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Publication Date

1952-12-17

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Richard Madey, Kenneth C. Bandtel, and Wilson J. Frank

December 17, 1952

Berkeley, California

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ABSTRACT

The full energy spread-out bremsstrahlung beam of the Berkeley synchrotron was shown to be emitted in sharp bursts with the 47.7 megacycle frequency of the electrons in the doughnut. A two-counter telescope, located about 20 inches away from a one-fourth inch lead target and about 20 degrees from the photon beam direction, was used to measure an accidental coincidence counting rate as a function of the length of delay line in one input to the coincidence circuit. When the length of delay cable was chosen equal to integral multiples of the period of the radiofrequency oscillator, the coincidence counting rate was of the order of 100 counts per unit of integrated beam; on the other hand, when the length of delay line was chosen to be equal to one-half of odd integral multiples of the period of the radiofrequency oscillator, the coincidence counting rate was less than about two counts per unit of integrated beam.

An estimate of the width of a single pulse of photons can be obtained from the measurement by unfolding a Gaussian resolution function of the coincidence circuit. If the time variation of the intensity of the photon beam is assumed to be a Gaussian function, then the r.m.s. value of that Gaussian is found to be less than 1.5×10^{-9} seconds.

A calculation by E. M. McMillan* indicates that the full energy photon beam should exhibit fine structure if the amplitude of the azimuthal phase oscillations at full energy is less than about one radian. The experimental result is another confirmation of the theory of phase stability.

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* E. M. McMillan. Private communication.

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The theory of the synchrotron[1,2] yields the result that the electrons in or near the synchronous orbit are accelerated in phase stable bunches. That is, the motion of the electrons can be described in terms of stable oscillations about a synchronous orbit. In Fig. 1, r_o denotes the radius of the synchronous orbit, $(\Delta r)_{max}$ = the maximum amplitude of the radial phase oscillations for a given energy electron in the synchronous orbit and $(\Delta \phi)_{max}$ = the maximum amplitude of the azimuthal phase oscillations for a given energy electron in the synchronous orbit. The electrons spiral inward to strike the target when the radiofrequency oscillator is turned off because of the energy loss of the radiating electrons.

If the oscillator is turned off suddenly, the time for one of these electron bunches to cross the target has been observed, on an oscilloscope, to be of the order of 8 to 10 microseconds. Within this 8 to 10 microsecond pulse, a fine structure will exist, with about the 2.1×10^{-8} second period of the electrons in the doughnut, provided the original phase stable bunch at the synchronous orbit does not spread out and fill the entire 360 degrees of the doughnut by the time the electrons have collapsed to the target radius.

E. M. McMillan[3] has considered the question of what happens to the size of the phase bunch of electrons after the oscillator loses control. As the electrons spiral inward from the synchronous orbit to the target, only the relative change in phase of the particles in the bunch will affect the size of the bunch. The relative change in phase of two electrons in the bunch depends, not on the entire distance to the target through which the electrons must collapse, but rather on the difference in the positions of the two electrons from the target. In collapsing to the target, the change in phase of an electron at maximum amplitude $(\Delta r)_{max}$ relative to that for one at the synchronous orbit is given by the following integral:

$$\delta\phi_2 - \delta\phi_1 = \int_{(\Delta r)_{max}}^{0} \frac{d\phi}{dt} \frac{1}{v} \, d(\Delta r) \qquad (1)$$

where $\frac{d\phi}{dt}$ = the rate of change in the phase of one electron relative to the other,

v = the velocity or rate at which the electrons collapse along a radius vector to the target.

Now

$$\frac{d\phi}{dt} = \Delta\omega = \Delta(\frac{c}{r}) = -\frac{c}{r_o^2} \Delta r \qquad (2)$$

where ω is the angular frequency of an electron that is moving at nearly the speed of light c in an orbit of radius r. The radiation loss of such an electron causes it to spiral inward toward the target with a radial velocity of collapse

$$v = \frac{c}{2\pi(1-n)} \frac{1}{W} \frac{dW}{dN} \qquad (3)$$

where W = the electron energy at the synchronous orbit,

$\frac{dW}{dN}$ = the radiation energy loss per turn of an electron at the synchronous orbit, and

$n = -d(\ln B)/d(\ln r)$, where

B = the magnetic field at the synchronous orbit.

