UC Berkeley
Other Recent Work

Title
Explicit Tests of Contingent Claims Models of Mortgage Defaults

Permalink
https://escholarship.org/uc/item/3df5357v

Author
Quigley, John M.

Publication Date
1993-11-01

Peer reviewed

UNIVERSITY OF CALIFORNIA AT BERKELEY

Department of Economics

Berkeley, California 94720

Explicit Tests of Contingent Claims Models
of Mortgage Defaults

John M. Quigley

Department of Economics
University of California at Berkeley

November 1993

Key words: contingent claims, homeowner equity, mortgage default, options models

Abstract

This paper provides explicit and powerful tests of contingent claims approaches to modelling mortgage default. We investigate a model of "frictionless" default (i.e., one in which transactions costs, reputation costs and moving costs play no role) and analyze its implications -- the relationship between equity and default, the timing of default, its dependence upon initial conditions, and the severity of losses. Absent transactions costs and other market imperfections, economic theory makes well-defined predictions about these various outcomes.

The empirical analysis is based upon two particularly rich bodies of micro data: one indicating the default and loss experience of all mortgages purchased by the Federal Home Mortgage Corporation (Freddie Mac); and a large sample of all repeat sales of single family houses whose mortgages were purchased by Freddie mac since 1976.

JEL Classification: G1, D1

equilibrium condition for M (a second order partial differential equation), specifying that the expected return on the security (that is, the coupon return plus capital gains) must equal the risk-free rate of return plus a risk adjustment, or, alternately, that the coupon plus risk adjusted capital gains equal the risk-free rate. This condition applies to any claim that is contingent on the underlying state variables; again it has the interpretation that the value of the mortgage equals the risk-adjusted expected present value of its net cash flows.

To simplify matters and to isolate the default option, assume that interest rates are non-stochastic (so that the only source of risk is house price volatility). Assume that house price changes are continuous with an instantaneous mean μ (which need not be constant) and a standard deviation σ. Let ρ be the imputed rent payout ("dividend") rate. It is well known that the arbitrage model implies that the value of the mortgage M satisfies

(1) $(1/2)M^2\sigma^2(\partial^2 M/\partial V^2) + M(i-\rho)(\partial M/\partial V) + (\partial M/\partial t) + C = iM,$

where i is the instantaneous interest rate and C is the coupon payment on the mortgage (which depends on the coupon rate, c). This follows almost directly from the analysis of Black and Scholes [1973]. Kau, et al. [1991] provide a generalized version of this equation in the case where

interest rates are stochastic. Equation (1) states that the coupon return plus the risk-adjusted expected capital gains, where the risk-adjusted mean price is (i-p), must equal the risk free rate.

Note that the expected appreciation rate μ of the house does not appear in equation (1), nor does the risk premium λ for holding the asset. In general, if the underlying state variables are traded assets, then arbitrage leads to a risk-neutral interpretation of the price of a contingent claim on an asset relative to the price of that asset, and the value of the option is the expected present value of the outcome, where prices are projected to grow at a mean rate of $i-\rho$ (and variance $\sigma^2 t$) and are discounted at the risk-free rate. This is equivalent to assuming risk neutrality (See Brennan and Schwartz [1985] and Kau *et al.* [1986] for applications to mortgages, Smith [1976] for a general discussion and Cox, Ingersoll and Ross [1985] for a proof for the expected present value interpretation).

An infinite number of functions satisfy (1) (depending on boundary conditions), which reflects the infinite number of ways that coupon plus capital gain can equal the required return. By incorporating the optimal exercise strategies, the function appropriate for a particular mortgage is determined.

In the "frictionless" model, which we define as one in which there are no costs to default other than losing the

house, the optimal default strategy, given t, is characterized simply by the house value $V_t{}^*$, at which default takes place. The optimal $V_t{}^*$ minimizes the value of the mortgage (this maximizes the borrower's net worth), subject to the condition that $V_t{}^*$ equal the value of the remaining balance when the option is exercised.

Figure 1 (adapted from Quigley and Van Order [forthcoming]), illustrates the optimal strategy. This strategy is represented by the lowest curve which satisfies equation (1) and is not above the 45 degree line (where the remaining balance equals the value of the house). If the solution is an interior one, it is represented by the tangency depicted in the figure. The curve must also be below the horizontal line M, which gives the value of a riskless mortgage. The curve approaches M asymptotically as V increases. The tangency determines $V_t{}^*$, the default "strategy." The entire curve gives the market relationship between mortgage values and house values. The distance X between M and the mortgage value is the value of the default option, the premium for insurance that a competitive mortgage insurer would charge. At $V_t{}^*$ the distance S (=X) represents the extent to which the option must be in the money before default. The distance is also the amount lost by the lender or mortgage insurer (absent transactions costs) from selling the house after foreclosure.

