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# Combining a Connectionist Type Hierarchy with a Connectionist Rule-Based Reasoner\*

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#### Abstract

This paper describes an efficient connectionist knowledge representation and reasoning system that combines rule-based reasoning with reasoning about inheritance and classification within an IS-A hierarchy. In addition to a type hierarchy, the proposed system can encode generic facts such as 'Cats prey on birds' and rules such as 'if x preys on y then y is scared of x' and use them to infer that Tweety (who is a Canary) is scared of Sylvester (who is a cat). The system can also encode qualified rules such as 'if an animate agent walks into a solid object then the agent gets hurt'. The proposed system can answer queries in time that is only proportional to the length of the shortest derivation of the query and is independent of the size of the knowledge base. The system maintains and propagates variable bindings using temporally synchronous i.e., in-phase - firing of appropriate nodes.

#### Introduction

In Shastri & Ajjanagadde 1990a, Shastri & Ajjanagadde 1990b, Ajjanagadde & Shastri 1991], Ajjanagadde and Shastri have described a solution to the variable binding problem [Lange & Dyer 1989, Smolensky 1987] and shown that the solution leads to the design of a connectionist reasoning system that can represent systematic knowledge involving n-ary predicates and variables, and perform a broad class of reasoning with extreme efficiency. The time taken by the reasoning system to draw an inference is only proportional to the length of the chain of inference and is independent of the number of rules and facts encoded by the system. The reasoning system maintains and propagates variable bindings using temporally synchronous - i.e., inphase - firing of appropriate nodes. The solution to the variable binding problem allows the system to maintain and propagate a large number of bindings simultaneously as long as the number of distinct entities participating in the bindings during any given episode of reasoning, remains bounded. Reasoning in the proposed system is the transient but systematic flow of rhythmic

patterns of activation, where each phase in the rhythmic pattern corresponds to a distinct constant involved in the reasoning process and where variable bindings are represented as the synchronous firing of appropriate argument and constant nodes. A fact behaves as a temporal pattern matcher that becomes 'active' when it detects that the bindings corresponding to it are present in the system's pattern of activity. Finally, rules are interconnection patterns that propagate and transform rhythmic patterns of activity.<sup>1</sup>

In this paper we describe how the above reasoning system may be combined with an IS-A hierarchy. Such an integration allows the occurrence of types (categories) as well as instances in rules, facts, and queries. This has the following interesting consequences. First, the reasoning system can combine rule-based reasoning with inheritance and classification. For example, such a system can infer that 'Tweety is scared of Sylvester', based on the generic fact 'Cats prey on birds', the rule 'If x preys on y then y is scared of x and the IS-A relations 'Sylvester is a Cat' and 'Tweety is a Bird'. Second, the integrated system can use category information to qualify rules by specifying restrictions on the type of argument fillers. An example of such a rule is:  $\forall x:animate, y:solid-obj[walk-into(x,y) \Rightarrow hurt(x)]$ , which specifies that the rule is applicable only if the two arguments of 'walk-into' are of the type 'animate' and 'solid-object', respectively.

An overview of the rule-based reasoning system is followed by a description of the IS-A hierarchy realization and its interface with the reasoning system. A detailed discussion of the reasoning system and the type hierarchy interface may be found in [Shastri & Ajjanagadde 1990b] and [Mani & Shastri 1991] respectively.

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<sup>&</sup>lt;sup>1</sup>It may be worth stating that the system does not require a central controller or a global clock.

<sup>&</sup>lt;sup>2</sup>Observe that this kind of reasoning combines 'relational inheritance' [Fahlman 1979] with rule-based reasoning: While relational inheritance can support the inference 'Sylvester preys on Tweety' by using the *IS-A* relationships on the generic fact 'Cats prey on birds', it cannot support the inference 'Tweety is scared of Sylvester', because doing so also requires the use of the rule 'If x preys on y then y is scared of x'.

