, \theta')$ , we have

$$\min_{\hat{y} \in Y(a, a', \theta, \theta')} \hat{y}_1(a) + \hat{y}_2(a') = u_1(a', \theta) - u_1(a, \theta).$$

Using the definition of the set  $W$  (recall Expression 2 on page 16), any given  $w \in W$  can be written in terms of the trade actions and transfers that support it. We have

$$w(\theta) = u(\hat{a}(\theta), \theta) + \hat{y}(\hat{a}(\theta))$$

and

$$w(\theta') = u(\hat{a}(\theta'), \theta') + \hat{y}(\hat{a}(\theta')),$$

where  $\hat{a}$  gives player 1's choice of trade action as a function of the state and  $\hat{y}$  is the transfer function that supports  $w$ .

For any state  $\tilde{\theta}$  and trade action  $\tilde{a}$ , define  $R(\tilde{a}, \tilde{\theta})$  to be the renegotiation surplus if, without renegotiation, player 1 would select  $\tilde{a}$ . That is,  $R(\tilde{a}, \tilde{\theta}) = U(a^*(\tilde{\theta}), \tilde{\theta}) - U(\tilde{a}, \tilde{\theta})$ . Combining the expressions for  $w$  in the previous paragraph with Fact 1 and the definition of ex post renegotiation outcomes, we obtain:

**Fact 4:** Consider any two states  $\theta, \theta' \in \Theta$  and let  $\alpha$  be any number. There is an ex post renegotiation outcome  $z \in Z$  that satisfies  $z_1(\theta) + z_2(\theta') = \alpha$  if and only if there are trade actions  $a, a' \in A$  and a transfer function  $\hat{y}$  such that  $(a, a') \in E(\theta, \theta')$ ,  $\hat{y} \in Y(a, a', \theta, \theta')$ , and

$$\alpha = u_1(a, \theta) + \hat{y}_1(a) + \pi_1 R(a, \theta) + u_2(a', \theta') + \hat{y}_2(a') + \pi_2 R(a', \theta'). \quad (5)$$

In the last line, the first three terms are  $w_1(\theta)$  plus player 1's share of the renegotiation surplus in state  $\theta$ , totaling  $z_1(\theta)$ . The last three terms are  $w_2(\theta')$  plus player 2's share of the renegotiation surplus in state  $\theta'$ , totaling  $z_2(\theta')$ .

Finding the best (minimum) punishment value for states  $\theta$  and  $\theta'$  means minimizing  $\hat{z}_1(\theta) + \hat{z}_2(\theta')$  by choice of  $\hat{z} \in Z$ . For now, holding fixed the trade actions  $a$  and  $a'$  that player 1 is induced to select in states  $\theta$  and  $\theta'$ , let us minimize the punishment value by choice of  $\hat{y} \in Y(a, a', \theta, \theta')$ . To this end, we can use Fact 3 to substitute for  $\hat{y}_1(a) + \hat{y}_2(a')$  in Expression 5. This yields the punishment value for trade actions  $a$  and  $a'$  in states  $\theta$  and  $\theta'$ , respectively, written

$$\lambda(a, a', \theta, \theta') \equiv u_1(a', \theta) + \pi_1 R(a, \theta) + u_2(a', \theta') + \pi_2 R(a', \theta'). \quad (6)$$

Next, we consider the step of minimizing the punishment value by choice of the trade actions  $a$  and  $a'$ , which gives us a useful characterization of  $P^{\text{EP}}(\theta, \theta')$ . Assumption 2 guarantees that  $\lambda(a, a', \theta, \theta')$  has a minimum.

**Fact 5:** The minimum punishment value in the setting of ex post renegotiation is characterized as follows:

$$P^{\text{EP}}(\theta, \theta') = \min_{(a, a') \in E(\theta, \theta')} \lambda(a, a', \theta, \theta').$$

We obtain a similar characterization of the minimal punishment value for the setting in which attention is restricted to forcing contracts. The characterization is exactly as in Fact 5 except with the additional requirement that  $a = a'$  because forcing contracts compel the same action in every state.

**Fact 6:** The minimum punishment value for the setting of forcing contracts and ex post renegotiation is characterized as follows:

$$P^{\text{EPF}}(\theta, \theta') \equiv \min_{a \in A} \lambda(a, a, \theta, \theta').$$

Recall that proving Theorem 2 requires us to establish that  $P^{\text{EP}}(\theta, \theta') > P^{\text{EPF}}(\theta, \theta')$  for some pair of states  $\theta, \theta' \in \Theta$ . Appendix B finishes the analysis by exploring how one can depart from the optimal forcing specification in a way that strictly reduces the value  $\lambda(a, a', \theta, \theta')$ .



