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Journal
Geotechnical and Geological Engineering, 36(1)

ISSN
0960-3182

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Publication Date
2018-02-01

DOI
10.1007/s10706-017-0316-7

Peer reviewed

APPLICATION OF HYSTERETIC TRENDS IN THE PRECONSOLIDATION STRESS OF UNSATURATED SOILS

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ABSTRACT: This paper involves an evaluation of a relationship describing the evolution in yield stress of unsaturated soils during hydraulic hysteresis, and an application of this relationship in an elasto-plastic framework to predict the compression curves of unsaturated soils under drained (free outflow of air and water with constant suction) or undrained (constant water content with no outflow of water and varying suction) conditions. The yield stress was quantified as the apparent mean effective preconsolidation stress obtained from compression tests reported in the literature on specimens that had experienced different hydraulic paths. It was observed that the preconsolidation stress does not follow a hysteretic path when plotted as a function of matric suction, but does when plotted as a function of the degree of saturation. Accordingly, an existing logarithmic relationship between the preconsolidation stress and matric suction normalized by the air entry suction was found to match the experimental preconsolidation stress results. This same relationship was also able to satisfactorily predict the trends in preconsolidation stress with degree of saturation by substituting the hysteretic soil-water retention curve (SWRC) into the place of the matric suction. The relationship between preconsolidation stress and suction was combined with an elasto-plastic framework to predict the compression curves of soils during drained compression, while the wetting-path relationship between preconsolidation stress and degree of saturation was

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combined with the framework to predict the compression curves of soils during undrained (constant water content) compression. A good match was obtained with experimental data from the literature, indicating the relevance of considering the hysteretic SWRC and preconsolidation relationships when simulating the behavior of unsaturated soils following different hydro-mechanical paths.

INTRODUCTION

Several hydro-mechanical, elasto-plastic frameworks have been developed to simulate the impacts of changes in mean total or effective stress and matric suction on the volume change of unsaturated soils. The ability of these frameworks to predict the hydro-mechanical behavior of unsaturated soil is influenced by the stress state definition. While early studies extended the critical state framework to unsaturated soils using independent state variables (Alonso et al. 1990; Wheeler and Sivakumar 1995; Cui and Delage 1996), other studies used the generalized effective stress concept (Loret and Khalili 200; Gallipoli et al. 2003; Wheeler et al. 2003; Tamagnini 2004; Romero and Jommi 2008; Khalili et al. 2008; Della Vecchia et al. 2013). An advantage of using the generalized effective stress concept is that a smaller number of material properties may be needed to simulate the complex volume change behavior of unsaturated soils, and the yield surface will always be concave and thermodynamically consistent (Khalili et al. 2008). Further, the effects of hydraulic hysteresis may be considered in an elasto-plastic framework by incorporating the soil-water retention curve (SWRC) (Wheeler et al. 2003; Tamagnini 2004) or the air entry suction (Khalili et al. 2008) in the definition of the generalized effective stress concept. Using the generalized effective stress concept, this study involves an evaluation of how the SWRC can also be incorporated into the definition of the yield stress in hydro-mechanical frameworks to consider the effects of hydraulic hysteresis on this parameter.

A common feature of most hydro-mechanical elasto-plastic frameworks is the evolution in yield stress with matric suction, commonly referred to as the loading-collapse (LC) curve. The Barcelona Basic Model (BBM) presented by Alonso et al. (1990) is one of the most widely used models for unsaturated soils in terms of independent stress state variables. Although the BBM is capable of considering suction hardening behavior (e.g., increase in the preconsolidation stress with suction increasing) and expansion and collapse phenomena, problems have been encountered in matching the model to experimental data (Wheeler et al. 2002), and there may be issues considering the behavior of soils under suctions less than the air entry suction. Despite the fact that the concept of the LC curve is well-established, limited experimental data sets are available to thoroughly evaluate the influence of suction and degree of saturation on the mean effective preconsolidation stress of unsaturated soils. Specifically, knowledge of the SWRC or monitoring of the degree of saturation and/or suction during compression is needed to estimate the mean effective preconsolidation stress (Salager et al. 2008; Uchaipichat and Khalili 2009; Uchaipichat 2010; Coccia 2016; Khosravi et al. 2016). This study seeks to develop and validate a simple hysteretic relationship between the mean effective preconsolidation stress p'_c, matric suction ψ, and degree of saturation S_r using the limited sets of data that are available in the literature. Following this, the new relationship is employed to as part of a simple hydro-mechanical framework to predict the volume change behavior of unsaturated soils under different drainage conditions.

BACKGROUND

The LC curve is used to indicate the yield limit transition from an elastic to an elasto-plastic volumetric soil response during compression of unsaturated soils. Under isotropic stress states, the yield limit of soils is typically assumed to be equal to the mean effective preconsolidation stress

(Wood 1990), although for unsaturated soils it has been defined in terms of the mean net preconsolidation stress $p_{c,net}$ or the mean effective preconsolidation stress p'_c depending on the choice of stress state variables. This paper is focused on isotropic loading, so all further discussion of preconsolidation stress are for the mean preconsolidation stress. In general, most LC curves indicate an increase in preconsolidation stress with increasing matric suction, an effect commonly referred to as "suction hardening", albeit with different shapes for this relationship. Analytical expressions to characterize the LC curve for unsaturated soils have been proposed in the literature in p_{net} vs. suction (ψ) space (Alonso et al. 1990), p' vs. ψ space (Salager et al. 2008; Tourchi and Hamidi 2015), p' vs. degree of saturation (S_r) space (Gallipoli et al. 2003; Romero and Jommi 2008), p' vs. modified suction space (i.e., suction multiplied by the porosity $\psi \cdot n$) (Wheeler et al. 2003), and p' vs. effective saturation (S_e) space (Zhou et al. 2012b). The shapes of some typical LC curves are shown in Figure 1, along with their analytical expressions. Several studies have defined the evolution of p'_c with changes in matric suction (Lloret et al. 2003; Geiser et al. 2006; Salager et al. 2008; Uchaipichat and Khalili 2009; Mun and McCartney 2015, 2016; Khosravi et al. 2016), which have all confirmed an increasing trend between p'_c and matric suction during drying. Alsherif and McCartney (2016) found that a similar increasing trend may also be present at high suction magnitudes. Although the suction is clearly related to p'_c, the evolution in p'_c may also be related to changes in soil structure during application of suction to the soil or due to different distributions in water throughout the soil during wetting and drying (hydraulic hysteresis).

