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Modeling Orientation Effects in Symmetry Detection: The Role of Visual Structure

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Abstract

Symmetry detection is a key part of human perception. One incompletely understood aspect of symmetry detection concerns *orientation effects*. The best-known orientation effect is the *preference for vertical symmetry*, where symmetry around a vertical axis is detected more quickly and accurately than symmetry at other orientations. Current symmetry detection models have difficulty explaining this effect. Using MAGI (Ferguson, 1994), we show how orientation effects may be caused by interactions between the perceived visual relations and the current reference frame. As evidence for this explanation, we simulate several orientation characteristics, including the preference for vertical symmetry and Wiser's (1981) theory of "intrinsic axes". Finally, we successfully simulate the results of a classic study by Palmer and Hemenway (1978) which explores the relationship between the preference for vertical symmetry, multiple symmetries, and inexact symmetry. Collectively, these results show that orientation effects may be due to characteristics of detected visual relations rather than either exact point-to-point equivalencies or the bilateral symmetry of the visual system.

Introduction

Symmetry detection is a core mechanism in perception, shape recognition, and perceptual organization. Yet the processes underlying symmetry detection are only partially understood. Studies of symmetry detection have revealed psychological characteristics more complex than previously assumed even a few decades ago.

One such set of characteristics are *orientation effects*: interactions between symmetry detection and the visual reference frame. Orientation effects are interesting because they separate human performance in judging symmetry from symmetry's geometric definition. In geometric terms, symmetry is orientation-invariant, yet human symmetry detection depends critically on a figure's orientation. In addition, under certain circumstances symmetric figures also

influence the visual reference frame.

Orientation effects can be placed into three broad categories: the preference for vertical symmetry, the preference for multiple symmetries, and the effect of symmetry on a figure's object-centered reference frame.

Preference for vertical symmetry. Bilateral symmetry is more quickly and accurately detected when the symmetry axis is vertical (Attneave & Olson, 1967; Bornstein & Krinsky, 1985; Chipman & Mendelsohn, 1979; Corballis & Roldan, 1975; Goldmeier, 1936/1972; Julesz, 1971; Mach, 1893/1986; Palmer & Hemenway, 1978). In most cases, vertical symmetry is easier than horizontal symmetry, which in turn is easier than diagonal symmetry.

A longstanding explanation for the preference for vertical symmetry is that it depends on the human visual system's own vertically bilateral structure. In this framework, originally suggested by Mach (1893/1986), human vision provides better and faster results for symmetries aligned with its own symmetric structure. Several visual subsystems have been proposed as this effect's locus, from eye placement (Mach, 1893/1986) to the corpus collosum (Braitenberg, 1984; Herbert & Humphrey, 1996). However, most of these explanations focus on the retina and structures just beyond it (Corballis & Roldan, 1975; Jenkins, 1982; Julesz, 1971). Thus, these explanations are known as *retinocentric models*.

Retinocentric models, while theoretically elegant, fail to explain a key result: vertical symmetry is still preferred when the retina is *misaligned* with the symmetry axis. Rock and Leaman (1963) showed that the preference for vertical symmetry is still present when a figure is vertical with respect to the gravitational reference frame, but the subject's head is tilted 45° away from vertical.

Symmetry in figures with intrinsic axes. The preference for vertical symmetry disappears or is greatly attenuated for

