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Essays on the Relationship Between Competition and Innovation

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by

Zhuozhen Zhao

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ABSTRACT OF THE DISSERTATION

Essays on the Relationship Between Competition and Innovation

by

Zhuozhen Zhao

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Dr. Urmee Khan, Co-Chairperson
Dr. Ruoyao Shi, Co-Chairperson

This dissertation consists of three essays that investigate the relationship between competition and firms' innovation. Chapter 1 provides an introduction of this dissertation. In Chapter 2, I construct a structural model of dynamic duopoly for durable goods to examine how competition affects firms' endogenous innovation with optimal releasing time. Firms dynamically decide pricing, how much to invest in R&D, and when to release new versions. I estimate my model using data of the personal computer microprocessor industry with Intel-AMD duopoly. Comparing the baseline market structure of duopoly and the counterfactual without AMD present, I find that firms' average investment is 12.4% less and the frontier quality upgrading rate is 0.9% lower in monopoly, but the average new product releasing probability is slightly higher. Moreover, the industry profit is decreased by 10% and consumer surplus is reduced by 4.7% without competition.

In Chapter 3, I construct another structural model of dynamic duopoly with multi-product firms to examine how competition affects firms' strategic innovations on high-end and low-end product lines. I identify and estimate the model using data of the US-brand

large SUV industry with GM-Ford duopoly, in the circumstance where firms' R&D choices between product lines are unobserved. I find that in the case where the competitor devotes itself to upgrading only high-end products, the firm chooses to compete face-to-face with its competitor: keeping investments in high-end products to remain competitive, but reducing investments in low-end ones to save effort. In the case where the competitor puts more effort into the low-end product line, the firm reacts by investing predominantly in both lines to build up market power thoroughly.

In Chapter 4, I construct and numerically analyze a structural model of dynamic duopoly with multi-product firms to examine how competition affects firms' strategic high-end product innovation and low-end process innovation. I find that firms' high-end product innovation is nearly nineteen times higher with the competitor present, but the low-end process innovation is 10% lower. Firms invest comparably in the high-end and low-end product lines in duopoly, but the monopolist only focuses on the low-end process innovation which demands less effort. The results demonstrate the dramatically positive effect of competition on the quality advancement, product differentiation, and social surplus.

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Chapter 1

Introduction

The relationship between competition and innovation has been long studied theoretically and empirically by economists to inform policies and enhance economic growth. Shumpeter (1942) firstly proposed that severe competition impedes firms from innovating. Arrow (1962) suggested an opposite viewpoint that more competitors encourage innovation, while Scherer (1967) argued that the relationship exhibits an inverse-U shape. The theoretical debate motivated extensive empirical literature which contributed mixed supports to these hypotheses. Most empirical studies employ reduced-form econometrics techniques and regress measures of innovation on a measure of market structure (e.g., Cohen and Levin (1989), Blundell et al. (1999), Aghion et al. (2005)) to conclude the relationship between competition and innovation. Other studies construct structural models which allow mutual effects between competition and innovation and the implementation of counterfactual experiments.

This dissertation consists of three essays that investigate this relationship from three unique and novel practical viewpoints by constructing dynamic structural models. They provide sharp insights of understanding firms' developing trend and meaningful policy implications regarding antitrust and economic growth especially in prevailing economical situations.

Chapter 2 examines the effect of competition on firms' product improvement as well as optimal release timing and meanwhile comprise good durability. In many R&D intensive product categories, firms heavily invest in R&D to improve product quality and periodically release new versions into market. It's important for them to optimize both decisions because of the cost-benefit trade-off. Furthermore, it is important to consider goods durability because consumers essentially have an outside option of using owned products they have purchased before. Firm competes with its rivals as well as previous goods that consumers owned. Therefore, firms need to take consumers' ownership into account when they optimize product development strategies.

I estimate this model using data of the personal computer microprocessor industry with Intel-AMD duopoly. Comparing the baseline market structure of duopoly and the counterfactual monopoly without AMD present, I find a positive relationship between competition and innovation: firms' average investment is 12.4% more and the frontier quality upgrading rate is 0.9% higher in duopoly. However, because of the business stealing effect, the average new product releasing probability is slightly lower when the competitor presents.

Different from existing papers typically assuming that each firm sells one normalized product due to data availability and computational burden (e.g., Goettler and Gordon (2011), Hashmi and Biesebroeck (2016), Borkovsky (2017), Igami and Uetake (2019)). Chapter 3 and 4 contribute to the literature by incorporating multi-product firms to emphasize how the competitor's strategies for differently developing each product line and market structure affect the firm's internal allocation of research and development (R&D) investment in a dynamic structural setting. These two papers focus on the multi-product firms with vertical differentiation who offer different product lines across various qualities, specifically high-end and low-end product lines. The results greatly help explain why it is more and more common for firms to create and develop a high-end product line in many industries regardless of their market positions, and emphasize the deterministic effect of competition to the high-end product advancement, product differentiation, and social surplus.

More specifically, Chapter 3 studies how a competitor's behavior affect the firm's product innovation strategies on the high-end and the low-end product lines. Identifying and estimating this dynamic structural model when the firm's choices are private information and unobserved constitute another main issue of this paper. To numerically study this question, I apply the model and empirical framework to the US large SUVs industry with a new data set. The most attractive finding shows when the competitor devotes itself to upgrading high-end products, the optimal responses are keeping regular investments in high-end products to remain competitive but reducing investments in low-end ones to save effort. However, when the competitor puts more effort into the low-end product line, the firm responds by investing predominantly in both lines to build up market power thoroughly.

Chapter 4 extends Chapter 3 by modeling firms' high-end product innovation which develops new and more-advanced-quality products and low-end process innovation which reduces production cost and gain pricing advantage. Comparing the baseline duopoly and the counterfactual monopoly, I find that firms devote comparable efforts in high-end product innovation and low-end process innovation in duopoly, but the monopolist only focuses on the low-end process innovation which needs less effort but with higher success probability. This confirms the importance of competition on the high-end product advancement, product differentiation, and social surplus.

Chapter 2

Does the Competitor Stimulate the Firm to Innovate More and Release Sooner?

2.1 Introduction

In this paper, I focus on understanding the relationship between firms' competition and innovation incorporating optimal release timing. In many R&D intensive product categories, such as electronics, video games and software, firms heavily invest in R&D to improve product quality and periodically release new versions into market. It's important for them to optimize decisions related to the development of product improvements and the timing of version releases. Releasing new products generates costs: for example, firms pay the fees related to licensing and retail/wholesale permits of selling; they spend on marketing to promote the products and enhance consumers' purchase; and they pay the fees associated with packaging, shipping, and storing. Because of these inescapable costs, firms must balance the expense and benefit when releasing a new product. Some firms choose to upgrade one step at a time, releasing new version every period no matter how large the product improvement is. This helps them always lead, or not fall much behind the leader in the industry, but still causes downside: consumers are less interested in purchasing because of the small quality improvement. On the contrary, other firms accumulate their technological achievements, incorporate them together into new product, and release once even though this process passes several periods. The product with qualitative leap may attract more consumers and earn generous profits, but firms must take a risk of stagnating and being left behind while competitors keep releasing new products. Therefore, when to release and how much to innovate in R&D with release timing taken place are important decisions to make. Following Borkovsky (2017), I allow firms to accumulate R&D stock—the stock of product improvements that firms developed since last version release, when they make strategic decisions.

I construct a structural model of dynamic duopoly with two firms' strategic interactions for durable goods to analyze the product development and competition. My model is within the context of Ericson and Pakes (1995) which provides a framework for numerically analyzing dynamic models of oligopolistic competition. In this framework, each firm optimizes behavior with rational expectations of competitors' actions, forming a Markov-perfect industry equilibrium. My model differs from earlier dynamic oligopoly models not only by incorporating optimal release timing and allowing accumulation of R&D success, but also by comprising goods durability. Due to durability, firm competes with its rivals as well as previous goods that consumers have already purchased. Consumers essentially have two outside options: one is in the normal sense that they can access products through public paths; the other is using owned ones they purchased before. I model this feature by incorporating consumers' ownership distribution—the proportion of consumers owning products of different qualities. Therefore, firms must take consumers' ownership into account when they optimize product development strategies.

In addition, firms' pricing strategies are dynamically determined due to durability. This setting captures that current price will affect consumers' future demand: they are not likely to repurchase in the near future when they have bought today. To reduce computational burden, I follow most empirical IO literature to assume naive consumers who only compare current-period utilities when making purchase decisions. I will consider rationally-expectant consumers who make dynamic upgrade decisions with anticipating product improvements and price declines in my future works.

To numerically study the effect of competition on firms' innovation and releasing, I apply my model to a particular industry: personal computer (PC) central processing unit (CPU). The reasons why I study microprocessor industry are as follows. First, this industry is important to the whole economy: according to Jorgenson et al. (2010), computer equipment manufacturing industry generated 25 % of U.S. productivity growth from 1960 to 2007. Second, CPUs are durable goods which satisfy my model setting. Then, CPU industry is essentially a duopoly, with Intel and AMD selling almost 95% in the market. Furthermore, two firms invest substantially in R&D department. The average R&D expenditure to revenue ratio is approximately 11% for Intel and 20% for AMD over 1993-2004 span of my data. Innovations happen frequently with new products being introduced nearly every one to two quarters. Finally, R&D accumulation and strategic releasing exist in this industry. For example, Intel didn't obtain quality improvements for almost three quarters before releasing Pentium with MMX technology, the product with one of the largest performance gains over my data span, in the first quarter, 1997. Similarly, AMD accumulated technological achievements and chose to release all at once when it progressed hugely from K5 to K6.

In addition, AMD and Intel cross-license each other's technologies, so that neither firm falls too behind the other. From my data, even though AMD's average R&D investment is only a quarter of Intel, its products have comparable qualities and even lead in some periods. So I follow Goettler and Gordon (2011) to model innovation as stochastic gains on a quality ladder where success is more likely to happen with more R&D investments and the laggard can benefit from the spillover effect.

I apply simulated minimum distance (SMD) approach, which matches model-simulated moments and empirical moments, to estimate consumer preferences and firms' innovation efficiencies. These parameters are key determinants of consumer demand, firms' pricing, R&D investment and releasing strategies. I then compare market outcomes and firm's policy functions under baseline duopoly scenario and counterfactual with AMD removed from the industry to conclude the effect of competition on innovation and releasing.

I find that in Intel-AMD duopoly, the industry profit is 10% higher than Intel monopoly, and consumers earn 4.7% more surplus. The CPU industry benefits from firms' competition and generates more profit because consumers bring on higher demand. Consumers benefit from competition as well since they can access products with higher quality but lower price.

Regarding firms' upgrading behavior, I find that in duopoly, the average investment is 12.4% higher than monopoly; the frontier quality upgrading rate is 0.9% higher; but the average new product releasing probability is slightly lower. My results support the hypothesis of Arrow (1962) who proposes a positive relationship between competition and innovation. When introducing advanced products, firms need to balance the benefit generated by leading in the industry and the loss caused by business stealing. My setting that firms can accumulate R&D achievements motivates them to invest more and advance the frontier product more often even though a competitor exists. Because they can stock technological achievements and wait for a best time to release to avoid business stealing to the maximum extent. The observations of releasing probability in two scenarios support my analysis as well: firms are more likely to accumulate innovation outcomes and release less frequently when competing with others, compared to monopoly.

Furthermore, the average price in duopoly is 12.4% lower than monopoly and the total sales are 21.1% greater. But Intel's market share[1] is 4.9% larger in monopoly. Because of the product durability, over 40% consumers choose no purchase and remain to use owned goods, no matter which market structure they are at. In monopoly, even though Intel gains

[1] In duopoly, total market share (100%) equals market share of Intel plus AMD and plus no-purchase; while in monopoly total market share equals Intel plus no-purchase.

more market share, the penetration rate is still lower than duopoly. I can say that the durability to some degree helps relieve consumers' benefit loss caused by monopoly. In summary, we cannot ignore the positive effect of competition and the contribution of AMD to the total sales, product developments and social surplus.

My paper is related to the literature studying impact of competition on R&D activity. As I mentioned before, for three main theoretical hypotheses, Shumpeter (1942) first proposes a positive relationship between market concentration and innovation. He thinks large firms are more common in the industry with high market concentration and they are better able than small ones to fund large R&D projects. On the contrary, Arrow (1962) argues that market concentration doesn't spur innovation, but competition does. A monopolist has less incentives than competitor to make a dramatic product improvement because he does not get much additional business given the truth that he already owned most of the business. If a competitor had come up with the same innovation, by contrast, he would earn more because he would expect to take away much of the business previously conducted by rival firms. Scherer (1967) combines the above two hypotheses and brings a inverse-U relationship up: high concentration tends to coincide with rich technological opportunity, but when concentration ratio exceeds 50%, additional market power has negative effect on technological improvement. Later works develop models to justify these conclusions: Cohen and Levin (1989) and Blundell et al. (1999) both empirically study the effect of competition (in the sense of market power or market share) on innovation through a reduced-form estimation across industries; Aghion et al. (2005) adds evidence of inverse-U relationship between Lerner index (competition measure) and patent production (innovation measure) across U.K. industries; Vives (2008) investigates the relationship across firms selling non-durable goods by different competition measures: firms innovate less if facing larger market size, but innovate more if there is greater product substitutability. A good survey of literature investigating this relationship in different industries can be found in Holmes and Schmitz Jr (2010). Recent works mostly develop an empirical dynamic structural model to analyze the relationship (see Goettler and Gordon (2011), Thurk (2018), Igami and Uetake (2019) and Hollenbeck (2020)), but none of them consider new version releases.

My paper relates to literature incorporating new product release timing as well. The analytic theory literature tends to focus on new product introductions and therefore restricts attention to a single release decision per product (see Wilson and Norton (1989), Moorthy and Png (1992), and Bhaskaran and Ramachandran (2011)). Empirically, Ofek and Sarvary (2003) construct an analytic model (with a one-dimensional state indicating whether a firm is leader or follower) to study dynamic competition in markets in which firms invest in R&D to develop next-generation products. They mainly explore the implications of different advantages that a leader might possess in terms of innovative ability, reputation, and advertising effectiveness, but not endogenous version releases. Ramachandran and Krishnan (2008) consider repeated releases of the same product, but they do so in a monopoly setting, whereas Morgan et al. (2001) and Aizcorbe (2005) treat the behavior of rival firms as exogenous. Borkovsky (2017) has the most similar model setting as mine, but his model is for non-durable goods and optimal price is statically determined.

In summary, this is the first paper to study the relationship between competition and innovation with firms' optimal release timing within the framework of structural model of dynamic duopoly for durable goods. The paper proceeds as follows. Section 2.2 describes the data I use to do the empirical analysis and presents summary statistics. Section 2.3 develops the theoretical model of firms and consumers behavior and discusses the equilibrium. Section 2.4 and 2.5 presents my estimation strategy and main empirical results. Section 2.6 concludes the paper.

2.2 Data

2.2.1 Data Description

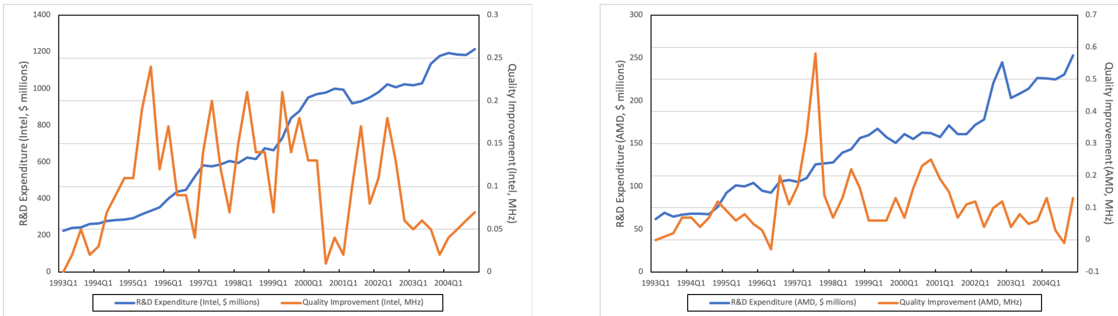
I use quarterly data from 1993 Q1 to 2004 Q4 of Intel and AMD. One feature of CPU product is that I can measure innovation directly by quality improvement instead of indirect measure such as patent applications. In my case, I use average processing speed to present CPU quality [2]. I use the data of manufacturer's average selling price to measure product's price; unit shipments to calculate market share and average production cost to present marginal cost[3]. Firms' R&D investment is measured by R&D expenditure; and revenue they collect is measured by quarterly total revenue[4]. Consumers' ownership distribution is measured by the fractions of consumers who own CPUs with different qualities; for non-owners, I use quality of public access to represent their ownership[5].

[2]The data is collected from two websites: www.cpuscorecard.com and www.cpubenchmark.net; Intel and AMD quarterly reports.

[3]These three data sets are obtained from the attachment, Goettler and Gordon (2011).

[4]These two data sets are collected from quarterly financial reports of Intel and AMD.

[5]This data is attached with Goettler and Gordon (2011).



(a) Intel R&D and Quality (b) AMD R&D and Quality

Figure 2.1: Firms' R&D and Quality

2.2.2 Summary Statistics

Table 2.1[6] provides summary statistics for my data span. One can observe that two firms' product qualities are neck and neck; in some periods, Intel's quality is more advanced than AMD (e.g. the minimum quality of Intel is 0.3117 better than that of AMD), while in others, AMD produces higher quality products (e.g. the maximum quality of AMD is 0.1436 better than that of Intel). Therefore, the competition between Intel and AMD are fierce and leader firm changes hands. The second and third rows summarize firms' quality improvements between periods and R&D investment to revenue ratio. These two indexes provide evidence that firms' releasing problem indeed exists. To clearly observe this pattern, I subtract these two indexes and draw a line chart as shown in figure 2.1[7]. In some periods (e.g. Q2, 1993; Q3, 2000; Q2, 2004 for Intel and Q2, 1993; Q3, 1996; Q2, 2002; Q1, 2003; Q3, 2004 for AMD), there is almost no quality improvement between two-period products, but the corresponding R&D expenditure is still very high and on the normal trend. Also,

[6] I use average quality over all CPUs of a firm currently sold in the market to represent quality index. Negative number of quality improvement exists may because high-quality product quit the market that period.

[7] In the line chart, I use absolute R&D expenditure amount instead of R&D-revenue ratio.

Table 2.1: Summary Statistics

	Intel					***AMD***				
	Mean	Std	Min	Max	Obs	Mean	Std	Min	Max	Obs
Quality (Mhz)	6.7301	1.6059	4.2167	8.9900	48	6.5010	1.8224	3.9050	9.1336	48
Quality Improvement (Mhz)	0.1016	0.0596	0.0120	0.2357	47	0.1112	0.0997	-0.0352	0.5813	47
R&D Expenditure/ Rev.	0.1138	0.0248	0.0769	0.1620	48	0.2034	0.0631	0.1247	0.4347	48
Unit Price ($)	219.7269	40.2702	157.1200	291.3600	48	99.5897	15.0699	79.9805	130.4873	48
Release Frequency	0.7708	0.4247	0	1	48	0.7708	0.4247	0	1	48

	Mean	Std	Min	Max	Obs
Ave. Quality Consumers Own (Mhz)	4.8360	1.7552	2.1200	7.6000	48
Market Share (Intel)	0.8463	0.0428	0.7397	0.9485	48

the quality improvement (the orange line in figure 2.1) doesn't move consistently with R&D investment (the blue line). The reason for this inconsistency is possibly the stochastic outcomes of R&D investment, say the investment may not always be successful to improve quality and I incorporate this feature in my model. Another hypothesis is that firms may stock technological improvements in some periods and release a new version with qualitative leap[8]. The fluctuation of quality difference: in the near future of the point without quality improvement, there is always a big jump, supports my hypothesis about the existence of firms' releasing problem.

I treat quality improvements less than a threshold value 0.05 as firms not releasing new versions and generate release frequencies (0 is not releasing, and 1 is releasing). Presented in row 5, table 2.1, firms release almost 75% over time and have 25% probability not to release. The second last line shows that the average quality owned by consumers has a gap compared to new products currently offered in the market. It means that consumers seldom own frontier products and many of them still use old ones they bought before. Hence, considering consumers' ownership in the study of durable goods cannot be neglected. Finally, observing the market share and unit price, Intel dominates in the CPU industry and AMD always plays a role of catching-up.

[8]One possibility is that some R&D projects need more than one period to finish, which causes mismatch between R&D expenditure and quality improvement. I categorize this case to release timing problem as well because firms choosing large R&D projects is equivalent to accumulate technological progress and release new versions all at once.

2.3 Model

2.3.1 Basic Setup

I construct a dynamic model of duopoly for a durable good with discrete time t and infinite horizon. Two firms, each denoted by $j \in \{1, 2\}$, sells a single product each period, invests to improve its quality, and decides whether to release the new version into market with paying a releasing cost. I define a R&D stock ladder as $r_{jt} \in \{0, \delta, 2\delta, \cdots, R\}$, with R as the upper bound. If the innovation is successful, firm's R&D stock improves by one ladder (δ), otherwise unchanged. Meanwhile, if firm decides to release the new version, the quality of its product could improve r_{jt}, but at the same time, its R&D stock resets to zero. If a firm doesn't release, its R&D stock remains to the next period and can be accumulated. For example, two firms are currently with ($q_1 = \delta, q_2 = 0, r_1 = 2\delta, r_2 = \delta$). If firm 1 releases, q_1 becomes 3δ and r_1 resets to 0; if firm 2 doesn't release, q_2 and r_2 keep unchanged, and if firm 2 innovates successfully, r_2 can be accumulated and becomes 2δ. In a word, firms' releasing decision changes quality and innovation decision changes R&D stock. The releasing cost is private, systematically and randomly following specific distribution $G(\cdot)$.

A key feature of demand for durable goods is that the utility of no-purchase option is endogenous because it depends on previous purchases. Each period, consumers decide whether to buy a new product or continue using old ones. Therefore, referring to Goettler and Gordon (2011), consumers' ownership distribution—fractions of consumers who are owning products with quality k at time t, denoted by $\Delta_t = (\Delta_{\underline{q}_t,t}, \cdots, \Delta_{k,t}, \cdots, \Delta_{\bar{q}_t,t})$—affects current demand. To help bounding the state space, I assume a lower bound and upper bound of consumers' ownership.

Firms are forward looking and take the optimal dynamic behaviors of its competitor into account when deciding actions. The state variables, which can be observed by both agents, are firms' qualities $\boldsymbol{q}_t = (q_{1t}, q_{2t})$, R&D stocks $\boldsymbol{r}_t = (r_{1t}, r_{2t})$ and consumers' ownership distribution Δ_t. These state variables constitute the state space according to which firms simultaneously choose optimal price $p_{jt} \in \mathbf{R}^+$, investment $x_{jt} \in \mathbf{R}^+$ and releasing $\eta_{jt} \in \{0, 1\}$. Consumers choose purchasing product 1, product 2 or no purchase, given the quality of her currently owned product $\tilde{q}_t$, firms' current offerings $\boldsymbol{q}_t$, and prices. I don't consider entry, exit and second-hand market.

2.3.2 Model Timeline

The timeline of firms' decisions each period is as follows. The state transits twice in one period due to firms' updating strategies. I omit the time subscript for conciseness:

1. At the start of each period, each firm observes state $s = \{q_1, q_2, r_1, r_2, \Delta\}$ and learns how much it will cost to release a new version of its product. The releasing cost $\phi_j \in \Phi$ is drawn from a distribution $G(\cdot)$.

2. Firms simultaneously choose optimal prices of products with quality $\{q_1, q_2\}$, sell in the market[9], and earn profit; meanwhile they also simultaneously choose optimal updating strategies which include two actions: releasing and investment.

3. For updating strategies, releasing and investment happen sequentially:

[9] Consumers' ownership distribution updates.

(1) At the beginning of sub-step 1, firms simultaneously decide whether to release new products;

(2) Releasing decisions are implemented. The state transits to $s' = \{q'_1, q'_2, r'_1, r'_2, \Delta'\}$ in which $\boldsymbol{q}'$ is the quality of products sold in the next-period market. $s = s'$ if neither firm releases.

(3) At the beginning of sub-step 2, firms simultaneously decide how much to invest in R&D after observing $\boldsymbol{q}'$;

(4) The outcomes of investments in R&D stocks are realized. The state transits from s' to s'', which is also the state at the beginning of next period.

2.3.3 Consumers

I model consumers as owning no more than one product at a time. Utility for a consumer i from firm j's new product with quality q_{jt} and price p_{jt} is given by

$$u_{ijt} = \gamma q_{jt} - \alpha p_{jt} + \xi_j + \varepsilon_{ijt}, \tag{2.1}$$

where γ is the taste for quality, α is the marginal utility of price, ξ_j represents brand preference for firm j, and ε_{ijt} captures idiosyncratic variation which is independently and identically distributed as standard type 1 extreme value across consumers, products and periods. Utility from no-purchase option is

$$u_{i0t} = \gamma \tilde{q}_{it} + \varepsilon_{i0t}, \tag{2.2}$$

where no price and brand preference effect act. $\tilde{q}_{it}$ is the quality of outside options: the quality of most recent purchase for previous purchasers; or the quality available through other means for non-purchasers.

As I mentioned before, to facilitate bounding the state space, I assume that $\tilde{q}_{it}$ is within $\bar{\delta}_c$ of the frontier quality $\bar{q}_t = max(q_{1t}, q_{2t})$. That is, $\underline{q}_t \equiv \bar{q}_t - \bar{\delta}_c \leq \tilde{q}_{it} \leq \bar{q}_t$. I also define the ownership distribution $\Delta_t = (\Delta_{\underline{q}_t,t}, \cdots, \Delta_{k,t}, \cdots, \Delta_{\bar{q}_t,t})$, where $\Delta_{k,t}$ is the fraction of consumers whose outside option has quality $\tilde{q}_{it} = q_{k,t}$.

Each consumer maximizes her utility, yielding the conditional choice probabilities of firm j's product for a consumer currently owning product $\tilde{q}$:

$$D_{j|\tilde{q}} = \frac{exp(u_j - \varepsilon_j)}{\Sigma_{k\in\{0,1,2\}} exp(u_k - \varepsilon_k)},$$

where $k = 0$ denotes no-purchase option. Integrating over the distribution of $\tilde{q}$ yields the aggregate demand for product j as market share:

$$D_j(\boldsymbol{p}, \boldsymbol{q}, \Delta) = \Sigma_{\tilde{q}\in\{\underline{q},\cdots,\bar{q}\}} D_{j|\tilde{q}} \Delta_{\tilde{q}}$$

Following Goettler and Gordon (2011), I determine the law of motion for Δ as below: if $\bar{q}$ is unchanged,

$$\Delta'_k(\Delta, q, p|\bar{q}' = \bar{q}) = D_{0|k}\Delta_k + \Sigma_{j=1,2} D_j \mathbb{1}(q_j = k); \tag{2.3}$$

the fraction of consumers owning product with quality k at the start of next period equals to the share of consumers who maintain product k plus who bought new products offered by any firm with quality k. If the quality frontier advances at the end of the current period, I shift the interim Δ' in the above equation via $\Gamma(\cdot)$, a shift operator defined on $y = (y_1, y_2, \cdots, y_L)$ as $\Gamma(y) = (y_1 + y_2, y_3, \cdots, y_L, 0)$. I do so because consumers' ownership is defined relative to the frontier quality. Hence,

$$\Delta'(\Delta, q, p) = \mathbb{1}(\bar{q}' = \bar{q})\Delta'(\Delta, q, p|\bar{q}' = \bar{q}) + \mathbb{1}(\bar{q}' > \bar{q})\Gamma(\Delta'(\Delta, q, p|\bar{q}' = \bar{q})). \tag{2.4}$$

2.3.4 Firms

Each period t, firm j makes dynamic pricing, investment and releasing decisions. The period profit function, excluding investment and releasing costs, for firm j is

$$\pi_j(\boldsymbol{p}, \boldsymbol{q}, \Delta) = M \times D_j(\boldsymbol{p}, \boldsymbol{q}, \Delta)(p_j - mc_j(\boldsymbol{q})), \tag{2.5}$$

where $\boldsymbol{p} = (p_j, p_{-j})$, $\boldsymbol{q} = (q_j, q_{-j})$, M is the market size, and $mc_j(\boldsymbol{q})$ is constant marginal cost of production. I follow Goettler and Gordon (2011) to set marginal cost related to quality level:

$$mc_j(\boldsymbol{q}) = \lambda_0 + \lambda_1 \left(\bar{q} - q_j\right). \tag{2.6}$$

λ_1 is negative to capture that the marginal production cost is lower for laggard firm and higher for the firm with frontier quality.