A well-known LC curve is that of Alonso et al. (1990), which was formulated to link the relationship between the preconsolidation stress and suction with the slope of the virgin compression line in net mean stress space. The LC curve proposed by Alonso et al. (1990) with a hypothetical hydro-mechanical loading path for an overconsolidated soil subjected to an increase in matric suction followed by an increase in mean net stress is shown in Figure 1(a) in $p_{net} - \psi$ space. As matric suction increases, the soil is assumed to behave elastically along path O→A while the stress state remains within the LC curve. During this increase in suction, an apparent increase in p_c will also occur as defined by the LC curve (Figure 1(a)). An increase in net stress along path A→B will also result in an elastic response until reaching the LC curve at point B. Further increases in mean net stress result in an elasto-plastic response, along with expansion of the LC curve due to stress-induced hardening. Although this loading path is the focus of this study, plastic strains may also be generated along other paths in the elasto-plastic frameworks for unsaturated soils that consider coupling between hydraulic and mechanical loading paths (Wheeler et al. 2003; Romero and Jommi 2008). The other LC curves presented in Figure 1 function in a similar way to that of Alonso et al. (1990), but interpret LC curve in terms of the mean effective preconsolidation stress. The relationship of Wheeler et al. (2003) also permits coupling between mechanical and hydraulic loading paths to be considered by incorporating suction increase (SI) and suction decrease (SD) curves that bound the region of elastic response.

An advantage of the framework of Alonso et al. (1990) is that it can predict wetting-induced swelling or collapse of unsaturated soils. In this case, the slope of the virgin compression line in mean net stress space decreases with suction in order to predict collapse. However, this change in the slope of the virgin compression line with suction is not necessarily observed when interpreting soil behavior in terms of generalized effective stress (Uchaipichat and Khalili 2009), and the decrease in slope may lead to problems when trying to predict the phenomenon of pressurized saturation during drained compression to high mean stresses (Mun and McCartney 2015, 2016). Mun and McCartney (2016) found that the slope of the virgin compression line of an unsaturated soil is different than that of a saturated soil, and that it will gradually converge with the virgin compression line of saturated soil at high stresses with an intersection point dependent on the initial degree of saturation. Nonetheless, similar to Uchaipichat and Khalili (2009), Mun and McCartney (2016) found that the virgin compression lines for specimens with different initial degrees of saturation in effective stress space were relatively parallel for mean effective stresses less than 3 MPa, which is in the stress range relevant to most geotechnical applications. Instead of changing the slope of the virgin compression lines for unsaturated soils, Khalili et al. (2004) found that the collapse phenomenon may be captured by considering the relative changes in preconsolidation stress and effective stress with changes in degree of saturation or suction. This may imply that a linkage between the LC curve and the compression curve may not be necessary in all effective stress-based elasto-plastic frameworks.

A remaining issue in the definition of the LC curve is confirmation of the role of hydraulic hysteresis on the yield stress evolution during wetting and drying, and understanding the situations in which it needs to be considered. In field applications such as slopes and pavements, for example, wetting and drying may lead to hysteretic changes in stiffness that may lead to temporal variations in deformations. Romero and Jommi (2008) noted that irreversible strains could be observed following a wetting-drying path using a suction-controlled oedometer under isotropic conditions and Khosravi and McCartney (2012) showed that hysteretic changes in the small strain shear modulus during drying and wetting may be linked to hysteretic changes in the preconsolidation stress. Further, hydraulic hysteresis may be encountered when loading soil under undrained drainage conditions, where a change in degree of saturation and suction are expected during compression, as will be discussed later in this paper. Although there have been several hydro-mechanical frameworks proposed to account for changes in the yield stress during wetting or drying (Wheeler et al. 2003; Tamagnini 2004; Sun et al. 2008, Sheng et al. 2004), they have not been used to evaluate the impact of hydraulic hysteresis for soils compressed under different drainage conditions. Accordingly, the objective of this paper is to provide a simple approach to investigate the evolution of the mean effective preconsolidation stress with degree of saturation using a linkage to the shape of the SWRC.

CALIBRATION OF THE PRECONSOLIDATION STRESS RELATIONSHIP

The hysteretic preconsolidation stress relationship proposed in this study couples the LC curve relationship of Salager et al. (2008) with the van Genuchten (1980) SWRC relationship to predict a non-linear relationship between the preconsolidation stress and the matric suction normalized by the air entry suction. The SWRC during hydrualic hysteresis is typically defined using a primary drying path that governs the drainage process from a water-saturated soil, a primary wetting curve that governs the wetting process from an air-dry soil, and scanning curves that transition between the primary wetting and drying curves depending on the initial state of an unsaturated soil undergoing a wetting or drying process. The van Genuchten (1980) SWRC model for the primary drying path is given as follows:

$$S_r = S_{res} + (1 - S_{res})\left[\frac{1}{1 + (\alpha_{vG} \cdot \psi)^{n_{vG}}}\right]^{\frac{1}{1-n_{vG}}} \quad (1)$$

where S_{res} is the residual degree of saturation and α_{vG} and n_{vG} are fitting parameters. The approach used to simulate hydraulic hysteresis in this study was to fit a modified version of the van Genuchten (1980) SWRC to the wetting path data from the experimental studies, which includes a scanning curve and a portion of the primary wetting path. The equation used in this case is:

$$S_r = (S_{\max} - S_{\min}) \cdot \left[\frac{1}{1 + (\alpha_{vG} \cdot \psi)^{n_{vG}}} \right]^{\frac{1}{1-n_{vG}}} \tag{2}$$

where S_{max} is the maximum degree of saturation on the primary wetting curve and S_{min} is the degree of saturation on the drying curve at the point of wetting. This approach provides a good fit to measured wetting path data but does not provide a generalized model for predicting any hysteretic path. Alternative approaches to capture the hysteretic SWRC are to assume a piece-wise log-linear SWRC (Wheeler et al. 2003), linear scanning curve connecting the primary drying and wetting paths (Tamagnini 2004), or modification of the parameters from the primary drying path to predict the wetting path (Kool and Parker 1987). Nonetheless, the approach using Equation (2) permits a simple evaluation of linkages between the SWRC and the preconsolidation stress that is the focus of this study.

It should be noted that hydraulic hysteresis can be influenced by the initial conditions. For example, Della Vecchia et al. (2013) reported a different primary wetting path for different void ratios. Although differences may exist between the SWRC (and the air entry suction) at different stresses during compression or for soils with different initial densities, the effect of these factors are neglected in this study for simplicity. As compression of soils will change the pore structure and thus the SWRC, the stress-dependent SWRC model of Zhou and Ng (2014) can be incorporated into a relationship for the preconsolidation stress. However, there is insufficient data to apply the relationship from this Zhou and Ng (2014) at the current time.

