Lawrence Berkeley National Laboratory
Recent Work

**Title**
TWO CLASSES OP DUAL MODELS WITH SPONTANEOUS SYMMETRY BREAKING

**Permalink**
https://escholarship.org/uc/item/3gr4q7d0

**Author**
Schvarz, John H.

**Publication Date**
1972-07-01

eScholarship.org Powered by the California Digital Library
University of California

RECEIVED
LAWRENCE
RADIATION LABORATORY

Submitted to Nuclear Physics

LBL-1013
Preprint

LIBRARY AND
DOCUMENTS SECTION

# TWO CLASSES OF DUAL MODELS WITH SPONTANEOUS SYMMETRY BREAKING

John H. Schwarz

July 21, 1972

AEC Contract No. W-7405-eng-48

For Reference

Not to be taken from this room

LBL-1013

## DISCLAIMER

This document was prepared as an account of work sponsored by the United States Government. While this document is believed to contain correct information, neither the United States Government nor any agency thereof, nor the Regents of the University of California, nor any of their employees, makes any warranty, express or implied, or assumes any legal responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by its trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof, or the Regents of the University of California. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof or the Regents of the University of California.

# TWO CLASSES OF DUAL MODELS WITH SPONTANEOUS SYMMETRY BREAKING *

John H. Schwarz †

Lawrence Berkeley Laboratory
Berkeley, California 94720

and

Joseph Henry Laboratories
Princeton University
Princeton, New Jersey 08540

July 21, 1972

## ABSTRACT

Two new classes of dual models are presented. One required technique is the introduction of spontaneous symmetry breaking by means of assigning a nonvanishing vacuum expectation value to the dual model operator $L_0$. Another is the use of "noncanonical" spaces--spaces that cannot be generated by use of the ordinary a and b type oscillators. The first class of models is charaterized by two nonnegative integers (M,N) and has a ground-state mass $\mu_0$ given by $\alpha'\mu_0^2 = \frac{3}{2}M + 2N - 1$. These models possess L-type (Virasoro) gauge operators.

---

* Research supported in part by the U.S. Atomic Energy Commission under contract AT (11-1)-3072 and in part by the U.S. Atomic Energy Commission under Contract W-7405-eng-48.

† Participating guest at the Lawrence Berkeley Laboratory. Address after Sept. 1, 1972: California Institute of Technology, Pasadena, California 91109.

The (0,0) model is the usual Veneziano model with intercept one, while the (0,1) model is probably equivalent to the intercept-minus-one model recently discovered by Gervais and Neveu. The second class of models is characterized by a single nonegative integer $M$ and has a ground state mass $\mu_0$ given by $\alpha'\mu_0^2 = \frac{3}{2}M - \frac{1}{2}$. These models possess G-type gauges and each of them can be extended to include fermions. The $M = 0$ case corresponds to the dual pion model. Even though the new models have large gauge algebras, all of them contain ghost states. A loose end in the presentation is the absence of a general proof of cyclic symmetry for the n-point amplitudes.

# I INTRODUCTION

Well known ghost-free dual models, such as the generalized Veneziano model (VM) and the dual pion model (DPM), have leading Regge intercept $\alpha(0) = 1$. This fact has been recognized for some time to be a troublesome restriction, inasmuch as a realistic model of nonstrange mesons should probably have $\alpha(0) = \frac{1}{2}$. The first progress reported in overcoming this barrier was made in a recent paper of Gervais and Neveu [1], which presents the construction of a model with $\alpha(0) = -1$. Their paper motivated further study of these problems and the discovery of two infinite classes of models.

The existence of an isospin-one massless vector meson is implied by $\alpha(0) = 1$. Shifting the intercept is therefore essentially equivalent to making the vector meson massive. There has been considerable discussion of similar problems in the context of quantum field theory, where it is now well understood how to give mass to a vector meson while maintaining a renormalizable theory [2]. The basic idea is to introduce a suitably coupled scalar field with a nonvanishing vacuum expectation value--the so-called spontaneous symmetry breaking mechanism. It is natural to look for an analog to this procedure for dual models, but it is not immediately evident on what level the analogy should take place. One widely held hope is that it may be possible to formulate dual models in terms of ordinary space-time Lagrangians constructed from locally interacting infinite-component fields. In such a framework it would probably be clear how to proceed. This has not been done, however, and it seems unlikely that it will be.

