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Publication Date

2024-05-17

DOI

10.1007/s10857-024-09635-5

Peer reviewed

eScholarship.org



Examining teachers' relational noticing: promoting equity through positive interactions in mathematics education

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Accepted: 16 April 2024 © The Author(s) 2024

Abstract

How teachers attend to and interpret positive relational interactions shapes how they enact instructional practices for equity. We draw on frameworks from equitable mathematics instruction, relational interactions, and teacher noticing to conceptualize mathematics teachers' relational noticing. Using noticing interview and classroom observation data from a research collaborative between secondary mathematics teachers and university-based teacher educators, we document the range and diversity of ten teachers' relational noticing. We use this analysis to examine how teachers' relational noticing supports enacting equitable instructional practices. Our findings indicate five themes of teachers' relational noticing that are informed by their personal histories, understanding of dominant narratives of mathematics education, and their local sociopolitical school context. Additionally, teachers enacted a range of practices for creating positive relational interactions, with attending to student thinking being the most enacted practice. Our findings suggest that mathematics teachers' relational noticing can support the three axes of equitable instruction.

Keywords Teacher noticing \cdot Relational interactions \cdot Equitable instructional practices \cdot Relational noticing \cdot Mathematics education

Introduction

Black, Hispanic, and Indigenous students in the US have historically experienced mathematics classrooms as emotionally charged spaces (Gutiérrez, 2018; Martin, 2009; Milner, 2013). Despite engaging in similar behaviors as their white peers, Black and Latino students are more likely to receive disciplinary action from teachers (Skiba et al., 2011). Battey and Neal (2018) document various relational interactions that mathematics teachers have with their Black and Latino students and find that addressing behavior tends to be a negative experience for students. Other relational interactions that shape students' learning includes how teachers attend to students' language, culture, and the overall emotional tone of the classroom (Battey & Neal, 2018). These interactions become building blocks for

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teacher-student relationships, yet they are not typically prioritized to support students from diverse backgrounds.

These issues are not unique to the US. Internationally, gender shapes how students experience mathematics, with boys having more positive attitudes toward mathematics and higher participation in higher levels of mathematics courses (Forgaz & Leder, 2017; Hall, 2012). Gender also commonly intertwines with other social and political factors. For instance, Palestinian women who attend Arab-sector schools in Israel experience structural and psychological harm in mathematics education that are related to religious, racial, and linguistic group marginalization (Rubel & Ehrenfeld, 2020). Similarly, decades of apartheid in South Africa, a product of colonialism, created different educational systems for Black and White students (Spaull, 2013; van der Berg, 2007), leading to a generation of students who associated math learning with belittling and humiliating treatment from teachers (Askew & Venkat, 2017). Thus, mathematics education is experienced as emotionally challenging internationally.

Research finds that teachers' attention to the relational aspects of classroom interactions is as important to the intellectual aspects when pursuing educational equity for culturally diverse students (Bartell, 2013; Bartell et al., 2017; Rubel, 2017). Positive student–teacher relationships promote higher levels of behavioral engagement and academic outcomes for students (Decker et al., 2007; Roorda et al., 2017), a higher sense of belonging, and feeling that their intellectual contributions matter (Maloney & Mathew, 2020; Sakiz et al., 2012). It is therefore important to attend to multiple axes of equitable instruction.

We situate the importance of cultivating positive relationships within the teacher noticing literature. This line of research identifies the role of teachers' attention to and interpretation of classroom interactions as consequential for developing student-teacher relationships (Hand, 2012; Kang, 2022). Specifically, teachers who interpret student mathematical thinking from a strengths-based perspective cultivate environments more conducive to positive student-teacher relationships (Bartell, 2011; Battey et al., 2016; Hinsdale, 2016). However, few studies have sought to characterize the noticing that supports teachers' enactment of positive relational interactions. This study builds on this research by exploring the nature of teachers' noticing in mathematics classrooms that supports enacting affirming and equitable instructional practices. Specifically, this study is guided by two questions:

RQ1 What is the nature of teachers' noticing of positive relational interactions?

RQ2 What is the relationship between teachers' noticing of positive relational interactions and their enactment of equitable instructional practices?

Theoretical perspective and related literature

We draw on two lines of research to situate our inquiry into teachers' noticing related to relational interactions. First, research argues for expanding theories of equity in mathematics to more centrally integrate affect and emotion (Bartell, 2011; Kokka, 2022; Martínez-Sierra & García González, 2014). Because relational interactions are more salient for racial, cultural, and linguistic minority groups (Battey et al., 2022; Valoyes-Chávez & Darragh, 2022), teachers must recognize how students experience these interactions and how their practices contribute to disrupting inequitable learning opportunities for minoritized youth. We bring this literature in conversation with sociopolitical perspectives of

teacher noticing (Hand, 2012; Louie, 2018) to investigate the nature of teachers' awareness and understanding of the historical, cultural, racialized, and political nature of mathematics education schooling and its relationship to cultivating affirming and equitable classrooms (Louie et al., 2021; van Es et al., 2017, 2022). Drawing from these areas of research, we assert that teacher noticing, as a practice of historical beings, is inherently relational and can further our understanding of classroom interactions.

Integrating the relational dimension with theories of equitable mathematics instruction

Existing models of equity focus primarily on students' academic achievement and their developing identities as mathematics learners (Nasir, 2002; Rubel, 2017), which is referred to as the dominant and critical axes of equitable mathematics education (Gutiérrez, 2011). The dominant axis includes expanding students' access to rigorous mathematics tasks and courses and supporting their academic success (Berry, 2008; Wilson et al., 2019). The critical axis recognizes issues related to power such as how culture shapes students' mathematical experiences and their potential to develop positive identities (Martin, 2009; Rubel, 2017). Kokka (2022) proposes a third axis related to teachers' affective pedagogical goals that includes teachers' awareness of students' well-being, how students experience mathematics, and schooling as a dominant practice (Battey & Neal, 2018; Hinsdale, 2016). These conceptualizations of equitable mathematics teaching recognize that broadening participation (dominant axis) and expanding students' opportunities to develop positive identities and to shape the mathematics in which they engage (critical axis) are inextricably linked to student well-being and the relationships that are cultivated in the mathematics classroom (affective axis).

To understand how teachers advance these axes in their classroom, we draw on the *relational interactions framework* (Battey & Neal, 2018) that identifies five dimensions to promote positive relationships: addressing behavior, framing ability, acknowledging student contributions, attending to language and culture, and setting the emotional tone. These dimensions are reflected in research that conceptualizes equitable instructional practices (Bartell et al., 2017; Berry, 2020). Thus, we see the relational interactions framework as advancing all three axes of equitable instruction (see Table 1).

The dominant axis is realized through teachers' *framing ability* and *acknowledging students' contributions*. This is similar to the literature that identifies "attending to student mathematical thinking" as an important aspect of equitable mathematical teaching (Bartell et al., 2017). How teachers understand students' "abilities" can be consequential for students who may not demonstrate their knowledge in traditional forms of assessment, such as quizzes or tests. Teachers' framing of students' abilities is particularly important for marginalized students whose teachers have been found to perceive them from deficit perspectives (Horn, 2007; Jackson et al., 2017). Framing students' abilities also impacts how teachers *acknowledge students' contributions* and further support or hinder access and achievement (Nasir et al., 2013). When teachers recognize the contributions of racially and linguistically diverse students from a strength-based perspective, they are more likely to value and use their ideas to move students' learning forward. Bringing these dimensions together with the dominant axis illuminates how supporting students' access and achievement can be relational.

Table 1 Relational in	nteractions for equitable instruction	
Equitable instruction axes	Relational interactions	Example teacher comment
Dominant	Framing ability Acknowledging students' contributions	"My honors students don't struggle with this unit as much as my regular students" "Timmy brings up a good point"
Critical	Acknowledging students' contributions Attending to language and culture	"Maria, that was a really interesting idea. Can you share it again for the whole class to hear?" "I heard him beatboxing so I wanted to use that to show a nonlinear function"
Affective	Addressing behavior Setting the emotional tone	"See, you've got this Juan, you can do this!" "The ready." activity. I know you all are ready."
	Attending to language and culture	"I want my students to see a Hispanic math teacher so they can see that it's possible to have a future in math."