When interpreting the preconsolidation stress, the mean effective stress was evaluated using the definition of generalized effective stress proposed by Bishop and Blight (1963), as follows:

$$p' = p_{net} + \chi \psi \tag{3}$$

where p_{net} is mean net stress, ψ is the matric suction equal to the difference between the pore air pressure u_a and the pore water pressure u_w, and χ is the effective stress parameter assumed to be equal to the degree of saturation S_r for simplicity. Many other choices are available for the definition of the value of x, including the effective saturation (Bolzon and Schrefler 1995; Lu et al. 2010), a function of the air entry suction (Khalili and Khabbaz 1998), and experimental shear strength data (Lu and Likos 2006; Khalili and Zargarbashi 2010).

Salager et al. (2008) proposed a semi-logarithmic relationship to capture the association between p'_c and ψ as follows:

$$\frac{p'_c(\psi)}{p'_c(\psi=0)} = \begin{cases} 1 & if\ \psi \le \psi_{ae} \\ 1+\gamma_\psi \log\dfrac{\psi_0}{\psi_{ae}} & if\ \psi > \psi_{ae} \end{cases} \quad (4)$$

where $p'_c(\psi)$ and p'_c $(\psi = 0)$ are the values of mean effective preconsolidation stress at matric suctions of ψ and zero, respectively, ψ_{ae} is the air entry suction, and γ_ψ is a material parameter that defines the impact of matric suction on p'_c. The relationship assumes a piecewise log-linear relationship for the SWRC where no change in degree of saturation occurs until the applied matric suction has surpassed the air entry suction. Therefore, no changes in p'_c will occur until $\psi > \psi_{ae}$. The relationship of Salager et al. (2008) has been adopted by several constitutive frameworks to define the LC curve in the p′ - ψ space (François and Laloui 2008; Bellia et al. 2015).

A potential drawback of the relationship of Salager et al. (2008) involves the “if” statement required to maintain a constant value for p'_c while $\psi \le \psi_{ae}$, and the fact that there is not data showing the evolution in p'_c for suctions less than the ψ_{ae}. In order to predict the trends in preconsolidation stress and degree of saturation, the model of Salager et al. (2008) was modified to relate p'_c and S_r as follows:

$$\frac{p'_c(S_r)}{p'_c(S_r=1)} = \begin{cases} 1 \quad if\ \psi \le \psi_{ae} \\ 1+\gamma_\psi \log\left[\left(\frac{1}{\alpha_{vG}}\left\{\frac{1}{(S_e)^{1-n_{vG}}}-1\right\}^{\frac{1}{n_{vG}}}\right)\times\frac{1}{\psi_{ae}}\right] \quad if\ \psi > \psi_{ae} \end{cases} \tag{5}$$

where γ_ψ is the same soil parameter from the relationship between suction and p'_c, and S_e is the effective degree of saturation defined as (S_r-S_{res}/1-S_{res}).

Data sets available in the literature that involve information on the role of suction and degree of saturation on the hardening response of unsaturated soils were selected to calibrate the hysteretic preconsolidation stress relationships in Equations (4) and (5). The calibration was restricted to studies that involved the drained, isotropic compression of soils that had experienced different hydraulic testing paths before loading. Uchaipichat and Khalili (2009) performed suction controlled isotropic loading tests on a compacted silt for different values of matric suction (0, 100, and 300 kPa) while Uchaipichat (2010) investigated the influence of hydraulic hysteresis on the compression curve and preconsolidation stress for different values of matric suction. Coccia (2016) investigated the impact of degree of saturation on the mean effective preconsolidation stress of compacted silt using a high pressure thermal isotropic cell. Khosravi et al. (2016) performed a series of isotropic compression tests to represent the impact of suction-induced hardening on the dynamic shear modulus of unsaturated soils. The data of Salager et al. (2008) was not included in this evaluation because of the large variability observed in the reported preconsolidation stress trends with suction. Other studies such as Mun and McCartney (2015, 2016) present drained compression curves for specimens with different initial suction values, but the specimens were prepared using different compaction efforts and may have different soil structures that may influence the preconsolidation stress.

The first step in evaluating the hysteretic preconsolidation stress model was to determine the parameters α_{vG} and n_{vG} for different soils from the literature using least squares minimization. Drying-path SWRCs collected from the literature are shown in Figure 2(a). Using the method of least squares, the preconsolidation stresses from Equation (5) calculated from the value of degree of saturation at the point of yielding are compared with the measured values of p'_c for each test in Figure 2(b). A good match between the experimental preconsolidation stress values and the results from Equation (5) is obtained in this figure. The evolution of p'_c as a function of the initial suction (ψ_0) and normalized initial suction (ψ_0/ψ_{ae}) for these soils calculated using Equation (4) are shown in Figures 2(c) and 2(d), respectively. The relationship also provides a reasonable match to the preconsolidation stress in terms of suction for the four soils.

The preconsolidation stress values for compacted Kaolinite specimens that had experienced a wetting and drying path before undergoing isotropic, drained compression reported by Uchaipichat (2010) are investigated in Figure 3. To identify the effect of S_r on p'_c by incorporating the hysteretic SWRC, the van Genuchten (1980) SWRC model parameters were determined for the primary drying and wetting paths shown in Figure 3(a). The changes in p'_c with S_r obtained from Equation (5) are shown in Figure 3(b), and changes in p'_c with ψ_0 and ψ_0/ψ_{ae} obtained from Equation (4) are shown in Figures 3(c) and 3(d), respectively. The values of p'_c were not observed to exhibit any significant change with matric suction during hydraulic hysteresis in Figures 3(c) and 3(d), while a clear hysteretic trend between p'_c and S_r was observed in Figure 3(b) for specimens that had previously experienced drying or wetting paths. Although the comparison in Figure 3 indicates that it may be simpler to use the relationship between the preconsolidation stress and suction (Fig. 3(c)) in hydro-mechanical frameworks due to the lack of hysteresis in this relationship, there are hydro-mechanical paths where the suction and degree of saturation may both change, such as undrained compression (i.e., compression under constant water content conditions). In this case, it may be easier to track or estimate changes in the degree of saturation, and use the hysteretic relationship in Equation (5) to predict the preconsolidation stress. In this case, the relationship in terms of degree of saturation in Figure 3(b) may be more useful. Although not shown here, Khosravi et al. (2016) also presented a single preconsolidation stress value for a specimen that had been wetted following a drying path, and similar observations to those in Figure 3 were drawn.

There are several other studies that have evaluated the compression of unsaturated soils under undrained (or constant water content) conditions (e.g., Jotisankasa et al. 2007; Sun et al. 2008; Della Vecchia et al. 2013). In these cases, the suction and degree of saturation need to be monitored or inferred during compression in order to perform an effective stress analysis, as both variables can change during undrained compression. Of the studies provided above, Jotisankasa (2005) and Jotisankasa et al. (2007) provided sufficient information to perform this analysis and evaluate the hysteretic preconsolidation stress relationship. The measured drying and wetting path SWRCs from Jotisankasa (2005) and Jotisankasa et al. (2007) along with van Genuchten (1980) SWRC fitting parameters are shown in Figure 4(a), while a comparison between the preconsolidation stress data from their undrained compression tests and the fitted relationship from Equation (5) is shown in Figure 4(b). A value of γ_ψ of 19.0 was identified for the compacted silty clay using least squares minimization, which was greater than that observed in the other studies presented in Table 1 and likely reflects soil- and test-specific conditions. Nonetheless, a good fit is observed between the experimental data and the model. The trends in preconsolidation stress with ψ and ψ/ψ_{ae} from Equation (4) are shown in Figure 4(c) and 4(d), and a good match is also observed with the model.