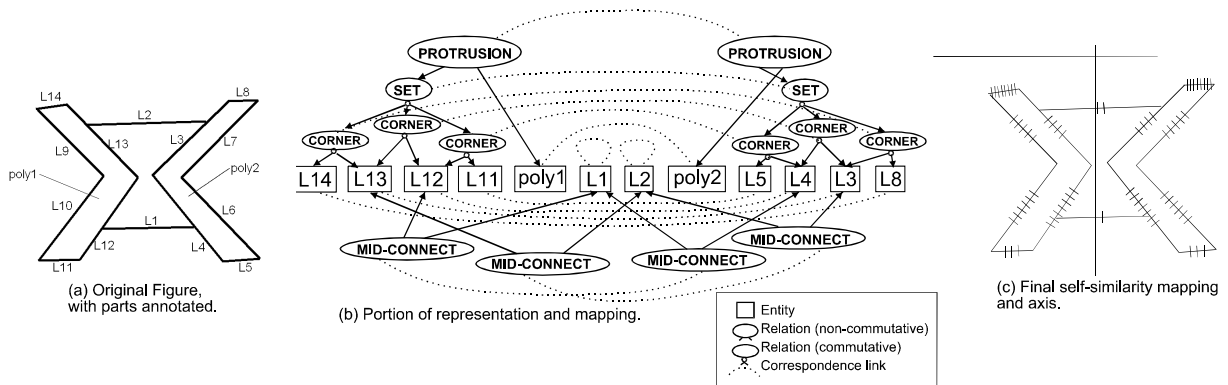


Figure 1: MAGI detects symmetry by aligning visual relations. Figure (a) shows a line drawing given to MAGI as a vector graphics file, with its vector elements labeled. Figure (b) shows a subset of the figure's visual relations (12 of 18 entities, 14 of 118 spatial relations) generated for those visual elements. Dotted lines indicate mapping links produced by MAGI. Note that two line segments, L1 and L2, map to themselves. Figure (c) indicates the full set of entity correspondences (using hash marks) and the axis produced by MAGI.

some kinds of figures. Figures with a good "intrinsic axis" (Palmer, 1983; Wiser, 1981) apparently impose their own reference frame, allowing recognition at any orientation.

Preference for multiple symmetries. Symmetry is also judged more quickly and accurately when a figure contains multiple symmetries (Royer, 1981; Wagemans, Van Gool, & d'Ydewalle, 1991). The preference for multiple symmetries is separate from the preference for vertical symmetry, and can produce additive results (Humphrey & Humphrey, 1989; Palmer & Hemenway, 1978).

Orientation effects pose significant challenges for cognitive models of symmetry detection, which have difficulty modeling interactions between symmetry detection and the visual reference frame. Some symmetry detection models, such as the so-called "brushfire" models (Blum & Nagel, 1978; Brady, 1983), do not use the reference frame at all. Other models use the reference frame in a limited sense – for example, utilizing it to find horizontally-aligned dots to link in symmetric dot patterns (Jenkins, 1983; Wagemans et al., 1991). These latter models can partially explain the preference for vertical symmetry by positing that some fixed set of orientations are tried until symmetry detection succeeds. At the same time, these models apply only to dot patterns, and cannot easily be extended to orientation effects found in more complex stimuli, such as polygons. More problematic, however, is that these models cannot explain how figures with good intrinsic axes eliminate the preference for vertical symmetry, nor why the order of preferences is first vertical, then horizontal, then diagonal (instead, it is typically assumed that this set of orientations results from either natural selection or perceptual learning in a world rich in vertically-symmetric objects). Finally, because these models assume a fixed orientation for each symmetry-detection attempt, and require exact symmetry, they have difficulty detecting even minor deviations from the assumed set of orientations (e.g., symmetry at a 38° angle).

A clue to resolving this quandary may be found in recent evidence that perceptual relations, such as connectivity relations and boundary characteristics, play a role in symmetry detection. Baylis and Driver (1994) provide evidence that symmetry detection in polygons may depend in part on curvature minima along figure boundaries. Ferguson, Aminoff & Gentner (1996) showed that specific qualitative differences, such as concavity or number-of-vertices mismatches, contributes to the speed and accuracy of symmetry judgments. Wagemans' bootstrap model (Wagemans et al., 1991) uses sets of conjoined "virtual quadrilaterals" to add higher-order structure to symmetric dot patterns, allowing the model to detect skewed symmetry.

If perceptual relations play a role in symmetry detection, they may be linked to orientation effects. Some have suggested (Goldmeier, 1936/1972; Rock, 1983) that the preference for vertical symmetry may be rooted in the phenomenological reversibility (or commutativity) of left-right spatial relations, which is not true of above-below relations. In other words, the preference for vertical symmetry is a product of how spatial relations, rather than symmetry-detection processes, depend on visual orientation. For our purposes, we term this the *horizontal commutativity conjecture*.

In this paper, we use MAGI (Ferguson, 1994; in preparation), our model of symmetry detection, to show why and how the horizontal commutativity conjecture may be true. The resulting explanation avoids at least three problematic assumptions of previous models: 1) that the symmetry detection process must use a set of fixed orientations; 2) that symmetry must be exact; or 3) that symmetry-detection is retinocentric.

This paper is arranged as follows. First, we briefly describe the MAGI model. Then, MAGI is used to explain the preference for vertical symmetry and the effect for intrinsic axes. We then perform an in-depth simulation of a classic study of the orientation effects for multiple and near symmetries (Palmer & Hemenway, 1978). We conclude by discussing the implications of these results, the model's limitations, and possible future research.

