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ABSTRACT

A short review is given of a number of phenomena in solid state physics which display nonlinear dynamics. Two examples are discussed in more detail: plasma waves in germanium crystals and spin waves in ferrites. Solids are often well characterized and have diverse properties; they are quite interesting from the viewpoint of experimental and theoretical nonlinear dynamicists. A fundamental understanding of their dynamics will have significant bearing on solid state device technology and applications.

1. Introduction: Perspectives

From my viewpoint as an experimentalist, these are some of the outstanding issues in nonlinear dynamics:

A. Identification of universal features: How well and under what conditions can the overall temporal behavior of a real nonlinear system be viewed as *composed of elementary recognizable elements* (e.g., period doubling, quasiperiodicity, entrainment, chaos, intermittency, specific power spectra, ...), characterized by scaling relations and universal numbers computed from elementary models, usually maps. How useful is this approach?

B. Spatio-temporal behavior: This is the most general problem and requires simultaneous measurement of temporal behavior at a large number of spatial elements of an extended system, e.g., a real fluid or a plasma. Of special interest is the transition to weak turbulence (chaos) and then to strong turbulence. Are there optimum data-taking schemes and analysis procedures? How can spatio-temporal chaos be characterized? What to do when the fractal dimension becomes intractably large?

C. Evolution of structures in complex systems. I refer here to relatively slow evolution such as dendritic crystal growth; development of biological structures; and emergent properties of extended collective systems, e.g., multidimensional arrays of nonlinear elements.

The above issues are, of course, very general. I focus now on the large field of solid state physics and list, in Section 2, some specific physical systems of interest. Westervelt in his contribution to this volume presents an extensive list of nonlinear phenomena in semiconductors.

2. Some Solid State Systems

Perhaps the simplest model of a solid is a set of coupled nonlinear oscillators. If the dissipation is large enough, these may be modelled by a set of coupled maps. The next level of modelling might be a set of nonlinearly coupled modes, roughly applicable to plasma waves, spin waves, and acoustic waves in crystals. As a simple example consider three modes of waves of amplitude C_1, C_2, C_3, bilinearly coupled:

$$\frac{dC_1}{dt} = -\gamma_1 C_1 + M_1 C_2 C_3 \qquad (1a)$$

$$\frac{dC_2}{dt} = -\gamma_2 C_2 + M_2 C_1 C_3^* \qquad (1b)$$

$$\frac{dC_3}{dt} = -\gamma_3 C_3 + M_3 C_1 C_2^* \qquad (1c)$$

where γ_i and M_i are damping and coupling constants, respectively. Numerical computations have shown that such equations can have a Poincare section that reduces to a one-dimensional map and hence display a period doubling cascade to chaos [1].

We now list some solid state systems that i) have been studied successfully from the viewpoint of contemporary nonlinear dynamics of the last five years; or ii) have been reported earlier to display various instabilities, not really understood, but that can now be fruitfully reexamined, experimentally and theoretically; or iii) systems that will probably display interesting nonlinear dynamics.

A. Plasmas: Helical electron-hole plasma density waves in Ge rods show period doubling, quasiperiodicity, loss of spatial coherence [2-4]. Electro-acoustic interactions in GaAs show subharmonic generation [5]. Other good candidates for study appear to be helicon and Alfvèn waves [6]; the two-stream instability [7]; and the magnetic pinch effect [8]. See general references [9-12].

B. Spin waves: Period doubling cascade to chaos observed in spin waves in yttrium iron garnet spheres excited by ferromagnetic resonance [13]. Similar phenomena observed by parallel pumping [14, 15]. Route to chaos by "irregular periods" observed by parallel pumping ferromagnets [16]. General references [17-19].

C. Charge density waves: Materials such as TaS_3, $NbSe_3$, driven by ac or dc currents display period doubling, chaos, and quasiperiodicity [20-22].

D. Acoustic waves: Strongly driven Rochelle crystals show a period cascade to chaos at a temperature near the ferroelectric transition [23].