APPLICATION OF THE PRECONSOLIDATION STRESS MODEL IN PREDICTING COMPRESSION RESPONSE UNDER DIFFERENT DRAINAGE CONDITIONS

The compression of soils under either undrained or drained conditions can be considered to elucidate how the hysteretic preconsolidation stress model may be used in different situations. During drained compression, the suction will remain constant while the degree of saturation will increase as the voids are compressed. The only way for the suction to remain constant but for the degree of saturation to increase would be for the shape of the SWRC to change, as shown in Figure 5(a) for a hypothetical soil. In this case, it may be simplest to estimate the preconsolidation stress in terms of suction using Equation (4), as shown in Figure 5(b). During undrained compression, it is expected that the degree of saturation will increase and the suction will decrease during undrained compression, as shown in Figure 5(c) for a hypothetical soil. The degree of saturation is likely proportional to the change in volume of the soil and may be easy to estimate using a model such as that of Zhou et al. (2012b). However, the change in suction may not be simple to estimate as the shape of the SWRC is likely changing. In addition to this, the model in Equation (4) does not capture the preconsolidation stress variation during the increase in degree of saturation during compression, so Equation (4) may not be suitable to predict the evolution in preconsolidation stress during undrained compression. In this case, it may be best to estimate the preconsolidation stress defined in Equation (5) using estimates of the degree of saturation during compression, as shown in Figure 5(d). It is acknowledged that the magnitude of changes in degree of saturation and suction are not as significant before the applied mean stress reaches the preconsolidation stress as they may be for higher stresses (Wheeler et al. 2003). Nonetheless, the nonlinear trends between preconsolidation stress and degree of saturation from the proposed relationship indicate that this still may be important to consider.

To evaluate the situations described in the previous paragraph quantitatively, the proposed preconsolidation stress models in Equations (4) and (5) can be utilized alongside a simple elasto-plastic model to predict the compression curves of soils during drained compression using the relationship between preconsolidation stress and suction, and to predict the compression curves of soils during undrained (constant water content) compression using the wetting-path relationship between preconsolidation stress and the degree of saturation. The compression curves from the experiments indicate that the unsaturated soils exhibit elastic behavior until reaching a mean effective preconsolidation stress p'_c. In the elastic region, changes in void ratio (e) up p'_c can be expressed as follows:

$$\Delta e = \kappa \cdot \ln \frac{p'_f}{p'_0} \qquad p'_f < p'_c \tag{6}$$

where p'_0 and p'_f are the initial and final mean effective stresses, and p'_c is the mean effective preconsolidation stress, which can be predicted by using either Equations (4) or (5). The value of κ is assumed to be constant regardless of the initial suction or initial degree of saturation.

After reaching the mean apparent preconsolidation stress, the unsaturated specimens are assumed to decrease in volume irrespective of suction magnitudes or degree of saturation. The compression response of unsaturated soils for stresses greater than p'_c can be calculated as follows:

$$\Delta e = \lambda \cdot \ln \frac{p'_f}{p'_0} \qquad p'_c < p'_0 < p'_f \tag{7}$$

where λ is the slope of the VCL for unsaturated soil and is assumed to be constant for unsaturated soils over the stress range of interest to geotechnical problems following the observations of Uchaipichat and Khalili (2009) and Mun and McCartney (2015). Although the slope λ for unsaturated soils is assumed to be the same regardless of the suction, it is assumed to be greater than or equal than the slope λ_0 for saturated soil in the mean effective stress range relevant to geotechnical engineering problems. This follows the observations of several experimental studies (i.e., Sivakumar 1993; Uchaipichat and Khalili 2009; Mun and McCartney 2015).

Zhou et al. (2012a) introduced an expression to predict the change in the effective saturation caused by suction and stress changes by using a hydro-mechanical interaction function (D_e). However, the model of Zhou et al. (2012a) has a limitation in the evaluation of compression behavior with the change of suction or degree of saturation because of the complexity in calculation methods. Sun et al. (2008) also suggested an analytical expression to predict the change of degree of saturation and suction through inclusion of hydraulic hysteresis during undrained compression. This study employed a simplified approach to predict the change of degree of saturation during undrained compression based on the assumption that the variation in the degree of saturation of unsaturated soil is directly related to the changes in void ratio, an approach used by Jotisankasa (2005) and Jotisankasa et al. (2007). Changes in degree of saturation during initial compression in the elastic region can be calculated as follows:

$$\Delta S_r = \kappa \cdot \ln \frac{p'_f}{p'_0} \quad p'_f < p'_c \tag{8}$$

The changes in degree of saturation can be used in an incremental form to calculate changes in the mean effective preconsolidation stress using Equation (5). Although not as critical to this study but still relevant for calculation of the mean effective stress using Equation (3), the degree of saturation for stresses greater than p'_c can be calculated as follows:

$$\Delta S_r = \lambda \cdot \ln \frac{p'_f}{p'_c} \quad p'_c < p'_0 < p'_f \tag{9}$$

VALIDATION OF THE PRECONSOLIDATION STRESS MODEL IN PREDICTING COMPRESSION CURVES

In order to verify the simple elasto-plastic framework together with the hysteretic preconsolidation stress model, comparisons between the simulated and measured compression curves for drained compression tests presented by Uchaipichat (2010) and undrained compression tests presented by Jotisankasa et al. (2007) are shown in Figures 6(a) and 6(b), respectively. In addition to the values of γ_ψ defined in Figures 3 and 4 for each of the two studies respectively, the parameters of the framework were calibrated to predict the compression curves of the two unsaturated soils under different initial conditions, and are summarized in Table 2. Further, the actual initial conditions (e.g., $S_{r,0}$, e_0, p') from the experiments shown in Table 2 were used as inputs. The simulated compression curves appear to match well with the experimental compression behavior in Figures 6(a) and 6(b), and capture the different suction hardening effects for the specimens tested in both drained and undrained conditions.