A firm simultaneously choose optimal price and update (i.e. releasing and investment) strategies every period. For updating, because releasing and investment happen sequentially, I follow Borkovsky (2017) to first discuss firm's optimal investment condition in sub-step 2 and then derive optimal releasing action in sub-step 1 with optimal investment plugged into. Firms invest $x_j \in \mathbf{R}^+$ in R&D aiming to improve R&D stock and consequentially introduce higher-quality product. To obtain a closed form of optimal investment, I restrict investment outcome τ_j to be either δ or 0:

$$\tau_j = r''_j - r'_j = \begin{cases} 0, & \text{not success,} \\ \delta, & \text{success.} \end{cases} \tag{2.7}$$

The probability of success is given by the form:

$$\chi_j(\tau = \delta | x, \boldsymbol{q}) = \frac{a^j(\boldsymbol{q})x}{1 + a^j(\boldsymbol{q})x}, \tag{2.8}$$

where

$$a^j(\boldsymbol{q}) = a_{0,j} \max\left[1, a_1\left(\frac{\bar{q}-q_j}{\delta}\right)^{1/2}\right] \tag{2.9}$$

represents firm j's investment efficiency. $a_{0,j}$ captures firm's investment capacity, while $a_1 > 0$ captures spillover effect. Therefore, the laggard firm is more likely to innovate successfully because of the technology spillover. The spillover effect of investment depends on two firms' qualities at the beginning of current period no matter whether firms choose to release a new version within this period. The probability of innovation failure is

$$\chi_j(\tau = 0|x, \boldsymbol{q}) = 1 - \chi_j(\tau = \delta|x, \boldsymbol{q}). \tag{2.10}$$

At the beginning of sub-step 2, the industry is in state s' after firms' releasing decisions. The expected net present value of firm j in state s' brought by investment x_j is denoted by U_j:

$$\begin{aligned} U_j(s') \equiv \max_{x_j \geq 0}\Big\{ -x_j + \beta \int_{\phi'_j} \Sigma_{\tau_j, r''_{-j}, \Delta'} V_j((q_j, q_{-j}, r_j + \tau_j, r''_{-j}, \Delta'), \phi'_j) \\ \times \chi_j(\tau_j|x_j, \boldsymbol{q}) k_{f_j}(r''_{-j}|r_{-j}, \Delta) h_{f_j}(\Delta'|\Delta, \boldsymbol{q}, \boldsymbol{p}) dG(\phi'_j)\Big\}, \end{aligned} \tag{2.11}$$

where β is the discounted factor, $k_{f_j}(\cdot|\cdot)$ is firm j's belief about competitor's future R&D stock, and $h_{f_j}(\cdot|\cdot)$ is his belief about the transition of consumers' ownership given current prices and qualities. I also integrate the expected future value over the density of future releasing cost.

Maximizing U_j by choosing x_j, I can derive firm j's optimal investment condition as follows:

$$x_j^* = max\left\{0, \frac{\{\beta a_j(\boldsymbol{q})[EU^+(s') - EU^-(s')]\}^{(1/2)} - 1}{a_j(\boldsymbol{q})}\right\}, \tag{2.12}$$

and

$$EU^{+}(s') = \int_{\phi'_j} \Sigma_{r''_{-j},\Delta'} V_j((q_j, q_{-j}, r_j + \delta, r''_{-j}, \Delta'), \phi'_j) k_{f_j}(r''_{-j}|r'_{-j}, \Delta)$$

$$\times h_{f_j}(\Delta'|\Delta, \boldsymbol{q}, \boldsymbol{p}) dG(\phi'_j)\},$$

$$EU^{-}(s') = \int_{\phi'_j} \Sigma_{r''_{-j},\Delta'} V_j((q_j, q_{-j}, r_j + 0, r''_{-j}, \Delta'), \phi'_j) k_{f_j}(r''_{-j}|r'_{-j}, \Delta)$$

$$\times h_{f_j}(\Delta'|\Delta, \boldsymbol{q}, \boldsymbol{p}) dG(\phi'_j)\}. \tag{2.13}$$

$EU^{+}(s')$ represents the expected future profit with successful innovation ($\tau_j = \delta$); while $EU^{-}(s')$ represents the case with innovation failure ($\tau_j = 0$). If $EU^{-}(s') > EU^{+}(s')$ or r_j achieves its upper bound ($= R$), $x_j = 0$.

Having solved for firm's optimal condition of investment, I can induct backward to derive expected value function at sub-step 1 with state s to solve for optimal releasing decision. Let $W_j^{None}(s)$ denotes the expected net present value for firm j if none of two firms release new product; $W_j^{j}(s)$ denotes it if firm j releases but competitor doesn't; $W_j^{-j}(s)$ denotes that competitor releases but firm j doesn't; and lastly, $W_j^{Both}(s)$ is for the case where both firms release. Period profit and releasing cost are not incorporated in W_j. Specifically,

$$W_j^{None}(q_j, q_{-j}, r_j, r_{-j}, \Delta) \equiv U_j(q_j, q_{-j}, r_j, r_{-j}, \Delta); \tag{2.14}$$

$$W_j^{j}(q_j, q_{-j}, r_j, r_{-j}, \Delta) \equiv U_j(q_j + r_j, q_{-j}, 0, r_{-j}, \Delta); \tag{2.15}$$

$$W_j^{-j}(q_j, q_{-j}, r_j, r_{-j}, \Delta) \equiv U_j(q_j, q_{-j} + r_{-j}, r_j, 0, \Delta); \tag{2.16}$$

$$W_j^{Both}(q_j, q_{-j}, r_j, r_{-j}, \Delta) \equiv U_j(q_j + r_j, q_{-j} + r_{-j}, 0, 0, \Delta). \tag{2.17}$$

Then, I can derive firm j's bellman equation with it simultaneously choosing optimal releasing strategy η_j and price p_j:

$$
\begin{aligned}
V_j\left(s, \phi_j\right) = \max_{\eta_j \in \{0,1\}, p_j \geq 0} \Bigg\{ & \pi_j(\boldsymbol{p}, \boldsymbol{q}, \Delta) \\
& + (1-\eta_j)\{g_{f_j}(\eta_{-j}(s, \phi_{-j}) = 0) W_j^{None}(s) \\
& + g_{f_j}(\eta_{-j}(s, \phi_{-j}) = 1) W_j^{-j}(s)\} \\
& + \eta_j\{-\phi_j + g_{f_j}(\eta_{-j}(s, \phi_{-j}) = 0) W_j^{j}(s) \\
& + g_{f_j}(\eta_{-j}(s, \phi_{-j}) = 1) W_j^{Both}(s)\} \Bigg\},
\end{aligned} \tag{2.18}
$$

where η_j indicates whether firm j chooses to release $(=1)$ or not $(=0)$; g_{f_j} is firm j's belief whether its rival chooses to release $(\eta_{-j} = 1)$ or not $(\eta_{-j} = 0)$; and ϕ_j is firm j's releasing cost in this period.

Firm j chooses to release only if the expected future profit brought by releasing is larger than not releasing. Substituting x_j^* obtained from equation 2.12 into equation 2.11; $U_j(s')$ from equation 2.11 into equations 2.14—2.17; and W_js from 2.14—2.17 into Bellman equation 2.18, I can determine firm j's optimal releasing decision with drawn ϕ_j. Specifically,

$$
\begin{aligned}
\eta_j^* = \mathbb{1}\Big\{ & -\phi_j + g_{f_j}(\eta_{-j} = 0) W_j^{j}(s) + g_{f_j}(\eta_{-j} = 1) W_j^{Both}(s) \\
& \geq g_{f_j}(\eta_{-j} = 0) W_j^{None}(s) + g_{f_j}(\eta_{-j} = 1) W_j^{-j}(s) \Big\}
\end{aligned} \tag{2.19}
$$

with

$$
\begin{aligned}
g(\eta_j^* = 1) = G\Big(& \left\{ g_{f_j}(\eta_{-j} = 0) W_j^{j}(s) + g_{f_j}(\eta_{-j} = 1) W_j^{Both}(s) \right\} \\
& - \left\{ g_{f_j}(\eta_{-j} = 0) W_j^{None}(s) + g_{f_j}(\eta_{-j} = 1) W_j^{-j}(s) \right\} \Big);
\end{aligned} \tag{2.20}
$$

$$
g(\eta_j^* = 0) = 1 - g(\eta_j^* = 1).
$$

$G(\cdot)$ is the CDF of releasing cost ϕ. Firm j also chooses price to satisfy the first-order condition where

$$\frac{\partial V_j}{\partial p_j} = \frac{\partial \pi_j(\boldsymbol{p}, \boldsymbol{q}, \Delta)}{\partial p_j} + (1-\eta_j)\Big\{ g_{f_j}(\eta_{-j}=0)\frac{\partial W_j^{None}(s)}{\partial p_j} + g_{f_j}(\eta_{-j}=1)\frac{\partial W_j^{-j}(s)}{\partial p_j}\Big\}$$
$$+ \eta_j\Big\{ -\phi_j + g_{f_j}(\eta_{-j}=0)\frac{\partial W_j^{j}(s)}{\partial p_j} + g_{f_j}(\eta_{-j}=1)\frac{\partial W_j^{Both}(s)}{\partial p_j}\Big\} = 0. \tag{2.21}$$

The reason why W_js are related to p_j is that current price of durable goods will affect consumer's future ownership, so I have $\frac{\partial h_{f_j}(\Delta'|\Delta, \boldsymbol{q}, \boldsymbol{p})}{\partial p_j}$ inside $\frac{\partial W_j}{\partial p_j}$ to capture this effect.

2.3.5 Equilibrium

I consider pure-strategy Markov-perfect Nash equilibrium (MPNE) of this dynamic duopoly game. This refinement guarantees the equilibrium where players' strategies only depend on current state variable values and firms optimize behaviors in each state and each sub-game. The uncertainty of firms' releasing strategy makes it as though a mixed strategy but this uncertainty actually comes from the randomness of releasing cost. Doraszelski and Satterthwaite (2010), proposition 2, proves that randomness of releasing cost is a necessary condition for existence of MPNE.

I consider asymmetric equilibrium by allowing different firms' brand fixed effects, production costs, and innovation efficiencies for different firms. In the equilibrium, each firm has rational expectations about competitor's policy functions for price, releasing and investment and about the evolution of the ownership distribution. Formally, an MPNE in this model is the set $\{V_j^*, \eta_j^*, p_j^*, x_j^*, g_{f_j}^*, k_{f_j}^*, h_{f_j}^*\}_{j=1,2}$, which includes firms' equilibrium value functions, policy functions, beliefs about rival's future R&D stocks and qualities, and beliefs about future ownership distribution. The expectations are rational in that $g^*_{f_j}$ satisfies optimal releasing rules in equation 2.20; $k^*_{f_j}$ is generated from innovation outcome functions χ; and $h^*_{f_j}$ follows Δ transition rule revealed in equation 2.4.

To help guarantee the uniqueness of MPNE in my model, I apply "limit-of-finite" approach to refine the equilibrium, for which I use backward induction to solve for an equilibrium of a T-period game and then let $T \to \infty$. In each period and each state, I compute firms' equilibrium policy functions according to the optimal conditions I derived for pricing, releasing and investment. Then I use backward induction to iterate value functions with initial values of 0 and obtain equilibrium of my model. I relegate the algorithm details in appendix A.1.

2.4 Empirical Application

In this section, I apply the dynamic duopoly model to the CPU industry. I first present my strategies of bounding the state space to reduce computational burden. Then I estimate consumers' preference and firm's innovation efficiencies using simulated minimum distance estimation method.

2.4.1 Bounding the State Space

Two firms' product qualities, R&D stocks, and consumers' ownership distribution constitute the state space of my model. Formally, $s = \{q_1, q_2, r_1, r_2, \Delta\}$. Because product qualities can improve without bound so that the equilibrium is not computable, I follow Goettler and Gordon (2011) to transform $\{q_1, q_2\}$ and consumers' ownership $\tilde{q}$ to relative values to the frontier quality $\bar{q}$ in each period. This shifting has no effect on firms' payoffs and consumers' demand. Because (1) log quality is linear in the utility function, so adding any constant to the utility of each alternative doesn't change consumers' choices; (2) this shifting doesn't change the form of $mc(\cdot)$ and $\pi(\cdot)$; (3) innovations (also R&D stocks) are governed by $\chi(\cdot)$, which is independent of absolute quality levels; and (4) Δ is already in relative terms because it is defined as the ownership of the products within $\bar{\delta_c}$ of the frontier.

To further reduce dimension of my state space, I assume the upper bound R of $\{r_1, r_2\}$ to be 2δ, which means the most R&D stock firms could accumulate before releasing is two-step on the quality ladder. This assumption on the one hand reduces my computational burden, on the other hand captures the characteristic that R&D improvements are time-effective and become obsolete rapidly especially in high-technological industry. Therefore, firms will not wait long to release new versions.

2.4.2 Estimation

I refer Goettler and Gordon (2011) to first estimate $\hat{\boldsymbol{\lambda}}$ in the cost function (equation 2.6) by linear regression. Then, I estimate the dynamic parameters θ, including $(\gamma, \alpha, \xi_{\text{Intel}}, \xi_{\text{AMD}}, a_{0,\text{ Intel}}, a_{0,\text{AMD}}, a_1)$, which consist of consumers' preference and firms' investment efficiencies. I use simulated minimum distance (SMD) estimation method which minimizes the distance between empirical moments observed in my data and their simulated counterparts generated from my model. Given one group of candidate θ, I solve for the equilibrium where firms' value functions achieve convergence; simulate model for T periods and S times with state transitions; obtain simulated model moments, $m_{S,T}(\theta; \hat{\lambda})$; match them with empirical moments obtained from data (m_T); and get $\hat{\theta}_T$ by finding the best match. Formally,

$$\hat{\theta}_T = \arg\min_{\theta \in \Theta} \left[m_{S,T}(\theta; \hat{\lambda}) - m_T \right]' A_T \left[m_{S,T}(\theta; \hat{\lambda}) - m_T \right], \tag{2.22}$$

where A_T is the inverse of covariance matrix of empirical moments to guarantee the efficiency of estimator.

The data exhibit an increasing trend in sales, revenue and R&D expenditure, therefore I choose stationary moments in both data and model to get rid of the time trend effect. For example, I use the moment of invest per revenue instead of absolute investment level. My moment vector, m_T, consists of the following 16 moments:

- two firms' average prices and the coefficients from linearly regressing price on a constant, $q_{Intel,t} - q_{AMD,t}$, and $q_{Intel,t} - \bar{\Delta}_t$, where $\bar{\Delta}_t = \sum_{k=\underline{q}_t}^{\bar{q}_t} k\Delta_{kt}$ is the mean log quality currently owned by consumers in time t;
- coefficients from linearly regressing Intel's market share on a constant and $q_{Intel,t} - q_{AMD,t}$;
- two firms' mean updating(innovation and releasing) rates, defined as $(1/T)[(q_T - q_0)/\delta]$;
- mean $(q_{Intel,t} - q_{AMD,t})$ and the probability of $(q_{Intel,t} > q_{AMD,t})$;
- mean investment per unit revenue for each firm;
- mean releasing frequency for each firm.

These moments from data and the fitted values from model are listed in table 2.2.

The moments I choose to match are closely associated to the structural parameters θ. The demand-side parameters $(\gamma, \alpha, \xi_{\text{Intel}}, \xi_{\text{AMD}})$ are primarily identified by the price equations and the Intel share equation. Consumers' preference of price, quality and brand sharply affect firms' pricing and market share. The supply-side parameters $(a_{0,\text{ Intel}}, a_{0,\text{AMD}}, a_1)$ are primarily identified by observed updating rates, quality differences, investment levels and releasing strategies. The parameters of investment efficiencies are optimized to match the observed updating rates; the spillover parameter is chosen to match quality differences: even though AMD's R&D expenditure is only a quarter of Intel, its product quality is still comparable to Intel. Furthermore, I calibrate the upper bound and lower bound (50 and 100 for Intel; 0 and 50 for AMD) of the releasing cost distribution $G(\cdot)$ in order to match the observed releasing frequencies. I will estimate the releasing cost instead of calibration in my future works.

I set a few model setup parameters before estimation. Referring to Goettler and Gordon (2011), I set log quality step δ to 0.1823 and $\bar{\delta}_c$ (the difference between consumers' $\tilde{q}$ and the frontier) to 29δ. The high enough $\bar{\delta}_c$ enables consumers to rarely reach the lowest bound of $\tilde{q}$ and meanwhile satisfies acceptable computation time. The upper bound of quality difference between two firms is set as 6δ, which exceeds the observed 5.2δ in the data. I set discount rate β as 0.9 and market size M as 400 millions.

I report the parameter estimates in table 2.3. The model fits (in table 2.2, column 3) given these 7 parameters match the 16 moments kind of well. Several moments have larger difference since the real world is too complicated for a simulated model to recover perfectly.

Table 2.2: Empirical and Simulated Moments

Moment	**Actual**	**Actual Standard Error**	**Fitted**
Intel price equation:			
Average Intel price	219.7	40.3	183.2
coef. of $(q_{Intel,t} - q_{AMD,t})$	47.4	17.6	9.0
coef. of $(q_{Intel,t} - \bar{\Delta}_t)$	94.4	31.6	2.9
AMD price equation:			
Average AMD price	99.6	15.1	149.1
coef. of $(q_{Intel,t} - q_{AMD,t})$	-8.7	11.5	-12.9
coef. of $(q_{AMD,t} - \bar{\Delta}_t)$	16.6	15.4	0.8
Intel share equation:			
Constant	0.834	0.007	0.784
coef. of $(q_{Intel,t} - q_{AMD,t})$	0.055	0.013	0.009
Mean updating rates:			
Intel	0.557	0.047	0.724
AMD	0.610	0.079	0.752
Relative qualities:			
Mean $q_{Intel,t} - q_{AMD,t}$	1.257	0.239	0.681
Mean $\mathbb{1}(q_{Intel,t} > q_{AMD,t})$	0.833	0.054	0.673
Mean R&D/ revenue:			
Intel	0.114	0.025	0.005
AMD	0.203	0.063	0.008
Mean releasing freq.:			
Intel	0.771	0.425	0.640
AMD	0.771	0.425	0.664

Notes: simulated moments are averages over 500 simulations of 48 quarters of data.

Table 2.3: Parameter Estimates

Parameter	Estimates
Price, α	0.01
Quality, γ	0.3
Intel fixed effect, ξ_{Intel}	-0.4
AMD fixed effect, ξ_{AMD}	-2.0
Intel innovation capacity, $a_{0,\text{Intel}}$	0.01
AMD innovation capacity, $a_{0,\text{AMD}}$	0.02
Spillover, a_1	4.0
Marginal cost regression:	
Constant, λ_0	44.5
$(\bar{q} - q_j), \lambda_1$	-19.7

2.5 Empirical Results

In this section, I use the estimated parameters in table 2.3 to solve the industry equilibrium and characterize the equilibrium behaviors of firms and consumers. Furthermore, I do a counterfactual to remove AMD from the market and the CPU industry becomes Intel monopoly. Then I compare surplus, policies and market share in these two scenarios: (1) AMD-Intel duopoly and (2) Intel monopoly. Scenario (1) is the factual model using all the estimates in table 2.3, while scenario (2) uses Intel's parameters for the monopolist. In each scenario, I solve the optimal policies and simulate a 48-period model for 500 times, starting from the initial state in my data. And then I calculate the mean of simulations to obtain observations of interest.

2.5.1 Firm Behaviors in Equilibrium

Figure 2.2 and 2.3 present value function, equilibrium price, R&D expenditure, and releasing probability for monopoly and AMD-Intel duopoly at select states. In figure 2.2[10], the first column compares value and policy functions for monopolist with consumers' average ownership quality changing from oldest to newest, while the second column compares the counterparts for duopoly[11]. Outcomes are separately presented for leader and laggard in the columns of duopoly. As expected, when consumers own relatively new products, demand is low, firms' value functions decrease and they tend to set lower prices. Monopolist invests

[10] Column 1 corresponds to the Intel monopoly; column 2 and 3 are for duopoly. In column 1 and 2, the x axis is the average quality of consumers' ownership Δ with other state variables fixed. In column 3, the x axis is the quality difference between the leader and laggard, ranging from 6δ to zero step.

[11] For Intel-monopoly, I choose the state with R&D stock δ. For AMD-Intel duopoly, I fix the state with both firms produce tie quality and own δ R&D stock. Keeping both firms' other state variables the same, one can best compare observations with only Δ differing. For label consistency, in column 2, I label Intel as leader, and AMD as laggard, even though they are tied in the industry.

more as Δ is newer, introducing more advanced products to trigger purchases. However, its releasing strategies are not affected by consumers' ownership. Interestingly, in duopoly, both firms invest slightly less when Δ is newer. Because firms need to invest even more to obtain advantage when consumers have already owned relatively newer products. This means the leader's spillover effect to laggard will be more and the laggard can catch up more easily. Therefore, both firms invest less to avoid business stealing. The releasing strategies are opposite for both firms: when consumers own older ones and demand is high, leader accumulates R&D and release more advanced product to earn more expected profit, while laggard seizes the chance to release and take profit away from leader; when consumers own newer ones, leader releases often to prompt demand, while laggard is disadvantaged in this competitive circumstance and accumulates instead of releasing.

In the third column, I fix the mean of Δ to be 4δ away from the frontier quality[12] and vary the leader's quality advantage on the x axis. The laggard's quality is 6δ behind the leader at the leftest and they are tied at the rightest. The leader's value function, price, and investment declines (to avoid business stealing) when its advantage lessens, whereas the laggard's counterparts raise when it catches up. The leader's releasing strategies are not affected by the quality difference. But the laggard's releasing probability has a "concave" shape: when the quality gap is too large, the laggard has less incentive to release because releasing is costly and it's hard to catch up; when the laggard gets closer, it releases frequently to avoid to be left much behind again; when the laggard is almost tied with the leader, it chooses to accumulate and tries to surpass the leader at one time.

[12]The trend of firms' value function and policy functions are similar when the mean of Δ takes other values.

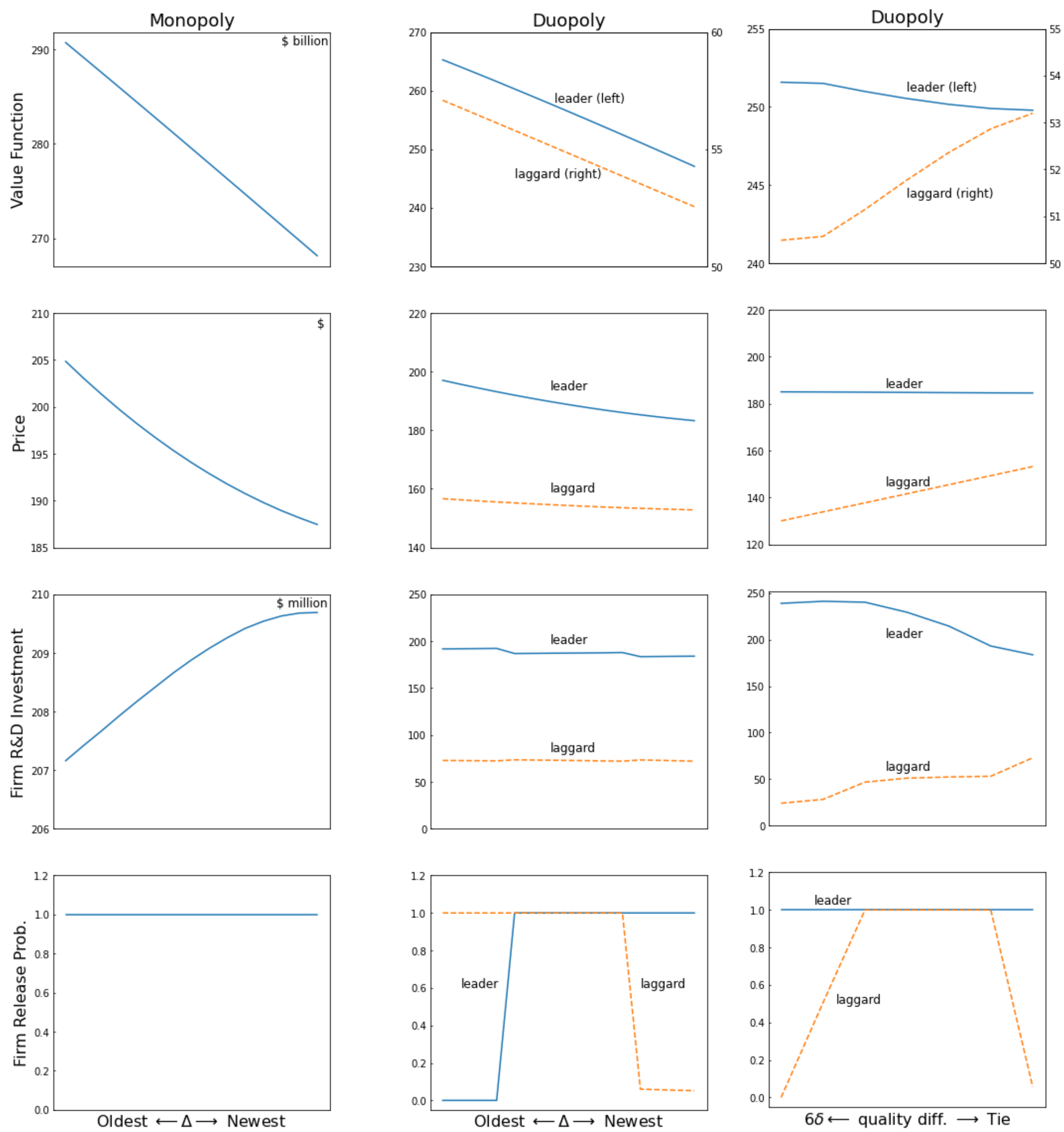


Figure 2.2: Value and Policy Functions for Monopoly and Duopoly: Ownership and Quality Difference Vary

In figure 2.3[13], column 1[14] presents monopolist's value function, price, investment and releasing probability given different R&D stock levels. Column 2 and 3[15] shows the counterparts for duopolists given different leader or laggard R&D stock levels[16]. For monopolist, value function increases as it accumulates more technology, whereas price, investment and release are not affected much by varying R&D stocks. In duopoly, firm has less value if competitor has more R&D stocks and their prices keep almost the same. Two firms both choose to release and invest more in R&D when the leader has more R&D stock (column 2), which also means the leader has potentiality to produce an even-higher-quality good. The leader invest more to surpass itself and so does the laggard because it has no second choice if it aims to catch up. On contrary, in column 3, the leader doesn't release to refrain from the business stealing when the laggard owns most R&D stock. And the leader invest less because it doesn't release and has already owned R&D achievements, while the laggard also invests less given it has already gained big quality improvements.

