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CINR Difference Analysis of Optimal Combining Versus Maximal Ratio Combining

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***Abstract*—The statistical gain differences between two common spatial combining algorithms: optimum combining (OC) and maximal ratio combining (MRC) are analyzed using a gain ratio method. Using the receive carrier-to-interference plus noise ratio (CINR), the gain ratio $\mathrm{CINR_{OC}/CINR_{MRC}}$ is evaluated in a flat Rayleigh fading communications system with multiple interferers. Exact analytical solutions are derived for the probability density function (PDF) and the average gain ratio with one interferer. When more than one interferer is present, the PDF of the gain ratio is illustrated using Monte Carlo simulations and its mean value is shown in basic integral form. An upper bound to the gain ratio is derived providing a simple means to determine when OC will exhibit significant gains over MRC.**

***Index Terms*—Antenna arrays, diversity methods, fading channels, interference suppression.**

I. Introduction

THE REALIZABLE performance enhancement using an antenna array to combat specific channel and interference conditions in wireless communication systems is determined by the algorithm chosen to combine the antenna element outputs. While maximal ratio combining (MRC) is optimal in white spatial noise, optimal combining (OC) can provide a higher receive carrier-to-interference plus noise ratio (CINR) in spatially colored interference [1]. Determination of the relative gain using OC versus MRC is desirable as OC demands additional complexity to implement. The average bit-error rate (BER) is the most common metric used to compare different combining algorithms and system configurations. OC and MRC BER were first analyzed for a single interferer and simulated with multiple interferers in [2]. OC BER with multiple interferers was presented in [3] in integral form and recently in [4] as a simplified expression with finite sums. The BER for MRC with multiple interferers is analytically derived and described in [5].

The OC and MRC CINR statistics are also a useful metric for explaining the performance of a given system configuration. In [6], the CINR expression using OC is derived and shown to be dependent on the transformed sum of eigenvalues of a Wishart matrix. An exact expression for the MRC CINR distribution with multiple interferers is obtainable and given in [7]. It was further shown in [8] that a CINR ratio difference analysis provides a valuable metric when comparing two specific combining algorithms such as equal gain combining or MRC to selection combining. In this paper, the CINR ratio method is extended to obtain the relative performance of OC to MRC. The comparison is based upon a computation of the *gain ratio* and is defined equal to $\mathrm{CINR_{OC}/CINR_{MRC}}$.

Initial results analyzing the gain ratio were presented in [9] for the single interferer case and applied to a code-division multiple access (CDMA) voice-data OC versus MRC capacity analysis in [10]. In this paper, the results of [9] are extended by deriving and illustrating more exact expressions for the gain ratio with both single and multiple interfering users in a Rayleigh faded channel. The PDF and mean of the gain ratio are analytically tractable with one interferer. When more than one interferer is present, the PDF of the gain ratio is illustrated using Monte Carlo simulations and the mean gain ratio is shown in basic integral form. OC is illustrated to approach MRC in the limit of a large number of receive antennas. An expression for the gain ratio upper bound is derived providing a simple means to determine when OC will significantly outperform MRC.

The general system model and gain ratio are defined in Section II. The statistical analysis of the gain ratio is developed and discussed in Section III.

II. System Model and Definition of the Gain Ratio

A signal and interference user model is defined in this section for a flat fading multi-user communication system employing a $m = 1 : M$ receive antenna array. Users are assumed uncorrelated with each other and modeled on a power basis to calculate the receive signal CINR.

