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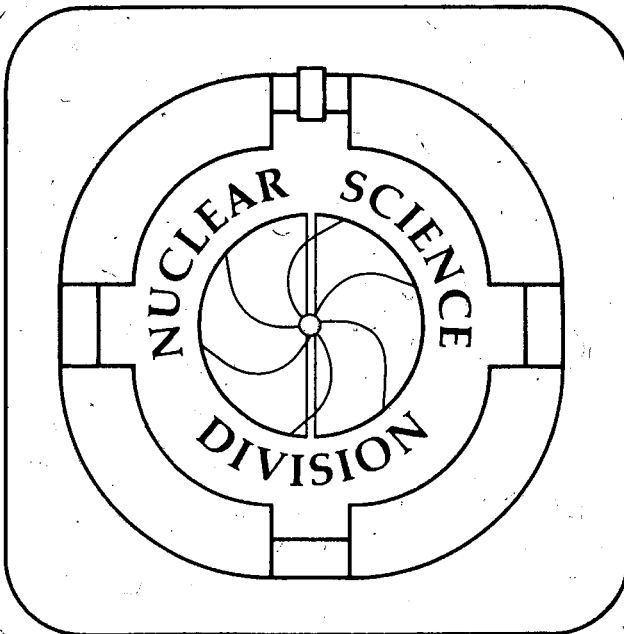
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Crystalline Structure in the Confined-Deconfined Mixed Phase

N.K. Glendenning and S. Pei

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Crystalline Structure in the Confined-Deconfined Mixed Phase[†]

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ABSTRACT

We review the differences in first order phase transition of single and multi-component systems, and then discuss the crystalline structure expected to exist in the mixed confined deconfined phase of hadronic matter. The particular context of neutron stars is chosen for illustration. The qualitative results are general and apply for example to the vapor-liquid transition in subsaturated asymmetric nuclear matter.

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Crystalline Structure in the Confined-Deconfined Mixed Phase

N. K. Glendenning and S. Pei

1 Introduction

First order phase transitions are very familiar only in one-component substances such as water. As is well known, on an isotherm the pressure remains constant as do all internal properties such as density and chemical potential, for all proportions of the two phases, gas and liquid, in equilibrium. These characteristic properties of *all* single-component substances are unique to them, and are not at all general. What is not familiar is that the precise converse of the above properties holds when the substance has more than one independent component. This has unique consequences in certain situations, such as in the presence of a gravitational field. More than that, when one of the independent components is electrically charged, the two phases in equilibrium may form a Coulomb lattice of the rare phase immersed in the dominant one. We have proven these properties in great detail and generality elsewhere [1]. Our aim here is to briefly recapitulate the physical reason for the different behavior of a first order phase transition in single- and multi-component substances, and then to compute the varying geometry of the crystalline structure as a function of proportion of the phases in equilibrium. We shall do this in the context of the confined-deconfined phase transition in neutron star matter, – matter that is charge neutral and in equilibrium with respect to all baryon and quark species. The results would be qualitatively similar for the liquid-vapor transition in sub-saturated nuclear matter.

2 Degrees of Freedom in Multi-component System

We stated above that in the mixed phase of a multi-component substance all internal properties of each phase and their common pressure vary as the proportion of the phases. Let us see why this is so, first by considering the physics rather than the mathematics. Consider a substance composed of two conserved ‘charges’ or independent components, – Q of one kind, B of the other. In the case of a neutron star, these could denote the net electric charge number (in units of e) and baryon charge

number. Let the substance be closed and in a heat bath. Define their concentration,

$$r = Q/B. \quad (1)$$

Is this ratio fixed? One would certainly think so since Q and B are fixed. But the ratio is fixed *only* as long as the system remains in one pure phase or the other! When in the mixed phase the concentration in each of the regions of one phase or the other may be different and they are restricted *only* by the conservation on the total numbers,

$$r_1 = Q_1/B_1, \quad r_2 = Q_2/B_2, \quad (Q_1 + Q_2 = Q, \quad B_1 + B_2 = B). \quad (2)$$

If the internal forces can lower the energy of the system by rearranging the concentration, they will do so. The essential point is that conservation laws in chemical thermodynamics are global, not local.