To advance the critical axis, teachers can *acknowledge students' contributions* and *attend to language and culture*. Teachers carry more power in a classroom, thus allowing them to create or shift classroom dynamics by whether and how they acknowledge (or ignore) student contributions (Moschkovich, 2011). For example, Philip et al. (2016) describe a classroom lesson in which students analyze graphs that reflect movie-viewing patterns by neighborhood. One student, William, interpreted the data from a racial solidarity lens. Despite William's multiple attempts to advance his claim, his teacher did not recognize his contribution from this lens, resulting in William disengaging from further discussions. By not recognizing and valuing William's contribution, the teacher removed William's opportunity to develop a mathematical identity. Conversely, acknowledging student contributions is a positive relational interaction (Battey & Neal, 2018) and can give agency to students whose ideas are typically unappreciated.

By attending to students' language and culture, teachers can better connect their curriculum and students' identities. Lunney Borden (2013) describes how a veteran teacher of indigenous Canadian children (Mi'kmaq), Richard, revises the order of the curriculum—teaching fractions before multiplication-based on students' language containing various words dealing with the concept of 'sharing' equally. Other teachers resisted reordering these concepts in the curriculum, but also acknowledged their resistance was rooted in an assumption that they should teach the way they learned. This example illustrates how teachers' awareness of students' linguistic resources can support students' learning and how prevailing mathematics practices constrain teachers' instruction and ability to cultivate caring relationships.

The affective axis includes *addressing behavior, setting the emotional tone,* and *attending to language and culture.* Addressing behavior is commonly found to be a negative experience for teachers and students (Battey et al., 2016). Research finds that some teachers interpret students' negative display of emotion as an inability to self-regulate and decide to punish the student (Schonert-Reichl, 2017). Yet, when a caring relationship exists, teachers are more likely to interpret students' display of emotion as prior trauma with mathematics content, frustrations with previous math educators, or simply feeling a disconnection to mathematics (Stephens, 2021). Thus, building positive classroom relationships can reduce disciplinary actions.

Similarly, a classroom's emotional tone communicates to students acceptable behaviors and recognizes tensions students may experience by challenging normative mathematical practices (Eynde et al., 2006). Teachers can also communicate acceptable social behavior by attending to language and culture, as they relate to mathematical learning. Connecting to students' cultural identities can help teachers promote positive relationships and enable students to feel that they are valued contributors of mathematics. For example, Newcomer (2018) documents students' increased sense of safety and support when their teachers understood the process of learning a second language. These findings suggest that teachers' awareness and interpretations of students' emotions and identities can shape how they interact with students to advance their affective pedagogical goals.

Overall, this literature suggests that teachers' equitable instruction is influenced by their awareness of mathematics education as a racialized practice. We turn to sociopolitical perspectives on teachers' noticing to theorize how teachers' understanding of narratives and histories of minoritized students, mathematics education, and schooling shape their noticing to cultivate positive relational interactions.

Teacher noticing of the relational aspects of classroom

Early research characterized teacher noticing as consisting of two broad dimensions attending and interpreting—that are informed by teachers' knowledge of content, students, and broader principles of teaching and learning (van Es & Sherin, 2008). This literature was less considerate of social and cultural aspects of math education that shape students' classroom experiences. Recently, sociopolitical perspectives illuminate how teachers' classroom noticing is shaped by their histories, their students' histories, and the sociopolitical and historical nature of mathematics education (Kalinec-Craig, 2017; Jessup, 2018; Louie et al., 2021; Mendoa et al., 2021; Schack et al., 2017; van Es et al., 2022). This perspective of noticing makes visible how teachers' in the moment attention and interpretation can perpetuate or disrupt inequitable learning opportunities for youth.

First, sociopolitical perspectives find that achievement and identity narratives shape teachers' classroom noticing (Louie, 2018; Zavala & Hand, 2019). These narratives influence which students are assigned mathematical competence, how competence is characterized, and whose participation is valued. Research contends these narratives serve to uphold White normative practices and routines that position Black and Brown students from deficit perspectives to maintain status differentials (Martin, 2019; Shah, 2017). Louie (2018), for example, illustrates how a teacher, Amanda Pepper, sought to disrupt narratives of "smartness" in the mathematics classroom by adopting asset-based lenses to interpret student participation. Despite Amanda's commitment, she struggled to disrupt narratives related to students' fixed mathematical abilities because of how deeply ingrained this is in mathematics education. These findings suggest that creating positive relational interactions is supported by teachers' awareness of prevailing narratives, and how these shape their attention and understanding of classroom phenomena.

Secondly, teachers' noticing is integrated with their understanding of students and themselves as historical beings (Mendoza et al., 2021; Penteado & Skovsmose, 2011). van Es et al. (2022), for example, provide two cases of secondary mathematics teachers' noticing for equity. They found that these teachers' awareness of students' contributions, behavior, and emotions was shaped by their understanding of students' prior mathematical experiences. Specifically, they recognized that students from racial, social, and linguistic minority groups were offered limited mathematically rigorous opportunities and were positioned from deficit perspectives. One teacher, Raymond, observed how students expressed themselves and sought to integrate that with mathematical learning to foster a sense of belonging. This example suggests the importance of teachers' attention to their students' rich social histories to promote positive interactions.

Finally, sociopolitical noticing perspectives recognize how teachers' understanding of the historically racialized nature of schooling shapes their noticing practices. For example, a teacher might interpret a student not having a writing utensil or entering a classroom late as oppositional and confront the student for being unprepared and lacking punctuality. Alternatively, a teacher might recognize how school policies that prioritize punishment are inherently inequitable (Yang, 2009) and can interpret these same behaviors from an affirming lens and instead assist a student with starting the task to avoid potential confrontation. Hand (2012) describes how a teacher interacts with a group of students who explain they cannot work because of an invisible wall between them. The teacher asks the students "How are we going to break the wall? That's what I want to know" (p. 240). Here, the teacher engaged with the students' metaphor to bring them toward the content, rather than interpreting through a discipline lens. Thus, teachers' awareness of White normative practices and how they position students is crucial to cultivating positive interactions that affirm students' humanity.

We draw on these findings to investigate how teachers' relational noticing is shaped by their awareness and understanding of sociopolitical contexts of mathematics education that maintains inequitable learning opportunities for culturally, racially, and linguistically diverse students. In doing so, we aim to advance a framework for relational noticing, while also investigating if and how their relational noticing supports equitable mathematics instruction. We now turn to describe the research context, methods, and data collection and analytic procedures.

Research design

Study context

This study draws on data from a multi-year research collaborative between university-based teacher educators, community educators, and secondary mathematics teachers. Informed by community-based design and participatory action research (Bang & Vossoughi, 2016; Zeichner, 2010), the research collaborative aimed to theorize teachers' noticing for equity and develop a professional learning model to support mathematics teachers' learning to notice for equity. The research partnership is located in a school district in the western US that served a predominantly Hispanic/Latinx population (70% across the district) at the time of data collection. This district has a long-standing partnership with the authors' university and is committed to increasing students' access toward post-secondary education. Additionally, during the time of data collection, the district offered several professional development programs to support teachers' shifting their instruction to elevate student voice and agency.

The first cohort of participants included secondary mathematics teachers, community educators, and university researchers who participated in a series of professional learning meetings focused on equitable mathematics instruction, classroom observations and associated interviews, and monthly team meetings that consisted of viewing and discussing video segments of classroom instruction. All but one teacher had over five years of experience teaching mathematics and were nominated by the district mathematics coordinator for enacting equity-oriented practices. The community youth educators were identified and recruited by the research team because their affiliated organizations advocated for local youth toward social activism. Importantly, this collaborative developed a professional learning model organized around six themes (see Appendix A) for advancing equitable mathematics teaching and noticing for equity. Three teachers from the first cohort took on leadership roles within the group to plan and facilitate professional development, video clubs, and noticing interviews with a second cohort.

