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ABSTRACT

Predictions of the Fermi and Lepore-Neuman statistical theories are compared with experimental results for pion production in 2.7- and 5.3-Bev p-p collisions, and for antiproton annihilation. The Fermi theory cannot be made to agree with all experiments, but a choice of 0.26 for the Lepore-Neuman cutoff parameter gives good agreement, subject to uncertainties in both theory and experiment.

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INTRODUCTION

The statistical model employed here is the one proposed by Lepore and Neuman.[1] This model differs from the Fermi[2] model in that a smeared-out interaction volume with an energy-dependent radius replaces the constant interaction volume of the Fermi theory. Thus in the Fermi theory the probability of a reaction leading to a final state containing n particles is proportional to

$$\Omega^{n-1} \int \prod_{i=1}^{n} d^3p_i \ \delta\Big(\sum_{j=1}^{n} \vec{P}_j\Big) \ \delta\Big(E - \sum_{k=1}^{n} \epsilon_k\Big) , \qquad (1)$$

and in the Lepore-Neuman theory is proportional to

$$\int \prod_{i=1}^{n} d^3p_i \ d^3x_i \ e^{-\lambda_i G_i^2 x_i^2} \ \delta\Big(\sum_{j=1}^{n} \vec{P}_j\Big) \ \delta\Big(E - \sum_{k=1}^{n} \epsilon_k\Big) \ \delta\Big(\sum_{\ell=1}^{n} \epsilon_\ell x_\ell\Big) \qquad (2)$$

In both Eqs. (1) and (2) the total energy is E, and total momentum is zero; in Eq. (1) the interaction volume is designated by Ω. In Eq. (2) the configuration space integrals are taken over all space for the coordinates of each of the particles, subject to the condition imposed by the last delta function, which requires that the center of mass remain at rest. The weight function $e^{-\lambda_i \epsilon_i^2 x_i^2}$ determines the effective interaction volume for the ith particle. The linear dimensions of this volume are of order $\lambda_i^{-\frac{1}{2}} \epsilon_i^{-1}$, so that each particle has its own effective volume, and the volume is energy-dependent. The parameters λ_i are assumed to be independent of energy, but may be different for different particles. They are regarded as adjustable parameters of the theory, and hence play a role analogous to Ω in the Fermi theory. We shall always work in a system of units in which $\hbar = c = 1$, so that energy has the dimensions of inverse length, and the λ_i are dimensionless.

Arguments in favor of the integrals in Eq. (2) may be found in Reference 1, as well as criticisms of the integrals of Eq. (1). Objections may be raised to both forms of the theory, and their relative merits must be decided by comparison with experiment. There is, however, one difficulty common to both models that should be discussed: no ready interpretation of the integrals relating to elastic scattering is available. To see this most clearly, imagine that the integrals have been performed for some very high energy, i.e., some energy well above threshold for a large number of processes. The elastic integral will then be small compared to the sum of the integrals corresponding to all other processes, which would indicate a vanishing elastic cross section. This cannot be true, however, because associated with the inelastic events there must be elastic diffraction scattering. The root of the trouble is obviously that the theory is essentially classical, dealing as it does only with probabilities, whereas the diffraction scattering is a purely quantum-mechanical effect.

From the last argument it is clear that the ratio of elastic to inelastic cross sections is not contained in the theory, therefore a separate assumption is required. For scattering at very high energies, it seems plausible to assume that the smallness of the elastic integral simply indicates that if the particles interact at all, the interaction is inelastic, so that the interaction region is "black", and the elastic cross section is equal to the total inelastic cross section.

At lower energies the elastic integral becomes comparable to the inelastic integrals, and can no longer be neglected. One could define in some way a diffraction amplitude and a reaction amplitude, but at most only the magnitudes of such quantities could be estimated, whereas their relative phases must also be known for prediction of a cross section. Therefore it seems that the most the statistical theory can say concerning elastic cross sections is that at high energies the elastic-to-inelastic ratio should approach unity; even this is only the simplest assumption to make, and not really a prediction.