Because in one period, firms make releasing decisions and then determine investments, their subsequent investment choices are closely related to releasing strategies. In table 2.4, I presents firms' investments in duopoly under four different releasing cases: neither releases, both release, only Intel releases, and only AMD releases[17]. Both firms invest least if neither firm releases under which firms own R&D stocks in hand and competitor's

[13]Column 1 corresponds to the Intel monopoly; column 2 and 3 are for duopoly. In column 1, the x axis is monopolist's R&D stock from 0 to 2δ with other state variables fixed. In column 2 and 3, the x axis is respectively the leader's and the laggard's R&D stock.

[14]Mean of Δ is 4δ away from the frontier quality.

[15]In column 2, I choose the state with Intel as the leader, AMD's quality is δ behind Intel, AMD has one step R&D stock, and mean of Δ is 4δ away from the frontier quality, whereas Intel has one step R&D stock in column 3.

[16]The lines are not smooth because in my model R&D stock can only take three candidate values, 0, 1, 2.

[17]As before, I choose the state where two firms have tie quality and R&D stocks and the mean of Δ is 4δ away from the frontier quality

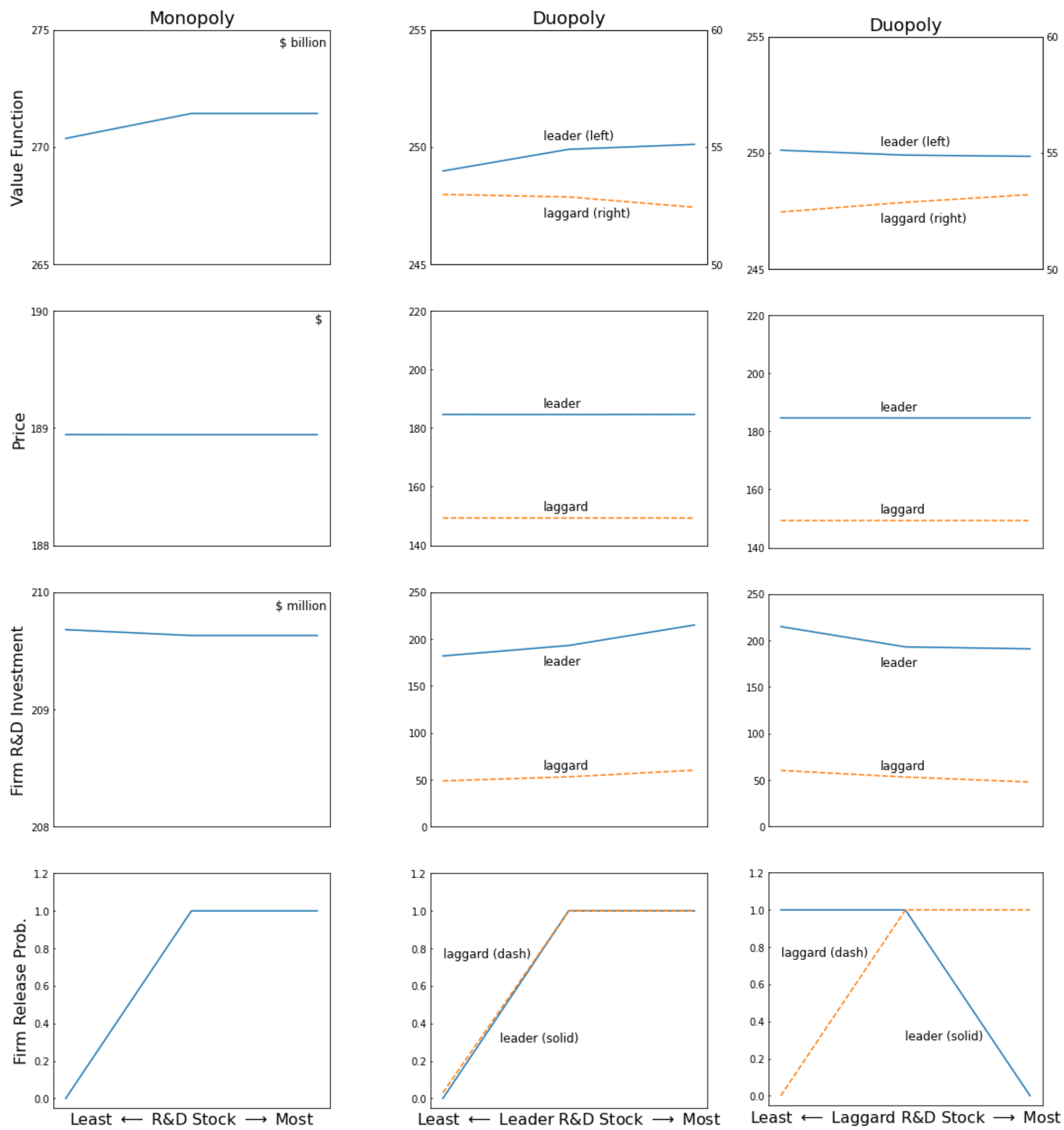


Figure 2.3: Value and Policy Functions for Monopoly and Duopoly: R&D Stocks Vary

Table 2.4: Firms' Investments Under Different Releasing Cases ($, million)

Investment	Neither Releases	Both Release	Only Intel Releases	Only AMD Releases
Intel	123.67	187.56	183.24	193.79
AMD	6.54	71.64	73.39	70.64

quality doesn't advance. They invest more if both release because they must generate new technology and compete with each other. The most interesting observation is that firm invests most among these four releasing cases if only competitor releases. Because in this case, the competitor's product upgrades and leads but the self doesn't. Firm must invest heavily and tries to catch up even exceeds in the next period.

2.5.2 Consumer Behaviors in Equilibrium

Figure 2.4 plots average choice probabilities over consumers' ownership distribution Δ at state where Intel is the leader, AMD's quality is δ behind Intel, and both firms have one step R&D stock. The x axis presents consumers' vintage relative to the frontier: for example, -5 means the quality of consumers ownership is 5δ behind the frontier quality in the market. Therefore, the lower a consumer's vintage, the more likely she is to purchase new product.

Figure 2.5 depicts the average ownership distributions for duopoly and monopoly of the states encountered in my simulation. Consumers in duopoly has a slightly newer distribution compared to monopoly because monopolist always charges a higher price. Besides, consumers have options to purchase non-frontier products (e.g. AMD's product) in a duopoly structure.

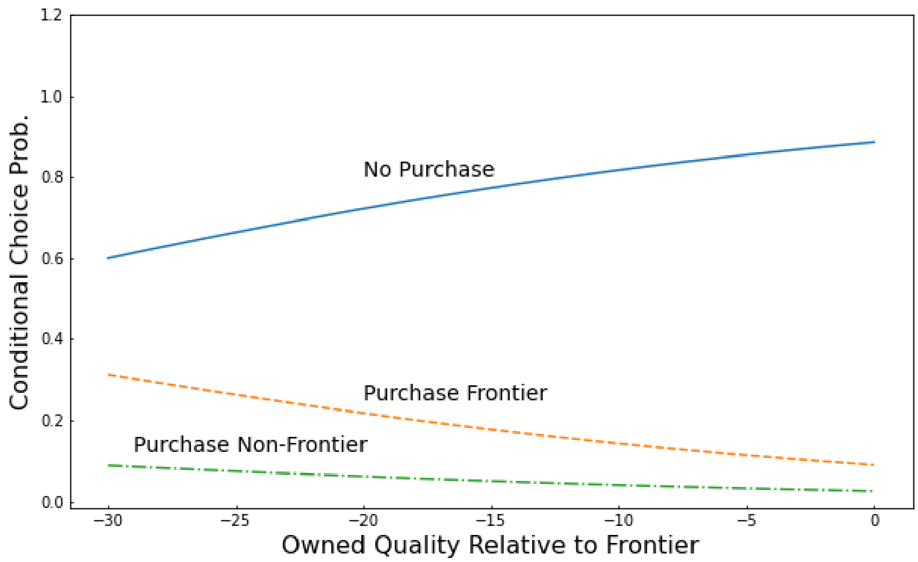


Figure 2.4: Choice Probabilities by Owned Quality Relative to the Frontier

2.5.3 Comparing Intel-AMD Duopoly and Monopoly

After analyzing firms and consumers' behaviors, I compare the estimated model (AMD-Intel duopoly) and counterfactual model (Intel monopoly with AMD removed from the market) in CPU industry. Table 2.5 reports the market outcomes in these two scenarios and the main results are listed below.

Observation 1. In duopoly, the industry profit is 10% higher than monopoly, and consumer surplus is 4.7% higher. Therefore, duopoly generates totally 6.8% higher social surplus than monopoly.

The first two rows of table 2.5 report aggregate discount industry profit and consumer surplus in AMD-Intel duopoly and Intel monopoly. The CPU industry benefits from firms' competition and generates more profit even though monopolist charges a higher price. Therefore, I can say that consumers' demand plays a crucial role when determining firms' surplus. In duopoly, firms compete with each other and make great efforts to produce

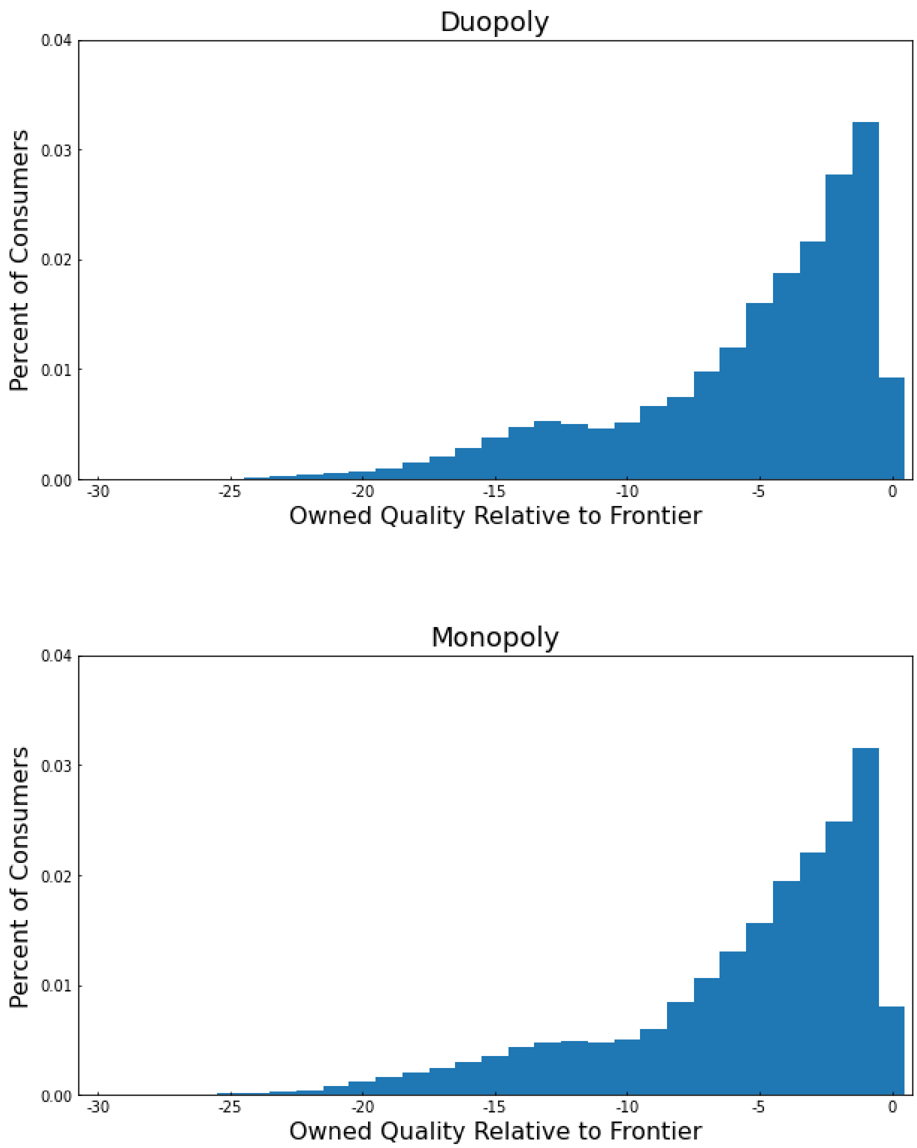


Figure 2.5: Average Ownership Distribution: Duopoly and Monopoly

pioneering goods with lower price, which to a large extent boosts more consumption. Moreover, consumers always have a second choice to purchase AMD's product with comparable quality but a much lower price. These two factors contribute a higher demand and a higher industry profit in duopoly. Consumers benefit from competition as well since they can access products with higher quality but lower price. In addition, consumer surplus respectively constitutes 65% and 67% of social surplus in duopoly and monopoly. Compared to firms' gain, consumers are the primary benefactors of industry innovation and new products introduction, regardless of market structure.

Observation 2. Regarding firms' upgrading behavior, I find that the average investment over time in duopoly is 12.4% higher than monopoly; the frontier quality upgrading rate is also 0.9% higher; but the average new product releasing probability is slightly lower.

My results support the hypothesis of Arrow (1962) which proposes a positive relationship between competition and innovation, but are different from Goettler and Gordon (2011) which finds monopolist invests more and innovates more successful advanced products. As I mentioned before, firms will prevent business stealing when making innovation decisions. Therefore, Goettler and Gordon (2011) argues that firms innovate less when a competitor exists since the loss due to business stealing exceeds the benefit generated by new goods. However, in my model, firms can stock technological achievements and choose a best time to release. This setting stimulates firms to invest more and advance the frontier more often even though a competitor exists. Because they can stock technological achievements and wait for a best time to release to avoid business stealing to the maximum extent. For example, Intel can release a new product in the period when AMD doesn't release. The

Table 2.5: Industry Outcomes Under Duopoly and Monopoly

	AMD-Intel Duopoly	Monopoly
Industry Profit ($, millions)	6311	5686
Consumer Surplus ($, millions)	11941	11403
Social Surplus ($, millions)	18252	17089
Price ($)	166.13	189.55
Industry Investment ($, millions)	236	210
Releasing Prob.	0.652	0.680
Frontier Upgrading Rate	0.738	0.677
Intel or leader share	0.442	0.465
AMD or laggard share	0.121	-

observations of releasing probability in two scenarios support my analysis as well: firms are more likely to accumulate innovation outcomes and release less frequently when competing with others, compared to monopoly.

Observation 3. The average price in duopoly is 12.4% lower than monopoly and the total market share are 21.1% more. But Intel's market share is 4.9% less compared to monopoly.

Because of the product durability, over 40% consumers choose no purchase and still use owned goods, no matter which market structure they are at. I can say that the characteristics of durability in some degree helps reduce dead weight loss caused by monopoly. Even though Intel earns more market share in monopoly, the total penetration rate is still lower than duopoly and more consumers choose not to purchase. Therefore, we cannot ignore the positive effect of competition and the contribution of AMD to the total sales and social surplus.

2.6 Conclusion

In this paper, I construct a dynamic model of duopoly to examine how competition affects firms' endogenous pricing, innovation and releasing strategies for durable goods. I estimate my model using data in CPU industry with Intel-AMD duopoly and simulate this industry numerically. Comparing the baseline market structure of duopoly and the counterfactual of Intel monopoly, I find that industry innovation is higher with competition, but firms are more likely to accumulate innovation outcomes and release less frequently when competing with others, compared to monopoly. Being able to stock investment achievements and optimally release spur firms innovate even though spillover effect exists. Moreover, the whole industry and consumers are better off under a duopoly because of higher quality product, lower prices and higher demand.

Firms in duopoly have two incentives to innovate and release new versions: competition between firms for technologically leading in the industry and competition with already-purchased goods to induce consumers to replace, whereas monopolist faces only the latter. Therefore, the characteristics of durability in some degree helps reduce dead weight loss caused by monopoly.

My paper has limitations and many extensive directions. First, to reduce computation burden, my model assumes naive consumers whose policy function is merely determined by current period utility. It can be extended to rationally expecting consumers who anticipate future quality and price when making current purchasing choices. It's worthy to explore more about estimating consumers' demand for durable goods with existence of firms' releasing problem.

Second, to constrain my state space and save computation time, I assume firms can only stock two-step R&D achievements at most. This limited upper bound may not reveal the reality very accurately. Future works can expand this dimension and make observations more sensible and interesting.

Third, the data of firms' releasing frequency is speculated by observing quality improvements between two periods. It's worthy to pursue the data directly indicating firms' releasing behaviors. Furthermore, the releasing cost in my model is calibrated to match the observed releasing probability. One can pursue real data representing firms' releasing cost, such as quarterly advertising cost and quarterly marketing expenditure; or estimate it.

Finally, my model with firms innovation and releasing can be applied to other industries which lay more emphasis on release timing, such as mobile phone, fashion, movies, video games and software, if data is available. These studies will undoubtedly contribute to inform policies and enhance economic growth.

Chapter 3

Investing in High-end or Low-end? Competition and Innovation With Unobserved Choices of Firms

3.1 Introduction

In many industries, such as automobile, electronics, and fashion, firms produce multiple products with different characteristics to meet the heterogeneous demand of consumers. This paper focuses on the multi-product firms with vertical differentiation who offer different product lines across various qualities, specifically high-end and low-end product lines. For instance, in the automobile market, Toyota Motor Corporation owns a high-end brand Lexus and a low-end brand Toyota. In the smartphone industry, Huawei Technologies

Corporation has a brand Huawei that focuses on cutting-edge models and a brand Honor specializing in cost-effective series. Similar examples appear in many other industries, such as the laptop, fashion, makeup, software industry, and so on.

An increasing number of firms tend to develop multiple product lines with distinct qualities. On the one hand, establishing a high-end product line helps firms make more markups, build brand reputation, and attract high-income consumers with persistent willingness to purchase. On the other hand, firms develop low-end products sold at a lower price to expand market share and monopolize the market. When investing and developing new and higher-quality products, firms optimally allocate R&D to two product lines and maximize expected profits, especially when competitors are present. While a single-product firm can only choose to innovate or not under the competitor's pressure, a multi-product firm is able to choose which product line to innovate. How does a competitor's behavior affect a firm's innovation strategies on both product lines? I study this question by constructing a dynamic structural model of a duopoly[1].

The theoretical model is within the context of Ericson and Pakes (1995) which provides a framework for numerically analyzing dynamic models of oligopolistic competition. Each firm optimizes behavior with rational expectations of competitors' actions in this framework, forming a Markov-perfect equilibrium. The model in this paper differs from earlier dynamic oligopoly models not only by incorporating multi-product firms but also by comprising strategic resource allocation and quality upgrading.

[1]The model can also be applied to an oligopoly.

The model assumes the product lines are independent of each other and follow individual developing processes and there is no spillover effect between the product lines within a firm[2]. Therefore, a firm's investment efficiencies on innovating both lines, governing the probabilities of the quality of each line upgrading, are different. To identify and estimate the investment efficiencies, the data of the firm's R&D expenditures on each product line and the actual upgrading history are needed. However, firms do not report separate R&D expenditures but only publish total R&D data. Therefore, identifying and estimating this dynamic structural model when the firm's choices are private information and unobserved[3] constitute another main issue of this paper. I provide an empirical framework to recover the dynamic parameters which govern the state transition probabilities, using observed total R&D and state variables, product qualities. According to the model setting, firms' optimal choices that can be represented by a closed-form function of the parameters and the observables, a larger number of observed periods together with the assumption of time-invariant investment efficiencies, as well as the nonlinearity of the innovation success probabilities, all facilitate the identification.

[2] "The production of high-end products often refers to more advanced technology, whereas that of low-end ones only uses relatively mature technology" (Lu et al. (2018)). In reality, it can be the case that the technology adopted in the low-end product line is partially the technology adopted in the high-end product line several years ago. This point is important if I aim to characterize the technology sharing or versioning within a firm. However, this paper focuses on the effect of competition on the firms' development strategies of both product lines. The spillover effect within a firm doesn't play a crucial role. Moreover, it is also assumed that there is no spillover effect between the firms for the computational simplicity. If this spillover effect is incorporated, I need to identify and estimate more dynamic parameters: the spillover of the high-end product line and the spillover of the low-end one. This exponentially increases my optimization iteration and greatly complicates my computation, especially in this multi-product and unobserved choices setting.

[3] A firm's choices of R&D expenditures on each product line are unobserved by the econometricians and the competitor.

I follow most empirical IO literature[4] to assume the consumers only compare current-period utilities when making purchase decisions and leave the market when finishing purchases[5]. Therefore, the consumers' demand and the firms' prices are statically determined and can be estimated outside the firms' dynamic interactions. To estimate dynamic parameters (i.e., firms' investment efficiencies of both product lines), I apply the simulated minimum distance (SMD) approach, which matches model-simulated moments and empirical moments.

To numerically study the effect of competition on firms' strategic innovations on different product lines, I apply the model and empirical framework to the large SUVs industry in the US. The potential consumers in the large SUVs market are fixed and they do not intend to purchase other vehicle types instead because of the seven-seats capacity[6]. Moreover, I focus on the American brands of the large SUVs, which consist of two firms, General Motors Company (GM thereafter) and Ford Motor Company (Ford thereafter), who are confronted with the fierce competition of each other. They constantly occupy the first and second place of sales in my 16-year sample period, accounting for more than 60% of total large SUV sales[7]. Other minor firms also act in the market of large SUVs, such as Nissan, Toyota, BMW, and Mercedes-Benz, but their market shares are all insignificant and below 10%. I categorize them as the outside option and assume GM and Ford constitute

[4]Examples can be found in Hashmi and Biesebroeck (2016), Borkovsky (2017), Igami and Uetake (2019), and Yang (2020).

[5]The purpose is to reduce the computational burden, especially in a context of multi-product firms.

[6]The most important unsubstitutability of the large SUVs is the capacity. Because the large SUVs have seven seats, but other smaller vehicles or SUVs usually have five seats. So for large households, they do not consider other vehicles as close substitutes.

[7]The data are collected from `https://www.goodcarbadcar.net/` and `https://www.carsalesbase.com/`.

a duopoly to make the model computable. I will extend the application to the whole large SUV industry in my future work in which the method of approximating agents' expected value will be adopted to reduce the computational burden.

Another reason I study the large SUV industry is because GM and Ford own high-end and low-end product lines of large SUVs, which satisfies my model setting well: GM owns high-end make Cadillac and low-end make Chevrolet; while Ford owns high-end make Lincoln and low-end make Ford. Most importantly, the two firms invest substantially in the R&D department. Taking the average of the data sample, they annually spend 7 billion dollars in R&D, which constitutes almost 5% of their total revenues.

It is worthy of mentioning that the data set used in this paper is very new, between the years 2005 and 2020, and unique, collected automatically from `https://www.thecarconnection.com/` by building a Python crawler. Because of the characteristic that product lines for an automobile firm are individual brands, the annual sales data of each product line (i.e., car make), typically the most difficult data to collect, can be accessed. Additionally, the data of car characteristics used to represent product qualities, retail prices, firms' total R&D expenditures, and total revenues are available to implement the empirical application.

Regarding firms' upgrading behaviors in equilibrium, I compare three counterfactual scenarios which help us thoroughly understand the effect of competition, coming from the behaviors of the competitor, on firms' innovations. The results mostly support the hypothesis of Arrow (1962) who proposes a positive relationship between the competition and the innovation but from a unique perspective of strategic innovations on both product lines. In the first scenario, I find that substantially investing in the high-end product line is always the firm's choice to catch up when there is one product line, no matter if it is high-end or low-end, largely falling behind the competitor. This conclusion demonstrates the importance of developing a high-end product line in earning profits and leading the market.

Second, in the cases where both firms possess neck-and-neck qualities, firms are mostly inspired to innovate when their high-end qualities are comparable. This incentive is weakened when they share the same low-end qualities. These findings highlight the importance of studying multi-product firms because a firm's best responses are different even though both product lines are in the same competitive circumstance. Besides, compared to Ford, GM relies more on its high-end product line to earn profits, while Ford earns more balanced value from both lines.

Moreover, when the competitor devotes itself to upgrading only the high-end products, the firm's optimal responses are keeping investments in the high-end products to remain competitive, but reducing investments in the low-end ones to save effort. In this case, the firm chooses to compete face-to-face with the competitor. On the other hand, when the competitor puts more effort into the low-end product line, the firm reacts by investing predominantly in both lines to build up market power thoroughly. It proves again that developing the high-end product line is always a strong defense against falling behind the competitor, but expenditures on the low-end line can be adjusted due to the competitive environment or resource constraint.

Finally, I characterize the states in which firms only develop one product line. Neither Ford nor GM ignores the high-end section in any circumstance due to the importance of developing the high-end line in earning profit and leading the market. In the large SUV market, Ford, typically playing the role of the laggard in terms of market share and revenue, constantly invests in both lines to thoroughly upgrade products and attract more consumers. GM is more likely to invest only in the high-end product line when its low-end products have a lower level of qualities.

This is the first paper to identify and estimate a dynamic structural model of multi-product duopoly when choices are unobserved to study the effect of competition on innovation. This paper is related to the existing literature on the relationship between competition and R&D activity. The three main theoretical hypotheses I mentioned before all focus on the competition in the form of market concentration, whereas I define competition as the level of product differentiation, also product substitutability, of each product line.

Later works develop models to justify the three main hypotheses: Cohen and Levin (1989) and Blundell et al. (1999) both empirically study the effect of competition (in the sense of market power or market share) on innovation through a reduced-form estimation across industries; Aghion et al. (2005) adds evidence to inverse-U relationship between Lerner index (competition measure) and patent production (innovation measure) across UK industries; Vives (2008) investigates the relationship across firms by different competition measures: firms innovate less if facing larger market size, but innovate more if there is greater product substitutability. They all develop the reduced-form models, but I study this classic question using a fully-specified structural model. Recent works which also develop an empirical dynamic structural model to analyze the relationship (see Goettler and Gordon (2011), Hashmi and Biesebroeck (2016), Borkovsky (2017), Thurk (2018), Igami and Uetake (2019) and Hollenbeck (2020)) all incorporate single-product firms, whereas in this paper, multi-product firms and strategic resource allocation within a firm are considered.

This paper relates to the literature studying competition of multi-product firms as well. The analytic theory literature tends to focus on the firm's optimal R&D portfolio and therefore restricts attention to a static game (see Lin and Zhou (2013)). Empirically, Sweeting (2013) incorporates a dynamic game in the commercial radio industry to study the effect of fees for musical performance rights; Wollmann (2018) constructs a two-stage model to analyze the equilibrium product characteristics for commercial vehicles; Fan and Yang (2020) explore the effect of competition on product proliferation in the smartphone market. They all concentrate on optimal product offerings, but none of them involve innovation or quality improvements.