The approach described in Figures 5(a) and 5(b) was used to simulate the drained compression data of Uchaipichat (2010) while the approach described in Figures 5(c) and 5(d) was used to simulate the undrained compression data of Jotisankasa et al. (2007). Uchaipichat (2010) did not report the change of degree of saturation during drained compression, which required an estimated trend in S_r to calculate the mean effective stress. Although Equations (8) and (9) are meant for use in estimating changes in degree of saturation in undrained compression tests, they were used to predict the change of degree of saturation during compression under constant suction conditions, as shown in Figure 6(c). On the other hand, Jotisankasa et al. (2007) reported changes in degree of saturation and suction during undrained compression, and the data in Figure 6(d) indicates that the degree of saturation calculated using Equations (8) and (9) matches the data well. Although the degree of saturation only increases by about 2% during the elastic loading for the different specimens, this small change still causes a relevant decrease in preconsolidation stress of about 6 to 10% due to the steep shape of the wetting path curve in Figure 4(b). The suction values used in the simulation of the data of Uchaipichat are shown in Figure 6(e), and are constant as the test is drained. The suction values used in the simulation of the data from Jotisankasa et al. (2007) were estimated from the calculated values of S_r using the SWRC for simplicity, and the comparison in Figure 6(f) indicates a good fit with most of the measured suction values in the tests.

The values of preconsolidation stress for the drained tests of Uchaipichat (2010) obtained from the suction values in Figure 6(e) using Equation (4) lead to a good fit to the experimental preconsolidation stress values as shown in Figure 6(g). As expected, the preconsolidation stress is constant during drained compression. In Figure 6(g), the point of yielding is reflected by the intersection between the preconsolidation stress line and the 1:1 line. The trends in preconsolidation stress for the undrained tests of Jotisankasa et al. (2007) obtained from the degree of saturation trends in Figure 6(d) using Equation (5) are shown in Figure 6(h). A decreasing trend in preconsolidation stress during compression is observed due to the reduction in S_r. The slope of the trend in preconsolidation stress increases with increasing suction due to the nonlinearity in the hysteretic preconsolidation stress relationship.

The comparison in Figure 6 shows conceptually how the hysteretic preconsolidation stress relationship described by Equations (4) and (5) can be applied for different hydro-mechanical paths. Although the changes in degree of saturation in the elastic zone are expected to be the same due to the use of the same value of κ in Equation (8), the shape of the SWRC may lead to different impacts of this change in degree of saturation on the predicted preconsolidation stress relationships. This also indicates that the shape of the SWRC for a given soil may have an impact on how important hydraulic hysteresis may be in predicting the compression curve in undrained conditions.

CONCLUSIONS

This paper proposes a mean effective preconsolidation stress relationship that describes evolution in this variable with the suction and degree of saturation during hydraulic hysteresis, which was validated based on available data sets in the literature. The relationship was then used in a simple, effective stress-based elasto-plastic framework to predict the compression curves of soils during drained compression (constant suction) using the relationship between preconsolidation stress and suction, and to predict the compression curves of soils during undrained (constant water content) compression using the wetting-path relationship between preconsolidation stress and the degree of saturation. The proposed relationship was found to satisfactorily capture the relative changes in preconsolidation stress with suction and degree of saturation, and was shown to be useful to simulate the compression curves of unsaturated soils under different drainage conditions and corresponding hydro-mechanical paths.

ACKNOWLEDGEMENTS

The authors appreciate the support from National Science Foundation grant CMMI-1054190. The views in this paper are those of the authors alone.

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TABLE 1: Parameters of the preconsolidation stress relationship for different soils

Study	Soils investigated	Drying-path SWRC model parameters (van Genuchten 1980)				p'_c relationship parameter
		α_{vg} (kPa^{-1})	n_{vg}	$S_{r,min}$	ψ_{ae} (kPa)	γ_ψ
Coccia (2016)	Compacted Bonny silt	0.16	1.38	0.03	2	0.765
Khosravi et al. (2016)	Compacted Bonny silt	0.02	2.60	0.06	10	0.700
Uchaipichat (2010)	Compacted Kaolinite	0.002	1.49	0.00	25	0.700
Uchaipichat & Khalili (2010)	Compacted Bourke silt	0.03	3.15	0.03	18	0.197

TABLE 2: Calibrated parameters for the simulated compression curves of soils loaded in drained and undrained conditions

Variable	Drained condition (Uchaipichat 2010)	Undrained condition (Jotisankasa 2005; Jotisankasa et al. 2007)
Initial suctions evaluated, ψ_0 (kPa)	0 / 50 / 100 / 200 / 300	0 / 75 / 134 / 525 /936
Initial degrees of saturation evaluated, $S_{r,0}$	1.00 / 0.97 / 0.96 / 0.95 / 0.91	1.00 / 0.54 / 0.49 / 0.40 / 0.36
Initial void ratios evaluated, e_0	1.05 / 1.04 / 1.04 / 1.03 / 1.03	0.71 / 0.71 / 0.71 / 0.70 / 0.69
λ_0 (Saturated)	0.065	0.086
λ (Unsaturated)	0.070	0.178
κ	0.005	0.007
ψ_{ae} (kPa)	25.0	45.0
α_{vg} (kPa^{-1})	0.019	0.095
n_{vg}	3.60	1.65
$S_{r,max}$	0.994	0.982
$S_{r,min}$ (the same for all tests)	0.912	0.365
p'_c relationship parameter, γ_ψ	0.7	19.0