PHASE SPACE INTEGRALS

In the Fermi[3] theory the relative probability for producing a final state containing n particles is[4]

$$P_n^F = \Omega^{n-1} (2\pi)^{-3(n-1)} S_n \, T_n \, I_n \, . \tag{3}$$

The corresponding probability of producing the same state according to the Lepore-Neuman theory is

$$P_n^{LN} = (2\pi)^{-3(n-1)} E^3 S_n \, T_n \, J_n' \, . \tag{4}$$

The total energy in the center-of-mass rest system is E, and T_n is the i-spin weight factor. If the state contains several kinds of identical particles, say v_1 of the first kind, v_2 of the second, etc., we have

$$S_n = (v_1! \; v_2! \; \ldots)^{-1} \, . \tag{5}$$

If a and b are two members of an isotopic multiplet, the states ab and ba are treated as indistinguishable in calculating T_n, therefore members of an isotopic multiplet must be treated as identical particles in calculating S_n. Finally

$$I_n = \int \prod_{i=1}^{n} d^3 p_i \; \delta\Big(\sum_{j=1}^{n} \vec{P}_j\Big) \; \delta\Big(E - \sum_{k=1}^{n} \epsilon_k\Big) \tag{6}$$

and

$$J_n' = \int \prod_{i=1}^{n} d^3 p_i \; d^3 x_i \; e^{-\lambda_i \epsilon_i^2 x_i^2} \; \delta\Big(\sum_{j=1}^{n} \vec{P}_j\Big) \; \delta\Big(E - \sum_{k=1}^{n} \epsilon_k\Big) \times \delta\Big(\sum_{\ell=1}^{n} \epsilon_\ell \vec{x}_\ell\Big) \, . \tag{7}$$

The space integrals in Eq. (7) are easily performed, and yield

$$J_n' = \pi^{3(n-1)/2} \prod_{i=1}^{n} \lambda_i^{-3/2} \left[\sum_{j=1}^{n} \lambda_j^{-1} \right]^{-3/2}$$

$$\int \prod_{i=1}^{n} d^3p_i \; \epsilon_i^{-3} \; \delta\Big(\sum_{j=1}^{n} \vec{p}_j\Big) \, \delta\Big(E - \sum_{k=1}^{n} \epsilon_k\Big) \; .$$

If we define

$$\Lambda_n = \prod_{i=1}^{n} \lambda_i \sum_{j=1}^{n} \lambda_j^{-1} \tag{8}$$

and

$$J_n = \int \prod_{i=1}^{n} d^3p_i \; \epsilon_i^{-3} \; \delta\Big(\sum_{j=1}^{n} \vec{p}_j\Big) \; \delta\Big(E - \sum_{k=1}^{n} \epsilon_k\Big) \; , \tag{9}$$

then Eq. (4) becomes

$$P_n^{LN} = (4\pi)^{-3(n-1)/2} \; E^3 \; S_n \; T_n \; \Lambda_n^{-3/2} \; J_n \; . \tag{10}$$

Tables of the integrals I_n have been prepared by Fialho,[5] who used a saddle-point approximation. All I_n values used here are taken from Fialho's tables. The integral J_2 can be performed analytically, while J_3 and J_4 have been reduced to single and double integrals, respectively, and an exact numerical integration over the remaining variables performed. Only integrals describing four particles or fewer have been evaluated exactly. Since higher multiplicities are involved in both annihilation and nucleon-nucleon collisions at several Bev, an approximation to the higher-multiplicity integrals is desirable. A method for finding a lower limit to these integrals is described below.[6]

The integrals I_n and J_n differ only by the factor of $\prod_i \epsilon_i^{-3}$ in the denominator of the integrand of J_n. This factor is a maximum for a fixed total energy when all the ϵ_i are equal. For nucleon-nucleon collisions, this equipartition may imply an energy greater than the total available energy, because of the large nucleon rest mass. In this case the product is a maximum when the kinetic energy is divided equally among the light particles only. A lower limit on J_n can be found by replacing $\prod_i \epsilon_i^{-3}$ by it maximum value. We define

$$K_n = (\bar{\epsilon})^{-3\nu_\ell} \; I_n \; , \tag{11}$$

where ν_ℓ is the number of light particles, and

$$\bar{\epsilon} = \frac{E - \nu_N M_N}{\nu_\ell} \tag{12}$$

with ν_N the number of nucleons, and M_N the nucleon rest mass. Then as shown above, we have

$$J_n > K_n \; . \tag{13}$$

Values of S, T, I, $\bar{\epsilon}$, and K for a number of process are presented in Tables I and II.