Another stream of the literature regarding the multi-product firms is versioning (i.e., vertical differentiation or market segmentation) of the information goods such as software, music, movies and video games. The firm can create the highest version of a product and then create degraded versions or versions with fewer features or functionality by removing functions from the flagship product at little additional cost (Bhargava and Choudhary (2008), Wei and Nault (2014)). But in my paper, the firms' high-end and low-end product lines are independently developed but not simply different versions. In other words, the cost of developing low-end products is comparable to the high-end ones[8]. In this body

[8]The stylized facts in the automobile industry (physical goods) support my model setting: materials and technologies applied to the high-end brand are greatly different from the low-end brand. It's not simply the case that the low-end technologies plus premier features and get high-end technologies.

of literature, two major themes that emerge are the factors that influence/determine the versioning decision of a firm (Chen and Seshadri (2007), Bhargava and Choudhary (2008), Lahiri and Dey (2013), Wei and Nault (2014), August et al. (2014), Niculescu and Wu (2014), Chellappa and Mehra (2018)) and the firm's optimal pricing/second-degree price discrimination of different versions (Bhargava and Choudhary (2001), Chellappa and Shivendu (2005), Bhargava and Choudhary (2008), Anderson and Dana Jr (2009), Linde (2009), Cox (2017), Man and Zuo (2019), Kim (2019)). However, I focus on the firms' endogenous innovation decisions of different product lines in a competitive circumstance[9].

This paper also relates to the econometrics literature investigating identification and estimation of the dynamic structural model when agent's choices are unobserved. To my best knowledge, only Hu and Xin (2019) study this issue in the framework of a general discrete choice model. However, this paper pays attention to the continuous choice and develops a specific model with firms' competition and innovation.

The rest of this paper proceeds as follows. Section 3.2 develops the theoretical model of firms' and consumers' behaviors and discusses the equilibrium. Section 3.3 presents the estimation and identification strategies on demand, marginal cost, and investment efficiencies. Section 3.4 describes the large SUV industry and introduces data I use to do the empirical analysis. The results of the estimation and the equilibrium behaviors of firms are presented in Section 3.5. And Section 3.6 concludes the paper.

[9] The existing papers mostly constructed a static model or implemented a reduced-form regression. But I construct a structural model and incorporate a dynamic game among the firms.

3.2 Model

3.2.1 Basic Setup

I construct a dynamic model of a differentiated-products duopoly with discrete time t and infinite horizon. Two firms, each denoted by $f \in \{1, 2\}$, sell two products, high-end and low-end, denoted by $j \in \{H, L\}$, and invest in each product line to improve qualities. I define a quality ladder as $q_{fjt} \in \{0, \delta_j, 2\delta_j, \cdots, R\}$, with R as the upper bound. If innovation is successful, the firm's quality of product j improves by one fixed step (δ_j); otherwise, it is unchanged. High-end and low-end products own independent developing tracks. Therefore their quality steps are different ($\delta_H \neq \delta_L$).

Firms are forward-looking and take the optimal dynamic behaviors of its competitor into account when choosing investment strategies. Both agents observe the vector of firms' qualities of both products $\boldsymbol{q}_t = (q_{1Ht}, q_{1Lt}, q_{2Ht}, q_{2Lt})$ which constitutes the state space. According to the state, firms simultaneously choose optimal investments $x_{fjt} \in \mathbf{R}^+$ for both product lines. Consumers are heterogeneous and compare period utilities by purchasing the high-end or the low-end products, from firm 1 or firm 2, or no purchase, given the quality of outside option, firms' current offerings $\boldsymbol{q}_t$, and prices.

I assume the consumers leave the market after purchase and only non-owners (new consumers and owners who exhaust, lose or break their previous purchases) re-enter the market. Therefore, consumers' ownership does not affect their purchases, and firms' prices are statically determined by maximizing period profit. I restrict consumers to be "myopic" because the computational burden of allowing rational expected consumers is prohibitive: consumers' preference becomes dynamic, and firms' optimal pricing and innovation are both dynamic decisions, affected by consumers' current ownership and future demand. The computation of equilibrium and optimization will become substantially more complex, especially in the context of multi-product firms. Accounting for rational expected consumers would be essential if my focus were on consumer upgrades or product developing trends of higher quality and lower price. However, my dynamic model captures the market features most relevant to my focus on endogenous innovation: multi-product firms strategically allocate resources to different product lines given competitor's behaviors.

I do not consider entry or exit because they rarely happen in the large SUV industry, especially for GM and Ford.

3.2.2 Model Timeline

The timeline of firms' decisions in each period is as follows. I omit the time subscripts for conciseness and instead use variables without a prime sign to represent the current period and with a prime sign to represent the next period:

1. At the start of each period, each firm observes state $s = \{q_{1H}, q_{1L}, q_{2H}, q_{2L}\}$, competes in the product market, sets prices and earns profits.

2. Firms simultaneously decide how much to invest in R&D for both product lines: high-end and low-end.

3. The outcomes of investments in R&D are realized. The state transits from s to $s' = \{q'_{1H}, q'_{1L}, q'_{2H}, q'_{2L}\}$, which is the state observed at the beginning of next period.

3.2.3 Consumers

I model consumers as heterogeneous and owning no more than one product at a time. Utility for a consumer i from firm f's product j with quality q_{fjt} and price p_{fjt} is given by a random-coefficient discrete choice model:

$$u_{ifjt} = \alpha_i q_{fjt} - \gamma p_{fjt} + \xi_{fjt} + \epsilon_{ifjt}. \tag{3.1}$$

q_{fjt} represents the quality of firm f's product j at period t. The random coefficient α_i captures consumers' heterogeneous tastes for quality and is assumed to follow a normal distribution with mean α and variance σ^2. So $\alpha_i = \alpha + \nu_i$, where $mean(\nu_i) = 0$ and $var(\nu_i) = \sigma^2$. γ represents the marginal disutility of price[10], ξ_{fjt} represents unobserved demand shock, and ε_{ifjt} captures idiosyncratic variation which is independently and identically distributed as standard type 1 extreme value across consumers, products and periods. I normalize the mean utility from the outside option to be 0 and therefore $u_{i0t} = \epsilon_{i0t}$.

Each consumer maximizes her utility, yielding the conditional choice probabilities of firm f's product j

$$s_{fjt}(\boldsymbol{q}_t, \boldsymbol{p}_t, \boldsymbol{\xi}_t) = \int \frac{\exp\left(\alpha_i q_{fjt} - \gamma p_{fjt} + \xi_{fjt}\right)}{1 + \sum_{j' \in \mathcal{J}_t} \exp\left(\alpha_i q_{fj't} - \gamma p_{fj't} + \xi_{fj't}\right)} d\Phi\left(\alpha_i\right). \tag{3.2}$$

where $\mathcal{J}_t$ denotes the set of all available products in period t, $\boldsymbol{q}_t = (q_j, j \in \mathcal{J}_t) = (q_{1Ht}, q_{1Lt}, q_{2Ht}, q_{2Lt})$, and $\boldsymbol{p}_t$ and $\boldsymbol{\xi}_t$ are analogously defined. Lastly, $\Phi\left(\alpha_i\right)$ represents the cumulative distribution function of the random coefficient α_i.

[10]The heterogeneity in γ (marginal disutility of price) can also be allowed. But due to the computational simplicity, I follow Berry et al. (1995) and many other empirical literature to assume that the heterogeneity of consumers only come from their different tastes for product qualities. Furthermore, α_i (heterogeneous taste for quality) is correlated to consumers' heterogeneous demographics, including their income. So α_i in some degree captures consumers' heterogeneity in valuing "money".

I define the mean utility of firm f's product j in period t as

$$\eta_{fjt} = \alpha q_{fjt} - \gamma p_{fjt} + \xi_{fjt} \tag{3.3}$$

and recover it based on equation (3.2) following Berry et al. (1995).

3.2.4 Firms

Each period, firms compete in the product market and determine optimal prices by maximizing period profit and then make dynamic investment decisions.

Static Decisions

The period profit function, excluding investments, for firm f is

$$\pi_f(\boldsymbol{q}, \boldsymbol{p}) = \max_{p_{fj}, j \in \{H,L\}} \Sigma_{j \in \{H,L\}} (p_{fj} - mc_{fj}(q_{fj})) \times M \times s_{fj}(\boldsymbol{q}, \boldsymbol{p}, \boldsymbol{\xi}), \tag{3.4}$$

where M is the market size, and mc_{fj} is constant marginal cost of production related to qualities:

$$mc_{fj}(q_{fj}) = \lambda_0 + \lambda_1 q_{fj}. \tag{3.5}$$

I omit subscript t for conciseness.

The first-order condition allows me to invert out mc_{fj} as

$$mc_{fj} = p_{fj} + [\Delta_f^{-1} \boldsymbol{s}_f]_{fj}, \tag{3.6}$$

where Δ_f represents a 2×2 matrix whose (j, j') element is $\frac{\partial s_{fj'}}{\partial p_{fj}}$, and $\boldsymbol{s}_f = (s_{fH}, s_{fL})$. Combining equation (3.5) and (3.6), I can bring to data for estimation and obtain equilibrium prices of both products and period profit.

Dynamic Decisions

Each firm has access to R&D processes that govern its ability to produce higher-quality products of two product lines and chooses an investment level $x_{fj} \in \mathbf{R}^+$ for both lines. To obtain a closed form of optimal investments, I follow Goettler and Gordon (2011) and restrict investment outcomes τ_{fj} to be either δ_j or 0:

$$\tau_{fj} = q'_{fj} - q_{fj} = \begin{cases} 0, & \text{not success,} \\ \delta_j, & \text{success.} \end{cases}$$

High-end products, regardless of firms, share the same upgrading step δ_j. So analogously do low-end products.

The success probabilities are given by

$$\chi_{fj}(\tau_{fj} = \delta_j | x_{fj}, a_{fj}) = \frac{a_{fj} x_{fj}}{1 + a_{fj} x_{fj}}, \tag{3.7}$$

where a_{fj} captures firm f's investment efficiency on each product line. And the probabilities of innovation failure are therefore $\chi_{fj}(\tau_{fj} = 0 | x_{fj}, a_{fj}) = 1 - \chi_{fj}(\tau_{fj} = \delta_j | x_{fj}, a_{fj})$.

Each firm maximizing its expected discounted profit with simultaneously choosing optimal investments x_{fH} and x_{fL}, I can derive the Bellman equation as:

$$\begin{aligned} V_f\left(\boldsymbol{q_f}, \boldsymbol{q_{-f}}\right) = \max_{x_{fH}, x_{fL} \geq 0} \Big\{ & \pi_f(\boldsymbol{q}, \boldsymbol{p}) - x_{fH} - x_{fL} + \beta \sum_{\boldsymbol{\tau_f}, \boldsymbol{q'_{-f}}} V_f(\boldsymbol{q_f} + \boldsymbol{\tau_f}, \boldsymbol{q'_{-f}}) \\ & \times \chi_f(\boldsymbol{\tau_f} | x_{fH}, x_{fL}, \boldsymbol{a_f}) \times h_f(\boldsymbol{q'_{-f}} | \boldsymbol{q_f}, \boldsymbol{q_{-f}}) \Big\}, \end{aligned} \tag{3.8}$$

where $\boldsymbol{q_f} = \{q_{fH}, q_{fL}\}$, $\boldsymbol{q_{-f}} = \{q_{-fH}, q_{-fL}\}$, $\boldsymbol{\tau_f} = \{\tau_{fH}, \tau_{fL}\}$, and $\boldsymbol{a_f} = \{a_{fH}, a_{fL}\}$. β is the discount factor and $h_f(\cdot|\cdot)$ is firm f's belief about the competitor's future qualities. I assume that the same firm's two lines have independent innovation outcomes conditional on investments, so that

$$\chi_f(\boldsymbol{\tau_f}|x_{fH}, x_{fL}, \boldsymbol{a_f}) = \chi_{fH}(\tau_{fH}|x_{fH}, a_{fH}) \times \chi_{fL}(\tau_{fL}|x_{fL}, a_{fL}),$$

and

$$h_f(\boldsymbol{q'_{-f}}|\boldsymbol{q_f}, \boldsymbol{q_{-f}}) = h_f(q'_{-f,H}|\boldsymbol{q_f}, q_{-f,H}) \times h_f(q'_{-f,L}|\boldsymbol{q_f}, q_{-f,L}).$$

Firms simultaneously choose investments on both product lines to satisfy the first-order conditions:

$$\begin{aligned}
\frac{\partial V_f}{\partial x_{fH}} &= -1 + \beta \sum_{\boldsymbol{\tau_f}, \boldsymbol{q'_{-f}}} V_f(\boldsymbol{q_f} + \boldsymbol{\tau_f}, \boldsymbol{q'_{-f}}) h_f(\boldsymbol{q'_{-f}}|\boldsymbol{q_f}, \boldsymbol{q_{-f}}) \times \frac{\partial \chi_f(\boldsymbol{\tau_f}|x_{fH}, x_{fL}, \boldsymbol{a_f})}{\partial x_{fH}} = 0; \\
\frac{\partial V_f}{\partial x_{fL}} &= -1 + \beta \sum_{\boldsymbol{\tau_f}, \boldsymbol{q'_{-f}}} V_f(\boldsymbol{q_f} + \boldsymbol{\tau_f}, \boldsymbol{q'_{-f}}) h_f(\boldsymbol{q'_{-f}}|\boldsymbol{q_f}, \boldsymbol{q_{-f}}) \times \frac{\partial \chi_f(\boldsymbol{\tau_f}|x_{fH}, x_{fL}, \boldsymbol{a_f})}{\partial x_{fL}} = 0.
\end{aligned} \tag{3.9}$$

Given logistic form of the χ functions, I refer to Goettler and Gordon (2011) to simplify the first-order conditions to

$$x^*_{fj} = max\left\{0, \frac{\{\beta a_{fj}[EV^+(\boldsymbol{q}) - EV^-(\boldsymbol{q})]\}^{(1/2)} - 1}{a_{fj}}\right\}, \tag{3.10}$$

where

$$EV^+(\boldsymbol{q}) = \sum_{q'_{f,-j}, \boldsymbol{q'_{-f}}} V_f(q_{fj} + \delta_j, q'_{f,-j}, \boldsymbol{q'_{-f}}) \chi_{f,-j}(q'_{f,-j}|q_{f,-j}) h_f(\boldsymbol{q'_{-f}}|\boldsymbol{q_f}, \boldsymbol{q_{-f}})$$

and

$$EV^-(\boldsymbol{q}) = \sum_{q'_{f,-j}, \boldsymbol{q'_{-f}}} V_f(q_{fj} + 0, q'_{f,-j}, \boldsymbol{q'_{-f}}) \chi_{f,-j}(q'_{f,-j}|q_{f,-j}) h_f(\boldsymbol{q'_{-f}}|\boldsymbol{q_f}, \boldsymbol{q_{-f}})$$

are the expected future values conditional on innovation success or failure, respectively. I use *max* function in equation (3.10) to restrict investments to be non-negative.

3.2.5 Equilibrium

I consider pure-strategy Markov-perfect Nash equilibrium (MPNE) of this dynamic duopoly game. The refinement guarantees the equilibrium where players' strategies only depend on current state variable values and firms optimize behaviors in each state and each sub-game. In the equilibrium, each firm has rational expectations about competitor's policy functions for investments on high-end and low-end product line. Formally, an MPNE in this model is the set $\{V_f^*, x_{fH}^*, x_{fL}^*, h_f^*\}_{j=1,2}$, which includes firms' equilibrium value functions, policy functions, and beliefs about rival's future qualities. The expectations are rational in that $h_f^* = \prod_{j=H,L} \chi_{fj}(\tau_{fj} = q'_{-fj} - q_{-fj} | x_{-fj}, a_{-fj})$.

An MPNE for this model can be shown to exist following Doraszelski and Satterthwaite (2010). For a discussion of potential multiplicity, see Doraszelski and Pakes (2007). According to Hollenbeck (2020), generally, "there is no way to fully rule out the possibility of multiple equilibria or to find all possible equilibria." Borkovsky et al. (2012) show multiplicity in a quality ladder model, although they conclude that "the differences between equilibria tend to be small and may matter little in practice."

While there is no way to rule out different equilibria completely, to help reduce the negative effect of multiplicity in my model, I apply the "limit-of-finite" approach to refine the equilibrium, for which I use backward induction to solve for an equilibrium of a T-period game and then let $T \to \infty$. In each period and each state, I compute firms' equilibrium policy functions according to the optimal conditions I derived for investments. Then I use backward induction to iterate value functions with initial values of 0 and obtain convergence. I relegate the algorithm details in appendix B.1.

3.3 Estimation

This paper has two important components: a theoretical component that develops a dynamic model of multi-product firms with innovation and an empirical component that estimates the model when firms' choices are unobserved. In this section, I first present the estimation process of static consumer preferences and marginal cost in the framework of Berry et al. (1995). Then I demonstrate the identification and estimation strategies on the dynamic parameters when firms' choices, respective R&D expenditure on each product line, are unobserved.

3.3.1 Static Estimation

In many industries, the data of product qualities are not directly measured. So I follow Fan and Yang (2020) to define q_{fjt} in equation (3.1) as a quality index depending on the observable product characteristics $\boldsymbol{y_{fjt}}$. Therefore, $q_{fjt} = \boldsymbol{y_{fjt}}\boldsymbol{\theta}$ and $\boldsymbol{\theta}$ are the parameters to be estimated. This functional form allows me to estimate consumers' heterogeneous preferences for all important product characteristics even if some of them lack variation so that random coefficient variances cannot be estimated precisely. It is worthy of mentioning that in the static estimation process, I use the continuous variables of q_{fjt}. But in the section 3.3.3, q_{fjt} are discretized to comply with the quality ladder model and for the purpose of dynamic estimation.

Because $\alpha, \sigma^2, \boldsymbol{\theta}$ enter the utility function in the form of $\alpha\boldsymbol{\theta}$ and $\sigma^2\boldsymbol{\theta}$,[11] I normalize the first element of $\boldsymbol{\theta}$ to be 1 in order to separately identify α and σ^2. Hence, all the parameters need to be estimated in the demand side are $\boldsymbol{\Theta} = \{\alpha, \sigma^2, \boldsymbol{\theta}, \gamma\}$, respectively, mean

[11] $\alpha_i \sim N(\alpha, \sigma^2)$, so $\alpha_i\boldsymbol{\theta} \sim N(\alpha\boldsymbol{\theta}, \sigma^2\boldsymbol{\theta})$.

and variance of random coefficients, preferences of product characteristics, and marginal utility of price. I follow Berry et al. (1995) and Nevo (2000) to approximate the integral in equation (3.2) and rewrite the market share of product fj as $s_{fj}(\eta(q,p,\xi),\boldsymbol{q},\boldsymbol{\Theta})$ with combining equation (3.3). Given a candidate $\hat{\boldsymbol{\Theta}}$ and the observed market share in the data, $\tilde{s}_{fj}$, I solve for each market the implicit system of equations

$$\tilde{s}_{fj} = s_{fj}(\eta(q,p,\xi),\boldsymbol{q},\hat{\boldsymbol{\Theta}}).$$

Then, I have 4 equations with 4 unknowns $\eta_{1H}(\hat{\boldsymbol{\Theta}}),\eta_{1L}(\hat{\boldsymbol{\Theta}}),\eta_{2H}(\hat{\boldsymbol{\Theta}}),\eta_{2L}(\hat{\boldsymbol{\Theta}})$ in each market t. For the resulting η s, I estimate via Generalized Method of Moments (GMM) with the population moment conditions $E(\xi\boldsymbol{Z})=0$, where $\boldsymbol{Z}$ are the instruments aiming to resolve the problem of price endogeneity. Following the literature, my instrumental variables are based on the charateristics of other products of the same firm and the products of the competing firm. Then I estimate $\hat{\boldsymbol{\Theta}}$ by minimizing the sample moment conditions in each market

$$m(\hat{\boldsymbol{\Theta}}) \equiv \frac{1}{4}\Sigma_{f\in\{1,2\},j\in\{H,L\}}\left(\eta_{fj}(\hat{\boldsymbol{\Theta}}) - \hat{\alpha}q_{fj} + \hat{\gamma}p_{fj}\right)\boldsymbol{Z}_{fj}.$$

The detailed computation process is discussed in Nevo (2000).

The second step of static estimation is to estimate marginal cost of each product, which are calculated based on equation (3.6) where $p,\Delta,\boldsymbol{s}$ are either observed in data or inferred by the demand estimates. With dependent variables mc_{fj} obtained, the parameters $\{\lambda_0,\lambda_1\}$ in the marginal cost function (equation (3.5)) can be recovered by a linear regression.

Preparing for the dynamic estimation, it is necessary to compute equilibrium prices of each product and period profit of each firm given a state $s = \{q_{1H}, q_{1L}, q_{2H}, q_{2L}\}$. The main issue to be taken care of is the heterogeneity of consumers. So I calculate expected period profit over the demand shocks ξ s. I first draw the shocks from its empirical distribution given by $\hat{\mathbf{\Theta}}$, compute the pricing equilibrium and calculate the resulting period profit for each draw, and then take the average of these profits across all draws. In this process, pricing equilibrium is obtained by a fixed point iteration: given an initial price vector, a given state, and demand shocks, utilities of heterogeneous consumers are obtained; then, I derive the market share of each product by equation (3.2); together with the estimated marginal costs from equation (3.5), equilibrium prices given by equation (3.6) are acquired; iterate this process until input prices and output prices converge to each other.

3.3.2 Identification of Dynamic Parameters

The dynamic parameters in my model are $\boldsymbol{a} = \{a_{1H}, a_{1L}, a_{2H}, a_{2L}\}$ which consist of firms' investment efficiencies of high-end and low-end product lines. They play crucial roles in determining the probabilities of innovation success or failure, and also state transition probabilities. To identify and estimate investment efficiencies, I need to observe each product line's quality improvements and R&D expenditures. However, regardless of industries, almost all firms do not publish R&D expenditures on specific product lines, but only the total amount. Therefore, the question becomes how to identify and estimate the dynamic parameters when agents' choices are unobserved.

Even though firms' choices, the investments on each product line, are unobserved by econometricians, the outcomes governed by the investments, the product qualities, can be observed. According to my model setup, each firm's investments $\{x^*_{fH}, x^*_{fL}\}$, as well as the state transitions χ s are precisely the functions of dynamic parameters and state variables. Therefore, I can utilize the observed quality improvements to invert the state transition probabilities and thus the dynamic parameters.

Referring to Hu and Xin (2019), I consider the identification of dynamic parameters for a fixed state s and utilize the first-order conditional moments of the observed state variable at $t+1$. Given a state $s = \{q_{1H}, q_{1L}, q_{2H}, q_{2L}\}$, I define the first-order conditional moments of the observed state variable at $t+1$ of firm f as

$$\mu_{f,t+1} = \mathrm{E}_{t+1}\left[s_{f,t+1} \mid s_t = s\right], \tag{3.11}$$

where $s_{f,t+1} = \{q_{fH,t+1}, q_{fL,t+1}\}$. Notice that this conditional moment can be directly estimated from the data and is thus treated as known constant for identification purpose. According to my model setting, firms' next-period qualities either improve one step or keep unchanged, so I can expand equation (3.11) as

$$\begin{aligned} \mu_{1,t+1} &= \begin{pmatrix} \mathrm{E}_{t+1}[q_{1H,t+1} \mid s] \\ \mathrm{E}_{t+1}[q_{1L,t+1} \mid s] \end{pmatrix} \\ &= \begin{pmatrix} q_{1H} \times \chi_1(0, H) + (q_{1H} + \delta_H) \times \chi_1(\delta_H, H) \\ q_{1L} \times \chi_1(0, L) + (q_{1L} + \delta_L) \times \chi_1(\delta_L, L) \end{pmatrix}, \end{aligned} \tag{3.12}$$

where

$$\chi_1(0,H) \equiv \chi(\tau_{1H} = 0 \mid x^*_{1H}(s;\boldsymbol{a}), a_{1H}),$$

$$\chi_1(\delta_H,H) \equiv \chi(\tau_{1H} = \delta_H \mid x^*_{1H}(s;\boldsymbol{a}), a_{1H}),$$

$$\chi_1(0,L) \equiv \chi(\tau_{1L} = 0 \mid x^*_{1L}(s;\boldsymbol{a}), a_{1L}),$$

$$\chi_1(\delta_L,L) \equiv \chi(\tau_{1L} = \delta_L \mid x^*_{1L}(s;\boldsymbol{a}), a_{1L}).$$

Similarly for firm 2,

$$\begin{aligned}\mu_{2,t+1} &= \begin{pmatrix} \mathrm{E}_{t+1}[q_{2H,t+1} \mid s] \\ \mathrm{E}_{t+1}[q_{2L,t+1} \mid s] \end{pmatrix} \\ &= \begin{pmatrix} q_{2H} \times \chi_2(0,H) + (q_{2H} + \delta_H) \times \chi_2(\delta_H,H) \\ q_{2L} \times \chi_2(0,L) + (q_{2L} + \delta_L) \times \chi_2(\delta_L,L) \end{pmatrix},\end{aligned} \tag{3.13}$$

where

$$\chi_2(0,H) \equiv \chi(\tau_{2H} = 0 \mid x^*_{2H}(s;\boldsymbol{a}), a_{2H}),$$

$$\chi_2(\delta_H,H) \equiv \chi(\tau_{2H} = \delta_H \mid x^*_{2H}(s;\boldsymbol{a}), a_{2H}),$$

$$\chi_2(0,L) \equiv \chi(\tau_{2L} = 0 \mid x^*_{2L}(s;\boldsymbol{a}), a_{2L}),$$

$$\chi_2(\delta_L,L) \equiv \chi(\tau_{2L} = \delta_L \mid x^*_{2L}(s;\boldsymbol{a}), a_{2L}).$$

Based on equation (3.12) and (3.13), together with the functional form of χ s and $\chi_f(0,\cdot) + \chi_f(\delta.,\cdot) = 1$, I have the following equation system connecting $\boldsymbol{a}$ and the observables:

$$\begin{aligned}\chi_1(\delta_H,H) &= \frac{\mu_{1,t+1}(1) - q_{1H}}{\delta_H} = \frac{a_{1H}x^*_{1H}(s;\boldsymbol{a})}{1 + a_{1H}x^*_{1H}(s;\boldsymbol{a})}, \\ \chi_1(\delta_L,L) &= \frac{\mu_{1,t+1}(2) - q_{1L}}{\delta_L} = \frac{a_{1L}x^*_{1L}(s;\boldsymbol{a})}{1 + a_{1L}x^*_{1L}(s;\boldsymbol{a})}, \\ \chi_2(\delta_H,H) &= \frac{\mu_{2,t+1}(1) - q_{2H}}{\delta_H} = \frac{a_{2H}x^*_{2H}(s;\boldsymbol{a})}{1 + a_{2H}x^*_{2H}(s;\boldsymbol{a})}, \\ \chi_2(\delta_L,L) &= \frac{\mu_{2,t+1}(2) - q_{2L}}{\delta_L} = \frac{a_{2L}x^*_{2L}(s;\boldsymbol{a})}{1 + a_{2L}x^*_{2L}(s;\boldsymbol{a})},\end{aligned} \tag{3.14}$$

where $\mu_{f,t+1}(y)$ represents yth row of $\mu_{f,t+1}$. In equation system (3.14), μ, q, and δ are known; firms' optimal choices x^* s are functions of observed s and four unknown dynamic parameters $\boldsymbol{a}$ based on equation (3.10). Therefore, I have four equations and four unknowns in this system which identifies $\boldsymbol{a}$.

