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ABSTRACT

A previously proposed interaction is examined as to its consequences for a neutral K in an rf field tuned to the K_S-K_L mass difference. Substantial K_1-K_2 conversion can be achieved by reasonable field strengths.

Of the numerous mechanisms proposed to account for the phenomena that have been attributed to CP violation practically all have by now been put to rest. The one that seemed to have gained the longest general acceptance, the superweak model, is now threatened by the experimental result [1] on the upper limit for the decay $K_L \to 2\mu$.

From the beginning, many of these proposals have been unconventional, some involving external fields called into play for the sole purpose of accounting for the phenomena in question. It therefore seems prudent, on empirical grounds alone, to pursue the possibility that the anomalous effects may be due to the action of a more mundane

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agent, the magnetic field [2], even though it takes a seemingly unconventional interaction [3] of this field to do the job. As discussed earlier [2], in the presence of a static magnetic field this coupling can lead to an observable K_1 component at large distances from the target, thus simulating CP violation.

The purpose of this note is to examine some consequences of this interaction for the behavior of a neutral K in an rf field tuned to the K_L-K_S mass difference. The coupling, which was proposed some time ago [3] on the basis of other considerations [4] and is invariant under C and P separately, is

$$H = G(K^2 e^{i\phi} + \bar{K}^2 e^{-i\phi})\underset{\sim}{E}\cdot\underset{\sim}{B} , \qquad (1)$$

where K and $\bar{K}$ are the neutral kaon field operators, $\underset{\sim}{E}$ and $\underset{\sim}{B}$ are the electromagnetic field operators, and ϕ is a phase which is not arbitrary once a convention is adopted as to the effect of CP on the K^0. I will use the phase convention $CP|K^0\rangle = |\bar{K}^0\rangle$, $CP|\bar{K}^0\rangle = |K^0\rangle$, which entails $\cos\phi = 0$. This interaction leads to creation of kaon pairs, e.g., by Coulomb scattering of hard photons, as well as to K^0-$\bar{K}^0$ interconversion in the presence of suitable external fields. In addition to the matrix elements bilinear in the external electromagnetic fields that are obvious from its structure, it leads to matrix elements linear in the external field [3]. Such linear terms can arise because one of the two photons emitted at the vertex at which K^0-$\bar{K}^0$ interconversion takes place can couple internally to the neutral K by virtue of its charge structure. The decisive factor that allows for a slowly (macroscopically) varying magnetic field to cause K^0-$\bar{K}^0$ interconversion is the degeneracy between these two states.

The electromagnetic-field dependence of the matrix elements linear in the external field is given by the pseudoscalar

$$\epsilon_{\mu\nu\rho\sigma} f_{\mu\nu}(k) p_\rho p'_\sigma , \qquad (2)$$

where p and p' are kaon momenta, and $k = p - p'$ is the momentum exchange with the external field, whose Fourier transform is $f_{\mu\nu}(k)$. Of the three kinds of terms contained in (2), the one of interest for the purpose of this note is proportional to the magnetic-field component $B_{||}$ along the K beam. One of the other two terms vanishes because the magnetic field is divergenceless and the one involving the electric field cannot have come into play in the experiments in question.

As mentioned previously [5], the regeneration effect in a static field may saturate. Experiments carried out at different values of the magnetic field, but above that corresponding to saturation, will then give the same value of the admixture parameter ϵ, which seems to be the case experimentally. The possibility of saturation has to do with the contingent fact that the static field, in exchanging momentum with the neutral K, acts as a refractive medium. As the momentum exchange becomes larger than the momentum spread of the packet, the phase of the regenerated packet begins to oscillate rapidly in the range over which the packet would otherwise be appreciable, resulting in destructive interference. The situation is similar in some respects to that of a spin-$\frac{1}{2}$ particle in a static magnetic field. Apart from the instability of the neutral K, which plays a decisive role, K_1 regeneration from K_2 by a static field $B_{||}$ is formally equivalent to a transition between two spin states with spin projections in say the x-direction, due to crossed static magnetic