Methods and data sources

We draw on data collected from two cohorts of teachers (all of whom are given pseudonyms) who participated in the research collaborative in 2018–2019 and 2021–22. Cohort 1 consisted of four middle and secondary mathematics teachers and Cohort 2 consisted of nine teachers, two from Cohort 1 and seven additional teachers, for a total of 11 teachers. Five members of the research team (the second author, two post-doctoral researchers,

and two graduate student researchers) observed and video recorded in each teachers' classroom. The researcher who observed the lesson conducted a noticing interview within one week of the observation to gain insight into teachers' classroom noticing practices. Noticing interviews are similar to clinical interviews, which seek to elicit reasoning processes that drive the actions observed in the classroom episodes (Hunting, 1997). From the observation, the researcher identified segments where teachers appeared to be enacting affirming and equitable instructional practices (Bartell et al., 2017; Berry, 2020). They then shared these segments with the research team and together identified up to six segments for the teacher to view and discuss in the noticing interview. The interview aimed to elicit what teachers were focusing on during the selected segments, as well as how they interpreted the interactions they observed. Teachers were briefed prior to starting the interviews that the goal was to capture what they were thinking during these moments and that they could also select moments that they found noteworthy. After viewing each clip, the researcher asked open-ended questions, such as: What kinds of things were you paying attention to during this interaction? What stood out to you? Why was that important to you? What were you feeling in that moment? Teachers were also invited to discuss why they chose to pursue particular students' ideas or behaviors; why they responded to students in a particular way; how they decided which students to engage with; and to discuss how the students' experiences in the teachers' classroom, at home, and the community might be relevant. For each interview, teachers viewed and discussed between two and six of the selected segments. In turn, each video segment that the teachers viewed elicited between one and six comments. On average, teachers made 9.5 comments over the course of the entire noticing interview.

Teachers in Cohort 1 participated in up to six observations and noticing interviews. The first three observations and interviews took place in spring 2018 and an additional three the following academic year. The interviews with Cohort 1 were conducted in teachers' class-rooms or in a researcher's office. Interviews lasted approximately one hour and were video recorded. These interviews were sent to an external source (e.g. Otter) for initial transcription. Teachers in Cohort 2 participated in the professional development in 2021–22 and were observed and interviewed three times from January to March of 2022. The noticing interviews with this cohort were conducted within one week of the observation and video recorded using Zoom, which generated initial transcripts. The first author reviewed and cleaned each interview transcript for both cohorts. We excluded five of the Cohort 1 interviews from the data corpus due to technical difficulties, missing video segments, or the discussions in the interviews not focusing on the observed classroom lesson.

Data analysis

Data analysis was qualitative in nature and followed an iterative analytic process for both research questions (Miles et al., 2014). First, to examine whether and how teachers characterized their noticing practices to promote positive relational interactions, we analyzed the noticing interview videos and transcripts using Transana. To answer research question two, we turned to the full recording of the classroom observation data to consider how the teachers' relational noticing supported their enactment of equitable and affirming instructional practices more broadly (van Es et al., 2022). For both questions, we used a top-down, bottom-up approach that was both informed by research, while also accounting for nuance and variation across teachers' noticing and practice (Erickson, 1985). We provide a detailed description of our analytic process for each question below.

Teachers' noticing of relational interactions

Analysis for research question one was conducted over three phases. Similar to van Es et al. (2017, 2022), the first phase involved open coding three interviews for the four teachers in Cohort 1 to generate a list of what they attended to in each clip, how they interpreted what they saw, and how they explained the instructional practice observed in the video segment. We segmented teachers' comments during noticing interviews based on when teachers raised new topics of focus. Each unique comment became a unit of analysis (Jacobs & Morita, 2002). The first author then returned to the open coding to identify teachers' comments related to noticing the affective and relational aspects of classroom interactions. Sometimes teachers commented directly on students' feelings, emotional well-being, and relationships. In other instances, they commented on other aspects of the interactions, such as how students worked together or how a student participated in class the previous day or week leading up to the observation. However, their interpretations of these observations often included reference to students' feelings, wellbeing, and relationships. We then used the relational interactions framework to code idea units that included comments related to the five dimensions (Battey, 2013; Battey & Neal, 2018). An additional dimension emerged that captured teachers' attention to their own emotions. We included this sixth code because teachers often referenced their own emotions and embodied awareness as they narrated their classroom noticing. We then excluded comments that were not coded as one of the six relational interactions, which ranged from 20 to 50% of comments for Cohort 1 teachers.

In the second analytic phase, we coded an additional three noticing interviews per Cohort 1 teacher using our process and the six codes from phase one, for a total of 24 interviews. We then created data matrices that summarized and categorized each teachers' comments by what they attended to, their interpretations of the interactions, and the related instructional responses (Miles et al., 2014). Here, we illustrate an example of how we created the data matrix from interview transcripts. The following is one idea unit from a teacher's (Alondra) interview.

Interviewer: How are you thinking about group members interacting with each other?

Alondra: They should be walking, working with each other and that's something that I noticed has been hard [for students] to work with each other. So one of the things I need to do is, I think I could do a better job of it too, is okay, I know, you know it, but make sure that he knows it and she knows it, like explain it to them and go around that way. But can he explain it? Can she explain it? He knows how to do it. Now explain it to me.

Interviewer: What I hear you saying now is sort of like, can they explain it to each other? Can they explain it to you? And then like, do you also have a layer where you try to get them to explain it across the class, to each other so that there's this like accountability across the class?

Alondra: Actually, if they're done with everything and depending on if I don't have an extension, and if they're done, I will have them as peer tutors. Because I don't have a student teacher, I don't have TAs, so I will tell them, "Can you please help me and go around and help me help your classmates?"

For this example, the data matrix noted Alondra's attending as "which students are done faster with the math task"; the interpretation was recorded as "students that finish faster can help their classmates"; and the instructional response was captured as "positioning students as teaching assistants." Alondra's discussion in this example was coded as relational noticing because it characterized one of the six dimensions (*framing math ability*). We then reviewed the data matrices for each teacher and wrote analytic memos that highlighted patterns in how they described objects of their attention and how they made sense of these interactions while teaching. Using analytic memos, the first author looked for patterns within and across teachers and identified six emerging themes that characterized their relational noticing (Miles et al., 2014).

Using interaction analysis methods (Jordan & Henderson, 1995), the authors shared emerging findings with their research group. The first author selected segments from three teachers' interviews. The group then read through the interview data along with our proposed themes to ensure that they were consistent across a larger data set. Because our analysis was largely interpretive in nature, this process focused on ensuring trustworthiness in interpretations (Nowell et al., 2017). This process also raised discussion related to how historicity and narratives were embedded in teachers' comments. The first author returned to the data matrices and analytic memos to further nuance the emergent themes from the sociopolitical perspective of mathematics education and schooling. This process led to consolidating the themes into five categories and refining the language used to define them. To ensure that emergent themes were grounded in the data, we then returned to the data to connect different teachers' comments onto our five themes (Nowell et al., 2017).

Finally, the third phase of analysis was largely concerned with ensuring reliability and trustworthiness in our emerging framework (Nowell et al., 2017). We included one interview per teacher from Cohort 2, for an additional seven interviews, to confirm and refine our characterizations of teachers' relational noticing. The first author selected the noticing interview by examining field notes conducted by the research team that indicated lessons that featured significant relational interactions. We applied the coding process from phase two to these interviews. We excluded one teacher's interview because her noticing of relational interactions was largely focused on students' misbehavior and classroom management—practices that cultivate negative relational interactions. We also wrote analytic memos to capture additional nuance in teachers' noticing. We used this analysis to confirm our inferences and further refine our characterizations of teachers' relational noticing.

Relational noticing that supports teachers' enactment of equitable mathematics teaching

Our second research question asked how teachers' relational noticing supported their enactment of equitable instructional practice, which we analyzed in two phases. In the first phase, we selected one classroom observation for each of the ten teachers. For Cohort 1, we selected the classroom observation that featured the greatest amount of relational noticing. For Cohort 2, we analyzed the classroom observation from the associated noticing interview from the previous analyses. We analyzed the entire lesson, as opposed to only the segments the teachers viewed in the noticing interviews, because the segments varied in length across the interviews and not all teachers commented on all segments due to time constraints. Since the lessons' duration were roughly equivalent, making the lesson the unit analysis provided a more comprehensive analysis of practice.

Because we theorized that relational noticing supports the three axes of equitable instruction, we developed a coding scheme consisting of 11 practices that would be sensitive to those three dimensions (Bartell et al., 2017; Kokka, 2022). Specifically, we

reviewed the literature to identify recurring instructional practices that overlap with the five relational interactions framework (see Table 2).

This allowed us to identify practices in more fine grain detail that supported teachers creating positive and affirming interactions. We used five-minute time sampling to code for the 11 teaching practices. We first viewed the classroom observation videos and noted which, if any, of these practices were evident in each segment. When teachers enacted more than one practice within a five-minute segment, we counted each practice as a separate piece of evidence. To characterize the type of equitable teaching practices for each teacher, we tallied the total number of instances of each practice they enacted in the lesson. All teachers enacted at least one equitable instructional practice in at least 80% of the five-minute segments.

