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Physics Division

Presented at the 7th Topical Workshop on
Proton-Antiproton Collider Physics,
Fermilab, Batavia, IL, June 20–24, 1988

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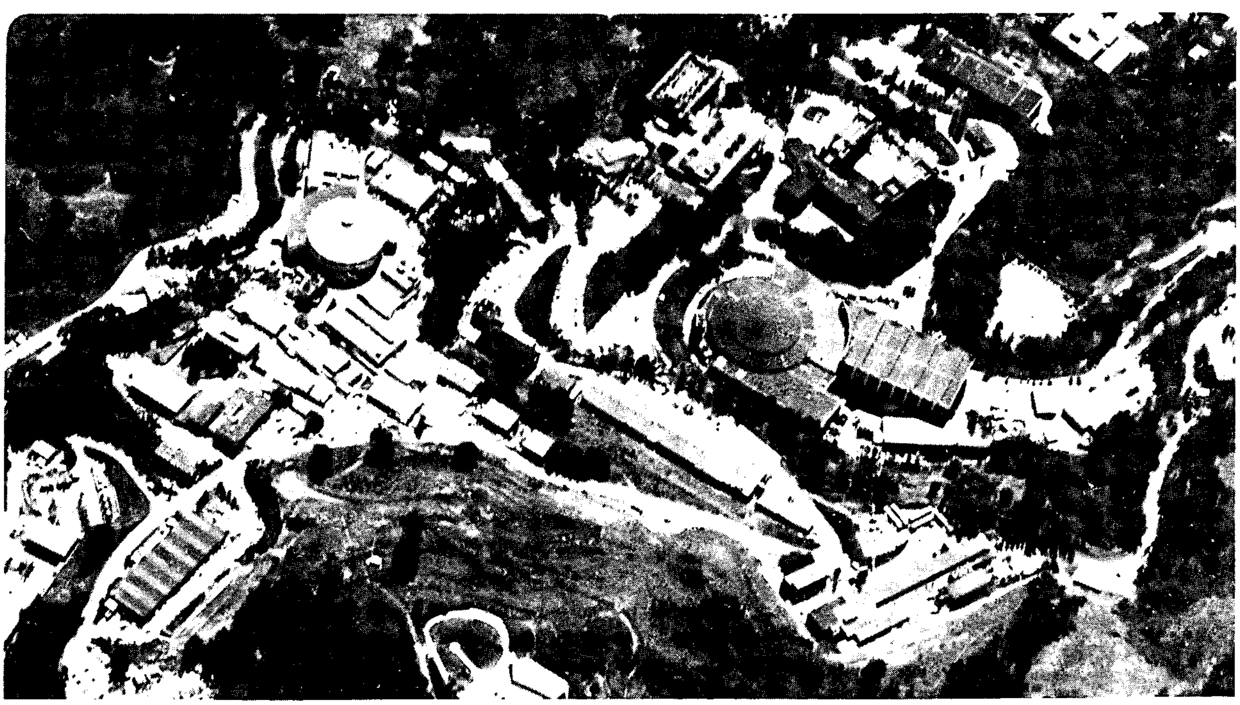
July 1988



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Overview of σ, ρ and $d\sigma/dt$ in High Energy Proton-Antiproton Scattering *

Presented at 7th Topical Workshop on
Proton-Antiproton Collider Physics
Fermilab, June 20-24, 1988

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Abstract

Recent results on the total cross section, real-to-imaginary ratio of the forward scattering amplitude, and the slope parameter are discussed. The ability of future Tevatron data to distinguish between alternative explanations of the large value of ρ measured by UA-4 is emphasized.

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Discussions of total cross sections and the like often fail to excite audiences of high energy physicists. This is not, I believe, because the subject is intrinsically uninteresting but because we are too ignorant. One day we may wake up and learn that Ed Witten has abandoned strings and has just discovered an ingenious approximation that makes possible predictions to 1% for non-perturbative QCD. Extra dimensions will evaporate before our eyes and everyone from Gordy Kane to Sidney Coleman will be calculating pp total cross sections.

While we await that great day is there nothing exciting to say about forward $p\bar{p}$ scattering? Although we don't know a lot, we know enough to realize that confirmation of the large value for the ratio of the real to the imaginary part of the forward scattering obtained by the UA-4 Collaboration at the Sp$\bar{\text{p}}$S would mean that the usual view of hadronic scattering is dramatically wrong.

One thing we do know is that scattering amplitudes come from analytic functions. (For a simple review, see Ref. 1.) And we know a good deal experimentally about pp and $p\bar{p}$ total cross sections. At high energies the subject is simple. There is no need for dispersion relations or even derivative dispersion relations. We simply write down the answers.