It can be the case that the equation system (3.14) has multiple solutions which invalidates my identification. Intuitively, based on the "ax" part in the χ functions, it's possible that the observed innovation outcomes from a lower investment efficiency and a higher investment are the same to a higher investment efficiency and a lower investment. However, the available data of total R&D of the firms, a larger number of observed periods together with the assumption of time-invariant investment efficiencies, as well as the nonlinearity of the innovation success probabilities, overcome this problem.

Table 3.1: Firms' Investments

Period	Total R&D	High-end investment	Low-end investment
1	T_1	x_{1H}	$T_1 - x_{1H}$
2	T_2	x_{2H}	$T_2 - x_{2H}$

Suppose for a firm, we have two periods (total R&D and investments on both product lines are shown in Table 3.1). According to my model setting of the success probability of innovation ($\chi = ax/(1 + ax)$), if we have another high-end investment efficiency $(a_H + \omega)$ which also generates the observed outcomes in data, then in period 1, $(a_H + \omega)x'_{1H} = a_H x_{1H}$. So another pair of investments in the high-end and the low-end product line is $(x'_{1H} = \frac{a_H x_{1H}}{a_H+\omega}, x'_{1L} = T_1 - \frac{a_H x_{1H}}{a_H+\omega})$. Therefore, $a'_L x'_{1L} = a_L(T_1 - x_{1H})$ and I can derive another low-end investment efficiency $a'_L = \frac{(T_1 - x_{1H})a_L(a_H+\omega)}{T_1(a_H+\omega) - a_H x_{1H}}$. Similarly, in period 2, I have $a'_L = \frac{(T_2 - x_{2H})a_L(a_H+\omega)}{T_2(a_H+\omega) - a_H x_{2H}}$. Because of the assumption of time-invariant investment efficiencies, a_L' s derived in both periods should be equal, so I can have $\frac{1}{a_H+T_1\omega/x_{1L}} = \frac{1}{a_H+T_2\omega/x_{2L}}$. Therefore, if $T_1/x_{L1} \neq T_2/x_{L2}$ based on data and the optimal policies derived from my model, I have a contradiction and the case that the equation system (3.14) has multiple solutions is overcomed. Therefore, the investment efficiencies can be identified with observed total R&D and innovation success probabilities.

3.3.3 Dynamic Estimation

Based on my identification strategies, I use simulated minimum distance (SMD) estimation method which minimizes the distance between empirical moments observed in my data and their simulated counterparts generated from my model. I incorporate the moments of quality improvements of each product and total R&D expenditures of both firms. To summarize, parameters $\boldsymbol{a}$ makes quality improvements observed in data most likely to happen, and meanwhile, the total R&D investments generated from the model are as close to data as possible.

Given one group of candidate $\boldsymbol{a}$, I solve for the equilibrium (see details in appendix B.1) where firms' value functions achieve convergence; simulate model for T periods and S times with state transitions; obtain simulated model moments, $m_{S,T}(\boldsymbol{a})$; match them with empirical moments calculated from data (m_T); and get $\hat{\boldsymbol{a}}_T$ by finding the best match. Formally,

$$\hat{\boldsymbol{a}}_T = \arg\min_{\boldsymbol{a}\in A} \left[m_{S,T}(\boldsymbol{a}) - m_T\right]' W_T \left[m_{S,T}(\boldsymbol{a}) - m_T\right], \tag{3.15}$$

where W_T is the inverse of covariance matrix of empirical moments to guarantee the efficiency of estimator.

The data exhibit an increasing trend in R&D expenditures, so I choose stationary moments in both data and model to eliminate the time trend effect. For example, I use the moment of investment per revenue instead of the total investment. My moment vector, m_T, consists of the following six moments:

- Firm 1's high-end and low-end product line's mean innovation rates, defined as $(1/T)[(q_{1jT} - q_{1j0})/\delta_j]$;
- Firm 2's high-end and low-end product line's mean innovation rates, defined as $(1/T)[(q_{2jT} - q_{2j0})/\delta_j]$;
- Mean total investment per-unit revenue for each firm.

3.4 Empirical Application

In this section, I apply my model and estimation method to the US-brand large SUV industry, which is well suited to investigate the interaction between competition and innovation in the context of multi-product duopoly. I first introduce the industry background and then summarize the data. The empirical results will be manifested in section 3.5.

3.4.1 Automobile Industry: US-Brand Large SUV

I restrict my attention to the US-brand large SUV industry, which consists of two firms, GM and Ford, constituting a duopoly. Several reasons facilitate my constraint on the study of these two firms. First, as mainstays of the large SUV industry in the US, two firms constantly occupy the first and second place of sales in my 16-year sample period, accounting for more than one 60% market share of large SUV total sales[12]. In the role of old-established American automobile brands, they are constantly confronted with the fierce competition of each other. Other firms (e.g., Toyota, Nissan, etc.) are also active in this industry but not included in my application. Except their market shares are insignificant, incorporating oligopoly, especially in the framework of multi-product firms, significantly increases the state space and substantially complicates the optimization within each state. I will extend the application to the whole large SUV industry in my future work, in which I will adopt the method of approximating agents' expected value to reduce the computational burden.

Another significant reason is that both firms own high-end and low-end product lines of large SUVs, which satisfies my model setting: GM owns high-end make Cadillac and low-end make Chevrolet; while Ford owns high-end make Lincoln and low-end make Ford. Because of the characteristic that product lines for an automobile firm are individual brands, the annual sales data of each product line (i.e., car make), typically the most challenging data to collect, can be accessed. Additionally, the data of car characteristics used to represent product qualities, retail prices, firms' total R&D expenditures, and revenues are available to implement the empirical application. The details of data are specified in section 3.4.2.

Finally, in the automobile industry, innovation is an important source of product differentiation as firms improve their competitive position through higher product quality, great reliability, and the introduction of new product features (Hashmi and Biesebroeck

[12]The data is obtained at `https://www.goodcarbadcar.net/`.

(2016)). Producing and upgrading automobiles is an exceedingly research-demanding activity. According to OECD ANBERD (Analytical Business Enterprise R&D) data, the automobile industry is continually in the top five places in terms of R&D expenditures in the United States from the year 2005. As for the US-brand large SUV firms I focus on in my application, GM and Ford, they also invest substantially in R&D. Taking the average of my data sample, they spend 7 billion dollars in R&D annually, which constitute almost 5% of their total revenue.

3.4.2 Data

My data set covers large SUVs of GM and Ford sold in the US between 2005 and 2020. Because quality upgrading occurs yearly in the automobile industry, I collect annual data of car characteristics, retail prices, sales, firms' total R&D expenditures, and revenues. It is empirically feasible to use data of 16-period to obtain the entire set of state transition probabilities. Because the product upgrading cycles in the automobile industry are relatively fixed, it does not affect much if I use a short-period of data[13]. The data of car characteristics and retail prices are collected automatically from `https://www.thecarconnection.com/` by building a Python crawler. I include the two most important indices consumers value when purchasing a large SUV: horsepower by weight and fuel efficiency (miles per gallon on the highway). I take the average of horsepower by weight, miles per gallon, and retail prices over all specific models under four makes (Cadillac, Chevrolet, Lincoln, and Ford). Prices are deflated using CPI from the US Bureau of Labor

[13] In addition, to my best knowledge, the earliest available data is from the year 2005. Furthermore, in the literature studying the innovation in the automobile industry such as Hashmi and Biesebroeck (2016), their data set also covers a short period, 25 years, because of the feature that the innovation happens typically once a year in the automobile industry.

Statistics based on the price in 2005. Total sales of 4 makes and 16 years are manually collected from `https://www.goodcarbadcar.net/`. Total R&D expenditures and revenues are manually collected from 16-year annual financial reports of GM and Ford.

Table 3.2 summarizes the primary data set, which consists of three parts corresponding to three steps of my estimation procedure introduced in section 3.3. Panel A presents each firm's annual data of horsepower per weight, fuel efficiency, list price, and sales of both product lines that I use to estimate demand in the first step. Panel B shows the firm-level market share at the yearly frequency, which is the ratio between each firm's sales and the total sales of large SUVs in the US. I use demand estimates and Panel B to invert out the marginal cost function. Panel C includes firms' R&D-revenue ratio and quality indices that are derived from demand estimates. I use these dynamic moments data to estimate the investment efficiencies as well as the state transition probabilities. One can observe from the statistics that two firms confront intense competition in that the average product characteristics (also product qualities), prices, and R&D investments are remarkably close to each other. Even though the two firms are both vanguards in the large SUV industry, the market share data reveals that GM slightly leads, and Ford plays a role of catching-up.

Each data panel is paired with a model element in section 3.2 and an empirical method in section 3.3 to estimate demand, marginal costs and dynamic investment efficiencies. Table 3.3 provides an overview of such model-method-data pairing to summarize the correlations among section 3.2, 3.3 and 3.4.

Table 3.2: Summary Statistics

	GM					**Ford**				
	Mean	Std	Min	Max	Obs	Mean	Std	Min	Max	Obs
Panel A. Demand										
Horsepower/ weight (hp/thousand lb)	65.8965	6.6201	56.0495	77.3716	32	58.6316	8.2945	48.4514	76.6791	32
Fuel efficiency (miles/gallon)	20.4612	1.5943	17	22.5	32	19.5040	1.4781	17	22.6667	32
Price (thousand $)	51.9411	11.8839	34.5924	68.3667	32	48.2521	8.0902	34.9221	65.1943	32
Sales (thousand)	62.7097	44.1119	12.592	161.491	32	36.8194	28.9013	8.018	114.137	32
Panel B. Marginal cost										
Market share	0.1801	0.1170	0.0477	0.3318	32	0.1016	0.0650	0.0303	0.1999	32
Panel C. Investment efficiencies										
Quality index	72.1240	5.4400	65.1928	82.6593	32	65.4712	12.1128	55.9859	81.4444	32
R&D/ Revenue	0.0486	0.0050	0.0386	0.058	32	0.0458	0.0047	0.0388	0.0558	32

Table 3.3: Summary of Empirical Procedure

Step	Model	Method	Data	Section
Demand	Random coefficient discrete choice	IV GMM	Panel A	2.3 & 3.1
Marginal cost	Random coefficient discrete choice	First order condition	Panel B	2.4.1 & 3.1
Investment efficiency	Dynamic continuous choice	Simulated minimum distance	Panel C	2.4.2 & 3.3

3.4.3 Other Details

I calibrate a few model setup parameters before estimation. The quality step δ_H is set to be 7, δ_L to be 4.5, based on the observed quality improvements. The upper bounds of both product lines' quality to be $5\delta_j$. The high enough $\bar{\delta}_j$ s cover the observed quality improvements in data and meanwhile allow acceptable computation time. The discount factor β is set as 0.9, satisfying the annual interest rate. Market size M is set as 350 thousand by taking the average of the observed total sales of large SUVs in data[14]. I follow Goettler and Gordon (2011) to assume the market size is fixed for the purpose of stationarity. To get rid of the increasing time trend, I also choose stationary moments in both the model and the data in the dynamic estimation.

3.5 Empirical Results

This section first presents the results of demand, marginal cost, and dynamic estimation using data in the large SUV industry. Then, with the estimated parameters, I solve for the industry equilibrium and characterize the counterfactual equilibrium behaviors of the firms.

[14]In my future work, I will use the number of large households in the US as market size to do the robustness check.

Table 3.4: Demand and MC Estimates

	Parameter	Standard Error
Panel A. Demand		
Quality coefficient ($\boldsymbol{\theta}$):		
Horsepower/weight(thousand lb)	0.784	0.709
Miles/gallon	1 (Normalization)	
Quality random coefficient (α_i)		
Mean	0.044	0.041
Standard deviation	0.100	0.131
Price (γ)	-0.113	0.020
Panel B. Marginal cost		
Constant (λ_0)	-7.7682	10.8143
Quality (λ_1)	0.5796	0.1563

3.5.1 Demand and Marginal Cost

Based on the empirical method described in section 3.3, the instrumental variables I use to estimate demand are characteristics of midsize sedans of each make and large SUV of the competing firm. Table 3.4 reports the estimation results on demand and marginal cost. The demand estimation results indicate that consumers, on average, favor products with higher horsepower and fuel efficiency. For example, one horsepower/weight increase is equivalent to a price decrease of $305 for an average consumer. Similarly, one-mile increase per gallon in fuel efficiency is equivalent to a price drop of $389. The estimated standard deviation of consumers' preference for quality is about 90% of the average taste, suggesting consumers' great heterogeneity in their willingness to pay for quality. On the supply side, I find that marginal cost increases in product quality. A negative constant term λ_0 reduces the amount of marginal cost and guarantees positive markups.

Table 3.5: Dynamic Estimates

	Parameter
GM (high-end) innovation efficiency (a_{1H})	0.0103
GM (low-end) innovation efficiency (a_{1L})	0.0400
Ford (high-end) innovation efficiency (a_{2H})	0.0299
Ford (low-end) innovation efficiency (a_{2L})	0.0717

3.5.2 Dynamic Estimation

I use the SMD estimator in equation (3.15) to estimate the dynamic parameters $\boldsymbol{a}$ given the demand and MC estimates. Table 3.5 reports the structural estimates. GM's innovation efficiencies of both product lines are less than Ford's, which explains why Ford invests relatively less but can still produce large SUVs of the same qualities. Moreover, a firm's innovation efficiency of high-end products is less than that of low-end products, reflecting that upgrading high-end products are more difficult and investments-demanding. Table 3.6 presents the model's fits with demand, MC, and structural parameters. I solve the optimal policies and simulate a 16-period model 500 times, starting from the initial state in my data. And then, I calculate the mean of simulations to obtain these six moments. We can see that the model generated moments fit the empirical moments reasonably well.

3.5.3 Counterfactual: Firm Behaviors in Equilibrium

In this section, I use the estimated parameters in table 3.4 and 3.5 to solve the industry equilibrium and characterize the counterfactual equilibrium behaviors of both firms. These counterfactuals help us thoroughly understand the effects of competition on innovation when firms own the high-end and the low-end product lines. I explore firms' behaviors

Table 3.6: Empirical and Simulated Moments

Moment	**Actual**	**Actual Standard Error**	**Fitted**
Mean updating rates:			
GM, high-end	0.1662	0.1085	0.2495
GM, low-end	0.1153	0.1036	0.0480
Ford, high-end	0.2126	0.1481	0.3085
Ford, low-end	0.2143	0.1601	0.2454
Mean R&D/ revenue:			
GM	0.0486	0.0012	0.0506
Ford	0.0458	0.0012	0.0386

Notes: simulated moments are averages over 500 simulations of 16 years of data.

in three counterfactual scenarios where there is a significant quality gap between itself and the competitor; both firms are of similar qualities; the competitor concentrates on developing one product line. I also analyze in what circumstance firms prefer to develop one product line or upgrade both lines in parallel.

Counterfactual Scenario 1: When There is a Big Gap Between Firms

How two firms' profits and investment strategies are different when there is a big quality gap between firms' products? Figure 3.1, 3.2, 3.3, and 3.4 present value function and equilibrium R&D investments for GM-Ford duopoly at corresponding states.

Figure 3.1 compares value and investments when the quality of GM's high-end product line is far ahead of Ford[15]. Outcomes are separately presented for GM and Ford in both columns. GM's value function is almost triple Ford's and increases (decreases) with a higher quality of its (rival's) low-end products. When severely left behind, Ford invests substantially in the high-end product line, while GM's high-end investments are slight and

[15] I choose the states where difference between GM's and Ford's high-end qualities are larger or equal to 4. And take the average of the outcomes over all satisfied states.

even less (a little more) when its (rival's) low-end line is more developed. Ford's low-end investments are also larger than GM to better catch up, and both firms' R&D expenditures in low-end exhibit an inverse-U shape with its own quality as $x-$axis.

In Figure 3.2, the leader changes hand, and Ford's high-end is ahead of GM[16]. Similarly, Ford's value function is almost triple GM's and increases (decreases) with a higher quality of its (rival's) low-end products. GM focuses on developing the high-end product line when severely left behind, and the investments are almost \$160 million, even more than Ford in the last case (almost \$100 million). Hence, GM, which is always the leader in the market, has more incentive to catch up with and even surpass the competitor. As expected, Ford's high-end investments are slight and even more minor (a little more) when its (rival's) low-end is more developed. GM's low-end investments are also larger than Ford's, and both firms' R&D expenditures in low-end exhibit an inverse-U shape with its own quality as $x-$axis.

Figure 3.3 and 3.4 depict firms' value and policy functions when their gaps of low-end products are large[17]. Unlike the last two cases, firms' value functions have no apparent disparity, demonstrating the dominance of high-end products in earning profits. Because of this finding, when a firm's low-end leads, the competitor still chooses to invest heavily in high-end products, which is more efficient to catch up. Moreover, firms do not reduce investments in both lines when low-end leads but high-end has no advanced development.

[16] I choose the states where the difference between Ford's and GM's high-end qualities are larger or equal to 4. And take the average of the outcomes over all satisfied states.

[17] In figure 3.3, I choose the states where GM's low-end qualities are at least four grids larger than Ford's. And take the average of the outcomes over all satisfied states. In figure 3.4, I choose the states where Ford's low-end qualities are at least four grids larger than GM's. And take the average of the outcomes over all satisfied states.

The results can be extended to the real world and help explain why an increasing number of firms add and develop high-end product series to survive when they are at a disadvantage in the competition. For example, Hyundai Motor Company of South Korea officially divided Genesis as its high-end brand in 2015, under the pressure of American, Japanese, and German luxury automobile brands. Beyond the automobile industry, Xiaomi Inc., a Chinese smartphone company, officially upgraded its original brand Xiaomi to concentrate on developing high-end models in 2019 after experiencing the trough from 2015 and severe competition from Apple, Samsung, and Huawei. H&M company added a high-end brand COS in 2007 when its fast-fashion brand was left behind the industry giant Zara.

Counterfactual Scenario 2: When Firms are Neck and Neck

Another interesting question is naturally brought up: how do firms' profits and investments change when their qualities are neck and neck? Figure 3.5[18] presents both firms' value and policy functions when they have comparable qualities. From column 1[19] where two firms own the same qualities of high-end products, we can see that Ford's value is larger than GM until the qualities of high-end products exceed the fourth grid. It indicates that compared to Ford, GM relies more on its advantage of high-end products in earning profits, while Ford earns more balanced profit from both lines. As for the policy functions, both firms invest heavily in high-end products, aiming to surpass the opponent when their

[18]Column 1 corresponds to the same qualities of high-end, and the x axis is high-end quality, ranging from the lowest to highest. Columns 2 and 3 correspond to the counterparts of the same qualities of low-end and same both. The x axis of column 3 is the quality difference of high-end and low-end.

[19]I choose the states where GM's and Ford's high-end qualities are the same. And take the average of the outcomes over all satisfied states. The x-axis is the high-end quality grids from the lowest to the highest.

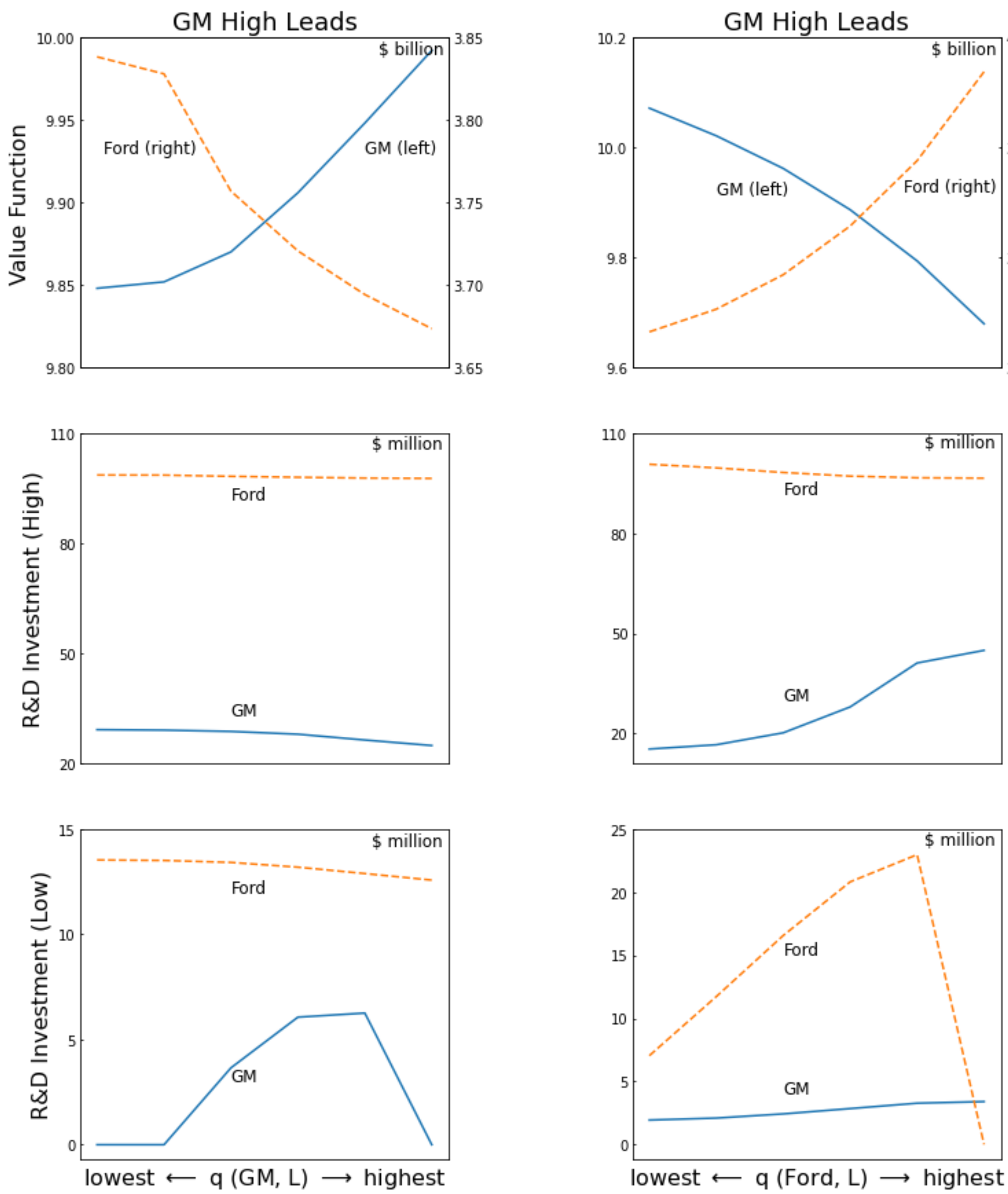


Figure 3.1: Value and Policy Functions When GM High-end Leads: Low-end Quality Varies

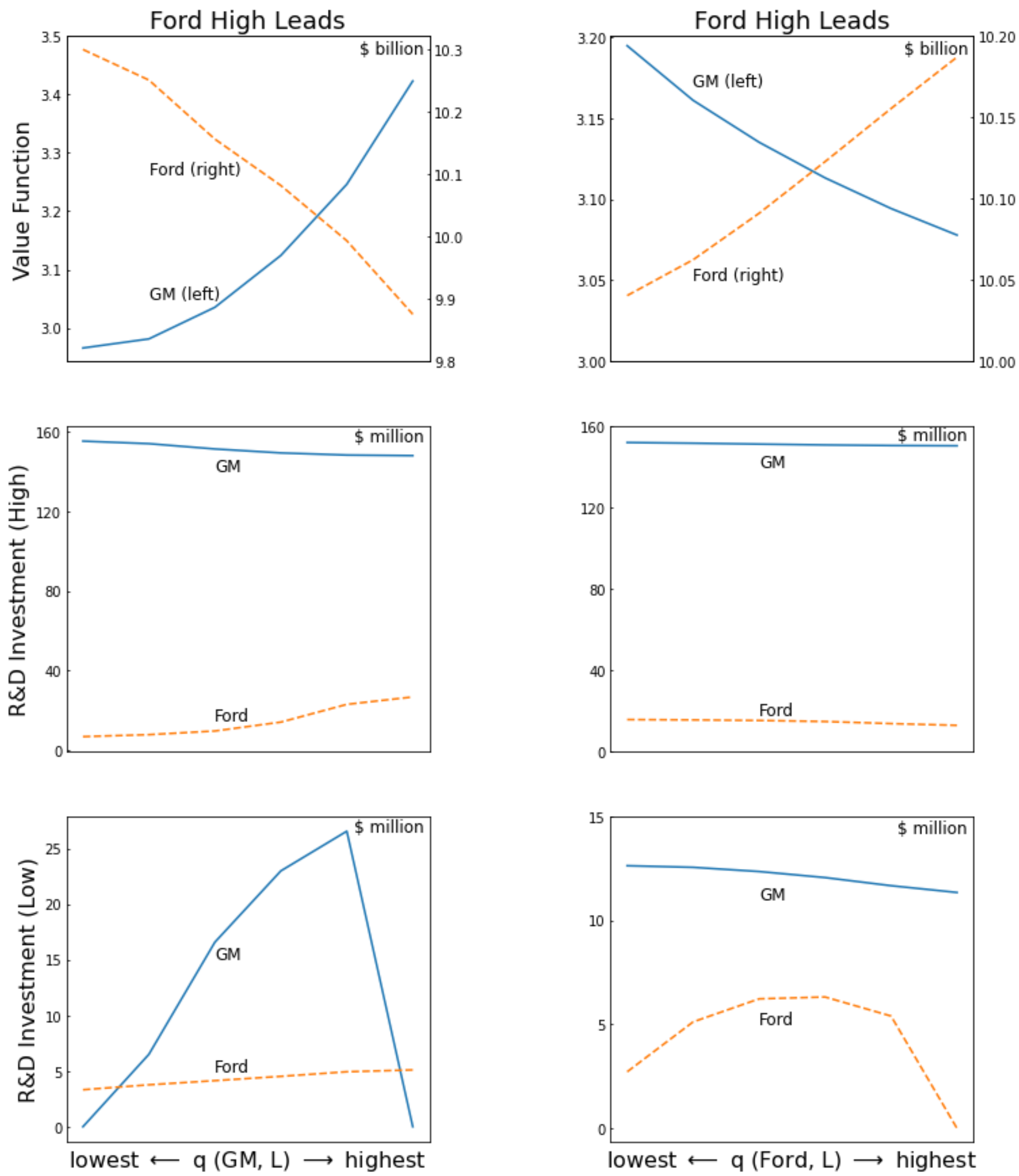


Figure 3.2: Value and Policy Functions When Ford High-end Leads: Low-end Quality Varies