The above observations allow us to prove easily that all properties of each phase in equilibrium with the other will vary according to the proportion of the phases. Consider the system at the density or pressure where the neutron star matter has just begun to condense some quark matter. There is little scope for the internal forces to optimize the concentrations, r_1, r_2 , in the two phases, since the small quantity of quark matter can neither receive nor donate much of either charge. However, at higher density or pressure, the proportion of the two phases will become more comparable, and the internal forces now have more scope to optimize the concentrations in the two phases, always consistent with overall conservation of the two charges. From this observation, we learn: *For a first order phase transition in a multi-component system, the nature of each phase in equilibrium changes with the proportion of the phases and since the total energy is now the volume proportion of the energy density of the two phases, each of which varies with the proportion, the derivative with respect to volume is no longer a constant. Therefore the pressure also varies as the proportion of phases!*

The mathematical proof of the above properties is not nearly so illuminating as the physical verbal proof above, but we give it for completeness.

The Gibbs condition for phase equilibrium is that the chemical potentials μ_b, μ_q corresponding to B and Q , temperature T and the pressures in the two phases be equal,

$$p_1(\mu_b, \mu_q, T) = p_2(\mu_b, \mu_q, T) \quad (3)$$

As discussed, the condition of *local* conservation is stronger than required. We apply the weaker condition of *global* conservation,

$$\langle \rho \rangle \equiv (1 - \chi)\rho_1(\mu_b, \mu_q, T) + \chi\rho_2(\mu_b, \mu_q, T) = B/V, \quad (4)$$

$$(1 - \chi)q_1(\mu_b, \mu_q, T) + \chi q_2(\mu_b, \mu_q, T) = Q/V, \quad \chi = V_2/V. \quad (5)$$

Given a temperature, the above three equations serve to determine the two independent chemical potentials and V for a *specified* volume fraction χ of phase '2' in equilibrium with phase '1'. We note that the condition of global conservation

expressed by (4) and (5) is compatible, together with (3), with the number of unknowns to be determined. It would *not* be possible to satisfy Gibbs conditions if *local* conservation were demanded, for that would replace (5) by two equations, such as $q_1(\mu_b, \mu_q, T) = Q_1/V_1$, $q_2(\mu_b, \mu_q, T) = Q_2/V_2$, and the problem would be over determined.

In systems possessing only one conserved charge, the pressure equation defines uniquely the corresponding chemical potential for phase equilibrium. In that case the energy densities of each phase are also determined as unique values and like the pressure are *independent* of the proportion of the phases in equilibrium. In contrast with this, for two or more conserved charges and corresponding chemical potentials, the situation is quite different. Through (3,4,5) the chemical potentials obviously depend on the proportion, χ , of the phases in equilibrium, and hence so also all properties that depend on them, the energy densities, baryon and charge densities of each phase. and the *common* pressure. This remarkable and little known property of first order phase transitions with more than one conserved charge and the role played by the microphysics or internal forces is discussed in detail elsewhere [1, 2]. It will be observed that the above discussion is completely general, and must apply to many systems, in particular, to the confined-deconfined phase transition at high density and equally to the so-called liquid-vapor transition in nuclear matter at sub-saturation density. For both systems the symmetry energy is the driving force, and clearly the results here for two component systems hold when $Z \neq N$. For the *special* case of equality however, the driving force is absent, – both phases are already symmetric, – and the pressure would be constant throughout the mixed phase. But only when $N = Z$!

3 Internal Forces

By the above discussion we understand that the internal force(s) can exploit the degree(s) of freedom available in rearranging concentrations of conserved quantities while conserving them globally and lowering the energy. Let us look now at a specific example, neutron star matter which is charge neutral and in chemical equilibrium. Stars must be neutral because they are bound by gravity and net charge would reduce their binding, it being also long-ranged. Since pure neutron matter is beta unstable, neutron star matter will be composed of various particles of different charges, – neutrons, protons, leptons, perhaps hyperons and quarks. The star is born with a definite number of baryons, and soon becomes neutral. There are two conserved charges, therefore, – electric charge and baryon number, – and two corresponding independent chemical potentials.

