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## Authors

Noland, Robert B.
Small, Kenneth A.
Koskenoja, Pia Maria
et al.

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# Simulating Travel Reliability 

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Robert B. Noland ${ }^{1}$<br>Kenneth A. Small ${ }^{2}$<br>Pia Maria Koskenoja ${ }^{3}$<br>Xuehao Chu ${ }^{4}$

# ${ }^{1}$ U.S. Environmental Protection Agency, Office of Policy, Planning and Evaluation 401 M Street SW, Washington, DC 20460, U.S.A. <br> ${ }^{2}$ Department of Economics and Institute of Transportation Studies, University of California, Irvine Irvine, CA 92697-5100, U.S.A., ksmall@uci.edu <br> ${ }^{3}$ National Institute of Statistical Sciences, 200 Park Offices Drive Research Triangle Park, NC 27709, U.S.A. <br> ${ }^{4}$ Center for Urban Transportation Research, University of South Florida; Tampa, FL 33620, U.S.A. 

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#### Abstract

We present a simulation model designed to determine the impact on congestion of policies for dealing with travel time uncertainty. The model combines a supply side model of congestion delay with a discrete choice econometric demand model that predicts scheduling choices for morning commute trips. The supply model describes congestion technology and exogenously specifies the probability, severity, and duration of non-recurrent events. From these, given traffic volumes, a distribution of travel times is generated, from which a mean, a standard deviation, and a probability of arriving late are calculated. The demand model uses these outputs from the supply model as independent variables and choices are forecast using sample enumeration and a synthetic sample of work start times and free flow travel times. The process is iterated until a stable congestion pattern is achieved. We report on the components of expected cost and the average travel delay for selected simulations.


The corresponding author is Robert B. Noland, U.S. Environmental Protection Agency, Office of Policy, Planning and Evaluation, 401 M St. SW (2126), Washington, DC 20460. email: noland.bob@epamail.epa.gov, fax: 202-260-0512

## 1. INTRODUCTION

One of the great rediscoveries of transportation analysis is the importance of reliability in travel times. Based on instinct and direct statements of travelers (Prashker 1979), travel demand analysts have long suspected that reliability should be an important component of travel demand models and that unreliability is one of the primary costs of road congestion. Yet only very recently have such models succeeded in finding measurable effects (Small, 1992, pp. 35-36).

Stated-preference techniques have opened the way to some solid empirical estimates of travel time reliability. Most of these measure how much people are deterred by a higher standard deviation of travel times, relative to a higher mean travel time (Bates, 1990; Black and Towriss, 1993; Abdel-Aty et al., 1994).

These measurements, however, do not necessarily distinguish among different reasons for aversion to unreliability. There are at least two such reasons. The first, which we call expected scheduling cost, is the desire to lower the likelihood of arriving at the destination at an inconvenient time; many authors have focused specifically on the probability that one will be late for an activity with a definite starting time. The second, which we call planning cost, is the pure nuisance of not being able to plan one's activities precisely because of uncertainty about when a trip will be completed. These two reasons have quite different implications for how people respond to various policies. For example, measures that change the degree of reliability, such as quick-response teams to clear up accidents, might simply make people happier by reducing planning costs, or they might induce complex changes in the timing of people's trips by changing the scheduling calculus. The same is true of measures, such as advanced traveler information systems, that improve information about travel conditions.

The possibility of people changing the timing of their trips leads to a further question of just how a highway corridor or network will re-equilibrate when reliability or information changes. A sophisticated literature has developed to analyze equilibria when people choose their trip schedules endogenously, but it seldom addresses reliability. Furthermore, most of this literature assumes greatly simplified models on both the supply and demand sides; usually the supply side is one or a few bottlenecks and the demand side is deterministic cost minimization with one or a few types of travelers. Exceptions on the supply side are Ben-Akiva et al. (1986)
and Vythoulkas (1990), who consider networks of some complexity. Exceptions on the demand side are De Palma et al. (1983) and Chu (1993). Chu applies a discrete choice model of travel choice that is rich in specification and that allows for unlimited heterogeneity of preferences. Noland (1997) simulates the impact of information provision on expected costs using theoretical relationships but lacking the heterogeneity of the approach followed here.

In this paper, we propose and empirically estimate a demand model whose components include both scheduling costs and planning costs. The estimation is carried out using stated preference data collected by Koskenoja (1994). We then develop an equilibration model in which travel schedules, congestion at each point in time, and the degree of reliability in travel times are all derived endogenously, given exogenous parameters fixing the probability, severity, and duration of an incident that reduces capacity. This equilibration model is an extension of Chu's, and therefore retains the advantage of a rich demand-side specification.

The analysis of the stated preference questions leads to a behavioral model of schedule choice in the face of uncertainty. Implicit in this model are estimates of the costs of various characteristics of the travel schedule: mean travel time, average "schedule delay early" (defined as number of minutes spent at work prior to the preferred work start time), average "schedule delay late" (defined as number of minutes that an arrival is later than that preferred work start time), probability of being late, and standard deviation of travel time. The results show that, consistent with prior research, people are moderately averse to arriving early at work and more averse to arriving late, with a substantial discrete penalty to being late at all. The results show that once these scheduling costs are taken into account, there is little additional residual cost to uncertainty per se, i.e., the planning cost is negligible.

In order to assess the practical importance of these findings, we perform simulations in which travel-time uncertainty results from "non-recurrent congestion" caused by incidents of capacity reduction. Both recurrent and non-recurrent congestion is generated by a set of hypothetical commuters making their scheduling choices according to the demand model just described. In these simulations, congestion results from insufficient capacity and uncertainty results from random "incidents" that reduce capacity in a specified manner. We find that slightly less than half of the increase in travel costs caused by incidents is due to increased travel time;
the rest is due to scheduling costs, primarily increased probability of arriving late. The latter occurs despite a small tendency for people to adjust to increasing uncertainty by leaving for work earlier, which of course imposes its own cost according to our model. These results have policy implications for decisions on the cost effectiveness of options for responding to recurrent and non-recurrent congestion; such as whether an incident management system is more effective than adding additional capacity.

Following a systematic review of the literature, we describe in turn our theoretical model of traveler choice, a stated-preference survey we conducted to obtain empirical measurements of its parameters, the full simulation model, and simulation results.

## 2. PRIOR LITERATURE

There are three strands of literature which we combine here: empirical scheduling choice, reliability, and equilibrium models of congestion with endogenous scheduling. We briefly review each, indicating how the present paper draws from the literature.

### 2.1 Scheduling Choice

Many authors have postulated traveler cost functions that include costs of arriving either earlier or later than a preferred arrival time such as a work start time. Only a few have been estimated empirically. Small (1982) estimates how commuters who have an official work start time choose their usual travel schedules from among twelve possible five-minute intervals. The logit specification postulates a fixed penalty (disutility) for arriving later than 2.5 minutes prior to the work start time. It also assumes additional per-minute penalties for arriving at work either early (schedule delay early, SDE) or late (schedule delay late, SDL). Small finds these penalties to vary systematically with personal and occupational characteristics; on average, the per-minute disutility of SDL is greater than that of SDE, and the fixed lateness penalty is equivalent to about 5 minutes of travel time (Small, 1982, model 1). Our demand model uses Small's specification applied to expected rather than known values of the variables.

Cosslett (1977), Hendrickson and Plank (1984) and Mannering and Hamed (1990) provide additional empirical measurements, the latter using time periods that are larger than five
minutes. Mannering and Hamed's study is unique in focusing on the trip from work to home rather than the opposite.

### 2.2 Reliability

A number of authors have focused on the fact that unreliable travel times create the possibility of being late for work (Gaver, 1968; Abkowitz, 1981; Polak, 1987; Bates, 1990). Typically they specify scheduling costs similarly to Small (1982), as described above, although usually including only a schedule-delay or a lateness penalty rather than both. They then derive conditions for how travelers adjust their scheduling choices to differing amounts of reliability. For example, Gaver (1968) postulates utility to be a linear function of travel time, schedule delay early, and schedule delay late; he then derives the scheduling choice that maximizes the expected value of this utility given a particular distribution of uncertain travel time. Polak (1987) also takes this approach, but adds a concave transformation to utility in order to represent risk aversion. ${ }^{1}$

The theoretical model used in this paper is based on that in Noland and Small (1995). Noland and Small's model is an extension of the models of Gaver (1968) and Polak (1987). Its key innovations are (1) the addition of a discrete lateness penalty; (2) accounting for the timevarying pattern of congestion over the course of the morning commute; and (3) deriving the form of the resulting expected costs of a morning commute trip, when the commuter chooses a schedule according to the model. To this expected cost function, we postulate in the present paper an additional term which we call the planning cost, a function of the standard deviation of travel times.

