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Authors

Rasmussen, John O.

Ma, Chin W.

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NUMBER-DISPLACEMENT DEGREES OF FREEDOM IN NUCLEAR ROTATIONAL THEORY*

John O. Rasmussen

Lawrence Berkeley Laboratory
University of California
Berkeley, California 94720

and

Chin W. Ma

Physics Department
Indiana University
Bloomington, Indiana 47401

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Abstract:

A new degree of freedom associated with shifting of particles between the even-parity orbitals system and the odd-parity orbitals system is introduced in the study of nuclear rotational problem. It is found that this new mode contributes significantly to the deviation of the $I(I + 1)$ rule with correction terms comparable to the Coriolis-anti-pairing effect.

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In theoretical calculations of nuclear rotational spectrum, several specific collective degrees of freedom have been considered as influencing deviations from the $I(I + 1)$ energy spacings. There are the proton and neutron pairing correlations involved in the Coriolis anti-pairing (CAP) effect. There

are the beta- and gamma-vibrational degrees of freedom involved in shape change effects. There is also a fourth-order cranking effect that may be expressed in terms of an additional collective degree of freedom.

The total energy E of a deformed rotating system may be expressed in terms of these collective degrees of freedom as¹

$$E = \sum_i \frac{1}{2} C_i (X_i - X_{i0})^2 + \frac{I(I+1)}{2J(X_i)} \quad (1)$$

where X_i ($i = 1, 2, 3, \dots$) describes the various degrees of freedom, C_i is the spring constant associated with the i -th degree of freedom and $J(X_i)$ is the moment-of-inertia. The rotational solution may be obtained by minimizing the total energy of Eq. (1) with respect to each of X_i at a given angular momentum I .

In this note we shall identify a new degree of freedom which will affect both the moment-of-inertia and the potential energy of Eq. (1). We first express the moment-of-inertia J in terms of the cranking formula of Inglis² and Belyaev³

$$J = 2\hbar^2 \sum_{\Omega_\alpha > 0} \frac{\langle \alpha' | j_x | \alpha \rangle}{E_{\alpha'} + E_\alpha} (U_{\alpha'} V_\alpha - U_\alpha V_{\alpha'})^2 \quad (2)$$

where $|\alpha\rangle$ is the deformed single particle state with α denoting the appropriate quantum numbers, Ω_α is the projection of the angular momentum on the symmetry axis, E_α is the quasi-particle energy and U_α, V_α are the probability coefficients

$$\left. \begin{matrix} U_\alpha^2 \\ V_\alpha^2 \end{matrix} \right\} = \frac{1}{2} \left[1 \pm \frac{\epsilon_\alpha - \lambda}{\sqrt{(\epsilon_\alpha - \lambda)^2 + v^2}} \right] \quad (3)$$

where ϵ_α is the single particle energy, v is the pairing correlation parameter and λ , the chemical potential. In usual practice, the chemical potential takes

two values, λ_p for the proton system and λ_n for the neutron system. What interests us here is the fact that the even-parity orbital systems and odd-parity orbital systems make separable contributions to the moment-of-inertia, since the j_x operator has even parity. Thus, it is possible that the nuclear system could change its moment-of-inertia by shifting nucleons between the two orbital systems of opposite parity. In other words, the nuclear system might assume unequal chemical potentials (λ_+ and λ_-) for the different-parity orbital systems in order to minimize the total energy for a given angular momentum.

The potential energy term corresponding to the separation of λ_+ and λ_- is easily derived in the limit of the continuous model, where the discrete distributions of Nilsson orbital energies are replaced by a density function ρ . Let ρ_+ and ρ_- be the number of Nilsson orbitals per energy interval for positive parity and negative parity, respectively. The number of nucleons transferred for a given change of chemical potential λ is by definition of ρ

$$dn = 2\rho d\lambda \quad , \quad (4)$$

noting that Nilsson orbitals are doubly degenerate. Thus, conservation of particle number gives the condition

$$\rho_+ d\lambda_+ = -\rho_- d\lambda_- \quad (5)$$

The new degree of freedom will be characterized by σ , the difference of the chemical potentials.

$$\sigma \equiv \lambda_+ - \lambda_- \quad . \quad (6)$$

The fundamental definition of the chemical potential is

$$\lambda_{\pm} = \frac{dE}{dn_{\pm}}$$

Integrating in the continuous model, using Eqs. (4) - (6), we obtain

$$\begin{aligned} E &= \int \lambda_{+} dn_{+} + \int \lambda_{-} dn_{-} = \int (\lambda_{+} - \lambda_{-}) dn_{+} \\ &= 2\rho_{+} \int \sigma d\lambda_{+} = \frac{2\rho_{+}\rho_{-}}{\rho_{+} + \rho_{-}} \int \sigma d\sigma \\ &= \frac{2\rho_{+}\rho_{-}}{\rho_{+} + \rho_{-}} \frac{1}{2} \sigma^2 = \frac{1}{2} C_{\sigma} \sigma^2 \end{aligned} \quad (7)$$

In the continuous model with the constant G pairing assumption the pairing energy terms proportional to G will not be altered. We see that the restoring potential against non-zero σ is harmonic and dependent solely on level densities. The desired spring constant C_{σ} associated with the number displacement degree of freedom is thus

$$C_{\sigma} = \frac{2\rho_{+}\rho_{-}}{\rho_{+} + \rho_{-}} \quad (8)$$

The effect of the new degree of freedom can be estimated by calculating its contribution to the correction term $BI^2(I+1)^2$ in an expansion of the rotational energy.^{1,4,5} The general form of the coefficient B is given by¹

$$B = - \sum_i \frac{1}{8C_i J_i^4} \left(\frac{\partial J}{\partial X_i} \right)^2 \quad (9)$$

Eq. (9) shows that each degree of freedom makes additive contribution which is inversely proportional to the spring constant C_i and directly proportional to the square of the corresponding partial derivative of the moment-of-inertia.