A clue as to how to introduce spontaneous symmetry breaking into dual models in a different manner is provided by examining Halpern's proof [3] that the "photon persists" if the vacuum is SU(1,1)

invariant. We interpret this theorem to imply that if the photon is to be avoided the vacuum must not be SU(1,1) invariant. In particular, we will assign a nonzero vacuum expectation value to $L_0$.

Since the $L_0$ operator of the VM or the DPM does annihilate the vacuum it is necessary to supplement it with an additional piece that does not. If we hope to achieve ghost-free models this requires the construction of new representations of the Virasoro algebra [4]. It was only after a long period of frustration seeking such representations that we came upon the sneaky idea of representing the algebra by itself! The details are spelled out in Sec. II, together with certain conjectures about the structure of the corresponding spaces.

In Section III we present a class of models labelled by a pair of nonnegative integers (M,N), for which the ground-state mass $\mu_0$ is given by $\alpha'\mu_0^2 = \frac{3}{2}M + 2N - 1$. These models have L-type gauge operators, the VM corresponding to the (0,0) model. In Sec. IV we construct another class of models labelled by a nonnegative integer (M), for which the ground-state mass $\mu_0$ is given by $\alpha'\mu_0^2 = \frac{3}{2}M - \frac{1}{2}$. These models have G-type gauge operators, the DPM corresponding to the M = 0 model. These models can be extended to include fermions. In Sec. V we construct the spectrum generating algebra for each of the models and show that they contain ghost states. Sec. VI presents some concluding remarks.

## II NONCANONICAL SPACES

The Virasoro algebra

$$\left[L_m \quad L_n\right] = (m - n)L_{m+n} \quad + \quad C_m \, \delta_{m,-n} \tag{2.1}$$

plays an important role in the construction of ghost-free dual models. In the VM this algebra is realized in terms of quadratic forms in harmonic oscillator operators $\left\{a_m^{\mu}\right\}$. The DPM also employs a representation involving anticommuting operators $\left\{b_m^{\mu}\right\}$. Clearly, the construction of new models must be closely linked to discovery of new representations of the Virasoro algebra. However, the algebra itself is not the whole story; more information is required to describe the space that the algebra generates. For example, it seems generally desirable to require that the spectrum of $L_0$ is bounded below. This means that there should be a lowest state $|0\rangle$ for which

$$L_m \, |0\rangle = 0 \quad , \quad m > 0. \tag{2.2}$$

We call a representation, and the space it generates, <u>canonical</u>* if the operators can be expressed entirely in terms of the usual a and b type operators. Representations and spaces that cannot be described in terms of a's and b's we call <u>noncanonical</u>. Although it is not critical to our subsequent development, we conjecture that the only canonical representations satisfying (2.1) and (2.2) are the well-known ones. If this conjecture is correct then the construction of genuinely new models requires the use of noncanonical spaces.

As an example of what we have in mind, consider the space generated by operators $\left\{L_m\right\}$ satisfying

---

* This terminology was suggested by M. Halpern.

$$\left[\mathcal{L}_m, \mathcal{L}_n\right] = (m-n)\mathcal{L}_{m+n} \tag{2.3a}$$

$$\mathcal{L}_m |0\rangle = 0 \ , \ m > 0 \tag{2.3b}$$

$$\mathcal{L}_0 |0\rangle = \lambda |0\rangle \tag{2.3c}$$

$$\mathcal{L}_{-m} = \mathcal{L}_m^+ \ . \tag{2.3d}$$

These equations provide a complete and consistent specification of an abstract space. For example, the first excited state is $\mathcal{L}_{-1}|0\rangle$, with norm

$$\langle 0|\mathcal{L}_1 \mathcal{L}_{-1}|0\rangle = 2\lambda. \tag{2.4}$$

This state has negative norm if $\lambda < 0$. In fact, it is shown in the appendix that as a consequence of (2.3a) not containing an anomaly term, states of negative norm occur for any choice of $\lambda$.