E. Oscillatory conduction in semiconductors: Low temperature photoconductivity studies of pure Ge crystals show period doubling, chaos, and quasiperiodicity [24]. Qualitatively similar behavior is found in GaAs [25]. In the post-breakdown regime in p-Ge, spontaneous oscillations and chaos are found [26]. Oscillatory and chaotic states are found in the conductivity of barium sodium niobate at temperatures ~600°C [27]. See [28] for general references to older experimental work on semiconductor instabilities before the development of contemporary nonlinear dynamics theory; many of these results could be reexamined and now understood. An understanding of semiconductor instabilities has important applications in present technology.

F. Josephson junctions: Intermittent chaos is observed in resistively shunted Josephson junctions [29]. Much simulation work has also been reported, including the effects of fractal boundaries of the basins of attraction on the low frequency power spectra [30].

G. Discrete nonlinear solid state oscillators: Driven resonators composed of a linear inductance and the nonlinear charge storage properties of p-n junctions have been extensively studied, and in the simplest cases of large damping display a period doubling cascade to chaos and periodic windows [31]. If driven harder, and with less damping, the system displays more complex behavior due to the extreme asymmetry pf the effective restoring force: a period adding sequence [32-34]. Intermittency [35], effects of added noise [36], and crises of the attractor [37] have also been studied. If two or more resonators are coupled, the system displays a Hopf bifurcation to quasiperiodicity, entrainment horns, and breakup of the invariant torus [34]. This simple real physical system displays much of the behavior of complex driven passive nonlinear systems.

Studies of a forced symmetric nonlinear self-oscillator (using a saturable inductor with hysteresis) show a rich behavior: symmetry breaking, quasiperiodicity, entrainment horns, and homoclinic bifurcations [38]. Through direct observation of both stable and unstable manifolds, the behavior near points of strong and weak resonance is found to correspond with V. I. Arnold's theory of versal deformations of the plane [39].

3. Chaos and Turbulence in an Electron-Hole Plasma in a Ge Crystal

As an example of nonlinear dynamics in a solid, we review the experiments of Held et al. [2-4] at Berkeley on the spatial and temporal behavior of chaotic instabilities of an electron-hole plasma in a germanium rod. The plasma is produced by injecting both electrons and holes into a rod-shaped crystal of germanium at liquid nitrogen temperatures; the crystal is placed in a magnetic field B_0 parallel to its axis, and an adjustable electric field is also applied along the length of the sample. The plasma can absorb energy from the applied fields and, beyond some threshold (typically a few volts/cm at a few kilogauss), an unstable travelling helical density wave develops within the plasma. Several nonlinearly coupled modes can be excited within the boundaries of the crystal.

Experimentally we measure the total current I(t) through the crystal and the potential across it, V(t), as the driving parameter V_{dc} is increased. By also recording the voltages $V_i(t)$ across pairs of probe contacts formed along the length of the sample, we can observe spatial variations in the plasma density. At the onset of the helical instability, spontaneous current oscillations are observed. As V_{dc} is increased further, we find that this simple physical system exhibits complex nonlinear dynamics, including a period doubling route to chaos when only one mode is excited. More generally, when more modes are excited, we observe quasiperiodicity; self-entrainment; temporal chaos; and a partial loss of spatial coherence -- indicating the spatial breakdown of the helical density wave and the onset of "turbulence" in this solid state system.

Our experiments are, of course, related to some hydrodynamic experiments on fluids, e.g., Rayleigh-Benard convection and Coette-Taylor flows, as well as other experiments on nonlinear dynamical systems. Such experiments are partly motivated by the conjecture that in dissipative nonlinear media the dynamics may be modelled by a strange attractor of relatively low dimension, in contrast to a very large number of degrees of freedom associated with ergodic systems. Coherent oscillations of the type we study were originally observed by Ivanov and Ryvkin [40] in Ge and were subsequently studied both theoretically [41,42] and experimentally [42,43] in a number of other semiconductors. It is possible that chaotic behavior was observed earlier but not recognized as such, owing to the lack of mathematical framework now available. Our physical system is well characterized, and the equations of motion well known [2]. In the simplest case the equations can be approximated by Eq. (1). This appears to be a good system for detailed study of spatio-temporal plasma turbulence and, in fact, it is the first plasma system found to exhibit the universal period doubling and quasiperiodic transitions to chaos.