Empirical measurements have typically not distinguished between expected scheduling costs and planning costs. Black and Towriss (1993) assume that expected travel cost is simply a linear function of mean travel time and its standard deviation, and estimate the parameters. A similar approach is taken by Abdel-Aty et al. (1994). Both sets of authors estimate this cost function from stated-preference data. Here we estimate a model that separates these components

[^0]by estimating the underlying cost function prior to taking its expectation over the distribution of uncertain travel times and prior to the commuter's optimization over schedules. Our stated preference survey questions specify departure time from home as well as a distribution of travel times. We are therefore able to estimate separate coefficients on schedule delay, probability of being late, and standard deviation of travel times (which we have hypothesized is related to planning costs).

### 2.3 Equilibrium Modeling with Endogenous Scheduling

There is an extensive literature on modeling equilibria or dynamic adjustment paths using a simple deterministic demand structure. On the supply side, most such papers use a bottleneck model that is basically that of Vickrey (1969), except usually simplified by making everyone's desired arrival times identical. An example is Arnott et al. (1990). These models are reviewed by Small (1992).

One author, Henderson (1981; 1985), uses instead a supply model which applies a conventional static speed-flow curve to each cohort of travelers. Chu (1995) demonstrates that it is essential to define that cohort by their arrival time at the end of the bottleneck rather than by their departure times from home; otherwise, anomalous possibilities occur and equilibria do not exist. Chu also shows that the Vickrey bottleneck model appears as a limiting case of the Henderson model in which the speed-flow curve becomes kinked, so that the elasticity of travel time with respect to traffic volume is zero for volumes less than capacity and becomes infinite at volumes equal to capacity.

Chu (1993) investigates equilibrium behavior in a model with a Henderson-type supply side, and in which the simple deterministic demand-side specifications just discussed are replaced by a discrete-choice model of scheduling very similar to that of Small (1982). Here, we extend Chu's model to incorporate reliability. This is done by making two changes. First, we substitute the demand model estimated from Koskenoja's data for that used by Chu. Second, we make capacity depend on the occurence, severity and duration of random incidents. In this way travel times become stochastic; equilibration then occurs with people responding to the entire distribution of travel times.

## 3. THEORETICAL DEMAND MODEL

We begin with a theory of scheduling costs under uncertain travel times. As in Noland and Small (1995), we build upon prior work by Gaver (1968), Polak (1987), and Bates (1990) by postulating a cost function for a commuter with a particular preferred arrival time at work, which empirically is taken to be the official work start time. This scheduling cost function, $C_{\mathrm{S}}$, is that of Small (1982):

$$
\begin{equation*}
\mathrm{C}_{\mathrm{S}}=\alpha \mathrm{T}+\beta(\mathrm{SDE})+\gamma(\mathrm{SDL})+\theta \mathrm{D}_{\mathrm{L}} \tag{1}
\end{equation*}
$$

where T is travel time, SDE and SDL are schedule delay early and late, respectively, according to the definition in Small (1982), and $\mathrm{D}_{\mathrm{L}}$ is equal to 1 when $\mathrm{SDL}>0$ and 0 otherwise. $\alpha$ is the cost of travel time, $\beta$ and $\jmath$ are the costs per minute of arriving early and late, respectively, and $\theta$ is an additional discrete lateness penalty. ${ }^{2}$

We define three elements of total commute time, T. The first is the free-flow travel time, $\mathrm{T}_{\mathrm{f}}$, which occurs if the highway has no congestion. Next is the minimum extra travel time due to congestion, $\mathrm{T}_{\mathrm{x}}$, also known as recurrent congestion and which we assume the commuter expects will occur daily. Finally, the added time due to non-recurrent (or unpredictable) congestion, due for example to incident-related delays (Lindley, 1987; Schrank et. al, 1993), is defined as $\mathrm{T}_{\mathrm{r}}$, a random variable. ${ }^{3}$ We specify a probability distribution for $\mathrm{T}_{\mathrm{r}}$.

For simplicity we assume the underlying events causing non-recurrent congestion are independent of the amount of recurrent congestion and of the time of day of travel. This simplification is more realistic than many people think. There is a general assumption that as congestion increases, non-recurrent events will also increase. If one considers accidents as a proxy for non-recurrent events then the limited research available indicates that the relationship between increasing traffic levels and accidents is not so simple. Zhou and Sisiopiku (1997)

[^1]suggest that when volume / capacity ratios are low, accident rates are actually higher than under more congested conditions. Shefer and Rietveld (1997) also discuss these effects. Major incidents, while perhaps less frequent when traffic volumes are low, could tend to result in longer delays than more frequent minor incidents. Satterthwaite (1981) reviewed the literature and found no compelling reason to conclude that there is any relationship between accidents and traffic volumes. Our model does produce a correlation between incident delays and congestion because a given reduction in capacity results in a non-linear increase in travel time.

The official work start time, $t_{w}$, is assumed to be exogenous. In contrast, the home departure time, $\mathrm{t}_{\mathrm{h}}$, will be chosen by the commuter. Let us define the maximum early arrival time, $\mathrm{T}_{\mathrm{e}}$, as,

$$
\begin{equation*}
T_{e}=t_{w}-t_{h}-T_{f}-T_{x} \tag{2}
\end{equation*}
$$

This is the "head start" time of Gaver (1968). These definitions enable us to rewrite the cost function as follows:

$$
\begin{equation*}
\mathrm{C}_{\mathrm{S}}\left(\mathrm{~T}_{\mathrm{r}}, \mathrm{~T}_{\mathrm{e}}\right)=\alpha\left[\mathrm{T}_{\mathrm{f}}+\mathrm{T}_{\mathrm{X}}+\mathrm{T}_{\mathrm{r}}\right]+\beta\left(1-\mathrm{D}_{\mathrm{L}}\right)\left[\mathrm{T}_{\mathrm{e}}-\mathrm{T}_{\mathrm{r}}\right]+\gamma \mathrm{D}_{\mathrm{L}}\left[\mathrm{~T}_{\mathrm{r}}-\mathrm{T}_{\mathrm{e}}\right]+\theta \mathrm{D}_{\mathrm{L}} . \tag{3}
\end{equation*}
$$

Note that this cost depends on $T_{e}$, which is subject to the commuter's choice through choice of $t_{h}$ in equation (2); however we postpone describing that choice until section 6.1.

Work start times are explicitly assumed to be exogenous. This simplifying assumption may not be true in all cases as employers and employees may select working arrangements that account for the probability of arriving late. These arrangements, such as flexible working hours, would presumably increase the utility of both employees and employers given existing uncertain travel times. However, they may still be sub-optimal for the employer who would prefer coordinated work schedules for all employees and thus different work schedules would still impose a cost as modeled by Henderson (1985) and measured empirically by Wilson (1989). Our model does not attempt to consider employer costs and how this may affect work start times.