The most general noncanonical space to be employed in this paper involves M sets of abstract operators analogous to the gauge operators of the DPM as well as M + N sets of L-type operators. It is specified by the following requirements, believed to be complete and consistent:

$$\left[\mathcal{L}_m^{(i)}, \mathcal{L}_n^{(j)}\right] = (m-n)\mathcal{L}_{m+n}^{(i)}\delta_{ij} \qquad i,j = 1,\ldots,M+N \tag{2.5a}$$

$$\left\{\mathcal{G}_m^{(i)}, \mathcal{G}_n^{(j)}\right\} = \mathcal{L}_{m+n}^{(i)}\delta_{ij} \qquad i,j = 1,\ldots,M \tag{2.5b}$$

$$\left[\mathcal{G}_m^{(i)}, \mathcal{L}_n^{(j)}\right] = (m - \tfrac{n}{2})\mathcal{G}_{m+n}^{(i)}\delta_{ij} \qquad i = 1,\ldots,M \quad j = 1,\ldots,M+N \tag{2.5c}$$

$$\mathcal{G}_m^{(i)}|0\rangle = 0 \ , \ m > 0 \qquad i = 1,\ldots,M \tag{2.6a}$$

$$\mathcal{L}_m^{(i)}|0\rangle = 0 \ , \ m > 0 \qquad i = 1,\ldots,M+N \tag{2.6b}$$

$$\mathcal{L}_0^{(i)}|0\rangle = \lambda_i |0\rangle \qquad i = 1,\ldots,M \tag{2.6c}$$

$$\mathcal{L}_0^{(i)} |0\rangle = \lambda_2 |0\rangle \qquad i = M+1,\ldots,M+N \tag{2.6d}$$

$$\mathcal{G}_{-m}^{(i)} = \mathcal{G}_m^{+(i)} \qquad i = 1,\ldots,M \tag{2.7a}$$

$$\mathcal{L}_{-m}^{(i)} = \mathcal{L}_m^{+(i)} \qquad i = 1,\ldots, M+N \tag{2.7b}$$

The subscripts on the $\mathcal{L}$'s assume integral values, whereas the ones on the $\mathcal{G}$'s are half integers. We conjecture that the space described in this way is well defined for any choice of $\lambda_1$ and $\lambda_2$. It will turn out that we are particularly interested in $\lambda_1 = \frac{3}{2}$ and $\lambda_2 = 2$. With these choices the space defined by (2.5)-(2.7) is isomorphic to a subspace of the space of physical states for the models to be constructed. Therefore the lack of positivity of the noncanonical spaces is a key element in the occurrence of ghost states.

## III DUAL MODELS WITH L GAUGES

New models are constructed by using both the usual harmonic oscillator operators $\left\{a_m^{\mu}\right\}$ and the abstract noncanonical operators $\left\{\mathcal{Y}_m^{(i)}, \mathcal{L}_m^{(i)}\right\}$ discussed in Sec. II. We require that

$$\left[a_m^{\mu}, \mathcal{Y}_n^{(i)}\right] = \left[a_m^{\mu}, \mathcal{L}_n^{(i)}\right] = 0. \tag{3.1}$$

The gauge operators for the new models are

$$L_m = L_m^{(0)} + \sum_{i=1}^{M+N} \mathcal{L}_m^{(i)} , \tag{3.2}$$

where $L_m^{(0)}$ represents the usual expression constructed from the a oscillators. The problem then is to construct vertices which transform suitably under these gauges.

The first step is to construct fields

$$\mathcal{Y}^{(i)}(z) = \sum_{m=-\infty}^{\infty} \mathcal{Y}_m^{(i)} z^{-m} \tag{3.3a}$$

$$\mathcal{L}^{(i)}(z) = \sum_{m=-\infty}^{\infty} \mathcal{L}_m^{(i)} z^{-m} , \tag{3.3b}$$

and observe that they transform with conformal spin $\frac{3}{2}$ and 2, respectively.

$$\left[L_m, \mathcal{Y}^{(i)}(z)\right] = z^m \left[z \frac{d}{dz} + \frac{3}{2} m\right] \mathcal{Y}^{(i)}(z) \tag{3.4a}$$

$$\left[L_m, \mathcal{L}^{(i)}(z)\right] = z^m \left[z \frac{d}{dz} + 2m\right] \mathcal{L}^{(i)}(z). \tag{3.4b}$$