Perhaps the single feature most useful in characterizing the plasma is the power spectrum of the current, $|I(\omega)|^2$, from which we can detect the onsets of spontaneous oscillations, period doubling, quasiperiodicity, and chaos. However, observation of only power spectra does not enable us to distinguish between deterministic chaos and stochastic noise; both result in broadened spectral peaks. To uniquely identify the observed spectral broadening as deterministic chaos, we observe in real time the phase portrait, a plot of V(t) vs. I(t); and the first return map, a plot of I_n vs. I_{n+1}, where $\{I_n\}$ is the set of local current maxima. The return map is topologically equivalent to a Poincaré section of the attractor. When the return map does not fill an entire area within 2-dimensional space, the motion of the system is confined to a low-dimensional strange attractor. However, a system in which the return map does fill an entire area within 2-dimensional space may still be characterized by low-dimensional chaos (with attractor dimension typically $\gtrsim 2.5$). In these cases even a return map cannot distinguish between chaos and stochastic noise, and one must consider more quantitative measures of the dimensionality of the system.

The fractal dimension [44] provides just such a quantitative measure and thus an approximate measure of the number of degrees of freedom needed to characterize the plasma at any instant of time. We use the following procedures [45] to measure the fractal dimension d of our plasma instabilities: we begin by recording a data set of N values of the current at uniformly spaced time intervals [i.e., $I(t+mT) \rightarrow I(mT)$, $m = 1,2,\ldots,N$] using a fast 12-bit analog-to-digital converter and an LSI-11/23 computer. From the data set $\{I(T), I(2T), \ldots, I(NT)\}$ we construct $N - D + 1$ vectors $\vec{G}_m = [I(mT), I((m+1)T), \ldots, I((m+D-1)T)]$ in a D-dimensional phase space; D is referred to as the embedding dimension of the reconstructed phase space $\vec{G}$. Next, we compute the number of points on the attractor, $N(\varepsilon)$, which are contained within a D-dimensional hypersphere of radius ε centered on a randomly selected vector $\vec{G}_m$.

One expects scaling of the form

$$N(\varepsilon) \propto \varepsilon^{d} \tag{2}$$

where d is the fractal dimension of the attractor. Thus, a plot of $\log \overline{N(\varepsilon)}$ vs. $\log \varepsilon$ is expected to have a slope d, where $\overline{N(\varepsilon)}$ is the average for hyperspheres centered on many different $\vec{G}_m$. This procedure is carried out for consecutive values of D = 2, 3, 4, ..., until the slope has converged. This is done to ensure that the embedding dimension chosen is sufficiently large (important if the dimension of the phase space is not known), and to discriminate against high dimensional stochastic noise, not of deterministic origin.

To determine whether or not a plasma density wave is spatially coherent, we compare the fluctuations in plasma density at different points along the sample. We obtain a crude measure of the degree of coherence by using a fast two-channel digital storage oscilloscope and comparing the voltages $V_i(t)$ across pairs of contacts located at different positions along the z-axis of the sample. If the temporal behavior of I(t) is periodic, we observe only a phase shift in $V_i(t)$ along z, which indicates a coherent travelling wave.

To obtain a more quantitative measure of the degree of spatial coherence, applicable for nonperiodic behavior, we calculate a spatial correlation function C(r), defined as

$$C(r) = \frac{2}{N}\left| \sum_{n=1}^{N} V_i(nT)V_j(nT) \right| , \tag{3}$$

where $V_i(t)$ and $V_j(t)$ are the voltages across two pairs of contacts separated by a distance r, T is the sampling time interval, and N is a number large enough that C(r) has converged, typically 20,000. We find that C(r) is independent of T.