FIGURE 1

Optimal Default
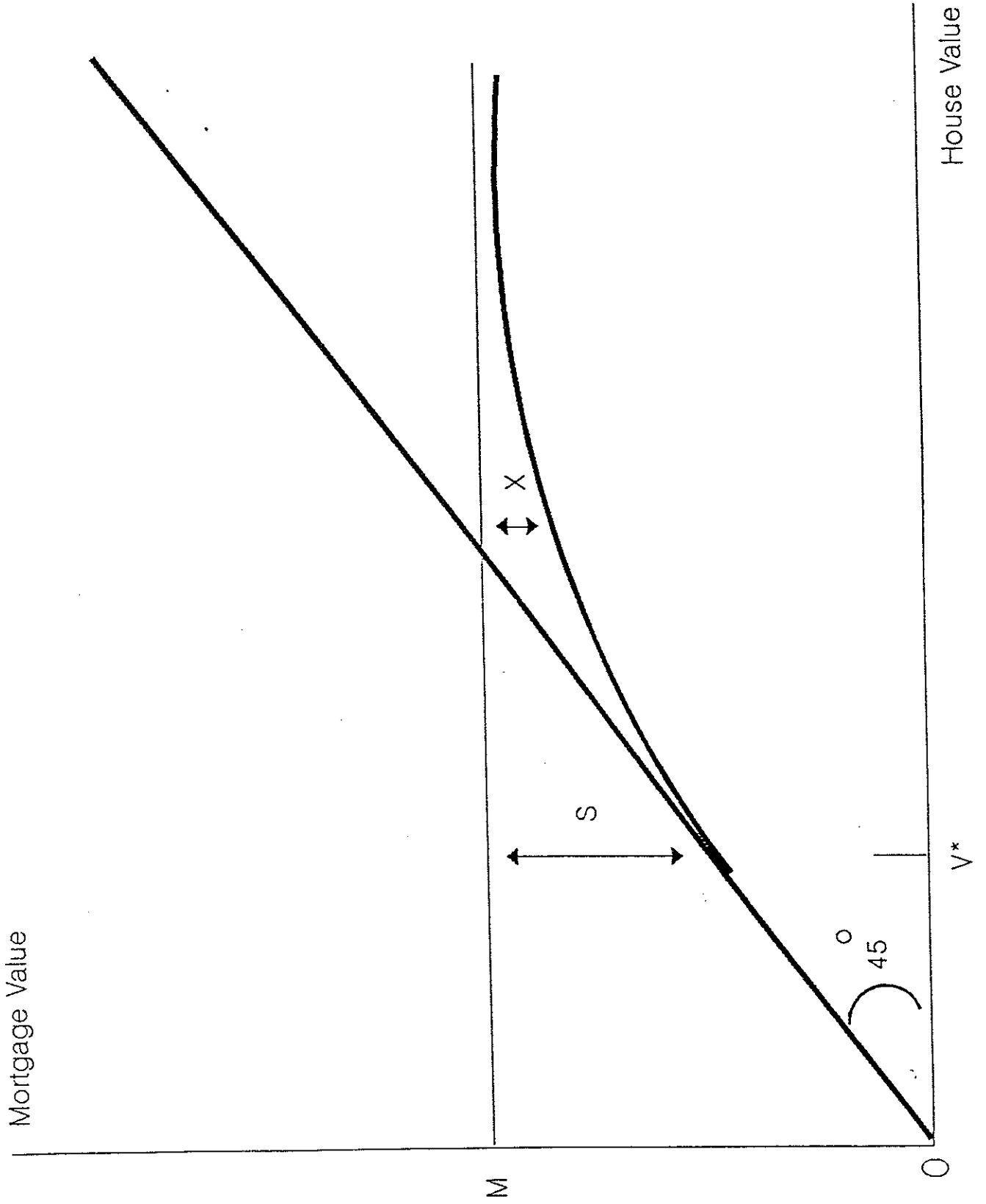
The virtue of the contingent claims approach is its simplicity. The default option is exercised at V_t^*, which depends only on the variables in (1) and on the boundary and tangency conditions. The equilibrium condition has the property that the mean price change of any traded asset as well as the risk premium are irrelevant in pricing the option or in exercising it. Thus, circumstances under which default occurs depend only on i, ρ, σ, c, t, M and V; they are independent of the original house price, expected price appreciation, the original loan-to-value ratio (LTV) and the historic path of prices. Loss severities (measured by S in Figure 1) have these same properties.

The model also has implications about default frequencies. These are more complicated than those about loss severities. This is because, although the value of the default option is independent of expected inflation, estimating the probability of exercise also requires estimating expected inflation (See Kau et al. [1991]).

Numerical solutions to these differential equations reveal the optimal default frequency implied by the frictionless model.[3] For example, Kau et al [1991] show that it is typically optimal to wait until the default option is

---

3   The forward-looking aspects of the option pricing problem mean that the solution to the differential equation is solved numerically by working backwards from the terminal conditions.

well into the money before actually defaulting. Indeed, they present an example where S is more than 10 percent of the mortgage balance at optimal exercise. They also simulate cumulative expected default rates by LTV, given a variety of initial parameters. These simulations of optimal behavior are not substantially different from casual empiricism about observed default frequencies. Introducing transactions costs, in the form of a cost of exercising the option, appears to make default frequencies implausibly low. Thus, the authors conclude that research which rejects the frictionless model, simply because people with negative equity do not default frequently, is misleading.

We explicitly test the frictionless model by analyzing the predictions discussed above. We begin by estimating a hazard model which specifies default as a function of the extent to which the option is "in the money." The parameters of the model can be used to simulate default frequencies. We can thus test whether simulated default behavior differs from ruthless behavior -- in terms of variations in initial LTV and variations over time. We then analyze loss severities.

III. HOUSING EQUITY AND DEFAULT BEHAVIOR

The option approach focuses on equity as the major determinant of default. That default rates are higher for high LTV loans is one of the major proposition that default

research has investigated. Before turning to that we consider a second proposition of the option model: not only should high LTV loans default more frequently, they should default sooner.

Put simply: for any price-generating process it takes longer on average (*i.e.*, it takes more draws from the price distribution) for a low LTV loan to get into the money than for a high LTV loan. Conditional default rates (or hazard rates) are about zero shortly after loan origination (because there is virtually no chance that the option will be in the money after a few draws). They will tend to rise after origination. If there is inflation, the time profile of default rates will tend to peak, because after some period inflation will make negative equity quite improbable. If there is a peak in the curve, it should be earlier for high LTV loans. In any event, as discussed in Kau *et al* [1991], high LTV loans should have shorter average times to default.

To illustrate, suppose house prices are lognormally distributed with mean pt and variance $\sigma^2 t$. In this case,[4] the probability that a house has LTV above some critical level $\ell*$ is $F[(pt-\log\{\ell*\})/\sigma t^{1/2}]$ where F is the cumulative normal. Assuming that $\ell*$ is constant (i.e., that the loan is perpetual rather than self-amortizing) and differentiating reveals that the probability that LTV is less than $\ell*$ peaks when

---

4 Note that in our empirical analysis (reported subsequently in footnote 6), house prices are assumed to be lognormally distributed with variance a function of t and t^2.

t=[log{ℓ*}]/p.   For given price appreciation, the peak in the hazard rate is later for lower LTVs.   For p=0.035, t is about 5 years for an 85 percent LTV loan, (ℓ*=0.85), and is about 10 years for ℓ*=0.70.   The numerical analysis of Kau *et al* [1991] quantifies and qualifies this relationship and indicates that average time to default (i.e., expected duration conditional on default) varies inversely with initial LTV.

Figure 2 presents simple hazard functions by initial LTV, computed from unadjusted Freddie Mac data aggregated over origination years 1975 through 1983.   While the picture clearly shows that default depends on LTV, the timing of the peak hazard is quite similar for all LTV categories.   This latter behavior is inconsistent with the frictionless model.

We re-estimated the separate hazard models reported in Figure 2 to control for origination year as well as LTV.   We used these parameters to calculate average time to default (for loans that defaulted) by LTV, holding origination year constant.   The results were 7.5 years, 7.4 years and 7.6 years for initial LTVs of less-than-80 percent, 80-to-90 percent, and greater-than-90 percent, respectively.   Hence, while the simple analytics confirm the prediction that high LTV loans default more frequently, they also reject the hypothesis that high LTV loans default sooner.   We next turn to a more rigorous model of the determinants of default.