Our findings revealed that teachers' instruction was more focused on a limited number of practices and aligned with the dominant and affective axes. In this second phase of analysis, we wanted to understand whether and how teachers' relational noticing and instructional practices advanced these equitable axes. Now, with the findings from research question 1, we examined the relationship between patterns in their noticing and practice and the axes of equitable instruction they advanced. We selected two teachers from Cohort 1 (Javier and Sharron) as cases because their noticing of relational comments and amount of observed equitable instructional practices were greater than the other two teachers. The first author then wrote an analytic memo to understand the various aspects of teachers' noticing to identify similarities and differences across these two teachers' noticing patterns and instructional practices. Both authors reviewed the memo and identified strong

Relational interactions	Instructional practice
Address behavior	Center relationships and healing to support student well-being/ emotions Attend to students' mathematical thinking
Framing ability	Positioning students as capable Monitor how students position each other Press for academic success Attend explicitly to race and culture
Acknowledging student contributions	Attend to students' mathematical thinking Establish norms for participation Position students as capable Draw funds of knowledge Press for academic success Recognize discourse and language as a resource Attend explicitly to race and culture
Attending to language and culture	Draw funds of knowledge Recognize discourse and language as a resource Attend explicitly to race and culture
Setting the emotional tone	Establish norms for participation Center relationships and healing to support student well-being/ emotions Press for academic success Recognize discourse and language as a resource Support students identifying & processing emo- tions in relation to dominant math

Table 2 Coding framework of instructional practices related to relational interactions

similarities between the two teachers' relational noticing patterns but notable differences between their instructional practices. The first author returned to the data matrices for all teachers and identified *math aversion* as a recurring theme and then analyzed the two cases to understand how relational noticing interacted with particular instructional practices (see Table 2) to support different equitable axes.

Findings

Data analysis revealed that teachers' noticing of relational interactions was informed by their personal histories, dominant narratives related to mathematics learning, and school and district-level policies and practices. Their noticing of relational interactions supported the three different three axes of equitable instruction—dominant, critical, and affective. Across the teachers' noticing, we observed a connection between the relational and content goals for their students, demonstrating the advancement of the dominant and affective axes. This finding was reinforced by analyzing teachers' instruction, in which the most enacted equitable practice was *attending to students' mathematical thinking*. These findings suggest that teachers notice the relational aspects of their classroom and enact practices to support their students' academic success. We present our findings for each research question and then discuss how teachers' noticing supports teachers creating positive relational interactions for equitable aims.

Teachers' noticing of relational interactions

Our first research question sought to characterize the nature of teachers' noticing of positive relational interactions. We found that teachers' noticing of relational interactions fell into five main categories (see Table 3). These categories reflect how teachers drew on their personal histories, understanding of the schooling and mathematics practices, and dominant narratives about mathematics learning to inform what they focused on and how they understood these interactions.

Student well-being

Seven of the ten teachers' noticed toward student well-being, which captured their awareness of students' physical and emotional needs during instruction. Notably, teachers' attention to students' well-being was informed by their personal histories as students who received varying levels of care from teachers. Drawing on their relational interactions as former students, these teachers aspired to promote positive interactions with their students. They expressed a commitment to understanding how their students felt in their classrooms, by attending to students' body language, peer interactions, and levels of engagement. Leslie, for example, administered a Google survey at the start of class that provided a colorcoded output of students' feelings each day. Leslie considered herself to be a mother-figure for her students and explained that this tool allowed her to quickly assess which students might require more patience or additional support.

Teachers also shared their awareness of the high levels of home and food insecurity that students in their school district experienced. Thus, two teachers intentionally checked for signs that students might not have sufficient food or clothing. They connected access to basic needs with students' abilities to engage meaningfully in mathematics activity. Krystal

	Attending and interpreting	Instructional response	Teachers
Student well-being	Attend to students' physical and emotional well-being and the consequences for student learning	Set emotional tone by supporting students' basic needs, address students with respect, and avoid punitive interactions	Javier Sharron Alejandra Kim Luis Krystal Brian
Students' personal lives	Attend to students' cultural practices, local communities and home lives and how that shapes their classroom behavior and participation	Set emotional tone by responding to students with care and creat- ing an environment where mathematics can be fun and joyful for students	Javier Sharron Kim Alondra Leslie
Schooling practices	Attend to the history and evolution of school-imposed practices (e.g. timelines and curriculum) and how they constrict student learning	Attend to language and culture by connecting mathematics with students' interests and broadening traditional mathematics	Javier Sharron Leslie Alondra
Potential for conflict	Attend to students' misbehavior or disengagement and understand how a teacher's response could lead to conflict	Address student behavior by "not noticing" and minimizing and redirecting attention to interactions that could lead to further student-teacher tension	Javier Sharron
Math aversion	Attend to students' past mathematical experiences and how those might lead to adverse reactions to mathematics	Frame ability and acknowledge student contributions by praising students for progress and achievement	Javier Sharron Alejandra Kim Luis Leslie Brian John

Table 3 Types of teachers' relational noticing and associated instructional practices

The "Teachers" column only includes teachers who discuss attention to and interpretation of relational interactions

explained how she designated an area in her classroom that was stocked with granola bars, bread, and peanut butter which students could use without needing to draw much attention. These teachers' awareness of students' physical needs and contexts informed how they interacted with students to cultivate positive classroom contexts.

For some teachers, their attention to students' well-being was informed by their personal histories as math learners. Three Latinx/Hispanic identifying teachers—Luis, Alejandra, and Javier—shared experiences of university math instructors and classmates who positioned them as not "smart" enough to be mathematics majors, which resulted in feeling inadequate about their mathematical abilities. These harmful experiences made them aspire to create more welcoming environments, so they attended to students emotional experiences in their classrooms. Luis explained how impactful a teachers' tone of voice could be, stating that if a teacher speaks to students in a dismissive tone then it "takes the rug from beneath their feet, and they don't want to stand anymore". In other words, he recognized that a teacher's tone is capable of discouraging students from academic participation. These examples illustrate how teachers drew on their emotional experiences as former students to allow them insight into how they could create a supportive classroom environment to support their students' well-being.

Students' personal lives

Another category of teachers' relational noticing includes an awareness of how students' personal and home lives can shape their classroom participation. Teachers attended to students' friend groups, celebrations, and familial hardships, which helped them determine how to interact with students. For instance, when Jaiver commented about a student who asked for a pencil but was not doing any work, he shared his awareness that this student was experiencing conflict at home and that mathematics content was not a priority on this particular day: "For a student like her, she's in a space where she's not ready to be an active learner in the math class. So, my role becomes how do I get her to feel like she's safe just being there." Javier interpreted that this student's hardship would preclude her from meaningful participation and that pressuring this student to participate would yield a negative interaction. Thus, Javier prioritized offering grace in that moment over pressing her to engage with the mathematics. Informed by his own experiences with mathematics teachers who expressed confidence in him, he determined that this response was appropriate in the moment and would have a greater positive impact on the student over time, by building a trusting relationship.

By attending to students' peer dynamics, teachers also expressed the need to be mindful of student groupings. This attention was shaped by teachers' recognition of how important the social context of schooling could be on students' classroom participation.

Schooling practices

A third dimension of teachers' noticing relates to their awareness of and frustrations with schooling practices that constricted their ability to build quality relationships with students, such as standardized testing and the fast pace of the mathematics curriculum. Several teachers commented on the large amount of time that building relationships required and the importance of these relationships to students' year-long learning. During an interview, Javier discussed an instance when a student came into the classroom and Javier asked if another teacher sent him there. The student responded yes, and Javier directed him to sit

in an open chair next to his desk. Javier shared that he and the other grade level mathematics teachers agreed to send students to other teachers' classrooms if they did not abide by their teachers' behavioral expectations. He understood this policy to preclude teachers from building relationships with students who would academically and emotionally benefit the most from positive student–teacher relationships.

Javier experienced a tension in needing to focus on a small number of students who required more attention that precluded him from building substantive relationships with all of his students. He explained, "you're there to connect with kids, and you're forced to connect with five, not 30...proof of how our system isn't based on relationship building. It's based on numbers." Javier considered that successful teachers intentionally sought to develop positive relationships with students who were labeled by their peers and administrators as "disruptive." Thus, he kept these students in his class because removing them would reinforce students' beliefs that teachers did not care about them. When asked how he developed this orientation toward student–teacher relationships, Javier responded,

Why are we here? We're here to create human beings, right? Not these robots that know math and science that are gonna be working to make some money. And that's what I feel that we're heading toward. Creating these workers for this machine who's in it for profit. And they are preying on the kids in my community. Under the guise of 'we're trying to do what's best for them', you have no idea what's best for them because you don't care.