Upon substituting Eqs. (2) and (3) in the integral (1), the difference in the phase change of the two electrons under consideration is

$$\delta\phi_2 - \delta\phi_1 = \frac{\pi(1-n)}{r^2} \frac{W}{\frac{dW}{dN}} \left[(\Delta r)_{max}\right]^2 \qquad (4)$$

This expression can be written in terms of $(\Delta\phi)_{max}$ by using the result of Bohm and Foldy [(2)] that

$$\frac{(\Delta r)_{max}}{r} = \sqrt{\frac{eV}{2\pi(1-n)W}} \, (\Delta\phi)_{max} \qquad (5)$$

where eV = the maximum energy gain per turn that can be supplied by the oscillator to the given energy electron in the synchronous orbit.

Substitution of Eq. (5) into the expression (4) gives the relatively simple result

$$\delta\phi_2 - \delta\phi_1 = \frac{1}{2} \frac{eV}{\frac{dW}{dN}} \left[(\Delta\phi)_{max}\right]^2 \qquad (6)$$

For the full energy beam of the Berkeley synchrotron, the maximum energy gain per turn is about 2000 ev, and the energy loss per turn is about 1000 ev; therefore, for the Berkeley machine, the general result for the difference in the phase change of the two electrons is approximately

$$\delta\phi_2 - \delta\phi_1 \sim \left[(\Delta\phi)_{max}\right]^2. \qquad (6')$$

Hence, if the maximum value of the azimuthal phase amplitude at full energy is equal to about one radian (that is, if the phase stable bunch of electrons fills up about 120 degrees of the doughnut at the synchronous radius), then, when the electrons collapse to the target, a particle that was at maximum radial amplitude will differ in phase by one radian from a particle that was at the synchronous orbit.

The radio-frequency fine structure of the full energy spread-out photon beam of the Berkeley synchrotron was observed by measuring an accidental coincidence counting rate as a function of the length of delay line in one input to a fast coincidence circuit. The experimental arrangement is shown in Fig. 2. A scintillation counter telescope, consisting of two 2.4 gm/cm^2 by 2 inch square stilbene phosphors, was placed at about 15 to 20 degrees from the beam and at about 20 inches from a 1/4 in. lead target. The negative pulses from the 1P21 photomultiplier tubes were clipped to a pulse duration of 3×10^{-9} seconds, limited in pulse amplitude to about 2 volts, and inverted in polarity. These pulses were fed directly, without the aid of distributed amplifiers, into a new crystal diode coincidence circuit; the schematic circuit diagram is shown in Fig. 3.

The coincidence counting rate is plotted in Fig. 4 against the length of delay cable in one input. The number of counts per unit of integrated beam is plotted logarithmically on the vertical scale; the number of feet of delay cable is plotted linearly on the horizontal scale. Standard deviations are indicated for each point. When the length of delay cable in one input to the coincidence circuit was chosen to be equal to 10, 30 and 50 feet (that is, to one-half odd integral multiples of the period of the r.f. oscillator) the coincidence counting rate was less than about 2 counts per unit of integrated beam on the other hand, when the length of delay cable was equal to 20, 40 and 60 feet (that is to integral multiples of the period of the r.f. oscillator), the coincidence counting rate was on the order of 100 counts per unit of integrated beam. The zero delay point gave a real plus accidental counting rate of about 850 counts per unit of integrated beam; however, we believe that this zero delay counting rate may be higher because of a blocking loss in the scaler. From these data, we conclude that the modulation is at least 98 percent..

Figure 5 shows how the coincidence counting rate falls off on both sides of the peak at 40 feet of delay line. The lengths of delay cable were varied by an amount equal to the length of the clipped pulses (3×10^{-9} seconds). Since such accidental counting rates are quite sensitive to fluctuations in beam intensity, the plotted points were obtained in consecutive runs because we felt that the machine operation was such that the beam intensities are more nearly reproducible in this manner. The 40 feet delay counting rate that appears in Fig. 4 is about 100 counts per unit of integrated beam compared with about 360 counts per unit beam exposure in Fig. 5. The average beam intensities (in arbitrary units) for these two points were 0.8 per minute compared to 1.0 per minute.

