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Angular Distribution of Electromagnetic Bremsstrahlung from Heavy-Ion Collisions at Intermediate Energies

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Abstract: Recently measured spectra of hard photons emitted in collisions between two heavy ions at bombarding energies as low as 35 MeV/n have been reported by Alamanos et al. to show dipole-like angular patterns. In a schematic calculations it will become obvious that these photons may be interpreted as coherent bremsstrahlung arising from collisions with non-zero impact parameters where the axial symmetry, which usually leads to cancelation of the dipole contribution, is distorted.

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The discussion concerning the mechanism which governs the production of particles in collisions between heavy ions at bombarding energies as low as a few tens MeV/n has continued since it became obvious that some kind of collective or cooperative action [1,2] of nucleons within the colliding nuclei is necessary to explain for example the surprisingly large cross sections for the emission of pions [3] - [7]. If the nuclei or their substantial parts act as a whole then we also expect this fact to be reflected in the spectra of hard photons [8,9,7] or other particles [7].

In this note I shall show that the dipole-like angular distribution of photons, seen recently [10] by bombarding a nickel target with the 35 MeV/n nitrogen beam, can be understood within the classical bremsstrahlung picture, in which coherent electrodynamic radiation is generated by the deceleration of charged nuclei. In Ref. [7], where a calculation performed for head-on collisions exhibited a quadrupole-type behaviour of the angular distribution but underestimated the known experimental cross sections [9] by a factor of ten, it was argued that with increasing impact parameter, i.e., when the axial symmetry around the beam direction becomes distorted, the originally exact cancelation in transverse direction of the dipole radiation due to negative interference between the contributions from projectile and target should gradually be turned off. We shall see that with increasing size of the collision system the differential cross section indeed loses its quadrupole character, and thus confirm the expectation of Ref. [7]. The lesson we learn from this is that even substantial deviations of the experimental angular distribution from quadrupole patterns need not necessarily indicate admixtures of incoherent radiation to the photon spectra, as suggested for example in Ref. [14].

In order to work out the contribution of collisions with non-zero impact parameter in a transparent way and to allow for an entirely analytic solution of the problem, the collision kinematics will be treated only schematically. Let us therefore consider two nuclei in the centre-of-mass system which move towards each other along the beam (z -) axis and stop instantaneously when they "touch". Due to their non-vanishing spatial extension this will happen at some finite distance. The instant of impact (and stopping) is then chosen to be $t = 0$, and the final distance between the nuclear centers is determined by the transverse separation b perpendicular to the beam axis (impact parameter), and the finite longitudinal distance S which decreases with increasing internuclear penetration. It is, in general, a function of b . Hence the projectile's center stops at $z = 0$, say, and the target at $z = S$ (cf. Fig. 1). Besides its influence on the particles' trajectories the finite spatial distribution of charged nuclear matter within the nuclei also leads to the appearance of form factors in the cross section which act as cut-off functions in the momentum space of the photons [7] and determine the slope of the photon energy spectra. Since the choice of these form factors does not substantially affect the main objective of this paper, namely the question how finite impact parameters influence the angular distribution of electromagnetic bremsstrahlung, the form factors will simply be replaced by 1. This corresponds to point-like nuclei. Only for very asymmetric

Figure 1: *Two point-like charges stop suddenly at the transverse distance b and at the longitudinal distance S .*

systems, where the ratio of the form factors deviates considerably from unity, do we expect an observable additional distortion of the photon angular distribution (see below).

We do not of course expect such a simple model to account for all other features of the photon spectra like their magnitudes or slopes. This is a task which can be completed only in a more complex and therefore less transparent calculation [11], where the explicit time and impact parameter dependence of the deceleration process is properly taken into account. Nevertheless, the influence of the spatial extension on the kinematics, which I wish to point out here, is incorporated schematically by explicitly assuming that the collision leads to instantaneous stopping even for non-zero impact parameters.