Javier connects this specific schooling practice that undermines relationship building with his interpretation of schooling as a broader institution that dehumanizes students from his community.

Another teacher, Sharron, explained that she heavily prioritized developing relationships in the first month of school and gradually elevated the math content, because she too felt the school-imposed time constraints. She also shared her awareness that some students have had negative mathematical experiences or have had difficult lives outside of school that required her to spend several months building trusting relationships to promote student learning. These teachers interpreted the schooling system to not prioritize relational aspects of teaching and learning and thus took it upon themselves to center relationship building in their classes.

Potential for conflict

How students' bodies and movements are attended to and interpreted are located within dominant narratives of acceptable classroom behavior and narratives of managing students. Two teachers' noticing was shaped by a recognition that students' behaviors can be interpreted differently—as playful or disruptive, as constructive or distracting, etc. These teachers recognized that how they interpreted students' behavior and whether or not they chose to address it could consume valuable instructional time. By redirecting his attention to students' mathematical thinking, Javier chose not to focus on behavior that could lead to further tension. For instance, in a lesson when students were provided with rulers, he explained,

I'm not looking for it, because it's gonna happen. I expect them to play around with the rulers... they'll snap them in half, and they'll write on them... I already know that, but if I make the point of addressing it at the beginning, they're more inclined to

do it more. If I say don't snap your rulers, don't break [them], don't write on them... I'm just gonna waste my day talking about the rulers.

Javier's interpretation of students' playing with rulers comes from prior experiences where he prioritized classroom management over content learning. Thus, by shifting his attention away from students playing with rulers, he began avoiding potential moments of conflict that would otherwise interfere with the content learning. Similarly, Sharron observed that a student had used a pen to complete his assignment and had marked out his multiple attempts to solve a problem. Instead of chastising him for not using a pencil that would allow him to erase mistakes, she offered him a new paper so that he had a clean surface to work through his thinking. Sharron shared that she was taught to believe a narrative that mathematics required the use of pencils, and that unlearning this narrative was a long process, but that this change ultimately reduced student–teacher conflicts. These examples represent teachers shifting away from dominant narratives of acceptable classroom behavior and appropriate tools used in mathematics in order to reduce conflict.

Math aversion

Finally, six teachers shared that over the years, they came to notice how some students entered their classrooms with a negative disposition toward mathematics. Many of these teachers were aware of a dominant narrative rooted in inherent ability about the type of students capable of doing math. They attended to students who expressed statements that reflected this narrative, and instead of adopting deficit lenses, shifted to acknowledge and elevate these students' contributions. Kim, for instance, shared how she allowed students to retake problems they got wrong during a test and earn half credit. She connected this practice to an awareness of how test taking can be a stressful process as well as her personal history with feeling pressure while doing mathematics:

I think not everybody is a good test taker. I mean certainly under the time pressure and almost all my kids feel pressure or stress. Only very few feel confident in taking the test. ...when I take a test, I'm sure I probably make a lot of mistakes...[even with] you in the classroom, I was self-conscious, [even though] I wasn't taking a test.

Kim attended to students' confidence levels and her own experiences doing math in a highpressure environment and interpreted that allowing students to reattempt questions they got wrong on an assessment would alleviate some of this burden. Furthermore, she shared how this practice was also intended to support students' mathematical learning: "There's certain kids they just want some earn some points, but I said that's not the point. The point is for you to go back and relearn the concept". Like Kim, other teachers commented on observing that students were "turned off to math," that "math is emotional" for many students, and that many students "feel pressure and stress because they don't feel confident." Teachers attended to students' math aversion, which they interpreted as an obstacle to students' learning and this awareness supported them attempting to reduce this anxiety through their interactions with students.

In summary, drawing on their own personal and professional experiences, and their understanding of narratives in mathematics that position students from a deficit perspective, teachers noticed how positive relational interactions with students helped promote more engagement in the classroom. Teachers noticed students as developing youth who needed encouragement and felt it was their responsibility to help students focus on the mathematics content via positive relational interactions. This noticing aimed to disrupt

Table 4	Instances	of eq	uitable	instructional	practices
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Instructional practice	Total observed instances
Attend to students' mathematical thinking	63
Establish norms for participation	31
Center relationships and healing to support student well-being	27
Position students as capable	25
Monitor how students position each other	18
Draw funds of knowledge	16
Press for academic success	15
Recognize multiple forms of discourse and language as a resource	9
Support students Identifying & processing emotions in relation to dominant math	7
Attend explicitly to race and culture	0
Support development of sociopolitical disposition	0

the dominant narrative that some children are inherently math learners and others are not by considering how an environment with positive student-teacher interaction could invite more student participation.

Enactment of equitable mathematics teaching practices

Our second question concerns the relationship between teachers' relational noticing and their instructional practices for equity. Recall, we expanded our analysis to a broader collection of practices because we theorized that advancing the relational dimensions of teaching also advances the dominant and critical axes. We found they enacted all but two of the dimensions of equitable mathematics instruction (see Table 4).

The most enacted equitable practice was attention to *students' mathematical thinking*, which occurred in twice as many instances compared to the second most enacted practice. We also observed the teachers enacting a range of equitable instructional practices that together supported them creating positive relational interactions, such as *establishing norms for participation, centering relationships*, and *positioning students as capable*. Notably, we did not observe any instances of teachers *attending explicitly to race and culture*. Our findings suggest that teachers coordinated the practices we observed to advance access and achievement in a supportive classroom environment for their students.

We now turn to provide cases of teachers enacting equitable practices and their associated relational noticing. To do so, we use cases of two teachers that focus on the noticing themes of attending to math aversion and students' academic success. One teacher, Sharron, notices math aversion and enacts practices to support the affective axis of equitable mathematics teaching. In the second case, Javier also notices math aversion and enacts practices to support the dominant axis of equitable instruction. These findings illuminate the multidimensional nature of teachers' noticing as they construct positive relational interactions, as well as how their noticing, in concert with different practices, can advance different equity aims.

Noticing students' math aversion to advance the affective axis

Teachers' relational noticing supported the enactment of practices outlined in the affective axis (Kokka, 2022). We begin with a vignette to illustrate Sharron's noticing related to math aversion and how it supported her praising a student. The class we observed, Mathematics, Engineering, Science, and Art (MESA), is a year-long project-based class where students apply mathematics and science knowledge to complete engineering and design tasks. For this specific lesson, the students were working in groups to find the perimeter and area of flat shapes and whether those measurements correlated with the area of 3D shapes. This lesson was a continuation of a task they started the previous day. Most of the students were working in groups, with some students seated throughout the room at individual desks. In the first five minutes, Sharron moved across the classroom, asking groups and individual students what they needed to get started on the task. As students started working, Sharron walked around the classroom asking if students were ready to "be checked." She reviewed students' work, asked them to explain their thought process, and identified next steps for students. As Sharron walked away from one group, she approached a student who was sitting by himself toward the front of the room and immediately raised her hand to offer him a high-five. The student reciprocated the high-five and Sharron exclaimed "Dude, I love this...Are you feeling more successful?" After a brief conversation about what had been helping the student make progress, Sharron told the student, "I'm really proud of you. You've been working really hard. I see it."

During the noticing interview, we asked Sharron what she noticed that communicated to her that this student was working hard. She responded by sharing that this student had a history of coming into class, sitting down, and not doing any work. In this moment, she noticed how this student is "really feeding off of that praise and that recognition for the work that he's doing" and that she was "so impressed because it feels like a *big* switch for him" and wanted "him to feel the success of the switch and that the effort he's putting in has a positive payout." She interpreted the joy that she saw the student experience in the interaction as being connected to her recognition—her noticing of his shifts in participation—and the praise she offered. She understood how important this feeling of accomplishment could be: "And for him right now, that positive payout is not necessarily content knowledge. He's not like, 'Woo-hoo. I know math.' He's getting that positive pay out from the emotional side of the encouragement from the adults in the room." Sharron's noticing, then, is not only shaped by how she understood the student's feelings in that moment but also by her awareness of this particular student's history of not engaging meaningfully in mathematics, as well as her awareness of her responses to him in that moment and over time. That is, by continuously recognizing this student's hard work and offering words of encouragement, she communicated that he is valued in the class and community and positioned him as capable of doing math.

