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NON-LINEAR HEAT TRANSFER ANALYSIS OF AXISYMMETRIC SOLIDS

by

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APRIL 1971

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1. INTRODUCTION

Several approximate methods of solution for linear heat transfer problems, which are based on the finite element method, have been introduced in the past decade (1, 2, 5). The generality of the finite element method with respect to arbitrary boundary conditions and material property variations has allowed for the solution of many complex problems.

The advantages of the method, as compared to other numerical approaches, are numerous. The method is completely general with respect to geometry and material properties; complex bodies composed of many different materials are easily represented. Nonlinearity of material and boundary conditions can be treated by the finite element method easily. Most numerical procedures convert the continuous governing differential or integral equations of heat transfer problems to a set of linear algebraic equations. In the finite element method, the linear equations produce a symmetric positive-definite matrix in band form which is readily solved with a minimum of computer storage and time.

The finite element method of analysis as applied to nonlinear heat transfer can represent, with good approximation, many factors which previously have been neglected. Therefore, the purpose of this investigation is to modify the finite element method as applied to heat transfer problems, and to develop a technique for the evaluation of the temperature in two-dimensional axisymmetric bodies. Based on this approach a general digital computer program is developed for two-dimensional axisymmetric systems of arbitrary geometry. The nonlinearity effects of conduction, readiation, convection, and cooling pipe forms of heat transfer are considered by the program.

2. HEAT FLOW EQUATION OF EQUILIBRIUM

A. Introduction

Previous applications of the finite element method to heat transfer analysis (1, 2) have been based on the assumption that the conductance of the material is not a function of temperature. The codes which were developed have been restricted to solution of special cases. A variational approach (1) and a physical approach (2, 3) were developed to treat the steady and transient states of the heat transfer problem. Here the study done by Wilson (3) is extended to include some important nonlinearities which arise during the transfer of heat within a body.

The basic heat equilibrium equation which is developed at each node of the discrete finite element representation of the structure is of the following form:

At any time "t"

Rate at which heat is stored in elements adjacent to the node	+	Rate at which heat flows from elements adjacent to the node	=	Rate at which external heat enters the node

If all nodes are considered, the above heat equilibrium equation can be written in matrix form as a set of first order nonlinear differential equations

$$\underline{C}\,(\rho)\,\dot{\underline{T}}\,(t) \;+\; \underline{K}\,(T)\,\underline{T}\,(t) \;=\; \underline{Q}\,(t) \qquad (2.1)$$

where

$\underline{C}$ is defined as the heat capacity matrix which is dependent on the density ρ of the material,

$\underline{K}$ is defined as the conductivity matrix which is dependent on the temperature T of the body,

$\underline{T}$ (t) is a vector of the nodal point temperatures,

$\underline{\dot{T}}$ (t) is a vector of the time rate of change of the nodal point temperatures, and

$\underline{Q}$ (t) is a vector of the external heat flows which are supplied at the nodes (heat which is generated within the elements can be considered in this vector).

In equation 2.1 $\underline{C}$, $\underline{K}$ and $\underline{Q}$ are known matrices at each instant of time. Therefore, the equation can be solved by a desirable numerical integration scheme. Next sections contain the development of $\underline{K}$ and $\underline{C}$ matrices.

B. Conductivity Matrix

i. Conductive element

The complete conductivity matrix for a body is developed by assembling the conductivity matrices of all elements of the body. For a two-dimensional axisymmetric body, the conductivity matrix of the i^{th} element is defined by (1):

$$K_i = \int_{v_i} K_i \left(\frac{\partial \phi}{\partial x} \frac{\partial \phi}{\partial x}^T + \frac{\partial \phi}{\partial y} \frac{\partial \phi}{\partial y}^T \right) dv_i \qquad (2.2)$$

where k_i is the conductance of the material and is assumed to be a function of temperature only.

ϕ is the vector of the interpolation functions relating the temperature of any point of the element to the nodal temperatures, and

v_i is the volume of the element.

The basic element (subdivision) used in the idealization of the two-dimensional body is a triangle of arbitrary shape (figure 2.1). The development of the conductivity matrix for this element is presented in Appendix A. To increase the capability of the code, four triangular elements are combined to form a quadrilateral element as shown in figure 2.2. The heat capacity term associated with the nodal point 5 is assumed to be distributed to the four adjacent nodes. The 5 x 5 element conductivity matrix is reduced to a 4 x 4 matrix by the assumption that there is no external heat flow at node 5. Therefore, the typical term of the 4 x 4 quadrilateral conductivity matrix is given by

$$\overline{K}_{ij} = K_{ij} - \frac{K_{i5}\ K_{5j}}{K_{55}} \qquad (2.3)$$

This reduction procedure is similar to the "static condensation" method in structural analysis.

ii. <u>Radiative element</u>

Consider two radiative black surfaces (figure 2.3). The net flow transferred from one surface to another is given by Stafen-Boltzmann law.

$$Q = a\ \sigma\ (T_1^4 - T_2^4) \qquad (2.4)$$

where

σ = .1713 x 10^{-8} Btu/ft^2/hr/(deg. R)^4,

T_1 is the absolute temperature of surface A_1, and

T_2 is the absolute temperature of surface A_2.

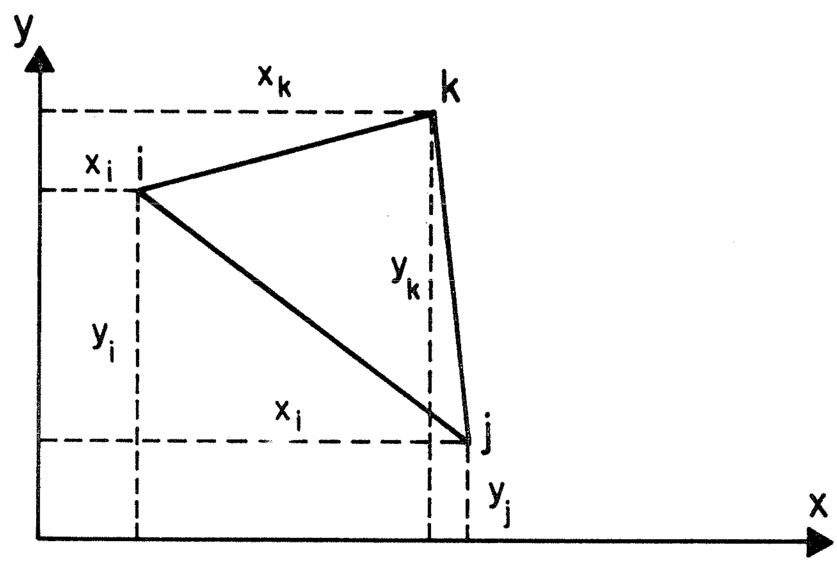

FIGURE 2.1 TYPICAL TRIANGULAR ELEMENT

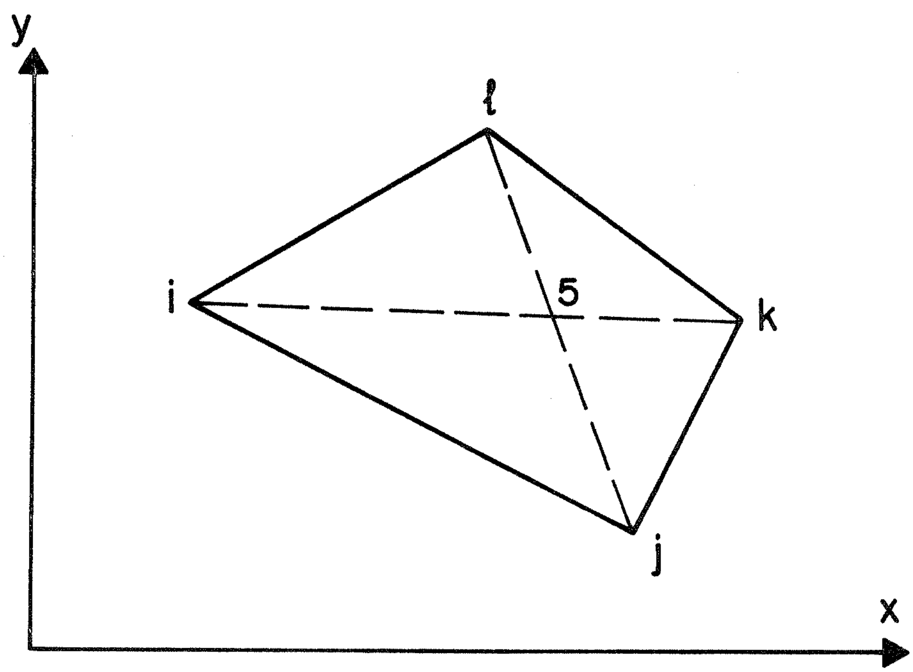

FIGURE 2.2 TYPICAL QUADRILATERAL ELEMENT

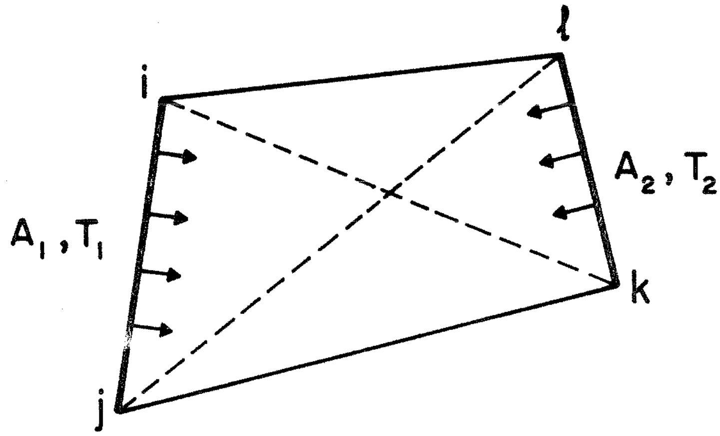

FIGURE 2.3 RADIATION OF TWO BLACK SURFACES

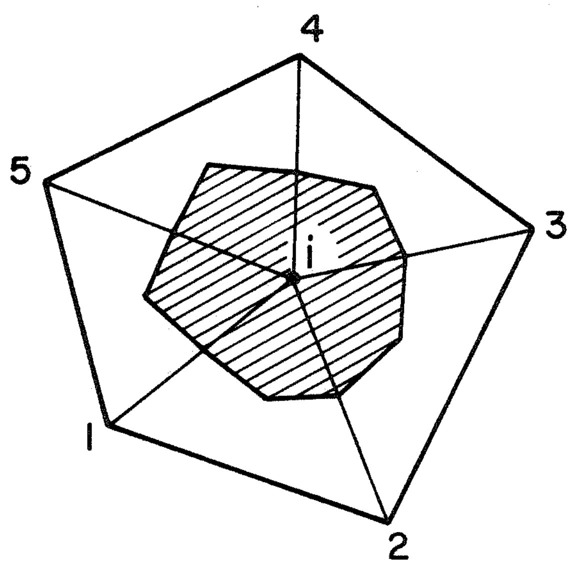

FIGURE 2.4 CONTRIBUTION OF NODE i TO HEAT CAPACITY

To obtain "a" the following discussion is made. Consider the enclosure of cross-section represented by the solid lines in figure 2.3. It is desired to formulate radiant-heat interchange between the heavy-lined surfaces A_1 and A_2. It is evident that the direct radiant interchange between A_1 and A_2 is the same regardless of whether they are connected by the solid-line surfaces or the corresponding dotted-line surfaces, since no part of the field of view of either A_1 or A_2 is affected by the substitution.

It can be shown that (4)

$$a = A_1 \gamma = [\, \overline{j\ell} + \overline{ik} - \overline{i\ell} - \overline{jk}]/2 \tag{2.5}$$

in which γ is a number between 0 and 1 which depends on the nature of the surface and A_1 is the area of the first surface.

If the difference between T_1 and T_2 is small, equation 2.4 is approximated by

$$Q = a \, \sigma \, T^3_{ave} \, (T_1 - T_2) \tag{2.6}$$

in which

$$T_{ave} = (T_1 + T_2)/2 + 460.$$

Substitution of

$$T_1 = (T_i + T_j)/2$$

$$T_2 = (T_k + T_\ell)/2$$

into equation 2.6 yields

$$A = K_r (T_i + T_j - T_k - T_\ell) \qquad (2.7)$$

where

$$K_r = \frac{a\sigma}{2} T^3_{ave}$$

It is observed that equation 2.7 forms a pseudo-conductivity matrix. One way to write the conductivity matrix is presented below.

$$\overline{\underline{K}} = K_r \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \qquad (2.8)$$

K_r is the pseudo-conductance of the element. As it is seen K_r is nonlinear of the third degree.

The conductivity matrix $\underline{K}$ of the body is given by the direct summation of the element conductivities $\overline{\underline{K}}_i$ or

$$\underline{K} = \sum_i \overline{\underline{K}}_i \qquad (2.9)$$

C. Capacity Matrix

Similar to conductivity matrix, the capacity matrix of the body is given by the direct summation of the element capacity matrices $\underline{C}_i$ or:

$$\underline{C} = \sum_i \underline{C}_i$$

The element capacity matrix $\underline{C}_i$ is given by (1):

$$\underline{C}_i = \int_{V_i} \rho_i \phi C_i \phi^T dv_i \tag{2.10}$$

in which C_i and ρ_i are the specific heat and the density of the material. For a quadrilateral element, equation 2.10 gives a 4 x 4 coupled matrix. Although equation 2.10 defines the capacity matrix accurately, it adds to the computational effort. A physical interpretation suggests the lumped capacity matrix. Assume that for a unit rate of change of temperature at node m ($\dot{T}_m = 1$), only the material up to mid-way on each side of the node contributes in storing heat in the body (hashed area in figure 2.4). This assumption eliminates the coupling between the rate of change of temperature of adjacent nodes and yields a diagonal capacity matrix. The concept of lumping the head capacity of the element at the four nodes of the element is equivalent to the lumped mass idealization in vibration problems.

If the volume of the m^{th} element is V_m and the specific heat (C_m) and the density (ρ_m) of the material are constant, then the total heat capacity of the element is given by:

$$A_m = V_m \rho_m C_m \tag{2.11}$$

This quantity is lumped at the four nodes of the element, such that the magnitude of the capacity at each node is proportional to its tributary volume (Appendix B).