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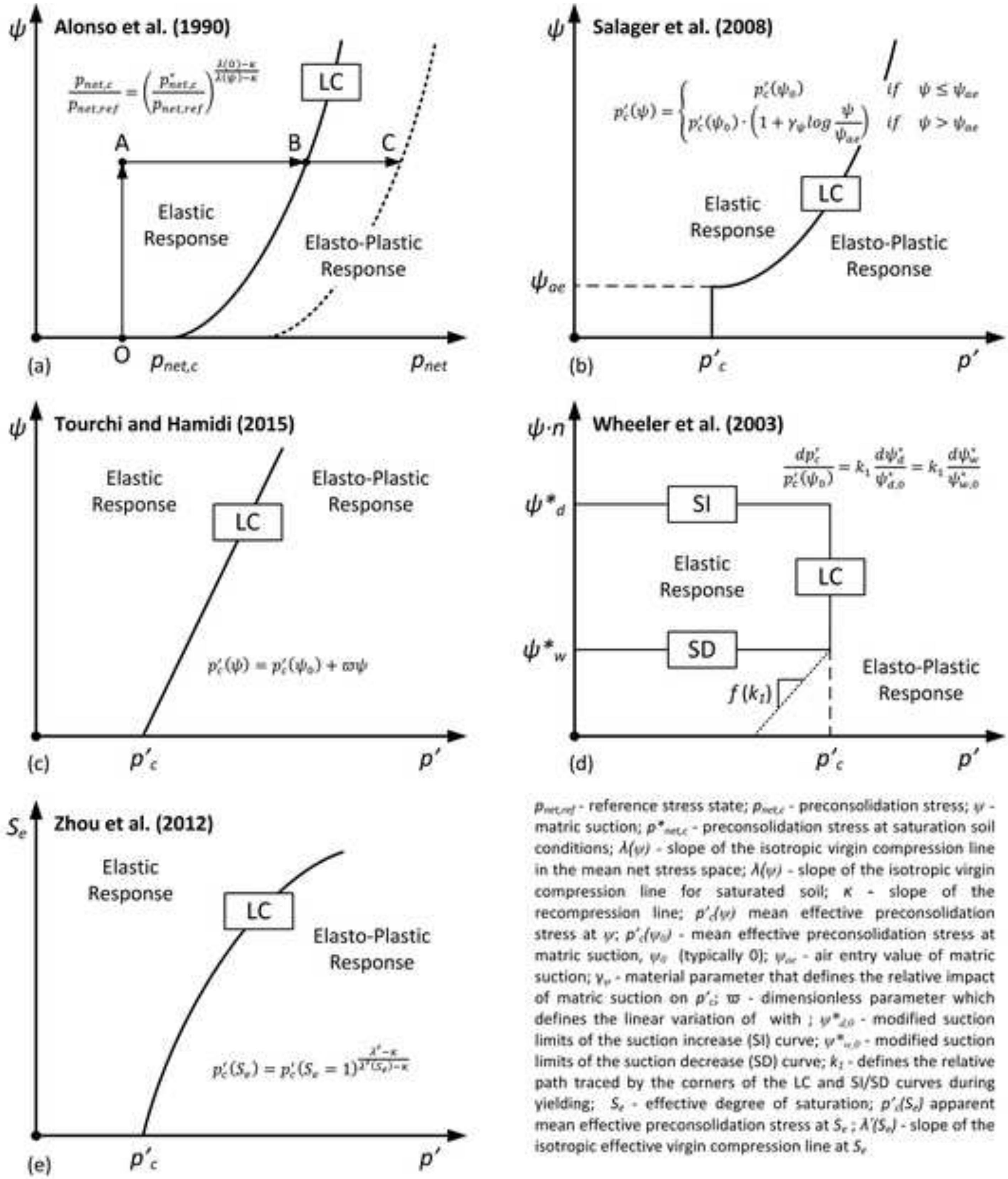

$$\frac{p_{net,c}}{p_{net,ref}} = \left(\frac{p^*_{net,c}}{p_{net,ref}}\right)^{\frac{\lambda(0)-\kappa}{\lambda(\psi)-\kappa}}$$

$$p'_c(\psi) = \begin{cases} p'_c(\psi_0) & if \quad \psi \le \psi_{ae} \\ p'_c(\psi_0)\cdot\left(1+\gamma_\psi \log\frac{\psi}{\psi_{ae}}\right) & if \quad \psi > \psi_{ae} \end{cases}$$

$$\frac{dp'_c}{p'_c(\psi_0)} = k_1\frac{d\psi^*_d}{\psi^*_{d,0}} = k_1\frac{d\psi^*_w}{\psi^*_{w,0}}$$

$$p'_c(S_e) = p'_c(S_e = 1)^{\frac{\lambda'-\kappa}{\lambda'(S_e)-\kappa}}$$

$p_{net,ref}$ - reference stress state; $p_{net,c}$ - preconsolidation stress; ψ - matric suction; $p^*_{net,c}$ - preconsolidation stress at saturation soil conditions; $\lambda(\psi)$ - slope of the isotropic virgin compression line in the mean net stress space; $\lambda(\psi)$ - slope of the isotropic virgin compression line for saturated soil; κ - slope of the recompression line; $p'_c(\psi)$ mean effective preconsolidation stress at ψ; $p'_c(\psi_0)$ - mean effective preconsolidation stress at matric suction, ψ_0 (typically 0); ψ_{ae} - air entry value of matric suction; γ_ψ - material parameter that defines the relative impact of matric suction on p'_c; ϖ - dimensionless parameter which defines the linear variation of with ; $\psi^*_{d,0}$ - modified suction limits of the suction increase (SI) curve; $\psi^*_{w,0}$ - modified suction limits of the suction decrease (SD) curve; k_1 - defines the relative path traced by the corners of the LC and SI/SD curves during yielding; S_e - effective degree of saturation; $p'_c(S_e)$ apparent mean effective preconsolidation stress at S_e ; $\lambda'(S_e)$ - slope of the isotropic effective virgin compression line at S_e

Figure 2

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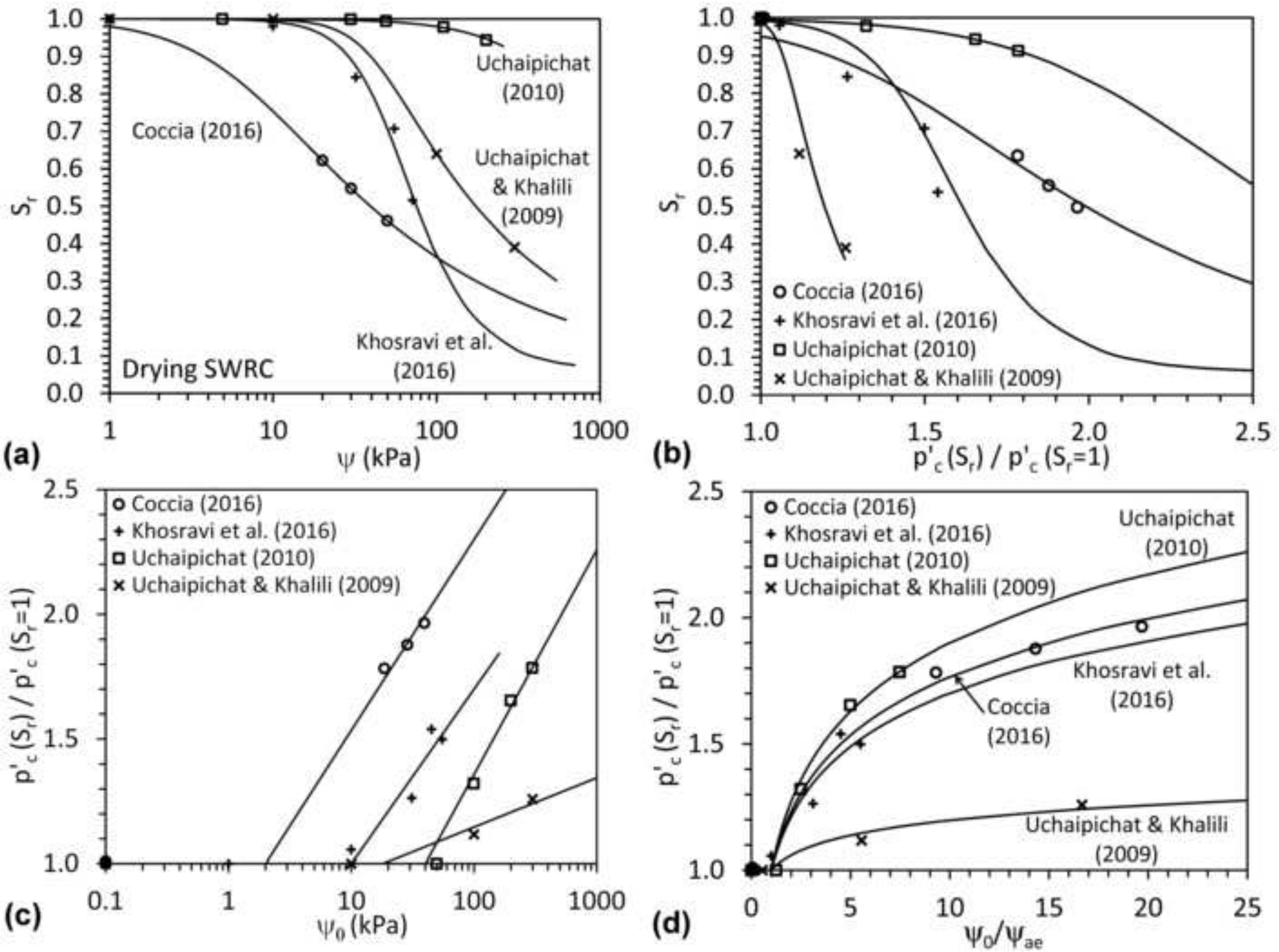