The resolution function of the coincidence circuit is a measure of the "transmission" of the double coincidence circuit as a function of the time displacement of the two input pulses. That is, the resolution function gives the probability of a coincidence count if the second input pulse arrives at a time t after the arrival of the first input pulse. Each input pulse is clipped, by means of a delay line, to 3×10^{-9} seconds at the base.

Now, the fold of the coincidence circuit resolution function r(t) and the function f(t) that represents the time variation of the intensity of the photon beam from the synchrotron is given by

$$P(s) = \int_{-\infty}^{+\infty} f(t)\, r(s-t)\, dt = \left[f*r\right](s) \qquad (7)$$

Since the rate of production of particles in the target is proportional to the photon beam intensity f(t), the probability of arrival of the first pulse at one input to the coincidence circuit is also proportional to f(t). The fold $[f*r]$ (s) is now a measure of the "transmission" of the double coincidence circuit for a distribution of initiating pulses; that is, it gives the probability of a coincidence count if the second pulse arrives at a time s, measured from any arbitrary point on the distribution function of the first pulse, after the arrival of the first input pulse.

In the actual experiment, coincident pulses were generated at different times. If T is a fixed time interval between the generation of the pulses, then the measured counting rate is proportional to

$$\int_{-\infty}^{+\infty} f(T-t)\ P(t)\ dt = [f*P]\ (T) = [f*f*r]\ (T) \qquad (8)$$

An estimate of the width of a single pulse of photons can be obtained from the data of Fig. 5 by unfolding the resolution function of the coincidence circuit. Figure 6 is a plot of the double coincidence counting rate versus the length of delay line in one input to the circuit when cosmic radiation is incident on the stilbene phosphors. These points can be fitted by a Gaussian resolution function of the form

$$r(\ell) \sim e^{-\ell^2/2\beta^2} \qquad (9)$$

if the r.m.s. value of the distribution β is 1.80 ± 0.17 feet of cable.

Similarly, the data of Fig. 5 can be fitted by a Gaussian function of the same form $e^{-\ell^2/2\gamma^2}$ if the r.m.s. value γ is 2.48 ± 0.27 feet of cable.

Now, if the time variation of the intensity of the photon beam is assumed to be a Gaussian function $f(\ell) \sim e^{-\ell^2/2\alpha^2}$ with an r.m.s. paramenter α, then

$$e^{-\ell^2/2\gamma^2} \sim f*f*r \sim e^{-\ell^2/2(2\alpha^2 + \beta^2)} \qquad (10)$$

Thus, the squares of the characteristic parameters of the three Gaussian functions are related by

$$\alpha^2 = \frac{1}{2} (\gamma^2 - \beta^2) \qquad (11)$$

Substitution of the measured values of γ and β into Eq. (11) gives

$$\alpha_\ell = 1.20 \pm 0.09 \text{ feet of cable, or}$$

$$\alpha_t = 1.28 \pm 0.10 \times 10^{-9} \text{ seconds.}$$

The value $\sqrt{2}\alpha$ may be taken as an estimate of the maximum amplitude of the azimuthal phase oscillations for full energy electrons at the synchronous orbit provided the original phase stable bunch of electrons is assumed not to spread out during the time the electrons collapse to the target radius. For $t = \sqrt{2}\alpha_t$, the amplitude of the Gaussian function that represents the time variation of the intensity of the photon beam is reduced to a value equal to $\frac{1}{e}$ times the maximum amplitude. This criterion gives 31.0 ± 2.4 degrees for the maximum amplitude of the azimuthal phase oscillations for full energy electrons at the synchronous orbit. The size of the phase stable bunch is twice this maximum amplitude.