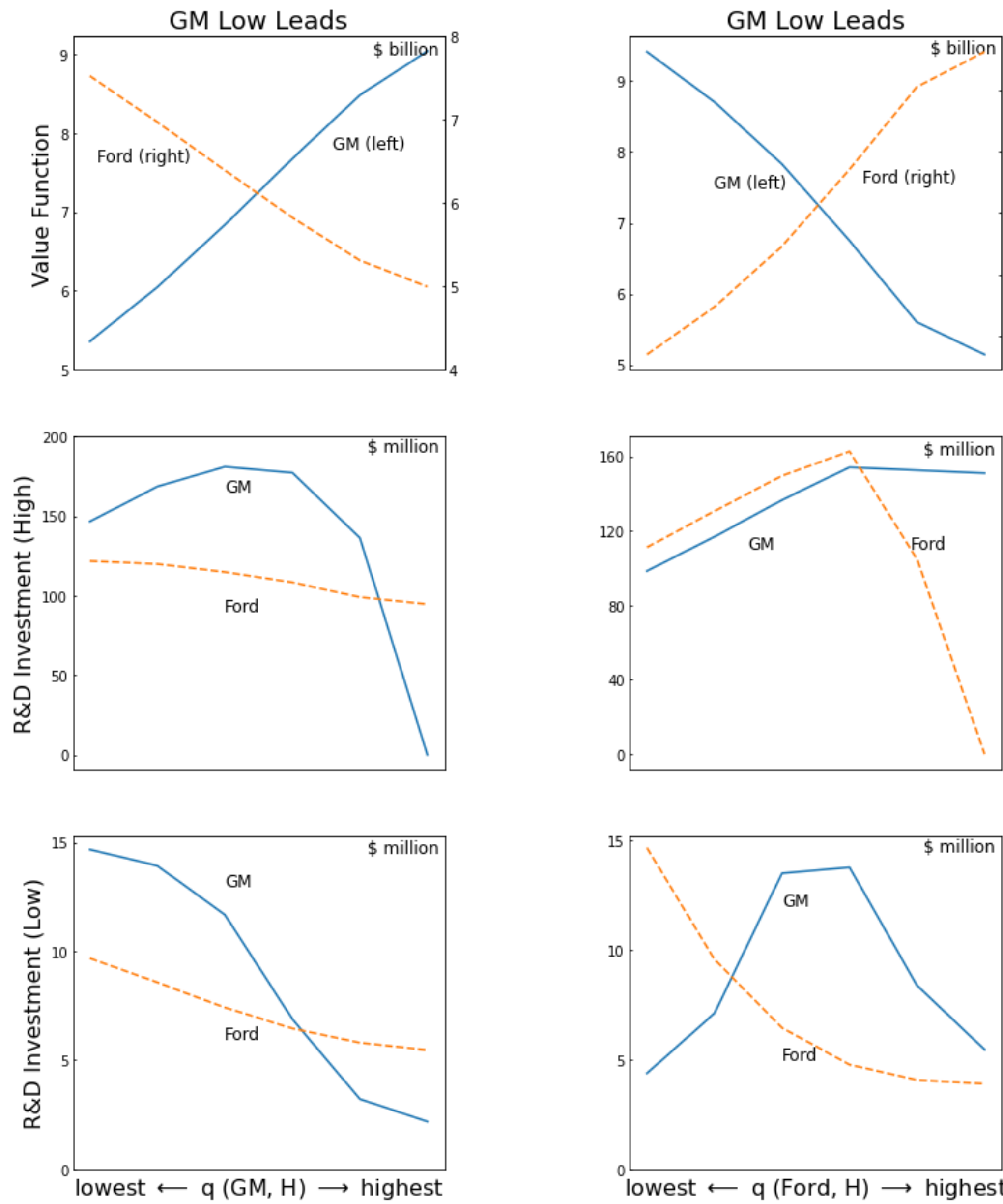


Figure 3.3: Value and Policy Functions When GM Low-end Leads: High-end Quality Varies

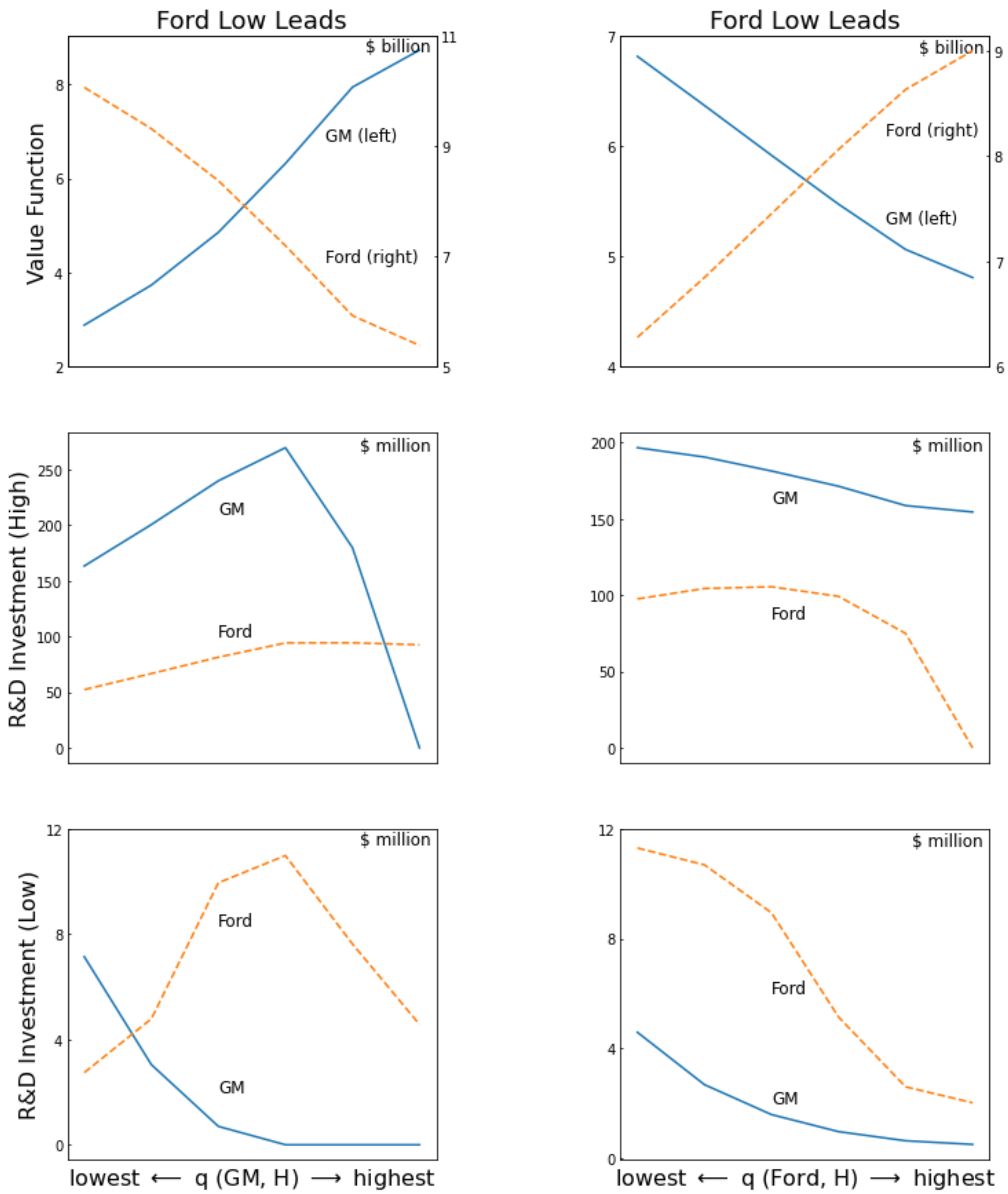


Figure 3.4: Value and Policy Functions When Ford Low-end Leads: High-end Quality Varies

corresponding product qualities are at the same level. They are mostly inspired to innovate when the high-end qualities are in the middle, not too low that pressure of innovation is not prominent, or too high that investments have diminished marginal returns. On the other hand, firms' low-end investments decline as high-end qualities are improved.

Column 2[20] shows the counterparts when firms' low-end products are of the same qualities. Ford gaining higher value than GM confirms that equality in low-end products brings GM to a disadvantageous situation, while Ford earns substantial profit only if its low-end products do not lag. Compared to column 1, sharing the same low-end qualities lessen firms' innovation stress. Both firms' high-end investments are less than column 1 and decrease with higher low-end qualities. Their low-end investments exhibit an inverse-U shape, and they are mostly inspired to innovate low-end products when the corresponding qualities are at the right of center. In summary, compared to owning the same qualities of low-end products, firms are struggling with the more fierce competition of innovating both lines when their high-end products are comparable.

The third column[21] shows the results when both firms own totally the same product qualities and how the outcomes change when the quality differences between both lines are changing. Both firms exhibit similar strategies of innovation: investments in high-end products are negatively related to the absolute value of quality differences between high-end and low-end products. In other words, developing a high-end product line is particularly important when a firm's both lines are balanced. Besides, investing in the low-end product

[20] I choose the states where GM's and Ford's low-end qualities are the same. And take the average of the outcomes over all satisfied states. The x-axis is the low-end quality grids from the lowest to the highest.

[21] I choose the states where GM's and Ford's both-end qualities are the same. And take the average of the outcomes over all satisfied states.

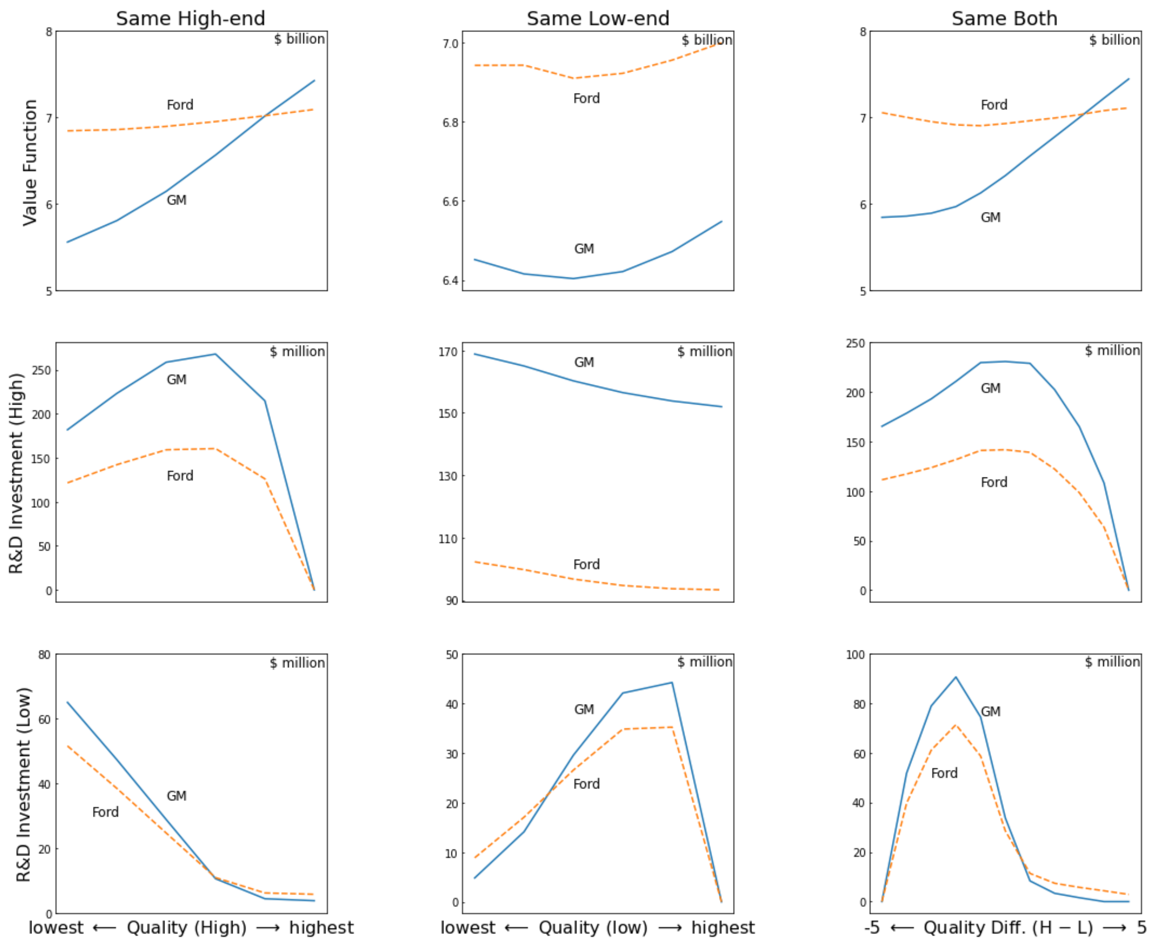


Figure 3.5: Value and Policy Functions When Two Firms Have Comparable Qualities

line is optimal when the low-end quality grid is similar and slightly ahead of the high-end. The observations suggest that each firm attempts to avoid cannibalization by investing in one specific line when its two lines are located close to each other.

Table 3.7: Investments When Rival Focuses on One Line

Investments ($million)	GM			Ford		
	General	Ford: H >L	Ford: L >H	General	GM: H >L	GM: L >H
High-end	157.3544	156.2322	170.9491	95.8054	96.3695	104.5316
Low-end	17.5228	9.2727	24.9070	16.4270	10.5772	22.0919

Notes: General investments are calculated by taking average of outcomes over all states. For the column "Firm: A > B", I choose states where Firm's A-end quality is at least three grids greater than B-end quality. And take average of outcomes over all satisfied states.

Counterfactual Scenario 3: When the Competitor Concentrates on One Product Line

We are also curious about firm's optimal response when competitor concentrates on developing only one product line. Table 3.7 characterizes firms' investments under the circumstance that the competitor places emphasis on its high-end or low-end section. I also include the firm's average investments over all states as a reference. For GM and Ford, when competitors devote themselves to upgrading high-end products, the optimal responses are keeping regular investments in high-end products to remain competitive but reducing investments in low-end ones to save effort. In this case, the firm and competitor share the same development track and compete face-to-face.

Differently, when the competitor puts more effort into the low-end product line, the firm responds by investing predominantly in both lines to build up market power thoroughly. Therefore, developing the high-end product line is always a strong defense against falling behind the competitor, but expenditures on the low-end line can be adjusted according to competitive environment or resource limit.

When Does a Firm Invest in Only One Line?

This section aims to characterize the states in which firms are inclined to develop one specific product line. Observing the equilibrium investments of both firms, excluding the states with the upper bound of grids[22], there is no situation where Ford invests in only one line, or GM invests in only low-end line. In the large SUV market, Ford typically plays the role of a laggard in market share and revenue. It must invest in both lines to thoroughly upgrade products and stimulate more demand. Moreover, due to the importance of developing the high-end line in earning profit and leading the market, neither Ford nor GM ignores the high-end section in any circumstance.

Hence, I only report the states where GM, the leader in market share and revenue, invests in only the high-end product line. The first graph in Figure 3.6 represents GM's quality grids, while the second graph represents Ford's counterparts. Darker color displays a higher frequency of state occurrence. One can observe that GM is more likely to invest in only the high-end product line when its low-end products have a deficient level of quality. GM has no advantage in competing with Ford in low-end products, so it concentrates on the high-end section. Also, GM is more probably to specialize in the high-end section when rival's qualities of both lines are more advanced, proving again the importance of developing the high-end line in earning profit and leading the market.

[22]States with the upper bound of grids have qualities equal to 5. Based on my model, when quality achieves the upper bound, no matter how much a firm invests, the probability of quality improvement is 0. Therefore, a firm's optimal investment should be 0 no matter which state it is in. Thus I exclude these states with "border".

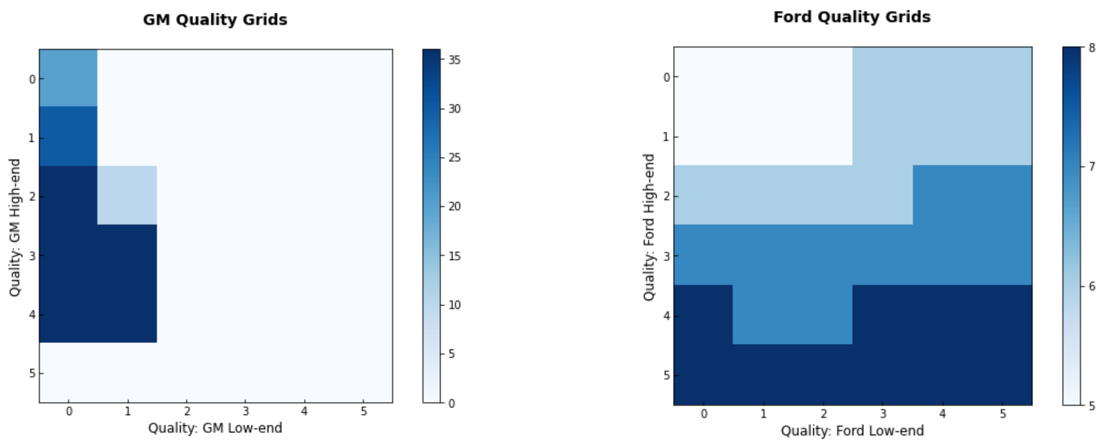


Figure 3.6: Frequencies of States Where GM Invests in Only High-end Product Line.

3.6 Conclusion

In this paper, I construct a dynamic model of multi-product duopoly to examine how competition affects firms' endogenous innovation on different product lines. I identify and estimate the model when firms' choices are private information and therefore unobserved. Using data in the US-brand large SUV industry with GM-Ford duopoly, I simulate the industry and solve the equilibrium numerically.

In the equilibrium, I find that developing the high-end product line is always a firm's first choice to catch up no matter the high-end or low-end product line falls behind. Second, firms are substantially inspired to innovate both product lines when their high-end qualities are neck and neck. This incentive is weakened when they share the same low-end qualities. Moreover, when the competitor devotes itself to upgrading high-end products, the optimal responses are keeping regular investments in high-end products to remain competitive but reducing investments in low-end ones to save effort. However, when the competitor puts more effort into the low-end product line, the firm responds by investing predominantly in both lines to build up market power thoroughly. Finally, there is no situation where Ford invests in only one line, or GM invests in only the low-end line. GM only invests in the high-end product line when its low-end products have a shallow level of quality, and Ford has leading advantages of both lines.

The results demonstrate the importance of developing the high-end product line in earning profits and leading the market. A solid high-end product line is always a strong defense against falling behind the competitor, but expenditures on the low-end line can be adjusted according to competitive environment or resource limit. The findings explain why it is more and more common for firms to create and develop a high-end product line in many industries regardless of their market positions.

My paper has limitations and many extensive directions. First, to reduce computation burden, I assume naive consumers whose policy function is merely determined by current period utility. It can be extended to rationally expecting consumers who anticipate future quality and price when making current purchasing choices. It is worthy of exploring more about estimating consumers' demand for durable goods in the framework of multi-product firms.

Second, I focus on firms' innovation strategies but do not pay much attention to the pricing. According to the assumption of naive consumers, firms' optimal pricing is statically determined by maximizing flow profit. However, the prices can be dynamically determined due to the durability of large SUVs and therefore the existence of used goods and secondary markets.

Finally, I restrict my attention to the US-brand large SUV industry with GM-Ford duopoly in the empirical application to reduce the computation burden. However, the large SUV industry has other brands (e.g., Toyota, Nissan, etc.) incorporated. A more comprehensive application with all existing firms will be involved in my future works. A time-saving computation method of value function approximation will be implemented.

Chapter 4

Product or Process Innovation? The Effect of Competition on Choices of Multi-Product Firms

4.1 Introduction

In many industries, firms produce multiple products with different characteristics to meet the heterogeneous demand of consumers. This paper focuses on the multi-product firms with vertical differentiation who offer different product lines across various qualities, specifically high-end and low-end product lines. For instance, in the automobile market, Toyota Motor Corporation owns a high-end brand Lexus and a low-end brand Toyota. In the smartphone industry, Huawei Technologies Corporation has a brand Huawei that focuses on cutting-edge models and a brand Honor specializing in cost-effective series.

There are plentiful firms tending to develop two product lines with different strategies. On the one hand, firms always invest in product innovation R&D to develop new and more-advanced-quality products in the high-end sector to make more markups, build brand reputation, and attract high-income consumers with persistent willingness to purchase. On the other hand, firms develop low-end products sold at a lower price to achieve abundant sales and expand market share. They focus on process innovation to reduce production cost and gain pricing advantage. When investing and developing different product lines, firms optimally allocate R&D between them and maximize expected profits, especially when the competitor is present. While a single-product firm can only choose to innovate or not, a multi-product firm is able to choose which product line to innovate, also product or process innovation. How does a competitor's behavior affect a firm's innovation strategies on both product lines? I study this question by constructing a dynamic structural model of a duopoly[1].

The theoretical model in this paper is within the context of Ericson and Pakes (1995) which provides a framework for numerically analyzing dynamic models of oligopolistic competition. Each firm optimizes behavior with rational expectations of competitors' actions, forming a Markov-perfect equilibrium. The model in this paper differs from earlier dynamic oligopoly models not only by incorporating multi-product firms but also by comprising strategic resource allocation and product-line-wise innovation mechanisms.

Because the product lines are independent of each other and follow individual developing processes, there is no spillover effect between the product lines within a firm. Moreover, it is also assumed that there is no spillover effect between the firms for the

[1]The model can be naturally extended to an oligopoly.

computational simplicity. Therefore, a firm's investment efficiencies on innovating both lines, respectively governing the probabilities of quality upgrading of the high-end line, and production cost reduction of the low-end line, are different. Two firms' investment efficiencies constitute the dynamic parameters of this model.

I follow most empirical IO literature[2] to assume the consumers only compare current-period utilities when making purchase decisions and leave the market when finishing purchases[3]. Therefore, the consumers' demand and the firms' prices are statically determined outside the firms' dynamic interactions.

By numerically analyzing the model, I study firms' equilibrium innovation behaviors in three counterfactual scenarios. This helps us thoroughly understand the effect of competition, coming from the behaviors of the competitor, on firms' innovations. The results mostly support the hypothesis of Arrow (1962) who proposes a positive relationship between the competition and the innovation but from a unique perspective of strategic product and process innovations on vertically differentiated product lines. In the first scenario, I find that when the firm is severely left behind the competitor, no matter in the high-end or low-end qualities, its efforts in innovating both product lines are relatively balanced. The laggard conduct substantial product innovation as well as process innovation to catch up.

Second, in the case where both firms possess neck-and-neck qualities, firms have more incentive to innovate the high-end/ low-end product line when they share similar high-end/ low-end qualities, but their investments in the other product line are reduced. In summary, firms are struggling with a more fierce competition of innovating one product

[2]Examples can be found in Hashmi and Biesebroeck (2016), Borkovsky (2017), Igami and Uetake (2019), and Yang (2020).

[3]The purpose is to reduce the computational burden, especially in a context of multi-product firms.

line when the product qualities of that specific line are comparable. Besides, compared to the market leader, which is the firm with higher investment efficiencies of both lines, the laggard relies more on its high-end product line to earn profits, while the leader earns more balanced value from both lines.

Moreover, when the competitor devotes itself to upgrading one product line, the leader's optimal response is increasing investment in that product line but reducing investment in the other one to save effort. The leader always chooses to compete face-to-face with the competitor. Differently, when the leader puts more effort into the high-end product line, the laggard is faced with severe pressure and responds by reducing investments in both lines to save effort for future use. On the other hand, the laggard reacts by investing predominantly in both lines to build up market power thoroughly when the leader focuses on its low-end product line.

I then compare market outcomes and firm's policy functions under baseline duopoly scenario and the counterfactual with the competitor (the laggard with lower investment efficiencies of both lines) removed from the industry to understand the effect of market structure on multi-product firms' innovations. I find that in duopoly, the industry profit is 8% higher than monopoly, and consumer surplus is double. Therefore, duopoly generates totally 69.3% higher social surplus than monopoly. The industry benefits from firms' competition and generates more profit because consumers bring on higher demand. Consumers benefit from competition as well since they can access products with either higher quality or lower price.

Regarding firms' upgrading behavior, I find that the average investment over time in duopoly is more than nine times higher than monopoly; the average high-end quality upgrading rate is nearly nineteen times higher; but the average low-end process upgrading rate is 10.0% lower. My results generally support the hypothesis of Arrow (1962) which proposes a positive relationship between competition and innovation, because either industry investment or total upgrading rate in duopoly is higher than monopoly. However, firms' innovative strategies on both product lines vary in different market structures. In duopoly, firms devote comparable efforts in high-end product innovation and low-end process innovation, but the monopolist only focuses on the low-end process innovation which needs less effort but with higher success probability. Therefore, we cannot ignore the deterministic effect of competition and the contribution of the market laggard to the high-end product advancement, product differentiation, and social surplus.

Furthermore, the average price in duopoly is 1.8% lower than monopoly and the total market share is 13.4% more. But the market leader's share is 63.0% less compared to monopoly. Even though the leader earns significantly more market share in monopoly, the total penetration rate is still lower than duopoly and more consumers choose not to purchase. Besides, a large proportion (95%) of the market share in monopoly is gained by the low-end sales, which indicates the monopolist has rare incentive to develop and sell the high-end products without the existence of a competitor. This explains why consumer surplus in monopoly is much less than duopoly because consumers have limited access to the products with advanced qualities in a monopolistic market.

I then conduct comparative statics by changing the market laggard to a high-end laggard but low-end leader to compare market outcomes in duopoly and monopoly with one-specific-line laggard removed. I find that in the high-end leader monopoly, the industry profit and consumer surplus are even decreased, so as the social surplus. The industry investment and product upgrading rates slightly expand, showing that the high-end leader has more incentive to innovate the high-end products even without a competitor. But the even higher price cancels out the positive effect of this product enhancement. On the contrary, if the low-end leader is the monopolist, it invests all R&D in the low-end process innovation. Compared to the duopoly scenario where the low-end leader generates 0.305 high-end upgrading rate, we are convinced again of the positive effect of competition on firms' innovations, especially the high-end product innovation.

This is the first paper to study the effect of competition on innovation of multiple-product firms with product innovation on the high-end product line and process innovation on the low-end product line. This paper is related to the existing literature on the relationship between competition and R&D activity. The three main theoretical hypotheses I mentioned before all focus on the relationship between market concentration and innovation. Later works develop models to justify the three main hypotheses: Cohen and Levin (1989) and Blundell et al. (1999) both empirically study the effect of competition (in the sense of market power or market share) on innovation through a reduced-form estimation across industries; Aghion et al. (2005) adds evidence to inverse-U relationship between Lerner index (competition measure) and patent production (innovation measure) across UK industries; Vives (2008) investigates the relationship across firms by different competition measures: firms innovate less if facing larger market size, but innovate more if there is greater product substitutability. They all develop the reduced-form models, but I study this classic question using a fully-specified structural model. Recent works which also develop an empirical dynamic structural model to analyze the relationship (see Goettler and Gordon (2011), Hashmi and Biesebroeck (2016), Borkovsky (2017), Thurk (2018), Igami and Uetake (2019) and Hollenbeck (2020)) all incorporate single-product firms, whereas in this paper, multi-product firms and strategic resource allocation within a firm are considered.

This paper relates to the literature studying competition of multi-product firms as well. The analytic theory literature tends to focus on the firm's optimal R&D portfolio and therefore restricts attention to a static game (see Lin and Zhou (2013)). Empirically, Sweeting (2013) incorporates a dynamic game in the commercial radio industry to study the effect of fees for musical performance rights; Wollmann (2018) constructs a two-stage model to analyze the equilibrium product characteristics for commercial vehicles; Fan and Yang (2020) explore the effect of competition on product proliferation in the smartphone market. They all concentrate on optimal product offerings, but none of them involve innovation, quality improvement, or cost reduction.