3. BOUNDARY CONDITIONS

A series of boundary conditions, which are generally nonlinear, accompany equation 2.1. The exact treatment of the boundary conditions is complex and numerically inefficient. In the next sections an approximation is sought for each type of boundary condition.

A. Convection Boundary Condition

i. Free convection

The previously developed heat flow equilibrium equations can be modified to reflect surface heat transfer. The rate of heat flow (Q) across a boundary layer at the surface of the body, for the free convection case, is given by:

$$Q = a\, h_c\, (T_e - T_s)^{\alpha}(T_e - T_s) \tag{3.1}$$

where

- "a" is the area of the surface,
- h_c is the convection coefficient for the surface,
- T_e is the temperature of the external environment,
- T_s is the temperature of the surface of the body, and
- α is a non-zero real number which is determined by experiment

If we consider a surface element between nodes i and j, the rate at which heat is transferred to the nodes will be approximately

$$Q_i = b_c (T_e - T_i) \tag{3.2}$$

$$Q_j = b_c (T_e - T_j) \tag{3.3}$$

where

$$b_c = \frac{\ell t h_c}{2} \left[T_e - \frac{T_i + T_j}{2} \right]^{\alpha} \tag{3.4}$$

t is the thickness of the element, and

ℓ is the distance between nodes i and j.

In order to satisfy total heat flow equilibrium at these boundary nodes equations 3.2 and 3.3 are added to equation 2.1. This will result in the following modifications to the conductivity and the external heat flow matrices.

$$\begin{aligned} K^*_{ii} &= K_{ii} + b_c \\ K^*_{jj} &= K_{jj} + b_c \\ Q^*_i &= Q_i + b_c T_e \end{aligned} \tag{3.5}$$

This procedure can be applied repeatedly for all the free convection boundary elements.

ii. <u>Forced convection boundary condition</u>

Another type of heat transfer by convection is "forced convection." It occurs whenever a body is placed inside a flowing liquid. A general relation for the heat flow transferred to the body across its exposed surface is (4).

$$Q = a h_f \, p^{\beta}(t) \cdot (T_e - T_s) = \ell b_f \tag{3.6}$$

where

h_f is the forced convection coefficient,

$p\ (t)$ is a time-dependent function which is dependent on the nature of flow and the geometry of the body, and

β is a real number which is determined by experiments.

Similar to the free convection case, in order to account for the heat flow transferred to the body through surface $i - j$, the following modifications to the conductivity matrix and the external heat flow vector must be made:

$$Q_i^* = Q_i + b_f T_e \quad , \quad K_{ii}^* = K_{ii} + b_f$$
$$Q_j^* = Q_j + b_f \quad , \quad K_{jj}^* = K_{jj} + b_f \qquad (3.7)$$

B. Radiation

The Stefan-Boltzman relation for the heat radiation of a black surface is

$$Q = a\sigma\ [(T_e + 460)^4 - (T_s + 460)^4] \qquad (3.8)$$

in which σ is the radiation coefficient equal to $.1713 * 10^{-8}$ $BTU/ft^2/hr/(deg\ R)^4$

Equation can be written as

$$Q = a\ h_r\ (T_e - T_s) \qquad (3.9)$$

where

$$h_r = \sigma\ [(T_e + 460)^2 + (T_s + 460)^2]\ (T_e + T_s + 920) \qquad (3.10)$$

It is observed that equations 3.9 and 3.1 are comparable. Therefore, in order to account for the radiation boundary condition the procedure which was used for the free convection case, can be employed.

C. Cooling Pipe

The heat transferred to the body by a cooling pipe can be approximately treated as a free convection boundary condition. Consider a surface element between nodes k and ℓ which is cooled by a pipe (figure 3.1). Suppose that the corresponding nodal points which represent the segment of the pipe in contact with surface ℓ - k are i and j. The approximate temperature of external environment is:

$$T_e = (T_i + T_j)/2 \qquad (3.11)$$

in which T_i and T_j are both known quantities. The rate of heat flow which is transferred across the boundary ℓ - k is approximately given by:

$$Q = a\, h_c\, (T_e - T_s) \qquad (3.12)$$

where h_c is the heat transfer coefficient which depends on the condition of the contact between the pipe and the body, and the surface condition of the pipe and the body. It is observed that similar to the free convection and radiation boundary condition, the heat flow equilibrium equation (2.1) can be modified to account for the heat transferred to the body by the cooling pipe. The procedure is complete when the following method is applied for evaluation of the new magnitude of temperature at nodes i and j. For the closed system ijkl the equation of heat flow equilibrium is given by:

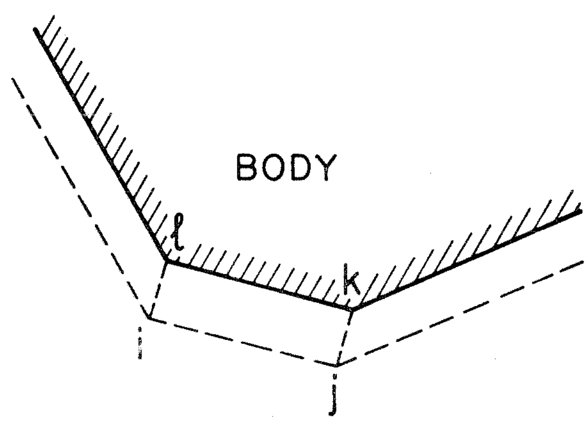

FIGURE 3.1 COOLING PIPE

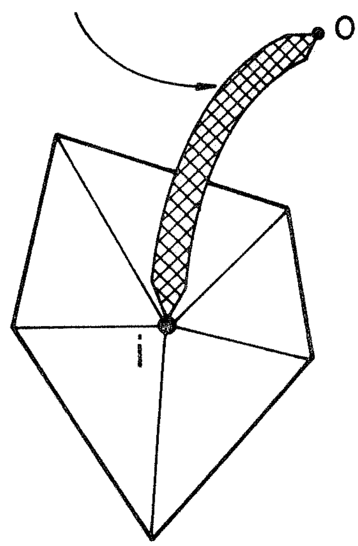

FIGURE 3.2 REPRESENTATION OF THE AUGMENTED BODY

$$\rho c v \ (T_i - T_j) + a h_c \ (T_s - T_e) = 0 \qquad (3.13)$$

in which v is the liquid flow inside the pipe (length/time), and

$$T_s = (T_k + T_\ell)/2 \qquad (3.14)$$

Equation 3.13 assumes that the heat flow due to conductivity is negligible as compared to the one due to mass liquid transfer.

D. Temperature and Flow

If flow is specified on surface i - j, using the tributary area method, it can be lumped at nodes i and j. The lumped flow quantities contribute to the flow vector Q.

The temperature boundary condition is analogous to the displacement boundary condition in structural analysis. One way of treating this type of boundary condition is as follows: Consider node i of a discretized body (figure 3.2), at which the temperature is known (T^0). Assume that an imaginary element with large volume and conductance is connected to that node only. Then the corresponding term in the conductivity matrix $\underline{K}$ is

$$K_{ii} = e$$

where e is a large number. It is suggested that

$$e = \sum_{j=1}^{N} (K_{jj} \times 10^6)$$

where N is the number of nodal points. The temperature boundary condition is now converted to the flow boundary condition, simply, by specifying the flow at node i:

$$Q_i^o = e\, T_i^o$$

4. NUMERICAL INTEGRATION

A. Numerical Scheme

Equation 2.1 is a set of nonlinear differential equations which can be solved by a step-by-step method. Equation 2.1 is valid at any instant of time including time "t", i.e.,

$$\underline{C}\,\dot{\underline{T}}_t + \underline{K}\,\underline{T}_t = \underline{Q}_t \qquad (4.1)$$

Similarly at time $t + \Delta t$ we have,

$$\underline{C}\,\dot{\underline{T}}_{t+\Delta t} + \underline{K}'\,\underline{T}_{t+\Delta t} + \underline{Q}_{t+\Delta t} \qquad (4.2)$$

where Δt is the time increment and $\underline{K}'$ is the conductivity matrix at temperature $T_{t+\Delta t}$. Here we assume that within each time increment the conductivity matrix $\underline{K}$ is constant. This assumption leads to a linear approximation of the flow-temperature curve as shown in figure 4.1. Subtraction of equation 4.1 from 4.2 yields:

$$\underline{C}\,(\dot{\underline{T}}_{t+\Delta t} - \dot{T}_t) + \underline{K}\,(\underline{T}_{t+\Delta t} - T_t) = \underline{Q}_{t+\Delta t} - \underline{Q}_t \qquad (4.3)$$

To simplify equation 4.3 one more assumption must be made. Suppose the variation of T within each time increment is linear (figure 4.2). Then,

$$\underline{T}_{t+\Delta t} = \underline{T}_t + \Delta t\,\dot{\underline{T}}_{t+\Delta t} \qquad (4.4)$$

or

$$\dot{\underline{T}}_{t+\Delta t} - \dot{\underline{T}}_t = (\underline{T}_{t+\Delta t} - \underline{T}_t)/\Delta t - \dot{\underline{T}}_t$$

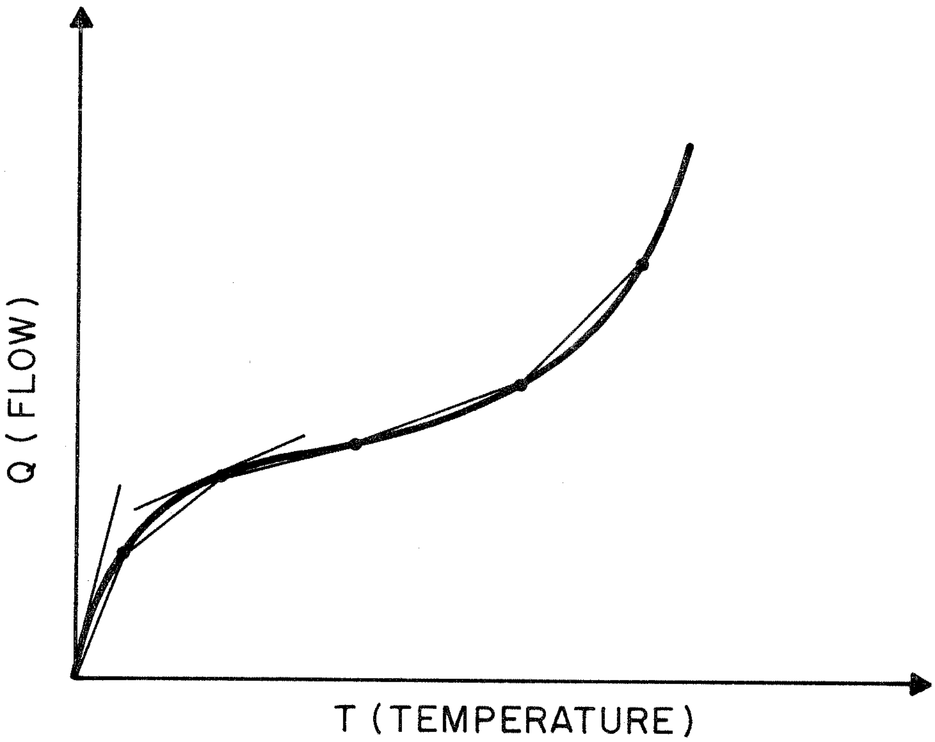

FIGURE 4.1 FLOW-TEMPERATURE CURVE

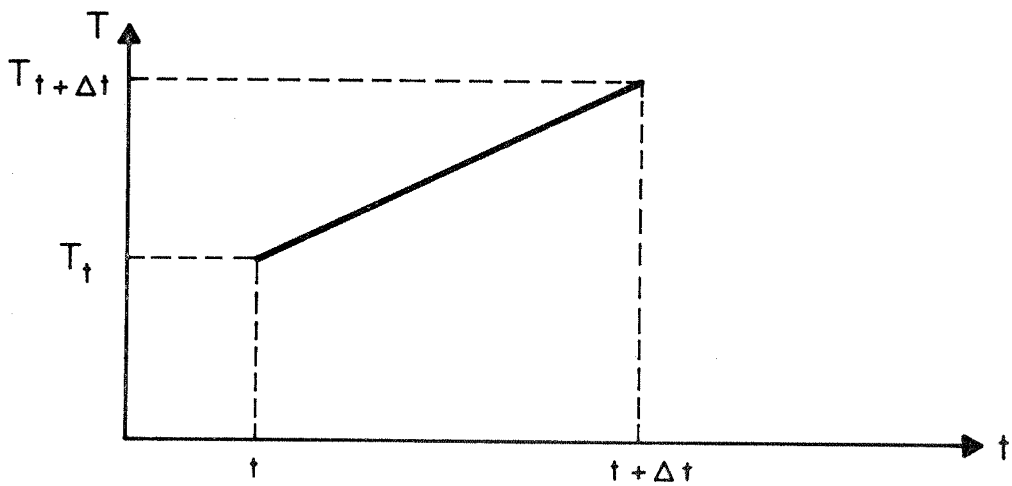

FIGURE 4.2 VARIATION OF T VS. TIME t

Substitution of equation 4.4 into equation 4.3 yields

$$\left(\frac{\underline{C}}{\Delta t} + \underline{K}\right) \Delta \underline{T}_t = \underline{C}\,\dot{\underline{T}}_t + \Delta \underline{Q}_t \qquad (4.5)$$

where

$$\Delta \underline{T}_t = \underline{T}_{t+\Delta t} - \underline{T}_t$$
$$\Delta \underline{Q}_t = \underline{Q}_{t+\Delta t} - \underline{Q}_t \qquad (4.6)$$

If $\dot{T}_t = \frac{\Delta T_{t-\Delta t}}{\Delta t}$, then equation 4.5 becomes

$$\underline{K}^* \Delta \underline{T}_t = \underline{Q}^*_t \qquad (4.7)$$

where

$$\underline{K}^* = \frac{C}{\Delta t} + \underline{K}$$

$$\underline{Q}^*_t = \frac{C}{\Delta t} \Delta \underline{T}_{t-\Delta t} + \Delta \underline{Q}_t$$

Equation 4.7 can now be solved directly for the temperature increments at the end of the time increment. The temperature quantities at the end of the time increment are calculated from equation 4.6. $\underline{K}^*$ is a function of time and may be computed at selected intervals where the change in magnitude of the elements of the matrix becomes appreciable. The complete solution procedure is summarized in Table I.