Notice that an anomaly term in (2.[illegible]a) would destroy the form of (3.4b). We plan to prove the Ward identities in the usual way, namely by means of the identities

$$(L_m - L_0 + 1)\frac{1}{L_0-1} = \frac{1}{L_0+m-1}(L_m - L_0 - m + 1). \qquad (3.5a)$$

$$(L_m - L_0 - m + 1)\, V(k) = V(k)\,(L_m - L_0 + 1). \qquad (3.5b)$$

Therefore we need a vertex $V(k, z)$ that transforms with conformal spin one.

$$\left[L_m, V(k, z)\right] = z^m\left[z\frac{d}{dz} + m\right] V(k, z). \qquad (3.6)$$

This may be achieved by taking

$$V(k, z) = \left(\prod_{i=1}^{M} \mathcal{U}^{(i)}(z)\right)\left(\prod_{j=M+1}^{M+N} \mathcal{X}^{(j)}(z)\right) V_0(k, z)\ , \qquad (3.7)$$

where $V_0(k, z)$ represents the usual expression involving a oscillators, provided that the emitted mass $\mu_0$ (in units with $\alpha' = 1$) satisfies

$$\mu_0^{\,2} = -k^2 = \tfrac{3}{2}M + 2N - 1. \qquad (3.8)$$

The n-point amplitude is then given by

$$A_n = \langle 0; k_1 |\, V(k_2)\frac{1}{L_0-1} V(k_3)\ldots\frac{1}{L_0-1} V(k_{n-1}) | 0; k_n \rangle. \qquad (3.9)$$

(Our notation is $V(k) \equiv V(k, 1)$, etc.)

It is evident from (3.5a) and (3.5b) that the $L_n$'s are gauge operators provided only that

$$(L_0 - 1)\,|0; k_n\rangle = 0. \qquad (3.10)$$

Using (2.6c) and (2.6d) this reduces to

$$-k_n^{\,2} = \mu_0^{\,2} = M\lambda_1 + N\lambda_2 - 1. \qquad (3.11)$$

Comparing with (3.8) we find as a solution

$$\lambda_1 = \frac{3}{2} \text{ and } \lambda_2 = 2. \qquad (3.12)$$

Some further insight into the models may be gained by calculating specific amplitudes. For example, the four-point amplitude is

$$A_4^{(M,N)} = \left(\frac{3}{2}\right)^M 4^N \int_0^1 x^{2(M+N-1)-s} (1-x)^{2(M+N-1)-t} \times (1-x+x^2)^N dx \quad (3.13)$$

The model with $M = N = 0$ is clearly just the usual VM. The four-point amplitude with $M = 0$ and $N = 1$ is the same as in the model of Gervais and Neveu. In fact, the five-point amplitude has also been calculated and found to be the same as theirs. It therefore seems safe to suppose that the $(0,1)$ model is the Gervais-Neveu model, although the present formulation is sufficiently different from theirs that it is not evident how to construct a general proof of the equivalence. The four-point amplitudes with $M = 0$ and $N$ arbitrary are all of the type suggested by Mandelstam [5]. These amplitudes have no odd daughter trajectories, but in a sense this is a fluke, since higher amplitudes do contain odd daughters.

The models presented here contain a vacuum that is not SU(1,1) invariant. One consequence is that the usual proofs of Möbius invariance and cyclic symmetry do not apply. The techniques developed by Corrigan and Olive [6] in their study of fermion vertices are probably useful in this connection. For example, a Möbius - invariant form of the n-point amplitude is

$$A_n = \int d\mu(z) \langle 0; k_1 | \frac{e^{L_1/z_1}}{z_1^2} V(k_2, z_2) \ldots V(k_{n-1}, z_{n-1}) \times e^{z_n L_{-1}} | 0; k_n \rangle, \quad (3.14)$$

where $d\mu(z)$ is the usual invariant measure. Further technical developments appear to be required for a general proof of cyclic symmetry.

0 0 4 0 3 8 0 3 3 5 5



The symmetry is undoubtedly present since the four and five-point amplitudes have it. Still, the construction of a general algebraic proof remains an important unsolved problem.