The internal force that can exploit the degree of freedom made available by allowing neutrality to be achieved *globally* and which is closed to one in which *local* neutrality is artificially enforced, is the isospin restoring force experienced by the confined phase of hadronic matter. It is embodied in the isospin symmetry energy in the empirical mass formula of nuclei and nuclear matter. The hadronic regions of the mixed phase can arrange to be more isospin symmetric (closer equality in proton and

neutron number) than in the pure phase by transferring charge to the quark phase in equilibrium with it. Symmetry energy will be lowered thereby at only a small cost in rearranging the quark Fermi surfaces. Electrons play only a minor role when neutrality can be realized among baryon charge carrying particles. Thus the mixed phase region of the star will have *positively* charged regions of nuclear matter and *negatively* charged regions of quark matter.

4 Structure in the Mixed Phase

The Coulomb interaction will tend to break the regions of like charge into smaller ones, while this is opposed by the surface interface energy. Their competition will be resolved by *forming a lattice of the rare phase immersed in the dominant one whose form, size and spacing will minimize the sum of surface and Coulomb energies*. In other words, a crystalline lattice will be formed. Since all internal properties of the two phases in equilibrium with each other vary with their proportion, so will the geometrical structure. When quark matter is the rare phase immersed in confined hadronic matter, it will form droplets. At higher proportion of quark matter, the droplets will merge to form strings and then sheets, and then the role in the geometric structure of confined and deconfined phases will interchange [1].

We consider a Wigner-Seitz cell of radius R containing the rare phase object of radius r and an amount of the dominant phase that makes the cell charge neutral. The whole medium can be considered as made of such non-interacting cells, under the usual approximation of neglecting the interstitial material. As we shall see, the size of these cells is of the order of tens of fermis or less. The variation of the metric over such small regions is completely negligible (see ref. [3] for the radial behavior of the metric in typical neutron star models), so they are locally inertial regions and our discussion of them as if gravity is absent is justified by the equivalence principle. The solution to problems involving a competition between Coulomb and surface interface energies is universal. We may adapt the results of [4] to write for the radius of the rare phase immersed in the other and the minimum of the sum of Coulomb and surface energies, in the case of three geometries, slabs, rods and drops,

$$\frac{1}{r^3} = \frac{4\pi[q_H(\chi) - q_Q(\chi)]^2 e^2 f_d(x)}{\sigma d}, \quad d = 1, 2, 3, \quad (6)$$

$$\frac{E_C + E_S}{V} = 6\pi x \left(\frac{[\sigma d (q_H(\chi) - q_Q(\chi)) e]^2 f_d(x)}{16\pi^2} \right)^{1/3}, \quad (7)$$

where, q_H , q_Q are the charge densities of hadronic and quark matter (in units of e) at whatever proportion χ being considered. We have denoted the volume fraction of quark matter V_Q/V by χ . The ratio of droplet (rod, slab) to cell volume is called,

$$x = (r/R)^d. \quad (8)$$

It is related to χ by,

$$\chi = (r/R)^d \equiv x \quad (\text{hadronic matter background}), \quad (9)$$

when hadronic matter is the background (ie. dominant) phase. The quark droplets (rods, slabs) have radius r and the spacing between centers is R , with $d = 1, 2, 3$ corresponding to slabs, rods and drops, respectively. In the case of drops or rods, r is their radius and R the half distance between centers while for slabs, r is the half thickness. In the opposite situation where quark matter is the background,

$$1 - \chi = (r/R)^d \equiv x \quad (\text{quark matter background}), \quad (10)$$

is the fraction of hadronic matter which assumes the above geometric forms.