A. Signal Description and Spatially Combined CINR of the Desired User

The $M \times 1$ received signal vector $\vec{u}(t)$ is defined in (1) as the sum of the desired user signal $s_0(t)$, N interfering user signals, $s_{i=1:N}(t)$, and background noise vector $\vec{n}(t)$. Uncorrelated zero mean complex Gaussian (Rayleigh) $M \times 1$ channel fading vectors with variance equal to one, $\vec{c}$, are assumed for the desired and N interfering user signals

$$\vec{u}(t) = s_0(t)\vec{c}_0 + \sum_{i=1}^{N} s_i(t)\vec{c}_i + \vec{n}(t). \tag{1}$$

The background noise term $n(t)$ is distributed per antenna as a zero-mean complex Gaussian random variable with variance

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σ^2. The desired user signal power is $E[s_0(t)\cdot s_0(t)^*]=\sigma_0^2$. All $i=1:N$ interference users are assumed equal power with $E[s_i(t)\cdot s_i(t)^*]=\sigma_y^2$. The vector and matrix complex conjugate transpose and conjugation operators are defined as $[\cdot]^H$ and $*$.

The CINR of the received signal in (1) is defined in (2) with the interference signal spatial correlation matrix R_I described in terms of C and R_n in (3). C is defined equal to the M antenna row by N interference user column matrix and R_n, the noise spatial correlation matrix, is defined equal to σ^2 times the identity matrix, i.e., $R_n=\sigma^2\cdot I$

$$\text{CINR}=\frac{\vec{w}^H\cdot E\left[s_0(t)\vec{c}_0\vec{c}_0^H s_0(t)^*\right]\cdot\vec{w}}{\vec{w}^H R_I\vec{w}}=\frac{\sigma_0^2\cdot\vec{w}^H\vec{c}_0\vec{c}_0^H\vec{w}}{\vec{w}^H R_I\vec{w}} \quad (2)$$

$$R_I=E\left[\left(\sum_{i=1}^{N}s_i(t)\vec{c}_i+\vec{n}(t)\right)(\cdot)^H\right]=R_n+\sum_{i=1}^{N}\sigma_y^2\vec{c}_i\vec{c}_i^H \equiv\sigma^2\mathrm{I}+\sigma_y^2CC^H. \quad (3)$$

B. Defining the Gain Ratio $Z=CINR_{\mathrm{OC}}/CINR_{\mathrm{MRC}}$

The MRC and OC spatial combining weights $\vec{w}_{MRC}=\vec{c}_0$ and $\vec{w}_{OC}=R_I^{-1}\vec{c}_0$, as defined in [1], are used to specify Z as the ratio of CINRs using OC and MRC weights in (4) as

$$\mathrm{Z}=\frac{\text{CINR}_{\text{OC}}}{\text{CINR}_{\text{MRC}}}=\vec{\tilde{c}}_0^H R_I^{-1}\vec{\tilde{c}}_0\cdot\vec{\tilde{c}}_0^H R_I\vec{\tilde{c}}_0 \quad (4)$$

where $\vec{\tilde{c}}_0$ is the normalized channel vector, $\vec{\tilde{c}}_0=\vec{c}_0/\|\vec{c}_0\|$, and $\|\cdot\|$ is the 2-norm.

Defining the singular value decomposition definition of $C=V\cdot\Sigma\cdot U^H$ and applying the matrix inversion lemma [11] to (4), Z is rewritten to separate terms with common indexes as

$$\mathrm{Z}=1+\frac{1}{b^2}\sum_{i=1}^{N}\left[\frac{\lambda_i^2\cos^2\left(\phi_{\vec{c}_0-\vec{v}_i}\right)\sin^2\left(\phi_{\vec{c}_0-\vec{v}_i}\right)}{b^2+\lambda_i}-\frac{\lambda_i\cos^2\left(\phi_{\vec{c}_0-\vec{v}_i}\right)}{b^2+\lambda_i}\cdot\sum_{\substack{j=1\\j\neq i}}^{N}\lambda_j\cos^2\left(\phi_{\vec{c}_0-\vec{v}_j}\right)\right] \quad (5)$$

where the ratio of antenna noise power to interferer power b^2 and the angle between any two normalized channel vectors, $\vec{c}_i$ and $\vec{c}_j$, are defined in (6) and (7) as

$$b^2=\frac{\sigma^2}{\sigma_y^2} \quad (6)$$

$$\cos\left(\phi_{\vec{c}_i-\vec{c}_j}\right)\equiv\frac{\vec{c}_i^H\vec{c}_j}{\|\vec{c}_i\|\cdot\|\vec{c}_j\|}=\vec{\tilde{c}}_i^H\vec{\tilde{c}}_j. \quad (7)$$