The total cross section is $1/s$ times the imaginary part of the forward scattering amplitude

$$\sigma = \frac{1}{s}\,\mathrm{Im}\,F. \tag{1}$$

Defining

$$F_{\pm} = \frac{1}{2}(F^{pp} \pm F^{\bar{p}p}) \tag{2}$$

F_{+} is an even function of s and F_{-} is odd. The variable s is taken to be complex and the F's have cuts along the real axis but between the left-hand and right-hand branch points the F's are real. Prototypical examples are

$$F_{\pm} = (s_{th} - s)^{\alpha} \pm (s_{th} + s)^{\alpha}. \tag{3}$$

The physical region is above the real axis and there we find

$$F_{\pm} = e^{-i\pi\alpha}\,|s_{th} - s|^{\alpha} \pm |s_{th} + s|^{\alpha} \tag{4}$$

so, far above threshold

$$F_{+} \;\cong 2\,|s|^{\alpha}\,e^{-i\pi\alpha/2}\cos\pi\alpha/2,$$

$$F_- \cong -2i\,|s|^\alpha\, e^{-i\pi\alpha/2} \sin \pi\alpha/2. \tag{5}$$

Thus in the especially interesting limiting cases of $\alpha \to 1$, the even function is is and the odd function is s. Another important even function is

$$\frac{1}{2}\left[\ln(s_{th} - s) + \ln(s_{th} + s)\right] \tag{6}$$

which above the right hand cut is, for $s \gg s_0$

$$\ln(s/s_0) - i\pi/2. \tag{7}$$

At lower energies, *i.e.* below the $Sp\bar{p}S$ and Tevatron collider it is important to keep the odd amplitude that behaves roughly as $s^{1/2}$ and which gives $\Delta\sigma = \sigma_{pp} - \sigma_{p\bar{p}} \propto s^{-1/2}$. At very high energies we can ignore this and consider

$$F = F_+ = iAs + iBs\left[\ln(s/s_0) - \frac{i\pi}{2}\right]^2 \tag{8}$$

so

$$\sigma = \frac{1}{s}\,\mathrm{Im}\,F = A + B\left[\ln^2(s/s_0) - \frac{\pi^2}{4}\right] \tag{9}$$

Now this really isn't good enough for the ISR region but it will do for our purposes. The minimum of σ occurs at s_0, let say $\sqrt{s_0} = 20$ GeV and

$$\sigma_{\min} = A - B\frac{\pi^2}{4} \approx 40 \text{ mb}. \tag{10}$$

Let's take

$$\sigma(\sqrt{s} = 546 \text{ GeV}) = 62 \text{ mb} \tag{11}$$
$$= A + B \cdot 41.3$$

Together these determine A and B

$$A = 41.3 \text{ mb} \qquad B = 0.50 \text{ mb}$$

Now the phase of the forward scattering amplitude can be measured by observing interference between the hadronic and coulombic scattering amplitudes. The results are expressed as

$$\rho = \frac{\mathrm{Re}\,F}{\mathrm{Im}\,F}. \tag{12}$$

For our simple amplitude

$$\rho = \frac{\pi B \ln s/s_0}{A + B\left[\ln^2(s/s_0) - \frac{\pi^2}{4}\right]}. \tag{13}$$

The maximum value of ρ occurs when

$$\ln^2(s/s_0) = \frac{A}{B} - \frac{\pi^2}{4}. \tag{14}$$

For our choice of A and B this gives $\sqrt{s} = 1800$ GeV and

$$\rho_{max} = \frac{\pi B\sqrt{\frac{A}{B} - \frac{\pi^2}{4}}}{2A - \pi^2 B/2} \approx \frac{\pi}{2}\sqrt{\frac{B}{A}} \approx 0.173 \tag{15}$$

Other choices of parameters give very similar results. At $\sqrt{s} = 546$ GeV we actually find $\rho = 0.168$.

The excitement comes from the measurement by the UA-4 collaboration at $\sqrt{s} = 546$ GeV,[2]

$$\rho = 0.24 \pm 0.04. \tag{16}$$

Not only is this far above our sample estimate of $\rho = 0.17$, more careful estimates[1,3] predict $\rho = 0.14$ - 0.15. Of course, the discrepancy is little more than two standard deviations and cynics are entitled to dismiss the whole issue. If, however, we take the reported central value seriously, the consequences are dramatic.