The function $f_d(x)$ is given in all three cases by,

$$f_d(x) = \frac{1}{d+2} \left[\frac{1}{(d-2)} (2 - dx^{1-2/d}) + x \right], \quad (11)$$

where the apparent singularity for $d = 2$ is well behaved and has the correct value,

$$\lim_{d \rightarrow 2} \frac{1}{(d-2)} (2 - dx^{1-2/d}) \rightarrow -[1 + \ln x]. \quad (12)$$

We have supposed that the electrons are uniformly distributed throughout the mixed phase whether quark or hadronic regions, and hence they do not appear in the above. In fact, we shall find that electrons are almost totally absent from the mixed phase.

What can we say of the surface tension? This is a very difficult problem to solve. Obviously it should be self-consistent with the two models of matter, quark and hadronic, in equilibrium with each other. This latter feature arises because of the fact that, unlike simple substances like water and vapor, the densities of each phase change as their proportion does [1, 5]. So the surface energy is not a constant. Following our deduction that a Coulomb lattice should exist in the mixed hadron-quark phase [1, 5], Heiselberg, Pethick and Staubo have investigated the dependence of the geometrical structure on the surface tension [6]. They adopted a selection of values from various sources, none of them computed self-consistently, for this is an extremely hard problem.

Gibbs studied the problem of surface energies, and as a gross approximation, one can deduce that it is given by the difference in energy densities of the substances in contact times a length scale typical of the surface thickness [7], in this case of the order of the strong interaction range, $L = 1$ fm. In other words, the surface interface energy should depend on the proportion of phases in phase equilibrium, just as everything else does.

$$\sigma = \text{const} \times [\epsilon_Q(\chi) - \epsilon_H(\chi)] \times L, \quad (13)$$

where χ is the volume proportion of quark phase. The constant should be chosen so that the structured phase lies below the unstructured one. Heiselberg et al found this energy difference to be about 10 MeV. We choose the constant accordingly.

It will be understood from the formulae written above that the structure size, whether drops, rods or slabs, and the sum of surface and Coulomb energies scale with the surface energy coefficient as $\sigma^{1/3}$ independent of geometry. Therefore the location in the star where the geometry changes from one form to another is independent of σ .

5 Bulk Description of the Phases

The geometrical structure of the mixed phase occurs against the background of the bulk structure, at least to good approximation. The energy and pressure are of course dominated by the bulk properties of matter. We outline briefly how to handle this part of the problem. It has been discussed in detail elsewhere.

For the confined hadronic phase we use the covariant Lagrangian,

$$\begin{aligned}
\mathcal{L} = & \sum_B \bar{\psi}_B (i\gamma_\mu \partial^\mu - m_B + g_{\sigma B} \sigma - g_{\omega B} \gamma_\mu \omega^\mu - \frac{1}{2} g_{\rho B} \gamma_\mu \boldsymbol{\tau} \cdot \boldsymbol{\rho}^\mu) \psi_B \\
& + \frac{1}{2} (\partial_\mu \sigma \partial^\mu \sigma - m_\sigma^2 \sigma^2) - \frac{1}{4} \omega_{\mu\nu} \omega^{\mu\nu} + \frac{1}{2} m_\omega^2 \omega_\mu \omega^\mu \\
& - \frac{1}{4} \boldsymbol{\rho}_{\mu\nu} \cdot \boldsymbol{\rho}^{\mu\nu} + \frac{1}{2} m_\rho^2 \boldsymbol{\rho}_\mu \cdot \boldsymbol{\rho}^\mu - \frac{1}{3} b m_n (g_\sigma \sigma_0)^3 - \frac{1}{4} c (g_\sigma \sigma_0)^4 \\
& + \sum_{e^-, \mu^-} \bar{\psi}_\lambda (i\gamma_\mu \partial^\mu - m_\lambda) \psi_\lambda.
\end{aligned} \tag{14}$$

We regard it as an effective theory to be solved at the mean field level, and with coupling constants adjusted, as described below, to nuclear matter properties. The baryons, B are coupled to the $\sigma, \omega, \boldsymbol{\rho}$ mesons. The sum on B is over all the charge states of the lowest baryon octet, ($p, n, \Lambda, \Sigma^+, \Sigma^-, \Sigma^0, \Xi^-, \Xi^0$) as well as the Δ quartet. However the latter are not populated up to the highest density in neutron stars, nor are any other baryon states save those of the lowest octet for reasons given elsewhere [3]. The last term represents the free lepton Lagrangians. How the theory can be solved in the mean field approximation for the ground state of charge neutral matter in general beta equilibrium (neutron star matter) is described fully in ref. [3].