From Table II K_n is seen to be always smaller than J_n , as it must be. In annihilation, $K \sim J/2$ for both three-pion and four-pion annihilation. We expect the ratio of J to K to increase with increasing multiplicity, as it does in all cases where a comparison is possible. Therefore we assume that J can be estimated for five-, six-, and seven-pion annihilation also by $J \sim 2K$, and we expect this to

Table I. Values of S and T for various states. Nucleons are denoted by N, K and Λ, and Σ are assumed to have i spin $\frac{1}{2}$, 0, and 1 respectively.

State	S	T $i = 0$	T $i = 1$
2N	1/2	1	1
$2N + \pi$	1/2	1	2
$N + K + \Lambda$	1	1	1
$N + K + \Sigma$	1	1	2
$2N + 2\pi$	1/4	2	4
2N + 2K	1/4	2	3
$2N + 3\pi$	1/12	4	9
2π	1/2	1	1
2K	1/2	1	1
3π	1/6	1	3
$2K + \pi$	1/2	1	2
4π	1/24	3	6
$2K + 2\pi$	1/4	2	4
5π	1/120	6	15
6π	1/720	15	36

Table II. Values of I_n, J_n, and K_n for various states. Units are such that $M_\pi = 1$, except for collision values of J_n and K_n where $M_n = 1$. J_n (at 5.3 Bev) has been obtained by interpolation. *Approximate values of J_n obtained from K_n by Eqs. (14), (15), and (16).

	I_n			K_n		
State	2.7 Bev	5.3 Bev	Annihilation at rest	2.7 Bev	5.3 Bev	Annihilation at rest
2N	5.76×10^2	10^3				
2Nπ	2.01×10^5	1.66×10^6		1.95×10^1	1.07×10^1	
NKΛ						
NKΣ						
2N2π	8.88×10^6	3.01×10^8		6.41×10^1		
2N2K						
2N3π		1.54×10^{10}			6.37×10^2	
2π			3.15×10^2			
2K						
3π			1.30×10^5			1.62×10^{-1}
2Kπ						
4π			1.29×10^7			5.45
2K2π						
5π			3.23×10^8			9.85×10^1
6π			5.36×10^9			2.09×10^3
7π			8.89×10^9			7.66×10^3

(Continued)

Table II, Continued.

	J_n							
							Annihilation	
State	0.8 Bev	1.5 Bev	2.7 Bev	4.5 Bev	5.3 Bev	6.3 Bev	At Rest	500 Mev
2N	1.69	1.30	8.13×10^{-1}	4.53×10^{-1}	3.63×10^{-1}	2.91×10		
$2N\pi$	1.18×10^{11}	2.16×10^{1}	2.34×10^{1}	1.88×10^{1}	1.60×10^{1}	1.45×10^{1}		
$NK\Lambda$	0	0	3.48	6.56	6.69	6.70		
$NK\Sigma$	0	0	2.26	5.58	5.8	6.03		
$2N2\pi$	3.66	1.10×10^{2}	4.28×10^{2}	4.31×10^{2}	4.28×10^{2}			
2N2K	0	0	5.15×10^{-2}	1.76	3.4	4.63		
$2N3\pi$					*3.2×10^{3}			
2π							2.91×10^{-3}	1.83×10^{-3}
2K							2.51×10^{-3}	1.64×10^{-3}
3π							2.67×10^{-1}	1.80×10^{-1}
$2K\pi$							8.32×10^{-2}	6.86×10^{-2}
4π							1.05×10^{1}	7.93
$2K2\pi$							1.09	1.27
5π							*2 $\times10^{2}$	
6π							*4 $\times10^{3}$	

underestimate J. In a similar way we find $K \sim .6\ J$ for single pion production, and $K \sim J/5$ for double pion production. We assume as before that $J \sim 5K$ gives a lower limit on J for triple pion production. The approximations used for higher-multiplicity integrals are then

$$J(2N\ \ 3\pi) \sim 5K(2N\ \ 3\pi) \tag{14}$$

for production, and

$$J(5\pi) \sim 2K(5\pi) \quad , \tag{15}$$

$$J(6\pi) \sim 2K(6\pi) \quad , \tag{16}$$

$$J(7\pi) \sim 2K(7\pi) \tag{17}$$

for annihilation. Results obtained from Eqs. (14), (15), and (16) are contained in Table II.