One way to explain the experimental result is to include an odd amplitude that persists at high energies. While the old Pomeranchuk theorem stated that if σ_{pp} went to a constant at high energies then $\sigma_{pp} - \sigma_{p\bar{p}} \to 0$ (provided $\rho/\ln s \to 0$), the revised Pomeranchuk theorem simply says that if $\sigma_{pp} + \sigma_{p\bar{p}} \propto (\ln s)^\gamma$ then $|\sigma_{pp} - \sigma_{p\bar{p}}|$ cannot grow faster than $(\ln s)^{\gamma/2}$. While the data for the difference $\sigma_{pp} - \sigma_{p\bar{p}}$ is consistent with the behaviour $s^{-1/2}$, in principle we can consider contributions of the form[4]

$$F_- = Cs\left[\ln^2(s/s_0') - \frac{i\pi}{2}\right]^2. \tag{17}$$

This will contribute to σ_{pp} a piece $-\pi C(\ln s/s_0')$ and an opposite amount to $\sigma_{p\bar{p}}$. These must not be large in the ISR region. In addition, $\rho_{p\bar{p}}$ will receive a contribution roughly

$$\Delta\rho_{p\bar{p}} = -\frac{C\left[\ln^2(s/s_0') - \frac{\pi^2}{4}\right]}{\sigma_{p\bar{p}}}. \tag{18}$$

If we somewhat arbitrarily take $s_0' = s_0$

$$\Delta\rho_{p\bar{p}} \approx -\frac{C[\sigma_{pp} - A]}{B\,\sigma_{pp}} \approx -\frac{C}{2B} \tag{19}$$

where we use the values near $\sqrt{s} = 546$ GeV. If $\Delta\rho_{p\bar{p}} \approx 0.06$ say, then $C \approx -$ 0.06 mb. This will make $\sigma_{pp} > \sigma_{p\bar{p}}$!

$$\begin{aligned} \sigma_{pp} - \sigma_{p\bar{p}} &\approx -2\pi C \ln(s/s_0) \\ &\approx 2.5 \text{ mb @ } \sqrt{s} = 546 \text{ GeV} \\ &\approx 3.4 \text{ mb @ } \sqrt{s} = 1.8 \text{ TeV} \end{aligned} \tag{20}$$

and larger values would result if we required, say, $\Delta\rho_{p\bar{p}} = 0.10$. Similar results have been obtained by Leader.[5] There is no hope of making a direct measurement of the difference, but a measurement of ρ at the Tevatron should find $\rho_{p\bar{p}} > 0.24$.

An alternative approach has been suggested by Andre Martin,[6] and by Hadjitheodoridis and Kang.[7] They suppose that the odd amplitude is negligible but the even amplitude has a threshold around $\sqrt{s} = 546$ GeV. Martin's version gives a very large cross section at the Tevatron, but a modest value of ρ, 0.17.

A specific model for a rapidly increasing total cross section has been proposed by Margolis et al.[8] They attribute a rising cross section to increased gluonic interactions of the sort manifested in minijets. Their model has $\rho = 0.19$ at the CERN collider and $\rho = 0.18$ at the Tevatron Collider. It is typical of these threshold models that the large value of ρ persists only for a limited range of energy.

A very rough guide to some possibilities for the Tevatron Collider results is given in Table 1.

Table 1

	Odderon	Threshold	Cynic
$\sigma_{p\bar{p}}$ (1.8 TeV)	75 mb	95 mb	75 mb
$\rho_{p\bar{p}}$ (1.8 TeV)	0.25	0.17	0.15

Tevatron results should conclusively discriminate between the alternatives. It must be borne in mind that the UA-4 result has a large uncertainty, so a first step at the Tevatron should be a measurement of ρ at an energy comparable to that of the CERN Collider.

So far from the Tevatron we have only a measurement of the slope parameter

$$B = \frac{d}{dt}\left[\ln \frac{d\sigma}{dt}\right] . \qquad (21)$$

This measures, in some sense, the size of the proton. In an impact parameter picture B is one-half the average value of the square of the impact parameter. Block and I made predictions[9] for B as a function of s using the Chou-Yang model. Our fits to σ and ρ were inputs to the Chou-Yang model. In this way we predicted $B(1.8\text{ TeV}) \cong 15.0\text{ GeV}^{-2}$ at $t = 0$. A fit to the existing data for the slope at lower energies extrapolated to $B(1.8\text{ TeV}) \approx 16.5\text{ GeV}^{-2}$. The new value[10] for the Tevatron Collider gives $17.2 \pm 1.3\text{ GeV}^{-2}$. This is probably not a useful clue to understanding the UA-4 data.

We can all look forward to new Tevatron Collider results on total cross sections and elastic scattering. Should the central value reported by UA-4 be confirmed perhaps my theoretical colleagues will decide it's time to worry about some new puzzles in an old field.

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