Another stream of the literature regarding the multi-product firms is versioning (i.e., vertical differentiation or market segmentation) of the information goods such as software, music, movies and video games. The firm can create the highest version of a product and then create degraded versions or versions with fewer features or functionality by removing functions from the flagship product at little additional cost (Bhargava and Choudhary (2008), Wei and Nault (2014)). But in my paper, the firms' high-end and low-end product lines are independently developed but not simply different versions. In other words, the cost of developing low-end products is comparable to the high-end ones[4]. In this body of literature, two major themes that emerge are the factors that influence/determine the versioning decision of a firm (Chen and Seshadri (2007), Bhargava and Choudhary (2008), Lahiri and Dey (2013), Wei and Nault (2014), August et al. (2014), Niculescu and Wu (2014), Chellappa and Mehra (2018)) and the firm's optimal pricing/second-degree price discrimination of different versions (Bhargava and Choudhary (2001), Chellappa and Shivendu (2005), Bhargava and Choudhary (2008), Anderson and Dana Jr (2009), Linde (2009), Cox (2017), Man and Zuo (2019), Kim (2019)). However, I focus on the firms' endogenous innovation decisions of different product lines in a competitive circumstance[5].

Lastly, this paper also relates to the literature investigating firms' optimal choices between product and process innovations. Earlier literature focus on developing a theoretical framework to analyze firm's, especially a monopolist's profit-maximizing innovation portfolios (Utterback and Abernathy (1975), Athey and Schmutzler (1995), Lambertini and Orsini (2000), Lambertini (2003), Lin (2004), Lambertini (2004), Mantovani (2006), Lambertini and Mantovani (2009), Lambertini and Orsini (2015)). Other literature either examine the effect of consumers' preferences or behaviors on firms' choices between product and process innovations (Rosenkranz (2003), Saha (2007), and Li and Li (2022)), or a firm's product and process innovation with corresponding knowledge accumulation resulting from

[4]The stylized facts in the physical goods industry support my model setting: materials and technologies applied to the high-end brand are greatly different from the low-end brand. It's not simply the case that the low-end technologies plus premier features and get high-end technologies.

[5]The existing papers mostly constructed a static model or implemented a reduced-form regression. But I construct a structural model and incorporate a dynamic game among the firms.

learning-by-doing (Li and Ni (2016) and Pan and Li (2016)).[6] The stream of literature that most closely relates to this paper is about the relationship between competition and firms' product and process innovations. More specifically, Klepper (1996) studies the effect of increasing number of producers, while Bonanno and Haworth (1998) and Lin and Saggi (2002) study firms' innovative behaviors under Cournot and Bertrand competitions. However, none of them provide empirical analysis, or incorporate vertically differentiated product lines.

The rest of this paper proceeds as follows. Section 4.2 develops the theoretical model of firms' and consumers' behaviors and discusses the equilibrium. Section 4.3 presents the parameterization strategies on demand, production cost, and investment efficiencies. The counterfactual results are presented in Section 4.4, followed by the comparative statics in Section 4.5. And Section 4.6 concludes the paper.

4.2 Model

4.2.1 Basic Setup

I construct a dynamic model of a differentiated-product duopoly with discrete time t and infinite horizon. Two firms, each denoted by $f \in \{1, 2\}$, sell two products, high-end and low-end, denoted by $j \in \{H, L\}$. The firms invest in the high-end product line to improve qualities (product innovation) and invest in the low-end one to reduce production cost (process innovation). I define a quality ladder for the high-end products

[6] Moreover, Fritsch and Meschede (2001) analyzes different innovative strategies of firms with different size, Lambertini and Mantovani (2010) investigates the timing of adoption of product and process innovation, and Lambertini et al. (2017) implies that one may not expect to achieve both successful product and process innovations over the product life cycle.

as $q_{fHt} \in \{0, \delta_H, 2\delta_H, \cdots, R\}$, with R as the upper bound. If the high-end innovation is successful, the firm's quality of product H improves by one fixed step (δ_H), otherwise keeps unchanged. I also define a production cost ladder for the low-end products as $ln(c_{fLt}) \in \{P, P - \Delta_L, \cdots, 2\Delta_L, \Delta_L\}$. If the low-end innovation is successful, the firm's logarithm production cost of product L is reduced by one fixed step (Δ_L), otherwise keeps unchanged. c_{fLt} is always positive and the innovation has diminishing returns.

Firms are forward-looking and take the optimal dynamic behaviors of its competitor when choosing investment strategies. Both agents observe the vector of firms' qualities and production costs of both products and $\boldsymbol{s}_t = (q_{1Ht}, \bar{q}_{1Lt}, q_{2Ht}, \bar{q}_{2Lt}, \bar{c}_{1Ht}, c_{1Lt}, \bar{c}_{2Ht}, c_{2Lt})$ constitutes the state space. I make a simple assumption that the high-end production costs and the low-end product qualities for both firms are constant over time. Therefore, I simplify the state set $\boldsymbol{s}_t = (q_{1Ht}, q_{2Ht}, c_{1Lt}, c_{2Lt})$ by only including variables affected by firms' policies. According to the state, firms simultaneously choose optimal investments $x_{fjt} \in \mathbf{R}^+$ for both product lines. Consumers are heterogeneous and compare period utilities by purchasing the high-end or the low-end products, from firm 1 or firm 2, or no purchase, given the quality of outside option, firms' current offerings, and prices.

I assume the consumers leave the market after purchase and only non-owners (new consumers and owners who exhaust, lose or break their previous purchases) re-enter the market. Therefore, consumers' ownership does not affect their purchases, and firms' prices are statically determined by maximizing period profit. I restrict consumers to be "myopic" because the computational burden of allowing rational expected consumers is prohibitive: consumers' preference becomes dynamic, and firms' optimal pricing and innovation are both dynamic decisions, affected by consumers' current ownership and future demand. The computation of equilibrium and optimization will become substantially more complex, especially in the context of multi-product firms. Accounting for rational expected consumers would be essential if my focus were on consumer upgrades or product developing trends of higher quality and lower price. However, my dynamic model captures the market features most relevant to my focus on endogenous innovation: multi-product firms strategically allocate resources to different product lines given competitor's presence and behaviors.

I do not consider entry, exit, or second hand market.

4.2.2 Model Timeline

The timeline of firms' decisions in each period is as follows. I omit the time subscripts for conciseness and instead use variables without a prime sign to represent the current period and with a prime sign to represent the next period:

1. At the start of each period, each firm observes state $\boldsymbol{s}_t = (q_{1Ht}, q_{2Ht}, c_{1Lt}, c_{2Lt})$, competes in the product market, sets prices and earns profits.

2. Firms simultaneously decide how much to invest in R&D for both product lines: high-end and low-end.

3. The outcomes of investments in R&D are realized. The state transits from $\boldsymbol{s}$ to $\boldsymbol{s}'_t = (q'_{1Ht}, q'_{2Ht}, c'_{1Lt}, c'_{2Lt})$, which is the state observed at the beginning of next period.

4.2.3 Consumers

I model consumers as heterogeneous and owning no more than one product at a time. Utility for a consumer i from firm f's product j with quality q_{fjt} and price p_{fjt} is given by a random-coefficient discrete choice model:

$$u_{ifjt} = \alpha_i q_{fjt} - \gamma_i p_{fjt} + \xi_{fjt} + \epsilon_{ifjt}. \tag{4.1}$$

q_{fjt} represents the quality of firm f's product j at period t. The random coefficient α_i capture consumers' heterogeneous tastes for quality which is assumed to follow a normal distribution with mean α and variance σ^2. So $\alpha_i = \alpha + \nu_i$, where $mean(\nu_i) = 0$ and $var(\nu_i) = \sigma^2$. Another random coefficient γ_i represents consumers' heterogeneous marginal disutility of price and follow $N(\gamma, \iota^2)$. ξ_{fjt} represents unobserved demand shock, and ε_{ifjt} captures idiosyncratic variation which is independently and identically distributed as standard type 1 extreme value across consumers, products and periods. I normalize the mean utility from the outside option to be 0 and therefore $u_{i0t} = \epsilon_{i0t}$.

Each consumer maximizes her utility, yielding the conditional choice probabilities of firm f's product j

$$s_{fjt}(\boldsymbol{q}_t, \boldsymbol{p}_t, \boldsymbol{\xi}_t) = \int \frac{\exp\left(\alpha_i q_{fjt} - \gamma_i p_{fjt} + \xi_{fjt}\right)}{1 + \sum_{j' \in \mathcal{J}_t} \exp\left(\alpha_i q_{fj't} - \gamma_i p_{fj't} + \xi_{fj't}\right)} d\Phi\left(\alpha_i\right) d\Phi\left(\gamma_i\right). \tag{4.2}$$

where $\mathcal{J}_t$ denotes the set of all available products in period t, $\boldsymbol{q}_t = (q_j, j \in \mathcal{J}_t) = (q_{1Ht}, q_{\bar{1}Lt}, q_{2Ht}, q_{\bar{2}Lt})$, and $\boldsymbol{p}_t$ and $\boldsymbol{\xi}_t$ are analogously defined. Lastly, $\Phi\left(\cdot\right)$ represents the cumulative distribution function of the random coefficient α_i and γ_i.

I define the mean utility of firm f's product j in period t as

$$\eta_{fjt} = \alpha q_{fjt} - \gamma p_{fjt} + \xi_{fjt} \tag{4.3}$$

and recover it based on equation (4.2) following Berry et al. (1995).

4.2.4 Firms

Each period, firms compete in the product market and determine optimal prices by maximizing period profit and then make dynamic investment decisions.

Static Decisions

The period profit function, excluding investments, for firm f is

$$\begin{aligned}\pi_f(\boldsymbol{q}, \boldsymbol{p}) = \max_{p_{fj}, j\in\{H,L\}} & (p_{fH} - \bar{c_{fH}}) \times M \times s_{fH}(\boldsymbol{q}, \boldsymbol{p}, \boldsymbol{\xi}) \\ & + (p_{fL} - c_{fL}) \times M \times s_{fL}(\boldsymbol{q}, \boldsymbol{p}, \boldsymbol{\xi}),\end{aligned} \tag{4.4}$$

where M is the market size, $\bar{c_{fH}}$ is constant marginal cost of high-end production, and c_{fL} is production cost of low-end products affected by firms' investment. I omit subscript t for conciseness.

The first-order condition allows me to invert out p_{fj} as

$$p_{fj} = mc_{fj} - [\Delta_f^{-1} \boldsymbol{s}_f]_{fj}, \tag{4.5}$$

where Δ_f represents a 2×2 matrix whose (j, j') element is $\frac{\partial s_{fj'}}{\partial p_{fj}}$, and $\boldsymbol{s}_f = (s_{fH}, s_{fL})$.

Dynamic Decisions

Each firm has access to R&D processes that govern its ability to produce higher-quality high-end product and produce low-end product more efficiently. It chooses an investment level $x_{fj} \in \mathbf{R}^{+}$ for both product lines. To obtain a closed form of optimal investments, I follow Goettler and Gordon (2011) and restrict investment outcomes $\tau_{fH}(\tau_{fL})$ to be either $\delta_H(-\Delta_L)$ or 0:

$$\tau_{fH} = q'_{fH} - q_{fH} = \begin{cases} 0, & \text{not success,} \\ \delta_H, & \text{success.} \end{cases}$$

$$\tau_{fL} = c'_{fL} - c_{fL} = \begin{cases} 0, & \text{not success,} \\ -\Delta_L, & \text{success.} \end{cases}$$

High-end products, regardless of firms, share the same upgrading step δ_H. So analogously do low-end products.

The success probabilities are given by

$$\begin{aligned} \chi_{fH}(\tau_{fH} = \delta_H | x_{fH}, a_{fH}) &= \frac{a_{fH}x_{fH}}{1 + a_{fH}x_{fH}}, \\ \chi_{fL}(\tau_{fL} = -\Delta_L | x_{fL}, a_{fL}) &= \frac{a_{fL}x_{fL}}{1 + a_{fL}x_{fL}}, \end{aligned} \tag{4.6}$$

where a_{fj} captures firm f's investment efficiency on each product line. And the probabilities of innovation failure are therefore $\chi_{fj}(\tau_{fj} = 0 | x_{fj}, a_{fj}) = 1 - \chi_{fj}(\tau_{fj} = \delta_H(-\Delta_L) | x_{fj}, a_{fj})$.

Each firm maximizing its expected discounted profit with simultaneously choosing optimal investments x_{fH} and x_{fL}, I can derive the Bellman equation as:

$$
\begin{aligned}
V_f\left(q_{fH}, q_{-fH}, c_{fL}, c_{-fL}\right) = \max_{x_{fH}, x_{fL} \geq 0} \Big\{ & \pi_f(\boldsymbol{q}, \boldsymbol{p}) - x_{fH} - x_{fL} \\
& + \beta \sum_{\boldsymbol{\tau_f}, q'_{-fH}, c'_{-fL}} V_f(q_{fH} + \tau_{fH}, q'_{-fH}, c_{fL} + \tau_{fL}, c'_{-fL}) \\
& \times \chi_{fH}(\tau_{fH}|x_{fH}, a_{fH}) \times \chi_{fL}(\tau_{fL}|x_{fL}, a_{fL}) \\
& \times h_{fH}(q'_{-fH}|q_{fH}, q_{-fH}) \times h_{fL}(c'_{-fL}|c_{fL}, c_{-fL}) \Big\},
\end{aligned}
\tag{4.7}
$$

where β is the discount factor and $h_f(\cdot|\cdot)$ is firm f's belief about the competitor's future qualities or production costs.

Firms simultaneously choose investments in both product lines to satisfy the first-order conditions:

$$
\begin{aligned}
\frac{\partial V_f}{\partial x_{fH}} &= -1 + \beta \sum_{\boldsymbol{\tau_f}, q'_{-fH}, c'_{-fL}} V_f(q_{fH} + \tau_{fH}, q'_{-fH}, c_{fL} + \tau_{fL}, c'_{-fL}) \times \chi_{fL}(\tau_{fL}|x_{fL}, a_{fL}) \\
& \quad \times h_{fH}(q'_{-fH}|q_{fH}, q_{-fH}) \times h_{fL}(c'_{-fL}|c_{fL}, c_{-fL}) \times \frac{\partial \chi_{fH}(\tau_{fH}|x_{fH}, a_{fH})}{\partial x_{fH}} = 0; \\
\frac{\partial V_f}{\partial x_{fL}} &= -1 + \beta \sum_{\boldsymbol{\tau_f}, q'_{-fH}, c'_{-fL}} V_f(q_{fH} + \tau_{fH}, q'_{-fH}, c_{fL} + \tau_{fL}, c'_{-fL}) \times \chi_{fH}(\tau_{fH}|x_{fH}, a_{fH}) \\
& \quad \times h_{fH}(q'_{-fH}|q_{fH}, q_{-fH}) \times h_{fL}(c'_{-fL}|c_{fL}, c_{-fL}) \times \frac{\partial \chi_{fL}(\tau_{fL}|x_{fL}, a_{fL})}{\partial x_{fL}} = 0.
\end{aligned}
\tag{4.8}
$$

Given logistic form of the χ functions, I refer to Goettler and Gordon (2011) to simplify the first-order conditions to

$$
\begin{aligned}
x^*_{fH} &= max\left\{0, \frac{\{\beta a_{fH}[EV^+_H(q) - EV^-_H(q)]\}^{(1/2)} - 1}{a_{fH}}\right\}, \\
x^*_{fL} &= max\left\{0, \frac{\{\beta a_{fL}[EV^+_L(c) - EV^-_L(c)]\}^{(1/2)} - 1}{a_{fL}}\right\},
\end{aligned}
\tag{4.9}
$$

where

$$EV_H^+(q) = \sum_{q'_{-fH}, c'_{fL}, c'_{-fL}} V_f(q_{fH} + \delta_{fH}, q'_{-fH}, c'_{fL}, c'_{-fL}) \times \chi_{fL}(c'_{fL}|x_{fL}, a_{fL})$$

$$\times h_{fH}(q'_{-fH}|q_{fH}, q_{-fH}) \times h_{fL}(c'_{-fL}|c_{fL}, c_{-fL});$$

$$EV_H^-(q) = \sum_{q'_{-fH}, c'_{fL}, c'_{-fL}} V_f(q_{fH} + 0, q'_{-fH}, c'_{fL}, c'_{-fL}) \times \chi_{fL}(c'_{fL}|x_{fL}, a_{fL})$$

$$\times h_{fH}(q'_{-fH}|q_{fH}, q_{-fH}) \times h_{fL}(c'_{-fL}|c_{fL}, c_{-fL});$$

$$EV_L^+(c) = \sum_{q'_{fH}, q'_{-fH}, c'_{-fL}} V_f(q_{fH}', q'_{-fH}, c_{fL} + \Delta_{fL}, c'_{-fL}) \times \chi_{fH}(q'_{fH}|x_{fH}, a_{fH})$$

$$\times h_{fH}(q'_{-fH}|q_{fH}, q_{-fH}) \times h_{fL}(c'_{-fL}|c_{fL}, c_{-fL});$$

$$EV_L^-(c) = \sum_{q'_{fH}, q'_{-fH}, c'_{-fL}} V_f(q_{fH}', q'_{-fH}, c_{fL} + 0, c'_{-fL}) \times \chi_{fH}(q'_{fH}|x_{fH}, a_{fH})$$

$$\times h_{fH}(q'_{-fH}|q_{fH}, q_{-fH}) \times h_{fL}(c'_{-fL}|c_{fL}, c_{-fL});$$

are the expected future values conditional on innovation success or failure, respectively. I use *max* function in equation (4.9) to restrict investments to be non-negative.

4.2.5 Equilibrium

I consider pure-strategy Markov-perfect Nash equilibrium (MPNE) of this dynamic duopoly game. The refinement guarantees the equilibrium where players' strategies only depend on current state variable values and firms optimize behaviors in each state and each sub-game. In the equilibrium, each firm has rational expectations about competitor's policy functions for investments on the high-end and low-end product lines. Formally, an MPNE in this model is the set $\{V_f^*, x_{fH}^*, x_{fL}^*, \boldsymbol{h_f}^*\}_{j=1,2}$, which includes firms' equilibrium value functions, policy functions, and beliefs about rival's future qualities or production costs. The expectations are rational in that $h^*_{fH} = \chi_{fH}(\tau_{fH} = q'_{-fH} - q_{-fH}|x_{-fH}, a_{-fH})$ and $h^*_{fL} = \chi_{fL}(\tau_{fL} = c'_{-fL} - c_{-fL}|x_{-fL}, a_{-fL})$.

An MPNE for this model can be shown to exist following Doraszelski and Satterthwaite (2010). For a discussion of potential multiplicity, see Doraszelski and Pakes (2007). According to Hollenbeck (2020), generally, "there is no way to fully rule out the possibility of multiple equilibria or to find all possible equilibria." Borkovsky et al. (2012) show multiplicity in a quality ladder model, although they conclude that "the differences between equilibria tend to be small and may matter little in practice."

I discuss the computational algorithm details in Section 4.3.

4.3 Computation

In this section, I first present the baseline parameterization. Then I demonstrate the algorithm that I use to compute equilibrium and simulate the model.

4.3.1 Baseline Parameterization

The baseline parameterization indicated in Table 4.1 refers to the estimated consumer preference and investment efficiencies in chapter 2 of my dissertation. The parameters α and σ^2 suggest that consumers favor products with higher quality and show great heterogeneity in their willingness to pay for quality. Consumers' price disutility and heterogeneity in valuing money are captured by γ and ι^2. As for dynamic parameters, firm 1's innovation efficiencies of both product lines are less than firm 2's, in which sense firm 2 is defined as a

Table 4.1: Baseline Parameterization

Parameter	α	σ^2	γ	ι^2	a_{1H}	a_{1L}	a_{2H}	a_{2L}
Value	0.044	0.1	-0.113	0.02	0.01	0.04	0.03	0.07

market leader but firm 1 as a laggard. Moreover, a firm's innovation efficiency of high-end products is less than that of low-end products, reflecting that upgrading high-end products are more difficult and investments-demanding. I vary different values of consumer preference and investment efficiencies to capture different implications in the comparative statics section.

For other parameterization details, the quality step δ_H is set to be 7, Δ_L to be 0.6. The upper bound of high-end product line's quality is set as $5\delta_H$, and for low-end product line cost, the upper bound is $5\Delta_L$. The high enough $\bar{\delta_H}$ and $\bar{\Delta_L}$ are likely to cover the observed quality improvements/ cost reductions in the industries and meanwhile allow acceptable computation time. The constant high-end production cost is set as 25, which is higher than the low-end products in almost all states. The constant low-end quality is set as 59, lower than the high-end product's qualities. The discount factor β is set as 0.9, corresponds to a period length of one year, an interest rate of 10%. Market size M is set as 350 thousand and I follow Goettler and Gordon (2011) to assume the market size is fixed for the purpose of stationarity.

4.3.2 Computational Algorithm

Preparing for the dynamic equilibrium derivation, it is necessary to compute equilibrium prices of each product and period profit of each firm given a state $\boldsymbol{s}_t = (q_{1Ht}, q_{2Ht}, c_{1Lt}, c_{2Lt})$. The main issue to be taken care of is the heterogeneity of consumers. So I calculate expected period profit over the demand shocks ξ s. I first draw the shocks from its empirical distribution given by the demand-side parameters, compute the pricing equilibrium and calculate the resulting period profit for each draw, and then take the average of these profits across all draws. In this process, pricing equilibrium is obtained by a fixed point iteration: given an initial price vector, a given state, and demand shocks, utilities of heterogeneous consumers are obtained; then, I derive the market share of each product by equation (4.2), together with the marginal cost grids, equilibrium prices given by equation (4.5) are acquired; iterate this process until input prices and output prices converge to each other.

To help guarantee the uniqueness of MPNE in my model, I apply the "limit-of-finite" approach to refine the equilibrium, for which I use backward induction to solve for an equilibrium of a T-period game and then let $T \to \infty$. In each period and each state, I compute firms' equilibrium policy functions according to the optimal conditions I derived for investments. Then I use backward induction to iterate value functions with initial values $V^0 = 0$ and obtain the equilibrium of my model.

For each iteration $k = 1, 2, \cdots$, I follow the following steps.

1. Given consumers' choice probabilities and firms' flow profits calculated before the dynamic game, I solve firms' optimal policy functions according to equations 4.9 for $\{x^*_{fj}\}_{f=1,2;j=H,L}$ at each state given continuation values V^{k-1} for firms.

2. Evaluate V^k as the discounted payoffs given firms' current policies $\{x^*_{fj}\}_{f=1,2;j=H,L}$ and continuation value V^{k-1}.

3. Check for convergence that the maximum value of $|V^k - V^{k-1}|$ is within a tolerance of 1, in which case the industry equilibrium is solved. If convergence is not achieved, return to step 1.

4.3.3 Model Generated Moments

To simulate the equilibrium model, I first specify the initial state for the industry where both firms' high-end qualities and low-end costs are at the first grid. Then for each simulated period $t = 0, 1, \cdots, T$, I implement firms' optimal investments on both lines according to the equilibrium policy functions and the stochastic innovation outcomes according to χ_j and h_j. I simulate model for S times with state transitions and obtain simulated model moments by taking the average, $m_{S,T}$, for a counterfactual analysis.

The empirical results will be manifested in section 4.4.

4.4 Empirical Results

In this section, I first solve the industry equilibrium with the baseline parameters and characterize the equilibrium behaviors of firms. Furthermore, I conduct counterfactual practice to remove the laggard firm from the market and the industry becomes monopoly. Then I compare surplus, policies, prices and market shares in two market structures. Duopoly is the factual model using all the parameters in table 4.1, while monopoly scenario uses market leader's parameters only.

4.4.1 Firm Behaviors in Equilibrium

In this section, I use the baseline parameters in table 4.1 to solve the industry equilibrium and characterize the counterfactual equilibrium behaviors of both firms. These counterfactuals help us thoroughly understand the effects of competition on innovation when firms own the high-end and the low-end product lines. I explore firms' behaviors in three scenarios where there is a significant quality gap between itself and the competitor; both firms are of similar qualities; the competitor concentrates on developing one product line.

When There is a Big Gap Between Firms

How two firms' profits and investment strategies are different when there is a big quality gap between firms' products? Figure 4.1 and 4.2 present value functions and equilibrium R&D investments for a duopoly at corresponding states.

Figure 4.1 compares value and investments when the quality of leader's high-end product line is far ahead of the laggard's[7]. Outcomes are separately presented for two firms in both columns. The leader's value function is almost triple the laggard's and increases (decreases) with a higher quality of its (rival's) low-end products. The laggard focuses on developing the high-end product line when severely left behind, and the investments are almost $120 million. As expected, the leader's high-end investments are slight and even more minor (a little more) when its (rival's) low-end is more developed. The laggard's low-end investments are also generally larger than the leader's, achieving $50 million on average. And both firms' R&D expenditures in low-end exhibit a downward sloping with its own quality as $x-$axis.

Figure 4.2 depicts firms' value and policy functions when their gaps of low-end products are large[8]. Compared to the last case, firms' value functions have milder disparity, especially when the laggard's high-end quality is not left behind, demonstrating the importance of high-end products in earning profits. Moreover, when a firm's low-end leads, the competitor chooses to invest even more heavily in both lines to catch up (high-end: $140 million; low-end: $75 million), compared to the last case (high-end: $120 million; low-end: $50 million).

When the laggard is severely left behind no matter in the high-end or low-end qualities, its efforts in increasing investments in both product lines are relatively balanced. The laggard conduct substantial product innovation as well as process innovation to catch up. Interestingly, this result is different from my second chapter where two product lines are

[7]I choose the states where difference between leader's and laggard's high-end quality grids are larger or equal to 4. And take the average of the outcomes over all satisfied states.

[8]In figure 4.2, I choose the states where leader's low-end qualities are at least four grids larger than the laggard's. And take the average of the outcomes over all satisfied states.

both product innovations. In that setting, the laggard focuses on investing in the high-end product line to catch up, but reduces effort in the low-end innovation. The possible reason for this difference is that high-end product innovation and low-end process innovation are relatively independent and demands for different R&D technologies. The trade-off between both lines' R&D allocation is not significant enough.

When Firms are Neck and Neck

Another interesting question is naturally brought up: how do firms' profits and investments change when their qualities are neck and neck? Figure 4.3[9] presents both firms' value and policy functions when they have comparable qualities. From column 1[10] where two firms own the same qualities of high-end products, we can see that the leader's value is larger than the laggard's but the difference is much less than the last scenario. It indicates that the leader also earns more profit from its low-end product line compared to the laggard. As for the policy functions, both firms invest heavily in high-end products, aiming to surpass the opponent when their corresponding product qualities are at the same level. They are mostly inspired to innovate when the high-end qualities are in the middle, not too low that pressure of innovation is not prominent, or too high that investments have diminished marginal returns. On the other hand, firms' low-end investments decline

[9] Column 1 corresponds to the same qualities of high-end, and the x axis is high-end quality, ranging from the lowest to highest. Columns 2 and 3 correspond to the counterparts of the same qualities of low-end and same both. The x axis of column 3 is the quality difference of high-end and low-end.

[10] I choose the states where both firms' high-end qualities are the same. And take the average of the outcomes over all satisfied states. The x-axis is the high-end quality grids from the lowest to the highest.