The MAGI model of symmetry detection

The basis of the MAGI model (Figure 1) is that *symmetry is like analogy*. Specifically, symmetry may use the same cognitive processes found within other analogical reasoning such as analogy, similarity and memory access. As a result, symmetry may share the flexibility and domain-generality found in these other kinds of analogical reasoning.

MAGI models symmetry detection within the framework of structure mapping. MAGI creates a within-description mapping using the constraints of Structure Mapping Theory (Gentner, 1983) to align similar sets of relational structure. In other forms of analogical reasoning, such as similarity comparison and analogy, the mapping process aligns relations in base and target descriptions. In MAGI's symmetry detection, mapping is performed over a single relational description. MAGI also uses additional mapping constraints to maximize the self-similarity of the mapped portions.

For visual figures¹, MAGI works directly from a vector-based line drawing. To obtain a description of the visual relations in the drawing, MAGI uses GeoRep (Ferguson & Forbus, 2000), a spatial representation engine. GeoRep represents visual relations detected early in perception, including element connectivity (such as corners and intersections), parallel elements, horizontally- and vertically-oriented structure, and protrusions and indentations in the figure boundary. MAGI then performs a self-similarity mapping over this relational description (Figure 1 shows an example of GeoRep's representation and MAGI's mapping).

MAGI's algorithm (see Ferguson, 1994, in preparation) is very similar to the Structure Mapping Engine (SME; Falkenhainer, Forbus, & Gentner, 1989; Forbus, Ferguson & Gentner, 1994). MAGI's self-similarity mappings are created using a local-to-global mapping process that enforces a set of six mapping constraints. Four of these constraints are adopted from SME: 1) the *tiered identity* constraint, which allows only expressions with identical predicates to align; 2) the *one-to-one mapping* constraint; 3) the *parallel*

¹ MAGI can also be used on non-visual stimuli, such as story narratives (Ferguson, 1994) or diagrams containing conceptual as well as visual regularity (Ferguson & Forbus, 1998). However, here we concentrate on visual symmetry alone.

connectivity constraint, which mandates that any aligned expression must also align its arguments; and 4) the *systematicity constraint*, which prefers large interconnected mappings with deep relational structure to smaller or unconnected mappings.

MAGI's final two constraints are specific to symmetry detection. The *limited self-matching* constraint states that an expression or entity may map to itself (i.e., self-match) only when it is the argument of an expression that is not a self-match. In Figure 1, this allows entity L1 to map to itself, because two separate *mid-connect* expressions involving L1 are aligned. The *maximal individuation* constraint encourages mappings that maximize the interconnectivity of each of the two mapped parts, and minimize the interconnectivity of the mapped parts with one another. In Figure 1, this constraint distributes the mapped *mid-connect* expressions to provide maximum entity overlap with other mapped expressions, such as the mapped *protrusion* expressions.

These constraints, as enforced by MAGI, produce one or more symmetry mappings. Each mapping contains a set of aligned entities and expressions and a systematicity score.

In MAGI, as in SME, systematicity is measured using a "trickle-down" structural evaluation mechanism (Forbus & Gentner, 1989). This mechanism gives higher scores to deeper expression matches and to matched entities with many matched superexpressions. For MAGI, this score is an approximate measure of "how symmetric" an object seems. For example, visualize a square and the X-shaped figure from Figure 1. Both figures have perfect geometric symmetry, but to MAGI, the X-shaped figure will have higher systematicity than the square because mapped expressions in the former are deeper and more interconnected than in the latter. Similar effects could be found even if we controlled for equivalent figure size and the number of segments.

A mapping also produces candidate inferences (as in SME) by carrying over unmapped structure that intersects mapped structure. Candidate inferences often indicate qualitative differences between the sides of the figure.

Once MAGI has found a self-similarity mapping, it uses the set of aligned entities to determine the axis. Using a Hough transform voting algorithm (Duda & Hart, 1987), MAGI produces either an axis or an object-centered reference frame for the mapping.

The nature of analogical mapping provides MAGI with a number of useful characteristics not found in other symmetry models. MAGI's symmetry detection is extremely

robust in the face of minor asymmetries and distracters. Symmetry mappings can also indicate qualitative differences between otherwise symmetric figures by producing candidate inferences. Finally, MAGI can link perceptual and conceptual symmetries in diagrams (Ferguson & Forbus, 1998), showing how self-similarity is utilized in perceptual reasoning tasks.

