

UC Berkeley

Other Recent Work

Title

Monopolistic Behavior in a Market for Durable Goods

Permalink

<https://escholarship.org/uc/item/3dp780z5>

Authors

Douglas, A. J.
Goldman, S. M.

Publication Date

1967-08-01

COMMITTEE ON ECONOMETRICS AND MATHEMATICAL ECONOMICS

Working Paper No. 112

MONOPOLISTIC BEHAVIOR IN A
MARKET FOR DURABLE GOODS

by

A. J. Douglas and S. M. Goldman

August, 1967

Note: This working paper is duplicated for private circulation and should not be quoted or referred to in publications without permission of the authors.

INSTITUTE OF BUSINESS AND ECONOMIC RESEARCH

University of California, Berkeley

Consideration of the economic nature of the contractual arrangements entered into by the monopolistic manufacturer of a durable good gives rise to an interesting application of neo-classical capital theory to an important problem of industrial organization. This application is interesting, if not novel, in that the principles and techniques are rarely applied to partial equilibrium problems. An essential ingredient to an appropriately flavored economic analysis of the role of the rentals arrangement (the product is rented for a time to firms or households at an instantaneous rate) or of the sales arrangement (the product is sold to firms or households at a fixed, nondiscriminatory price) is the lack of complete information as to the future course of events on the part of all concerned. This aspect of the problem has been ignored in previous analyses [2, 3] of the economics underlying these contractual arrangements and is not fully treated here. First, we do not suppose that the relevant economic agents alter their actions in response to uncertainty; they act upon their expectations even though these expectations are continually being thwarted, and they are in no way influenced by the firmness, or lack of firmness (often quantified by some central moment of a subjective probability distribution held by the agent), of these expectations.

The analysis given below is not an application of the economics of risk, though it most certainly is not an application of the economics of perfect knowledge. In the absence of a more appropriate term, we

say that our analysis is of a description of economic behavior under ignorance. If the purchasers of the durable good behave according to a continuous version of the Cagan adaptive expectations model (even though these expectations are never fulfilled), we consider this to be an example of ignorant behavior. For, though adaptive expectations provide a good empirical explanation for the behavior of economic agents in a dynamic context, expectations are never realized and are not derived from postulates of rational behavior under uncertainty. Failure to form any expectations is neither more, nor less, rational than adaptive expectations--they are equally ignorant. Of course, the point of our analysis is that ignorance matters. This point is more paradoxical [1, 5] than those who are unacquainted with the mathematical treatments of expectations realize. For, in a sense, the purchasers of the durable good are continually forming mistaken expectations as to the future course of rentals prices and are ignorant at the margin. These expectations turn out to be asymptotically correct and are absolutely correct in a stationary state, but the precise formulation affects not only the dynamic path but the equilibrium solution as well.

The mathematical structure sketched out below is an investigation of the behavior of a monopolist who produces a durable good. The capital theoretic problems involved are handled with the optimization techniques developed by Pontryagin and his co-workers [4]. The mathematics are only briefly sketched; most of the written text is simply an exposition of the way in which the dynamic shadow price technique gives a deep and simple insight into partial equilibrium problems.

Our monopolist is described as wielding complete control over the production of new durables subject to an increasing marginal cost of production. He may then rent (or sell) these goods to competitive renters (or buyers). We shall analyze and compare his optimal production schedules subject to the following conditions on the nature of his market:

- I. The monopolist may only rent.
- II. The monopolist may only sell to buyers who in turn have perfect foresight regarding future quantities and prices.
- III. The monopolist may only sell to buyers who have stationary price expectations.
- IV. The monopolist may only sell to buyers who have adaptive expectations.

Clearly, the third and fourth specifications are somewhat different from the first and second in that they specify complete knowledge of future events for one side of the market (the sellers), while the other side (the buyers) slowly adjusts its predictions until, in the limit, they are correct. The novelty of our results is that in spite of this asymptotic convergence of expectations to reality, the impact of erroneous expectations leads to a sizable shift in the long-run equilibrium stock of capital.

I. The Monopolist Who Rents

The monopolist who may only rent will seek the equilibrium stock of durables such that the present value of the revenue time stream generated by an additional unit is exactly offset by the marginal cost of maintaining the stock.