Our empirical model of default is based upon the behavior

12

FIGURE 2

MEAN HAZARD RATE
LTV CLASSES 80, 90, 94, AND 95 PERCENT

SOURCE: DEV. CNTL (PLTHAZ3)
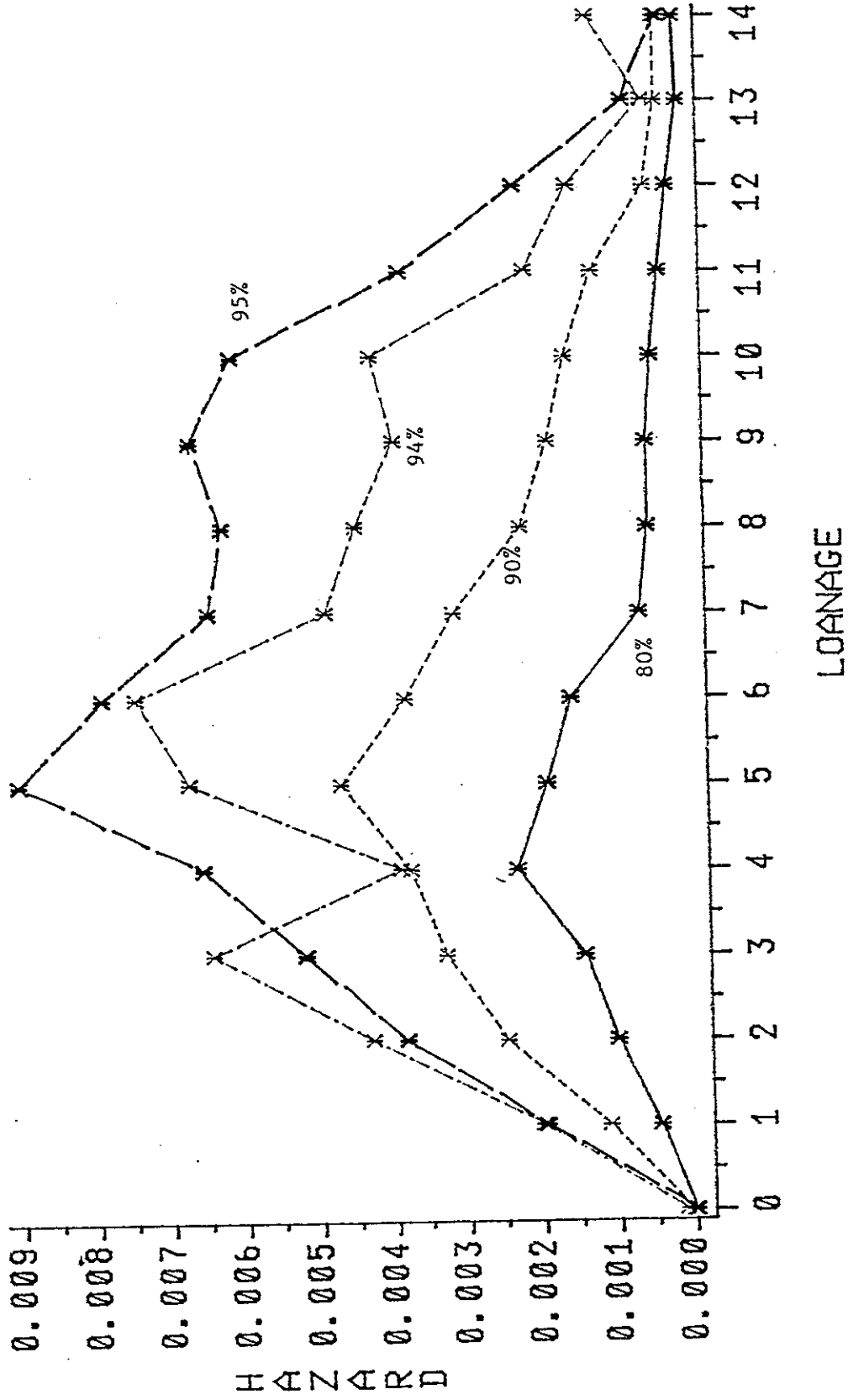
Our empirical model of default is based upon the behavior of a random sample of the holders of mortgage contracts, issued between 1976 and 1980 and bought by Freddie Mac, whose default experience is followed through 1990. This is an interesting period to consider. Loans originated in the early part of the period experienced high inflation, but loans originated the end of the period were exposed to a sharp recession. The statistical analysis is based upon a simple random sample of about five percent of these mortgages -- all fixed rate, level payment, fully amortizing loans, most with thirty year terms. For each mortgage we observe the year of origination, the housing value at origination (the purchase price of the property), the contractual terms, and the region in which the property is located.

We estimate hazard models of default,[5] where $H(d_t)$, the instantaneous default hazard at age t, is:

(7) $\quad H(d_t) = \alpha_t \exp[\Sigma \beta_i\ Y_i + \gamma E_t]$

In this formulation the Y's are fixed covariates, dummy variables indicating the year of mortgage origination, and E_t is a time-varying covariate reflecting homeowner equity when the mortgage is at age t. As the model is specified, the hazard is not proportional to t, but knowledge of the time

---

5   A fully developed model would analyze both prepayment and default simultaneously (as in Foster and Van Order [1985]) because the two decisions can be interrelated -- refinancing of mortgage debt is more difficult without positive equity.

13

profile of E determines the relative change in the hazard. Thus the parameters β_i and γ can be estimated by maximizing the likelihood function without reference to the parameters governing the baseline hazard α_t (See Kalbfleisch and Prentice, [1980]).

At any age of the mortgage t, the book equity of the mortgage holder E_t is :

(8) $E_t = V_{T+t} - D_t$

$\quad\quad = V_{T+t} - V_T L f(N,t,\omega)$

where the V_{T+t} is the current value of the house and D_t is the outstanding debt at age t. This unpaid balance depends upon the value of the house at the year of purchase T, V_T, and the loan-to-value ratio at origination, L. $f(N,t,\omega)$ is the outstanding fraction after t periods, on a fully-amortizing level payment loan written for N periods at contract rate ω:

$$(9)\quad f(N,t,\omega) = \frac{1 - 1/(1+\omega)^{N-t}}{1 - 1/(1+\omega)^N}$$

As equation (8) indicates, values of the key state variable are strongly affected by L, the initial loan to value ratio, as well as the course of housing prices after purchase. Figure 3 presents the distribution of L for mortgages purchased by Freddie Mac during this period. The mode is a mortgage loan for 80 percent of the purchase price of a