The columns of the square unitary matrices V and U are equal to the right and left singular vectors of C. Σ is the $M\times N$ diagonal matrix with singular values along its diagonal with rank $N\leq M$. The eigenvalues of CC^H are jointly distributed eigenvalues of a Wishart matrix [6], [12], [13]. The vector angle between the desired user channel vector direction and V is defined using (7) with $\vec{\tilde{c}}$ and $\cos(\phi)$ independent of $\|\vec{c}\|$ for Rayleigh fading channels [14].

As expected, OC is lower bounded by MRC in (5) with $Z_{\min}=1$ and $Z>1$ dependent upon b^2 and $\phi_{\vec{c}_0-\vec{v}_i}$. When $\vec{c}_0$ and $\vec{c}_i$ are all 0° or 90° apart, OC and MRC will perform equivalently (5).

III. Statistical Analysis of Z

The PDF and mean of Z (and its upper bound) given M, N, and b^2, are developed in this section. While the statistics of each variable component in the PDF of Z, $f_Z(z)$ will be identified, an exact closed form solution for $f_Z(z)$ is difficult to obtain due to the combination of its components. The mean of Z, $E[Z]$ depends upon the eigenvalues of a Wishart matrix and is, in general, left in integral form. Monte Carlo simulations are used, where applicable, to illustrate $f_Z(z)$ and $E[Z]$.

A. PDF of Z

The PDF of Z, $f_Z(z)$, is dependent upon the eigenvalue joint PDF and the PDF of $\cos^2(\phi_{\vec{c}_0-\vec{v}_i})$. In [9] and [14], the PDF of $\cos^2(\phi_{\vec{c}_0-\vec{c}_1})$, $f_\Phi^M(\phi_{\vec{c}_0-\vec{c}_1})$, is shown distributed as a Beta function with integer values $p=1$ and $q=M-1$, where the general Beta function PDF [15] is equal to

$$f_x^{p,q}(x)=\frac{(p-1)!\cdot(q-1)!}{(p+q-1)!}x^{p-1}\cdot(1-x)^{q-1}\quad\text{where } 0\leq x\leq 1. \quad (8)$$

Supporting both when $M>N$ and $M<N$ and using notation in [13], we define $N_{\min}=\min[M,N]$ and $N_{\max}=\max[M,N]$ to obtain the eigenvalue joint PDF [6], [12] as

$$f_\lambda\left(\lambda_1,\lambda_2\ldots\lambda_{N_{\min}}\right)=\frac{1}{\prod_{i=1}^{N_{\min}}(N_{\min}-i)!\cdot\prod_{i=1}^{N_{\min}}(N_{\max}-i)!}\cdot\prod_{i=1}^{N_{\min}}e^{-\lambda_i}\lambda_i^{N_{\max}-N_{\min}}\cdot\prod_{i=1}^{N_{\min}-1}\left(\prod_{j=i+1}^{N_{\min}}(\lambda_i-\lambda_j)^2\right). \quad (9)$$

Given $M\leq N$, CC^H is full rank with M Wishart distributed eigenvalues (9). When $N<M$, CC^H has N Wishart distributed eigenvalues plus $(M-N)$ zero (null) eigenvalues. Without loss of generality, $N<M$ is assumed for illustration in the analysis that follows.

Evaluating Z in (5) with $N=1$ yields $\mathrm{Z}_{N=1}$ with $\lambda_1=\|\vec{c}_1\|^2$ and $\vec{\tilde{c}}_1=\vec{v}_1$ as

$$\mathrm{Z}_{N=1}=1+\cos^2\left(\phi_{\vec{c}_0-\vec{c}_1}\right)\cdot\sin^2\left(\phi_{\vec{c}_0-\vec{c}_1}\right)\cdot\frac{\|\vec{c}_1\|^4}{b^2\cdot(b^2+\|\vec{c}_1\|^2)}\equiv 1+A\cdot B\cdot C. \quad (10)$$