The selection of the optimal stock is only part of the monopolistic problem. With an unchanging demand schedule for durable services, there is some optimal pattern of production for reaching this equilibrium and this dynamic behavior is of interest in its own right. Since the monopolist is faced with an ascending marginal cost of production schedule for current production, his desire to increase his capital stock (or, in this instance, the supply of durables) is partially offset by the rising cost of production. From our point of view, there is a more intimate connection between the optimal stock of capital and the optimal rate of investment than is customarily supposed in capital theory.

A precise specification of the model for the rentals case is relatively simple. Let R be that price which consumers are willing to pay for the services of a single unit of the durable good per unit of time. Thus, at time t , where $Q(t)$ is the existing stock of the durable good, the rental price is determined by $R(Q(t))$. Unless otherwise stated, marginal revenue is assumed to be positive and decreasing in Q , i. e.

$$(1) \quad \begin{aligned} R'(Q)Q + R(Q) &> 0 \\ R''(Q)Q + 2R'(Q) &< 0. \end{aligned}$$

Technological and pecuniary conditions of supply are summarized by the cost curve $g(q(t))$ where $q(t)$ is the rate of production of new machines. Marginal cost is positive and increasing in q , i. e.

$$(2) \quad \begin{aligned} g'(q) &> 0 \\ g''(q) &> 0. \end{aligned}$$

Capital is assumed to depreciate at a technologically constant rate δ , independent of its rate of usage. The net rate of change in capital is

given by

$$(3) \quad \dot{Q} = q - \delta Q$$

The firm's problem is to select a time path of output which will maximize the discounted value of the stream of profits, V_ρ , defined by the equation:

$$(4) \quad V_\rho = \int_0^\infty \{ R(Q(t))Q(t) - g(q(t)) \} e^{-rt} dt$$

subject to the constraints that

$$Q(0) = Q_0, \quad q(t) \geq 0 \quad \text{and} \quad Q(t) \geq 0 \quad \text{for all } t,$$

where r is a positive constant denoting the rate of interest and the revenue and cost curves have the shapes described above. The solution to this problem follows readily from the application of Pontryagin's Maximum Principle. The interpretation of the necessary conditions are of particular economic interest and are presented below.

Let ϕ be the shadow value to the monopolist of an additional unit of capital. Since we shall assume that the monopolist possesses perfect foresight, the capital gains plus the revenue from the marginal unit of capital must be equal to the sum of the interest and depreciation costs of holding it. Formally:

$$(5) \quad \frac{\dot{\phi}}{\phi} + \frac{R + R'Q}{\phi} = r + \delta.$$

At each moment in time the monopolist will produce up to that level where the shadow price of a unit of capital is equated to its marginal cost of production, hence,

$$(6) \quad g'(q) = \phi$$

At the optimal stationary stock of capital (or dynamic equilibrium) the stock of capital must be unchanging, so,

$$(7) \quad q = \delta Q.$$

From (6) it is clear that a change in the shadow value of capital would warrant a change in the rate of production as well, and thus for q to be stationary, it must be true that ϕ is also unchanging with time. Thus, the dynamic equilibrium solution for ϕ is derivable from (5), namely

$$(8) \quad \phi = \frac{R + QR'}{r + \delta}$$

Conditions (6) - (8) together imply that at the dynamic equilibrium the marginal cost of maintaining the stock is equal to the present value of the revenue stream generated by the marginal unit of capital, or

$$(9) \quad g'(\delta Q) = \frac{R + R'Q}{r + \delta}$$

Equation (9) uniquely determines the equilibrium capital stock since the LHS is increasing in Q and the RHS is decreasing. We assume that the ranges of marginal cost and revenue are sufficient to provide a solution.

II. Sale in a Market of Perfect Foresight

If the monopolist may sell his output, the problem becomes significantly more complicated since we must now specify the relationship between the demand curve for services and the demand curve for ownership. The question of expectations arises explicitly in this context.

We shall assume first that consumers possess perfect foresight with regard to the monopolist's future activities. Under these circumstances, the demand price for a unit of the durable must change in such a way as to equate the capital gains plus the value of services rendered per period with the interest and depreciation costs of ownership, or

$$(10) \quad \frac{\dot{P}}{P} + \frac{R}{P} = r + \delta.$$

Under these conditions of perfect foresight on the part of all concerned, there is no divergence between the behavior of the selling monopolist and the renter. The income stream from rentals has simply been converted into a lump sum payment with no change in the present value.