If E_{lab} is the bombarding energy in the laboratory system then in this case the projectile with mass number A_P moves in the centre-of-mass system with the velocity

$$\vec{v}_P = \theta(-t) (v_L - v_F) \vec{e}_z \quad (1)$$

along the beam axis pointing in z -direction (the units are such that $\hbar = c = 1$). For collisions below $100 \text{ MeV}/n$ we can employ the non-relativistic relations

$$v_L = \sqrt{2E_{lab}/m_n} \quad (2)$$

for the beam lab velocity, and

$$v_F = v_L \frac{A_P}{A_P + A_T} \quad (3)$$

for the final lab velocity of the compound system. It consists of both nuclei which stick together after a completely inelastic collision. In eq. (2) $m_n = 931.5 \text{ MeV}$ is the bound nucleon mass. In the centre-of-mass system the target nucleus with mass number A_T is then found to move with the velocity

$$\vec{v}_T = -\theta(-t) v_F \vec{e}_z . \quad (4)$$

In this system of reference both nuclei are at rest for times $t > 0$. Using the definition of the mass ratio

$$\xi = \frac{A_P}{A_T} , \quad (5)$$

we can then write the respective trajectories of the projectile and target as

$$\vec{R}_P = \theta(-t) \frac{1}{\xi + 1} v_L t \vec{e}_z , \quad (6)$$

$$\vec{R}_T = -\theta(-t) \frac{\xi}{\xi + 1} v_L t \vec{e}_z + \vec{b} + \vec{S} , \quad (7)$$

where b , the magnitude of $\vec{b}$, is the impact parameter and $\vec{S} = S \vec{e}_z$. S is the longitudinal distance between the two point particles after stopping.

The differential cross section for γ -bremsstrahlung [12,7] emitted in such a heavy-ion collisions is given by the expression

$$\frac{d^2\sigma_\gamma}{dE d\Omega} = \frac{\alpha}{4\pi^2 E} \int d^2b \left| Z_P \frac{\vec{n} \times \vec{v}_L}{1 + \xi - \vec{n} \cdot \vec{v}_L} - Z_T \xi \frac{\vec{n} \times \vec{v}_L}{1 + \xi + \xi \vec{n} \cdot \vec{v}_L} e^{-i(\vec{b} + \vec{S}) \cdot \vec{k}} \right|^2 . \quad (8)$$

Notice that in the second part of the integrand, generated by the target, ξ is replaced by $\frac{1}{\xi}$, as compared to the first term generated by the projectile. Exchanging projectile and target does not change, therefore, the final result if $\vec{n}$ is replaced by $-\vec{n}$. The photon momentum $\vec{k} = E\vec{n}$ is taken along the observation direction $\vec{n}$, $\alpha = \frac{1}{137}$ is the fine structure constant.

The integration over the impact parameter cannot be performed analytically if S is some general function $S(b)$. However, the following considerations suggest a tractable approximation. The anticipated qualitative appearance of $S(b)$ is that of a flat-topped function with zero derivative at the origin, as follows by symmetry. Its value at $b = 0$ would be the sum of the nuclear radii, diminished somewhat by a stopping distance, which at the relatively low energies in question would be rather small. With increasing b the function $S(b)$ would begin to fall off, until around some impact parameter R the assumption of sudden stopping would cease to be valid altogether, and the present schematic treatment would have to be supplemented by the consideration of only partially decelerated (and bent) trajectories. It would seem, however, that within the stopping regime, to which this model is restricted, it may be a fair approximation to replace $S(b)$ by a constant S representing some average value of $S(b)$ in the relevant range of impact parameters. This is what I shall do in the following in order to preserve the simplicity of the present calculation. The result is that there will be two length parameters in the theory, the longitudinal internuclear separation S and the transverse distance R at which the integral over b has to be cut off. Their significance will be discussed below.