<u>Results: Temporal routes to chaos</u>. In different regions of parameter space we observe different types of transitions to chaos. A sequence was taken with B_0 = 4 kG, as V_{dc} was increased from 0 to 25 V. The overall behavior of I(t) was found to be as follows: For V_{dc} < 6 V, I(t) has

only a dc component. At $V_{dc} = 6$ V, I(t) spontaneously becomes periodic. Regions of chaotic dynamics occur in the intervals $7.0 \leq V_{dc} \leq 7.4$ V; $10.0 \leq V_{dc} \leq 10.7$ V; and $14.9 \leq V_{dc} \leq 18$ V; otherwise, I(t) is periodic. The clearest of these three chaotic sequences starts at $V_{dc} = 10.0$ V: I(t) is oscillating at a fundamental frequency $f_0 \approx$ 118 kHz, i.e., at period 1. The phase portrait, I(t) vs. V(t), shows that the oscillation has a small spectral component at a harmonic of f_0. However, there is no subharmonic component. As V_{dc} is increased, I(t) shows a period doubling bifurcation: the emergence of a spectral component at $f_0/2$. At larger V_{dc}, another period-doubling bifurcation occurs with new spectral components at $f_0/4$, $3f_0/4$, $5f_0/4$, ... At slightly larger V_{dc} I(t) becomes nonperiodic and its power spectrum enters a region of broadband "noise". For further increases of V_{dc} there appear noise-free windows of periods 3, 4, 5, ..., within this region of broadband noise. This sequence ends at $V_{dc} = 10.7$ V with a return to period 1 oscillations.

A second type of transition which we have observed is the quasi-periodic route to chaos: as V_{dc} is increased, the onset of a quasi-periodic state is followed by a transition to chaos. In one such sequence taken at $B_0 = 11.15$ kG, at $V_{dc} = 2.865$ V, I(t) is spontaneously oscillating at a fundamental frequency $f_1 = 63.4$ kHz. At $V_{dc} = 2.907$ V, the system becomes quasiperiodic: a second spectral component appears at $f_2 = 14$ kHz, incommensurate with f_1. At $V_{dc} = 2.942$ V, the system is still quasiperiodic; however, the two modes are interacting and the nonlinear mixing gives spectral peaks at the combination frequencies $f = mf_1 + nf_2$, with m,n positive and negative integers. As V_{dc} is increased further, we observe a series of frequency lockings, i.e., (f_1/f_2) = rational number, until the onset of chaos is reached, indicated by a slight broadening of the spectral peaks. As V_{dc} is increased further, the spectra become even broader. This is followed by a return to quasiperiodicity at $V_{dc} = 3.125$ V and, subsequently, simple periodicity at $V_{dc} = 3.442$ V. For $V_{dc} = 3.058$ V we measure the fractal dimension of the attractor, as discussed above, finding $d \approx 2.6$; convergence is obtained both with respect to the embedding dimension D and the number of data points ($N \approx 10^4$).

Results: Spatial behavior. Turning attention to the question of spatial coherence within the instabilities, we ask whether the chaotic states we observe correspond to a temporally chaotic yet still spatially coherent or whether the onset of chaos corresponds to a breakup of spatial order within the density wave.

For our system we define a transition to "weak" turbulence to be one in which the transition from periodicity to chaos is followed by a transition back to periodicity as V_{dc} is increased further. The two scenarios discussed above both correspond to transitions to "weak" turbulence. For this case (data taken at B_0 = 11.15 V, $V_{dc} \approx$ 5 to 6 V) we calculate the correlation function C(r), Eq. (3), for the periodic state and find that it is fit by the correlation function for a travelling wave - not surprising. In addition, we find that the quasiperiodic and chaotic states both have correlation functions that follow the periodic case and so conclude that this weakly turbulent instability is chaotic in the temporal domain only. Even while exhibiting chaotic behavior it remains essentially a spatially coherent plasma density wave.

However, with sufficiently large applied electric and magnetic fields, we find we can drive the plasma into a turbulent state which will not become periodic again. Instead, all of the frequency peaks in the power spectrum merge into a single, broad, noiselike band. We classify this as a transition to "strong" turbulence. An example is found at B_0 = 11.15 kg as V_{dc} is increased from 10.0 V to 21.8 V. At 11.6 V the system becomes quasiperiodic, and chaotic at 12.1 V, with measured fractal dimension d = 2.6. At 12.9 V the measured fractal dimension has increased to $d \geq 8$: the fractal dimension plots do not show a convergence of slope for embedding dimensions as large as D = 18 and number of data points N = 884,000. Thus we can only set a lower limit to the value of d. At 13.8 V the power spectra are broad with a few peaks, and at 21.8 V, very broad with no peaks.