Given a probability distribution for $\mathrm{T}_{\mathrm{r}}$, we can generate expected values for each of the terms in equation (3), thereby determining the expected cost as a function of scheduling choice, $\mathrm{T}_{\mathrm{e}}$ :

$$
\begin{equation*}
\mathrm{EC}_{\mathrm{s}}\left(\mathrm{~T}_{\mathrm{e}}\right)=\alpha E(\mathrm{~T})+\beta E(\mathrm{SDE})+\gamma E(\mathrm{SDL})+\theta \mathrm{P}_{\mathrm{L}}, \tag{4}
\end{equation*}
$$

where $\mathrm{P}_{\mathrm{L}}=E \mathrm{D}_{\mathrm{L}}$ is the probability of arriving late. For example, if $\mathrm{T}_{\mathrm{r}}$ has a uniform distribution over $\left[0, T_{m}\right]$, then equation (4) takes the form,

$$
\begin{align*}
\mathrm{EC}_{\mathrm{S}}\left(\mathrm{~T}_{\mathrm{c}}\right) & =\alpha\left(\mathrm{T}_{\mathrm{f}}+\mathrm{T}_{\mathrm{x}}+\frac{\mathrm{T}_{\mathrm{m}}}{2}\right)+\frac{1}{\mathrm{~T}_{\mathrm{m}}} \int_{0}^{\mathrm{T}_{\mathrm{c}}} \beta\left(\mathrm{~T}_{\mathrm{e}}-\mathrm{T}_{\mathrm{r}}\right) \mathrm{d} \mathrm{~T}_{\mathrm{r}}+\frac{1}{\mathrm{~T}_{\mathrm{m}}} \int_{\mathrm{T}_{\mathrm{c}}}^{\mathrm{T}_{\mathrm{m}}}\left[\gamma\left(\mathrm{~T}_{\mathrm{r}}-\mathrm{T}_{\mathrm{c}}\right)+\theta\right] \mathrm{T}_{\mathrm{r}}  \tag{5}\\
= & \alpha\left[\mathrm{T}_{\mathrm{f}}+\mathrm{T}_{\mathrm{x}}+\frac{\mathrm{T}_{\mathrm{m}}}{2}\right]+\frac{1}{2 \mathrm{~T}_{\mathrm{m}}}\left[\beta \mathrm{~T}_{\mathrm{e}}^{2}+\gamma\left(\mathrm{T}_{\mathrm{m}}-\mathrm{T}_{\mathrm{e}}\right)^{2}\right]+\frac{1}{\mathrm{~T}_{\mathrm{m}}}\left[\theta\left(\mathrm{~T}_{\mathrm{m}}-\mathrm{T}_{\mathrm{c}}\right)\right] \tag{6}
\end{align*}
$$

provided $0 \leq T_{e} \leq T_{m} .{ }^{4}$
An underlying assumption is that people minimize the expected cost of travel. This amounts to assuming no risk aversion, or alternatively assuming that any risk aversion is already incorporated into the coefficients of the schedule-delay variables. Either interpretation is consistent with our empirical methodology for measuring the coefficients.

In this paper, the distribution of $\mathrm{T}_{\mathrm{r}}$ is determined by the possibilities presented in the stated preference survey questions for purposes of estimating the model's parameters; in the simulations of section 6, it is determined as part of the simulations. Here, equation (4) suffices for our purposes.

In addition to producing mis-matched schedules, travel time uncertainty may also impose an inconvenience due to the inability to plan one's activities exactly. We call this "planning cost", $C_{\mathrm{p}}$, and assume it is a function of the standard deviation $S$ of uncertain travel time $\mathrm{T}_{\mathrm{r}}$, with coefficient $\sigma$. Total expected cost is therefore

$$
\begin{align*}
\mathrm{EC}= & E C_{\mathrm{S}}+\mathrm{C}_{\mathrm{p}}  \tag{7}\\
& =\alpha E(\mathrm{~T})+\beta E(\mathrm{SDE})+\gamma E(\mathrm{SDL})+\theta \mathrm{P}_{\mathrm{L}}+\sigma f(S) \tag{8}
\end{align*}
$$

This is the basic model that we estimate in section 5. Our expectation is that $\beta<\alpha<\gamma$ and that all coefficients are positive.

Previous work, such as by Black and Towriss (1993), has considered a reduced form of (8) in which the traveler is allowed to optimize departure time. Noland and Small (1995) show that the parameters $\beta, \gamma$, and $\theta$ on the right-hand side then obtain an optimized value that may be approximated as proportional to the standard deviation, $S$, of travel time. Their effect may

[^2]therefore be subsumed in the last term, and the model specified purely as a tradeoff between mean travel time and its standard deviation. The advantage of our model is that we can estimate the parameters $\beta, \gamma$, and $\theta$, which are key to the shifts that form the heart of our simulation exercises. The disadvantage is that two of the three corresponding variables, namely $S D L$ and $P_{L}$, are unavoidably correlated with each other and with travel-time variability, $S$, in any realistic set of scenarios, making estimation more difficult.

In Noland and Small (1995), we permit head-start $\mathrm{T}_{\mathrm{e}}$ to be chosen to minimize expected scheduling cost, $\mathrm{EC}_{\mathrm{s}}$. However, here we assume instead that head-start is chosen from a random utility model in which disutility is proportional to EC. We find that this leads to similar qualitative behavior, such as a shift toward earlier schedules in response to increased standard deviation of travel time.

## 4. STATED PREFERENCE SURVEY AND DATA COLLECTION

In order to empirically estimate the trade-offs among reliability, mean travel time, and scheduling decisions a stated-preference (SP) survey was administered to a sample of more than 700 commuters in the Los Angeles region who had already taken part in a recent panel study (see Koskenoja (1994) and Small et al. (1995) for more details on the survey methodology). This strategy enabled us to take advantage of information already compiled about employer, work start time, and travel conditions, thereby providing information for calculating schedule delay and also allowing us to customize the survey to provide respondents with realistic situations corresponding to their particular commute. Through extensive follow-up we achieved an 80 percent response rate, ultimately resulting in 543 usable questionnaires. Nine stated preference choices were asked of each respondent; from this we obtained 4340 usable observations on binary choice. Koskenoja (1996) and Small et al. (1995) use the same data to investigate the impact of various socio-economic variables on scheduling choices. Transit ridership is extremely low in the area surveyed, so our results are not confounded by modal choice.

Each SP choice is between two alternative commutes to work, each with a specified distribution of travel times and a specified departure time from home. Departure time is
presented in minutes prior to the "usual arrival time," which was ascertained from a previous question about the commuter's actual situation. A sample question is shown in the Appendix; it is preceded by a number of questions that orient the respondent to the idea of travel-time variability, departure-time decisions, and scheduling constraints. A number of consistency checks suggested to us that respondents understood the SP question. ${ }^{5}$ Black and Towriss (1993) also tested different formulations and decided that this type of SP question was understood by respondents. The empirical demand model reported in the next section is estimated from the answers to the SP questions.

The question format is a compromise between the need to describe a travel time distribution that would be realistic to the respondent, and the need to keep the question simple enough to be understood. Based on the experience of Black and Towriss (1993), who studied different question formats with this tradeoff in mind, the travel time distribution in the current study is described as a five point discrete distribution, where each possible travel time has an equal probability. The possible travel times were determined by choosing a log-normal distribution with a given mean and standard deviation, then picking the 1 st, $3 \mathrm{rd}, 5 \mathrm{th}, 7 \mathrm{th}$, and 9 th decile points, each rounded to the nearest minute. The mean and standard deviation were chosen to be larger for those commuters whose current actual travel time was longer.

To represent a travel time distribution as a discrete distribution is clearly a simplification. Two aspects could be problematic. First, it restricts the domain of the probability distribution, creating an artificial certainty as to the maximum possible delay that could occur. Second, one may mis-represent the skewness of the underlying distribution by showing only five points. To counteract any hidden skewness effects, all the sets of travel times we presented to respondents are derived from distributions with the same skewness, which means we cannot study the effects of third or higher moments of the travel time distribution.

Since the respondent was shown only the five rounded travel times, all our subsequent computations of means, standard deviation, and expectations of derived quantities are based only on the discrete distribution.

[^3]Many potential sources of bias or inaccuracy have been identified in stated preference questions (Bonsall, 1985; Bradley and Kroes, 1990). In order to reduce some of these problems the questions were designed to be realistic and relevant to the respondents. For example, the distributions presented were customized so as not to deviate too far from the respondent's current mean travel time. In order to avoid "political bias," the questions were designed as abstract alternatives with no obvious way to promote any particular political philosophy through the answer. Finally, following a pilot study in which questions about tolls elicited responses that were clearly political statements, the price attribute was dropped from the design; this means that we can measure ratios of cost coefficients but not the actual costs.

Our design strategy for the independent variables was to define three levels (low, medium, high) for each of the three quantities subject to independent control: mean travel time and the standard deviation of the underlying log-normal distribution of travel times, and expected arrival time. This design results in 27 possible combinations of independent variables for each alternative. SP surveys allow one to minimize the correlations among the independent variables by design, although as previously noted we cannot prevent correlation among SDL, $\mathrm{P}_{\mathrm{L}}$, and S . Hensher and Barnard (1990) note that a factorial design drawn from all combinations of attribute levels will contain many pairs for which one alternative dominates another on each attribute. The dominated choice will not be informative and could possibly bore the respondent. Therefore, in forming our choice sets, we selected from the 27 possible combinations the largest possible subset within which no alternative dominates another. These seven attribute combinations are shown in Table 1. Randomly drawn pairs of these seven combinations were assigned to each individual to create nine repeated questions (but respondents were not presented the same pair twice).