fields B_x and B_y. Variation of B_y corresponds to that of $B_{||}$, and a constant B_x corresponds to the K_S-K_L mass difference. A physically more useful analogy is that of the neutral K with the Stern-Gerlach experiment, though the conditions under which the effects of interest are produced are reversed in the two cases. In the latter case, it is well known that if the magnetic-field inhomogeneity is strong enough to split the initial wave packet into nonoverlapping outgoing ones, then it will also destroy an initial polarization in any direction but that of the inhomogeneity. In that case the desired effect is thus not produced until the momentum exchange with the external field is larger than the momentum spread of the packet, thereby destroying coherence, while in the case at hand a regenerated K_1 packet is produced only as long as the momentum exchange is less than the momentum spread of the packet and coherence is maintained.

An rf field can be expected to overcome this difficulty because, roughly speaking, its effect is the same on each momentum component of the wave packet. The general advantages of resonance over static-field techniques for studying quantum systems have of course been recognized for decades.

We shall consider the case where the matrix elements for K^0-$\bar{K}^0$ interconversion involve the external field linearly rather than bilinearly because this eliminates much of the guess work as to the strength of interaction (1). In particular, a measure of the kind of rf field strength required will be seen to be provided by its value B_c corresponding to a critical potential defined in terms of the short-lived decay rate by

$$V_c \equiv \frac{1}{4} \Gamma_S \approx 1.8 \times 10^{-6} \text{ eV} . \qquad (3)$$

The estimate of an upper limit on B_C in the matrix elements linear in the external field can in effect be made model independent by appealing to the experimental results in static fields. To that end, we note that if the phenomena attributed to CP violation are due to an external static potential U, then the experimental value of the admixture parameter,

$$|\epsilon| \approx 2 \times 10^{-3} , \tag{4}$$

gives

$$U \approx 10^{-8} \text{ eV} . \tag{5}$$

It should also be noted that the CP-invariance condition $\cos \phi = 0$ leads to a phase of ϵ in agreement with the measured value. If the anomalous effects are indeed due to external magnetic fields coupled through interaction (1), the value (5) presumably corresponds to the magnetic field B_1 at saturation, which I take to be the lowest field in any of the experiments in question, $B_1 \lesssim 0.1$ gauss, although it could be orders of magnitude lower. Since the potential is linear in the external field, we have the upper bound

$$B_C = (V_C/U)B_1 \lesssim 18 \text{ gauss} . \tag{6}$$

A measurement of B_C in the rf experiment will give the value of the static field at which regeneration saturates and below which variation of the effects with the static field could be observed.

It is adequate for our purposes to approximate the short- and long-lived states K_S and K_L by the CP eigenstates K_1 and K_2 respectively. The equations for the K_1 and K_2 amplitudes a_1 and a_2 are then

$$i\gamma\dot{a}_1 = (m_S - \tfrac{1}{2} i\Gamma_S)a_1 + iVa_2 ,$$
$$i\gamma\dot{a}_2 = -iVa_1 + (m_L - \tfrac{1}{2} i\Gamma_L)a_2 , \tag{7}$$

where γ is the Lorentz factor, dot indicates differentiation with respect to laboratory time t, and $V = V(t)$ is the effective external potential, which includes the rf field.

To develop the consequences of tuning the rf field to the K_S-K_L mass difference, it is convenient to introduce the amplitudes b_1 and b_2, defined by

$$b_1 \equiv a_1 \exp(im_S t/\gamma) ,$$
$$b_2 \equiv a_2 \exp(im_L t/\gamma) . \tag{8}$$

For simplicity we will be concerned only with 2π decays, for which the b amplitudes are adequate. But if one is concerned with interference effects between K_1 and K_2, e.g., semileptonic decays, one must in effect return to expansion of the wave function in terms of the a's. The amplitudes b_j satisfy

$$\gamma\dot{b}_1 = -\tfrac{1}{2}\Gamma_S b_1 + V\exp(-i\Delta' t)b_2 ,$$
$$\gamma\dot{b}_2 = -V\exp(i\Delta' t)b_1 - \tfrac{1}{2}\Gamma_L b_2 , \tag{9}$$

with

$$\Delta' \equiv \Delta/\gamma ,$$
$$\Delta \equiv m_L - m_S > 0 . \tag{10}$$

Because the K beam is not monochromatic, the effective mass difference Δ' is not unique. If the applied rf field is to be tuned to Δ' for an appreciable fraction of the K's, the frequency spread of the field must overlap the corresponding spread in Δ'. I will assume that this important practical problem can be solved and will consider a single fixed value of Δ' and a monochromatic field.