We find Sharron's noticing noteworthy in this instance because her interpretation of this student's initial behavior in the classroom did not lead her to frame him as a perpetually disengaged student. Instead, she left open the opportunity for this student to become an active agent of his learning trajectory. She recognized that this student may not have gotten to this point without the emotional payout that comes from having a teacher recognize a student's work. In Sharron's words, she was one of the "adults in the room" and was aware of how meaningful her words could be for students. She sought the opportunity to support this student's academic trajectory by attending to his previous mathematics aversion. When asked explicitly about how she attended to emotions, she responded, "Human beings and junior high (students) are emotional beings. And math is so filled with negative emotions...." Sharron is aware of the narrative of mathematics as embodying negative emotions for students. She goes on to describe her own experiences where math elicited negative interactions. She shared that, "the only classes that I've ever been kicked out of were math classes, where the teachers were impatient because I didn't have the answer." She recalled how difficult it was to engage in class after teachers removed her. "I was still trying to process being kicked out of the classroom and how ashamed that made me feel." Reflecting on her negative relational interactions, Sharron learned to notice how her own students' disposition toward math could shape their engagement in her classroom and responded in a manner that supported a positive relational interaction. Moreover, Sharron's prior experiences shaped her desire to create positive interactions with her students.

I want them to have a positive interaction with me. I want that relationship when they think about how I speak to them and the way that I talk to them, that's a positive thing and not something that's scary or hurtful, or anything else, but that 'she really believes in me' and 'she wants to see good things from me' and maybe at some point there's this tiny part of them that wants to make me proud.

Sharron's interpretation of how positive interactions are consequential for future learning opportunities supports how she responds to students. In the case above, she sought to create a positive interaction with the student because she understood how negative interactions can build up to hinder students' success in mathematics. Sharron added that she was intentional about supporting students—"especially those who are not [always willing to engage]" and "really pump them up for it." Sharron's awareness of negative dispositions caused by their experiences with math led to her ability to *acknowledge students' contributions* to ultimately support student learning.

Noticing students' math aversion to advance the dominant axis

Teachers' relational noticing also supported equitable practices related to the dominant axis (Gutiérrez, 2011). Specifically, in the case of Javier, his noticing of students' math aversion was informed by a commitment to increase students' access and opportunities in mathematics. To illustrate, we draw on an interview segment with Javier where he shared his philosophy toward creating caring interactions with his students. In this video segment, the class was working in small groups. The overall objective for the lesson was for students to be able to compute the square and cube roots of numbers. One problem asked students to identify the length of a side of a treehouse given the area of each side to be 36 feet². Javier approached several groups as they worked on this problem. As he moved across the room, Javier walked over to a group where he heard laughter. He asked the students, "How are you guys doing, are you stuck?" In the noticing interview, Javier was asked to discuss what he noticed that prompted him to approach this group. He explained that he tries to "formulate [interactions] in a positive way" by asking questions such as "do you need my help?" rather than saying, "hey! you're not doing work! Get on task." We then asked Javier why interacting with students in this way was important to him. He responded that

Most of the students are turned off to math, they don't like it, they think it's hard if they can't do it. So, as a math teacher, you have to figure out a way to make the environment positive for them so they start peeling away at that negative response to math...so if you're telling them, 'hey you can't do this because you're stupid, and you can't perform, you're not smart enough, and this is really easy, and I can't believe you can't do it,' you're just perpetuating the narrative that's in their head.

Javier was aware of how highlighting students' disengagement or framing the mathematics as 'easy' could lead to further math aversion. Javier drew on his personal history as a math student to explain why he used this approach:

If I was them, how would I want people to respond to me? Because I was them. I did have teachers that responded very well to me, and that's kind of what connected me to math in the first place. So, I understand how powerful that can be because some of these kids are gonna become math teachers because of these interactions. Right? So that's the inspiration behind it. How can I pass on that type of interaction to them?

Javiers' noticing is informed by his former teachers who proactively addressed students' math aversion, which led him to want to become a mathematics teacher and emulate those interactions with his own students. Feeling grateful for having had responsive teachers, Javier now feels it is his duty to "pass on" positive interactions with his students. In this sense, he anticipated negative relational interactions to result from confronting his students about their work in a punitive manner. Javier interpreted how positive teacher–student interactions can have potentially long-lasting effects on how students relate to mathematics when he discussed his own academic journey. Using this lens, Javier was motivated to ensure that he responded to students by offering academic support instead of penalizing students. Javier thus understands how he can shape students' long-term goals and attitudes toward mathematics through his noticing practices.

These vignettes illustrate a pattern we observed across teachers' noticing and classroom practices to create positive relational interactions. Specifically, despite enacting practices that support different aims, teachers' shared awareness of math aversion—both demonstrated by students and from their own experiences—allowed them to attend to students' struggling with mathematics in the moment of instruction to foster positive relational interactions. This was also the case for other dimensions of teachers' noticing, such as awareness of students' well-being, schooling practices, and potential moments of conflict. Their interpretation of how these dimensions shape learning enables them to be on the lookout for these moments in teaching and enact practices that create supportive and affirming classroom environments. We now turn to discuss our findings.

Discussion

This study contributes to the growing interest in mathematics teaching for equity by documenting teachers' noticing and practice that supports creating positive relational interactions (Battey, 2013; Roth & Walshaw, 2019). Previous approaches to integrating emotions often have a limited account for factors beyond cognition (Eligio, 2017; Martinez-Sierra & Garcia Gonzalez, 2014). Thus, we theorized noticing from a sociopolitical perspective that considers how historical factors and dominant narratives contribute to teachers' creating positive relational interactions. Our findings are consistent with Kalinec-Craig (2017) who discusses how three preservice teachers learned to notice students' mathematics learning and position their students with agency by drawing on their own experiences as minoritized students. Specifically, Sharron's experience and reflections about mathematics as an emotional discipline are documented in the math anxiety literature (Maloney & Beilock, 2012). Her awareness of the effects that such negative experiences in mathematics can have on students allows her to attend to and interpret her students from a position of caring. In contrast, Javier's experiences with affirming mathematics educators helped him adopt antideficit, strengths-based lenses to interpret his students' behaviors and contributions. Noticing from an anti-deficit perspective has been documented as supporting teachers disrupting prevailing instructional practices that marginalize racially, culturally, and linguistically diverse students (Hand, 2012; Louie et al., 2021). Together, these findings suggest that teachers' relational noticing can support equitable learning experiences for their students.

We also found that teachers' awareness and understanding of dominant narratives in mathematics education and schooling informed their noticing to cultivate positive relational interactions. Specifically, two of the five noticing themes, *potential for conflict* and *math aversion*, were both informed by narratives about what it means to behave in mathematics and how one should participate to demonstrate being a math learner. These findings are consistent with research that documents how dominant narratives perpetuate inequities and how teachers' awareness of them can serve to disrupt inequitable practice (Gutierrez et al., 2023; Louie et al., 2021; Shah, 2017). By recognizing how these dominant narratives have socialized students into developing math aversion, for instance, teachers can be better equipped to avoid deficit perspectives toward their students.

Our findings also suggest that a teacher's own emotions and well-being inform their relational noticing. This became particularly salient when teachers described their interpretations of relational interactions through the lens of their personal experiences in mathematics and their awareness of dehumanizing practices of mathematics and school policies. Specifically, they described an ongoing tension they felt between moving through the curriculum, while also taking time to cultivate relationships with students. They also described frustrations with existing policies that allowed teachers to remove students from their classes who did not follow norms for appropriate behavior, instead of teachers being accountable to learn how to create caring classrooms for their students. These findings point to an area for future inquiry, specifically, how teachers can use insight into their own emotions to develop their noticing of mathematics as an oppressive space for students from marginalized communities (Askew & Venkat, 2017; Gutiérrez, 2018; Martin, 2009). Such work would make clearer the connection between school policies and teachers' affective experiences, both of which play a central role in teachers' relational noticing.

Our second research question sought to understand how teachers enact practices that support equitable teaching and how their relational noticing informed those practices. Given that the most prominent practice that teachers in this study enacted was *attending to students' mathematical thinking*, we argue that this practice can serve to divert attention away from behaviors that might otherwise be addressed through punitive practices. Not only does this advance affective pedagogical goals, but by focusing on students' thinking, the teachers in our study communicated to students that their ideas mattered and positioned them as academically capable. While the literature positions attending to student thinking suggest that they are integrated; that is, attending to student thinking is essential for creating positive relational interactions.