The actual values corresponding to low, medium, or high in forming these choice sets were different for each of five different groups of respondents, based on usual commute time from home to work. The three departure times were chosen to set expected arrival time at work equal to the usual arrival time minus zero, one, or two standard deviations of the log-normal distribution governing the travel-time distribution. For a complete listing of all possible combinations used in the survey see Table 4-3 of Small et al. (1995).

## 5. EMPIRICAL DEMAND MODEL

### 5.1 Analysis Methodology

In order to estimate equation (8) empirically, we assume that our survey respondents choose among the two alternatives A and B that are presented to them by combining their attributes as in (8), adding a random error term, and choosing the alternative with the lower cost. Thus (8) serves as the "systematic utility" in a discrete choice model (Ben-Akiva and Lerman, 1985). With an appropriate distribution for the random errors, the resulting choice model is binary logit, which can be estimated by maximum likelihood using standard software.

Stated preference designs are well suited to discrete choice binary logit models; the design provides the survey respondent with an actual choice of two hypothetical situations. One drawback in analyzing SP surveys is that we use a repeated measures approach (nine pairs of choices were presented to each individual) to obtain a large number of usable observations. This can result in a downward bias in estimated standard errors (Ouwersloot and Rietveld, 1996). Such bias results from the existence of unobserved preferences for one alternative over another that would characterize all the repeated observations from a given survey respondent. However, in our case the alternatives presented are abstract, literally choice " $A$ " versus choice " $B$ ". The specified characteristics are varied so that there is no reason to think a given respondent would systematically prefer alternative "A" over alternative "B". Alternative estimation methods, such as that in Ouwersloot and Rietveld (1996), or ad-hoc adjustments to the standard error as proposed by Louviere and Woodworth (1983), are inapplicable and unnecessary here, since any bias should be minor. ${ }^{6}$

To correspond with our analytical model (equation 8) we need to determine the expected schedule delay (both early and late) and the lateness probability from these questions. Schedule delay, early and late, as used in our model were based on the definition in Small (1982). We modified that framework slightly in order to match the format of the question shown in the Appendix, in which we used the words "usual arrival time" instead of "official work start time"

[^4]as the basis for representing people's most preferred arrival time. We did this to avoid having to make elaborate descriptions of how to count time in the elevator, walking through the office, and so forth. The two times coincide for 56 percent of the respondents who answered both questions (Small et al., 1995).

The stated-preference question format specifies "departure $\left[\mathrm{T}_{\mathrm{a}}\right]$ minutes before your usual arrival time", where a specific number is inserted for $T_{a}$; that number is therefore taken to be $a$ measure of $t_{w}-t_{h}$ in our notation above. (The notation $T_{a}$ indicates minutes ahead of desired arrival time.) The definition of early and late schedule delay can be formally defined as:

$$
\begin{align*}
& \operatorname{SDL}_{\mathrm{i}}= \begin{cases}\mathrm{T}_{\mathrm{a}}-\mathrm{T}_{\mathrm{i}}, & \text { if }>0, \\
0 & \text { otherwise } ;\end{cases}  \tag{9}\\
& \operatorname{SDE}_{\mathrm{i}}= \begin{cases}\mathrm{T}_{\mathrm{i}}-\mathrm{T}_{\mathrm{a}}, & \text { if }>0, \\
0 & \text { otherwise } .\end{cases} \tag{10}
\end{align*}
$$

$T_{a}$ and the five values of $T_{i}$ are stated directly in the question. The expectations of T, SDE and SDL are derived by averaging, over the five possible values. For example, using the sample question in Figure 1, for choice A we have three possibilities of arriving early since the departure time is 15 minutes before usual arrival time, (i.e. $\mathrm{T}_{\mathrm{a}}$ is 15 minutes) and travel times $\left(\mathrm{T}_{\mathrm{i}}\right)$ are 12 , $13,14,16$, and 20. In three cases one can arrive early, by 3,2 , and 1 minute. Therefore to calculate $\mathrm{E}(\mathrm{SDE})$ we sum the early arrivals $(3+2+1+0+0)$ and divide by 5 to get a value of $\mathrm{E}(\mathrm{SDE})$ equal to $6 / 5=1.2$ minutes. In choice $\mathrm{B}, \mathrm{E}(\mathrm{SDE})$ is 1.8 minutes. $\mathrm{E}(\mathrm{SDL})$ is calculated in a similar manner and would be 1.2 minutes in choice $A$ and 2 minutes in choice $B$. $E(T)$ is 15 minutes for choice A and 10.2 minutes for choice B .

The lateness probability is determined discretely by counting the number of possible travel times that will result in a late arrival and dividing by 5 . Using the sample question in Figure 1, choice A has 2 possibilities of arriving late ( 16 and 20 minute travel times) which result in a $40 \%$ lateness probability $\left(P_{L}=0.40\right)$. Choice $B$ also $P_{L}=0.40$. The design of the SP questions provided only three discrete levels of lateness probability: $0 \%, 20 \%$, and $40 \%$.

The standard deviation of the travel time is defined in the usual way as the square root of the variance, which in turn is the sum of five terms $\left[\mathrm{T}_{\mathrm{i}}-\mathrm{E}(\mathrm{T})\right]^{2}$ divided by 5 . In the sample question in the Appendix the standard deviation is 2.83 for choice A and 4.53 for choice B.

### 5.2 Estimation Results

Our first step in analyzing the results is to estimate a simple model that contains only the trade-off between mean travel time and the standard deviation of travel time. That is, we estimate equation (8) omitting the middle three terms on the right-hand side. This model could be regarded as a reduced-form of model (8) after the traveler is allowed to optimize over departure time; it would be appropriate if one believed that our respondents ignored the departure time specified in our question and instead chose their own preferred departure time. The result is shown in Table 2, column 1. Both attributes are highly significant in explaining choice and both estimated coefficients have the expected negative sign (i.e., the larger the travel time and/or the standard deviation, the less desirable the alternative). For comparison, column 2 shows the results of Black and Towriss (1993) for this same specification. They also include money in their estimations.

A useful comparison is the ratio of the coefficient of standard deviation to that of $E(T)$. We find the ratio to be 1.27 while Black and Towriss have a ratio of 0.55 . Thus our estimation indicates that each minute of standard deviation is $27 \%$ more costly than each minute of mean travel time. One of the key differences between our study and Black and Towriss is that they did not specify the head start time as we do; therefore it is natural that they found people less averse to travel time variation because in their survey people can anticipate its effects by adjusting their schedules. We can also take that into account, but to do so we need to estimate the full model of equation (8).

Table 3, column 1, shows the result of estimating the full model of equation (8) with planning cost assumed proportional to the standard deviation of travel time. Planning cost has a positive and significant coefficient, which is contrary to the theory. The coefficient for the mean travel time is also less than that for $\mathrm{E}(\mathrm{SDE})$, which implies that people prefer to be in traffic than to arrive early; this also seems implausible. As an alternative we specify planning cost as proportional to the coefficient of variation (standard deviation divided by mean travel time) and find that while this is statistically insignificant, it does have the appropriate sign. This is shown in Table 3, column 2, and henceforth will be referred to as our "Basic Model".

All coefficients in the basic model have the expected negative signs and acceptable relative magnitudes: $\mathrm{E}(\mathrm{SDE})$ is less onerous than $\mathrm{E}(\mathrm{T})$ which is less so than $\mathrm{E}(\mathrm{SDL})$. However, it is surprising that the coefficient of $\mathrm{E}(\mathrm{SDL})$ was not much greater than that of $\mathrm{E}(\mathrm{T})$. All four of the coefficients just mentioned are statistically significant.

It is possible that $\mathrm{E}(\mathrm{SDL}), \mathrm{P}_{\mathrm{L}}$, and the coefficient of variation are too highly correlated to distinguish their effects separately. Column 3 (Table 3) shows that when $P_{L}$ is removed the coefficients of the other two increase, and the coefficient of variation becomes significant.