## 6 Conclusion

In this paper, we have reported on the analysis of contractual relationships for a large class of trade technologies. We have provided general results on the relation between individual-action and public-action models of contractual relationships, showing that limiting attention to forcing contracts has significant implications for implementability and hence inefficiency. Further, we have shown that (by utilizing non-forcing contracts) the payoff of the party with the trade action can be neutralized so that the other party claims the full benefit of the investment, gross of investment costs. This result led to the key novel insight of our analysis for applications, which is to identify the distinction between the *divided* and *unified* cases of investment and trade actions. We find that, in the setting of cross investment, the hold-up problem can be averted (and efficiency obtained) in the divided case but generally not in the unified case.

Our results reinforce the message of Watson (2007) on the usefulness of modeling trade actions as individual, particularly in settings of cross investment. The results suggest revisiting some of the conclusions of public-action models in the existing literature. In particular, settings with cross investment are generally not as problematic as previous modeling exercises (Che and Hausch 1999, Edlin and Hermalin 2000, and others) have found. Efficient outcomes can be achieved in the case of divided investment and trade actions. Our results show the importance, for applied work, of differentiating between the cases of divided and unified investment and trade actions. This distinction may be just as important as the distinction between own- and cross-investment (on which the literature has focused until now).

In our model, the trading opportunity is non-durable in that there is a single moment in time when trade can occur. One might wonder if the results differ substantially in settings with durable trading opportunities (where if trade does not occur at one time, then it can still be done at a later date). This issue has been explored by Evans (2008) and Watson and Wignall (2007), both of which examine individual-action models. Evans' (2008) elegant model is very general in terms of the available times at which the players can trade and renegotiate. He constructs equilibria in which, by having the players coordinate in different states on different equilibria in the infinite-horizon trade/negotiation game, the hold-up problem is partly or completely alleviated. Evans' strongest result (in which the efficient outcome is reached) requires the ability of the players to commit to a joint financial hostage; that is, money is deposited with a third party until trade occurs, if ever. Without the joint financial hostage, the efficient outcome may not be achieved.

Watson and Wignall (2007) examine a cross-investment setting without the possibility of joint financial hostages, and their model is more modest in other dimensions. They show that the set of implementable post-investment payoff vectors in the setting of a durable trading opportunity is essentially the same as in the setting of a non-durable trading opportunity. This suggests that, in general, the results from the current paper carry over to the durability setting. Watson and Wignall also show that, in the divided condition, there are non-stationary contracts that uniquely support the efficient outcome.

Our modeling exercise, combined with the recent literature, suggests some broad conclusions about the prospect of efficient investment and trade in contractual relationships. First, the hold-up problem is not necessarily severe, and efficient outcomes can often be achieved. Durability of the trading opportunity does not worsen the hold-up problem and may soften it in some cases, but it depends on the investment and trade technologies. Inefficiency may be unavoidable in the following problematic cases:

- when there is cross investment and unified investment and trade actions, as identified herein;
- when trade involves “complexity/ambivalence” as described by Segal (1999), Hart and Moore (1999), and Reiche (2006);
- when multiple parties make cross/cooperative investments; and
- when the investment conveys a significant direct benefit (not requiring trade) on the non-investing party, in addition to any benefit contingent on trade.

On the last point, Ellman’s (2006) model provides intuition in terms of the notion of specificity.

In each of the cases above, the hold-up problem would be reduced if the parties have some way of creating joint financial hostages, as explored by Evans (2008) and Baliga and Sjöström (2008). Bull (2009) provides a cautionary note on the inability of such financial arrangements to withstand side-contracting.

Regarding extensions of our analysis here, it may be useful to examine different classes of trade technologies, in particular ones in which both parties take trade actions (either simultaneously or sequentially). We expect our results to extend to such settings.<sup>18</sup> Perhaps more interesting would be to examine settings with partially verifiable trade actions. For example, a court may observe whether a particular trade was made but have trouble identifying which party disrupted trade (in the event that trade did not occur).<sup>19</sup>

Finally, recall that in the modeling exercise here, we have assumed that each party’s productive actions are exogenously given. However, in some settings it may be possible to arbitrarily assign a particular task (such as delivering an object from one place to another) to an individual player. Our model indicates that the parties would have preferences over task assignment. Thus, it would be useful to determine whether physical trade actions are assignable in some real settings, and to develop a model of optimal assignment. One might imagine a theory of firm boundaries that is based on the optimal assignment of different types of tasks over time.