$\mathrm{Z}_{N=1}$ in (10) is described in terms of three independent random variables A, B, and C to allow determination of $f_{Z_{N=1}}(z)$ in terms of the PDFs of A, B, and C. Using (8), the PDFs of A and B are equal to $f_A(a)=f_X^{1,M-1}(x)$ and $f_B(b)=f_X^{M-1,M-1}(x)$. The PDF of C is defined setting $N=1$ in $f_C^N(c)=1/(c^2\cdot b^2\cdot(N\cdot M-1)!)\cdot(1/(A(c)+1))\cdot(2b^2/A(c))^{N\cdot M+1}\cdot\exp^{(-2b^2/A(c))}$ with $A(c)=-1\sqrt{1+4/c}$ ($f_C^N(c)$ is also used in Section III-C for Z^U).

Defining the variable $D=\ln(A)+\ln(B)+\ln(C)$, $f_{Z_{N=1}}(z)$ is solved for in (11) as

$$f_{Z_{N=1}}=\frac{1}{z-1}\cdot f_{D_{N=1}}\left(\ln(z-1)\right)\cdot\mathrm{u}(\mathrm{z}-1)$$
$$\text{where } \mathrm{u}(\mathrm{z}-1)\text{ is the unit step operator} \quad (11)$$

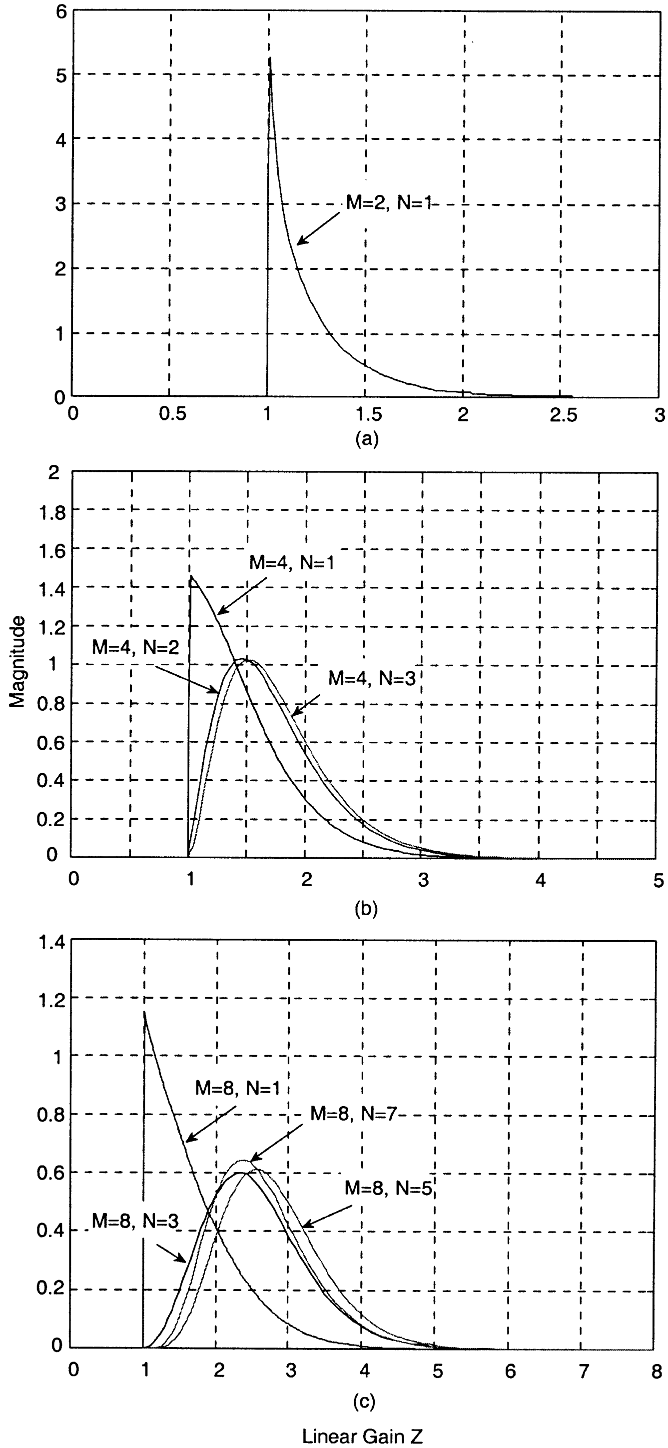

Fig. 1. PDF of Z for N interferers at $1/b^2 = 1.0$: (a) $M = 2$, (b) $M = 4$, and (c) $M = 8$.

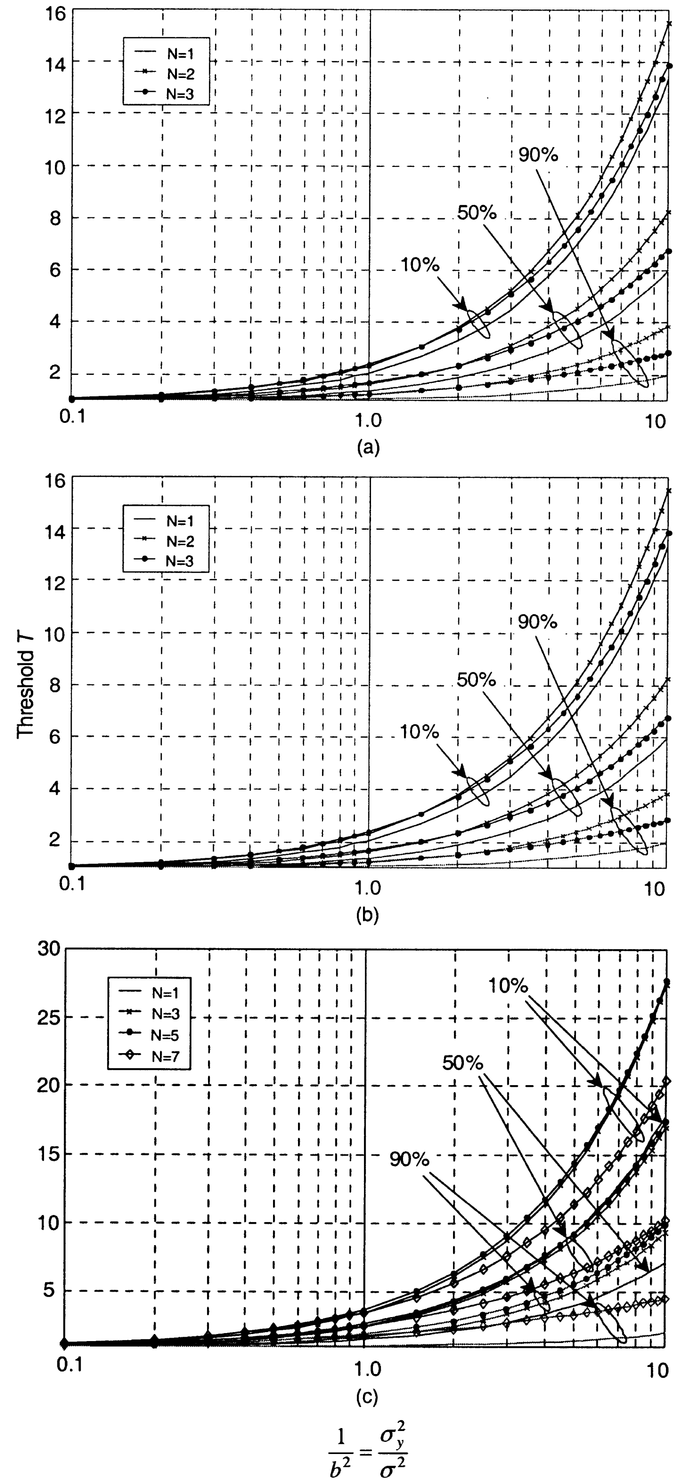

Fig. 2. $P_{\text{Outperform}}$: 10%, 50%, and 90% for threshold T versus $1/b^2$ with: (a) $M = 2$ and $N = 1$, (b) $M = 4$ and $N = 1, 2,$ and 3, and (c) $M = 8$ and $N = 1, 3, 5,$ and 7.

and $f_D(d) = \exp^d \cdot f_A(\exp^d) * \exp^d \cdot f_B(\exp^d) * \exp^d \cdot f_C(\exp^d)$ is determined using random variable transformation theory [16].