There are five constants here that are determined by the properties of nuclear matter, three that determine the nucleon couplings to the scalar, vector and vector, iso-vector mesons, $g_\sigma/m_\sigma, g_\omega/m_\omega, g_\rho/m_\rho$, and two that determine the scalar self-interactions, b, c . The nuclear properties that define their values are the saturation values of the binding energy, baryon density, symmetry energy coefficient, compression modulus and nucleon effective mass. The hyperon couplings are not relevant to the ground state properties of nuclear matter but information about them can be gathered from levels in hypernuclei, the binding of the Λ in nuclear matter, and from neutron star masses [8]. We shall assume that all hyperons in the octet have the same coupling as the Λ . They are expressed as a ratio to the above mentioned nucleon couplings,

$$x_\sigma = g_{H\sigma}/g_\sigma, \quad x_\omega = g_{H\omega}/g_\omega, \quad x_\rho = g_{H\rho}/g_\rho. \tag{15}$$

The first two are related to the Λ binding by a relation derived in [8] and the third can be taken equal to the second by invoking vector dominance. We adopt the value of $x_\sigma = 0.6$ and corresponding x_ω taken from [8].

The chemical potentials of all hadrons are given by,

$$\mu_B = b_B \mu_n - e_B \mu_e, \tag{16}$$

where b_B and e_B are the baryon and electric charge numbers of the baryon state B , and μ_n and μ_e are independent chemical potentials for unit baryon number and unit negative electric charge number (neutron and electron respectively).

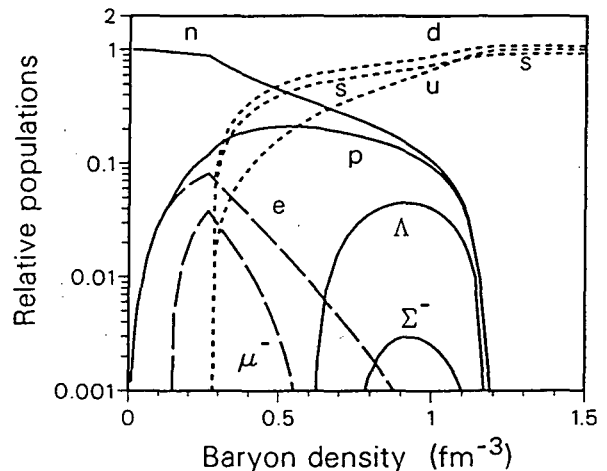
The values of nuclear matter properties are the binding, $B/A = -16.3$ MeV, saturation density, $\rho_0 = 0.153$ fm $^{-3}$, and symmetry energy coefficient, $a_{\text{sym}} = 32.5$ MeV, $K = 240$ MeV, $m_{\text{sat}}^*/m = 0.78$.

To describe quark matter we use a simple version of the bag model for finite quark masses and $T = 0$ [1]. Because of the long time-scale, strangeness is not conserved in a star. The quark chemical potentials for a system in chemical equilibrium are therefore related to those for baryon number and electron by

$$\mu_u = \mu_c = \frac{1}{3}(\mu_n - 2\mu_e), \quad \mu_d = \mu_s = \frac{1}{3}(\mu_n + \mu_e). \quad (17)$$

Solving the models of confined and deconfined phases, in both pure phases and in the mixed phase, we can compute the composition of charge-neutral, beta-stable neutron star matter. It is shown in Fig. 1. Note the saturation of the leptons as soon as quark matter appears. At this stage, charge neutrality is achieved more economically on baryon charge carrying particles, since the star has a definite baryon number. We note the transition from pure hadronic to mixed phase occurs at the rather low density of about $2\rho_0$, as was found also by several other authors [6, 9].