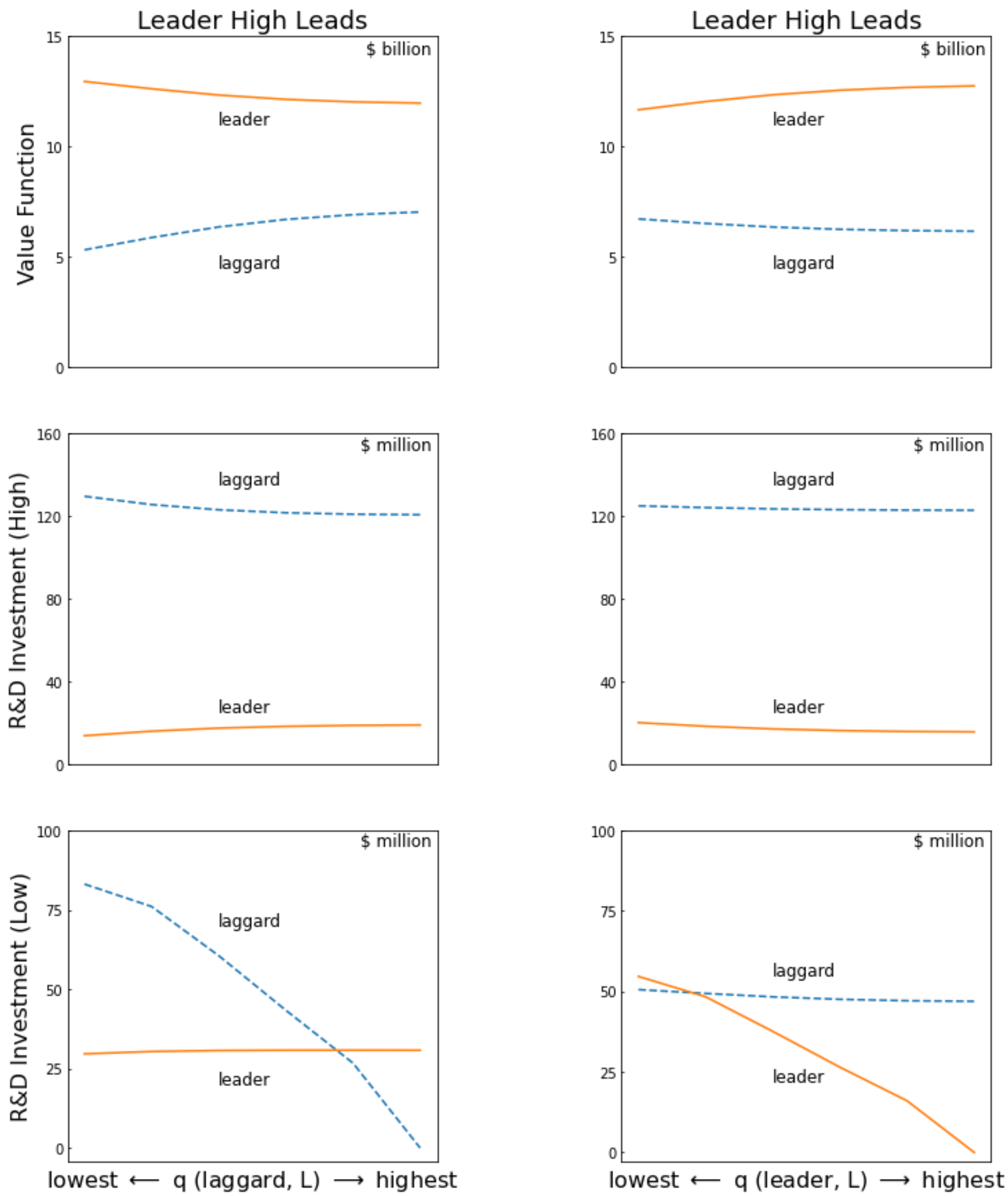


Figure 4.1: Value and Policy Functions When Leader's High-end Quality Leads: Low-end Qualities Vary

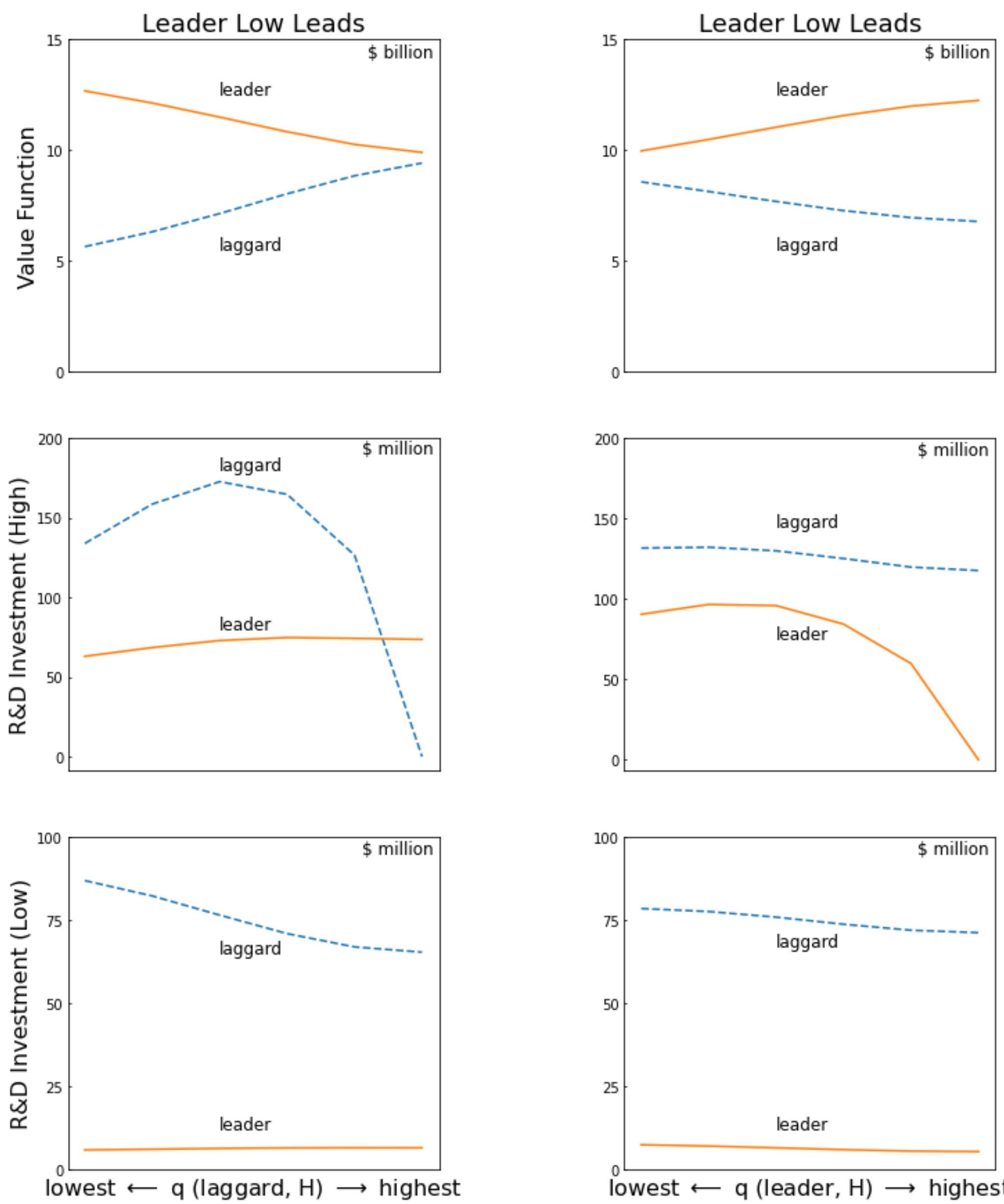


Figure 4.2: Value and Policy Functions When Leader's Low-end Leads: Low-end Qualities Vary

as high-end qualities are improved. The laggard's investments in both product lines are greater than the leader's, which indicates the laggard is always faced with more pressure in market competition.

Column 2[11] shows the counterparts when firms' low-end products are of the same qualities. The leader gaining higher value than the laggard confirms that equality in low-end products brings the laggard to a disadvantageous situation, while the leader earns substantial profit only if its low-end products do not lag. Compared to column 1, sharing the same low-end qualities lessen firms' high-end innovation stress. Both firms' high-end investments are less than column 1 and decrease with higher high-end qualities. On the contrary, their low-end investments are more than column 1 and decrease with higher low-end qualities. In summary, firms are struggling with a more fierce competition of innovating one product line when the product qualities of that specific line are comparable.

When the Competitor Concentrates on One Product Line

We are also curious about firm's optimal response when the competitor concentrates on developing only one product line. Table 4.2 characterizes firms' investments under the circumstance that the competitor places emphasis on its high-end or low-end section. I also include the firm's average investments over all states as a reference. For the market leader, when the competitor devotes itself to upgrading one product, the optimal response is increasing investment in that product line but reducing investment in the other one to save effort. The leader always chooses to compete face-to-face with the competitor.

[11] I choose the states where two firms' low-end qualities are the same. And take the average of the outcomes over all satisfied states. The x-axis is the low-end quality grids from the lowest to the highest.

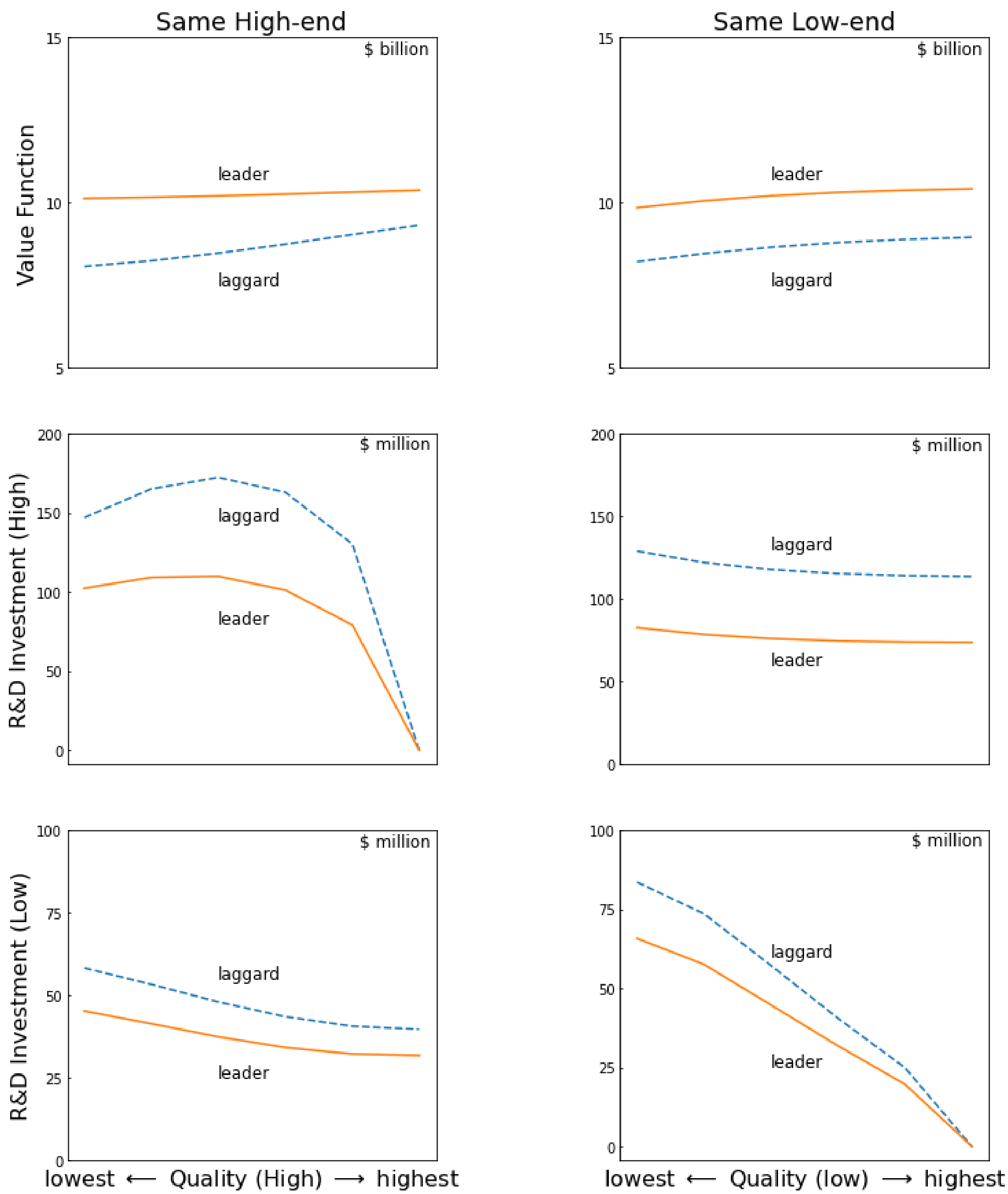


Figure 4.3: Value and Policy Functions When Two Firms Have Comparable Qualities

Table 4.2: Investments When Rival Focuses on One Line

Investments ($million)	Leader			Laggard		
	General	laggard: H >L	laggard: L >H	General	leader: H >L	leader: L >H
High-end	76.2	76.4	75.4	118.1	115.1	121.2
Low-end	35.9	35.8	36.4	45.6	45	46.4

Notes: General investments are calculated by taking average of outcomes over all states. For the column "Firm: A > B", I choose states where Firm's A-end innovation outcome is at least three grids greater than B-end. And take average of outcomes over all satisfied states.

Differently, when the leader puts more effort into the high-end product line, the laggard is faced with severe pressure and responds by reducing investments in both lines to save effort for future use. On the other hand, the laggard reacts by investing predominantly in both lines to build up market power thoroughly when the leader focuses on its low-end product line.

4.4.2 Comparing Duopoly and Monopoly

After analyzing firms' innovation behaviors in different states, I compare the duopolistic model and counterfactual model with laggard removed from the market. Table 4.3[12] reports the market outcomes in these two scenarios and the main results are listed below.

Observation 1. In duopoly, the industry profit is 8% higher than monopoly, and consumer surplus is double. Therefore, duopoly generates totally 69.3% higher social surplus than monopoly.

[12]Industry investment adds two firms' investments in two product lines together; high-end (low-end) upgrading rate takes average over two firms; leader (laggard) market share is the total share of high-end and low-end products.

The first two rows of table 4.3 report aggregate discount industry profit and consumer surplus in duopoly and monopoly. The industry benefits from firms' competition and generates more profit even though monopolist charges a higher price. Therefore, consumers' demand plays a crucial role when determining firms' surplus. In duopoly, firms compete with each other and make great efforts to produce pioneering high-end goods and lower low-end prices, which to a large extent boosts more consumption. Moreover, consumers always have a second choice to purchase the second firm's products with comparable quality and price. These two factors contribute a higher demand and a higher industry profit in duopoly. Consumers greatly benefit from competition since they can access products with either higher quality or lower price. In addition, consumer surplus respectively constitutes 77.5% and 64.7% of social surplus in duopoly and monopoly. Compared to firms' gain, consumers are the primary benefactors of industry innovation, regardless of market structure.

Observation 2. Regarding firms' upgrading behavior, I find that the average investment over time in duopoly is more than nine times higher than monopoly; the average high-end quality upgrading rate is nearly nineteen times higher; but the average low-end process upgrading rate is 10.0% lower.

In general, my results support the hypothesis of Arrow (1962) which proposes a positive relationship between competition and innovation, because either industry investment or total upgrading rate in duopoly is higher than monopoly. However, firms' innovative strategies on both product lines vary in different market structures. In duopoly, firms' innovations on both product lines are balanced, but the monopolist only focuses on

Table 4.3: Industry Outcomes Under Duopoly and Monopoly

	Duopoly	Monopoly
Industry Profit ($, millions)	872	807
Consumer Surplus ($, millions)	3003	1482
Social Surplus ($, millions)	3875	2289
Industry Investment ($, millions)	146	16
High-end Upgrading Rate	0.284	0.015
Low-end Upgrading Rate	0.281	0.309
Price ($, thousand)	23.17	23.58
Leader market share	0.362	0.590
Laggard market share	0.307	-

the low-end process innovation which needs less effort but with higher success probability. This action substantially hurts consumers' benefit so that they have little chance to access advanced products. This also explains why consumer surplus in monopoly is much less than duopolistic market.

Observation 3. The average price in duopoly is 1.8% lower than monopoly and the total market share is 13.4% more. But the market leader's share is 63.0% less compared to monopoly.

Even though the leader earns significantly more market share in monopoly, the total penetration rate is still lower than duopoly and more consumers choose not to purchase. Furthermore, a large proportion (95%) of the market share in monopoly is gained by low-end sales. The monopolist has rare incentive to develop vertically differentiated product lines. The high-end product line generates very little sales and quality improvements. Therefore, we cannot ignore the deterministic effect of competition and the contribution of the market laggard to the product differentiation, total sales, and social surplus.

4.5 Comparative Statics

In all the analysis above, I assume one firm as the market leader in both product lines, which means they have higher high-end and low-end investment efficiencies. In this section, I conduct comparative statics by changing it to a high-end-only leader and a low-end-only leader to see different outcomes in duopoly and monopoly with one-specific-line laggard removed.

4.5.1 Comparing Duopoly and Monopoly: With the High-end Laggard Removed

In this case, I set $a_{1H} = 0.01, a_{1L} = 0.07, a_{2H} = 0.03, a_{2L} = 0.04$. Firm 1 is the low-end leader and firm 2 is the high-end leader. I remove firm 1 from the market in monopoly. The market outcomes in duopoly are similar to 4.3. In the high-end leader monopoly, the industry profit and consumer surplus are even decreased, so as the social surplus. The industry investment and product upgrading rates slightly expand, showing that the high-end leader has more incentive to innovate the high-end products even without a competitor. But the even higher price cancels out the positive effect of this product enhancement.

Table 4.4: Duopoly and Monopoly (Keeps the High-end Leader)

	Duopoly	Monopoly
Industry Profit ($, millions)	873	799
Consumer Surplus ($, millions)	3003	1427
Social Surplus ($, millions)	3876	2226
Industry Investment ($, millions)	145	22.8
High-end Upgrading Rate	0.284	0.020
Low-end Upgrading Rate	0.280	0.310
Price ($, thousand)	23.13	23.73
Leader market share	0.359	0.588
Laggard market share	0.310	-

Table 4.5: Duopoly and Monopoly (Keeps the Low-end Leader)

	Duopoly	Monopoly
Industry Profit ($, millions)	873	807
Consumer Surplus ($, millions)	2997	1475
Social Surplus ($, millions)	3870	2282
Industry Investment ($, millions)	147	16
High-end Upgrading Rate	0.276	0.000
Low-end Upgrading Rate	0.280	0.310
Price ($, thousand)	23.18	23.59
Leader market share	0.337	0.590
Laggard market share	0.332	-

4.5.2 Comparing Duopoly and Monopoly: With the Low-end Laggard Removed

In this case, I set $a_{1H} = 0.01, a_{1L} = 0.07, a_{2H} = 0.03, a_{2L} = 0.04$. Firm 1 is the low-end leader and firm 2 is the high-end leader. I remove firm 2 from the market in monopoly. If the low-end leader is the monopolist, it invests all R&D in the low-end process innovation. Compared to the duopoly scenario where the low-end leader generates 0.305 high-end upgrading rate, we are convinced again the positive effect of competition on firms' innovations, especially the product innovations.

4.5.3 Consumers' Preference Varies

I also modify consumers' average taste for quality by setting $\alpha = 0.03, 0.06, 0.1$ and average disutility of price by setting $\gamma = -0.05, -0.08, -0.2$ to study the effect of consumers' preference on firms' product and process innovation strategies. The relationship is not significant or monotone, which indicates competition still dominates firms' strategic innovations and resource allocation between the high-end and the low-end product lines. The details are presented in appendix C.1.

4.6 Conclusion

In this paper, I construct a dynamic model of multi-product duopoly to examine how competition affects firms' endogenous innovation on different product lines. Firms incorporates product innovation on the high-end product line, while process innovation on the low-end line. With baseline parameterization, I simulate the industry and solve the equilibrium numerically.

Comparing the baseline market structure of duopoly and the counterfactual of monopoly, I find that firms' innovative strategies on both product lines vary in different market structures. In duopoly, firms devote comparable efforts in product innovation and process innovation, but the monopolist only focuses on the process innovation which needs less effort but with higher success probability. This action substantially hurts consumers' benefit so that they have little chance to access advanced products. The results demonstrate the dramatically positive effect of competition, also the existence of the market laggard, on the product differentiation and social surplus.

My paper has limitations and many interesting extensive directions. First, to reduce computation burden, I assume naive consumers whose policy function is merely determined by current period utility. It can be extended to rationally expecting consumers who anticipate future quality and price when making current purchasing choices. It is worthy of exploring more about estimating consumers' demand for durable goods in the framework of multi-product firms.

Second, I focus on firms' innovation strategies but do not pay much attention to the pricing. According to the assumption of naive consumers, firms' optimal pricing is statically determined by maximizing flow profit. However, the prices can be dynamically determined due to the durability of goods and therefore the existence of used goods and secondary markets.

Finally, my model can be applied to many industries which consist of a high-end product line with product innovation and a low-end product line with process innovation, such as smartphone, apparel, and so on. A more comprehensive empirical application with real-world data is worthy to be incorporated.

Chapter 5

Conclusions

In this dissertation, I construct dynamic models of duopoly to examine how competition affects firms' endogenous innovation strategies from three novel angles. Chapter 2 investigates the effect of competition on firms' endogenous pricing, innovation and releasing strategies for durable goods. I estimate my model using data in CPU industry with Intel-AMD duopoly and simulate this industry numerically. Comparing the baseline market structure of duopoly and the counterfactual of Intel monopoly, I find that industry innovation is higher with competition, but firms are more likely to accumulate innovation outcomes and release less frequently when competing with others, compared to monopoly. Being able to stock investment achievements and optimally release spur firms innovate even though spillover effect exists. Moreover, the whole industry and consumers are better off under a duopoly because of higher quality product, lower prices and higher demand.

Chapter 3 examines how competition affects firms' endogenous product innovations on vertically differentiated product lines. I identify and estimate the model when firms' choices are private information and therefore unobserved. Using data in the US-brand large SUV industry with GM-Ford duopoly, I simulate the industry and solve the equilibrium numerically. I find that when the competitor devotes itself to upgrading high-end products, the optimal responses are keeping regular investments in high-end products to remain competitive but reducing investments in low-end ones to save effort. However, when the competitor puts more effort into the low-end product line, the firm responds by investing predominantly in both lines to build up market power thoroughly. The finding explains why it is more and more common for firms to create and develop a high-end product line in many industries regardless of their market positions.

Chapter 4 extends by incorporating product innovation on the high-end product line, while process innovation on the low-end line. Comparing the baseline market structure of duopoly and the counterfactual of monopoly, I find that firms' innovative strategies on both product lines vary in different market structures. In duopoly, firms devote comparable efforts in product innovation and process innovation, but the monopolist only focuses on the process innovation which needs less effort but with higher success probability. This action substantially hurts consumers' benefit so that they have little chance to access advanced products. The results demonstrate the dramatically positive effect of competition, also the existence of the market laggard, on the product differentiation and social surplus.

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Appendix A

Appendix for Chapter 2

A.1 Solving and Simulating Industry Equilibrium

To help guarantee the uniqueness of MPNE in my model, I apply "limit-of-finite" approach to refine the equilibrium, for which I use backward induction to solve for an equilibrium of a T-period game and then let $T \to \infty$. In each period and each state, I compute firms' equilibrium policy functions according to the optimal conditions I derived for pricing, releasing and investment. Then I use backward induction to iterate value functions with initial values $W^0 = 0$ and obtain equilibrium of my model.

For each iteration $k = 1, 2, \cdots$, I follow the following steps.

1. After solving for consumers' choice probability, I solve firms' optimal policy functions in equations 2.12, 2.20 and 2.21 for $\{x_j^*, \eta_j^*, p_j^*\}_{j=1,2}$ at each state given continuation values W^{k-1} for firms.

2. Evaluate W^k as the discounted payoffs given firms' current policies $\{x_j^*, \eta_j^*, p_j^*\}_{j=1,2}$, continuation ownership distribution Δ' determined by equation 2.4, and continuation value W^{k-1}.

3. Check for convergence that the maximum value of $|W^k - W^{k-1}|$ is within a tolerance of 30, in which case the industry equilibrium is solved. If convergence is not achieved, return to step 1.

To simulate the equilibrium model, I first specify the initial state for the industry with AMD's quality is two-step behind Intel and each firm own δ R&D stock. Then for each simulated period $t = 0, 1, \cdots, T$, I implement firms' policy functions for optimal pricing, releasing and investment, consumers' choice probabilities, and the stochastic updating outcomes according to χ_j and g_j. I follow Goettler and Gordon (2011) to generate Δ using a distribution parameterized by its mean and choose the grid such that the mean qualities are 1.25 apart and range from 9 to 29 relative to $\bar{q} = 30$.

Appendix B

Appendix for Chapter 3

B.1 Solving and Simulating Industry Equilibrium

To help guarantee the uniqueness of MPNE in my model, I apply the "limit-of-finite" approach to refine the equilibrium, for which I use backward induction to solve for an equilibrium of a T-period game and then let $T \to \infty$. In each period and each state, I compute firms' equilibrium policy functions according to the optimal conditions I derived for investments. Then I use backward induction to iterate value functions with initial values $V^0 = 0$ and obtain the equilibrium of my model.

For each iteration $k = 1, 2, \cdots$, I follow the following steps.

1. Given consumers' choice probabilities and firms' flow profits calculated before the dynamic game, I solve firms' optimal policy functions according to equations 3.10 for $\{x^*_{fj}\}_{f=1,2;j=H,L}$ at each state given continuation values V^{k-1} for firms.

2. Evaluate V^k as the discounted payoffs given firms' current policies $\{x^*_{fj}\}_{f=1,2;j=H,L}$ and continuation value V^{k-1}.

3. Check for convergence that the maximum value of $|V^k - V^{k-1}|$ is within a tolerance of 1, in which case the industry equilibrium is solved. If convergence is not achieved, return to step 1.

To simulate the equilibrium model, I first specify the initial state for the industry with GM's high-end leads one quality step and Ford's low-end leads one quality step. Then for each simulated period $t = 0, 1, \cdots, T$, I implement firms' optimal investments on both lines according to the equilibrium policy functions and the stochastic innovation outcomes according to χ_j and h_j.

Appendix C

Appendix for Chapter 4

C.1 Comparative Statics: Consumers' Preference Varies

The baseline parameterized quality preference $\alpha = 0.044$, I modify it by setting $\alpha = 0.03, 0.06, 0.1$ to study the effect of consumers' preference of quality on firms' product innovation in duopoly. If consumers treasure advanced quality more, will firms innovate more in the high-end product line? As presented in table C.1, the average product innovation rate of two firms is not monotone to consumers preference of quality.

Table C.1: Product Innovation Rate ($\gamma = -0.113$)

α	0.03	0.044	0.06	0.1
Product Innovation Rate	0.2799	0.2835	0.2765	0.2804

Similarly, I modify the average disutility of price by setting $\gamma = -0.05, -0.08, -0.2$, besides the baseline $\gamma = -0.113$, to examine firms' process innovations in duopoly when consumers value money differently. As shown in table C.2, it is not the case when consumers value money more greatly, firms invest more in the process innovation.

Table C.2: Process Innovation Rate ($\alpha = 0.044$)

α	-0.05	-0.08	-0.113	-0.2
Process Innovation Rate	0.2855	0.2838	0.2810	0.2794

The relationship between consumers' preference and firms' innovations is not significant or monotone. Therefore, competition still dominates firms' strategic innovations and resource allocation between the high-end and the low-end product lines.