Determination of $f_Z(z)$ with arbitrary N is difficult to obtain in closed form. The statistics of each variable component in $f_Z(z)$ can be identified but the combination of these components complicates the analysis. Monte Carlo simulations are used, in general, to illustrate $f_Z(z)$ using the system model described in Section II. Each time index in the Monte Carlo simulation is a new Rayleigh fading channel occurrence. CINR samples using OC weights and MRC weights are generated and used to obtain simulated statistics for Z. The total number of iterations per simulation normally output 1–5 million data points to ensure accurate representation of true metrics. Simulations for the PDF of Z used at least 200–5000 data points per PDF bin.

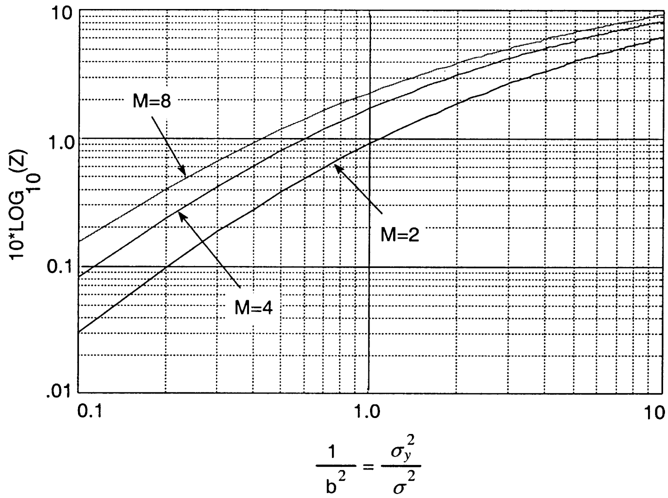

Fig. 3. E[Z] versus $1/b^2$ for $M = 2, 4$, and 8 with $N = 1$ (single interferer).

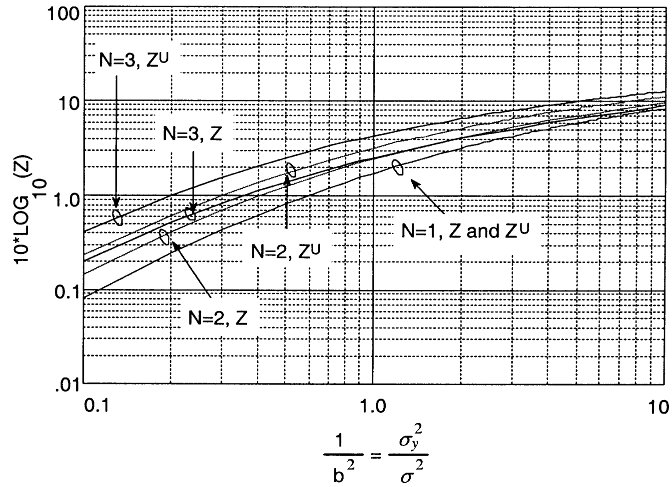

Fig. 4. $E[Z]$ and $E[Z^U]$ versus $1/b^2$ for $M = 4$ with $N = 1, 2$ and 3.