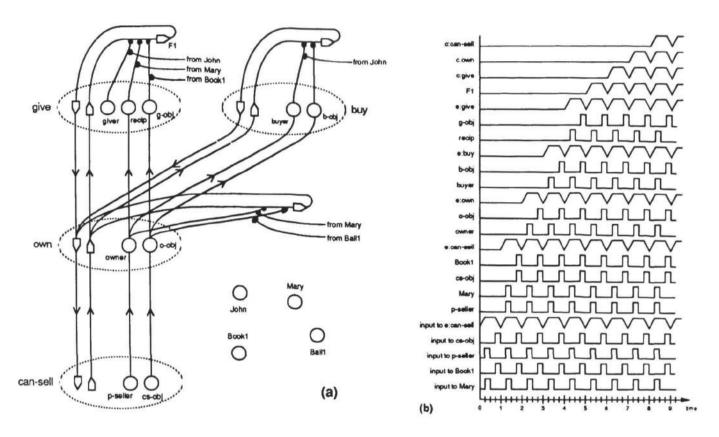


Figure 1: (a) Encoding rules and facts. (b) Activation trace for the query can-sell(Mary, Book1)?

### The rule-based reasoning system

Fig. 1a illustrates how long-term knowledge is encoded in the rule-based reasoning system. The network encodes the following rules and facts: i)  $\forall x, y, z$  [  $give(x,y,z) \Rightarrow own(y,z)$  ], ii)  $\forall x,y$  [  $buy(x,y) \Rightarrow own(x,y)$  ], iii)  $\forall x,y$  [  $own(x,y) \Rightarrow can\text{-sell}(x,y)$  ], iv) give(John,Mary,Book1), v) buy(John,x), and vi) own(Mary,Ball1).

The encoding makes use of two types of nodes:  $\rho$ -btu nodes (depicted as circles) and  $\tau$ -and nodes (depicted as pentagons). The computational behavior of these nodes is as follows: A  $\rho$ -btu is a phase-sensitive binary threshold unit. When such a node becomes active, it produces an oscillatory output in the form of a pulse train that has a period  $\pi$  and pulse width  $\omega$ . The timing (or the phase) of the pulse train produced by a  $\rho$ -btu node depends on the phase of the input to the node. A τ-and node acts like a temporal AND node. Such a node also oscillates with the same frequency as a  $\rho$ -btu node except that it becomes active only if it receives uninterrupted activation over a whole period of oscillation. Furthermore, the width of the pulses produced by a  $\tau$ and node equals  $\pi$ . The maximum number of distinct entities that may participate in the reasoning process equals  $\pi/\omega$  (assume integer divide). The encoding also makes use of *inhibitory modifiers* – links that impinge upon and inhibit other links. A pulse propagating along

an inhibitory modifier will block a pulse propagating along the link it impinges upon. In Fig. 1a, inhibitory modifiers are shown as links ending in dark blobs.

Each constant in the domain is encoded by a  $\rho$ -btu node. An n-ary predicate is encoded by a pair of  $\tau$ and nodes and  $n \rho$ -btu nodes, one for each of the narguments. One of the  $\tau$ -and nodes is referred to as the enabler and the other as the collector. As a matter of convention, an enabler always points upwards and is named e:redicate-name>. A collector always points downwards and is named c:credicate-name>. The enabler e:P of a predicate P becomes active whenever the system is being queried about P. Such a query may be posed by an external process or by the system itself during an episode of reasoning. On the other hand, the system activates the collector c:P of a predicate P whenever the system wants to assert that the current dynamic bindings of the arguments of P are consistent with the knowledge encoded in the system. A rule is encoded by connecting the collector of the antecedent predicate to the collector of the consequent predicate, the enabler of the consequent predicate to the enabler of the antecedent predicate, and by connecting the arguments of the consequent predicate to the arguments of the antecedent predicate in accordance with the correspondence between these arguments specified in the rule. A fact is encoded using a \u03c4-and node that receives an input from the enabler of the associated predicate. This input is modified by inhibitory modifiers from the argument nodes of the associated predicate. If an argument is bound to a constant in the fact then the modi-

<sup>&</sup>lt;sup>3</sup>Later we will introduce a third type of node, namely the  $\tau$ -or node. A  $\tau$ -or node becomes active on receiving any activation but its output is like that of a  $\tau$ -and node.

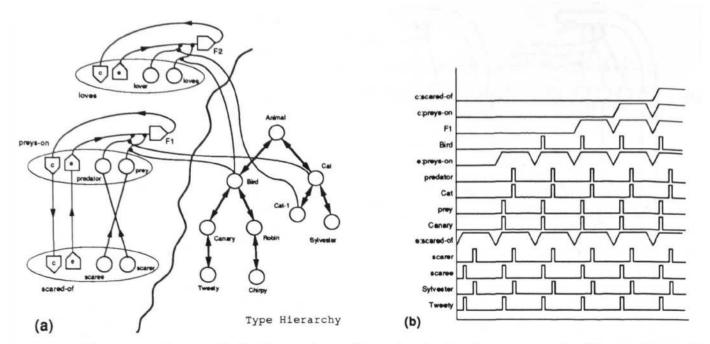


Figure 2: (a) An example network. (b) Trace of spreading activation for the query scared-of(Tweety, Sylvester)?

fier from such an argument node is in turn modified by an inhibitory modifier from the appropriate constant node. The output of the  $\tau$ -and node is connected to the collector of the associated predicate (refer to the encoding of the fact give(John, Mary, Book1) and buy(John, x) in Fig. 1a.)