III. Stationary Expectations

Let us now suppose that rather than possessing perfect foresight, consumers expect the current rentals rate to persist indefinitely and thus the demand price for a unit of capital is equated to the present value of services ($R(Q)$) discounted at the rate $r + \delta$, or

$$(11) \quad P = \frac{R(Q)}{r + \delta}.$$

Under this assumption of stationary expectations, the monopolist will seek to maximize the present value of all future sales less the cost of production, or

$$(12) \quad V_p = \int_0^{\infty} \left\{ \frac{R}{r+\delta} q - g(q) \right\} e^{-rt} dt$$

subject to $\dot{Q} = q - \delta Q.$

Equations (5) - (9) may now be replaced by:

$$(13) \quad \frac{\dot{\phi}}{\phi} + q \frac{R'(Q)}{\phi(r+\delta)} = r + \delta$$

$$(14) \quad g'(q) = \phi + \frac{R(Q)}{r+\delta}$$

$$(15) \quad q = \delta Q$$

$$(16) \quad \phi = q \frac{R'(Q)}{(r+\delta)^2}$$

$$(17) \quad g'(\delta Q) = \frac{R(Q) + \frac{\delta}{r+\delta} QR'(Q)}{r+\delta}$$

The interpretation of (13) is quite similar to that for (5). ϕ retains the significance of the shadow value to the firm of another unit of outstanding durable stock, $\frac{qR'(Q)}{r+\delta}$ is the additional revenue generated by the existence of another unit of Q and $\dot{\phi}/\phi$ denotes the capital gain for the firm. Thus, again, in order to sustain the perfect foresight assumption on the part of the monopolists, we equate the rate of return on the durable stock to the rate of interest plus depreciation.

Equation (14) indicates that the marginal cost of durable production must be equal to the value of the income stream generated by a unit of new output plus the benefit (or damage) to the firm from a higher stock.

Conditions (15) and (16) are entirely analogous to the previous case and do not warrant repetition.

The equilibrium description (17) states that at the desired durable stock, the marginal cost of production is exactly offset by the income generated by the sale of another unit of durables plus the effect of sale upon the revenue from other sales.

The differences between these results and those obtained from the preceding cases arise from the ability of the monopolist to receive a higher price in a growing market than would be feasible under either rentals or sale with perfect foresight. Thus, instead of taking account of the falling cost of rental services for durables, the consumer consistently offers "too high" a price and the monopolist is encouraged to expand his production of durables beyond the level indicated by (9). This expansion is evident from (17) since for the same marginal cost of maintaining the durable stock, the marginal revenue under stationary expectations is greater than under rentals.

IV. Adaptive Expectations

The effect of adaptive, rather than stationary, expectations leads to a still greater durable stock. Briefly, the following explanation indicates the underlying reasoning: A growing durable stock induces a

falling rentals price. Adaptive expectations, however, give weight to the preceding higher prices and thus exhibit a tendency toward a sale price greater than both the stationary expectations price and the perfect foresight price.

The consumer will be supposed to have some long-range notion of the rental cost for durable services. This expected cost, R^e , is subject to modification whenever it diverges from the current rate according to:

$$(18) \quad \dot{R}^e = \beta(R - R^e).$$

For the sake of computational convenience, we shall assume this adjustment is proportional. The sale price, at any moment in time, is given by

$$(19) \quad P = R^e / (r + \delta)$$

and the monopolist seeks to maximize his current worth indicated by the discounted sum of future net earnings or

$$(20) \quad V_\rho = \int_0^\infty \left\{ q \frac{R^e}{r + \delta} - g(q) \right\} e^{-rt} dt$$

subject to the previous assumptions on the nature of $g(q)$ and $R(Q)$.