Modeling the preference for vertical symmetry and intrinsic axes

Using the MAGI model, it is possible to test the horizontal commutativity conjecture. We begin by assuming that some visual relations are *orientation-dependent* (such as the *above* relations highlighted in Figure 2-A). Along with having orientation-dependent relations, we also can assume that vertically-oriented visual relations are directed, while horizontally-oriented relations are commutative. There is substantial evidence of just this dichotomy in human visual processing (Rock 1983). Humans often confuse left and right, but seldom confuse up and down.

Now we can see how mapping relational structure affects the produced mapping. Given (A), MAGI produces a vertical symmetry mapping. The vertical mapping is due to the alignment of many orientation-dependent visual relations, including the *above* relations. When the figure is rotated 45° (B) and then remapped, the set of orientation-dependent relations changes with it, and this affects the elements that MAGI aligns. Even though all the visual elements have moved relative to (A), MAGI's mapping of (B) is also vertical due to this new set of orientation-dependent relations. In other words, MAGI exhibits a preference for vertical symmetry.

Note that orientation-dependent visual relations do not dictate the mapping MAGI produces. Orientation-dependent relations are only part of the set of visual relations for any given figure, and for that reason, figures with sufficient visual structure can be mapped at many different orientations.

This explains why some figures may have good intrinsic axes that eliminate the preference for vertical symmetry. Figure 2-C shows MAGI's mapping of one of Wiser's (1981) example figures. Because the visual structure of this figure is distinctive enough to produce a symmetry mapping without orientation-dependent relations, this figure produces an axis at almost any orientation.

How symmetry can adjust the frame of reference

This demonstration, however, only partially answers questions about the nature of orientation effects. If this model is correct, then how does the visual system detect symmetry in figures that neither have a good intrinsic axis nor are oriented vertically? Does the system have to try many different orientations, either serially or in parallel?

No, it doesn't. Instead, MAGI can use the initial partial mapping of a figure to find a potential new reference frame, and then shift the frame of reference to obtain a new representation of the figure. With this new representation, it can then reconstruct the symmetry of the figure as if it was presented in a vertical orientation.

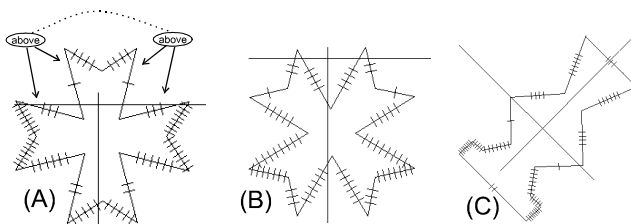


Figure 2: How orientation-dependent relations affect MAGI's symmetry mapping. Vertically-oriented relations in A and B enforce different mappings, even though the figures are identical. The preference for vertical symmetry can be overcome if there is sufficient structure when orientation-dependent relations are absent, as in (C), redrawn from Wiser (1981).

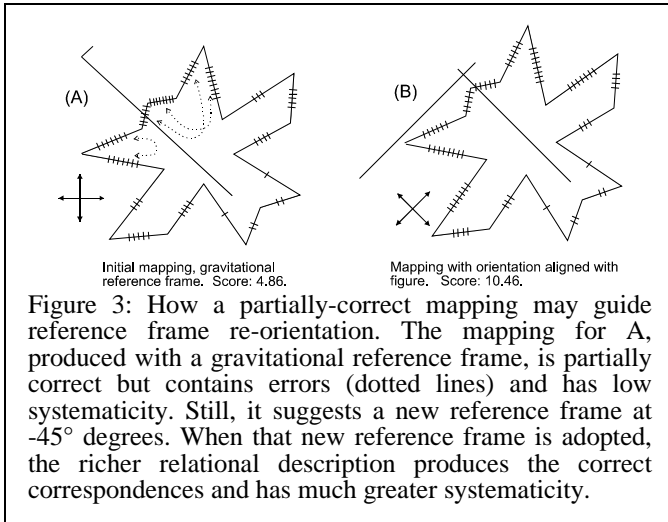


Figure 3: How a partially-correct mapping may guide reference frame re-orientation. The mapping for A, produced with a gravitational reference frame, is partially correct but contains errors (dotted lines) and has low systematicity. Still, it suggests a new reference frame at -45° degrees. When that new reference frame is adopted, the richer relational description produces the correct correspondences and has much greater systematicity.