Finally, our findings suggest that teachers' noticing and practice supported the dominant and affective dimensions of equitable teaching. Notably, we observed few instances of noticing and instruction associated with the critical axis. This is not surprising, particularly because of district-wide initiatives focused on increasing student achievement for increased college access and opportunity, which assigns mathematics prominent status in schooling and further perpetuates narratives of who can and should be in mathematics (e.g., Martin, 2013). This raises questions about how teachers negotiate district-level efforts with their pedagogical goals to enact more caring practices in an educational system that does not privilege the interpersonal relationships that teachers aspire to create.

Implications and limitations

While these findings are encouraging, the study is not without its limitations. First, the teachers in our study were selected because they expressed a commitment to and had been engaged in professional learning to advance equitable instruction. Thus, we are limited in understanding how teachers change their noticing and practice as they develop dispositions and commitments to disrupt mathematics inequities. Additionally, our teachers shared that their awareness and integration of dominant narratives with their noticing and instruction took place over many years. Thus, there is a need for expansive models of professional learning that accounts for the temporal dimension of teacher learning (Ehrenfeld, 2022), which would have important implications for how we understand where, how, and with whom professional development should take place.

Second, we studied a small group of teachers in a particular geographical region that represent a particular form of cultural, linguistic, and racial diversity. However, we recognize teaching and learning are situated processes and important questions concern whether our model of relational noticing would apply in different settings and how it would need to be adapted to account for the different ways students experience mathematics as inequitable in their local contexts. Moreover, we are cautious about claims that students experienced the classrooms in ways that teachers described. We propose that research on teacher noticing for relational instruction also include student perspectives. In fact, the participatory action research approach that informed our study points to the importance of those who are most disenfranchised as having a voice in shaping research (Bang & Vossoughi, 2016). Including students more centrally in the research process would further expand how we understand teachers' noticing that supports advancing the three axes of equitable math teaching.

Finally, these findings have implications for teacher professional development given the relationship between noticing positive relational interactions and enacting equitable instruction. A wide body of research emphasizes the importance of social and emotional well-being for student academic success (Battey, 2013; Hall, 2012; Hinsdale 2016). One of the shortcomings of this work is that it locates the problem with students, in relation to their behavior and ability to conform to white dominant practices. Our findings recognize the sociopolitical underpinnings of social and emotional well-being by locating them in the histories of mathematics education and schooling more broadly. Thus, our findings have implications for shifting the responsibility from students to empowering teachers to interrogate existing narratives and practices to cultivate teachers' relational noticing for positive and affirming classrooms.

Appendix A Site 2 CoATTEND professional development model

Introduction to noticing	Noticing is intentional, purposeful and happens all the time; Awareness and understanding = noticing; Awareness of self as a noticer becomes a way of being & doing
Historical self	Positionality; Personal histories and multiple aspects of our identities shape noticing; How our history helps/ limits noticing in-group diver- sity; Self-tied to systemic histories
Systemic history and inequities	Power structures dominate schooling practice; Class and profit; Testing and measurement system; Narratives of oppression shape noticing
Positioning and identities	Math maintains power by neutrality; Noticing influences student position- ing; Student positioning influences identities; Mathematical literacy can empower students
Organizing to notice	How are the space, students, and materials organized to help me notice? How can I change the space, organization of students and materials to notice differently?
Philosophy of practice	How do personal history, institutional structures, beliefs about students shape practice? How can we practice noticing these connections in the classroom?

Acknowledgements This study is based upon research supported by the National Science Foundation under Grant No. 1661164. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the author(s) and do not necessarily reflect the views of the National Science Foundation.

Declarations

Conflict of interest On behalf of all the authors, the corresponding author states that there is no conflict of interest.

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References

- Askew, M., & Venkat, H. (2017). "I Hate Maths": Changing Primary school teachers' relationship with mathematics. In Understanding emotions in mathematical thinking and learning (pp. 339–354). Academic Press.
- Bang, M., & Vossoughi, S. (2016). Participatory design research and educational justice: Studying learning and relations within social change making. *Cognition and Instruction*, 34(3), 173–193.
- Bartell, T. (2011). Caring, race, culture, and power: A research synthesis toward supporting mathematics teachers in caring with awareness. *Journal of Urban Mathematics Education*, 4(1), 50–74.
- Bartell, T. (2013). Learning to teach mathematics for social justice: Negotiating social justice and mathematical goals. *Journal for Research in Mathematics Education*, 44(1), 129–163.
- Bartell, T., Wager, A., Edwards, A., Battey, D., Foote, M., & Spencer, J. (2017). Toward a framework for research linking equitable teaching with the standards for mathematical practice. *Journal for Research in Mathematics Education*, 48(1), 7–21.

- Battey, D. (2013). "Good" mathematics teaching for students of color and those in poverty: The importance of relational interactions within instruction. *Educational Studies in Mathematics*, 82(1), 125–144.
- Battey, D., Amman, K., Leyva, L. A., Hyland, N., & McMichael, E. W. (2022). Racialized and gendered labor in students' responses to precalculus and calculus instruction. *Journal for Research in Mathematics Education*, 53(2), 94–113.
- Battey, D., & Neal, R. A. (2018). Detailing relational interactions in urban elementary mathematics classrooms. *Mathematics Teacher Education and Development*, 20(1), 23–42.
- Battey, D., Neal, R. A., Leyva, L., & Adams-Wiggins, K. (2016). The interconnectedness of relational and content dimensions of quality instruction: Supportive teacher–student relationships in urban elementary mathematics classrooms. *The Journal of Mathematical Behavior*, 42, 1–19.
- Berry, R. Q. (2020). Equitable mathematics, social justice, and actions [Webinar]. Colorado Council of Teachers of Mathematics. https://www.cctmath.org/event-archive
- Berry, R. Q. (2008). Access to upper-level mathematics: The stories of successful African American middle school boys. *Journal for Research in Mathematics Education*, 39(5), 464–488.
- Decker, D. M., Dona, D. P., & Christenson, S. L. (2007). Behaviorally at-risk African American students: The importance of student-teacher relationships for student outcomes. *Journal of School Psychology*, 45(1), 83–109.
- Ehrenfeld, N. (2022). Framing an ecological perspective on teacher professional development. *Educational Researcher*, 51(7), 489–495.
- Eligio, U. X. (2017). An overview of the growth and trends of current research on emotions and mathematics. Understanding Emotions in Mathematical Thinking and Learning, 8, 3–41.
- Erickson, F. (1985). *Qualitative methods in research on teaching* (pp. 119–162). Institute for Research on Teaching.
- Eynde, P. O. T., Corte, E. D., & Verschaffel, L. (2006). "Accepting emotional complexity": A socio-constructivist perspective on the role of emotions in the mathematics classroom. *Educational Studies in Mathematics*, 63, 193–207.
- Forgasz, H. J., & Leder, G. C. (2017). Persistent gender inequities in mathematics achievement and expectations in Australia, Canada and the UK. *Mathematics Education Research Journal*, 29, 261–282.
- Gutiérrez, R. (2018). Introduction: The need to rehumanize mathematics. In *Rehumanizing mathematics for Black, Indigenous, and Latinx students*. National Council of Teachers of Mathematics.
- Gutiérrez, R. (2011). Context matters: How should we conceptualize equity in mathematics education? Equity in discourse for mathematics education: Theories, practices, and policies (pp. 17–33). Springer.
- Gutiérrez, R., Myers, M., & Kokka, K. (2023). The stories we tell: Why unpacking narratives of mathematics is important for teacher conocimiento. *The Journal of Mathematical Behavior*, 70, 101025.
- Hall, J. (2012). Gender issues in mathematics: An Ontario perspective. *Journal of Teaching and Learning*, 8(1), 32.
- Hand, V. (2012). Seeing culture and power in mathematical learning: Toward a model of equitable instruction. *Educational Studies in Mathematics*, 80(1–2), 233–247.
- Hinsdale, M. 2016. Relational Pedagogy. Online Publication. https://doi.org/10.1093/acrefore/9780190264 093.013.28.
- Horn, I. S. (2007). Fast kids, slow kids, lazy kids: Framing the mismatch problem in mathematics teachers' conversations. *The Journal of the Learning Sciences*, 16(1), 37–79.
- Hunting, R. P. (1997). Clinical interview methods in mathematics education research and practice. *The Journal of Mathematical Behavior*, 16(2), 145–165.
- Jackson, K., Gibbons, L., & Sharpe, C. J. (2017). Teachers' views of students' mathematical capabilities: Challenges and possibilities for ambitious reform. *Teachers College Record*, 119(7), 1–43.
- Jacobs, J. K., & Morita, E. (2002). Japanese and American teachers' evaluations of videotaped mathematics lessons. *Journal for Research in Mathematics Education*, 33(3), 154–175.
- Jessup, N. A. (2018). Understanding teachers' noticing of children's mathematical thinking in written work from different sources. The University of North Carolina at Greensboro.
- Jordan, B., & Henderson, A. (1995). Interaction analysis: Foundations and practice. The Journal of the Learning Sciences, 4(1), 39–103.
- Kalinec-Craig, C. A. (2017). The rights of the learner: A framework for promoting equity through formative assessment in mathematics education. *Democracy and Education*, 25(2), 5.
- Kang, H. (2022). Teacher responsiveness that promotes equity in secondary science classrooms. *Cognition and Instruction*, 6, 1–27.
- Kokka, K. (2022). Toward a theory of affective pedagogical goals for social justice mathematics. *Journal for Research in Mathematics Education*, 53(2), 133–153.
- Louie, N. L. (2018). Culture and ideology in mathematics teacher noticing. *Educational Studies in Mathematics*, 97, 55–69.