When the coefficient of variation is removed from the model (Table 3, column 4) the other coefficients do not noticeably change. The coefficient on lateness probability does increase slightly, indicating that it is picking up some of the explanatory power of the coefficient of variation. This result seems to suggest that our hypothesized "planning cost" is not as important a variable in the commuters' choice as the other variables. Alternatively, we may not have specified an appropriate functional form for planning cost. In any case, it appears that much of the uncertainty inherent in unreliable commuting trips is better explained by the schedule delay and lateness probability variables.

The relative importance of the schedule delay variables with respect to travel time was first detected by Small (1982). We present his most comparable model in column 5. This model contained no uncertainty in travel time, and therefore no planning cost.

The bottom of Table 3 shows the ratios of the coefficients for $\mathrm{E}(\mathrm{SDE})$ and $\mathrm{E}(\mathrm{SDL})$ relative to that of travel time for each model. As can be seen, other than for the model in column 1, we get results similar to Small's. Column 1 has very high ratios, but this probably reflects a mis-specification. These results are generally encouraging and seem to indicate that the respondents to the questionnaire interpreted the trade-offs in the SP questions appropriately.

These models show that the components of the scheduling cost, $\mathrm{C}_{\mathrm{s}}$, in equation (8) are important determinants of the travel choices individuals make. We believe these are the underlying factors behind the aversion to travel time uncertainty found by other researchers, such as Black and Towriss (1993). Whether planning cost is a significant factor when scheduling variables are taken into account remains unproven due to the statistical insignificance of the
coefficient of variation in our basic model. It may be that it is a lesser factor whose importance is too small for us to measure.

The model with the highest $\bar{\rho}^{2}$ (and the largest likelihood function) which also matches our theoretical formulation is the "basic model" in column 2 of Table 3; for this reason we utilize this model in the simulations presented next.. The low value of $\bar{\rho}^{2}$ may indicate that we are not capturing all the variables that could explain an individual's choice, such as socioeconomic variables.

## 6. SIMULATIONS WITH ENDOGENOUS CONGESTION

We now combine the analysis of the basic demand model of Table 3, column 2, with a supply side model of a congested highway corridor. Our purpose is to simulate the effect of nonrecurrent events on actual congestion patterns, taking account of people's rescheduling decisions. This procedure will allow us to examine scheduling shifts due to exogenous changes such as a reduction in incident probabilities or an expansion of capacity. We also examine the resulting changes in the expected cost to commuters and its components.

First we discuss the basic simulation procedure and methodology. We then briefly present the travel conditions generated by the simulations. This is followed by an analysis of the pattern of scheduling shifts and the relative components of total travel costs.

### 6.1 Simulation Methodology

Our simulation model, adapted from that of Chu (1993), is an iterative algorithm combining the discrete-choice demand model discussed above with a simple supply model based on a standard relationship between volume-capacity ratio and travel time. We represent incidents directly by postulating randomly occurring reduction in capacity of specified levels and duration. The probability of an incident occurring during a given time interval is a key exogenous parameter which, when varied, allows us to investigate how the behavior of the system changes as incidents become more or less frequent, for example due to changes in vehicle safety inspection regulations, driving habits, or methods of clearing incidents. We also treat
capacity as a variable parameter so we can compare the effects of policies to expand capacity with policies to reduce incidents.

The demand model takes as given the entire distribution of travel times facing a commuter entering the hypothetical highway. It is applied to a synthetic sample of 5000 individuals, predicting for each the probability of choosing any of eleven possible departure times. Each individual is asigned a "work start time", $t_{w}$, which is really to be thought of as the time that individual would need to exit from the highway in order to arrive at work on time; it is drawn randomly from a normal distribution with mean equal to 8:00 a.m. and standard deviation equal to 60 minutes.

Although our respondents faced only two choices in the SP questions, we assume that when faced with a larger choice set they would apply a multinomial logit choice rule to that larger set using the same estimated utility function. In our simulation there are eleven choices, with expected schedule delay (the difference between actual and desired exit time from the highway) ranging from -20 to +20 minutes with intermediate values $\pm 15, \pm 10, \pm 5, \pm 3$, and 0 .

For each member of the synthetic sample, the demand model determines the probabilities of each of the eleven possible values of expected schedule delay, $E(\mathrm{SD})$. The probability that a given individual will travel during a specified time interval is then calculated by summing each schedule delay interval with that individual's unique "work start" time. This gives us for each individual the probability of travelling during a given time slot. For example, if one individual has a "work start" time of 8:35 a.m. the probability that $E(\mathrm{SD})$ is -20 minutes is equivalent to the probability that this individual travels in the time interval between 8:10 and 8:20 am. Sample enumeration (as described in Ben-Akiva and Lerman, 1985), which consists of summing the choice probabilities for each individual in the synthetic sample, allows us to determine the estimated traffic volume for each 10 minute time slot.

Our supply model applies the following simple speed-flow relationship to each time slot:

$$
\begin{equation*}
\mathrm{T}=\ell \cdot\left[\mathrm{T}^{0}+\mathrm{T}^{1}\left(\frac{\mathrm{~V}}{\mathrm{C}}\right)^{\varepsilon}\right] \tag{11}
\end{equation*}
$$

where T is the travel time in minutes, V is the number of vehicles leaving the highway per hour, C is the capacity of the facility, $\varepsilon$ is the elasticity parameter, $l$ is the length of the facility
(assumed to be equal to 5 miles), and $\mathrm{T}^{0}$ and $\mathrm{T}^{1}$ are constants. The supply model of equation (11) has a long history in transportation engineering and economics, dating back at least to the U.S. Bureau of Public Roads (1964). It was incorporated into the Urban Transportation Planning Process computer software used widely in the U.S. (Branston, 1976, p. 230) and has also been used in many economic models of congestion including Vickrey (1963), Mohring (1979), and Kraus (1981), with values of $\varepsilon$ ranging from 2.5 to 5 . Small (1992, pp. 70-73) finds that equation (11) fits quite well the data from a dynamic simulation of city streets in Toronto (with $\varepsilon=4.08$ ) and the data from an aggregate analysis of Boston express roads (with $\varepsilon=3.27$ ). Since the precise function is less important for our purposes than its general ability to measure rapidly increasing congestion, we forego an extensive empirical estimation and simply used the parameters of U.S. Bureau of Public Roads (1964), namely: $\varepsilon=4$ and $T^{1} / T^{0}=0.15$. The value of $\mathrm{T}^{0}$ is immaterial to our model; for convenience we set $\mathrm{T}^{0}=1.0$ minutes $/$ mile corresponding to a free-flow speed of 60 miles per hour. ${ }^{7}$

This supply model was first used to model endogenous scheduling equilibria by Henderson (1981; 1985). Chu (1995) shows that in order to assure equilibria and avoid "overtaking" situations (where an individual can depart later than another yet arrive earlier), one must assume that volume, V , in equation (11) is calculated at the point where the flow leaves the highway. The capacity is assumed equal to 1200 vehicles/hour except for random reductions due to incidents. ${ }^{8}$ It is these random capacity reductions that make T stochastic.

We assume that the probability of an incident is the same for every 10 minute increment of clock time. We also assume that each incident is independent of other incidents, except that for simplicity, we assume that only one incident can occur within a given time interval, and that no additional incidents occur during the time when the capacity is reduced. The probability of a

[^5]capacity reduction is assumed independent of traffic volume; although, as we will show, the resulting standard deviation of travel times varies and is higher over the peak period.

Three levels of incident severity were defined, given that an incident had occurred: namely capacity reductions of $50 \%, 30 \%$, and $10 \%$, occurring with conditional probabilities of $10 \%, 20 \%$, and $70 \%$, respectively. ${ }^{9}$

The duration of each reduction in capacity must also be specified. Incident durations have been determined to occur with a log-normal distribution (Giuliano, 1989; Golob et al., 1987). For simplicity, we instead set three levels based on the clock time intervals. The probability that the incident lasted for only 10 minutes ( 1 interval) was set to $50 \%$, for 20 minutes ( 2 intervals), $30 \%$, and for 30 minutes ( 3 intervals) $20 \%$. These durations imply that for a given 10 minute interval the probability of there being some capacity reduction is equal to 1.7 times the specified incident probability. Therefore, variation in incident durations are essentially equivalent to variations in the incident probability for a given time interval.