## IV DUAL MODELS WITH G GAUGES

In this section models are constructed that generalize the DPM [7] in the same sense that the models of Sec. III generalize the VM. Since we desire to have G-type gauges, it is necessary to include a $\mathcal{Y}$ for every $\mathcal{L}$. This forces us to take $N = 0$ in the algebra of (2.5). The gauge operators are then given by

$$G_m = G_m^{(0)} + \sum_{i=1}^{M} \mathcal{Y}_m^{(i)} \quad , \tag{4.1}$$

where $G_m^{(0)}$ represents the usual expression constructed from a and b oscillators. We require that in addition to (3.1)

$$\left[b_m^{\mu} , \mathcal{L}_n^{(i)}\right] = \left\{b_m^{\mu} , \mathcal{Y}_n^{(i)}\right\} = 0. \tag{4.2}$$

The construction proceeds in analogy with the $\mathcal{F}_2$ formulation of the DPM, [8] so that the $G_m$'s are the gauge operators, the propagator is $(L_0 - \frac{1}{2})^{-1}$, and the lowest-mass particle is described by the ground state of the Fock space. The vertex for emission of a ground-state particle is taken to be

$$V(k) = \left[G_m, \left(\prod_{i=1}^{M} \mathcal{Y}^{(i)}\right) V_0(k)\right]_{\pm} \quad . \tag{4.3}$$

The $\pm$ indicates that a commutator is used for $M$ even and an anticommutator for $M$ odd. Fortunately, (4.3) is independent of m. The vertex defined in this way satisfies (3.6) provided that

$$\mu_0^2 = -k^2 = \frac{3}{2} M - \frac{1}{2} \, . \tag{4.4}$$

As an example consider $M = 1$. In this case the vertex may be rewritten in the form

$$V(k) = (\mathcal{L} - \mathcal{Y} \, k \, .H) \, V_0(k) \, . \tag{4.5}$$

Requiring once again that $\lambda_1 = 3/2$, one finds for the four-point amplitude

$$A_4 = \int_0^1 x^{-s} (1-x)^{-t} \left[ \frac{9}{4} (1-x+x^2) - \frac{3}{4} x (1 - t) \right] dx. \quad (4.6)$$

This expression is s-t symmetric. Indeed, the cyclic symmetry of all these models undoubtedly holds, but a general proof is once again lacking. Notice that in contrast with the DPM (corresponding to M=0), the ground state of the M = 1 model must be a scalar with even G parity since it possesses a trilinear coupling. All the models with M odd have an additional subsidiary condition expressing the fact that the physical states have even G parity in the sense that they are formed from an even number of $\mathcal{Y}$'s and b's.

The models of this section can all be extended to include fermions. The techniques required are a completely straightforward extension of those used in the DPM case, [9] and shall not be discussed here. We simply remark that the fermion sector gauge conditions require that the meson emission vertices for the models with M even contain a factor of $\gamma_5$ while those for M odd do not, which is in accord with the comments about parity made above.

## V SPECTRUM GENERATING ALGEBRAS

The techniques recently developed for proving the no-ghost theorem in the VM[10-11] and the DPM [11-13] are easily extended to the models under consideration here. We shall follow the notation of Ref. 12, except that the parameter $\gamma$ introduced there to display Lorentz-frame dependence will be put equal to one. We recall that in the case of the VM the spectrum-generating algebra consisted of operators $\left\{A^i_m\right\}$ and $\left\{A^L_m\right\}$, where $i$ labels the d-2 transverse directions of d-dimensional space-time and L denotes "longitudinal." Brower showed that these operators have the algebra

$$\left[A^i_m, A^j_n\right] = \frac{m}{2}\,\delta_{ij}\,\delta_{m,\,-n} \tag{5.1a}$$

$$\left[A^L_m\,,\,A^L_n\right] = (n-m)A^L_{m+n} + 2\,m^3\,\delta_{m,-n} \tag{5.1b}$$

$$\left[A^L_m\,,\,A^i_n\right] = n\,A^i_{m+n}\ . \tag{5.1c}$$

Using this albgra it was an easy matter for him to prove the absence of ghosts for $d \leq 26$.