Necessary conditions for the maximization are derived

by the use of the Maximum Principle and yield the following relationships:

$$(21) \quad \frac{R^e}{r + \delta} + \phi_1 = g'(q)$$

that is, the marginal cost of production is equated to the sale price plus the value (loss) on future sales from the existence of a larger durable stock,

$$(22) \quad \begin{aligned} \dot{Q} &= q - \delta Q \\ \dot{R}^e &= \beta(R - R^e) \end{aligned}$$

and the perfect foresight conditions (efficiency) governing the dynamics of the shadow price of capital, ϕ_1 , and the shadow "price" of expectations, ϕ_2 :

$$(23) \quad \begin{aligned} \dot{\phi}_1 &= (r + \delta) \phi_1 - \beta \phi_2 R' \\ \dot{\phi}_2 &= (r + \beta) \phi_2 - q / (r + \delta). \end{aligned}$$

This last condition may be interpreted as the value to the firm of an exogenous upward shift in expectations at time t .

The above system of equations yields a unique saddle point solution so that in dynamic equilibrium the marginal cost of maintaining the durable stock, $g'(\delta Q)$, is equated with present value of future returns, i. e.

$$(24) \quad g'(\delta Q) = \frac{R(Q) + \left[\frac{\beta}{r + \beta}\right] \left[\frac{\delta}{r + \delta}\right] QR'(Q)}{r + \delta}$$

The Competitive or Socially Optimum Solution

Conditions of perfect competition on the parts of both producers and consumers along with a perfect capital market (and, of course, perfect

foresight) are sufficient to generate a socially optimum path of durable accumulation. In our simple framework, the flow of welfare produced at a moment in time by a durable stock Q is given by:

$$(25) \quad W(Q) = \int_0^{\infty} R(x) dx.$$

The maximization of the net social benefit, discounted by the interest rate, r , or

$$(26) \quad \int_0^{\infty} [W(Q) - g(q)] e^{-rt} dt$$

subject to the investment equation $\dot{Q} = q - \delta Q$ yields the following:

$$(27) \quad g'(q) = \phi$$

$$(28) \quad \dot{\phi} = (r + \delta)\phi - R(Q).$$

Interpreting ϕ as the shadow price of capital, equation (27) equates marginal cost and marginal social benefit; equation (28) indicates that the rate of capital gain plus earnings must equal depreciation plus interest costs. These efficiency conditions are exactly those implied by perfect competition.

The dynamic equilibrium stock of durables is the unique solution to the stationary equations, or

$$(29) \quad g'(\delta Q) = \frac{R(Q)}{r + \delta}.$$

The derived relationships determining the dynamic equilibrium durable stocks under the various assumptions, (9), (17), (24), (29), may

be exhibited simultaneously and compared. The solutions for Q are indicated by Figure 1, below.