REFERENCES

1. E. M. McMillan, Phys. Rev. 68, 143 (1945)
2. D. Bohm and L. Foldy, Phys. Rev. 70, 249, (1946)
3. E. M. McMillan. Private Communication

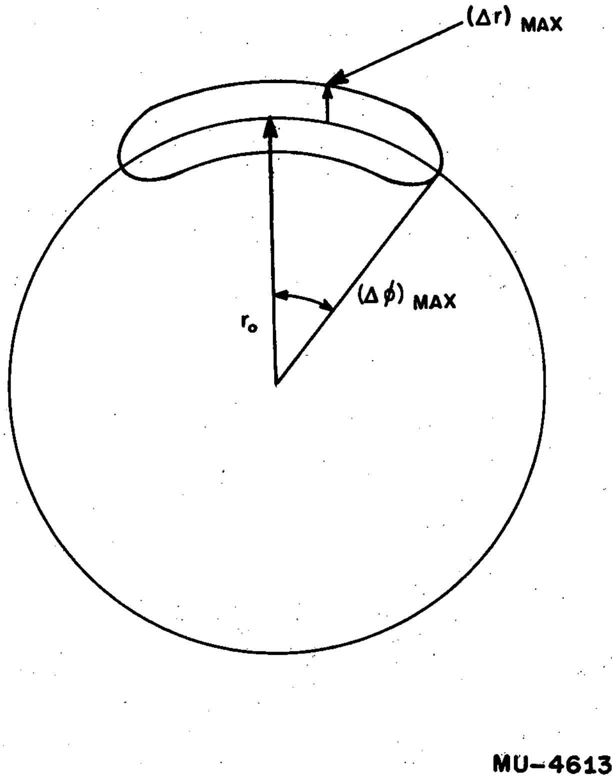

Fig. 1

A phase stable bunch of electron at the synchronous orbit.

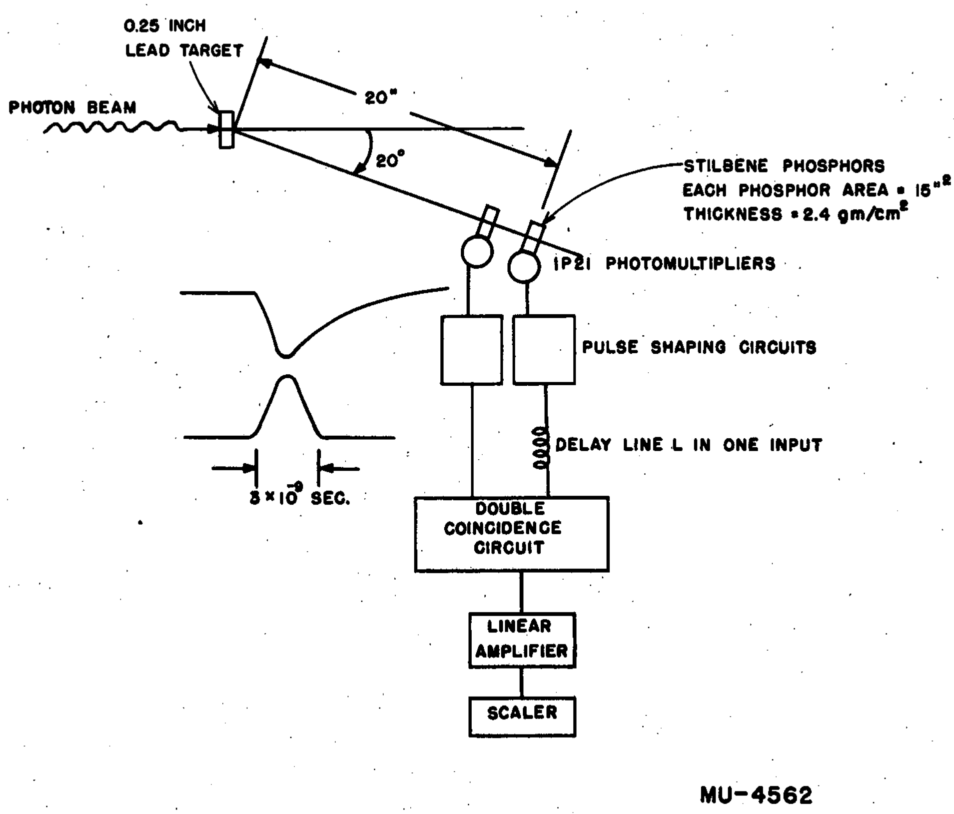

Fig. 2

The experimental arrangement for measuring the radiofrequency fine structure of the photon beam from the Berkeley synchrotron.