FIGURE 3

ORIGINAL LOAN TO VALUES
LOANS PURCHASED BY FREDDIE MAC
1976-1980
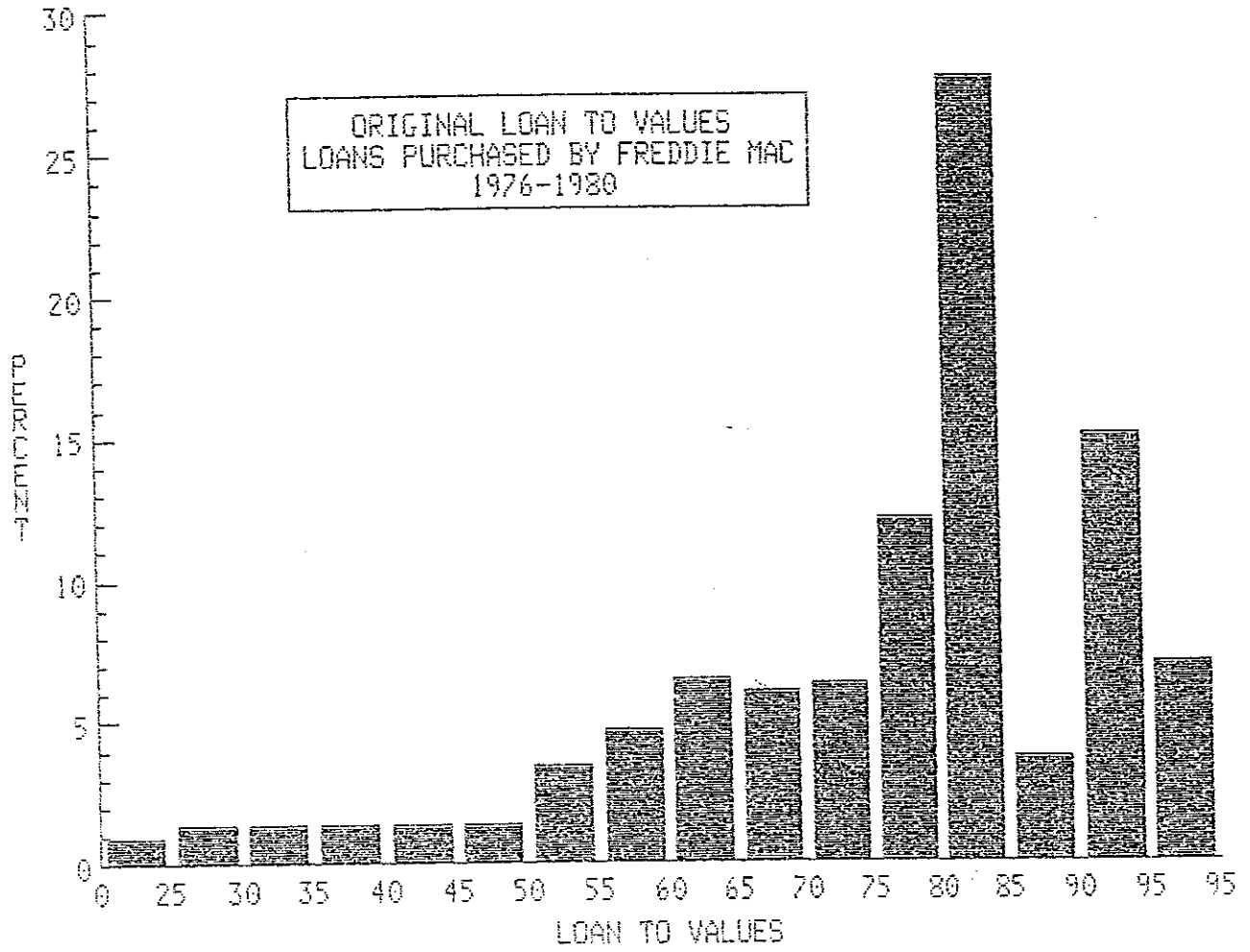
property, but there is considerable variation in these ratios. A substantial fraction of loans were for 70 percent or less of market value, and there were some loans for as much as 95 percent of value.

We do observe the purchase price of each house V_T, but we do not observe the subsequent course of price variation for individual houses in the sample. We do, however, have access to the prices of about 200,000 properties whose mortgages were purchased by Freddie Mac at least twice during the period 1970-1989. These data are sufficient to estimate, rather precisely, a quarterly weighted repeat sales (WRS) price index for each of five U.S. regions, using the methodology proposed by Case and Shiller [1987].

These indexes and the methodology which underlies them are discussed by Abraham and Schauman [1990].[6] Figure 4

---

6 These price indices are estimated according to the three stage regression procedure outlined in the appendix to Case and Shiller's 1987 paper, but they incorporate one slight extension. The model assumes that logarithm the housing price P_{it} in each region is given by

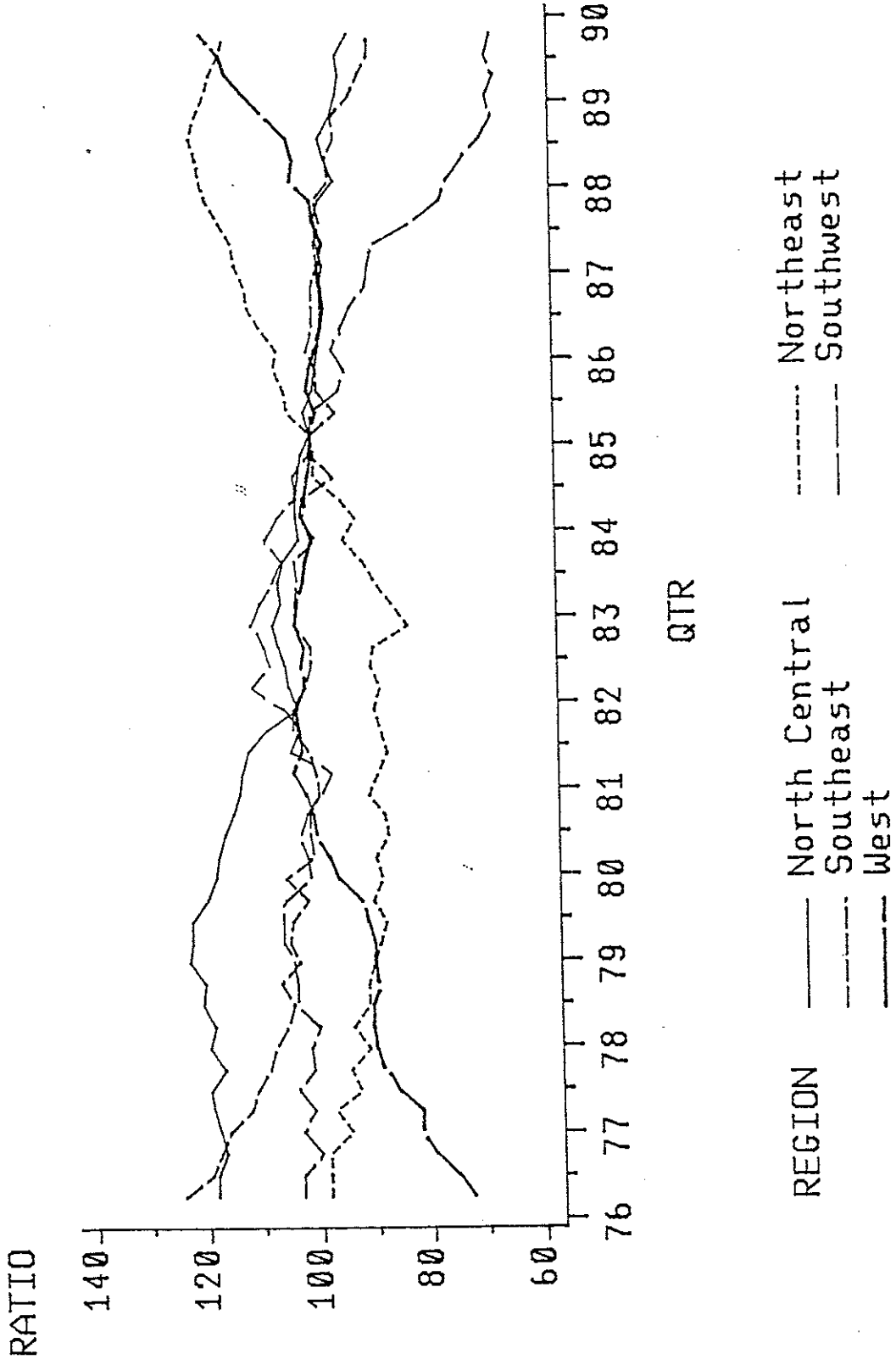$$P_{it} = I_t + H_{it} + N_{it} \, , \qquad (N-1)$$