Let us now consider the models with L gauges formulated in Sec. III. The spectrum generating algebra for these models contains $\left\{A^i_m, A^L_m\right\}$ as before. Clearly there must also be additonal operators corresponding to the $\mathcal{L}^{(i)}$ and $\mathcal{Y}^{(i)}$ degrees of freedom. The required operators that commute with the $L_n$'s are

$$X_m^{(i)} = \frac{1}{2} < \mathcal{L}^{(i)}\,P_+^{-1}\,V_m >_m\,, \quad i=1,\ldots,M+N \tag{5.2a}$$

$$Y_m^{(i)} = \frac{1}{\sqrt{2}} < \mathcal{Y}^{(i)}\,P_+^{-1/2}\,V_m >_m \quad i=1,\ldots,M. \tag{5.2b}$$

As usual, it is understood that the subscript on the X is an integer whereas the subscript on the Y takes half-integer values. These operators have the algebra

$$\left[X_m^{(i)}, X_n^{(j)}\right] = (m-n)\, X_{m+n}^{(i)}\, \delta_{ij} \tag{5.3a}$$

$$\left\{Y_m^{(i)}, Y_n^{(j)}\right\} = X_{m+n}^{(i)}\, \delta_{ij} \tag{5.3b}$$

$$\left[Y_m^{(i)}, X_n^{(j)}\right] = (m - \tfrac{n}{2})\, Y_{m+n}^{(i)}\, \delta_{ij} \tag{5.3c}$$

Note that the algebra of (5.3) is identical to that of (2.5). This is not an accident; in the infinite momentum limit $X_m^{(i)} \to \mathcal{L}_m^{(i)}$ and $Y_m^{(i)} \to \mathcal{Y}_m^{(i)}$. Thus in view of the discussion in the Appendix all of the new models (including the one of Gervais and Neveu) contain ghost states. Actually, since the $\mathcal{L}$'s and $\mathcal{Y}$'s enter into the vertices in a symmetrical fashion, the physical space only involves states built from symmetrical combinations of the X's and antisymmetrical combinations of the Y's.

The remainder of the spectrum generating algebra is

$$\left[A_m^i, X_n^{(j)}\right] = 0 \tag{5.4a}$$

$$\left[A_m^i, Y_n^{(j)}\right] = 0 \tag{5.4b}$$

$$\left[A_m^L, X_n^{(i)}\right] = (n-m)\, X_{m+n}^{(i)} \tag{5.4c}$$

$$\left[A_m^L, Y_n^{(i)}\right] = (n - \tfrac{m}{2})\, Y_{m+n}^{(i)}. \tag{5.4d}$$

Defining the operator

$$\tilde{A}_m = A_m^L + \sum_{i=1}^{M+N} X_m^{(i)}, \tag{5.5}$$

One finds that the algebra of $\left\{A_m^i, \tilde{A}_m\right\}$ is isomorphic to that of (5.1) with $\tilde{A}_m$ replacing $A_m^L$ and that

$$\left[\tilde{A}_m, X_n^{(i)}\right] = \left[\tilde{A}_m, Y_n^{(i)}\right] = 0. \tag{5.6}$$

Thus the algebra splits into two orthogonal pieces, one of which is positive definite for for $d \leq 26$, while the other depends on the non-canonical space and has ghosts.

The analysis of the models of Sec. IV proceeds in a similar fashion. The physical operators of the DPM $\left\{A^i_m\,,\ B^i_m\,,\ A^L_m\,,\ B^L_m\right\}$ still commute or anticommute with the $G_\ell$'s and therefore belong to the spectrum generating algebra. The additional operators in the spectrum generating algebra are

$$X^{(i)}_m = \langle e^{-i\ell\theta}\left\{G_\ell,\ \frac{1}{4}\left(3\mathcal{Y}^{(i)}P_+^{-1} - \mathcal{L}^{(i)}H_+P_+^{-2}\right)V_m\right\}\rangle_m$$

$$= \langle\left(\frac{1}{2}\mathcal{L}^{(i)}P_+^{-1} - \frac{3i}{8}\mathcal{Y}^{(i)}H'_+P_+^{-2} - \frac{3m}{4}\mathcal{Y}^{(i)}H_+P_+^{-1}\right.$$