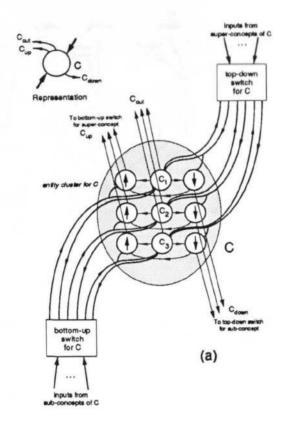
#### Inference Process

Posing a query to the system involves specifying the query predicate and the argument bindings specified in the query. In the proposed system this is done by simply activating the relevant nodes in the manner described below. Choose an arbitrary reference point in time—say,  $t_0$ —for initiating the query. We assume that the system is in a quiescent state just prior to  $t_0$ . The query predicate is specified by activating the enabler of the query predicate, with a pulse train of width and periodicity  $\pi$  starting at time  $t_0$ .

The argument bindings specified in the query are communicated to the network as follows: Suppose the argument bindings in the query involve n distinct constants  $c_1, ..., c_n$ . With each of these n constants, associate a delay  $\delta_i$  such that no two delays are within  $\omega$  of one another and the longest delay is less than  $\pi - \omega$ . Each of these delays may be viewed as a distinct phase within the period  $t_0$  and  $t_0 + \pi$ . Now the argument bindings of a constant  $c_i$  are indicated to the system by providing an oscillatory pulse train of pulse width  $\omega$  and periodicity  $\pi$  starting at  $t_0 + \delta_i$ , to  $c_i$  and all arguments to which  $c_i$  is bound. This is done for each constant  $c_i$   $(1 \le i \le n)$  and amounts to representing argument bindings by the in-phase or synchronous activation of the appropriate constant and argument nodes.

We illustrate the reasoning process with the help of an example. Consider the query can-sell(Mary, Book1)? (i.e., Can Mary sell Book1?) The query is posed by i) activating the enabler node e:can-sell ii) activat-

ing Mary and p-seller in the same phase (say, phase-1), and iii) activating Book1 and cs-obj in some other phase (say, phase-2). As a result of these inputs, Mary and p-seller will fire synchronously in phase-1 of every period of oscillation, while Book1 and cs-obj will fire synchronously in phase-2 of every period of oscillation. The node e:can-sell will also oscillate and generate a pulse train of periodicity and pulse width  $\pi$  (Fig. 1b). The activations from the arguments pseller and cs-obj reach the arguments owner and oobj of the predicate own, and consequently, starting with the second period of oscillation, owner and oobj become active in phase-1 and phase-2, respectively. At the same time, the activation from e:can-sell activates e:own. The newly created dynamic bindings for the arguments of own, in conjunction with the activation of e:own can be thought of as encoding the query own(Mary, Book1)? The  $\tau$ -and node associated with the fact own(Mary, Ball1) does not match the query and remains inactive. As the activation propagates from the arguments of own to the arguments of buy and give (Fig. 1b), new bindings for the arguments of give and buy get established which, in effect, encode two new queries: give(x, Mary, Book1)? (i.e., Did someone give Mary Book1?), and buy(Mary, Book1)? The τ-and node F1, associated with the fact give (John, Mary, Book1) becomes active as a result of the uninterrupted activation it receives from e:give. Observe that the inhibitory modifiers from recip and g-obj incident on the link from e:own to F1 are blocked by the in-phase inputs from Mary and Book1, respectively. The activation from F1 causes c:give, the collector of give, to become active and the output from c:give in turn causes c:own to become active and transmit an output to c:can-sell. Consequently c:can-sell, the collector of the query predicate can-sell, becomes active indicating an affirmative answer to the query can-sell(Mary, Book1)? (Fig. 1b).