Consider the potential to be of the form

$$V(t) = V_0[\exp(i\omega t) + \exp(-i\omega t)] , \qquad (11)$$

where V_0 is time independent and ω is the effective rf frequency, which will be different from the actual frequency if the spatial dependence of the cavity mode along the K beam is not constant. Suppose, for example, $B_{||}$ is of the form

$$B_{||} = B_0 J_0(hr) \cos(\omega_0 t - kz) ,$$

where ω_0 is the actual cavity frequency and the z-direction is along the K momentum. Then the effective frequency will be shifted to

$$\omega \equiv \omega_0 \pm kv ,$$

where v is the K speed.

In substituting Eq. (11) for $V(t)$ into (9) we now keep only the terms capable of resonance, i.e., drop $\exp(-i\omega t)$ in the equation for $\dot{b}_1$ and $\exp(i\omega t)$ in the equation for $\dot{b}_2$. The nonresonant terms in equations of the kind (9) due to a perturbing rf field, have been shown in connection with magnetic resonance [6] and the Lamb shift [7] to cause a shift in the resonance frequency. I will assume that any such shift is sufficiently small for the frequency band of the rf field to overlap the spread in Δ' even when the two are

matched without regard for the possible effect of the nonresonant terms.

The equations to be solved are then

$$\begin{aligned} \gamma \dot{b}_1 &= -\tfrac{1}{2}\Gamma_S b_1 + V_0 \exp[i(\omega - \Delta')t]b_2 , \\ \gamma \dot{b}_2 &= -\tfrac{1}{2}\Gamma_L b_2 - V_0 \exp[-i(\omega - \Delta')t]b_1 . \end{aligned} \qquad (12)$$

The general solution of these equations is a sum of two exponentials with coefficients determined by initial conditions. Because of its prime interest and for simplicity, I consider only the case of an initially pure K_2 beam. The result is

$$b_1 = \frac{V_0}{\gamma(\mu_2 - \mu_1)}\Big[\exp(-\mu_1 t) - \exp(-\mu_2 t)\Big] \qquad (13)$$

$$b_2 = \frac{\exp[-i(\omega - \Delta')t]}{\gamma(\mu_2 - \mu_1)} \times [(\tfrac{1}{2}\Gamma_S - \gamma\mu_1)\exp(-\mu_1 t) - (\tfrac{1}{2}\Gamma_S - \gamma\mu_2)\exp(-\mu_2 t)] ,$$

where

$$2\gamma\mu_{1,2} = \tfrac{1}{2}(\Gamma_S + \Gamma_L) + i(\Delta - \gamma\omega) \pm \{[\tfrac{1}{2}(\Gamma_S - \Gamma_L) + i(\gamma\omega - \Delta)]^2 - 4V_0^2\}^{\frac{1}{2}} . \qquad (14)$$

Consider now in more detail two limiting cases, characterized in terms of the critical potential V_C.

<u>Case A: $V_0 \ll V_C$</u>

In this case, neglecting Γ_L compared to Γ_S and to first order in

$$\rho \equiv \frac{4V_0^2}{\Gamma_S^2 + 4(\gamma\omega - \Delta)^2} ,$$

we have

$$|b_1|^2 \approx \rho\{\exp(-\Gamma_S t/\gamma) + \exp(-\Gamma_a t/\gamma) - 2\exp[-(\Gamma_S + \Gamma_a)t/2\gamma] \times \cos(\omega - \Delta')t\} ,$$

$$|b_2|^2 \approx (1 + 2\rho)\exp(-\Gamma_a t/\gamma) - 2\rho\{\cos(\Delta' - \omega)t + 2(\gamma\omega - \Delta)\Gamma_S^{-1}\sin(\Delta' - \omega)t\}\exp[-(\Gamma_S + \Gamma_a)t/2\gamma] , \qquad (15)$$

where $\Gamma_a \equiv \Gamma_L + \rho\Gamma_S$.