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<sup>18</sup>The contract could force one of the players to select a specific trade action and give the other player an option as studied here. It would be interesting to work out how Assumption 4 would have to be modified to generate the same results.

<sup>19</sup>Hart and Moore’s (1988) model has this feature. It is straightforward to incorporate partial verifiability into the modeling framework developed here. One can represent the external enforcer’s information about the trading game as a partition of the space of action profiles. One can then simply assume that the contracted transfers  $y$  must be measurable with respect to this partition.

## A Proof of Theorem 1

This appendix provides a proof of the first theorem. For any fixed  $k$ , consider the following contract. In the message phase (Date 4), player 2 must declare the state. Let  $\hat{\theta}$  denote player 2's announcement. If player 1 subsequently selects action  $a^*(\hat{\theta})$  then the enforcer is to compel a transfer of  $\hat{t} = (k - u_1(a^*(\hat{\theta}), \hat{\theta}), u_1(a^*(\hat{\theta}), \hat{\theta}) - k)$ . If player 1 selects action  $\underline{a}$  then the transfer is  $\underline{t} = (k, -k)$ . If player 1 chooses any other trade action, then the enforcer compels transfer  $(-\tau, \tau)$ , where  $\tau$  is set large enough so that player 1 is forced to choose between  $a^*(\hat{\theta})$  and  $\underline{a}$ . That is, regardless of  $\hat{\theta}$ , in no state will player 1 have the incentive to choose  $a \notin \{a^*(\hat{\theta}), \underline{a}\}$ .

Suppose that Date 6 is reached without renegotiation and that the state is  $\theta$ . Note that, by Assumption 1, player 1 would get a payoff of  $k$  if he chooses  $\underline{a}$ . Alternatively, his payoff would be

$$u_1(a^*(\hat{\theta}), \theta) + k - u_1(a^*(\hat{\theta}), \hat{\theta})$$

if he chooses  $a^*(\hat{\theta})$ . Thus, it is rational for player 1 to choose  $a^*(\hat{\theta})$  if  $u_1(a^*(\hat{\theta}), \theta) \geq u_1(a^*(\hat{\theta}), \hat{\theta})$  and to select  $\underline{a}$  otherwise, which we suppose is how player 1 will behave.

Consider next how player 2's payoff from Date 4 depends on  $\hat{\theta}$ . Let  $\theta$  be the actual state and divide the analysis into three cases. First, if player 2 declares  $\hat{\theta} = \theta$  then, under the original contract, player 1 would choose  $a^*(\hat{\theta})$  at Date 6 and there is nothing to be jointly gained by renegotiating at Date 5. In this case, the payoffs from Date 4 are  $k$  for player 1 and

$$u_1(a^*(\theta), \theta) + u_2(a^*(\theta), \theta) - k = \gamma(\theta) - k$$

for player 2.

Second, if player 2 were to instead declare the state to be some  $\hat{\theta} \neq \theta$  such that  $u_1(a^*(\hat{\theta}), \theta) < u_1(a^*(\hat{\theta}), \hat{\theta})$ , then the players anticipate that player 1 would select  $\underline{a}$  at Date 6 under the original contract. Incorporating the impact of renegotiation at Date 5, player 1's payoff from Date 4 would then be  $k + \pi_1 R(\underline{a}, \theta)$ , where  $R(\underline{a}, \theta)$  is the renegotiation surplus in state  $\theta$  if, without renegotiation, the players anticipate that  $\underline{a}$  will be the chosen trade action. Since  $R(\underline{a}, \theta) \geq 0$ , player 1's payoff from Date 4 weakly exceeds  $k$  and we conclude that player 2's payoff is weakly less than  $\gamma(\theta) - k$ .

Finally, suppose that player 2 were to declare the state to be  $\hat{\theta} \neq \theta$  such that  $u_1(a^*(\hat{\theta}), \theta) > u_1(a^*(\hat{\theta}), \hat{\theta})$ . In this case, the players anticipate that player 1 would select  $a^*(\hat{\theta})$  at Date 6 under the original contract. Incorporating renegotiation at Date 5, player 1's payoff from Date 4 would then be

$$u_1(a^*(\hat{\theta}), \theta) + k - u_1(a^*(\hat{\theta}), \hat{\theta}) + \pi_1 R(a^*(\hat{\theta}), \theta),$$

where  $R(a^*(\hat{\theta}), \theta)$  is the renegotiation surplus in state  $\theta$  if, without renegotiation, the players anticipate that  $a^*(\hat{\theta})$  will be the chosen trade action. The first and third terms sum to weakly more than zero, so the entire expression weakly exceeds  $k$ . This implies that player 2's payoff is weakly less than  $\gamma(\theta) - k$ .