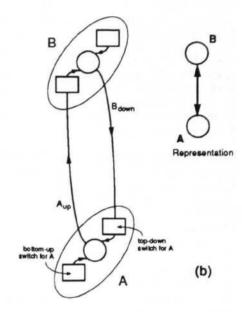


Figure 3: (a) Structure of the entity cluster for C, and its interaction with the bottom-up and top-down switches. The  $\uparrow$  and  $\downarrow$  nodes have a threshold  $\theta = 2$ . The multiple instantiation constant, k = 3. (b) Encoding of the *is-a* relation *is-a*(A, B). A bundle of k wires is represented by a single link.

## Combining the rule-based reasoner with an IS-A hierarchy

Fig. 2a gives an overview of the combined reasoning system. The rule-based part of the network encodes the rule  $\forall x,y \ [$  preys-on $(x,y) \Rightarrow scared$ -of $(y,x) \ ]$  (i.e., if x preys on y, then y is scared of x), and the facts  $\forall x:Cat$ ,  $y:Bird\ preys-on(x,y)$  and  $\exists x:Cat\ \forall y:Bird\ loves(y,x)$ . The former is equivalent to preys-on(Cat,Bird) and amounts to 'Cats prey on Birds'. The latter amounts to 'there is a cat that loves all birds'. The network on the right encodes the IS-A relationships: is-a(Bird, Animal), is-a(Cat, Animal), is-a(Robin, Bird), is-a(Canary, Bird), is-a(Tweety, Canary), is-a(Chirpy, Robin), and is-a(Sylvester, Cat).

Facts involving typed variables are encoded in the following manner: A typed, universally quantified variable is treated as being equivalent to its type. Thus  $\forall x:Cat$ ,  $y:Bird\ preys-on(x,y)$  is encoded as preys-on(Cat,Bird). A typed, existentially quantified variable is encoded using a unique subconcept of the associated type. Thus in Fig. 2a,  $\exists x:Cat\ \forall y:Bird\ loves(x,y)$  is encoded as loves(Cat-1,Bird), where Cat-1 is a unique instance of Cat. For now let us assume that i) each concept (type or instance) is encoded as  $a\ \rho$ -btu node ii) each conceptual IS-A relationship such as is-a(A,B) is encoded using two connectionist links -a bottom-up link from A to B and a top-down link from B to A, and iii) the top-down and bottom-up links can be enabled selectively by built-in control mechanisms (missing details are pro-

vided below).

The time course of activation for the query scared-of(Tweety,Sylvester)? is given in Fig. 2b. The query is posed by turning on e:scared-of and activating the nodes Tweety and Sylvester in synchrony with the first and second arguments of scared-of, respectively. The bottom-up links emanating from Tweety and Sylvester are also enabled. The net result of spreading activation (Fig. 2b), in the conceptual hierarchy and the rule-base is that the query scared-of(Tweety,Sylvester)? is transformed into the query preys-on(Cat,Bird)? The latter query matches the stored fact preys-on(Cat,Bird) and leads to the activation of c:preys-on. In turn, c:scared-of becomes active and signals an affirmative answer to the query.

#### Two technical problems

There are two technical problems that must be solved in order to integrate the conceptual hierarchy and the rule-based component. First, the encoding of the IS-A hierarchy should be capable of representing multiple instantiations of a concept. For example, in the query discussed above, we would like the network's state of activation to represent both 'the animal Tweety' and 'the animal Sylvester'. This is problematic because the node Animal cannot be in synchrony with both Tweety and Sylvester at the same time. Second, the encoding must provide built-in mechanisms for controlling the direction of propagating activation in the IS-A hierarchy so

as to correctly deal with queries containing existentially and universally quantified variables. Thus i) Activation originating from an instance or a concept that corresponds to a universally quantified variable in the query should propagate upwards to all its ancestors, and ii) If the IS-A hierarchy is a taxonomy, then activation originating from a concept C that corresponds to an existentially quantified variable in the query should propagate to the ancestors as well as descendents of C. If however, the IS-A hierarchy permits multiple inheritance then the activation must also propagate to the ancestors of the descendents of C. The following solution to these problems does not require an external controller to monitor and control the nodes in the network, during the reasoning process.