where I_t is the log of the price level, H_{it} is a Gausian random walk (i.e., $E[H_{i\tau} - H_{it}] = 0$; $E[H_{i\tau} - H_{it}]^2 = A[\tau-t] + B[\tau-t]^2$, and N_{it} is white noise (i.e., $E[N_{it}] = 0$; $E[N_{it}]^2 = C$). The first stage is the regression of the difference in log sale prices, for multiple sales of the same property, upon a set of dummy variables with values of zero for all quarters except those in which the two sales occurred:

$$P_{i\tau} - P_{it} = g(\tau,t) \, . \qquad (N-2)$$

FIGURE 4

Regional Price Indices

relative to a national housing price index

RATIO

140

120

100

80

60

76 77 78 79 80 81 82 83 84 85 86 87 88 89 90

QTR

REGION ——— North Central   ········· Northeast
        ——— Southeast   ——— Southwest
        ——— West

Source: Freddie Mac

summarizes the course of the price indices relative to the national average for the period 1976-1989. The figure reveals substantial regional variation about the national price trend.

A. A Crude Test of the Model

If we assume that all the houses in our sample appreciate at the average for the region as a whole,

$$(10) \quad V_{\tau+t} = V_\tau I_{r,\tau,\tau+t} \quad ,$$

where $I_{r,\tau,\tau+t}$ is the proportionate change in the WRS price index for region r between τ and $\tau+t$, then

$$(11) \quad E_t(\tau,L,N,\omega,r) = V_\tau I_{r,\tau,\tau+t} - V_\tau Lf(N,t,\omega)$$

can be calculated from sample information. In the empirical analysis, we compute E_t quarterly, and we also observe individual mortgage defaults and hence hazards quarterly.

The second stage is a weighted regression of the squared residuals upon an intercept, the elapsed time between sales, and its square, yielding estimates of A, B, and C:

$$(P_{it} - \hat{P}_{it})^2 = A [\tau-t] + B[\tau-t]^2 + C \quad . \qquad (N-3)$$

The third stage is a re-estimation of the stage one regression by generalized least squares (GLS) using the fitted values in the second stage as GLS weights. The incorporation of the square of elapsed time between sales in the second stage, not considered by Case and Shiller, reflects the expectation that the variance of prices does not increase at the same rate forever.

Table 1 presents coefficient estimates for these simple nonproportional hazard models.[7] Columns 1 and 2 present coefficients using the dollar value of expected equity as an independent variable; columns 3 and 4 report the results using equity as a fraction of current housing value as an independent variable.

As indicated by the pattern of dummy variables in Table 1, *ceteris paribus*, successive origination years had higher default rates. One explanation, consistent with the ruthless default model, emphasizes the effect of the interest rate cycle. Borrowers who took out low rate mortgages early in the period saw the market values of their liabilities fall over time, as interest rates rose, so that economic equity was larger than the book value of equity, reducing default behavior.[8]

---

7 One consequence of the reliance upon average housing prices in equation (10) is the measurement error thereby introduced into the variable representing individual homeowner equity. *Ceteris paribus,* we should expect a higher (lower) probability of default for those whose housing price appreciation is below (above) the average in any region. By ignoring the dispersion of housing prices around the regional average, we ignore the fact that positive equity for the average homeowner in a given region will coexist alongside negative equity and higher default risk for some homeowners.

8 An alternative explanation, not consistent with the frictionless model, emphasizes the rise in the cash flow costs of housing relative to incomes in the late 1970's (Housing prices increased faster than incomes, and mortgage interest rates rose substantially). Hence, those who took out fixed-rate mortgages earlier in the period were less likely to have had difficulty making repayments after the recession began in the early 1980s -- simply because their mortgage payments were relatively low. Finally, for reasons

TABLE 1

Hazard Models of Mortgage Default
(9,229 observations)

$$H(d_t) = \alpha_t \exp[\Sigma\beta_i Y_i + \gamma (E_t)]$$

Parameter	1	2	3	4
Origin year Dummies, β_i				
1976	-2.199 (7.37)		-1.417 (4.46)	
1977	-1.669 (8.43)		-0.878 (3.99)	
1978	-1.165 (7.51)		-0.718 (4.37)	
1979	-0.076 (6.01)		-0.826 (5.33)	
Equity, γ				
E (in thousands)	-0.076 (13.54)	-0.093 (16.39)		
E Ratio (E/V)			-9.352 (16.58)	-10.235 (22.26)
χ^2	451.9	243.6	632.2	511.7

Note: Asymptotic t ratios in parentheses.

When these temporal effects are not accounted for separately, the variables measuring equity are even more important. In any case, the coefficients of the variables measuring equity are highly significant and rather large in magnitude. For example, a decline in equity of ten thousand dollars multiplies the probability of default by at least exp[.76] or by about 214 percent. A decline in the equity ratio by 0.05 increases the conditional default rate by about 160 percent.

Not surprisingly, the option model passes the simple test: equity clearly matters a lot in the calculus of default.

B. A More Refined Test: The Distribution of Equity

The large sample of repeat sales which underlies the regional price indices can also be used to estimate the variance in individual house prices. Indeed, the procedure used to compute the WRS price indices provides a direct estimate of price dispersion. The course of individual housing prices is specified as a random walk, with variance

---

indicated in note 6, over time those mortgages that remain outstanding may tend to have less equity than average. For loans originated early in the period, this selectivity is not likely to be large (since the subsequent course of mortgage interest rates exceeded the coupon rates for these mortgages). Mortgages originated later in the period did eventually experience interest rates lower than their coupons. This is also consistent with monotonically increasing dummy variables for origination year.