We have shown that player 2 optimally tells the truth at Date 4; that is, she declares  $\hat{\theta} = \theta$ . The payoffs from Date 3 are thus  $k$  for player 1 and  $\gamma(\theta) - k$  for player 2, which means that the contract implements the desired value function. *Q.E.D.*

## B Proof of Theorem 2

In this appendix, we complete the proof of Theorem 2. We start with the comparison of  $V^{\text{EPF}}$  and  $V^{\text{EP}}$  and then provide the analysis for the comparison of  $V^{\text{EP}}$  and  $V^1$ .

### Completion of the Proof that $V^{\text{EPF}} \neq V^{\text{EP}}$

We pick up from the analysis at the end of Section 5. Consider a pair of states  $\theta^1, \theta^2$  that satisfies Assumption 5. That is, we have  $\theta^1 > \theta^2$  and either  $U(\underline{a}, \theta^2) < U(\bar{a}, \theta^2)$  or  $U(\underline{a}, \theta^1) > U(\bar{a}, \theta^1)$ . Let  $b^1$  denote a solution to the forcing-contract problem

$$\min_{a \in A} \lambda(a, a, \theta^1, \theta^2)$$

and let  $b^2$  denote a solution to the forcing-contract problem

$$\min_{a \in A} \lambda(a, a, \theta^2, \theta^1).$$

We shall demonstrate that either  $P^{\text{EP}}(\theta^1, \theta^2) < P^{\text{EPF}}(\theta^1, \theta^2)$  or  $P^{\text{EP}}(\theta^2, \theta^1) < P^{\text{EPF}}(\theta^2, \theta^1)$ , or both, which implies that  $V^{\text{EPF}} \neq V^{\text{EP}}$ .

Let us evaluate the minimum punishment value corresponding to the ordered pair of states  $(\theta^1, \theta^2)$ . Specifically, compare the optimal forcing contract punishment (forcing player 1 to select  $b^1$  in both states) with a non-forcing specification in which player 1 is induced to select  $b^1$  in state  $\theta^1$  and  $\underline{a}$  in state  $\theta^2$ . This is a valid non-forcing contractual specification because, by Fact 2,  $\theta^1 > \theta^2$  and  $b^1 \geq \underline{a}$  imply  $(b^1, \underline{a}) \in E(\theta^1, \theta^2)$ .

If  $V^{\text{EP}} = V^{\text{EPF}}$  then it must be that  $\lambda(b^1, b^1, \theta^1, \theta^2) \leq \lambda(b^1, \underline{a}, \theta^1, \theta^2)$ . Applying the definition of  $\lambda$ , this is

$$\begin{aligned} u_1(b^1, \theta^1) + \pi_1 R(b^1, \theta^1) + u_2(b^1, \theta^2) + \pi_2 R(b^1, \theta^2) \\ \leq u_1(\underline{a}, \theta^1) + \pi_1 R(b^1, \theta^1) + u_2(\underline{a}, \theta^2) + \pi_2 R(\underline{a}, \theta^2). \end{aligned}$$

Canceling the second term on each side and using the definition of  $R$ , we arrive at

$$u_1(b^1, \theta^1) + u_2(b^1, \theta^2) - \pi_2 U(b^1, \theta^2) \leq u_1(\underline{a}, \theta^1) + u_2(\underline{a}, \theta^2) - \pi_2 U(\underline{a}, \theta^2).$$

Substituting  $u_2(\cdot, \theta^2) = U(\cdot, \theta^2) - u_1(\cdot, \theta^2)$  on both sides, we have

$$\begin{aligned} u_1(b^1, \theta^1) + U(b^1, \theta^2) - u_1(b^1, \theta^2) - \pi_2 U(b^1, \theta^2) \\ \leq u_1(\underline{a}, \theta^1) + U(\underline{a}, \theta^2) - u_1(\underline{a}, \theta^2) - \pi_2 U(\underline{a}, \theta^2). \end{aligned}$$