$$\left. + \frac{i}{8}\mathcal{Y}'^{(i)}H_+P_+^{-2} + \frac{i}{4}\mathcal{L}^{(i)}H_+H'_+P_+^{-3}\right)V_m\rangle_m\,,\quad i=1,\ldots,M \tag{5.7a}$$

$$Y^{(i)}_m = -\frac{1}{\sqrt{2}}\langle e^{-i\ell\theta}\left[G_\ell,\ \mathcal{Y}^{(i)}H_+P_+^{-3/2}V_m\right]\rangle_m$$

$$= \frac{1}{\sqrt{2}}\langle\left(\mathcal{Y}^{(i)}P_+^{-1/2} - \mathcal{L}^{(i)}H_+P_+^{-3/2} - \frac{3i}{4}\mathcal{Y}^{(i)}H_+H'_+P_+^{-5/2}\right)$$

$$V_m\rangle_m \quad i=1,\ldots,M\,. \tag{5.7b}$$

These operators once again satisfy the algebra of (5.3). Furthermore, defining

$$\tilde{A}_m = A^L_m + \sum_{i=1}^{M} X^{(i)}_m \tag{5.8a}$$

$$\tilde{B}_m = B^L_m - \sum_{i=1}^{M} Y^{(i)}_m\,, \tag{5.8b}$$

the algebra again breaks up into two orthogonal pieces. One part

consists of $\left\{A^i_m , B^i_m , \tilde{A}_m , \tilde{B}_m\right\}$ and is isomorphic to the DPM algebra, and hence positive definite for $d \leq 10$. The other part is generated by $\left\{X^{(i)}_m , Y^{(i)}_m\right\}$, and contains ghost states.

## VI CONCLUSION

Two classes of new dual models have been formulated. One generalizes the VM and possesses L-type gauges, while the other generalizes the DPM and possesses G-type gauges. In each case we were unable to shift trajectories arbitrarily, but only in discrete steps of $\frac{3}{2}$ or 2. It appears unlikely that one could interpolate between these models to have a continuously variable intercept. For example, if we consider the class of L-type models with $M = 0$ and $N \neq 0$, the n-point functions can certainly be continued in N. However, it seems that when N is nonintegral, states on the second daughter trajectory and below become infinitely degenerate.

In this paper use was made of two new considerations. The first was the introduction of spontaneous symmetry breaking by means of assigning a nonvanishing vacuum expectation value to $L_0$. We saw that this choice could not be made arbitrarily, but instead was dictated by consistency requirements of the models. In addition to shifting the states, the introduction of spontaneous symmetry breaking resulted in the occurence of an $SU(1,1)$ noninvariant vacuum. This meant that the usual methods for proving cyclic symmetry of the n-point amplitudes were no longer applicable. We convinced ourselves that cyclic symmetry is present by checking special cases, but were unable to present a general proof. We feel that this is an important unsolved problem not only because it means that our discussion is incomplete, but also because it needs to be understood before one can expect to systematically explore the possibilities for constructing other models along similar lines. The second new consideration was the use of noncanonical spaces. We saw that such spaces are prone to

contain ghost states, but one may hope to be more fortunate in the future. The positivity proof is difficult in general for a noncanonical space since there is no natural orthonormal basis. The lack of such a basis also makes the computation of traces, required for the investigation of loop diagrams, more difficult.

## ACHNOWLEDGMENTS

I am grateful to A. Neveu for a letter describing the work of Ref. 1 and to M. Halpern for helpful discussions and a careful reading of the manuscript. The hospitality of the Aspen Center for Physics and the Lawrence Berkeley Laboratory are also gratefully acknowledged.

## APPENDIX

In Section II we introduced "noncanonical" spaces specified by algebras similar to those of the gauge operators. The important question of whether or not these spaces are positive definite was raised. In this appendix we display specific examples showing that ghosts are in fact present.