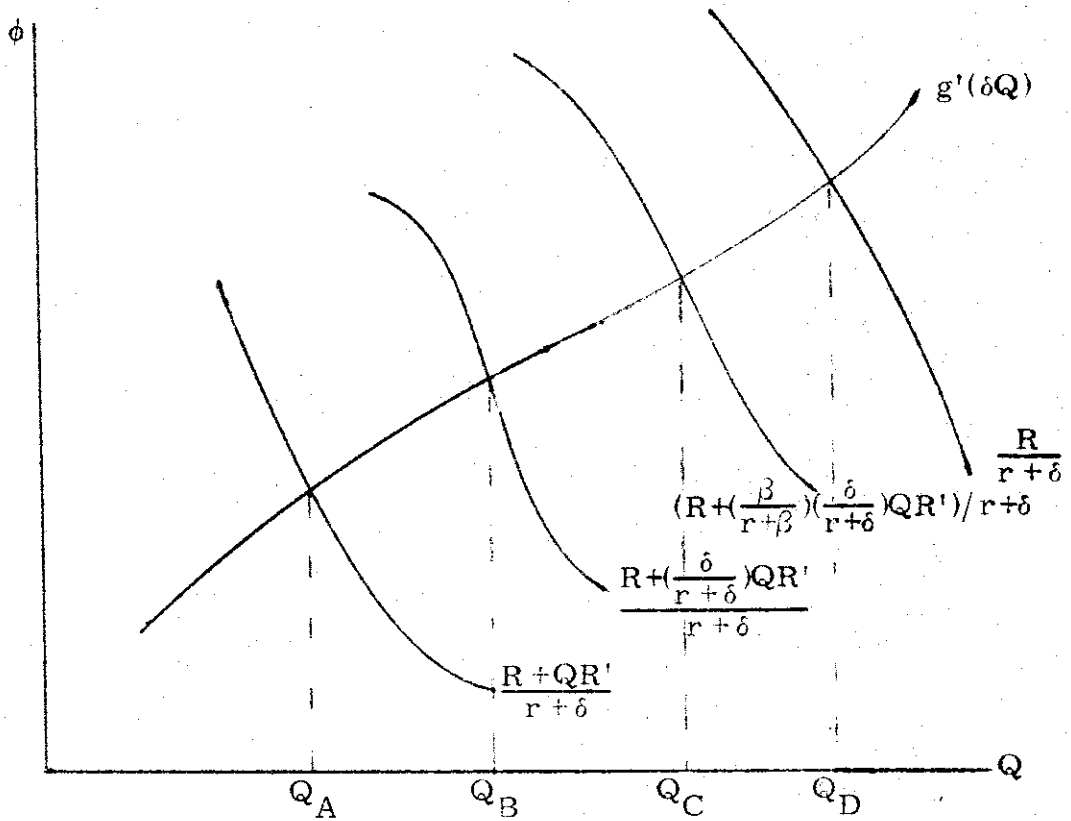


FIGURE 1

The ordering of the downward sloping curves is unambiguously derived from the negativity of $R'(Q)$ and the positivity of r , β , δ , and $R(Q)$. We see in this manner that:

- A. Rentals (or sale with perfect foresight) results in the smallest equilibrium stock of durables,
- B. Stationary expectations result in a larger stock,
- C. Adaptive expectations result in a still larger stock, and
- D. Under all of the expectational assumptions regarding the nature of consumer behavior, monopoly is restrictive in terms of the equilibrium stock; that is, the social optimum stock is greater than A - C.

The Relative Profitabilities of Rentals and Sales

We shall now address ourselves to the monopolist's query-- shall he rent or sell? Clearly, the revenue he can obtain through sales depends upon the manner in which consumers formulate expectations, but further, the relative profitabilities depend upon the size of the durable stock as well.

Before proceeding, let us note that since the behavior of a renting and selling monopolist, under conditions of perfect foresight on the part of purchasers, are identical in terms of the pattern of future production, the present value of future profits must also be identical (once the renter's stock of durables is capitalized). Thus, we are able to utilize the case of sales under perfect foresight as a surrogate for rentals.

Let us now proceed and compare the relative profitabilities of rentals and sales in a market with stationary expectations:

I. Suppose the durable stock is given by Q_A and expectations governed by stationarity. The monopolist has the opportunity to remain at Q_A and receive that revenue stream that he would obtain through sales in a market with perfect foresight (or rentals). Instead, he chooses to expand the durable stock to Q_B since stationary expectations permit him to charge a consistently higher price in an expanding market than would be possible under perfect foresight. Thus, it is more profitable to sell.

II. Suppose, instead, that the durable stock is given by Q_B and expectations are again stationary. The renting monopolist has the

opportunity to remain indefinitely at Q_B but chooses to diminish the durable stock to the level Q_A since under perfect foresight he may charge a higher price in a contracting market than under stationary expectations. Thus, rentals will yield the higher income.

The other market alternatives described in this paper may be compared in a similar fashion since we note that any stationary stock satisfies each of the expectations assumptions.

In conclusion, then, let us note the following:

I. The manner in which buyers formulate their expectations affects both the dynamic behavior and asymptotic behavior of a monopolistic durables market.

II. The relative profitabilities of renting and selling under various expectations mechanisms depend critically upon the existing stock of durables.

REFERENCES

- [1] Arrow, K. J. "Price Quantity Adjustments in Multiple Markets with Rising Demands," Mathematical Methods in the Social Sciences 1959. Stanford: Stanford University Press, 1960.
- [2] Douglas, A. J. "Monopoly and Market Organization," Working Paper No. 107. Berkeley: Institute of Business and Economic Research, University of California, May 1967.
- [3] Friedman, M. Price Theory. Chicago: Aldine, 1962, pp. 278-79.
- [4] Pontryagin, L. S. et al. The Mathematical Theory of Optimal Processes. New York: Interscience Publishers, 1962.
- [5] Solow, R. M. "Investment and Technical Progress," Mathematical Methods in the Social Sciences 1959. Stanford: Stanford University Press, 1960.