Finally, rearranging this expression a bit and using  $\pi_1 + \pi_2 = 1$ , we conclude that  $\lambda(b^1, b^1, \theta^1, \theta^2) \leq \lambda(b^1, \underline{a}, \theta^1, \theta^2)$  is equivalent to

$$u_1(b^1, \theta^1) - u_1(\underline{a}, \theta^1) - [u_1(b^1, \theta^2) - u_1(\underline{a}, \theta^2)] \leq \pi_1[U(\underline{a}, \theta^2) - U(b^1, \theta^2)]. \quad (7)$$

Similarly, ordering states  $\theta^1$  and  $\theta^2$  in the opposite way, we compare the optimal forcing contract punishment (forcing player 1 to select  $b^2$  in both states) with a non-forcing specification in which player 1 is induced to select  $b^2$  in state  $\theta^2$  and  $\bar{a}$  in state  $\theta^1$ . Note that  $\theta^2 < \theta^1$  and  $b^2 \leq \bar{a}$  imply  $(b^2, \bar{a}) \in E(\theta^2, \theta^1)$ . If  $V^{\text{EP}} = V^{\text{EPF}}$  then it must be that  $\lambda(b^2, b^2, \theta^2, \theta^1) \leq \lambda(b^2, \bar{a}, \theta^2, \theta^1)$ , which similar algebraic manipulation reveals to be equivalent to

$$u_1(\bar{a}, \theta^1) - u_1(b^2, \theta^1) - [u_1(\bar{a}, \theta^2) - u_1(b^2, \theta^2)] \leq \pi_1[U(\bar{a}, \theta^1) - U(b^2, \theta^1)]. \quad (8)$$

We have shown that if  $V^{\text{EPF}} = V^{\text{EP}}$ , then Expressions 7 and 8 hold. Assumption 4 then implies that the left sides of these inequalities are non-negative, which implies

$$U(\underline{a}, \theta^2) \geq U(b^1, \theta^2) \quad \text{and} \quad U(\bar{a}, \theta^1) \geq U(b^2, \theta^1).$$

Using Assumption 3, we obtain:

**Fact 7:** *If  $V^{\text{EPF}} = V^{\text{EP}}$  then  $U(\underline{a}, \theta^2) \geq U(\bar{a}, \theta^2)$  and  $U(\bar{a}, \theta^1) \geq U(\underline{a}, \theta^1)$ .*

Assumption 5 and the contrapositive of Fact 7 provide the contradiction that proves  $V^{\text{EPF}} \neq V^{\text{EP}}$ .

### Proof that $V^{\text{EP}} \neq V^{\text{I}}$

We next prove the claim about the relation between  $V^{\text{I}}$  and  $V^{\text{EP}}$ . Since forcing contracts are sufficient to construct  $V^{\text{I}}$ , we have:

**Fact 8:** *The minimum punishment value in the setting of interim renegotiation is characterized as follows:*

$$P^{\text{I}}(\theta, \theta') = \min_{a'' \in A} u_1(a'', \theta) + u_2(a'', \theta').$$

Remember that, by Result 2,  $V^{\text{I}} = V^{\text{EP}}$  if and only if  $P^{\text{EP}}(\theta, \theta') = P^{\text{I}}(\theta, \theta')$  for all  $\theta, \theta' \in \Theta$ . We can again compare the minimization problems to determine if this is the case.

Take  $\theta^1, \theta^2$  satisfying Assumption 5. Consider any solution to the minimization problem that defines  $P^{\text{EP}}(\theta^1, \theta^2)$  and denote it  $(b, b')$ . That is,  $(b, b')$  solves

$$\min_{(a, a') \in E(\theta^1, \theta^2)} u_1(a', \theta^1) + \pi_1 R(a, \theta^1) + u_2(a', \theta^2) + \pi_2 R(a', \theta^2).$$

Then  $P^{\text{EP}}(\theta^1, \theta^2) = P^{\text{I}}(\theta^1, \theta^2)$  is equivalent to

$$u_1(b', \theta^1) + \pi_1 R(b, \theta^1) + u_2(b', \theta^2) + \pi_2 R(b', \theta^2) = \min_{a'' \in A} u_1(a'', \theta^1) + u_2(a'', \theta^2).$$

Because  $R(\cdot, \cdot) \geq 0$ , we see that  $P^{\text{EP}}(\theta^1, \theta^2) = P^{\text{I}}(\theta^1, \theta^2)$  only if  $b'$  solves the minimization problem on the right side of the above equation and also  $R(b, \theta^1) = R(b', \theta^2) = 0$ .