Consider the space defined by

$$\left[L_m , L_n\right] = (m-n)\, L_{m+n} + c\, m(m^2-1)\, \delta_{m,-n} \qquad \text{(A. 1a)}$$

$$L_m \,|0\rangle = 0 \;,\; m > 0 \qquad \text{(A. 1b)}$$

$$L_0 \,|0\rangle = \lambda \,|0\rangle \qquad \text{(A. 1c)}$$

$$L_{-m} = L_m^+ \,. \qquad \text{(A. 1d)}$$

For the choice $c = 0$ and $\lambda = \frac{3}{2}$ or 2 this corresponds to a subspace of the spectrum generating algebra of any of the new models. In Sec. II we pointed out that the first excited state, $L_{-1}\,|0\rangle$, has positive norm if $\lambda > 0$. At the next level there are two states to be considered.

The combination

$$\left(A_1 L_{-1}^2 + A_2 L_{-2}\right)|0\rangle \qquad \text{(A. 2)}$$

has norm

$$\frac{\lambda}{2\lambda+1}\left[(4\lambda+2)A_1+3A_2\right]^2 + \frac{1}{2\lambda+1}\left[8\lambda^2 - 5\lambda+6c(2\lambda+1)\right]A_2^2 \,. \qquad \text{(A. 3)}$$

Therefore, the critical requirement for positivity is that the factor in front of $A_2^2$ be positive. It is easy to see that this factor is positive for all $\lambda \geq 0$ if and only if $c \geq \frac{1}{12}$ .

The value $c = \frac{1}{12}$ is canonical since it corresponds to a representation based on one set of a oscillators. In this case positivity of the entire space is easily proved. On the other hand we are now finding that when $c = 0$ positivity requires at least that $\lambda \geq \frac{5}{8}$.

Let us now consider the pair of state $L_{n-m} L_{-n} |0>$ and $L_{-m} |0>$, assuming for convenience that $m > 2n > 0$. Then it is easy to show that for $c = 0$ this two-dimensional subspace has positive norm only if

$$\lambda \geq \frac{n(2m-n)^2}{2m(m-n)} - n \quad . \qquad \text{(A. 4)}$$

Considering the limit $m \to \infty$ with $n$ fixed we see that $\lambda \geq n$ is required. However, $n$ is arbitrary, and therefore there is no value of $\lambda$ for which the entire space is positive definite. We conclude that $c > 0$ is critical for positivity.

REFERENCES

1. J-L Gervais and A. Neveu, Orsay preprint LPTHE 72/24 (1972).

2. P. W. Higgs, Phys. Letters 12 (1964) 132; Phys. Rev. 145 (1966) 1156. T. W. B. Kibble, Phys. Rev. 155 (1967) 1554.

3. M. B. Halpern, Phys. Rev. D5 (1971) 3068.

4. M. A. Virasoro, Phys. Rev. D1 (1970) 2933. S. Fubini and G. Veneziano, Ann. Phys. (N.Y.) 63 (1971) 12.

5. S. Mandelstam, Phys. Rev. Letters 21 (1968) 1724.

6. E. Corrigan and D. Olive, CERN preprint TH. 1464 (1972).

7. A. Neveu and J. H. Schwarz, Nucl. Phys. B31 (1971) 86.

8. A. Neveu, J. H. Schwarz, and C. B. Thorn, Phys. Letters 35B (1971) 529.

9. A Neveu and J. H. Schwarz, Phys. Rev. D4 (1971) 1109. C. B. Thorn Phys. Rev. D4 (1971) 1112.

10. R. C. Brower, MIT preprint CTP 277 (1972)

11. P. Goddard and C. B. Thorn, CERN preprint TH. 1493 (1972). P. Goddard, C. Rebbi, and C. B. Thorn, CERN preprint TH. 1500 (1972).

12. J. H. Schwarz, Princeton University, Preprint PURC 3072 - 06 (1972).

13. R. C. Brower and K. A. Friedman, MIT preprint CTP 288 (1972).

LEGAL NOTICE

*This report was prepared as an account of work sponsored by the United States Government. Neither the United States nor the United States Atomic Energy Commission, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness or usefulness of any information, apparatus, product or process disclosed, or represents that its use would not infringe privately owned rights.*

*TECHNICAL INFORMATION DIVISION*
*LAWRENCE BERKELEY LABORATORY*
*UNIVERSITY OF CALIFORNIA*
*BERKELEY, CALIFORNIA 94720*