We have calculated the various derivatives of moments-of-inertia with respect to chemical potentials λ_+ and λ_- for protons and neutrons for several nuclei in the rare earth region. The calculations were made with the new Nilsson et al.⁶ method with diagonalization over the space $0 \leq N \leq 12$ and with both quadrupole (ϵ) and hexadecapole (ϵ_4) deformation taken into account. The pairing force strengths, number of orbitals included in the BCS solution, and potential parameters κ and μ are as given by Nilsson et al.⁶ Table I gives for three nuclei the deformations assumed and the calculated inertial derivatives. It can be shown from Eqs. (5) and (6) that $\frac{\partial J}{\partial \sigma} = [\rho_- \frac{\partial J}{\partial \lambda_+} - \rho_+ \frac{\partial J}{\partial \lambda_-}] / \rho$. The trends of these derivatives are understandable from inspection of Fig. 1 of Ref. 1, where the contributions of various j shells to the moment-of-inertia are graphed. One expects large positive derivatives in the beginning of the deformed region (i.e., ^{152}Sm), especially from $i_{13/2}$ neutron orbitals, which are just beginning to fill. These derivatives decrease around mid-shell.

For the estimate of the spring constants in the σ mode we have used for total level density ρ a Strutinsky-type smeared average over the new Nilsson level diagram at $\epsilon = 0.25$ and $\epsilon_4 = 0$ with a smearing constant of $0.4 \hbar \omega_0$. The ratios ρ_+/ρ_- were roughly estimated from the total of shell model orbitals in the 50-82 shell for protons and the 82-126 shell for neutrons. That is, for protons $\rho_+/\rho_- = 10/6$ and for neutrons $\rho_+/\rho_- = 7/15$. With these level density values were calculated the spring constants according to Eq. (8). In turn were calculated the contributions to the $BI^2(I+1)^2$ term according to Eq. (9) where the moments-of-inertia were taken from the experimental 2^+ energy levels. These values are summarized in Table II along with comparison values of the total CAP contribution according to Marshalek⁷ and the experimental B value.

Including these new degrees of freedom (σ -effect) actually makes worse the disagreement with experiment in Refs. 1 and 7, where the CAP contribution (B_{Δ}) alone often exceeds experimental. However, the disagreement of cranking model calculations with experiment is a much broader question which we do not address here.

It is evident that the σ effect contributions due to the number displacement degree of freedom are very significant and cannot be neglected. Of course, there should be made more realistic calculations of the spring constant C_0 without using the continuous model, but we do not expect a qualitative change from the estimates here. The σ -degrees of freedom may alter the role of subshell irregularities in the Coriolis anti-pairing effect.

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FOOTNOTES AND REFERENCES

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1. C. W. Ma and J. O. Rasmussen, Phys. Rev. C2, 798 (1970).
2. D. R. Inglis, Phys. Rev. 96, 1059 (1954).
3. S. T. Belyaev, Kgl. Danske Videnskab Selskab, Mat.-Fys. Medd. 31, No. 11 (1959).
4. A. Bohr and B. R. Mottelson, Mat. Fys. Medd. Dan. Vid. Selsk. (1953).
5. V. G. Soloviev, Theory of Complex Nuclei, Science Press, Moscow (1971);
(in Russian) (English translation, Pergamon Press, in progress).
6. S. G. Nilsson et al., Nucl. Phys. A131, 1 (1969).
7. E. R. Marshalek, Phys. Rev. 158, 993 (1967).

Table I. Derivatives of Cranking Moment-of-Inertia with Respect to Chemical Potentials (All Units in MeV^{-2}).

Nucleus	ϵ	ϵ_4	$\frac{\partial 2J}{\partial \lambda_{+p}}$	$\frac{\partial 2J}{\partial \lambda_{-p}}$	$\frac{\partial 2J}{\partial \sigma_p}$	$\frac{\partial 2J}{\partial \lambda_{+n}}$	$\frac{\partial 2J}{\partial \lambda_{-n}}$	$\frac{\partial 2J}{\partial \sigma_n}$
^{152}Sm	0.202	-0.036	7.42	22.80	-11.5	44.32	19.40	24.0
^{162}Dy	0.256	-0.006	5.46	17.97	-9.17	37.16	17.38	19.8
^{178}Hf	0.250	0.052	0.30	8.83	-5.40	25.65	5.66	15.7

Table II. Number-Displacement Effect Calculations and Comparison of CAP Values.

Nucleus	ρ_p (MeV ⁻¹)	ρ_n (MeV ⁻¹)	C_{op} (MeV ⁻¹)	C_{on} (MeV ⁻¹)	B_{op} (MeV ⁻¹)	B_{on} (eV)	B_{exp} (eV)	B_{Δ} Marshalek ⁷ (eV)
¹⁵² Sm	2.32	3.40	1.09	1.48	10.2	33.1	193	33.7
¹⁶² Dy	2.20	3.03	1.03	1.32	1.33	4.86	10.8	13.1
¹⁷⁸ Hf	2.68	2.88	1.26	1.25	0.67	5.79	13.1	19.4

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