By Assumption 3,  $R(b', \theta^2) = 0$  if and only if  $b' = a^*(\theta^2)$ . Combining this with the requirement that  $b'$  must minimize  $u_1(\cdot, \theta^1) + u_2(\cdot, \theta^2)$ , we derive that

$$u_1(a^*(\theta^2), \theta^1) + u_2(a^*(\theta^2), \theta^2) \leq u_1(a'', \theta^1) + u_2(a'', \theta^2)$$

for all  $a''$ . In particular, the following inequality must hold:

$$u_1(a^*(\theta^2), \theta^1) + u_2(a^*(\theta^2), \theta^2) \leq u_1(\underline{a}, \theta^1) + u_2(\underline{a}, \theta^2).$$

Using the identity  $u_2 = U - u_1$  and rearranging terms, we see that this is equivalent to

$$\begin{aligned} u_1(a^*(\theta^2), \theta^1) - u_1(\underline{a}, \theta^1) - [u_1(a^*(\theta^2), \theta^2) - u_1(\underline{a}, \theta^2)] \\ \leq U(\underline{a}, \theta^2) - U(a^*(\theta^2), \theta^2). \end{aligned} \quad (9)$$

Similarly, ordering states  $\theta^1$  and  $\theta^2$  in the opposite way, it is necessary that  $a^*(\theta^1)$  must solve  $P^{\text{I}}(\theta^2, \theta^1)$  in order for  $P^{\text{EP}}(\theta^2, \theta^1) = P^{\text{I}}(\theta^2, \theta^1)$ . In particular, we must have

$$u_1(a^*(\theta^1), \theta^2) + u_2(a^*(\theta^1), \theta^1) \leq u_1(\bar{a}, \theta^2) + u_2(\bar{a}, \theta^1).$$

This inequality is equivalent to

$$\begin{aligned} u_1(\bar{a}, \theta^1) - u_1(a^*(\theta^1), \theta^1) - [u_1(\bar{a}, \theta^2) - u_1(a^*(\theta^1), \theta^2)] \\ \leq U(\bar{a}, \theta^1) - U(a^*(\theta^1), \theta^1). \end{aligned} \quad (10)$$

By Assumption 4, the left sides of Expressions 9 and 10 must be non-negative, which implies both  $U(\underline{a}, \theta^2) \geq U(a^*(\theta^2), \theta^2)$  and  $U(\bar{a}, \theta^1) \geq U(a^*(\theta^1), \theta^1)$ . From Assumption 3, we see that this is only possible if  $\underline{a} = a^*(\theta^2)$  and  $\bar{a} = a^*(\theta^1)$ . If this is the case, Assumption 3 also implies that  $U(\underline{a}, \theta^2) \geq U(\bar{a}, \theta^2)$  and  $U(\bar{a}, \theta^1) \geq U(\underline{a}, \theta^1)$ . Thus we obtain:

**Fact 9:** *If  $V^{\text{I}} = V^{\text{EP}}$  then  $U(\underline{a}, \theta^2) \geq U(\bar{a}, \theta^2)$  and  $U(\bar{a}, \theta^1) \geq U(\underline{a}, \theta^1)$ .*

The contrapositive of Fact 9 combined with Assumption 5 provides the contradiction that proves  $V^{\text{I}} \neq V^{\text{EP}}$ . *Q.E.D.*

## C Additional Analysis

In this appendix, we explore whether there are implementable value functions that hold player 2's payoff constant in the state, which would be required to extend the Corollary in Section 3 to the unified case. We also provide more analysis of cross-investment settings.

### Making Player 1 the Residual Claimant

To make player 1 the residual claimant, we need to implement a value function  $v$  satisfying, for some constant  $k$ ,  $v_2(\theta) = k$  and  $v_1(\theta) = \gamma(\theta) - k$  for all  $\theta \in \Theta$ . Consider two states  $\theta$  and  $\theta'$ , and order them so that  $\theta > \theta'$ . The conditions for implementation associated with these two states (for  $(\theta, \theta')$  and  $(\theta', \theta)$ ) are

$$v_1(\theta) + v_2(\theta') \geq P^{EP}(\theta, \theta')$$

and

$$v_1(\theta') + v_2(\theta) \geq P^{EP}(\theta', \theta).$$

Using Fact 5 from Section 5, these conditions are equivalent to the existence of trade actions  $a, a', b, b'$  such that  $(a, a') \in E(\theta, \theta')$ ,  $(b', b) \in E(\theta', \theta)$ ,

$$v_1(\theta) + v_2(\theta') \geq \lambda(a, a', \theta, \theta')$$

and

$$v_1(\theta') + v_2(\theta) \geq \lambda(b', b, \theta', \theta).$$

Substituting for  $v_1$  and  $v_2$  using the identities  $v_2(\theta) = k$  and  $v_1(\theta) = \gamma(\theta) - k$ , these two inequalities become:

$$\lambda(a, a', \theta, \theta') \leq \gamma(\theta) \tag{11}$$

and

$$\lambda(b', b, \theta', \theta) \leq \gamma(\theta'). \tag{12}$$

Summarizing, we have:

**Lemma:** *Consider any contractual relationship that satisfies Assumptions 2 and 4. Let  $k$  be any real number and define value function  $v$  by  $v_2(\theta) = k$  and  $v_1(\theta) = \gamma(\theta) - k$  for all  $\theta \in \Theta$ . Then  $v \in V^{EP}$  if and only if for all pairs of states  $\theta, \theta'$  with  $\theta > \theta'$ , there are trade actions  $a, a', b, b'$  such that  $(a, a') \in E(\theta, \theta')$ ,  $(b', b) \in E(\theta', \theta)$ , and Inequalities 11 and 12 hold.*

One can use these conditions to establish whether efficient investment can be obtained in specific examples with unified investment and trade actions, but sufficient conditions would be much stronger than are the assumptions we have made in this paper.

For an illustration of cases where the conditions of the Lemma fail, suppose that the strict version of Assumption 4 is satisfied, meaning  $u_1$  is strictly supermodular. Further suppose that Assumptions 2, 3, and 6 hold. Also suppose that  $U$  is strictly increasing in  $\theta$

and that  $U(\bar{a}, \bar{\theta}) > \gamma(\underline{\theta})$ . That is, the joint value of the highest trade action in the highest state exceeds the maximal joint value in the lowest state (gross of investment cost).

Using Equation 6,  $U = u_1 + u_2$ , and some algebra, we can rewrite Inequality 12 as:

$$\pi_1[U(b, \theta) - U(b', \theta')] \leq \pi_2[\gamma(\theta') - \gamma(\theta)] - [u_1(b, \theta') - u_1(b, \theta)].$$

Examining the case of  $\theta = \bar{\theta}$  and  $\theta' = \underline{\theta}$ , this becomes

$$\pi_1[U(b, \bar{\theta}) - U(b', \underline{\theta})] \leq \pi_2[\gamma(\underline{\theta}) - \gamma(\bar{\theta})] - [u_1(b, \underline{\theta}) - u_1(b, \bar{\theta})]. \quad (13)$$

Because  $u_1$  is strictly supermodular,  $b \geq b'$  is required. From Assumption 3, that  $U(\bar{a}, \bar{\theta}) > \gamma(\underline{\theta})$ , and that  $U$  is strictly increasing in  $\theta$ , we conclude that the left side of Inequality 13 is strictly positive and bounded away from zero.<sup>20</sup> We also have that the first bracketed term on the right side is strictly negative.

Thus, if  $|u_1(b, \underline{\theta}) - u_1(b, \bar{\theta})|$  is small relative to  $\pi_2|\gamma(\underline{\theta}) - \gamma(\bar{\theta})|$ , then Inequality 13 fails to hold and there is no way to implement value functions that make player 2's payoff constant in the state. In other words, in the case of unified investment and trade actions, the first-best level of investment generally cannot be induced.

### More on Cross-Investment

This part of the appendix continues the discussion of Section 3. Consider the unified case with ex post renegotiation and cross-investment, where player 2's gain from trade is  $B(a, \theta)$  and player 1's (the investor's) gain is  $C(a, \theta)$ . We focus on the unified case because the divided case is solved by Theorem 1. Suppose that the cross-investment element is strong relative to the own-investment element, so that Che and Hausch's (1999) result on forcing contracts holds—that is, the null contract is optimal among forcing contracts.

We look at two subcases: *pure cross-investment*, where  $C(a, \theta)$  is constant in the state  $\theta$ , and *near pure cross investment*, where  $C$  varies in  $\theta$  only slightly compared to how  $B$  varies in  $\theta$ .