Figure 3 shows how this may occur. In the original figure (A), the mapping created by MAGI is only partial, and the resulting mapping has low systematicity and some incorrect correspondences. This is because the figure has insufficient visual structure to produce the correct mapping at this orientation (i.e., it does not have a good intrinsic axis). However, this partial mapping is sufficient to produce a potential new orientation for the figure, based on the parts of the mapping that do correspond. When the reference frame for the figure is set at this new orientation (B), the figure can be mapped as if it were at the vertical orientation, producing a richer set of orientation-dependent relations, and an axis is produced. In other words, the partial symmetry mapping tells the system to "tilt its head," and when it does so, it is rewarded by a set of visual relations that lead to a much richer symmetry mapping.

Although we do not yet have a theory of what mapping characteristics lead the viewer to re-orient the visual reference frame given a partial mapping (it may depend on several factors, including the task demands), clearly it is possible for the viewer to shift the reference frame using these clues. As a result, it is possible to see symmetry at an angle without presuming that the symmetry detection process must choose a set of orientations beforehand. One possible characteristic allowing a reference frame shift might be the systematicity of the initial mapping, a factor we return to in the next section.

A Simulation in Depth

We now show the results of a simulation of an experiment (Palmer and Hemenway, 1978) testing both the preference for vertical symmetry and the effect of multiple symmetries.

Palmer and Hemenway's study used a set of 30 stimuli (Figure 4). The figures are 16-gons, containing five different symmetry types: single, double, and quadruple symmetry, rotational symmetry, and near symmetry. These figures were displayed at four different orientations: tilted left (-45°), vertical (0°), tilted right ($+45^\circ$), and horizontal ($+90^\circ$). In the first experiment, subjects had to judge whether the stimulus was mirror symmetric (requiring negative responses for rotational and near symmetry). Response latency and accuracy were measured.

	Quadruple	Double	Single	Near	Rotational
A					
B					
C					
D					
E					
F					

Figure 4: The stimuli used in Simulation 1 (redrawn from Palmer and Hemenway, 1978) arranged by symmetry type.

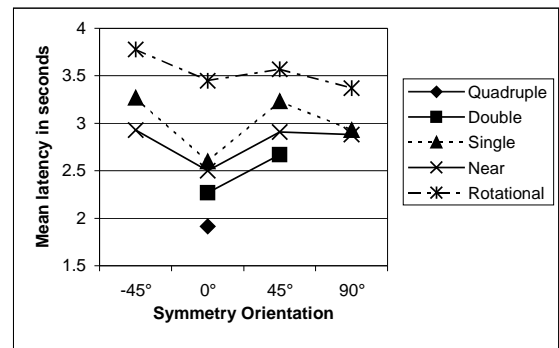


Figure 5: Palmer & Hemenway Experiment 1 results. Graph shows response time latency at four symmetry orientations. Redrawn from Palmer & Hemenway.

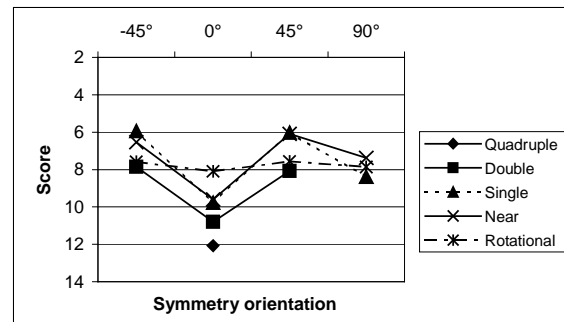


Figure 6: Results of simulating Experiment 1 using MAGI. Graph shows average systematicity score of each figure's best mapping (the Y-axis is inverted for easier comparison to Figure 5). Aside from the rotational symmetry results, MAGI duplicates the experimental results, with consistently higher systematicity scores for figures more quickly detected by human subjects.

Palmer and Hemenway's results (Figure 5) show a clear preference for vertical symmetry, with vertical better than horizontal, and horizontal better than diagonal (Figure 5 shows response latencies – accuracy results were similar). An effect was also found for multiple symmetries, with quadruple better than double, and double better than single symmetry.

For our simulation, the study's 30 stimulus figures were given to MAGI as line drawings. Each figure was presented at up to four orientations, as in the original study. We then used the systematicity score of MAGI's top mapping as a measure of the strength of the relational symmetry.

The results from MAGI are shown in Figure 6. With the exception of rotational symmetry, the results closely mirror those of Palmer and Hemenway, with vertical symmetry having the highest systematicity score, followed by horizontal symmetry and diagonal symmetry. Notably, these effects are reproduced separately for double, single, and near symmetries, as in the original study. MAGI's results also reproduce the effect for multiple symmetries, with quadruple symmetry producing the highest systematicity scores, followed by double symmetry, and then single and near symmetries. These latter two symmetry types produce roughly equal results, as in the original experiment.