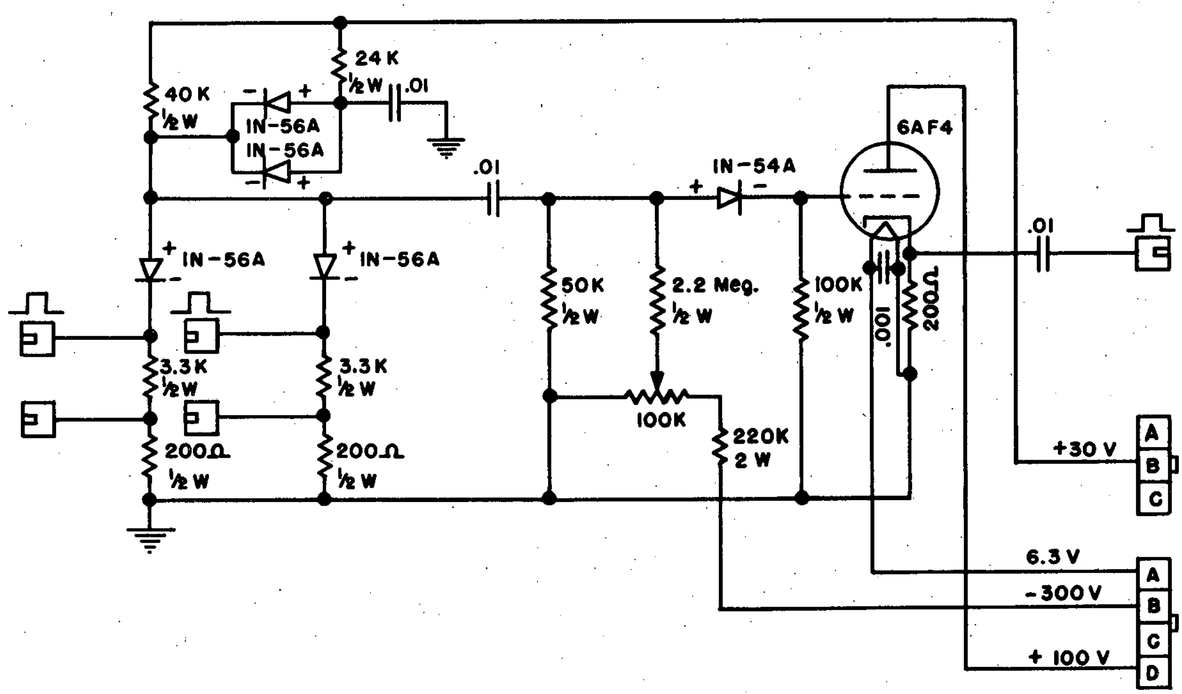

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Fig. 3
The crystal diode double coincidence circuit.