B. Stability of the Numerical Scheme

It can be proven that the suggested numerical scheme for solving the differential equations defined by equation 4.1 is unconditionally stable. Equation 4.7 is a difference equation in terms of $\Delta\underline{T}$. The solution of the equation is (4)

$$\Delta\underline{T}_n = \Delta\underline{T}^* \lambda^n \tag{4.8}$$

where

$$n = t/\Delta t$$

For simplicity assume that $\Delta\underline{Q}_t$ is zero. The stability characteristics of the scheme is not affected by this assumption.

Substitution of equation 4.8 into equation 4.7 gives

$$\lambda^{n-1} \left(\underline{K}^* \lambda - \frac{\underline{C}}{\Delta t}\right) \Delta\underline{T}^* = 0 \tag{4.9}$$

Existence of a non-trivial solution for $\Delta\underline{T}^*$ requires that

$$\lambda^{n-1} = 0$$

or

$$\underline{K}^* \lambda - \frac{\underline{C}}{\Delta t} = 0 \tag{4.10}$$

To prove the stability of the scheme it is necessary and sufficient to show that the absolute values of all roots of equations 4.10 are less than or equal to unity.

The roots of equations 4.10 are

$$\lambda = 0,$$

$$\lambda = \underline{K}^{*-1} \frac{\underline{C}}{\Delta t}$$

It is obvious that the second root is less than unity for every time increment Δt, except for $\Delta t = \infty$ which the root becomes unity. If Δt approaches infinity equation 4.7 becomes

$$\underline{K} \; \Delta \underline{T}_t = \Delta \underline{Q}_t$$

which is the steady state and stable solution.

TABLE I

SUMMARY OF STEP-BY-STEP SOLUTION METHOD

INITIAL CALCULATIONS

1. Form $\underline{C}$ and $\underline{K}$.

2. Modify for temperature boundary conditions.

3. Modify for radiation, convection, and cooling pipe boundary conditions.

4. Compute the initial resisting heat flow vector $\underline{E}$.

5. Compute

$$\underline{C}^* = \underline{C} / \Delta t$$

$$\underline{K}^* = \underline{K} + \underline{C}^*$$

FOR EACH TIME INCREMENT

1. Compute the effective heat flow vector

$$\underline{Q}_t^* = \underline{Q}_{t + \Delta t} - \underline{E}_t$$

2. Evaluate $\Delta\underline{T}_t$ by solving

$$\underline{K}^* \; \Delta\underline{T}_t = \underline{Q}_t^*$$

3. Calculate $T_{t + \Delta t}$ from

$$\underline{T}_{t + \Delta t} = \underline{T}_t + \Delta\underline{T}_t$$

4. Compute the resisting heat flow vector

$$\underline{E}_{t + \Delta t} = \underline{Q}_{t + \Delta t} - \underline{C}^* \; \Delta\underline{T}_{t + \Delta t}$$

5. Compute the nodal point temperatures of the liquid flowing inside the cooling pipe (if there is any).

6. Calculate $\underline{K}^*$ again if desired.

7. Repeat for the next time step.

5. APPLICATIONS

A. Introduction

The intention in this chapter is to show the applicability of the theories presented previously. For this purpose some examples are given. Before presenting the examples it is instructive to know the function of the computer program. The following steps define the operation of the program.

a. Input

1. Nodal point informations are received (or generated) by the computer (coordinates, adiabatic or isothermal condition, initial temperature, specified functions for flow or temperature at nodes).

2. Element informations are read (or generated) by the computer (nodal points of the element, specified functions for the heat generating units inside the elements).

3. Material informations, temperature, and time dependent functions are given to the computer. Also the boundary nodes and the corresponding boundary conditions are specified.

b. Computations

1. The conductivity and the heat capacity matrices are formed for each conductive element. The pseudo-conductivity matrix is computed for each radiative element. The corresponding matrices are assembled to form the system conductivity and heat capacity matrices.

2. The applied heat flow at each node is calculated and the various boundary conditions are satisfied.

3. For each time increment the program solves for the nodal point temperatures. Then, new conductivity and heat capacity matrices are formed and a new cycle will begin.

If nonlinearity of the material and the boundary conditions are weak so that the conductivity matrix of the system does not change appreciably in one time step, it might be advantageous to go through the whole process of computing the element and system characteristic matrices only at the beginning of the preassigned intervals. This option has been included in the program. A description of the input data, the computer program, and a sample investigation are presented in Appendices C, D, and E.

B. Examples

The time step used in all examples is $\Delta t = 1$ hr.

a. Free convection heat flow input

As the first example we consider a hollow cylinder a section of which is shown in figure 5.1. The initial temperature of the body is zero while the temperature of the external environment is $10000^{o}F$. Surface 2, 3, and 4 are insulated, i.e., the system is adiabatic at these surfaces. Figure 5.2 shows the finite element idealization of the cross-section of the cylinder. The variation of conductivity of the material with respect to temperature is presented in figure 5.3. The average temperature of the body is plotted versus time (figure 5.4). Also, the variation of temperature across the depth of the cylinder is presented in figure 5.5. Figure 5.4 demonstrates that the temperature

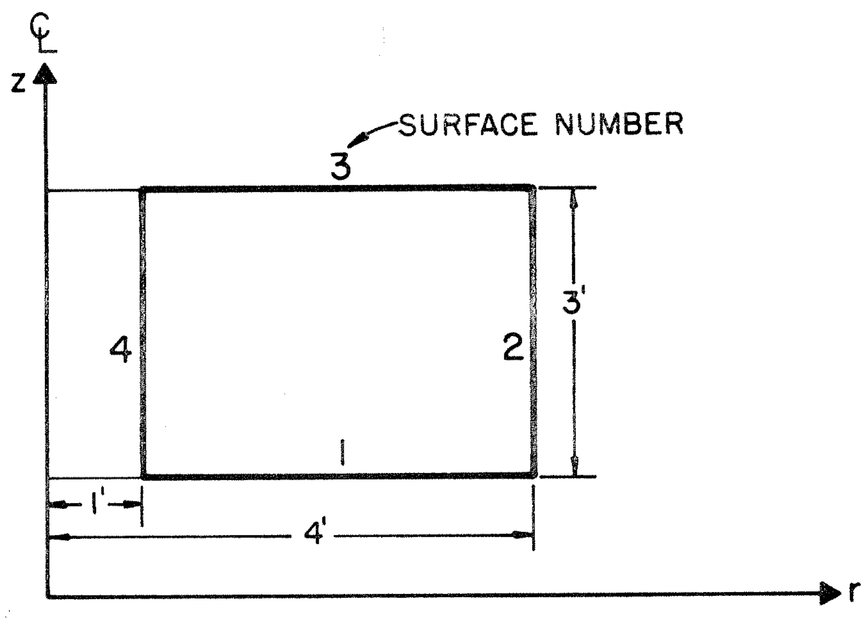

FIGURE 5.1 SECTION OF THE HOLLOW CYLINDER

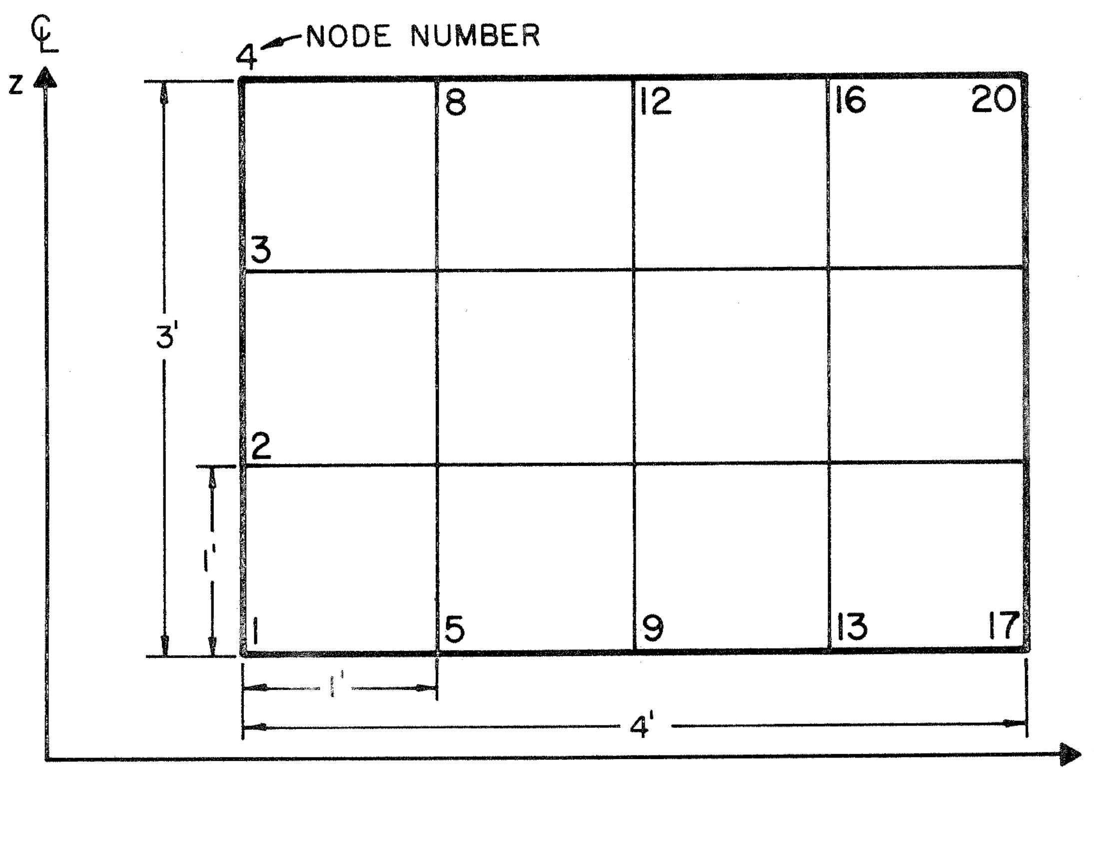

FIGURE 5.2 FINITE ELEMENT IDEALIZATION

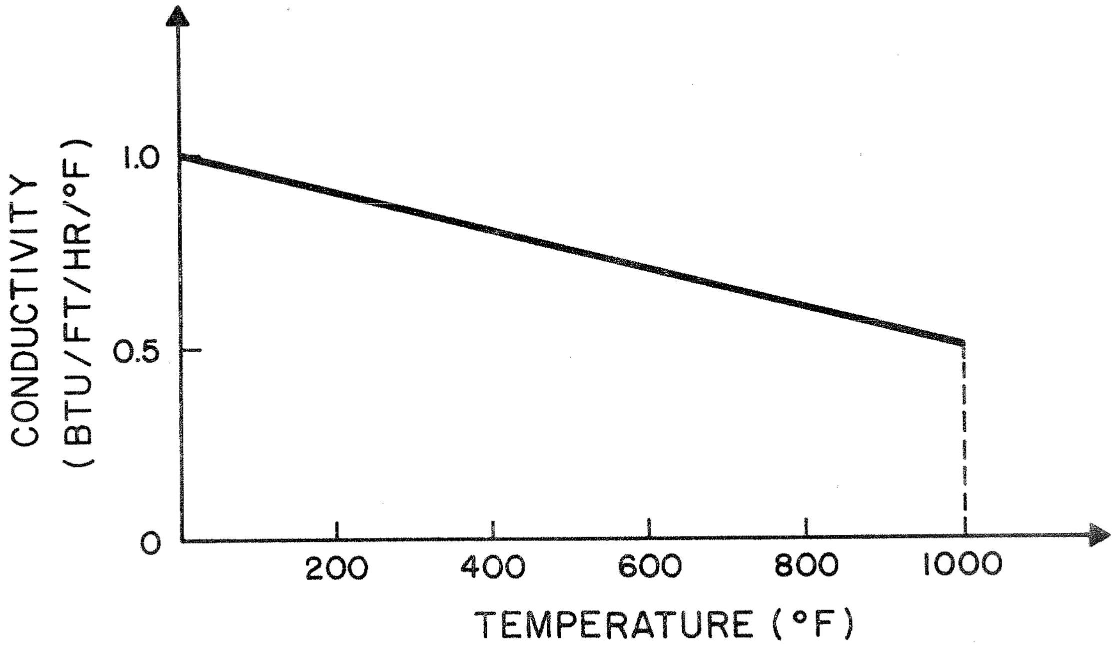

FIGURE 5.3 a VARIATION OF CONDUCTIVITY VS. TEMPERATURE

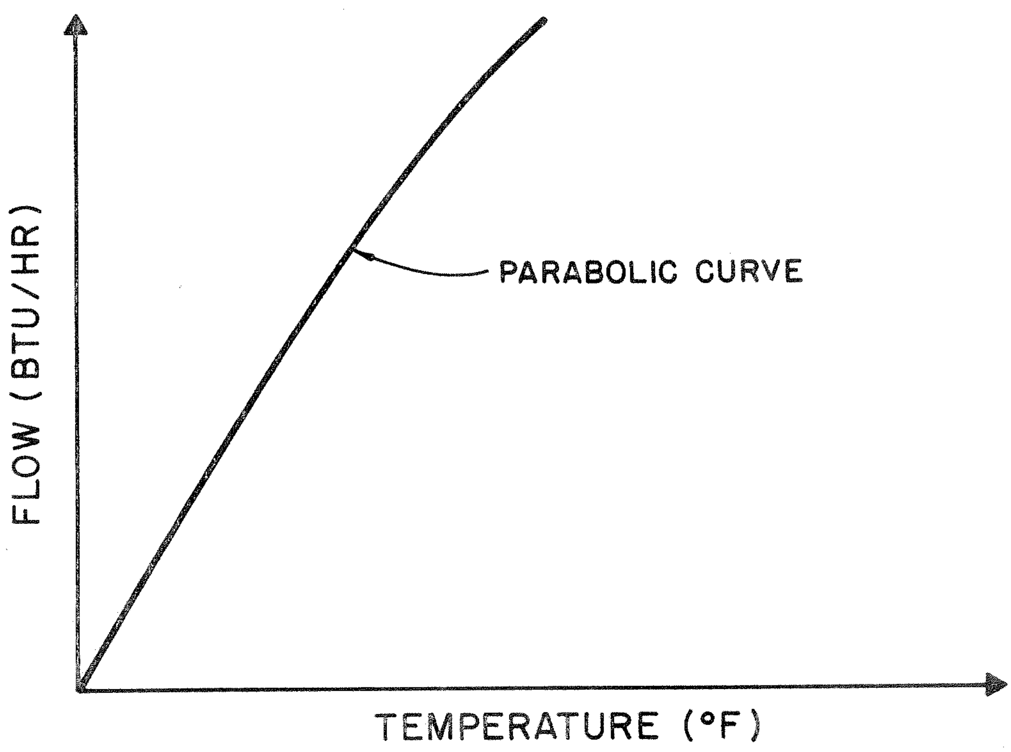

FIGURE 5.3 b VARIATION OF FLOW VS. TEMPERATURE

of the body is rising to the temperature of the external environment, and figure 5.5 emphasizes that the temperature of the surface, where the heat enters the body, is greater than other surfaces.

b. Forced convection-radiation heat flow input

The system used in the previous example is employed here to demonstrate the capabilities of the computer program in solving radiation and forced convection problems.