The result of the b -integration is now

$$\begin{aligned} \frac{d^2\sigma_\gamma}{dE d\Omega} = & \frac{\alpha v_L^2 \sin^2 \theta}{4\pi^2 E} \pi R^2 \left[\left(\frac{Z_P}{1 + \xi - v_L \cos \theta} \right)^2 + \left(\frac{Z_T \xi}{1 + \xi + \xi v_L \cos \theta} \right)^2 \right. \\ & \left. - \frac{2Z_P Z_T \xi}{(1 + \xi - v_L \cos \theta)(1 + \xi + \xi v_L \cos \theta)} \frac{J_1(RE \sin \theta)}{\frac{1}{2}RE \sin \theta} \cos(SE \cos \theta) \right]. \end{aligned} \quad (9)$$

Here θ is the angle between the direction of observation $\vec{n}$ and the z -axis. Let us first consider this equation for rather soft photons and neglect the factor involving S completely. For small arguments z the Bessel function $J_1(z)$ can be expanded in a power series [13] :

$$J_1(z) = \frac{1}{2}z - \frac{1}{16}z^3 + \frac{1}{384}z^5 + O(z^7). \quad (10)$$

The leading term gives the negative interference we expect for soft photons which, due to their long wave length, do not "see" the spatial extension of the collision system. The higher powers of $z = ER \sin \theta$ are corrections which become increasingly important with growing photon energy. This can be seen explicitly by inserting the first two terms of the expansion (10) into eq. (9):

$$\begin{aligned} \frac{d^2\sigma_\gamma}{dE d\Omega} = & \frac{\alpha v_L^2 \sin^2 \theta}{4\pi^2 E} \pi R^2 \left[\left(\frac{Z_P}{1 + \xi - v_L \cos \theta} - \frac{Z_T \xi}{1 + \xi + \xi v_L \cos \theta} \right)^2 \right. \\ & \left. + \frac{\frac{1}{4}Z_P Z_T \xi R^2 E^2 \sin^2 \theta}{(1 + \xi - v_L \cos \theta)(1 + \xi + \xi v_L \cos \theta)} + \text{higher orders} \right]. \end{aligned} \quad (11)$$

Notice that the photon cross section is proportional to the "geometrical" total cross section πR^2 !

In order to get a feeling for the dependence of the photon cross section on the parameter R , let us consider eq. (11) for symmetric systems, i.e. for $\xi = 1$ and $Z_P = Z_T$. In this case it reduces up to first order in E to

$$\begin{aligned} \frac{d^2\sigma_\gamma}{dE d\Omega} & \approx \frac{\alpha Z^2 v_L^2 \sin^2 \theta}{4\pi^2 E (4 - v_L^2 \cos^2 \theta)} \pi R^2 \left(\frac{v_L^2 \cos^2 \theta}{1 - \frac{1}{4}v_L^2 \cos^2 \theta} + \frac{1}{4}R^2 E^2 \sin^2 \theta \right) \\ & \approx \frac{\alpha Z^2 v_L^2 \sin^2 \theta}{16\pi^2 E} \pi R^2 \left(v_L^2 \cos^2 \theta + \frac{1}{4}R^2 E^2 \sin^2 \theta \right). \end{aligned} \quad (12)$$

The first term in the parentheses is the origin of the familiar quadrupole patterns. It dominates for soft photons and central collisions. On the other hand, for

$$\frac{1}{4}R^2 E^2 \sim v_L^2 \quad (13)$$

the sum in the parentheses in eq. (12) is ~ 1 , and we are confronted with a pure dipole distribution. In the region of bombarding energies under consideration we find $v_L \sim \frac{1}{4}$. Hence photons with energies of the order of

$$E \sim \frac{1}{2R} \quad (14)$$

will exhibit "dipole-like" angular dependence $\sim \sin^2 \theta$.