For the sake of exposition, we calculate the distribution of travel time values assuming 10,000 trips. This allows us to calculate travel time values given a range of incident probabilities and the specified severity probabilities. For example, if we assume a $2 \%$ incident probability, then there would be 200 trips with some reduction in capacity and $200 \times 1.7=340$ time intervals with a reduction in capacity. If for each incident there was a $50 \%$ chance of a $10 \%$ reduction in capacity, then 100 trips would have capacity reduced by this amount in the calculation of the travel time using the speed-flow equation (11) above. This was done for each 10 minute clock interval, resulting in a complete description of the the travel time distribution for that time interval, including the mean travel time and the standard deviation of travel time.

This distribution was then fed back into the demand model (see Figure 1 for a flowchart of the simulation process). This allows us to calculate a new distribution of expected schedule delays for each individual. The demand model also uses the ratio of standard deviation to the mean travel time, the latter also includes the free flow travel time for each individual. From this the demand model allocates each individual stochastically to a clock time interval and we enumerate over the entire synthetic sample. This process continues until the number of

[^6]individuals assigned to each time interval by the demand model remains essentially constant (or changes by a very small amount) from one iteration to the next. After convergence is achieved we evaluate the congestion profile, the average travel delay, and the total cost.

### 6.2 Travel Conditions Generated by Simulations

The travel conditions generated by the simulations are a function of our assumptions about incident probability levels, the severity of those incidents, the probability of a given level of severity occurring, and the incident duration. The travel conditions represent the equilibrium level of the system. Here we review these values and briefly discuss their realism.

Average travel times vary with the "work start" time. This results in a peak travel time between 11 and 15 minutes, depending on the probability of an incident occurring and an offpeak travel time of 5 minutes. Off-peak times do not vary because the capacity reduction does not result in any congestion during the off-peak periods. The travel delay, which occurs only because of non-recurrent congestion, ranges from peaks of about 2.5 minutes up to 5.5 minutes in our simulations.

The standard deviation of travel time and the coefficient of variation also vary over the peak. The maximum standard deviation and coefficient of variation occur at the most congested time. This is because any reduction in capacity at this time will have a much greater impact on travel times than a capacity reduction when traffic volumes are small. Incidents during off-peak hours will not have any impact on travel times since there is ample capacity, even after an incident causes a reduction in capacity. The increase in standard deviation (and the coefficient of variation) over the peak occurs despite modeling a constant incident probability for each clock interval. The coefficient of variation ranges up to about 0.14 which matches empirical measurements ranging from 0.08 to 0.2 as reported by Bates (1990).

The probability of arriving at work late for any given choice of schedule shows an expected pattern. As the probability of a capacity reduction increases, lateness probability increases. The simulations generate a maximum lateness probability of slightly over $40 \%$ in the on-time case (with a $25 \%$ incident probability); this is a good match to the range of lateness probabilities in our stated preference questions which had three levels of $0 \%, 20 \%$, and $40 \%$.

### 6.3 Travel Delay and Scheduling Shifts from Incident Reduction and Capacity Expansion

Reductions in non-recurrent delay (expressed as incident probabilities) can decrease average travel times. Increases in capacity can have a similar effect. Both may also have an impact on scheduling choices which may reduce the benefits of reductions in peak travel time by allowing more commuters to travel during peak hours. There may be reductions in scheduling costs associated with any shift to the peak.

Average travel delay is reduced as the incident probability decreases. Both total delay and delay due only to non-recurrent congestion increase with increasing incident probability. For a capacity level of 1200 , average travel time increases from about 2.2 minutes to about 4.5 minutes as the probability of an incident goes from $0 \%$ to $25 \%$. Obviously, policies that reduce the probability of an incident blocking capacity will result in a decrease in average travel times.

Figure 2 shows the effect on average travel delay of varying highway capacity. For an incident probability level of $20 \%$, increasing the capacity from 1200 vehicles per hour to about 1400 vehicles per hour reduces average delay by about 2.5 minutes per vehicle. This is comparable to the average travel time savings from eliminating incidents. While we don't know what the costs of reducing the incident probability would be, we do know that freeway capacity expansions are generally very costly. Therefore, if reducing travel delay is the only objective, this shows the relative trade-offs of two possible alternative strategies for reducing delay.

Scheduling costs involved in commuting decisions may be as important as travel time costs. Figure 3 shows that reducing the probability of an incident results in significant shifts in schedules: many commuters who previously planned to arrive early or late now choose to instead arrive at their desired work start time. Out of 5000 commuters, about 400 more choose to arrive with schedule delay of zero when incident probabilities are zero, compared to an incident probability level of $25 \%$. For comparison, a simulation using the values calculated by Small (1982) that does not account for variance in travel times resulted in a larger shift to earlier travel periods than the current model. This can probably be attributed to the relatively larger coefficient for schedule delay late in Small's model (see Table 3, column 5). Other than this difference the scheduling shifts of the current model show the same basic pattern.

Such shifts do not occur as a result of increasing capacity. Figure 4 shows the difference in schedule delay choices between capacity levels 1200 and 2400 for each incident probability level. The greatest shift occurs with an incident probability of $25 \%$, with a very small increase of about 50 (out of 5000 ) commuters choosing to arrive with no schedule delay. Increasing capacity does enable commuters to better fine-tune their schedules, but the effect is small even with a very large capacity increase.

Despite the scheduling benefits of incident reductions, the overall congestion profile does not really change. There is a slight increase in peak travel when there are no incidents, but it is essentially negligible and will have only a minor impact on increasing average travel delays; therefore, the scheduling cost reductions do not seem to be off-set by significantly more congestion at the peak.

### 6.4 Components of Total Travel Costs

The expected travel costs can be calculated using the demand model (Table 3, column 2) and the equilibrium travel conditions generated by the simulations. These are calculated for different incident probability levels and different capacity levels.

Table 4 shows the average total cost per trip which increases as the incident probability increases. The percent contribution from each component is also shown. These components are expected travel time, schedule delay early and late, lateness probability, and the "planning cost" (specified as proportional to the coefficient of variation of travel time). Schedule delay early costs are the largest cost component at small incident probabilities, but their proportion decreases with increasing incident probability. When incident probability is high, expected travel time costs become dominant because of the high level of non-recurrent congestion. The "planning cost" is relatively minor, showing an expected increase with increasing incident probability. The costs associated with the probability of arriving late also increase. The major reduction in total costs with decreasing probability of an incident can be attributed to decreases in costs of schedule delay early and lateness probability.

Table 5 shows a similar breakdown for simulations with increasing levels of capacity. Total costs decrease by about the same amount when capacity is doubled from 1200 to 2400 as
in the case when incident probabilities are reduced from $25 \%$ to $0 \%$. The source of the decrease in costs is, however, different. When capacity is increased the cost reduction comes overwhelmingly from reductions in the travel time costs associated with both recurrent and nonrecurrent congestion. Scheduling costs and lateness probability remain essentially the same. The "planning cost" is again negligible.

These cost calculations are averages over all the clock intervals of the simulations. Those choosing to travel at "peak" periods near 8:00 a.m., due perhaps to job or other constraints, will face higher total costs than those traveling at other hours. The relative contribution of the various components will also differ. Table 6 shows the cost components for a particular 10 minute period far from the peak, namely the clock interval 6:35 a.m. to 6:45 a.m.. Travel time costs due to non-recurrent capacity reductions are negligible. There is also very little variation in total costs as the incident probability increases. Most of the increase is due to the costs of the probability of arriving late increasing.

During a peak period, such as 7:55-8:05 a.m., travel time costs are a much larger fraction of the total costs, which increase significantly as the incident probability increases (see Table 7). Lateness probability costs also show an increase while scheduling costs do not change much and their total percent contribution decreases.

## 7. CONCLUSIONS

Our model provides a way to simulate complex interactions among scheduling choices, reliability of travel times, and congestion. It allows for heterogeneity of travelers, considerable complexity in the determinants of scheduling choices, and a variety of underlying processes for generating randomness in travel times. At the same time, the actual choices made and the resulting travel conditions are fully endogenous, though we do not endogenize the response of employers to congestion and how work schedules may change.