Suppose surfaces 1 and 3 are insulated while surface 2 receives heat by radiation and surface 4 receives heat flow by forced convection form of heat transfer. The temperature of the external environment is $1000^{o}F$. For the forced convection heat flow ($Q = f^{\beta}$), it is assumed that $\beta = 1$, and f is a function of time as shown in figure 5.6.

A finite element analysis is performed and the results are presented in figures 5.6 and 5.7. The variation of the temperature of the body is plotted versus time in figure 5.6. Figure 5.7 shows the contours of nodal point temperatures in the radial direction.

c. Cooling pipe heat flow input

In this example the cylindrical body which is shown in figure 5.1 is considered again. Assume that the body undergoes a state of initial temperature equal to $1000^{o}F$. On surface 3 water flows in a pipe at the rate of 2 feet per hour and cools the body. The temperature of water at the beginning of the cooling pipe is zero. A finite element analysis is carried out and the following informations are obtained. Figure 5.8 demonstrates that while water cools the body, it gains heat. The longitudinal variation of temperature is plotted in figure 5.9. The change of the average temperature of surface 3 vs. time is shown in figure 5.10.

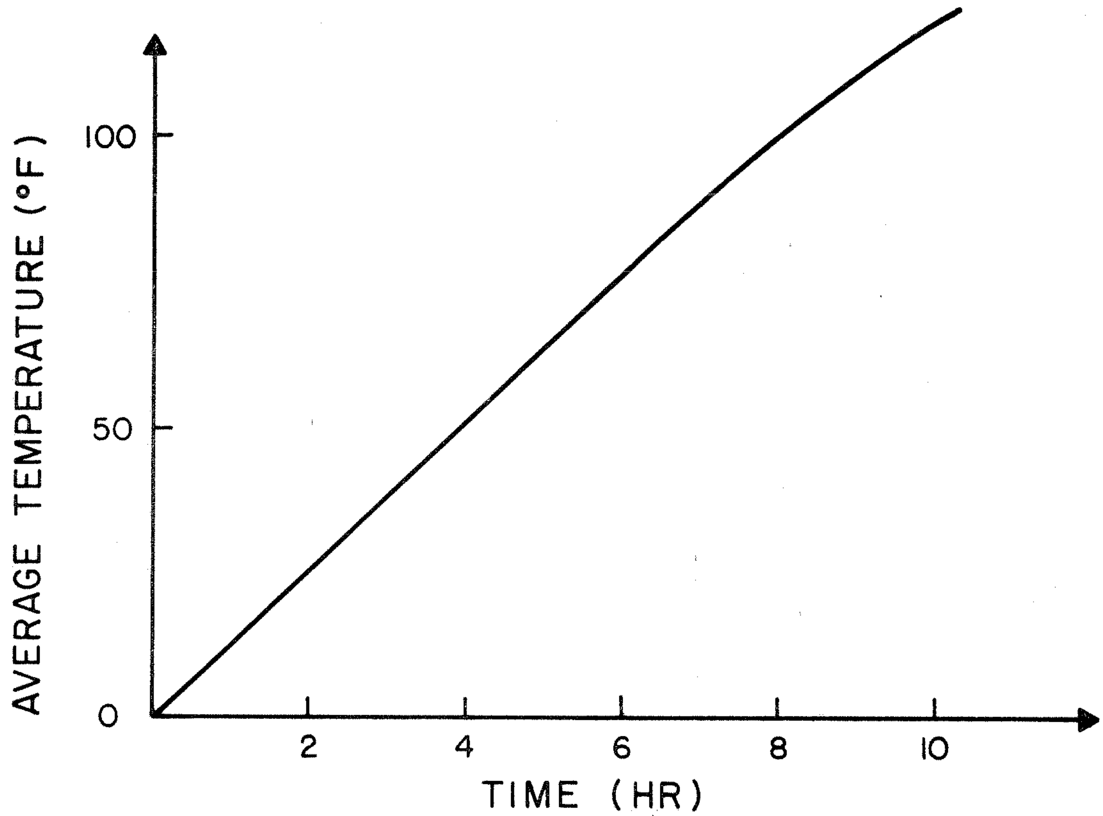

FIGURE 5.4 TEMPERATURE VS. TIME

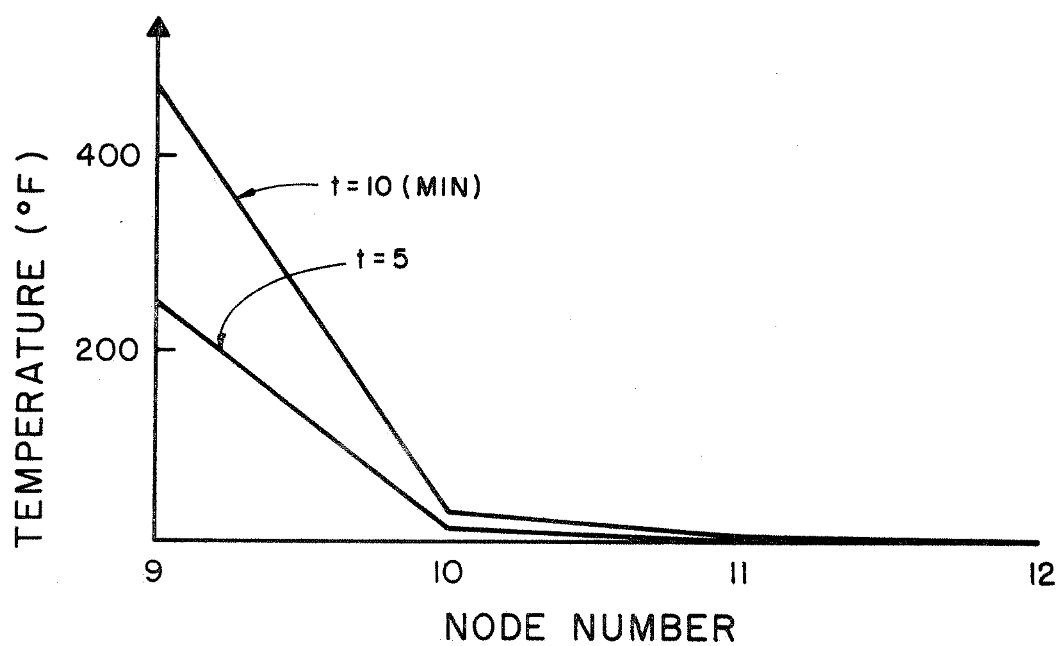

FIGURE 5.5 VARIATION OF TEMPERATURE ACROSS DEPTH

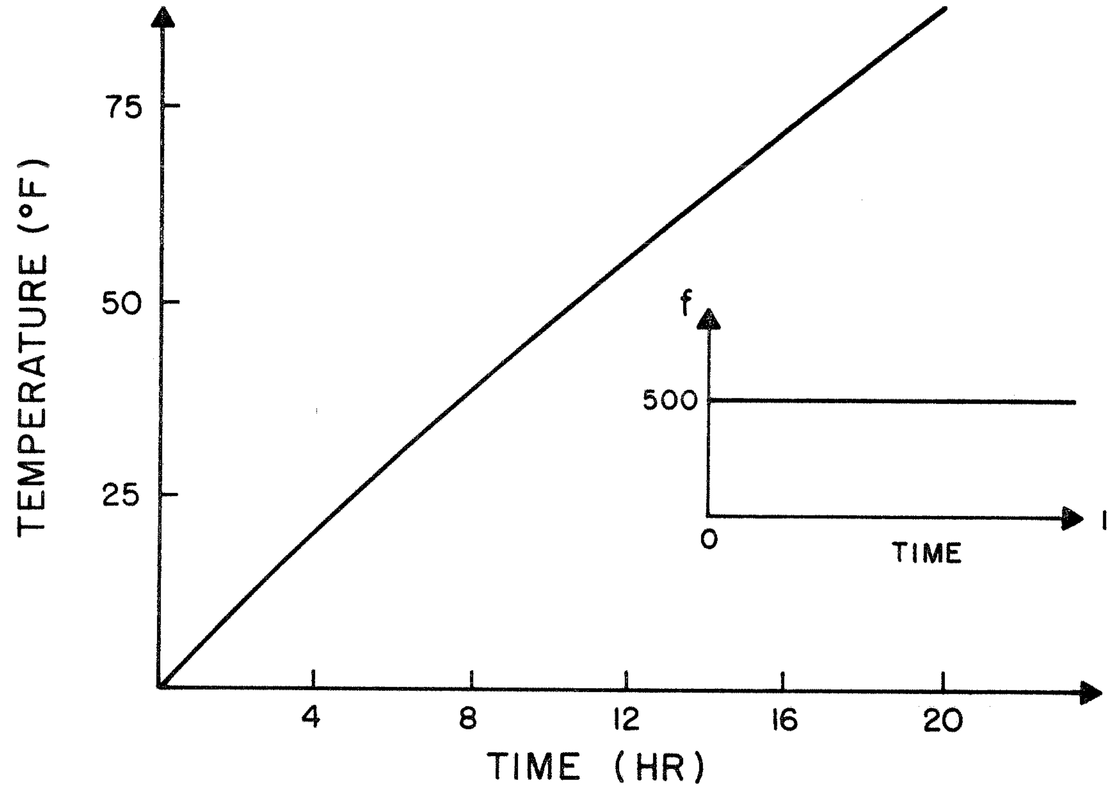

FIGURE 5.6 TEMPERATURE VS. TIME

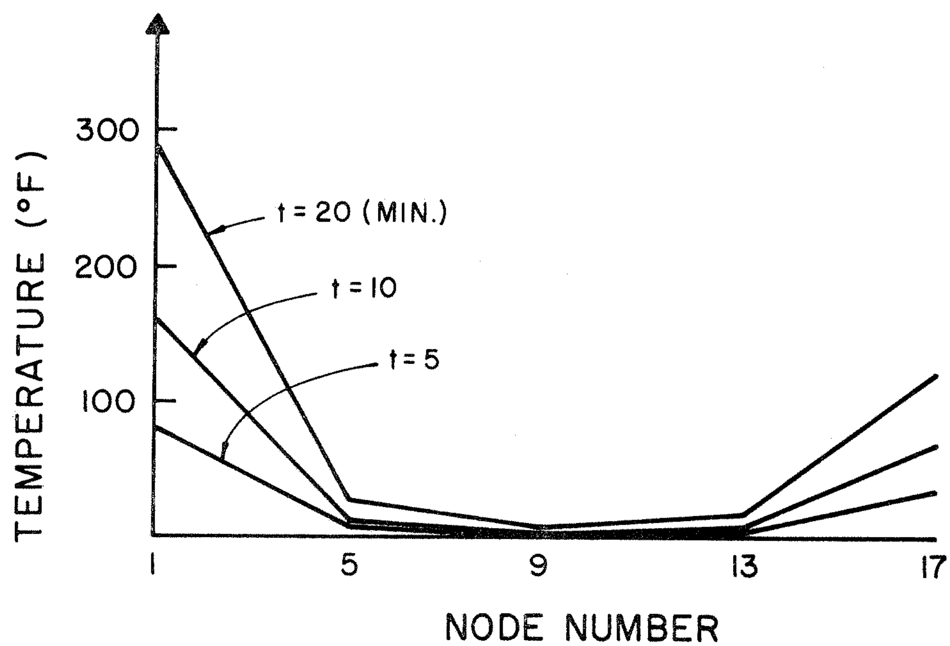

FIGURE 5.7 VARIATION OF TEMPERATURE VS. WIDTH

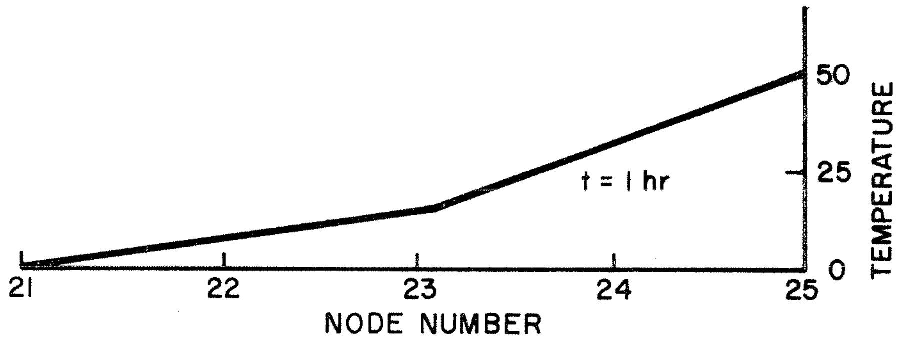

FIGURE 5.8 TEMPERATURE OF THE COOLANT

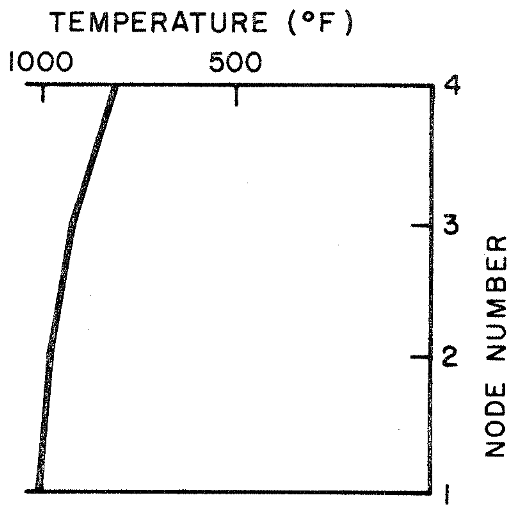

FIGURE 5.9 LONGITUDINAL VARIATION OF TEMPERATURE

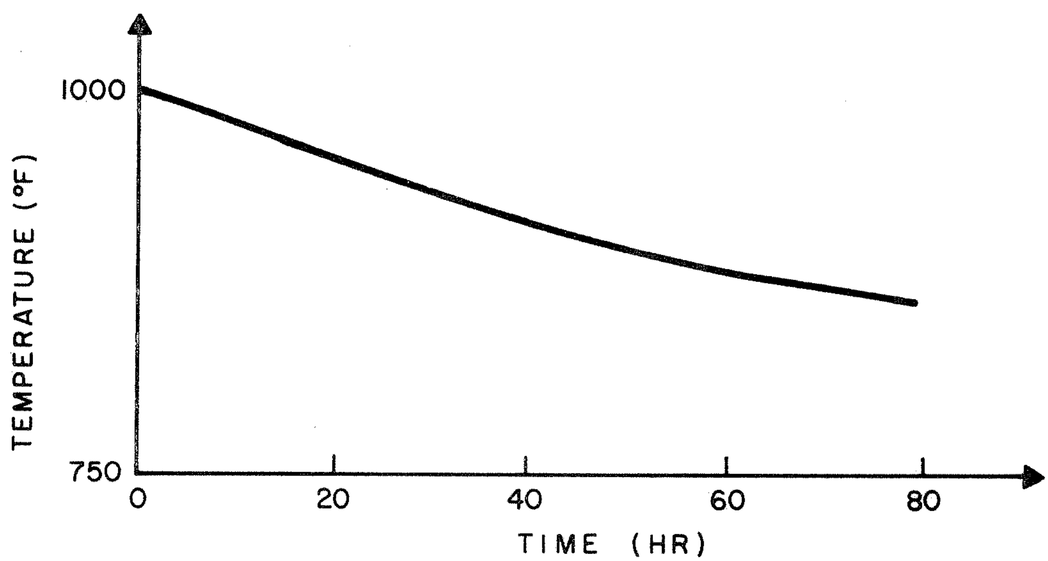

FIGURE 5.10 TIME VARIATION OF TEMPERATURE

C. Final Remarks

A computer program which is based on the method of analysis presented in this report has been developed. A description on the use of the program, a Fortran IV listing, and a sample investigation is given in the Appendices of this report. The program is developed with particular stress on nonlinearity of material and boundary conditions. Along with treating different convection forms of boundary conditions, the cooling pipe and radiation forms of heat transfer were considered. Linear, nonlinear, transient and steady state heat problems can all be treated.

6. REFERENCES

1. Wilson, E. L. and Nickell, R. E., "Application of the Finite Element Method to Heat Conduction Analysis," Nuclear Eng. and Design, Vol. 4 1966, pp. 276-286.

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5. Zienkiewicz, O. C., "The Finite Element Method in Structural and Continuum Mechanics," McGraw-Hill, New York, 1967.

APPENDIX A

CONDUCTIVITY MATRIX FOR A TRIANGULAR ELEMENT

APPENDIX A

CONDUCTIVITY MATRIX FOR A TRIANGULAR ELEMENT

A. Introduction

The purpose of this section is to evaluate the conductivity matrix for a triangular element of an axisymmetric body. The conductivity matrix for an element of an axisymmetric isotropic body is given by

$$\underline{\bar{k}} = \int_A k \left(\frac{\partial \phi}{\partial r} \frac{\partial \phi}{\partial r}^T + \frac{\partial \phi}{\partial z} \frac{\partial \phi}{\partial z}^T \right) r \, dA \qquad (A.1)$$

where

- k is the conductance of the material,
- A is the area of the element, and
- ϕ is the vector of the interpolating functions which relate the temperature of every point of the element to the nodal points of the element.

B. Interpolation Functions

For the temperature distribution within each triangular element (figure A.1) the following relation is assumed:

$$T\ (r,\ z) \ = \ \underline{\phi}^T \ \underline{T} \qquad (A.2)$$

Where $\underline{T}$ is the temperature vector of the nodal points of the triangle, and

$$\underline{\phi}^T = \frac{1}{2A} < 2A + er + dz \quad b_k r - a_k z - b_k r + a_j z > \qquad (A.3)$$

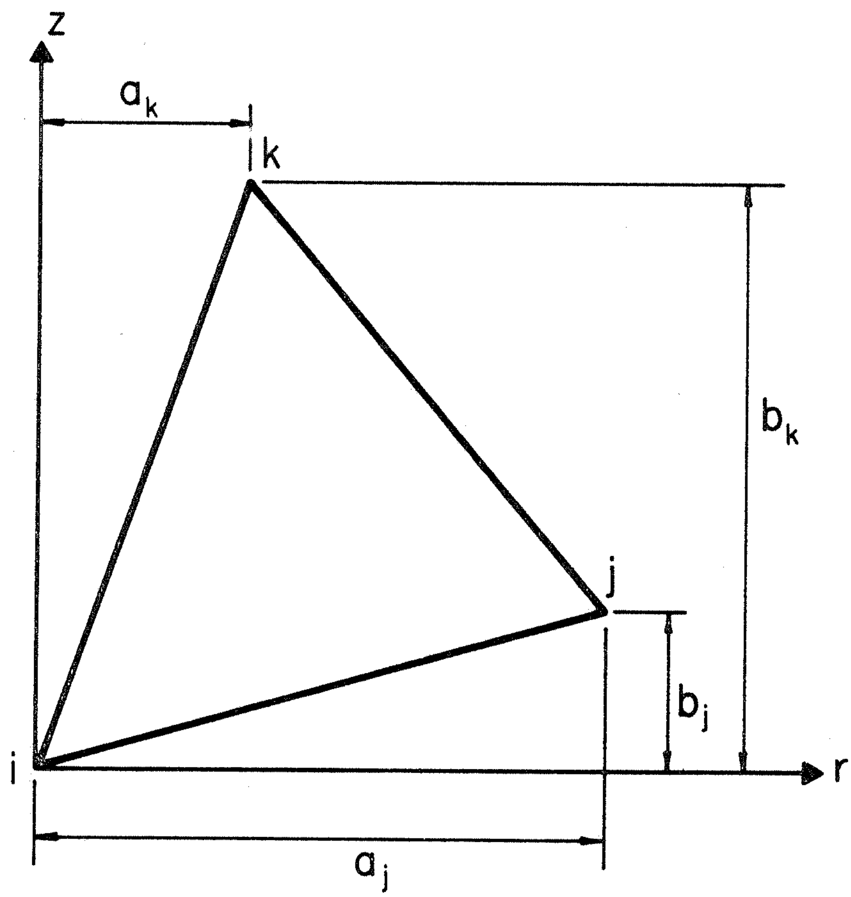


FIGURE A-1 TYPICAL TRIANGULAR ELEMENT

A is the total area of the triangle

$$e = b_j - b_k$$
$$d = a_k - a_j \qquad (A.4)$$

Equation A.3 yields

$$\frac{\partial \phi^T}{\partial r} = \frac{1}{2A} \langle \; e \;\; b_k \; - b_j \; \rangle$$
$$\frac{\partial \phi^T}{\partial z} = \frac{1}{2A} \langle \; d \; - a_k \;\; a_j \; \rangle \qquad (A.5)$$

Assume k is constant over the triangular element. Then, substitution of equation A.5 into equation A.1 and evaluation of the integral for one radian of the ring element yeilds

$$\underline{\bar{k}} = \frac{k \; r_c}{4A} \begin{bmatrix} e^2 + d^2 & b_k e - a_k d & -b_j e + a_j d \\ & b_k^2 + a_k^2 & -b_j b_k - a_k a_j \\ \text{SYMM} & & b_j^2 + a_j^2 \end{bmatrix} \qquad (A.6)$$

where r_c is the radius of the center of gravity of the triangle. In evaluating the integral in equation A.1 it is assumed that r is constant and equal to r_c. Equation 4.6 is the conductivity matrix of an axisymmetric triangular ring element.

APPENDIX B

HEAT CAPACITY MATRIX FOR A QUADRILATERAL ELEMENT

APPENDIX B

HEAT CAPACITY MATRIX FOR A QUADRILATERAL ELEMENT

The goal in this section is the computation of a lumped heat capacity matrix for an axisymmetric quadrilateral ring element. Figure B.1 shows the cross-section of a quadrilateral ring element. Connecting the mid-points of the opposite sides, the element is divided into four quadrilaterals. The common vertex of all quadrilaterals is located at a point which its radius is:

$$r_c = (r_i + r_j + r_k + r_\ell)/4 \tag{B.1}$$

The radius of the center of each quadrilateral is approximately:

$$\begin{aligned} r_{ci} &= (2\,r_i + \frac{r_j + r_\ell}{2} + r_c)/4. \\ r_{cj} &= (2\,r_j + \frac{r_k + r_i}{2} + r_c)/4. \\ r_{ck} &= (2\,r_k + \frac{r_j + r_\ell}{2} + r_c)/4. \\ r_{c\ell} &= (2\,r_\ell + \frac{r_k + r_i}{2} + r_c)/4. \end{aligned} \tag{B.2}$$

the total heat capacity of the element is given by

$$A = \rho\, c\, v \tag{B.3}$$

in which

ρ is the density of the material,

c is the specific heat of the material, and

v is the volume of the element.

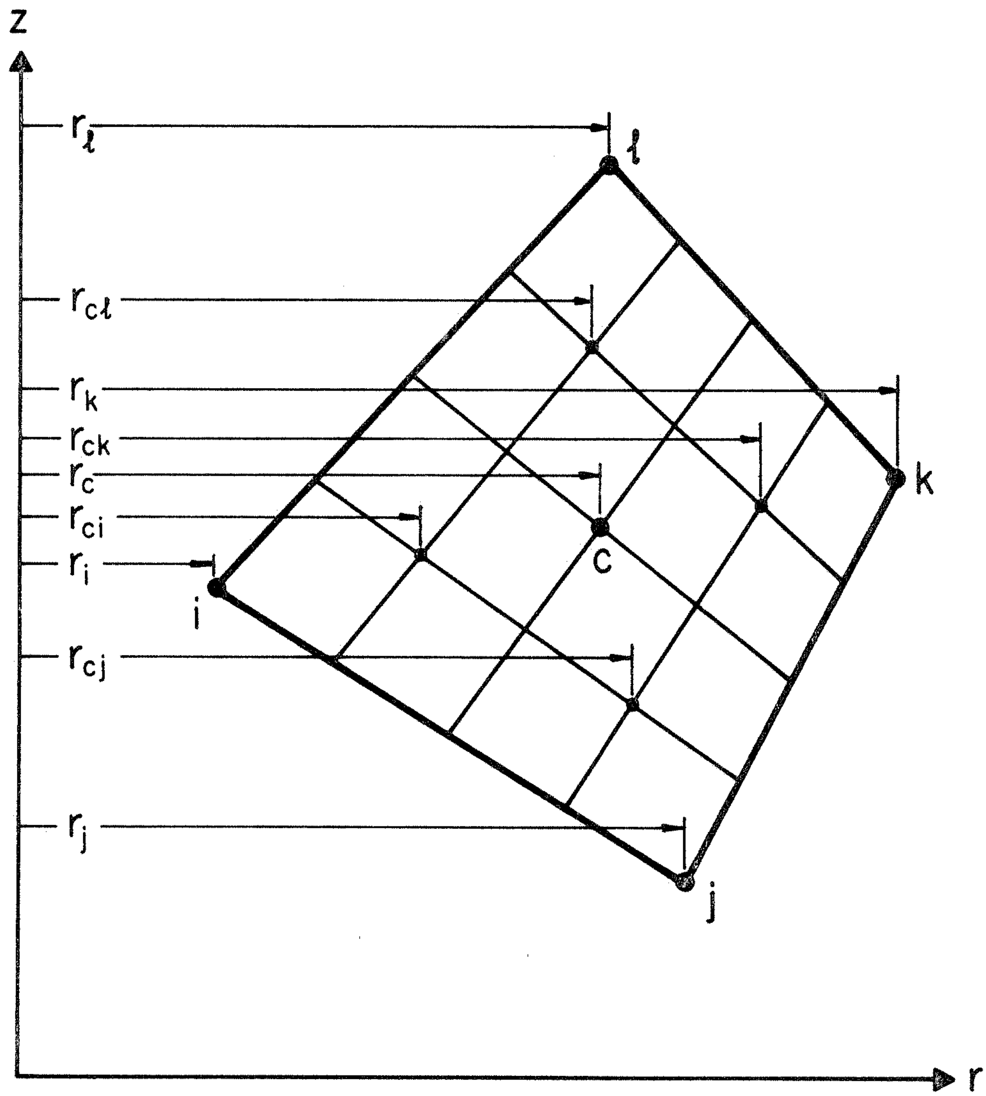


FIGURE B-1 TYPICAL QUADRILATERAL ELEMENT

The total heat capacity is lumped at each node proportional to the radius of the center of the quadrilateral subelement which contains that node. The diagonals of the lumped heat capacity matrix are:

$$c_{mm} = \alpha \, r_{cm} \qquad m = i, j, k, \ell \tag{B.4}$$

$$\alpha = \rho \, c \, v/4 \, r_c$$

APPENDIX C

INPUT SPECIFICATION FOR THE TWO-DIMENSIONAL NONLINEAR HEAT TRANSFER PROGRAM (NOHEAT)

APPENDIX C

INPUT SPECIFICATION FOR THE TWO-DIMENSIONAL NONLINEAR HEAT TRANSFER PROGRAM (NOHEAT)

A. PURPOSE

The purpose of this computer program is to determine nodal temperatures of a two-dimensional or axisymmetric body subjected to transient disturbances. The material may have temperature dependent conductance. The heat generation and temperature or flow due to any boundary condition may be time dependent. The input data must comply with the following description:

B. IDENTIFICATION CARD (72H)

Columns 1 to 72 of this card contain information to be printed with results.