Obviously, the angular dependence of the photons is sensitive to the effective range of the nucleus-nucleus interaction, i.e., to the value assigned the parameter R . If we set R equal to the geometrical maximum impact parameter

$$R_{max} = r_0 (A_P^{\frac{1}{3}} + A_T^{\frac{1}{3}}) \quad (15)$$

with $r_0 \approx 1.2 \text{ fm}$, the contribution of peripheral collisions would become unrealistically large (we know that nuclei do not stop in grazing collisions). Therefore, when comparing the results of this calculation with experiments we would expect R to be somewhat smaller than R_{max} . If we use eq. (14) to roughly estimate the frequency of "dipole photons", we observe that for an effective impact range as large as $R \sim 5 \text{ fm}$ it will be of the order of 20 MeV . This corresponds to the minimum measurable photon energy dictated by the properties of detectors used in actual experimental set-ups [10,9]. In view of this result it does not seem surprising any more that quadrupole-like angular dependence has not been seen for hard photons. Note that for the $N + Ni$ -system $R_{max} = 7.54 \text{ fm}$.

The angular dependence of the double differential photon number distribution calculated from formula (9) via

$$\frac{d^2 N_\gamma}{dE d\Omega} \equiv \frac{1}{\pi R^2} \frac{d^2 \sigma_\gamma}{dE d\Omega}, \quad (16)$$

is displayed in Fig. 2, as function of the transverse distance parameter R and with the longitudinal separation $S = 0$, for two different collision systems, an asymmetric one at lower bombarding energy (a), and a symmetric one at higher projectile velocity and for a higher photon frequency (b). Note that in the plot the distribution (16) has been transformed into the laboratory system. In both cases we find a gradual transition from a quadrupole type angular distribution for $R \lesssim 2 - 3 \text{ fm}$ over a dipole-like one for intermediate values of R to even steeper behaviour, if photon emission from nuclei stopping at peripheral impacts is dominating. Obviously, intermediate values of R are consistent with the angular distributions observed in experiments. Multiplying e.g. the curve with $R \sim 5 \text{ fm}$ in Fig. 2(a) by $\pi R^2 \sim 8 \times 10^5 \mu b$ does not only reproduce the angular dependence of the photon cross section from the reaction $N + Ni \rightarrow \gamma + X$ at $35 \text{ MeV}/n$ [10] quite well, but is even of the right order of magnitude of several $\mu b/\text{MeV sr}$! The latter is somewhat surprising in view of the uncertainties in the treatment of peripheral trajectories and due to absence of nuclear form factors in this simple model (perhaps for photon frequencies of 35 MeV the neglect of the form factors and the neglect of peripheral

Figure 2: Inclusive angular distributions (16) of the number of hard photons with lab frequencies of (a) 35 MeV from a $^{14}\text{N} + ^{58}\text{Ni}$ reaction at 35 MeV/n bombarding energy and of (b) 50 MeV from a $^{12}\text{C} + ^{12}\text{C}$ reaction with 84 MeV/n bombarding energy, respectively. In both cases the curves from bottom to top correspond to the choice of the parameter $R = .1, 1, 2, 3, 4, 5 \text{ fm}$. The top curves are plotted with $R = R_{\text{max}}$ as calculated from eq. (15). The longitudinal distance is $S = 0$.

trajectories accidentally cancel each other). Notice that the asymmetry between nitrogen and nickel is still small enough to retain the negligibility of the form factor ratio mentioned above. A simple estimate using normalized gaussian charge distributions [1] shows that for a photon energy of 35 MeV its implementation would effectively lead to a mere 8%-increase of the target charge.