Let us begin with the pure cross-investment case. Investment is best motivated by making  $v_1(\theta) - v_1(\theta')$  large for  $\theta > \theta'$ , requiring  $v_1(\theta') + v_2(\theta)$  to be low. Thus we want the punishment value  $\hat{z}_1(\theta') + \hat{z}_2(\theta) = \lambda(a', a, \theta', \theta)$  to be small. Here a non-forcing contract can provide better investment incentives if and only if there exists  $a, a'$  such that

$$\lambda(a', a, \theta', \theta) < \lambda(0, 0, \theta', \theta).$$

Expanding this inequality using Expression 6, we have

$$\begin{aligned} u_1(a, \theta') + \pi_1 R(a', \theta') + u_2(a, \theta) + \pi_2 R(a, \theta) \\ < u_1(0, \theta') + \pi_1 R(0, \theta') + u_2(0, \theta) + \pi_2 R(0, \theta). \end{aligned}$$

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<sup>20</sup>To see this, consider two cases. If  $U(a, \bar{\theta}) \geq U(\bar{a}, \bar{\theta})$ , because  $U$  is strictly quasiconcave in  $a$ , every point on the graph of  $U(\cdot, \bar{\theta})$  is above every point on the graph of  $U(\cdot, \underline{\theta})$  and so the result is immediate. If  $U(a, \bar{\theta}) < U(\bar{a}, \bar{\theta})$ ,  $U$  strictly increasing in  $\theta$  implies that the result holds over the range  $[\underline{a}, a^*(\bar{\theta})]$ . Over the range  $[a^*(\bar{\theta}), \bar{a}]$ , the problem reduces to the first case.



This reduces to

$$u_1(a, \theta') + \pi_1 R(a', \theta') + u_2(a, \theta) + \pi_2 R(a, \theta) < \pi_1 R(0, \theta') + \pi_2 R(0, \theta).$$

Using the definition of renegotiation surplus, this inequality becomes

$$\begin{aligned} u_1(a, \theta') + \pi_1 U(a^*(\theta'), \theta') - \pi_1 U(a', \theta') + u_2(a, \theta) + \pi_2 U(a^*(\theta), \theta) - \pi_2 U(a, \theta) \\ < \pi_1 U(a^*(\theta'), \theta') - \pi_1 U(0, \theta') + \pi_2 U(a^*(\theta), \theta) - \pi_2 U(0, \theta). \end{aligned}$$

Simplifying yields

$$u_1(a, \theta') - \pi_1 U(a', \theta') + u_2(a, \theta) - \pi_2 U(a, \theta) < -\pi_1 U(0, \theta') - \pi_2 U(0, \theta),$$

and further

$$u_1(a, \theta') + u_2(a, \theta) - \pi_2 U(a, \theta) < \pi_1 U(a', \theta') \tag{14}$$

Because player 1's utility is constant in the state (and thus not strictly supermodular), we are not bound by the constraint that  $a$  must be at least as high as  $a'$ . Therefore an implementable non-forcing contract that will ensure that Expression 14 holds is  $a' = a^*(\theta')$  and  $a = 0$ . This makes the left hand side zero, while the right hand side is positive by Assumption 3 as long as  $a^*(\theta') > 0$  and so a non-forcing contract can improve on the best forcing contract just as in the divided case.

Next take the case of near pure cross-investment, so that the investor's (player 1's) trade utility  $C(a, \theta)$  depends only a bit on the state  $\theta$ . For example, we could have  $C(a, \theta) = \varepsilon a \theta$  where  $\varepsilon$  is a constant that is close to zero. Results will depend on whether  $\varepsilon$  is positive or negative.

If the investment is such that the beneficiary receives more than the total benefit created by the investment (so  $\varepsilon < 0$ ), then the investor's trade utility becomes submodular in  $\theta$ , just as in our example. In the unified case, in order to satisfy Theorem 2's assumption of weak supermodularity, we simply reverse the action space: The investor's utility function is then weakly supermodular in  $(-a, \theta)$ . Just as in the case of pure cross investment, it is clear that there is a feasible option contract in which the investor will select  $a^*(\theta')$  in state  $\theta'$  and 0 in state  $\theta$ .<sup>21</sup> This implies that, for investment incentives, a non-forcing contract can improve on a forcing contract.

Next suppose that the beneficiary receives less than the total benefit created by the investment (so  $\varepsilon > 0$ ). The utility function of the investing party (player 1) is strictly supermodular, and so a non-forcing contract can only induce  $a \geq a'$  for two states  $\theta, \theta'$  with  $\theta > \theta'$ . Thus, it is not possible to induce player 1 to choose 0 in state  $\theta$  and  $a^*(\theta')$  in state  $\theta'$ . As a result, Expression 14 will not hold in general, and whenever this condition fails, the incentives provided by the forcing contract cannot be improved upon using a non-forcing contract.

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<sup>21</sup>Recall that  $(a^*(\theta), 0) \in E(\theta', \theta)$  is required.

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