#### Implementing the Type Hierarchy

Each entity (i.e., type or instance) C, is represented by a group of nodes called the entity cluster for C. Such a cluster is organized as shown in Fig. 3a. The entity cluster for C has k banks of  $\rho$ -btu nodes, where k, the multiple instantiation constant, refers to the number of dynamic instantiations a concept can accommodate. Each bank  $C_{B_i}$  consists of three  $\rho$ -btu nodes:  $C_i$ ,  $C_{i\uparrow}$ ,  $C_{i\downarrow}$ . Each  $C_i$  represents a distinct (dynamic) instantiation of C. If this instantiation is in phase  $\rho$ , then,  $C_i$  fires in phase  $\rho$ . The relay nodes  $C_{i\uparrow}$  and  $C_{i\downarrow}$  control the direction of propagation of the activation represented by  $C_i$ . The  $C_{i\uparrow}$  and  $C_{i\downarrow}$  nodes have a threshold  $\theta = 2$ . As shown in Fig. 3a,  $C_i$  is connected to both  $C_{i\uparrow}$  and  $C_{i\downarrow}$ .  $C_{i\downarrow}$  is linked to  $C_{i\uparrow}$ , but not vice versa. Directional control of propagating activation is exercised using a suitable modification of the relay-node scheme discussed in [Shastri 1988].

Every entity C is associated with two switches – a top-down switch and a bottom-up switch. The switches, both of which are identical in structure, control the flow of activation in the type hierarchy. Each switch has k outputs. Output; from the bottom-up switch connects to  $C_i$  and  $C_{ij}$  while the corresponding output from the top-down switch goes to the  $C_i$  and  $C_{ij}$  nodes,  $1 \le i \le k$ . There is also a feedback from the  $C_i$  nodes to both the switches (See Fig. 3a and Fig. 4.)

The interaction between the switches and the entity cluster (Fig. 3a) brings about efficient and automatic dynamic allocation of banks in an entity cluster, by ensuring that:

- Activation is channeled to the entity cluster banks only if the entity cluster can accommodate more instantiations; the maximum number of instantiations is therefore limited to k.
- Each C<sub>i</sub> picks up a unique phase; thus new instantiations are always in a phase not already represented in the entity cluster.

The architecture of the switch (with k=3) is illustrated in Fig. 4. The k  $\rho$ -btu nodes,  $S_1, \ldots, S_k$ , with their associated  $\tau$ -or nodes form the basic components of the switch. Every input to the switch makes two connections – one excitatory and one inhibitory – to each of  $S_2, \ldots, S_k$ ; as a result of these excitatory-inhibitory connections, all these nodes are disabled to begin with, and cannot respond to incoming activation. Input activation will have an effect only on the  $S_1$  node, since the

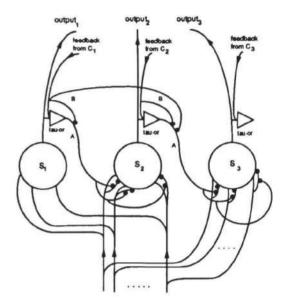


Figure 4: Architecture of the switch. The multiple instantiation constant k=3.

inputs to the switch directly connect to  $S_1$  (Fig. 4.) In keeping with the behavior of  $\rho$ -btu nodes,  $S_1$  becomes active in response to the first available input and continues to fire in phase with that input as long it remains active. As  $S_1$  goes active, the  $\tau$ -or node associated with  $S_1$  turns on, thereby enabling  $S_2$ . Inhibitory feedback from  $C_1$  ensures that  $S_2$  is not enabled during the phase  $\rho$  in which  $C_1$  is firing<sup>4</sup>. Thus  $S_2$  selects and starts firing in a phase other than  $\rho$ . Once  $S_2$  has made its selection,  $S_3$  gets its turn, and so on.

As instantiations are deputed to the entity cluster, the  $\rho$ -btu nodes in the switch are progressively enabled from left to right. If  $C_1, \ldots, C_{i-1}$  are firing in phases  $\rho_1, \ldots, \rho_{i-1}$ , then  $S_i$  always picks a distinct phase  $\rho \notin \{\rho_1, \ldots, \rho_{i-1}\}$ , since inputs in phases  $\rho_1, \ldots, \rho_{i-1}$  are inhibited by the feedback links from  $C_1, \ldots, C_{i-1}$ . At any stage, if  $C_i$ ,  $1 \le i \le k$ , picks up activation channeled by the other switch, feedback from  $C_i$  into the  $\tau$ -or node associated with  $S_i$  causes  $S_{i+1}$  to be enabled, even though  $S_i$  has not picked a phase. This ensures that at most k instantiations are selected jointly by the bottomup and top-down switches; hence, only k instantiations can be channeled to C, at worst.