Until now it was assumed that $S = 0$, i.e., that the nuclei penetrate each other completely and stop at $z = 0$. Now we go back to eq. (9) and look at the effects of non-vanishing mean longitudinal internuclear distance S in more detail. In Fig. 3 the calculation for the system considered already in Fig. 2(a) has been repeated with $R = 2 \text{ fm}$ and $R = 5 \text{ fm}$, respectively, but in the first case with $S = 0, 1$, and 3 fm , and in the second case in addition to these three values also with $S = R = 5 \text{ fm}$. We observe that with increasing S quadrupole-like patterns develop on top of the distributions calculated with $S = 0$. Thus the interplay between the two parameters, R and S , determines the actual form of the photon spectra, if we measure them in the only available unit of length $\frac{1}{E}$. Fortunately, the conclusions about the value of the transverse distance R , drawn in the preceding discussion from experimental results, are not altered substantially if we assume that S is less than R . On the other hand, it should be noticed here that the explicit appearance of the factor $\sin^2 \theta$ in the cross section (9) prevents it from being isotropic for any value of R , and thus seems to contradict the detection of isotropic photon angular distributions from the reaction $C + C \rightarrow \gamma + X$ at $84 \text{ MeV}/n$ reported in Ref. [14]. However, it is well known that different trajectories of the colliding particles give rise to different angular distributions of the emitted bremsstrahlung. If for example the nuclei behaved like charged hard spheres, coherent electromagnetic radiation would be emitted also in forward and backward directions [15]. The oversimplification of the particles' trajectories was employed here to stress the wiping out of the quadrupole angular dependence due to non-zero impact parameters, and not in order to study details of the deceleration process. If photons from peripheral impacts contributed more at higher bombarding energies the discrepancy between the present model and experiment could perhaps be resolved.

In conclusion the coherent classical bremsstrahlung picture has been schematically investigated to point out the influence of non-zero impact parameters on angular distributions of hard photons emitted from collisions between charged nuclei. The basic approximation in this calculation is the assumption of instantaneous stopping of point-like nuclei in the c.m. system. The interplay between R and S , the transverse and longitudinal internuclear distances, respectively, at which the nuclei stop, is seen to determine the main features of the angular dependence of electromagnetic bremsstrahlung.

The dipole-like angular distribution measured in the reaction $^{14}\text{N} + ^{58}\text{Ni} \rightarrow \gamma + X$ at $35 \text{ MeV}/n$ [10] finds a natural explanation in this model as being dominated by photons from impacts with intermediate values of b . If the range of impact

Figure 3: Inclusive angular distributions (16) of the number of hard photons in the centre-of-mass system. Photon frequency is 35 MeV, the reaction under consideration $^{14}\text{N} + ^{58}\text{Ni}$ at 35 MeV/n bombarding energy. Solid lines with $S = 0 \text{ fm}$ coincide with the corresponding lines in Fig. 2(a) for $R = 2 \text{ fm}$ and $R = 5 \text{ fm}$. Dotted lines are calculated with $S = 1, 2$, and 3 fm , for $R = 5 \text{ fm}$ also with $R = S$.

parameters relevant for the emission of hard photons is assumed to be $\lesssim 5 \text{ fm}$ and the longitudinal distance S between the nuclei after stopping $\lesssim 2 - 3 \text{ fm}$, then not only the shape but also the magnitude of the measured cross section [10] can be reproduced. On the other hand, the angular isotropy of photons found in earlier measurements at higher energies [9,14] cannot be understood within this simple approach. The discrepancy between the present calculation and the experiment [9,14] might be due to the oversimplification of the reaction dynamics, and could possibly be resolved if more realistic (peripheral) trajectories of the colliding nuclei were taken into account. Perhaps it is not too much to hope that the photon angular distributions could even serve as a tool to gain information about the details of the collision dynamics.

However, the fact, that pure quadrupole patterns with almost vanishing emission sideways will not be seen when looking at hard photons with frequencies as high as 20 MeV , is independent of these details. There is thus no reason to believe that even substantial deviations of the experimental photon angular distributions from the quadrupole type should necessarily indicate admixtures of incoherent radiation to the photon spectra that would be expected from independent nucleon-nucleon collisions.

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