- Louie, N., Adiredja, A. P., & Jessup, N. (2021). Teacher noticing from a sociopolitical perspective: The FAIR framework for anti-deficit noticing. ZDM, 53(1), 95–107.
- Lunney Borden, L. (2013). What's the word for...? Is there a word for...? How understanding Mi'kmaw language can help support Mi'kmaw learners in mathematics. *Mathematics Education Research Journal*, 25, 5–22.
- Maloney, E. A., & Beilock, S. L. (2012). Math anxiety: Who has it, why it develops, and how to guard against it. *Trends in Cognitive Sciences*, 16(8), 404–406.
- Maloney, T., & Mathews, J. S. (2020). Teacher care and students' sense of connectedness in the urban mathematics classroom. *Journal for Research in Mathematics Education*, 51(4), 399–432.
- Martin, D. B. (2009). Researching race in mathematics education. *Teachers College Record*, 111(2), 295–338.
- Martin, D. B. (2013). Race, racial projects, and mathematics education. Journal for Research in Mathematics Education, 44(1), 316–333.
- Martin, D. B. (2019). Equity, inclusion, and antiblackness in mathematics education. *Race Ethnicity and Education*, 22(4), 459–478.
- Martínez-Sierra, G., & García González, M. D. S. (2014). High school students' emotional experiences in mathematics classes. *Research in Mathematics Education*, 16(3), 234–250.
- Mendoza, E., Hand, V., van Es, E. A., Hoos, S., & Frierson, M. (2021). 'The ability to lay yourself bare': Centering rupture, inherited conversations, and vulnerability in professional development. *Professional Development in Education*, 47(2–3), 243–256.
- Miles, M. B., Huberman, A. M., & Saldaña, J. (2014). Qualitative data analysis: An expanded sourcebook (3rd ed.). Sage.
- Milner, H. R., IV. (2013). Analyzing poverty, learning, and teaching through a critical race theory lens. *Review of Research in Education*, 37(1), 1–53.
- Moschkovich, J. N. (2011). How equity concerns lead to attention to mathematical discourse. In B. Herbel-Eisenmann, J. Choppin, D. Wagner, & D. Pimm (Eds.), *Equity in discourse for mathematics education: Theories, practices, and policies* (pp. 89–105). Springer.
- Nasir, N. I. S. (2002). Identity, goals, and learning: Mathematics in cultural practice. *Mathematical Thinking and Learning*, 4(2–3), 213–247.
- Nasir, N. I. S., Snyder, C. R., Shah, N., & Ross, K. M. (2013). Racial storylines and implications for learning. *Human Development*, 55(5–6), 285–301.
- Newcomer, S. N. (2018). Investigating the power of authentically caring student-teacher relationships for Latinx students. *Journal of Latinos and Education*, 17(2), 179–193.
- Nowell, L. S., Norris, J. M., White, D. E., & Moules, N. J. (2017). Thematic analysis: Striving to meet the trustworthiness criteria. *International Journal of Qualitative Methods*, 16(1), 1609406917733847.
- Penteado, M. G., & Skovsmose, O. (2011). Ghettoes in the classroom and the construction of possibilities. *Mapping Equity and Auality in Mathematics Education*, 3, 77–90.
- Philip, T. M., Olivares-Pasillas, M. C., & Rocha, J. (2016). Becoming racially literate about data and data-literate about race: Data visualizations in the classroom as a site of racial-ideological microcontestations. *Cognition and Instruction*, 34(4), 361–388.
- Roorda, D. L., Jak, S., Zee, M., Oort, F. J., & Koomen, H. M. (2017). Affective teacher–student relationships and students' engagement and achievement: A meta-analytic update and test of the mediating role of engagement. *School Psychology Review*, 46(3), 239–261.
- Roth, W. M., & Walshaw, M. (2019). Affect and emotions in mathematics education: Toward a holistic psychology of mathematics education. *Educational Studies in Mathematics*, *102*(1), 111–125.
- Rubel, L. H. (2017). Equity-directed instructional practices: Beyond the dominant perspective. *Journal of Urban Mathematics Education*, 10(2), 66–105.
- Rubel, L. H., & Ehrenfeld, N. (2020). Palestinian/Arab Israeli women's experiences in mathematics education: An intersectional analysis. *International Journal of Educational Research*, 102, 101616.
- Sakiz, G., Pape, S. J., & Hoy, A. W. (2012). Does perceived teacher affective support matter for middle school students in mathematics classrooms? *Journal of School Psychology*, 50(2), 235–255.
- Schack, E. O., Fisher, M. H., & Wilhelm, J. A. (Eds.). (2017). Teacher noticing: Bridging and broadening perspectives, contexts, and frameworks. Springer.
- Schonert-Reichl, K. A. (2017). Social and emotional learning and teachers. The Future of Children, 6, 137–155.
- Shah, N. (2017). Race, ideology, and academic ability: A relational analysis of racial narratives in mathematics. *Teachers College Record*, 119(7), 1–42.
- Skiba, R. J., Horner, R. H., Chung, C. G., Rausch, M. K., May, S. L., & Tobin, T. (2011). Race is not neutral: A national investigation of African American and Latino disproportionality in school discipline. *School Psychology Review*, 40(1), 85–107.

- Spaull, N. (2013). South Africa's education crisis: The quality of education in South Africa 1994–2011. Johannesburg Centre for Development and Enterprise, 21(1), 1–65.
- Stephens, G. (2021). "It's because of community meeting:" Toward a responsive reconceptualization of social emotional learning. *Educational Action Research*, 6, 1–18.
- Valoyes-Chávez, L., & Darragh, L. (2022). Identity work, racialized emotions, and equity in mathematics education. *Journal for Research in Mathematics Education*, 53(5), 372–378.
- Van der Berg, S. (2007). Apartheid's enduring legacy: Inequalities in education. Journal of African Economies, 16(5), 849–880.
- Van Es, E. A., & Sherin, M. G. (2008). Mathematics teachers' "learning to notice" in the context of a video club. *Teaching and Teacher Education*, 24(2), 244–276.
- van Es, E. A., Hand, V., & Mercado, J. (2017). Making visible the relationship between teachers' noticing for equity and equitable teaching practice. Teacher noticing: Bridging and broadening perspectives, contexts, and frameworks, pp. 251–270.
- van Es, E. A., Hand, V., Agarwal, P., & Sandoval, C. (2022). Multidimensional noticing for equity: Theorizing mathematics teachers' systems of noticing to disrupt inequities. *Journal for Research in Mathematics Education*, 53(2), 114–132.
- Wilson, J., Nazemi, M., Jackson, K., & Wilhelm, A. G. (2019). Investigating teaching in conceptually oriented mathematics classrooms characterized by African American student success. *Journal for Research in Mathematics Education*, 50(4), 362–400.
- Yang, K. W. (2009). Focus on policy: Discipline or punish? Some suggestions for school policy and teacher practice. *Language Arts*, 87(1), 49–61.
- Zavala, M. D. R., & Hand, V. (2019). Conflicting narratives of success in mathematics and science education: Challenging the achievement-motivation master narrative. *Race Ethnicity and Education*, 22(6), 802–820.
- Zeichner, K. (2010). Rethinking the connections between campus courses and field experiences in collegeand university-based teacher education. *Journal of Teacher Education*, 61(1–2), 89–99.

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