Figure 3

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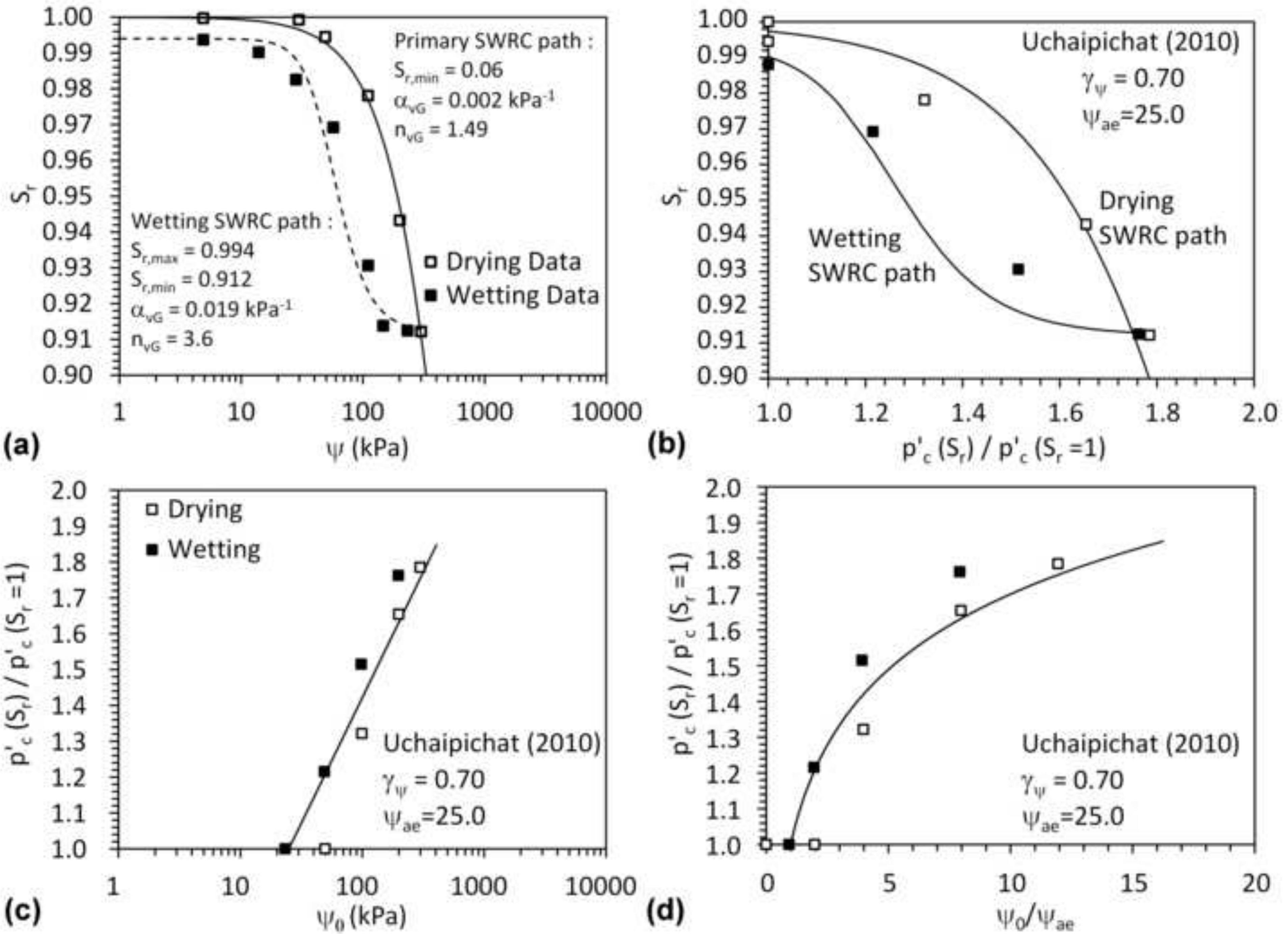