These models indicate an even more powerful relationship between homeowner equity and default probabilities. Columns 1 and 2 imply that a homeowner with negative equity is more than 81 times (*i.e.*, exp[4.4]) as likely to exercise the option as a homeowner with positive equity. Columns 3 and 4 again indicate that negative equity is strongly associated with higher default rates, but that low positive levels of equity are also associated with increased default probabilities.

The coefficients in column 3, for example, imply that households with a 15 to 30 percent equity stake in their houses are about 2.7 times as likely to default as those with larger equity stakes. Households with a zero to 15 percent equity stake are about 29 times as likely to default as those with at least 30 percent equity. Finally those with negative equity are more than 75 times as likely to default. The coefficients in column 4 are even more extreme.

The coefficients in columns 5 and 6 disaggregate negative equity into classes. Quite clearly, for realized equity ratios more negative than -0.1, default is essentially complete and "instantaneous" (but, with quarterly data, an instant is three months). Again, for small negative equity ratios (less than 0.1 in absolute terms), the probabilities of default are significantly larger. In column 5 the estimate is an increased default probability of 22 percent. In column 6, the estimate is very much larger indeed.

These results are quite consistent with the ruthless default model: higher probabilities of default for moderately negative equities (where the option has value) and instantaneous default for highly negative equities.

Some caution is required, however, in interpreting the default responses. The model imposes a particular exponential structure on the equity-default relationship. Moreover, the sample does not include many observations where the probability of negative equity is at all close to one. For these reasons, we have simulated default responses using various assumptions about the mean and variance of house price changes; we compare the responses with several versions of ruthless model.

C. Simulations

Table 3 summarizes a large number of simulations taking account of the stochastic nature of individual house prices. We compare three default models. The first, Model I, is based upon the numerical solution to the differential equation predicting default presented by Kau et al [1991]. The second is a simpler specification which assumes immediate default if the option is 10 percent in the money.[10] The third, a "behavioral" model, is based upon the hazard rate estimates in

---

10 The results are qualitatively similar for other variants of the ruthless model, for example one with immediate default at $(E/V) \leq -.2$.

model.   This finding is also consistent with the unadjusted raw default data reported in the Appendix.[12]

IV.   LOSS SEVERITY AND OPTIMAL EXERCISE

Severity rates should depend only on items in (1) and in the boundary conditions.   Important factors are the age of the mortgage, interest rates and the mortgage coupon.   If the coupon rate is high relative to current rates, the value of the mortgage exceeds par, which lowers the value of keeping the option alive.   This implies more rapid exercise of the option and therefore lower severity rates.   Similarly, the older is the mortgage (and therefore the closer it is to maturity) the less important is future option value, implying quicker exercise and lower loss severity.

In summary, the ruthless model of default implies four propositions about severity rates:

1.   *Ceteris paribus*, severity should be independent of initial LTV.   However, high LTV loans almost always have insurance if they are purchased by Freddie Mac.   The cost of insurance increases the effective coupon rate to the borrower for high LTV loans, but not the mortgage coupon rate measured in these

---

12   Appendix Table A1 presents actual unadjusted default experience on comparable Freddie Mac mortgages.   These data are consistent with the qualitative properties of the frictionless model but again, the ratio of high LTV to low LTV defaults is not as large as predicted by the frictionless model.

data. Thus for this data set, severity should fall as LTV increases.

2. *Ceteris paribus*, severity should be the same in regions with high default frequencies as in regions with low frequencies.

3. Severity should decrease with the age of the mortgage.

4. Severity should decrease as coupon rate minus the current interest rate increases.

Table 4 tabulates loss severity for all Freddie Mac defaults on loans purchased within two years of origination.[13] Loss severity, gross of any insurance payments, is measured by the difference between the mortgage balance and the value of the house for defaulted loans. It excludes all transactions costs and foregone interest. House values are measured in two ways. The first is based on an appraisal at the time a defaulted property is acquired by Freddie Mac. The second is the actual sale price when (about a year later) the house is sold from the Freddie Mac inventory. Neither is a perfect measure of the extent to which the option was in the money when the borrower chose to default. Nonetheless, there is no reason to believe there is a systematic bias by LTV, coupon rate, interest rate, or age.

---

13 Seasoned loans acquired by Freddie Mac were eliminated to avoid potential selectivity biases.

TABLE 4

Loss Severity by LTV as a Percent of Mortgage Balance*
1975-1990

Original LTV	Loss I	Loss II
A. All Loans		
51-60	-7.0%	2.0%
61-70	-0.3	8.7
71-75	0.9	10.0
76-80	2.1	11.7
81-90	5.2	15.1
91-95	14.6	24.3
B. Texas Loans		
51-60	-1.5%	6.9%
61-70	13.1	21.6
71-75	13.8	21.7
76-80	16.7	24.5
81-90	18.2	26.8
91-95	23.0	32.0

Notes:

* Average losses on all defaulted loans, 1975-1990, excluding defaults on seasoned loans purchased by Freddie Mac.

Loss I is computed as mortgage balance minus appraised value at acquisition.

Loss II is computed as mortgage balance minus actual sales price at time of sale.

Source: Freddie Mac.

Panel A presents loss severity as a fraction of loan balance for all loans defaulted from 1975-1990. The first column indicates the losses based on appraisal data at acquisition while the second column uses eventual selling prices. As expected, actual losses consistently exceeded appraised losses, by about 8 to 10 percent. In both columns, however, there is a strong effect of LTV. High LTV loans have much higher severity rates. This is not consistent with the first prediction of the ruthless model.

Panel B of the table presents similar calculations for Texas defaults during the same time period. The LTV effect remains, though it appears to be much smaller. However, the Texas losses are substantially higher. This is not consistent with the second prediction of the frictionless model.[14]

Of course, these static comparisons require holding other things constant. For instance, the table does not control for the age of the loan. High LTV loans will, *ceteris paribus*, have negative equity at younger ages than low LTV loans. Because younger loans have a larger option values than older ones, default rates should be lower, but severity should be higher. To control for this and for other factors, we

_____

14 Institutional differences, such as homestead provisions and state laws requiring delays in enforcing eviction, may cause average loss rates to vary among states. In general, however, Texas provides fewer protections against eviction than any other state. Thus based on institutional differences alone, Texas loss rates should be lower than elsewhere. See Clauretie and Herzog [1989].

column 3.   In contrast, the effect of LTV appears to be quite robust to changes in specification.16

IV.   CONCLUSION

This paper investigates the home mortgage default behavior of households, using micro data on household choice and a rich body of data on individual house prices.   The results document the close but highly nonlinear relationship between homeowner equity at the individual level and homeowner default probabilities.   "In the money" options are exercised, and the probability of default approaches one rapidly at moderate values of negative equity.

Of particular importance is the empirical finding that at low levels of negative equity the option is not exercised immediately (confirming that the option itself has value), but at higher levels of negative equity (above about .1 in absolute magnitude) default is essentially instantaneous.