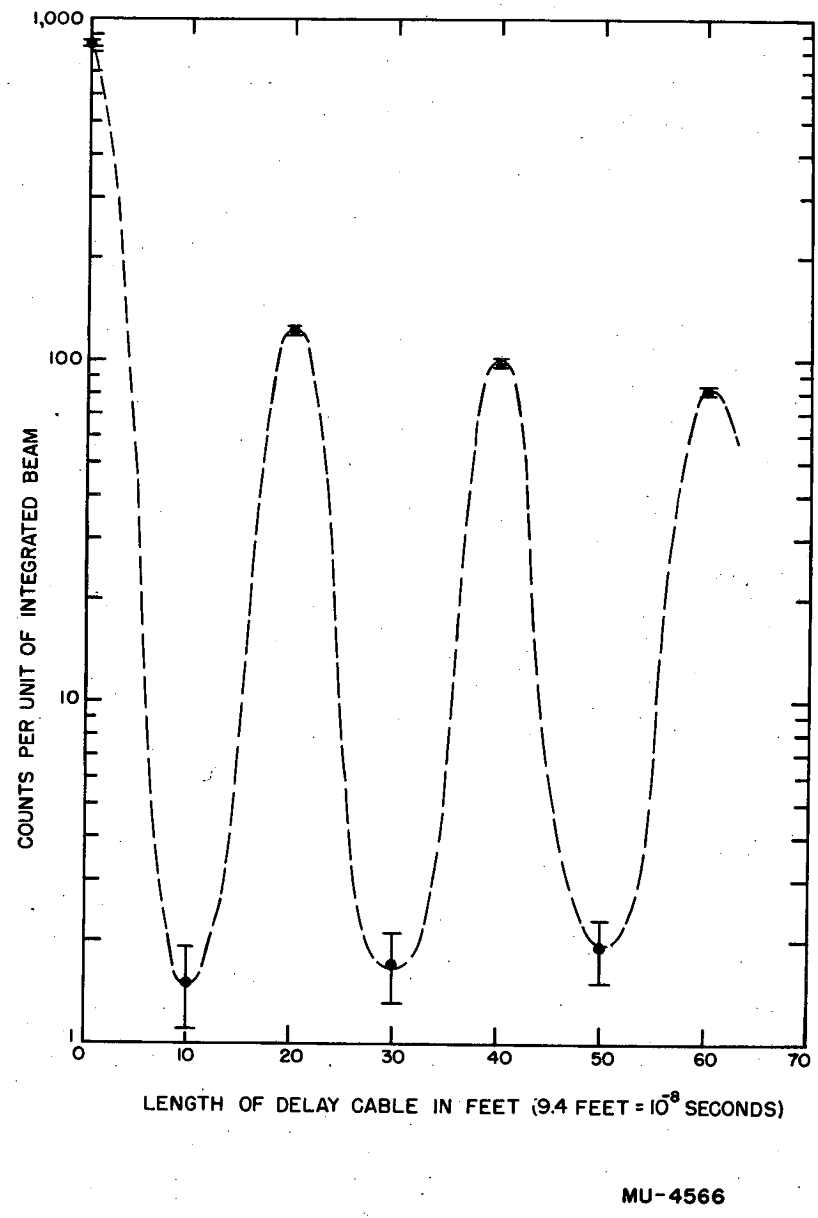

Fig. 4

The double coincidence counting rate versus the length of delay line in one input to the fast coincidence circuit when the delay line is varied by half integral multiples of the period of the radiofrequency oscillator.