This difficulty in calculating large fractal dimensions is a problem encountered whenever one works with a very chaotic system; the number of data points required for convergence increases exponentially with the fractal dimensions of the system. At present, although we know that our system experiences a large jump in dimensionality, we have not yet determined whether this onset is characterized by chaotic dynamics of an attractor of fractal dimension many orders smaller than the number of degrees of freedom of the particles in the system. In the same scenarios which shows the jump in dimension we also find a gradual loss in spatial coherence, as observed from the oscilloscope voltage traces between pairs of probe contacts and the measured correlation function C(r), which decreases with increasing V_{dc}.

4. Chaotic Dynamics of Spin Wave Instabilities in Ferrites

Noisy instabilities in ferromagnetic resonance saturation in some ferrites were experimentally discovered in the 1950's [46] and explained by Suhl [17] in a detailed theory of nonlinear coupling between the uniform precession mode of the magnetization vector and spin waves. The uniform mode can excite the spin waves which grow exponentially. Suhl recognized early that "...This situation bears a certain resemblance to the turbulent state in fluid dynamics...". In the simplest case the equations are similar to Eqs. (1), where C_1 is the amplitude of the uniform mode (wave vector k = 0); C_2 and C_3 are amplitudes of a pair of spin waves (wave vector k and -k); and a radio frequency driving term must be added to Eq. (1a), as well as higher order nonlinear terms of the form $C_1C_2C_3^*$, the so-called Suhl 2nd order terms.

We review below recent experiments at Berkeley on spheres of gallium yttrium iron garnet (YIG) which show that Suhl's 2nd order instability is a period doubling cascade to chaos [13]. The sample magnetization is now known to display temporal chaos; it is not yet known if it also displays spatial incoherence as in the plasma case. Parallel pumping experiments on YIG [14] and in copper salts [15] also show period doubling. In some salts a route to chaos through irregular periods without period doubling has been repeated [16]. Suhl's theory has recently been extended [47] by numerical computation, and a period doubling cascade to chaos is found. Similar computations have been carried out for the parallel pumping case [14,48].

The Berkeley experiments are carried out by mounting a highly polished sphere of Ga-YIG in a magnetic field $H_0 \parallel z$; pumping with a field $H_1 \parallel x$ at the ferromagnetic resonance frequency f_0; and observing a signal $V_s(t)$ which is the time derivation of the transverse magnetization. $V_s(t)$ has a strong component at f_0, which is the usual ferromagnetic resonance signal. In our case $H_s \approx 460$ Oe; $f_0 = 1.3 \times 10^9$ Hz; sample radius ≈ 0.047 cm and saturation magnetization $4\pi M_s = 300$ gauss; resonance line width ≈ 0.5 gauss. As the driving field H_1 is increased, there is a threshold value H_{1a} at which low frequency self-oscillations set in at $f_{1a} \approx 250$ kHz, corresponding to annihilation of two $(\omega_0, k=0)$ magnons and the creation of a spin wave pair (ω_k, k) and $(\omega_k, -k)$. The value of the frequency f_{1a} corresponds closely to the lowest standing wave mode in the spherical sample of a packet of such spin waves travelling parallel to H_0 [49]. We note that numerical solutions [47,14] of the coupled mode equations predict self-oscillations arising from a Hopf bifurcation to a limit cycle at a frequency determined by the coupling parameters and independent of the sample size. However, the calculated frequency does not yet agree with observation and the question is at present unresolved.

As H_1 is increased still further, there is another threshold H_{1b} at which a second self-oscillation sets in at $f_{1b} \approx 16$ kHz, which we interpret as the onset of creation of a spin wave pair with $k \approx 0$. From theory, the relaxation process becomes exponentially weaker as $k \to 0$ [50]; we observe long lifetimes for these low frequency self-oscillations, as well as a period doubling cascade to chaos and periodic windows. This is the first experiment to demonstrate the existence of chaotic dynamics in magnetic materials and much work remains to be done both experimentally and theoretically. Spin systems are well characterized and the macroscopic nonlinear parameters in the coupled mode equations can be calculated microscopically. This feature makes possible, in principle, the comparison between a nonlinear dynamics experiment and high level theory.

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