Figure 4

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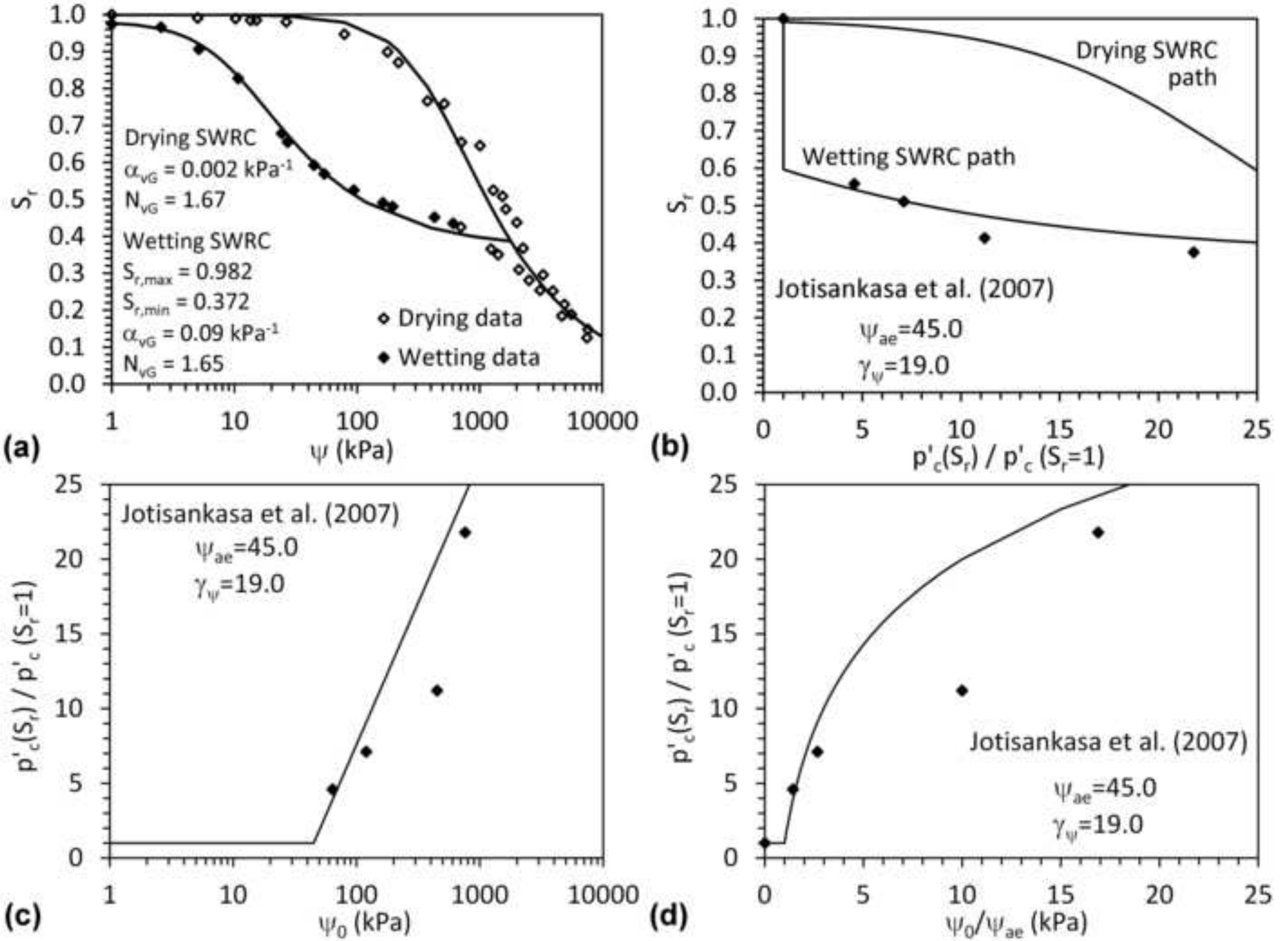

Figure 5

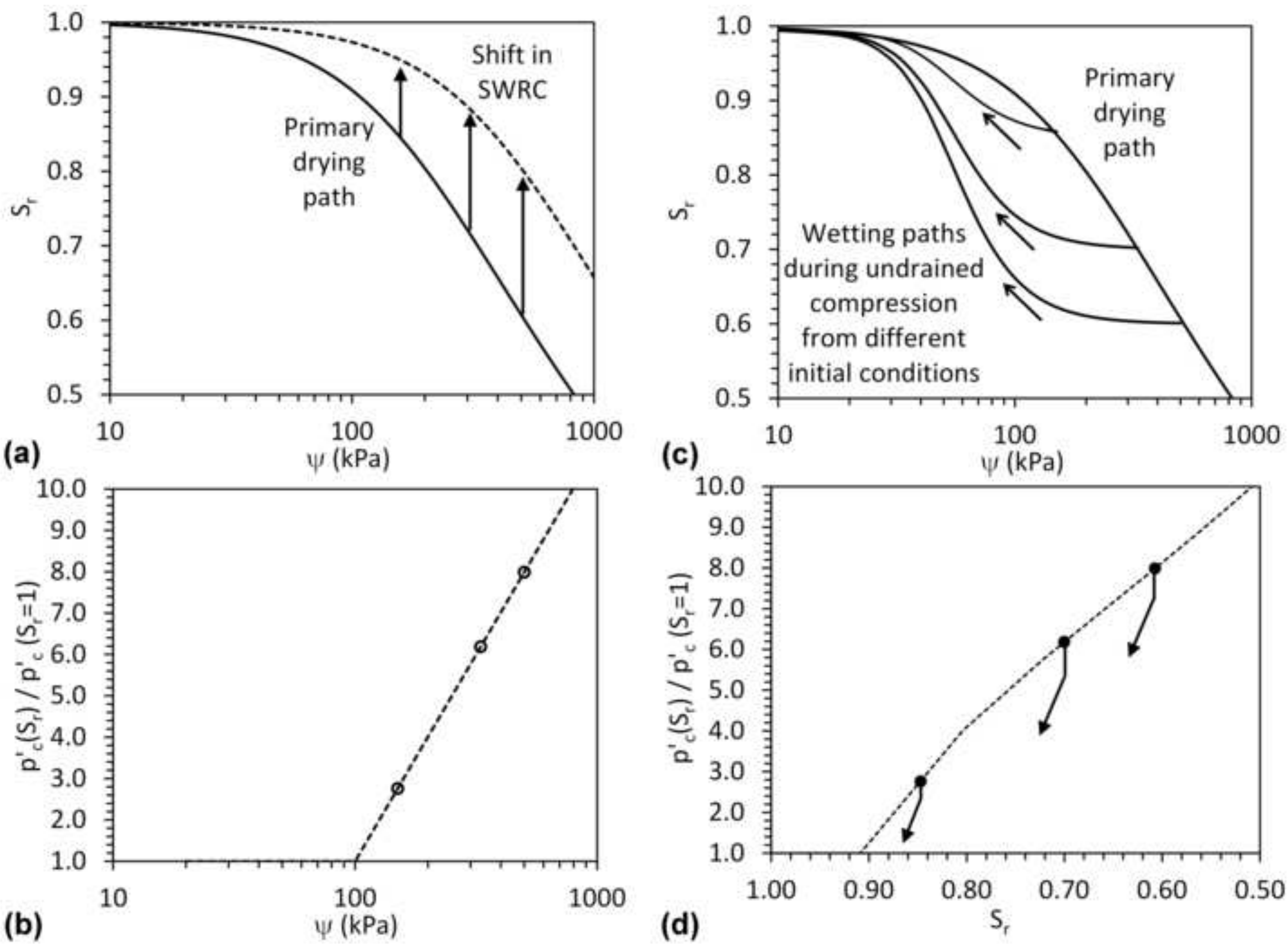
1.0
0.9
0.8
0.7
0.6
0.5
S_r
Shift in SWRC
Primary drying path
10
100
1000
(a)
ψ (kPa)
10.0
9.0
8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
$p'_c(S_r) / p'_c(S_r=1)$
(b)
Primary drying path
Wetting paths during undrained compression from different initial conditions
(c)
1.00
0.90
0.80
0.70
0.60
0.50
(d)

Figure 6

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