Whether a really "ruthless" zero-transaction cost model fully explains default is a more difficult question.   Our analysis does suggest that the frictionless model is qualitatively consistent with observed default data. Nonetheless, there are discrepancies and complications:

---

16 For a more detailed analysis of loss severities, see
   Lekkas, Quigley and Van Order (1993).

First, the frictionless model overstates the variations in the peaks over time in default rates for different initial LTVs. Empirically, the peaks in the average default rates over time are more similar for various initial LTVs than is predicted by the frictionless default model, and average durations do not vary by LTV. Second, loss severities increase significantly as a function of initial LTV, contrary to the theory. Third, there is some rather weak evidence that the ruthless model overstates the spread between default frequencies for high LTV and low LTV loans.

Transactions costs in the conventional form of costs of exercising the option do not explain these discrepancies (Indeed, these transactions costs alone imply improbably low default rates). However, we know that transactions costs are larger in housing markets than in most securities markets, and our empirical analysis suggests that a zero transaction cost model is not consistent with the data. What explains these apparently contradictory observations?

One explanation concerns the definition of transactions costs. Most options-based analyses, including those of the housing market (e.g., Cunningham and Hendershott [1984], and Kau et al [1991]) define transactions costs as the cost of *exercising* the option. It is clear intuitively that these costs will lower default rates. The cost of trading housing is another important transaction cost in this market. If it

costs 6 percent of house value to sell a house for cash, but 0 percent to sell it in exchange for the mortgage, then there is an extra incentive to default.

This latter transaction cost greatly complicates the neat arbitrage argument on which equation (1) is based. An important implication is that the value of the option will vary across people. On the one hand, households without liquidity constraints or without an "exogenous" reason for moving may well value the option in the manner described by equation (1) and the boundary conditions. If these households have costs of exercising the default option (e.g, bad credit ratings, etc.), then they will not exercise it until the option is well into the money. That is, they will have lower default frequencies than those implied by the frictionless model.

On the other hand, for liquidity-constrained households, or those with "exogenous" reasons for moving transactions costs can lead to different behavior. For example, consider a household forced to move for "exogenous" reasons (e.g., a divorce). For this household, the term of the mortgage is now short, and thus (if the mortgage is not assumable, as is the case with most of the loans in our sample) the value of keeping the option alive is small. Moreover, the cost of selling the house to the lender is less than selling it on the market. Hence, absent costs of exercising the option, the

household will rationally default with positive equity (as long as it is less than selling costs).

More broadly, households that are "in trouble" (e.g., those experiencing a lost job, a medical emergency) and are liquidity-constrained may well default when the option is out of the money, depending on their own costs of exercise. Suppose, for example, that households default only if they have negative equity and are "in trouble." Suppose that the period of time they can hold out (or the time that it takes the lender to foreclose) is random. Then:

1. Equity will still be a major factor in explaining default, but exercise will also depend on the personal characteristics of the borrower.

2. There will be a smaller spread between default frequencies for high and low LTV loans than is predicted by the frictionless model. High LTV loans will have lower default rates than predicted by the frictionless model (because only households in trouble are candidates for default). Low LTV loans will have higher default rates than predicted (because households in trouble will default when the option is less far in the money).

3. The random time to default generates the shape of the time profile of default which will be independent of initial LTV. Although 95 percent LTV loans will default more frequently than 85 percent LTV loans, the peak default years

will be the same for both (because the time to default is generated by the same random process).

4. High LTV loans will have greater loss severities than low LTV loans.

5. More depressed areas, like Texas in the 1980's, will have greater loss severities than less depressed areas.

Each of these propositions is consistent with our empirical findings. This leads to the conclusion that transactions costs, broadly defined, are important in default decisions. These costs include those of exercising the option, but more importantly, they include the costs of trading houses.

None of this really implies that transactions costs and other imperfections have a disproportionately large impact on the housing market. Indeed, as Case and Shiller observe "There is little hope of proving definitively whether the housing market is [or is not] efficient." (p.135) Our evidence suggests that in these respects, at least, the housing market is not fundamentally different from other markets, presumed to operate reasonably efficiently, though not with textbook perfection.

References

Abraham, Jesse and Bill Shauman, "Evidence on House Prices from FHLMC Repeat Sales," Federal Home Loan Mortgage Corporation, May 1990, mimeo.

Black, F. and M. Scholes, "The Pricing of Options and Corporate Liabilities," **Journal of Political Economy**, Vol. 81, 1973, pp. 637-659.

Buser, S.A. and P. Hendershott, "Pricing Default Free Mortgages," **Housing Finance Review**, Vol. 3, 1984.

Brennan M. and E.S. Schwartz, "Determinants of GNMA Mortgage Prices," **Journal of AREUEA**, Vol. 13, 1985, pp. 209-228.

Campbell T., and J. Dietrich, "The Determinants of Default on Insured Conventional Residential Mortgage Loans," **Journal of Finance**, December 1983, pp. 1569-1580.

Clauretie, T., and T. N. Herzog, "How State Laws Affect Foreclosure Costs," **Secondary Mortgage Markets**, Vol. 6, Spring 1989, pp. 25-28.

Cox, J., J. Ingersol, and S. Ross, "An Intertemporal General Equilibrium Model of Asset Prices," **Econometrica**, Vol. 53, 1985, pp. 363-384.

Cunningham, D. and Patric Hendershott, "Pricing FHA Default Insurance," **Housing Finance Review**, Vol. 3, #4, 1984, pp. 342-373.

Dunn, K. and J. McConnell, "Valuation of GNMA Mortgage Backed Securities," **Journal of Finance**, Vol. 32, 1983, pp. 613-623.

Foster, Chester and Robert Van Order, "FHA Terminations: A Prelude to Rational Mortgage Pricing," **Journal of AREUEA**, Vol. 13, 1985, pp. 273-291.

Green, Jerry and John B. Shoven, "The Effect of Interest Rates on Mortgage Prepayments," **Journal of Money, Credit and Banking**, Vol. 18, February 1986, pp. 41-50.

Jones, Lawrence D., "Deficiency Judgments and the Exercise of the Default Option in Home Mortgage Loans," Faculty of Commerce and Business Administration, University of British Columbia, June 1991, mimeo.

Kalbfleisch, J.D. and R.L. Prentice, **The Statistical Analysis of Failure Time Data**, New York: John Wiley and Sons, 1980.

Kau J., D. Keenan, W. Muller III, and J. Eppenson, "Option Theory and Fixed-Rate Mortgages," University of Georgia, 1986, mimeo.

Kau, James B., Donald C. Keenan and Taewon Kim, "Default Probabilities for Mortgages," University of Georgia, 1991, mimeo.

Lekkas, Vassilis, John M. Quigley, and Robert Van Order, "Loan Loss Severity and Optimal Mortgage Default," **AREUEA Journal**, forthcoming, 1993.