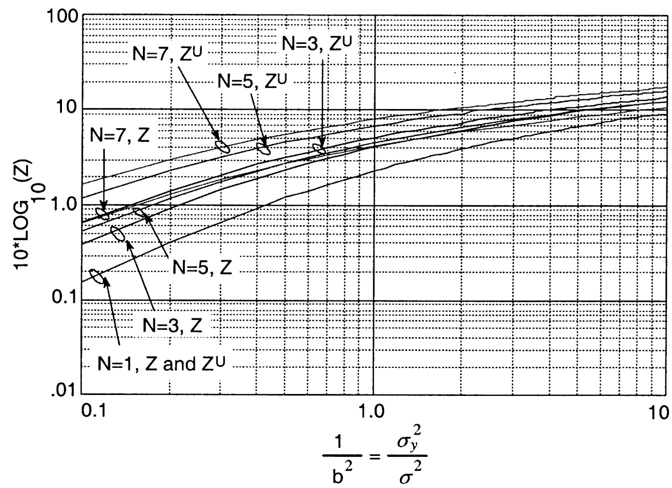

Fig. 5. $E[Z]$ and $E[Z^U]$ versus $1/b^2$ for $M = 8$ with $N = 1, 3, 5$, and 7.

$f_Z(z)$ is shown in Fig. 1 using Monte Carlo simulations for $M = 2, 4$, and 8 at $1/b^2 = 1.0$. $f_Z(z)$ exhibits a chi-square random variable χ^2_{2N}-like distribution dependent upon M and N. When $1/b^2 \ll 1$, $f_Z(z)$ was seen as characteristic of a delta function at $Z = 1$. Empirically, a near delta function-like region was seen for M equal to 2, 4, and 8 at $1/b^2$ less than 1.0, 0.8, and 0.5. The probability that Z is greater than or equal to a threshold T, i.e., $\int_T^\infty f_Z(z)dz$ is defined as $P_{\text{Outperform}}$ and illustrated in Fig. 2 for $M = 2, 4$, and 8 versus T and $1/b^2$ values. $P_{\text{Outperform}}$ illustrates that OC rarely outperforms MRC when $1/b^2 < 1$. When $1/b^2 > 2$, OC outperforms MRC by 3 dB at least 10% of the time. OC outperforms MRC 90% of the time only at large values for $1/b^2$.

B. Mean Value of Z

The mean of Z, $E[Z]$, is dependent upon the joint eigenvalue distribution (9) and the mean of $\cos^2(\phi_{\vec{c}_0-\vec{v}_i})$. The $E[\cos^2(\phi_{\vec{c}_0-\vec{c}_1})] = 1/M$ is obtained by evaluating the first moment of (8) illustrating that in the limit of large M, $\phi_{\vec{c}_0-\vec{c}_i} \Rightarrow 90°$ due to the higher vector space dimensionality.

The mean of Z at $N = 1$, $E[Z_{N=1}]$ is solved for in (12) as a function of $Ei(n, x)$ $\int_1^\infty (\exp(-x \cdot t)/t^n)dt$ as

$$E\left[Z_{N=1}(M, b^2)\right] = 1 + \frac{(M-1)}{b^2 \cdot (M+1)!} \cdot \left(\sum_{r=0}^{M} (-1)^r (b^2)^r (M-r)! + (-1)^{M+1} (b^2)^{M+1} \exp(b^2) \cdot Ei(1, b^2) \right). \quad (12)$$

Fig. 3 plots (12) for $M = 2, 4$, and 8 illustrating larger $E[Z_{N=1}]$ for increasing M and $1/b^2$.

The solution for $E[Z]$ with arbitrary N is described in integral form dependent upon the eigenvalue joint PDF in (13), located at the bottom of the page.

An upper bound to Z, $Z_{\max}$, is derived using Kuhn–Tucker conditions in Appendix A offering a simple expression to determine maximum possible gains using OC versus MRC. Additionally, an analytically tractable upper bound to $Z_{\max}$ is defined in Appendix A as Z^U.