A fact of the form is-a(A,B) is represented by (Fig. 3b): (i) connecting the  $A_{i\uparrow}$ ,  $i=1,\ldots,k$  nodes to the bottom-up switch for B; (ii) connecting the  $B_{i\downarrow}$ ,  $i=1,\ldots,k$  nodes to the top-down switch for A.

 $1, \ldots, k$  nodes to the top-down switch for A.

Consider a concept C in the type hierarchy. Suppose  $C_i$  receives activation from the bottom-up switch in phase  $\rho$ .  $C_i$  starts firing in synchrony with this activation. The  $C_{i\uparrow}$  node (with threshold  $\theta=2$ ) is now receiving two inputs in this phase (from the bottom-up switch and from  $C_i$ ; see Fig. 3a) and begins to fire in phase  $\rho$ . This causes activation in phase  $\rho$  to eventually

<sup>&</sup>lt;sup>4</sup>In general,  $C_i$  could receive input in two phases – one from the bottom-up switch for C, and another from its top-down switch.  $C_i$  being a  $\rho$ -btu node, picks one of these phases to fire in.

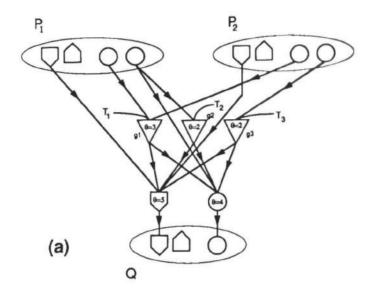


Figure 5: Network encoding the rule  $\forall x:T_1, y:T_2 \exists z:T_3$  [  $P_1(x,y) \& P_2(x,z) \Rightarrow Q(y)$  ] in a forward reasoning system. Links from  $T_1, T_2$  and  $T_3$  are actually bundles of k wires carrying the k instantiations of these constants.

spread to the super-concept of C. Hence, any upward traveling activation continues to travel upward – which is the required behavior when C is associated with a universal typed variable. Similarly, when  $C_i$  receives activation from the top-down switch in phase  $\rho$ , both  $C_i$  and  $C_{i\downarrow}$  become active in phase  $\rho$ .  $C_{i\uparrow}$  follows suit, because of the link from  $C_{i\downarrow}$  to  $C_{i\uparrow}$ , so that the whole bank  $C_{B_i}$  now fires in phase  $\rho$ . This mechanism allows a concept associated with an existential typed variable to eventually spread its activation to its ancestors, descendents and ancestors of descendents.

#### Typed Variables in Rules

The type hierarchy can be used to impose type restrictions on variables occurring in rules, for both forward and backward reasoning systems. To utilize this feature, we need to modify the implementation of rules: In a forward reasoning system, the rule is encoded by introducing a  $\tau$ -or node to perform the type checking for the argument under question. For example, in Fig. 5, which encodes the rule  $\forall x:T_1, \ y:T_2 \ \exists z:T_3 \ [P_1(x,y) \& P_2(x,z) \Rightarrow Q(y)], \ g_3$  would turn on if and only if the second argument of  $P_2$  and  $T_3$  are in synchrony – which is to say that the argument is bound to an object of type  $T_3$ . In the forward reasoner, typed variables are allowed only in the antecedent of the rule.

In a backward reasoner, the strategy is similar, except that i) type checking for a typed universally quantified variable is enforced by an inhibitory link from the concept representing the type of the concerned argument; ii) For a typed, existentially quantified variable, an inhibitory link derived from a unique subconcept of the associated type performs the type enforcement — in a manner similar to the interpretation of a typed existential variable in a fact [Mani & Shastri 1991]. In the backward reasoner, typed variables are allowed only in the consequent. Both in the forward and backward reasoners, a rule fires only if all typed arguments are firing

in synchrony with their respective types.

#### Conclusions

Adding a type hierarchy allows the connectionist reasoning system to handle rules, facts, and queries with typed variables. The simulator described in [Mani 1990] has been used to test the system. Several extensions to the system proposed here are being investigated. The current system assumes that any fact or query with both existentially and universally quantified variables is such that all the universal quantifiers are within the scope of the existential quantifiers. Work is being done on handling more general forms of facts and queries. We are also working on the design of an expanded system that would allow property-value attachments to concepts [Shastri 1988]. We also wish to combine the IS-A hierarchy with a reasoning system that allows multiple instantiation of predicates. Lastly, we hope to combine a forward and backward reasoner that can make use of both long-term and dynamic (temporary) facts during reasoning.

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