C. CONTROL CARD (10I5,F10.0)

Columns	1 - 5	Number of nodal points (N)
	6 - 10	Number of elements (M)
	11 - 15	Number of different materials (K)
	16 - 20	Number of time increments
	21 - 25	Print interval
	26 - 30	Number of boundary surfaces (B)
	31 - 35	Number of increments for which the conductivity matrix is assumed to be constant
	36 - 40	Number of temperature points at which the conductivity functions are specified (T)
	41 - 45	Number of time points at which the time functions are specified (L)
	46 - 50	Number of different time dependent functions (F)
	51 - 55	Number of different pipe segments (P)
	56 - 60	Total number of pipe nodes (NP)
	61 - 70	Time increment Δt
	71 - 80	Reference number to be added to all R coordinates

If $\Delta t = 0$ the program prints a suggested time increment.

If T = 0 the conductivity is not a function of temperature for all materials. If the reference number is large as compared to dimensions of the system, the problem approaches the two-dimensional case.

D. NODAL POINT CARDS (2I5,4F10.0,I5)

One card for each nodal point with the following information:

Columns	1 - 5	Nodal point number
	6 - 10	Type of the nodal point
	11 - 20	X-ordinate
	21 - 30	Y-ordinate
	31 - 40	Initial temperature
	41 - 50	Coefficient of time dependent function for flow or temperature at the node
	51 - 55	Identification number of the function

If KK = 0 external heat flow is specified, and
If KK = 1 external temperature is specified.
If the identification number is zero, the external heat flow is zero for all times. Nodal point cards must be in numerical sequence. If cards are omitted, the omitted nodal points are generated at equal intervals along a straight line between the specified nodal points. The boundary condition code (KK), the function multiplier and the function identification number for the generated nodes are set equal to the values of the previous node.

E. ELEMENT CARDS

One card for each element with the following information:

Columns	1 - 5	Element number
	6 - 10	Nodal point I
	11 - 15	Nodal point J
	16 - 20	Nodal point K
	21 - 25	Nodal point L
	26 - 30	Material identification number
	31 - 35	Identification number of the function for heat generation

Nodal point numbers (I, J, K, and L) must be in counter-clockwise order around each element. Element cards must be in element number sequence. If element cards are omitted, the program automatically generates the omitted information by incrementing the preceding I, J, K and L. The material and function identification number for the generated elements are set equal to the values of the previous elements. For radiation element nodal points I and J are on the radiative wall which is opposite to K and L.

F. MATERIAL PROPERTY CARDS - (I5,3F10.0)

One card for each material with the following information:

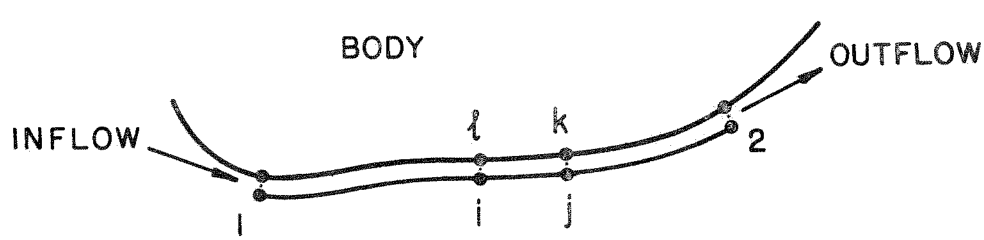
Columns	1 - 5	Material identification number
	6 - 15	Conductivity of material
	16 - 25	Specific heat of material
	26 - 35	Density of material

If columns 6 - 15 are blank, the conductivity of material must be given separately as a function of temperature. For radiative elements specific heat is not required to be specified. However, the conductivity of material is replaced by radiation coefficient (σ) and the density is set equal to zero.

G. PIPE INFORMATION CARDS (I5,4F10)

The cooling pipe is treated as follows:

Suppose a cooling pipe of constant cross-section is in contact with the body between points 1 and 2 as shown in Fig. 1.



The heat flow transferred to the body by the section of the pipe defined by nodes i, j, k, and l is:

$$Q = Ah\,(T - T_o) \qquad (C.1)$$

in which

A is the area of the surface of contact

h is the heat transfer coefficient

$T = (T_\ell + T_k)/2.$

$T_o = (T_i + T_j)/2.$

Equation C.1 and similar equations for other parts of the pipe are used to compute the nodal temperatures of the body.

The temperature T_j of node j is computed from the following recursive relation

$$T_j = [(1 - \gamma)\, T_i + \gamma\, (T_\ell + T_k)]/(1 + \gamma)$$

in which

$\gamma = Ah/2\rho vc$

ρ is the density of the liquid

c is the specific heat of the liquid, and

v is the flow of the liquid (volume/time).

The above equation assumes that the effect of the conductivity of the liquid on the heat equilibrium equation is negligible as compared to the heat transferred by the mass transfer of the fluid. One card must be supplied for each different pipe segment with the following information:

Columns	1 - 5	Pipe number
	6 - 15	Flow (volume/sec.)
	16 - 25	Specific heat of the liquid
	26 - 35	Density of the liquid
	36 - 45	Heat transfer coefficient from the pipe to the solid body

H. TIME-DEPENDENT FUNCTION CARDS (8F10.0)

Specified nodal temperatures on flow rates surface heat transfer coefficients may be a function of time. At each point in time the following information must be supplied:

First Card

Columns	1 - 10	Time (t)
	11 - 20	f_1 (t)
	21 - 30	f_2 (t)

	-----------------	
	71 - 80	f_7 (t)

Second Card - if required

Columns	1 - 10	f_8 (t)
	11 - 20	f_9 (t)

	-----------------	
	and so on	

For additional functions more cards may be used. The computer program uses linear interpolation to evaluate the functions between the specified time points.

I. BOUNDARY CONDITION CARDS (6I5,F10.0)

One card must be supplied for each boundary surface with the following information:

Columns	1 - 5	Nodal point number I
	6 - 10	Nodal point number J
	11 - 15	Function identification number for the external or radiation temperature or the temperature of the liquid at the beginning of the pipe
	16 - 20	Function identification number for free convection
	21 - 25	Function identification number for forced convection
	26 - 30	Function identification number for radiation factor (σ)
	31 - 40	Exponent of the forced convection boundary conditions

All the pipe boundary conditions must be input prior to any other types of boundary conditions. Each cooling surface boundary condition must be input in sequence starting from the node at which the flow starts. The starting node of each type of cooling pipe boundary condition is preceded by a negative sign. Columns 16 - 20 identify the pipe number which must be preceded by a negative sign.

J. TEMPERATURE FUNCTION CARDS (8F10.0)

Conductivity or radiation coefficient of each material may be a function of temperature. For each different temperature the following information must be supplied:

First Card (8F10.0)

Columns	1 - 10	Temperature
	11 - 20	Conductivity for material 1
	21 - 30	Conductivity for material 2
	------------------	------------------
	------------------	------------------
	71 - 80	Conductivity for material 7

Second Card - if required

Columns	1 - 10	Conductivity for material 8
	------------------	------------------
	------------------	------------------

If more materials are used additional cards with the same format must be supplied. For a given material, the time-dependent conductivity is used only if the conductivity given with the Material Property Information is left blank. The computer program uses linear interpolation to determine the conductivity at points between the specified temperatures.

K. PROGRAM LENGTH

The dimension of array A must not exceed NLIMIT

$$NLIMIT = (9 + \text{Band Width}) * N + 7M + 3K + 8B + 6P + NP + (F + 1) * L + (K + 1) * T$$