The Monte Carlo simulations used to analyze $f_Z(z)$ were also used to analyze $E[Z]$. Simulations for $E[Z]$ used at least 500–20 000 data points per calculated value. Increased E$[Z]$ and $E[Z^U]$ (14) versus $1/b^2$ is illustrated in Figs. 4 and 5 for $M = 4$ and 8 for different N. Note Z^U is equal to Z at $N = 1$. Given a fixed M, decreasing relative gains in $E[Z]$ are illustrated for increasing N. At small M, the increase in $E[Z]$ due to more antennas is greater than the loss associated with channel vector angle difference overlap $(E[\cos^2(\phi_{\vec{c}_0-\vec{c}_1})])$. In the limit of large

$$E[Z] = \int_0^\infty \int_0^{\lambda_1} \cdots \int_0^{\lambda_{N-1}} \left\{ 1 + \frac{1}{M \cdot b^2} \times \sum_{i=1}^{N} \left[\left(\frac{M-1}{M+1} \right) \frac{\lambda_i^2}{b^2 + \lambda_i} - \left(\frac{1}{M} \right) \frac{\lambda_i}{b^2 + \lambda_i} \cdot \sum_{\substack{j=1 \\ j \neq i}}^{N} \lambda_j \right] \right\} \cdot f_\lambda(\lambda_1, \lambda_2, \ldots \lambda_N) d\lambda_N \ldots d\lambda_2 d\lambda_1. \quad (13)$$

$$E\left[Z^U(M,N,b^2)\right] = 1 + \frac{(M-1)}{b^2 \cdot M \cdot (M+1) \cdot (N \cdot M - 1)!} \cdot \left(\sum_{r=0}^{N \cdot M} (-1)^r (b^2)^r (N \cdot M - r)! + (-1)^{N \cdot M+1} \times (b^2)^{N \cdot M+1} \exp(b^2) \cdot Ei(1,b^2) \right). \quad (14)$$

$M, \phi_{\vec{c}_0-\vec{c}_1} \Rightarrow 90^\circ$, and per (5) OC will on average perform near equal to MRC as $E[Z] \Rightarrow 1$.

IV. Conclusion

The ratio of CINRs, *gain ratio*, was analyzed for OC and MRC algorithms to determine their relative performance differences. A multiple receive antenna system with a flat Rayleigh faded channel and multiple interferers was assumed. The PDF and mean of the gain ratio were analyzed and exact solutions were obtained for a single interferer and the gain ratio upper bound. Simulations were used to illustrate the gain ratio with multiple interferers.

The statistical distribution of the difference of the desired and interference users' channel direction and also the interferer user to background noise power ratio determines OC versus MRC relative gains. OC rarely outperforms MRC when the ratio of interferer power to background noise power is less than one. When the ratio of interferer to background noise power is greater than two, OC outperforms MRC by 3 dB at least 10% of the time. OC outperforms MRC by 3 dB for more than 90% of the time only at very large ratios of interferer power to background noise power. In the limit of a large number of receive antennas, the performance of OC approaches that of MRC for all levels of interference. The derived gain ratio upper bound expression was shown to provide a simple means to determine the maximum OC versus MRC difference attainable.

Appendix A
Maximum Z and Upper Bound to $Z_{\max}$

The maximum of Z, $Z_{\max} = 1+(1/4)\cdot[\lambda_1^2/b^2\cdot(b^2+\lambda_1))]$, is derived using the Kuhn–Tucker conditions [17] with constraints $\sum_{i=1}^{M} \cos^2(\phi_{\vec{c}_0-\vec{v}_i}) = 1$ and $\cos^2(\phi_{\vec{c}_0-\vec{v}_i}) \geq 1$ where $\|\vec{c}_1\|^2 \leq \lambda_1 \leq \sum_{i=1}^{N} \|\vec{c}_i\|^2$. Given $\|\vec{c}_1\|^2 = 1$, $N = 1$, and $b^2 = 1$, for example, $Z_{\max}$ gives a 1.125 (0.51 dB) maximum gain using OC versus MRC.

$Z^U(\phi)$ is defined as an analytically tractable upper bound to $Z_{\max}$ at $\lambda_1 = \sum_{i=1}^{N} \|c_i\|^2$ with random angle $\phi_{\vec{c}_0-\vec{v}_1}$. The $E[Z^U]$ is defined in (14), located at the top of the page, with $f_{Z^U,N}(z)$ equal to (11) with N interferers for $f_C^N(c)$.

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