As expected, people shift their schedules earlier in response to increases in travel-time dispersion. The main motivation for doing this is to offset the increased probability of being late that would otherwise result from less predictability in travel time. This shift is not very large, and it does not seem to appreciably change the extent of peaking, so the amount of congestion on
those days when travel time is low (i.e., when recurring congestion is present) is barely affected. Our analysis did not consider how providing information to commuters may change their decision making but it does implicitly assume that commuters are responding to known information on average conditions. Noland (forthcoming) examines the ex-post costs of having or not having information.

Scheduling accounts for an important part of the costs of congestion and of unreliability. As the probability of an incident is increased in our model, commuters' total travel costs increase. Nearly half the increase ( 44 percent) is due to the extra travel time due to incidents, and almost as much ( 37 percent) is due to the extra probability of late arrival at work; the remaining 15 percent is due to other scheduling considerations such as spending more time at work before work begins.

Once the costs of non-optimal schedules are taken into account, uncertainty in travel time has only a very small additional cost. "Planning cost" or residual pure cost of uncertainty accounts for only 4.5 percent of the rise in costs as incident probability is increased. Therefore we are able to explain most of people's aversion to uncertainty in terms of costs of early or late arrival. This is an important finding because earlier studies measuring people's aversion to uncertainty have not distinguished among the causes.

The importance of lateness probability and schedule delay-late in our results suggests that one way to reduce the costs associated with unreliability is to encourage more flexible work schedules. Late arrival and adherence to strict schedules seem to be the greatest source of stress related to travel time unreliability (Small et al., 1995). Many occupations may require employees to have coordinated schedules; this is obviously dependent on the nature of the specific business or professional activity. It is therefore not advisable to mandate the loosening of strict work schedules, but it may be advantageous to encourage it.

While our analysis focuses on commuter travel behavior, it has implications for the freight industry and for service industries that rely on transportation during peak times. Productivity impacts on these sectors due to unreliability are probably large and it is likely that investment decisions are made based upon existing travel conditions. Additional research in this area is certainly warranted.

We have shown how reducing the probability of incidents and non-recurrent congestion can affect the costs of schedule delay. Policies aimed at reducing incident probability may be more effective at reducing social costs than policies increasing capacity. One area where better research is needed is exactly how traffic volumes and capacities affect the probability of incidents, their severity, and their duration. Furthermore, methodologies for determining the costs of specific policies and how they reduce travel time variance are needed in order to perform analysis of the costs and benefits of alternative methods for decreasing non-recurrent congestion. For example, what would be the effects on travel time variance, travel costs, and traveler benefits of a capacity expansion relative to a freeway service patrol that detects and removes incidents rapidly? (See Skabardonis, et al., (1995), for a detailed evaluation of Freeway Service Patrols in the San Francisco Bay Area).

We found that our hypothesized "planning cost" did not seem to account for a large fraction of the total costs of unreliable travel. However, if advanced traveler information systems become widely available, these costs could be affected because people will need to plan in order to use them. Future research could determine whether the benefits of these systems will exceed both the monetary costs and planning costs of using them.

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## APPENDIX

## SAMPLE STATED PREFERENCE QUESTION

Below are nine pairs of scenarios for your usual morning commute. In these scenarios you do not know what the exact travel time will be, but you know it will be one of the five listed travel times (each has an equal chance). The departure time is expressed in minutes before your usual arrival time at the work place. You can refer to question 21 for your usual arrival time.

Please consider how you feel about the time spent at home and in traffic, and how early or late you feel comfortable arriving at your work place. Then look at each pair and circle either A or B as the alternative you would most likely choose. It is possible that neither one of the alternatives describes your commuting in real life. In such a case circle the alternative you would be most likely to choose if you had no other alternatives.

$$
1^{\text {st }} \text { pair of scenarios }
$$

| Time | : minutes |  |  |  |
| :--- | :--- | :--- | :--- | :--- |
| 12 | 13 | 14 | 16 | 20 |


| Time |  |  | : minutes |  |
| :--- | :--- | :--- | :--- | :--- |
| 5 | 7 | 9 | 12 | 18 |

```
Departure 15 minutes
before your usual
arrival time
```

Departure 10 minutes before your usual arrival time

Please circle
your choice: A
B

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TABLE 1

Design of Attribute Levels for SP Questions

| mean travel time | Standard deviation of <br> travel time | departure time |
| :--- | :--- | :--- |
| high | medium | low |
| medium | high | low |
| high | low | medium |
| medium | medium | medium |
| low | high | medium |
| medium | low | high |
| low | medium | high |

TABLE 2

Simple Model Compared with Black and Towriss Model

|  | Simple <br> Model | Black and <br> Towriss <br> model (cars <br> only) (1993) |
| :---: | :---: | :---: |
|  | $(1)$ | $(2)$ |
| E(travel time) | -0.0996 | -0.0635 |
| T-Stat |  |  |
|  | $(-17.517)$ | $(-8.90)$ |
| Standard deviation | -0.1263 | -0.0352 |
| T-Stat | $(-12.669)$ | $(-3.17)$ |
| Money | - | -0.0082 |
| T-Stat | - | $(-6.34)$ |
| N | - | - |
| $L(\beta)$ | -2340 | - |
| $\bar{\rho}^{2}$ | 0.0598 | - |

Note: $\quad$ The measure of fitness was computed as $\bar{\rho}^{2}=1-(L(\beta)-K) / L(0)$, where K equals the number of estimated parameters, $L(\beta)$ is the log-likelihood value evaluated at the estimated parameters, and $L(0)=-3008.3$ is the log-likelihood value evaluated setting all coefficients equal to zero. Sample size is equal to N above..

TABLE 3
Results of Model Estimations

|  | with standard deviation <br> (1) | Basic Model | without lateness probability (3) | without coefficient of variation (4) | Small (1982), model $1^{*}$ <br> (5) |
| :---: | :---: | :---: | :---: | :---: | :---: |
| E(travel time) | -0.0556 | -0.1051 | -0.1285 | -0.0976 | -0.106 |
| T-Stat | (-4.656) | (-10.148) | (-15.451) | (-11.052) | (-2.79) |
| E(SDE) | -0.1311 | -0.0931 | -0.0966 | -0.0945 | -. 065 |
| T-Stat | (-11.386) | (-10.606) | (-11.004) | (-10.854) | (-9.29) |
| E(SDL) | -0.3036 | -0.1299 | -0.2807 | -0.1280 | -0.254 |
| T-Stat | (-5.085) | (-2.694) | (-10.594) | (-2.656) | (-8.47) |
| Lateness probability | -2.564 | -1.3466 | - | -1.529 | -0.58 |
| T-Stat | (-6.426) | (-3.704) | - | (-4.495) | (-2.76) |
| Coef. of variation |  | -0.3463 | -0.6674 | - | - |
| T-Stat |  | (-1.403) | (-2.908) | - | - |
|  | - |  |  |  |  |
| Standard deviation | 0.1510 | - | - | - | - |
| T-Stat | (5.098) | - | - | - | - |
| $\mathrm{N}=4340$ |  |  |  |  |  |
| $L(\beta)$ | -2747.3 | -2759.6 | -2766.5 | -2760.6 | - |
| $\bar{\rho}^{2}$ | 0.0851 | 0.0810 | 0.0790 | 0.0810 | - |
| Coefficient Ratio (eqn. 8)s |  |  |  |  |  |
| $\beta / \alpha$ | 2.36 | 0.89 | 0.75 | 0.97 | 0.61 |
| $\gamma / \alpha$ | 5.46 | 1.24 | 2.18 | 1.31 | 2.40 |

[^7]TABLE 4
Components of Total Cost by Incident Probability (capacity = 1200)

| Incident Probability | Average Cost (\$/trip) | Expected <br> Travel Time | $E$ (SDE) | E(SDL) | Lateness probability | Planning Cost |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | \$1.51 | 27.92\% | 43.88\% | 10.91\% | 17.29\% | 0.00\% |
| 0.1 | \$1.89 | 30.83\% | 36.86\% | 9.51\% | 21.67\% | 1.13\% |
| 0.15 | \$2.07 | 32.00\% | 34.54\% | 9.08\% | 22.97\% | 1.41\% |
| 0.2 | \$2.24 | 33.04\% | 32.68\% | 8.75\% | 23.96\% | 1.57\% |
| 0.25 | \$2.39 | 33.99\% | 31.17\% | 8.50\% | 24.69\% | 1.65\% |
| Difference between highest \& lowest incident probabilities | \$0.88 | 44.33\% | 9.52\% | 4.39\% | 37.30\% | 4.46\% |