The overall factor in the K_1 intensity in (15) satisfies $\rho \ll \frac{1}{4}$ in this case. The lower limit of field intensities for which the expression (15) for $|b_1|^2$ is of interest corresponds to values of ρ such that the 2π decays due to the effect of the rf field can be observed above those attributed to CP violation, so that we want $\frac{1}{4} \gg \rho \gg 4 \times 10^{-6}$. The exponentials in two of the three terms in $|b_1|^2$ in (15) are field dependent, whereas the other one involves the short-lived decay rate Γ_S. If $\rho\Gamma_S \ll \Gamma_L$, i.e., for $\rho \ll 1.5 \times 10^{-3}$, the decay rate of the second term is that of the long-lived state in the absence of the rf field. But as ρ increases, i.e., for $1.5 \times 10^{-3} \lesssim \rho \ll \frac{1}{4}$, this decay rate increases and can become appreciably larger than Γ_L; e.g., for $\rho \approx 6 \times 10^{-2}$, we have $\Gamma_a \approx 40\ \Gamma_L$. The interference term is characterized by a decay rate average between the other two and modulated by an oscillatory factor that becomes unity at resonance.

The K_2 intensity in this case consists of two terms, the field dependence of each of which can be observed in the allowed range of intensities.

Case B: $V_0 \gg V_C$

To focus attention on the salient features, the results in this case will be given only for resonance. This amounts to dropping terms in $(\gamma\omega - \Delta)/V_0$ compared to unity. In this approximation, and to first order in (Γ_S/V_0), the K_1 and K_2 intensities are

$$|b_1|^2 \approx \exp(-\Gamma_S t/2\gamma)\ \sin^2(V_0 t/\gamma) ,$$

$$|b_2|^2 \approx \exp(-\Gamma_S t/2\gamma) \times [\cos^2(V_0 t/\gamma) + (\Gamma_S/4V_0)\sin(2V_0 t/\gamma)] . \qquad (16)$$

Several qualitative features of these expressions are worth noting explicitly. (1) The distinction between short-lived and long-lived states has disappeared. Both the K_1 and K_2 have a lifetime equal to twice that of the short-lived state in the absence of the rf field. This result is actually not confined to the case $V_0 \gg V_C$ but is true more generally for $V_0 > V_C$, as can be seen from Eq. (14); the square root in the expressions for $\mu_{1,2}$ becomes pure imaginary for $V_0 > V_C$ at $\gamma\omega = \Delta$, whence $\mathrm{Re}\,\mu_1 = \mathrm{Re}\,\mu_2$. (2) Since in this case we have $V_0 \gg \frac{1}{4}\Gamma_S$, the harmonic factors oscillate many times over one lifetime of either the K_1 or K_2. Hence for sufficiently intense rf fields, the K_1 and K_2 intensities reach the same average values before the damping factor in either intensity changes too drastically. (3) The oscillatory factor in the K_1 intensity reaches its first maximum in a sufficiently short time so that the corresponding cavity need not be unreasonably long: $(c\pi\gamma/2V_0) \ll 6\pi\gamma$ cm. (4) Because both the K_1 and K_2 lifetimes are very short, the portion of the initially pure K_2 beam with Δ' within the frequency band of the rf field can be significantly depleted within a relatively short distance. For example, for $t = (8\pi/\Gamma_S)$, we have $|b_1|^2 + |b_2|^2 \approx 1.8 \times 10^{-2}$ for that portion of the beam. This is to be compared to $|b_2|^2 \approx \exp(-8\Gamma_L/\Gamma_S) \approx 1$ in the absence of the action of the rf field. Hence, a consequence of interaction (1) is that the wave guide can serve as an energy filter for neutral kaons.

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