Figure 1: Baryon, lepton and quark populations in charge neutral, beta-stable neutron star matter, as a function baryon density $\rho < \rho_0$. In the mixed phase region, the quark densities refer to their values averaged over the volume of a Wigner-Seitz cell and similarly for the baryons.



6 Varying Crystalline Structure

We are now in a position to compute the geometrical structures, their sizes and spacings as they vary from one radial point to another throughout the mixed phase region. Our purpose is to demonstrate the extreme dependence of the structure of the crystalline region as a function of proportion of phases or equivalently density or pressure.

In Fig. 2 we show some of the ingredients from the bulk calculation that enter the computation of the structure as laid down in section 4. It is noteworthy how the

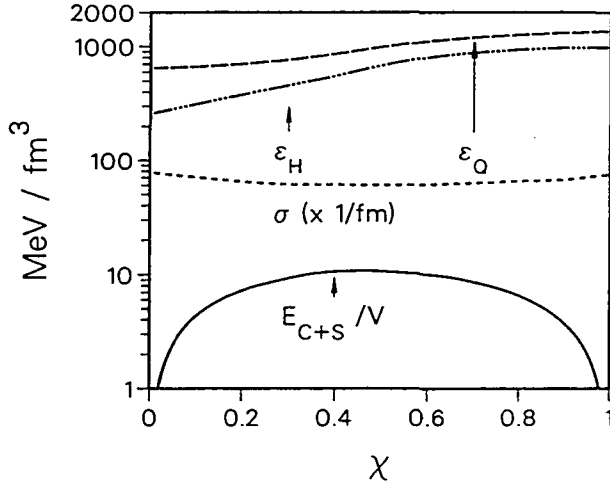


Figure 2: The bulk energy density of the hadronic and quark phases in equilibrium as a function of local volume proportion of the quark phase, $\chi = V_Q/V$, the surface energy coefficient, $\sigma(\chi)$, proportional to the difference of the above, and the sum of Coulomb and surface energies.

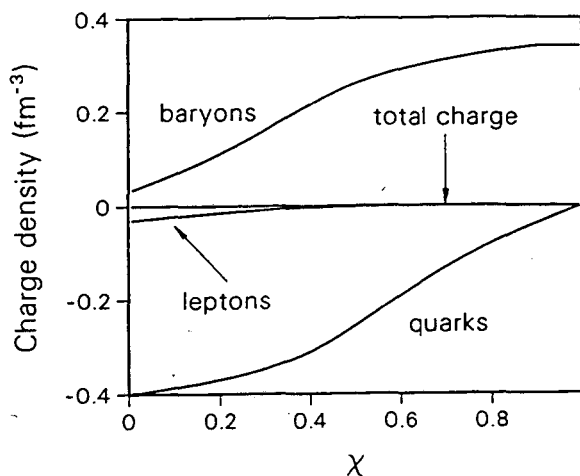
energy density, of each phase varies throughout the mixed phase region as a function of the volume fraction of quark matter, just as we showed above must be the case in general. Therefore the total energy density,

$$\epsilon(\chi) = (1 - \chi)\epsilon_H(\chi) + \chi\epsilon_Q(\chi), \quad (18)$$

is a *non-linear* function of proportion (or volume). As a consequence, the pressure varies throughout the mixed phase. This is in contrast to a simple substance, one with only one conserved charge, in which the density of each phase in equilibrium remains constant as well as the pressure. It is also worth noting that the bulk energy densities of the confined and deconfined phase are about two orders of magnitude greater than the sum of the energy densities of the Coulomb and surface interface energy. This justifies the two part approach to the problem, of computing the bulk properties and then against this background, the geometrical structure imposed by the surface and Coulomb energies. As already noted, the total charge in a Wigner-Seitz cell is zero, so the Coulomb force is shielded by the lattice arrangement of the rare phase immersed in the dominant. To illustrate the rearrangement of the electric charge concentration between the quark and baryonic regions of the mixed phase, we show the charge density in each region, and the electron charge density, assumed to be uniform throughout the Wigner-Seitz cell, as functions of the proportion of quark matter in Fig. 3. It is interesting to see that quark matter, which in the absence of baryonic matter ($\chi = 1$) is charge neutral, carries a high negative charge density when there is little of it and it is in equilibrium with baryonic matter. The latter acquires an ever increasing density as the quantity of quark matter, with which it can balance electric charge, grows. This illustrates how effectively the symmetry driving force acts to optimally rearrange charge.