TABLE 5

## Components of Total Cost by Capacity (Incident Probability $=\mathbf{0 . 2}$ )

| Capacity <br> (vehicles/hr) | Average <br> Cost <br> (\$/trip) | Expected <br> Travel <br> Time | $E$ (SDE) | $E$ (SDL) | Lateness <br> probability | Planning <br> Cost |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 1200 | $\$ 2.24$ | $33.04 \%$ | $32.68 \%$ | $8.75 \%$ | $23.96 \%$ | $1.57 \%$ |
| 1500 | $\$ 1.76$ | $18.08 \%$ | $41.67 \%$ | $10.75 \%$ | $28.49 \%$ | $1.02 \%$ |
| 1800 | $\$ 1.56$ | $10.10 \%$ | $47.20 \%$ | $11.90 \%$ | $30.18 \%$ | $0.61 \%$ |
| 2100 | $\$ 1.46$ | $5.90 \%$ | $50.37 \%$ | $12.55 \%$ | $30.81 \%$ | $0.37 \%$ |
| 2400 | $\$ 1.41$ | $3.60 \%$ | $52.03 \%$ | $12.91 \%$ | $31.23 \%$ | $0.23 \%$ |
| Difference between | $(\$ 0.83)$ | $83.68 \%$ | $-0.61 \%$ | $1.61 \%$ | $11.44 \%$ | $3.88 \%$ |
| highest \& lowest |  |  |  |  |  |  |
| capacity |  |  |  |  |  |  |

TABLE 6
Components of Total Cost During an "Off-peak" Interval (6:35-6:45 am), by Incident Probability (capacity $=1200$ )

| Incident Probability | Average <br> Cost <br> (\$/trip) | Expected <br> Travel <br> Time | $E(\mathrm{SDE})$ | $E(\mathrm{SDL})$ | Lateness <br> probability | Planning <br> Cost |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0 | $\$ 1.11$ | $2.86 \%$ | $60.09 \%$ | $14.19 \%$ | $22.86 \%$ | $0.00 \%$ |
| 0.1 | $\$ 1.27$ | $3.65 \%$ | $55.77 \%$ | $12.85 \%$ | $27.56 \%$ | $0.17 \%$ |
| 0.15 | $\$ 1.34$ | $4.03 \%$ | $54.44 \%$ | $12.39 \%$ | $28.91 \%$ | $0.23 \%$ |
| 0.2 | $\$ 1.40$ | $4.42 \%$ | $53.48 \%$ | $12.01 \%$ | $29.82 \%$ | $0.27 \%$ |
| 0.25 | $\$ 1.46$ | $4.81 \%$ | $52.79 \%$ | $11.70 \%$ | $30.38 \%$ | $0.31 \%$ |
| Difference between | $\$ 0.35$ | $11.05 \%$ | $29.42 \%$ | $3.73 \%$ | $54.49 \%$ | $1.30 \%$ |
| highest \& lowest |  |  |  |  |  |  |
| incident probabilities |  |  |  |  |  |  |

TABLE 7

Components of Total Cost During a "Peak" Interval (7:55-8:05 am), by Incident
Probability (capacity $=1200$ )

| Incident Probability | Average <br> Cost <br> (\$/trip) | Expected <br> Travel <br> Time | $E(\mathrm{SDE})$ | $E(\mathrm{SDL})$ | Lateness <br> probability | Planning <br> Cost |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0 | $\$ 2.02$ | $44.73 \%$ | $32.82 \%$ | $8.78 \%$ | $13.67 \%$ | $0.00 \%$ |
| 0.1 | $\$ 2.62$ | $46.86 \%$ | $26.40 \%$ | $7.60 \%$ | $17.54 \%$ | $1.61 \%$ |
| 0.15 | $\$ 2.89$ | $47.72 \%$ | $24.34 \%$ | $7.25 \%$ | $18.74 \%$ | $1.96 \%$ |
| 0.2 | $\$ 3.16$ | $48.47 \%$ | $22.72 \%$ | $6.99 \%$ | $19.69 \%$ | $2.13 \%$ |
| 0.25 | $\$ 3.40$ | $49.13 \%$ | $21.37 \%$ | $6.80 \%$ | $20.50 \%$ | $2.20 \%$ |
| Difference between | $\$ 1.38$ | $55.57 \%$ | $4.61 \%$ | $3.90 \%$ | $30.50 \%$ | $5.42 \%$ |
| highest \& lowest |  |  |  |  |  |  |
| incident probabilities |  |  |  |  |  |  |

## FIGURE 1

Flow chart of Simulation Procedure


FIGURE 2
Average Travel Delay, Incident Probability $=0.2$


FIGURE 3
Schedule Delay Distribution, by incident probability, capacity = 1200 (5000 commuters)


## FIGURE 4

Change in Schedule Delay, by incident probability, capacity $=1200$ relative to capacity $=2400$ (5000 commuters)



[^0]:    ${ }^{1}$ Polak (1987) developed some very detailed equations attempting to deal with risk aversion. Our approach is much simpler and results in the derivation of a "lateness probability" variable that could be interpreted as representing risk aversion and which also separates this measure from the value of time.

[^1]:    ${ }^{2}$ Small (1982) found that the parameters of (1) vary with measurable socioeconomic characteristics, but we prefer the simplicfication of constant parameters in order to keep the supply-demand simulations manageable. It might also seem that $\alpha$ and $\theta$ would be correlated across the population, but this is not necessarily the case. The value of travel time, $\alpha$, and SDL are probably related to an individual's wage rate while the lateness penalty is probably related to the rigidity of working conditions (i.e., habitual lateness may in some cases lead to the loss of a job or reduced opportunities for career advancement). In fact one could reasonably argue that those with low wage rates, and hence low values of $\alpha$ might have higher values of $\theta$.
    ${ }^{3}$ We simplify the real world somewhat by identifying all unpredictable congestion as due to incidents and giving it a positive value. There can also be pleasant surprises, due for example to unexpectedly low demand on a particular day, but we do not model these. Rather, our recurrent congestion is fully predictable while non-recurrent congestion is a positive random variable whose distribution is known but whose specific value on a given day is not.

[^2]:    ${ }^{4}$ See Noland and Small (1995) for the full derivation and for the case $T_{e}<0$ or $T_{e}>T_{m}$, as well as a derivation using an exponential distribution of $T_{r}$.

[^3]:    ${ }^{5}$ For example, in our pilot survey we gave some people a choice in which one alternative dominated the other on all criteria, and respondents always chose the former.

[^4]:    ${ }^{6}$ This was further confirmed by estimating a random-coefficients version of the model (see Revelt and Train, 1996) both with and without accounting for these repeated measures. We did this for the "basic model" of Table 3, column (2). Accounting for repeated measures increases the standard error estimates by only $0-10$ percent. For this work we adapted a Gauss program kindly provided by Kenneth Train.

[^5]:    ${ }^{7}$ For purposes of computing the expected travel time that forms the denominator of the coefficient of variation in the demand model, we approximated by drawing a fixed component of travel time (assumed to occur either off the highway or on uncongested parts of the highway) from a normal distribution with mean 20 minutes and standard deviation of 5 minutes. This is done because we need to represent the entire trip, not just that part on the congested highway.
    ${ }^{8}$ A free-flow speed of 60 mph and capacity of 1200 vehicles per hour may not correspond to "realistic" conditions.
    During the course of this analysis other volume/capacity ratios were tested. These did not produce substantively different results. The traffic volume level presented did give reasonable congestion levels for the number of travelers simulated and was chosen for computational reasons (i.e., faster convergence times). 5000 commuters provided a good distribution of individual choices. Higher capacity levels would need a higher level of commuter traffic to generate realistic congested travel times but would lead to longer computational times.

[^6]:    ${ }^{9}$ Analysis of variations in severity level and probabilities found no substantive differences to variations in incident probabilities, so are not shown in our results.

[^7]:    * Small's model also contains coefficients to adjust for rounding errors in reported measurements. The variable definitions are somewhat different also. The travel time, SDE and SDL variables are actual reported values as opposed to expected values; the lateness probability was a dummy variable for those choices involving actually arriving at work late, whose expectation would be the lateness probability in the context of the present paper.