APPENDIX D

FORTRAN IV PROGRAM LISTING

```
      PROGRAM NOHEAT (INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
      COMMON NNOD,NELM,NMAT,NPRINT,NT,XA,NP     ,MBAND,DELT,TM,KKK,NST,
     INTIM,NBC,NPIPE,IK,DT,NTEM,HED(12),A(30000)
C--------  READ AND PRINT GLOBAL INFORMATIONS.
  100 READ (5,1000) HED,NNOD,NELM,NMAT,NT,NPRINT,NBC,NST,NTEM,NTIM,NTF,
     1 NPIPE,NP,DELT,XA
      WRITE (6,2001) HED,NNOD,NELM,NMAT,NT,NPRINT,NBC,NST,NTEM,NTIM,NTF,
     1 NPIPE,NP,DELT
C--------  EVALUATE INITIAL ADDRESS OF EACH ARRAY.
      NTF=NTF+1
      N1=1
      N2=N1+NNOD
      N3=N2+NNOD
      N4=N3+NNOD+NP
      N5=N4+NNOD
      N6=N5+NNOD
      N7=N6+5*NELM
      N8=N7+NMAT
      N9=N8+NMAT
      N10=N9+NMAT
      N11=N10+NPIPE
      N12=N11+NPIPE
      N13=N12+NPIPE
      N14=N13+NPIPE
      N15=N14+NPIPE
      N16=N15+NPIPE
      N17=N16+(NMAT+1)*NTEM
      N18=N17+NBC
      N19=N18+NBC
      N20=N19+4*NBC
      N21=N20+NBC
      N22=N21+NTF*NTIM
      N23=N22+NNOD
      N24=N23+NBC
      N25=N24+NELM
      CALL DATAIN (A(N1),A(N2),A(N3),A(N4),A(N5),A(N6),A(N7),A(N8),A(N9)
     1 , A(N10),A(N11),A(N12),A(N13),A(N14),A(N15),A(N16),A(N17),A(N18),
     2 A(N19),A(N20),A(N21),A(N22),A(N23),A(N24),NTEM,NTF)
      N26=N25+NNOD
      N27=N26+NNOD
      N28=N27+NNOD
      N29=N28+NELM
      N30=N29+NNOD*MBAND
      IF( N30.LT. 30000) GO TO 200
      WRITE (6,2000)
      STOP
  200 DO 300 I=N25,N30
  300 A(I)=0.
      CALL SOLVE(A(N1),A(N2),A(N3),A(N4),A(N5),A(N6),A(N7),A(N8),A(N9),
     1 A(N10),A(N11),A(N12),A(N13),A(N14),A(N15),A(N16),A(N17),A(N18),
     2 A(N19),A(N20),A(N21),A(N22),A(N23),A(N24),A(N25),A(N26),
     3 A(N27),A(N28),A(N29),NNOD,NTEM,NTF)
      GO TO 100
 1000 FORMAT (12A6/12I5,2F10.0)
 2000 FORMAT(28H DIMENSION.OF.A.IS.EXCEEDED    )
 2001 FORMAT (1H1,12A6//
```

```
     1          35H NUMBER OF NODAL POINTS------------   I5 //
     2          35H NUMBER OF ELEMENTS----------------   I5 //
     3          35H NUMBER OF MATERIALS---------------   I5 //
     4          35H NUMBER OF CYCLES------------------   I5 //
     5          35H PRINT INTERVAL--------------------   I5 //
     6          35H NUMBER OF BOUNDARY SIDES----------   I5 //
     7          35H CONDUCTIVITY CHANGE INTERVAL------   I5 //
     8          35H NUMBER OF TEMPRATURE POINTS-------   I5 //
     9          35H NUMBER OF TIME POINTS-------------   I5 //
     0          35H NUMBER OF TIME FUNCTIONS----------   I5 //
     1          35H NUMBER OF VARIOUS PIPE SHAPES-----   I5 //
     2          35H TOTAL NUMBER OF PIPE NODES--------   I5 //
     3          35H TIME INCREMENT--------------------F10.6  )
      END
```

```
      SUBROUTINE DATAIN (X,Y,T,CF,ID,IX,XCON,CP,RO,PA,PL,PRC,V,PK,PC,
     1  TFUN,IBC,JBC,IDB,CBC,TIF,IDF,XL,IH,NTEM,NTF)
      DIMENSION X(1),Y(1),T(1),CF(1),ID(1),IX(5,1),XCON(1),CP(1),RO(1),
     1 TFUN(NTEM,1),IBC(1),JBC(1),IDB(4,1),CBC(1),TIF(NTF,1),IDF(1),
     2 XL(1),IH(1),PC(1),PA(1),PL(1),PRC(1),V(1),PK(1)
      COMMON NNOD,NELM,NMAT,NPRINT,NT,XA,NP      ,MBAND,DELT,TM,KKK,NST,
     1NTIM,NBC,NPIPE
C--------  READ AND GENERATE,AND PRINT NODAL POINT DATA
      WRITE (6,2000)
      L=0
   60 READ (5,1000) N,ID(N),X(N),Y(N),T(N),CF(N),IDF(N)
      X(N)=X(N)+XA
      IF(L.EQ.0) GO TO 85
      ZX=N-L
      DX=(X(N)-X(L))/ZX
      DY=(Y(N)-Y(L))/ZX
   85 NL=L+1
   70 L=L+1
      M=L-1
      IF (N-L) 100,90,80
   80 ID(L)=ID(M)
      X(L)=X(M)+DX
      Y(L)=Y(M)+DY
      T(L)=T(M)
      CF(L)=CF(M)
      IDF(L)= IDF(M)
      GO TO 70
   90 WRITE (6,2001) (K,ID(K),X(K),Y(K),T(K),CF(K),IDF(K),K=NL,N)
      IF (NNOD-N) 100,110,60
  100 WRITE (6,2002) N
      STOP
  110 CONTINUE
C--------  READ AND GENERATE,AND PRINT ELEMENT DATA
      WRITE (6,2003)
      N=0
      MBAND=0
  130 READ (5,1001) M,(IX(I,M),I=1,5),IH(M)
  140 N=N+1
      IF (M.EQ.N) GO TO 145
      K=N-1
      DO 142 I=1,4
  142 IX(I,N)=IX(I,K)+1
      IH(N)=IH(K)
      IX(5,N)=IX(5,K)
C--------  DETERMINE THE BAND WIDTH
  145 MB=0
      DO 160 I=1,4
      DO 160 J=I,4
      MM=IABS(IX(I,M)-IX(J,M))
      IF(MM.GT.MB) MB=MM
  160 CONTINUE
      MB=MB+1
      IF (MB.GT.MBAND) MBAND=MB
      WRITE (6,2004) N,(IX(I,N),I=1,5),MB,IH(N)
      IF (N.EQ.NELM) GO TO 700
      IF (N.EQ.M) GO TO 130
```

```
      GO TO 140
  700 CONTINUE
C--------  READ AND PRINT  MATERIAL PROPERTIES.
      READ (5,1002)(MTYPE,XCON(MTYPE),CP(MTYPE),RO(MTYPE),I=1,NMAT)
      WRITE (6,2005) (I,XCON(I),CP(I),RO(I),I=1,NMAT)
C--------  PIPE  ELEMENT IDENTIFICATIONS
      IF (NPIPE.EQ.0) GO TO 62
      READ (5,1006) (J,PA(J),PRC(J),V(J),PC(J),J=1,NPIPE)
      WRITE (6,2010) (J,PA(J),PC(J),V(J),PRC(J),J=1,NPIPE)
C--------  READ AND PRINT ALL TIME FUNCTIONS.
   62 IF (NTIM.EQ.0) GO TO 30
      DO 52 I=1,NTIM
   52 READ (5,1005) ( TIF(J,I),J=1,NTF)
      N=NTF+9
      NF=10
      NS=2
   40 IF (NF.GT.NTF) NF=NTF
      NN=NS-1
      NK=NF-1
      WRITE (6,2008) (I,I=NN,NK)
      DO 45 I=1,NTIM
   45 WRITE (6,2016) TIF(1,I),(TIF(J,I),J=NS,NF)
      NS=NF+1
      NF=NF+9
      IF (N.NE.NF) GO TO 40
C--------  READ AND PRINT EXPONENTS OF CONVECTION BOUNARIES.
   30 IF (NBC .EQ.0) GO TO 61
C--------  READ AND PRINT BOUNDARY CONDITION PROPERTIES.
      N=0
      DO 57 I=1,NBC
      READ (5,1004) IBC(I),JBC(I),( IDB(J,I),J=1,4),CBC(I)
      II=IBC(I)
      IJ=JBC(I)
      IF(IDB(2,I).GE.0) GO TO 57
      IF(II.GT.0) GO TO 53
      II=-II
      N=N+1
      K=IDB(1,I)+1
      T(NNOD+N)=TIF(K,1)
   53 N=N+1
      T(NNOD+N)=T(IJ)
   57 XL(I)=   SQRT((X(II)-X(IJ))**2+(Y(II)-Y(IJ))**2)*(X(II)+X(IJ))*.25
      WRITE(6,2007)(IBC(I),JBC(I),( IDB(J,I),J=1,4),CBC(I),I=1,NBC)
C--------  READ AND PRINT CONDUCTIVITY OF MATERIALS V.S. TEMPRATURE.
   61 IF (NTEM.EQ.0) RETURN
      M=NMAT+1
      DO 51 I=1,NTEM
   51 READ (5,1003) ( TFUN(I,N),N=1,M)
      N=M+9
      NF=10
      NS=2
   50 IF(NF.GT.M) NF=M
      NN=NS-1
      NK=NF-1
      WRITE (6,2006) (I,I=NN,NK)
      DO 55 I=1,NTEM
```

```
   55 WRITE (6,2016) TFUN(I,1),(TFUN(I,J),J=NS,NF)
      NS=NF+1
      NF=NF+9
      IF (N.NE.NF) GO TO 50
      RETURN
 1000 FORMAT (2I5,4F10.0,I5)
 1001 FORMAT (7I5)
 1002 FORMAT (I5,3F10.0)
 1003 FORMAT (8F10.0)
 1004 FORMAT (6I5,F10.0)
 1005 FORMAT (8F10.0)
 1006 FORMAT (I5,4F10.0)
 2000 FORMAT (72H1 NODE  TYPE     X          Y       TEMPRATURE    MULTIPLIE
     1R     FUNCTION NO.    /)
 2001 FORMAT (2I6,2F9.2,E15.5,F12.3,I5)
 2002 FORMAT ( 27H ERROR IN NODAL POINT DATA    I5   )
 2003 FORMAT (65H1 ELEMENT    I       J       K       L    MATERIAL  BAND
     1HEAT I.D.     /)
 2004 FORMAT (I5,5I7,I10,I5)
 2005 FORMAT (1H1,(10H MATERIAL=  I3/
     1 21H          CONDUCTIVITY = E12.5 //
     2 21H          SPECIFIC HEAT= E12.5 //
     3 21H          DENSITY      = E12.5 // ))
 2006 FORMAT (15H1    TEMPRATURE   ,I6,8I12)
 2007 FORMAT (1H1,32X,31H FUNCTION IDENTIFICATION NO.      /
     1 89H        I        J    EXTER. TEMP.  FREE CONVEC.  FORCED CONVEC.  R
     2AD. FACTOR  CONVECTION EXP. /(2I7,I9,3I15,E15.5))
 2008 FORMAT (13H1    TIME        I6,8I12    )
 2010 FORMAT (71H1 NUMBER    PIPE FLOW   PIPE CONVEC.  FLUID DENSITY  FLU
     1ID SPECIFIC HEAT    / (I6,4E14.4))
 2011 FORMAT (I5,5I7,I10,16H  PIPE NUMBER =  I5)
 2016 FORMAT (10E12.5)
      END
```

```
      SUBROUTINE FLOTEM(    IK  ,TAVE,TFUN,NT)
      COMMON / ELM / S(5,5),LM(5),MTYPE,COND,XX
      DIMENSION TFUN(NT,1)
C---------   FIND THE CONDUCTANCE OF THE MATERIAL
      M=MTYPE+1
  200 IF  (TAVE.GE.TFUN(IK,1).AND.TAVE.LE.TFUN(IK+1,1)) GO TO 300
      IF  (TAVE.GT.TFUN(IK+1,1))  IK=IK+1
      IF  (TAVE.LT.TFUN(IK,1)  )  IK=IK-1
      GO TO 200
  300 DT=TFUN(IK+1,1)-TFUN(IK,1)
      DC=TFUN(IK+1,M)-TFUN(IK,M)
      DTEM=(TAVE- TFUN(IK,1))/DT
      COND = TFUN(IK,M)+DTEM*DC
      RETURN
      END
```

```
      SUBROUTINE RADIAT (TAVE,X,Y)
      DIMENSION X(1),Y(1)
      COMMON / ELM / S(5,5),LM(5),MTYPE,COND,XX
      I=LM(1)
      J=LM(2)
      K=LM(3)
      L=LM(4)
      XX=(X(I)+X(J)+X(K)+X(L))/4.
      DO 100 I=1,4
      DO 100 J=1,4
  100 S(I,J)=0.
      AIL=SQRT((X(I)-X(L))**2+(Y(I)-Y(L))**2)
      AJL=SQRT((X(J)-X(L))**2+(Y(J)-Y(L))**2)
      AJK=SQRT((X(J)-X(K))**2+(Y(J)-Y(K))**2)
      AIK=SQRT((X(I)-X(K))**2+(Y(I)-Y(K))**2)
      A=(AIK+AJL-AIL-AJK)/4.
      COND=COND*A*(TAVE+460.)**3
      S(1,1)=COND
      S(2,2)= COND
      S(3,3)=-COND
      S(4,4)=-COND
      RETURN
      END
```

```
      SUBROUTINE CONDUC (IX,X,Y)
      COMMON / ELM / S(5,5),LM(5),MTYPE,COND,XX
      DIMENSION E(3,3),KX(3),X(1),Y(1),IX(5)
      DO 150 I=1,5
      LM(I)=IX(I)
      DO 150 J=1,5
  150 S(I,J)=0.
      I=LM(1)
      J=LM(2)
      K=LM(3)
      L=LM(4)
      LM(5)=I
      XX=(X(I)+X(J)+X(K)+X(L))/4.
      YY=(Y(I)+Y(J)+Y(K)+Y(L))/4.
      DO 152 K=1,4
      I=LM(K)
      J=LM(K+1)
      IF(I-J) 135,152,135
  135 AJ=X(J)-X(I)
      AK=XX-X(I)
      BJ=Y(J)-Y(I)
      BK=YY-Y(I)
      C=BJ-BK
      DX=AK-AJ
      XLAM=AJ*BK-AK*BJ
      IF (XLAM.LE.0.) RETURN
  136 COM =COND*(X(I)+X(J)+XX)/(6.*XLAM)
      E(1,1)=C**2+DX**2
      E(1,2)=BK*C-AK*DX