The one difference between the two graphs are the results for rotational symmetry. For both MAGI and humans, rotational symmetry results varied only slightly with respect to orientation. However, while rotational figures showed the worst latencies for humans, the systematicity scores MAGI produced are average relative to the other symmetry types. One explanation for this difference, as noted in Palmer and Hemenway's analysis, is that in the original experiments subjects were to accept only mirror-symmetric figures, and thus had to reject rotationally symmetric figures. This means that the high latencies in the original experiment may not be due to a low sense of the figures' symmetry, but because subjects' needed to avoid that sense to produce a negative response. MAGI was not constrained to judge only mirror symmetry, and so frequently found rotational mappings.

We briefly note a second result. In a second experiment, Palmer and Hemenway showed subjects the same 30 figures solely in the vertical orientation, meaning that subjects no longer had to look for symmetry at multiple orientations. This had the effect of greatly decreasing the average response latencies (from a mean of 2626 ms. to 1111 ms.). While accuracy and response time results for quadruple, double, and single symmetry maintained their previous ordering, the error rate for near symmetry shot up from 1.4% to 16.7% from the first to the second experiment, an error rate more than twice the rate for any other symmetry type, while the error rate for rotational symmetry decreased.

The MAGI model suggests a possible explanation. Because the experiment's demand characteristics reduced response time, and because only vertical symmetry was used, it would no longer be necessary to consider partial mappings as indicators of alternative symmetry orientations. Simpler factors, such as the lack of candidate inferences (indicating qualitative asymmetry) might suffice. This strategy is not problematic for quadruple, double, or single symmetries, since exact symmetries do not produce

candidate inferences. Nor is it a problem for rotational symmetries, which always produce candidate inferences. However, near-symmetric figures produce few or no candidate inferences in MAGI. When MAGI was run on the near-symmetric figures, each figure only produced a few candidate inferences and one (in Figure 4's row E) produced none. The relative scarcity of candidates inferences may have made asymmetry detection difficult for near-symmetric figures and lead to subjects' high error rate.

Conclusion

These results demonstrate that a structure-mapping model of symmetry detection can concisely explain orientation effects using a few simple assumptions: 1) that visual structure is at least partially orientation-dependent; 2) symmetry detection is performed by mapping visual structure; and 3) partial mappings are used to find potential mappings and suggest alternate frames of reference. Using this simple model, we simulated the preference for vertical symmetry, showing that the preference for vertical over horizontal symmetry, and for both over diagonal symmetry, was not the result of a pre-established list of potential orientations, but the natural result of a visual system where vertically-oriented relations are phenomenological different than horizontally-oriented relations (the horizontal commutativity conjecture). Similarly, we showed that the preference for multiple symmetries could be modeled with the same assumptions. We showed the correctness of this model by running it on the stimuli of Palmer & Hemenway (1978), which tested both of these effects, and MAGI reproduced the same general pattern of results. Finally, we showed why some figures with good "intrinsic axes" (Palmer, 1983; Wisner, 1981) do not show the same preference for vertical symmetry (an explanation currently beyond the capabilities of other models of symmetry detection). This defined conditions when the sense of symmetry is strong enough to overcome effects of orientation. These collective results suggest that a structure-mapping model of symmetry detection, such as MAGI, could provide a better analysis of a wide variety of symmetry-related phenomena.

There are several limitations with the current model, however. Because the relational mapping depends on the visual relations found in the figure, representation assumptions can drastically change MAGI's results. In the current study, we have attempted to minimize this effect by using GeoRep's default representation engine, which builds a set of relations based on the visual relations assumed to be built by Ullman's universal visual routines (Ullman, 1984). However, further research is needed to test the reliability of these assumptions. MAGI's dependence on spatial relations leaves open the question of exactly when quantitative differences (such as small differences in the angles of corresponding corners) are detected. When such differences exist, but these differences are not qualitative, MAGI does not detect them. Other limitations of GeoRep and MAGI precluded other possible simulations. Because GeoRep does not have a model of grouping, it was not possible to model orientation effects based on grouped items (Palmer, 1983).

This research also creates interesting new questions. The effect for multiple symmetries bears closer analysis. Initial results suggest that the effect is a result of the greater number of visual relations found in figures with multiple symmetries, as well as the greater systematicity of systems with many similar subparts. However, this result should be tested in another domain.

Acknowledgments

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