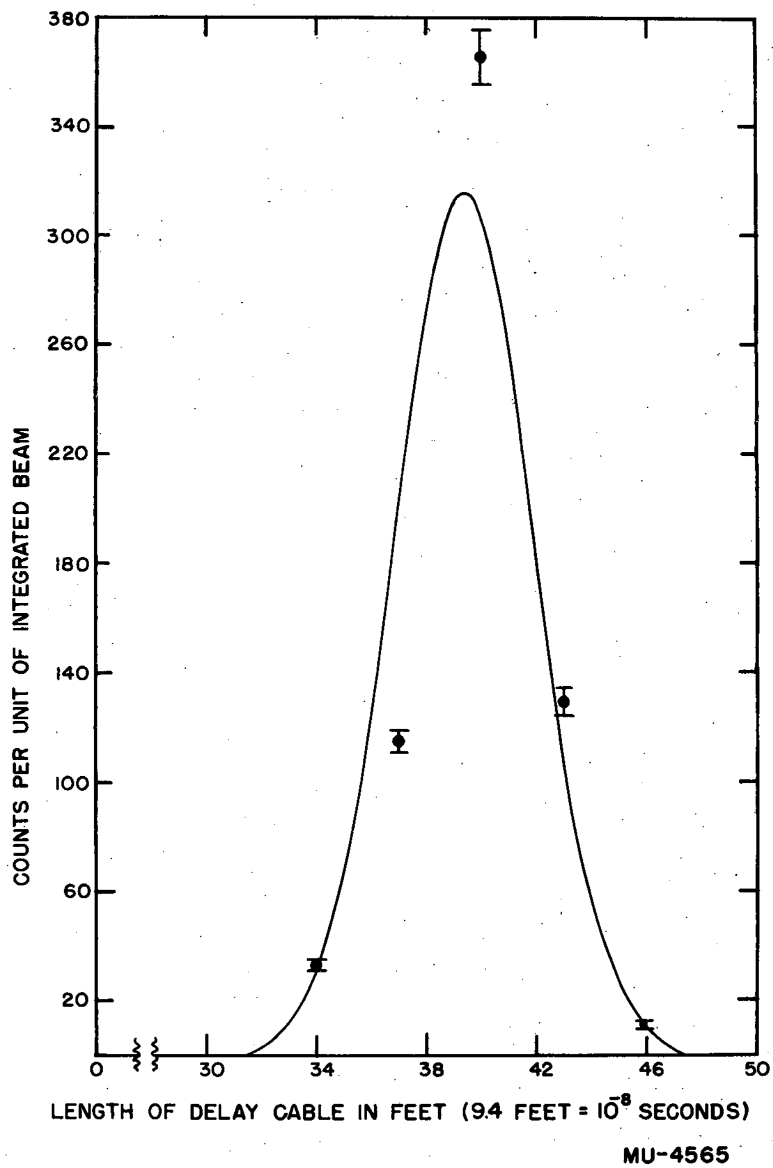

Fig. 5.

The double coincidence counting rate versus the double coincidence counting rate versus the length of delay line in one input to the fast coincidence circuit when the delay line is varied by amounts equal to the resolution time of the coincidence circuit.

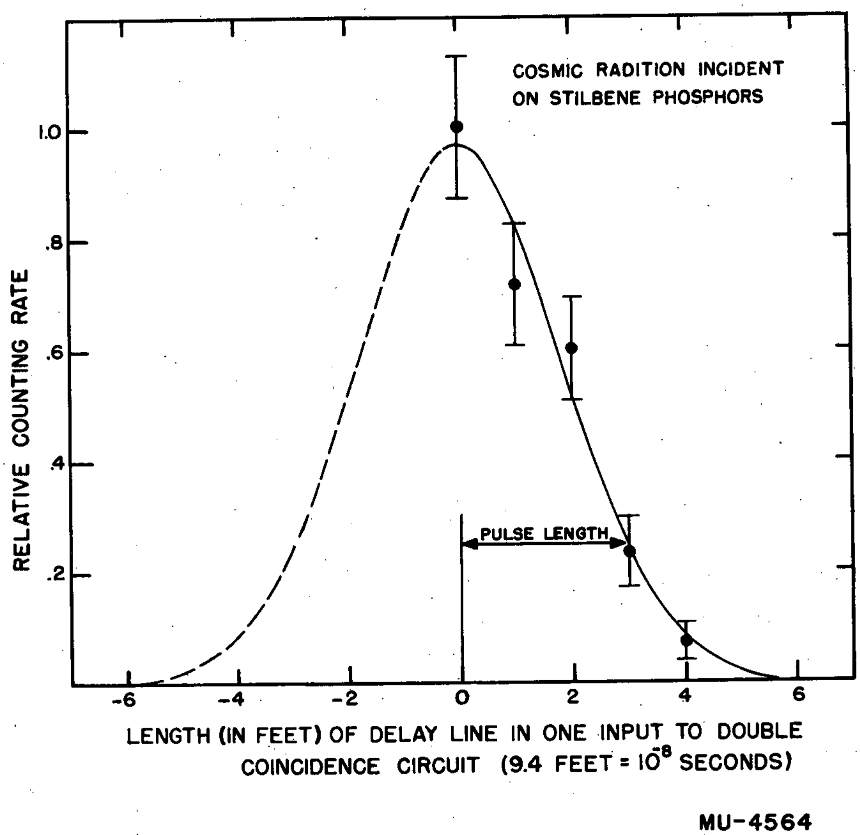

Fig. 6

The double coincidence counting rate versus the length of delay line in one input to the coincidence circuit when cosmic radiation is incident on the stilbene phosphors.