      E(1,3)=-BJ*C+AJ*DX
      E(2,1)=E(1,2)
      E(2,2)=BK**2+AK**2
      E(2,3)=-BJ*BK-AJ*AK
      E(3,1)=E(1,3)
      E(3,2)=E(2,3)
      E(3,3)=BJ**2+AJ**2
      KX(1)=K
      KX(2)=K+1
      IF (K-4) 145,140,145
  140 KX(2)=1
  145 KX(3)=5
      DO 151 I=1,3
      II=KX(I)
      DO 151 J=1,3
      JJ=KX(J)
  151 S(II,JJ)=S(II,JJ)+E(I,J)*COM
  152 CONTINUE
C--------   ELEMINATE THE CENTER NODE
      DO 155 I=1,4
      DO 155 J=1,4
  155 S(I,J)=S(I,J)-S(I,5)*S(J,5)/S(5,5)
      RETURN
      END
```

```
      SUBROUTINE ADCON (RO,IX,X,Y,A,NEQ,TAVE,N)
      COMMON / ELM / S(5,5),LM(5),MTYPE,COND,XX
      DIMENSION RO(1),IX(5),X(1),Y(1),A(NEQ,1)
C--------  FORM THE TATAL CONDUCTIVITY MATRIX A
      IF(RO(MTYPE).EQ.0.) GO TO 100
      CALL CONDUC (IX,X,Y)
      IF (S(1,1).NE.0.) GO TO 200
      WRITE (6,2003) N
      STOP
  100 CALL RADIAT (TAVE,X,Y)
  200 DO 175 L=1,4
      I=LM(L)
      DO 175 M=1,4
      J=LM(M)-I+1
      IF (J.GT.0) A(I,J)=A(I,J)+S(L,M)
  175 CONTINUE
      RETURN
 2003 FORMAT ( 27H ELEMENT WITH NEGATIVE AREA  I5)
      END
```

```
      SUBROUTINE FLOW(TIF,IDB,Q,A,T,NEQ,IBC,JBC,XL,CF,PC,IDF,NTF,CBC)
      DIMENSION  TIF(NTF,1),T(1),A(NEQ,1),Q(1), IDB(4,1),F(4),IBC(1),
     1 JBC(1),XL(1),CBC(1)    ,CF(1),IDF(1),PC(1)
      COMMON / ELM / S(5,5),LM(5),MTYPE,COND,XX
      COMMON NNOD,NELM,NMAT,NPRINT,NT,XA,NP      ,MBAND,DELT,TM,KKK,NST,
     1NTIM,NBC,NPIPE,IK,DT
      DO 700 I=1,NEQ
  700 Q(I)=0.
  100 TAU=TM
  200 IF (TAU.GE.TIF(1,IK).AND.TAU.LE.TIF(1,IK+1))GO TO 300
      IF (TAU.GT.TIF(1,IK+1))IK=IK+1
      IF (TAU.LT.TIF(1,IK)) IK=IK-1
      GO TO 200
  300 D=TIF(1,IK+1)-TIF(1,IK)
      DT=(TAU-TIF(1,IK))/D
      IF (NBC.EQ.0) GO TO 211
C--------   DUE TO FORCED AND FREE CONVECTION (S2,S3) AND RADIATION (S1)
C--------   FIND FLOW VECTOR  Q  AND CHANGE THE CONDUCTIVITY MATRIX  A
C--------   BOUNDARY CONDITIONS
      MM=0
      DO 215 N=1,NBC
      DO 400 M=1,4
      F(M)=0.
      K=IDB(M,N)
      IF( K.LE.0) GO TO 400
      K=K+1
      DH=TIF(K,IK+1)-TIF(K,IK)
      F(M)=TIF(K,IK)+DT*DH
  400 CONTINUE
      II=-IDB(2,N)
      I=IBC(N)
      J=JBC(N)
      TAVE=(T(I)+T(J))/2.
      S4=0.
      IF(II.LE.0) GO TO 57
C--------   COOLING PIPE
      SIG=0.
      S1=0.
      S2=0.
      IF(I.GT.0) GO TO 53
      I=-I
      MM=MM+1
      T(NNOD+MM)=F(1)
   53 MNOD=NNOD+MM
      S3=(T(MNOD)+T(MNOD+1))*PC(II)/2.
      MM=MM+1
      TC=PC(II)
      GO TO 58
   57 CONTINUE
      TC=F(2)
      IF(II.LT.0) CONV=CBC(N)
      IF(F(1).NE.TAVE. OR.F(2).NE.0.) TC=ABS(F(1)-TAVE)**CONV*F(2)
      IF(IDB(3,N).GT.0) BETA=CBC(N)
      IF(F(3).GT.0.) S4=F(3)**BETA
      S2=S4*F(1)
      S3=TC*F(1)
```

```
      F(1)=F(1)+460.
      TAVE=TAVE+460.
      SIG=(F(1)**2+TAVE**2)*F(4)
      S1=SIG*(F(1)**2-460.*TAVE)
   58 CONTINUE
      R=(S1+S2+S3)*XL(N)
      Q(I)=Q(I)+R
      Q(J)=Q(J)+R
      IF (KKK.EQ.0) GO TO 215
      TC=(TC+S4+SIG*TAVE)*XL(N)/2.
      A(I,1)=TC+A(I,1)
      A(J,1)=TC+A(J,1)
      K=J-I+1
      IF (K) 212,212,210
  210 A(I,K)=A(I,K)+TC
      GO TO 215
  212 K=I-J+1
      A(J,K)=A(J,K)+TC
  215 CONTINUE
  211 CONTINUE
C--------   CHANGE OF THE TEMPRATURE TO FLOW BUNDARY CONDITION
      DO 500 I=1,NNOD
      K=IDF(I)+1
      IF (K.EQ.1) GO TO 500
      DH=TIF(K,IK)+DT*(TIF(K,IK+1)-TIF(K,IK))
  600 Q(I)=Q(I)+CF(I)*DH
  500 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE SOLVE  (X,Y,T,CF,ID,IX,XCON,CP,RO,PA,PL,PRC,V,PK,PC,
     1 TFUN,IBC,JBC,IDB,CBC,TIF,IDF,XL,IH,Q,E,C,VOL,A,NEQ,NTEM,NTF)
      DIMENSION X(1),Y(1),T(1),CF(1),ID(1),IX(5,1),XCON(1),CP(1),RO(1),
     1 TFUN(NTEM,1),IBC(1),JBC(1), IDB(4,1),TIF(NTF,1),Q(1),E(1),C(1),
     2  A(NEQ,1),IH(1),VOL(1),IDF(1),XL(1),PA(1),PL(1),PC(1),CBC(1),
     3 PRC(1),PK(1),V(1)
      COMMON NNOD,NELM,NMAT,NPRINT,NT,XA,NP      ,MBAND,DELT,TM,KKK,NST,
     1NTIM,NBC,NPIPE,IK,DT
      COMMON / ELM / S(5,5),LM(5),MTYPE,COND,XX
C---------  INITIAL CONDUCTIVITY AND CAPACITY MATRICES (A,C)
      MNOD=NNOD+NP
      DO 375 N=1,NELM
      I=IX(1,N)
      J=IX(2,N)
      K=IX(3,N)
      L=IX(4,N)
      MTYPE=IX(5,N)
      IKT=1
      TAVE=(T(I)+T(J)+T(K)+T(L))/4.
      COND=XCON(MTYPE)
C---------  FIND CONDUCTANCE OF THE ELEMENT N
      IF (COND.EQ.0.) CALL FLOTEM (  IKT,TAVE,TFUN      ,NTEM)
  600 CALL ADCON (RO,IX(1,N),X,Y,A,NNOD,TAVE,N)
      VOL(N)=((X(I)-X(K))*(Y(J)-Y(L))-(X(J)-X(L))*(Y(I)-Y(K)))/8.*XX
      DIN=VOL(N)*RO(MTYPE)* CP(MTYPE)
      IF(ID(I).EQ.0) C(I)=C(I)+DIN
      IF(ID(J).EQ.0) C(J)=C(J)+DIN
      IF(ID(K).EQ.0) C(K)=C(K)+DIN
      IF(ID(L).EQ.0) C(L)=C(L)+DIN
  375 CONTINUE
      ENORM=0.
      CNORM=0.
      DO 220 J=1,NEQ
      CNORM=CNORM+C(J)
  220 ENORM=ENORM+A(J,1)
      ENORM=ENORM*1000000.
      DO 230 I=1,NEQ
      IF(ID(I).EQ.0) GO TO 230
      A(I,1)=ENORM
      CF(I)=CF(I)*ENORM
      IF(ID(I).EQ.1) ID(I)=0
      C(I)=0.
  230 CONTINUE
      IK=1
      KKK=1
      TM=TIF(1,1)
C---------  CHANGE OF THE CONDUCTIVITY MATRIX  A  DUE TO THE BOUNDARY
C---------  CONDITIONS
      CALL       FLOW(TIF,IDB,Q,A,T,NEQ,IBC,JBC,XL,CF,PC,IDF,NTF,CBC)
C---------  INITIAL RESISTANCE VECTOR  E
      DO 100 J=2,NEQ
      DO 102 I=2,MBAND
      JJ=I+J-1
  102 E(J)=E(J)+T(JJ)*A(J,I)
      K=0
      IF (J.GT.MBAND) K=J-MBAND
```

```
      L=J-K+1
      II=K+1
      DO 101 M=II,J
      L=L-1
      K=K+1
  101 E(J)=E(J )+T(M)*A(K,L)
  100 CONTINUE
      DO 200 I=1,MBAND
  200 E(1)=E(1)+T(I)*A(1,I)
  888 KKK=0
      IF(DELT.GT.0.) GO TO 300
      DELT=CNORM/(ENORM+ENORM) *1000000.
      WRITE (6,2000) DELT
      STOP
C-------- REVISE THE CONDUCTIVITY AND CAPACITY MATRICES  (A,C)
  300 DO 400 I=1,NEQ
      IF(C(I).NE.0.) GO TO 210
      E(I)=0.
      GO TO 400
  210 C(I)=C(I)/DELT
      A(I,1)=A(I,1)+C(I)
  400 CONTINUE
      CALL TRIA (NNOD,MBAND,A)
      NNN=0
      KK=0
      LL=0
C-------- START THE STEP BY STEP PROCEDURE
  500 NNN=NNN+1
      TM=TM+DELT
C-------- EVALUATE FLOW VECTOR  Q  AND REVISE THE CONDUCTIVITY MATRIX A
      CALL        FLOW(TIF,IDB,Q,A,T,NEQ,IBC,JBC,XL,CF,PC,IDF,NTF,CBC)
C-------- ADD THE SHARE OF INTERNAL HEAT GENERATION TO FLOW VECTOR  Q
  211 DO 217 I=1,NELM
      K=IH(I)+1
      IF(K.EQ.1) GO TO 217
      RQ=(TIF(K,IK)+DT*(TIF(K,IK+1)-TIF(K,IK)))*VOL(I)
      DO 216 J=1,4
      II=IX(J,I)
      IF(C(II).NE.0.) Q(II)=Q(II)+RQ
  216 CONTINUE
  217 CONTINUE
C-------- EFFECTIVE FLOW VECTOR  Q
      DO 460 I=1,NEQ
      TEMP=Q(I)
      Q(I)=Q(I)-E(I)
      E(I)=TEMP
  460 CONTINUE
C-------- SOLVE FOR THE INCREMENT OF TEMPRATURE VECTOR  Q
      IF (KKK.EQ.1) CALL TRIA (NNOD,MBAND,A)
      CALL BACKS (NNOD,MBAND,A,Q)
C-------- COMPUTE AND PRINT THE TEMPRATURE VECTOR  T  AT TIME  TM
      DO 480 I=1,NEQ
      IF(C(I).EQ.0.) GO TO 480
      T(I)=Q(I)+T(I)
      E(I)=E(I)-C(I)*Q(I)
  480 CONTINUE
```

```
      IF( NPIPE.EQ.0) GO TO 10
      N=0
      DO 11 I=1,NBC
      M=-IDB(2,I)
      IF(M.LE.0) GO TO 11
      II=IBC(I)
      JJ=JBC(I)
      IF(II.GT.0) GO TO 12
      II=-II
      N=N+1
      C2=PC(M)/(V(M)*PRC(M)*PA(M))*6.28318
   12 CO= XL(I)*C2
      T(NNOD+N+1)=((1.-CO)*T(NNOD+N)+CO*(T(II)+T(JJ)))/(1.+CO)
      N=N+1
   11 CONTINUE
   10 CONTINUE
      LL=LL+1
      IF (LL.NE.NPRINT ) GO TO 499
      LL=0
  421 WRITE (6,2006) TM
      MNOD=NNOD+NP
  482 WRITE (6,2008)(N,T(N),N=1,MNOD)
  499 IF (NNN.EQ.NT) RETURN
      KKK=0
      KK=KK+1
C--------- CHANGE THE CONDUCTIVITY MATRIX  A  IF DESIRED
      IF (KK.NE.NST) GO TO 500
      DO 497 N=1,NELM
      KKK=1
      IF (N.GT.1) GO TO 424
      DO 425 I=1,NEQ
      IF (A(I,1).NE.0.) A(I,1)=C(I)
      DO 425 J=2,MBAND
  425 A(I,J)=0.
  424 I=IX(1,N)
      J=IX(2,N)
      K=IX(3,N)
      L=IX(4,N)
      MTYPE=IX(5,N)
      TAVE=(T(I)+T(J)+T(K)+T(L))/4.
      COND=XCON(MTYPE)
      IF (COND.EQ.0.) CALL FLOTEM (   IKT,TAVE,TFUN       ,NTEM)
  550 CALL ADCON (RO,IX(1,N),X,Y,A,NNOD,TAVE,N)
  497 CONTINUE
      DO 498 I=1,NEQ
      IF (C(I).EQ.0.) A(I,1)=ENORM
  498 CONTINUE
      KK=0
      GO TO 500
 2000 FORMAT (18H1TIME INCREMENT =    E7.1)
 2006 FORMAT ( 6H1TIME=   F10.6/6(20H  NODE    TEMPRATURE    ))
 2008 FORMAT (6(I5,E15.5))
      END
```

```
      SUBROUTINE BACKS(NN,MM,A,B)
      DIMENSION A(1),B(1)

      MMM=MM-1
      N=0
  270 N=N+1
      C=B(N)
      IF(A(N).NE.0.0) B(N)=B(N)/A(N)
      IF(N.EQ.NN) GO TO 300
      IL=N+1
      IH=MINO(NN,N+MMM)
      M=N
      DO 285 I=IL,IH
      M=M+NN
  285 B(I)=B(I)-A(M)*C
      GO TO 270

  300 IL=N
      N=N-1
      IF(N.EQ.0) RETURN
      IH=MINO(NN,N+MMM)
      M=N
      DO 400 I=IL,IH
      M=M+NN
  400 B(N)=B(N)-A(M)*B(I)
      GO TO 300
      END
```

```
      SUBROUTINE TRIA(NEQ,M,A)
      DIMENSION A(1)
C
      NE=NEQ-1
      MN=M-1
      MM=MN*NEQ
      MK=NEQ-MN
      DO 300 N=1,NE
      NT=N-MK
      IF(NT.GT.0) MM=MM-NEQ
      IF(A(N).EQ.0.0) GO TO 300
      L=N
      IL=N+NEQ
      IH=N+MM
      DO 200 I=IL,IH,NEQ
      L=L+1
      J=L
   90 C=A(I)/A(N)
      DO 100 K=I,IH,NEQ
      A(J)=A(J)-C*A(K)
  100 J=J+NEQ
      A(I)=C
  200 CONTINUE
  300 CONTINUE
      RETURN
      END
```