Quigley, John M., "Interest Rate Variations, Mortgage Prepayments and Household Mobility, **Review of Economics and Statistics**, Vol. LXIX, #4, November 1987, pp. 636-643.

Quigley, John M. and Robert Van Order, "Efficiency in the Mortgage Market: The Borrower's Perspective," **Journal of AREUEA**, Fall 1990.

Quigley, John M. and Robert Van Order, "Defaults on Mortgage Obligation and Capital Requirements for U.S. Savings Institutions," **Journal of Public Economics**, Vol. 44, 1991, pp. 353-369.

Smith, Clifford W., "Option Pricing," **Journal of Financial Economics**, Vol. 3, Jan-Mar 1976, pp. 616-621.

APPENDIX TABLE A1

Default Rates By LTV and Origination Year for Freddie Mac Loans

Actual Cumulative Default Rates Through 1990*

Origination Year	LTV Categories				
	0-75%	76-80%	81-85%	86-90%	91-95%
1975	0.04%	0.11%	0.32%	0.33%	0.94%
1976	0.06	0.11	0.42	0.60	0.92
1977	0.09	0.22	0.66	0.80	1.89
1978	0.25	0.63	1.22	1.84	4.30
1979	0.50	1.25	1.75	3.29	7.42
1980	0.81	2.87	3.08	6.83	10.43
1981	1.00	4.81	4.99	11.43	12.04
1982	0.78	3.40	3.21	7.12	12.69
1983	0.33	1.43	2.74	4.19	8.21
1984	0.20	0.54	1.20	2.17	5.23
1985	0.10	0.49	0.67	1.09	3.48
1986	0.06	0.32	0.44	0.70	2.16
1987	0.03	0.17	0.21	0.36	0.75
1988	0.03	0.06	0.07	0.12	0.26
1989	0.02	0.02	0.00	0.04	0.07
Average	0.29	1.10	1.40	2.73	4.72

Note:

* Including only 30 year, fixed rate, single family mortgages.

Source: Freddie Mac

December 9, 1993

Working Paper Series
Department of Economics
University of California, Berkeley

92-187   "Shocking Aspects of European Monetary Unification." Tamim Bayoumi and Barry Eichengreen. January 1992.

92-188   "Is There a Conflict Between EC Enlargement and European Monetary Unification?" Tamim Bayoumi and Barry Eichengreen. January 1992.

92-189   "The Marshall Plan: Economic Effects and Implications for Eastern Europe and the Soviet Union." Barry Eichengreen and Marc Uzan. January 1992.

92-190   "Exploring the Relationship between R&D and Productivity at the Firm Level in French Manufacturing." Bronwyn H. Hall and Jacques Mairesse. February 1992.

92-191   "Three Perspectives on the Bretton Woods System." Barry Eichengreen. February 1992.

92-192   "Are Futures Margins Adequate?" Roger Craine. April 1992.

92-193   "Heterogeneity in Organizational Form: Why Otherwise Identical Firms Choose Different Incentives for Their Managers." Benjamin E. Hermalin. May 1992.

92-194   "Investment and Research and Development at the Firm Level: Does the Source of Financing Matter?" Bronwyn H. Hall. May 1992.

92-195   "The Determinants of Efficiency and Solvency in Savings and Loans." Benjamin E. Hermalin and Nancy E. Wallace. May 1992.

92-196   "Economics of Development and the Development of Economics." Pranab Bardhan. June 1992.

92-197   "The Core and the Hedonic Core: Equivalence and Comparative Statics." Suzanne Scotchmer and Greg Engl. July 1992.

92-198   "Incorporating Behavioral Assumptions into Game Theory." Matthew Rabin. July 1992.

92-199   "Incorporating Fairness into Game Theory and Economics." Matthew Rabin. July 1992.

92-200   "A Consumer's Guide to EMU." Barry Eichengreen. July 1992.

92-201   "Macroeconomic Adjustment Under Bretton Woods and the Post-Bretton-Woods Float: An Impulse-Response Analysis." Tamim Bayoumi and Barry Eichengreen. August 1992.

92-202   "Oligopoly Structure and the Incidence of Cigarette Excise Taxes." Paul G. Barnett, Theodore E. Keeler, Teh-wei Hu. September 1992.

92-203   "The Impact of 1989 California Major Anti-Smoking Legislation on Cigarette Consumption: Three Years Later." Teh-wei Hu, Jushan Bai, Theodore E. Keeler, Paul G. Barnett. September 1992.

92-204   "A Dynamic Simultaneous-Equations Model for Cigarette Consumption in the Western States." Hai-Yen Sung, Teh-Wei Hu, and Theodore E. Keeler. November 1992.

92-205   "Maastricht: Prospect and Retrospect." John M. Letiche. December 1992.

92-206   "Nonparametric Methods to Measure Efficiency: A Comparison of Methods." Richard F. Garbaccio, Benjamin E. Hermalin, Nancy E. Wallace. December 1992.

93-207   "The Value of Intangible Corporate Assets: An Empirical Study of the Components of Tobin's Q." Bronwyn H. Hall. January 1993.

93-208   "R&D Tax Policy During the 1980s: Success or Failure?" Bronwyn H. Hall. January 1993.

93-209   "Hospital Costs and Excess Bed Capacity: A Statistical Analysis." Theodore E. Keeler and John S. Ying. April 1993.

93-210   "Homothetic Preferences, Homothetic Transformations, and the Law of Demand in Exchange Economies." John K.-H. Quah. April 1993.

93-211   "Deviations, Dynamics and Equilibrium Refinements." Matthew Rabin and Joel Sobel. May 1993.

93-212   "Loss Aversion in a Savings Model." David Bowman, Debby Minehart, and Matthew Rabin. May 1993.

93-213   "Nonparametric Multivariate Regression Subject to Constraint." Steven M. Goldman and Paul A. Ruud. May 1993.

93-214   "Asset Prices and the Fundamentals: A Q Test." Roger Craine. May 1993.

93-215   "A Note on the Finiteness of the Set of Equilibria in an Exchange Economy with Constrained Endowments." Deborah Minehart. September 1993.

93-216   "Institutional Prerequisites for Economic Growth: Europe After World War II." Barry Eichengreen. September 1993.

93-217   "Industrial Research During the 1980s: Did the Rate of Return Fall?" Bronwyn H. Hall. September 1993.

93-218   "Efficient Standards of Due Care: Should Courts Find More Parties Negligent Under Comparative Negligence?" Aaron S. Edlin. October 1993.

93-219   "Classical Estimation Methods for LDV Models Using Simulation." Vassilis A. Hajivassiliou and Paul A. Ruud. October 1993.

93-220   "Perspectives on the Borchardt Debate." Barry Eichengreen. November 1993.

93-221   "Explicit Tests of Contingent Claims Models of Mortgage Defaults." John M. Quigley. November 1993.