As shown above, because one of the conserved quantities is the electric charge, having long range, an order will be established in the mixed phase, the size of the objects of the rare phase and their spacing in the dominant one, being determined by the condition for a minimum sum of Coulomb and surface energy. In Fig. 4 the diameter D and spacing S is shown by the lower and upper curves as a function of

Figure 3: The charge densities in the mixed phase carried by regions of quark and hadronic matter, as well as leptons which permeate all regions in our approximation. Multiplied by the respective volumes occupied, the total charge adds to zero.



proportion of quark phase. The discrete geometries are labeled and their content as quark or hadronic by 'q' or 'h'. The dotted line shows a continuous dimensionality interpolation. It is noteworthy that at the limit of the pure phases corresponding to $\chi = 0$ or 1; the spheres of rare phase are of finite diameter, but spaced far apart. The size of the objects is between 7 and 15 fm. As noted previously the location in χ of the geometries is independent of σ , but the size and spacings scale as $\sigma^{1/3}$.

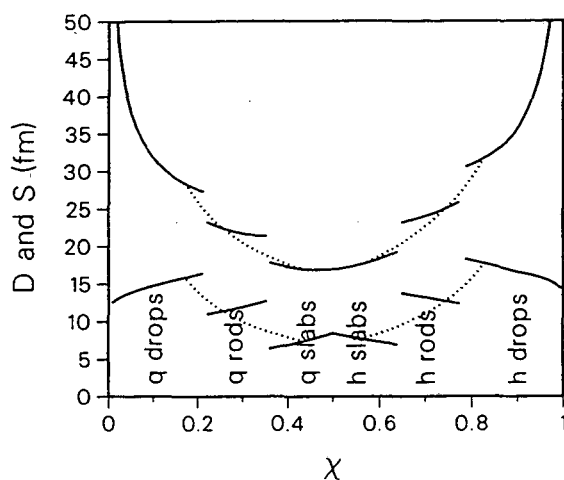


Figure 4: Diameter (lower curves) and Spacing (upper curves) of rare phase immersed in the dominant as a function of the proportion of quark phase. Geometries are identified as drops, rods, slabs, and composition as q (quark) or h (hadronic). Dots are a continuous dimensionality interpolation of the discrete shapes.

7 Summary

We have exhibited the crystalline structure of the mixed phase of confined and deconfined neutron star matter. What is of crucial importance is that the mixed phase,

if it had a constant pressure for all proportions, would be absent from the star, or any gravitational field. This is because a constant pressure region cannot support any overlaying material, and the pressure is monotonic in a star as it is in our atmosphere. This squeezing out of the mixed phase was an inadvertent feature of idealizations of all treatments of the phase transition in neutron stars until our work. The idealizations were either an assumption of purely neutron star [10], or an assumption of local charge neutrality [11]. Neither is a valid constraint.

It is almost certain that a solid region in a pulsar will play a role in the period glitch phenomenon, which is highly individualistic from one pulsar to another. We have suggested that this high degree of individual behavior may be due to the extreme sensitivity on stellar mass of the radial extent of the solid region and the particular geometrical forms and sizes of the objects at the lattice sites [12]. The sensitivity arises because of the rather flat radial profile of the pressure and energy density in neutron stars, so that a small change in central density and therefore a small change in stellar mass, moves a transition pressure a considerable distance in the radial direction in the star.

As remarked earlier, we have illustrated very general phenomena associated with first order phase transitions in multi-component systems. Whether geometric structures can develop on the time scale of collisions between nuclei is problematic, but the non-constant pressure in the mixed phase is likely to have consequences that may be observable. In particular the so called plateau behavior ascribed to phase transitions in nuclear collisions can be present only for $N=Z$ symmetric systems, since otherwise the symmetry energy will have scope to act in the mixed phase.

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