APPENDIX E

SAMPLE INVESTIGATION EMPLOYING NOHEAT

APPENDIX E

SAMPLE INVESTIGATION EMPLOYING NOHEAT

Although most of the operating procedures and program capabilities have been described, many points concerned with the operation of the NOHEAT program may have been overlooked. A sample investigation conducted by NOHEAT will be included in this report. The sample problem employed in this investigation is basically a simple case that could be treated by NOHEAT, and still requires the use of most subroutines available in the computer program.

1. Statement of the Problem

Consider an axisymmetric body a section of which is shown in figure E-1. The body is undergoing a heat transient.

The density and the specific heat of the material are:

$$\rho = 150 \text{ lb/ft}^3$$

$$c = .25 \text{ BTU/lb/oF}$$

The conductivity of the material is dependent on the temperature of the body as defined in the following table:

Temperature (^oF)	Conductivity (BTU/hr/ft/oF)
- 1000	.5
0	.5
2000	.3

At time zero the temperature of the body is 1000^{oF}. At later instances the temperature of nodes 5 and 13 are kept at zero $^{(oF)}$, while the temperature of the other nodes is allowed to vary freely. Heat generating units with linearly increasing capacity from 0 at time $t = 0$ to 1000, at time $t = 100$ (hr) are placed in elements 2, 5, 8, and 11.

Surface 1 is exposed to zero (oF) temperature, and exchanges heat in a free coneefficient and the exponent of the heat flow (equation 3.1 of the report) are .2 and .5 respectively. Surface 2 and its surrounding atmosphere, which is at 100 (of) temperature, exchange heat in radiation from. Surface 3 is cooled by a pipe line. Water flows in at a rate of 5 ft^3/hr and with zero (oF) temperature. Only 50 per cent of the total heat flow is exchanged between the pipe and the body. The density and the specific heat of water are 62.4 lb/ft^3 and 1 BTU/lb respectively. Finally surface 4 gains heat by the forced convection form of heat transfer. The base and the exponent of the heat flow (equation 3.6 of the report) are 100 BTU/hr and 1.

The goal of the analysis is to find the temperature distribution within the cylindrical body. The necessary input data is recorded on the next page.

AXISYMMETRIC BODY SUBJECTED TO VARIOUS HEAT INPUT.

```
20    12     1    10     1    14     2     3     2     5     1     5    .2     1.
 1                                1000.
 4                       3.       1000.
 5     1     1.                   1000.        1.          1
 6           1.          1.       1000.
 8           1.          3.       1000.
 9           2.                   1000.
12           2.          3.       1000.
13     1     3.                   1000.        1.          1
14           3.          1.       1000.
16           3.          3.       1000.
17           4.                   1000.
20           4.          3.       1000.
 1     1     5     6     2     1
 2     2     6     7     3     1     2
 3     3     7     8     4     1
 4     5     9    10     6     1
 5     6    10    11     7     1     2
 6     7    11    12     8     1
 7     9    13    14    10     1
 8    10    14    15    11     1     2
 9    11    15    16    12     1
10    13    17    18    14     1
11    14    18    19    15     1     2
12    15    19    20    16     1
 1               .25        150.
 1    5.          1.        62.4        .5
                                  .2     .1713E-08    100.
 100.                  1000.      .2     .1713E-08    100.
 4     8     1    -1
 8    12     1    -1
12    16     1    -1
16    20     1    -1
 1     5     2     3     0     0    .5
 5     9     2     3     0     0    .5
 9    13     2     3     0     0    .5
13    17     2     3     0     0    .5
17    18     1     0     0     4
18    19     1     0     0     4
19    20     1     0     0     4
 1     2     0     0     5     0    1.
 2     3     0     0     5     0    1.
 3     4     0     0     5     0    1.
-1000.      .5
            .5
2000.       .1
```