MULTIPLE PION PRODUCTION

The results of the last section will now be applied to multiple pion production at 2.7 and 5.8 Bev.[7,8] Since the Fermi theory contains only one adjustable parameter, a comparison of the predicted ratio of double to single production with experiment at two energies constitutes a test of the theory. The Lepore-Neuman theory depends on two parameters, λ_π and λ_N. We reduce the number of parameters by taking all the λ equal. This is the analogue of using a single interaction volume for all particles in the Fermi theory.

The relative probabilities of single, double, and triple production are given in Table III for p-p collisions. Brookhaven experiments at 2.75 Bev yield the ratios

$$P\pi : P2\pi : P3\pi = 2.25 : 3.25 : 1 .$$

The ratio of double to single production implies a volume of $8.2\ M_\pi^{-3}$ or an interaction sphere of radius 1.25 pion Compton wave lengths. With the same λ for all particles, the value $\lambda = 0.26$ is required. These parameters then imply ratios of double to single production of 4.8 and 2.5 for the Fermi and LN theories, respectively at 5.3 Bev. The Lorentz contraction factor $\frac{2\ M_N}{E}$ has been included in deriving the above ratio for the Fermi theory.

The p-p data at this energy are incomplete, but a ratio of 4.8 seems definitely too large, and even 2.5 may be too large. However, we have overlooked the effect of final-state interactions, which are believed to be important in the 2 to 3-Bev region.[9,10,11] If such effects exist, and become less important with increasing energy as one would expect,

Table III. Relative probabilities of single, double, and triple pion production in p-p collisions. A common factor E^3 has been omitted from the P^{LN}.

T_{LAB} (Bev)	$P^F(2N\pi)$	$P^F(2N2\pi)$	$P^F(2N3\pi)$	$P^{LN}(2N\pi)$	$P^{LN}(2N2\pi)$	$P^{LN}(2N3\pi)$
2.7	$3.29\Omega^2$	$0.578\Omega^3$		$1.19 \times 10^{-3}\Lambda_3^{-3/2}$	$3.59 \times 10^{-3}\Lambda_4^{-3/2}$	
5.3	$27.1\Omega^2$	$19.9\Omega^3$	$3.08\Omega^4$	$9.50 \times 10^{-3}\Lambda_3^{-3/2}$	$4.86 \times 10^{-3}\Lambda_4^{-3/2}$	$6.13 \times 10^{-4}\Lambda_5^{-3/2}$

smaller Ω and larger λ would be required for the 2.7-Bev data, and hence the predicted ratio at 5.3 Bev would be decreased. In any case, the available data seem to favor the LN theory over the Fermi theory for pion production.

ANTIPROTON ANNIHILATION

The results of the two theories for antiproton annihilation into pions are collected in Tables IV and V. In Table V the normalized probabilities for annihilation at rest are given for the parameters deduced from the 2.7-Bev data. The observed mean multiplicity is about 5.5.[12] Clearly with $\Omega = 8.2$ the Fermi theory predicts too small multiplicities. With $\lambda = 0.26$ the LN theory gives multiplicities of about the right order of magnitude. The probability of 7π is somewhat smaller than the value one would expect from extrapolation from lower multiplicities. This is believed to be caused by the decrease in accuracy of the approximation $J \sim 2K$.

In preparing Tables IV and V we have neglected the conditions imposed by invariance under charge conjugation.[13] If some special cases at low multiplicities are neglected, the main effect of this invariance property is the requirement that a completely specified state of a nucleon and antinucleon must decay into either an even or an odd number of pions, i.e., mixtures of even and odd numbers of pions are forbidden. This will tend to decrease the multiplicites relative to the values of Table V if the probabilities of seven- and eight-pion annihilation are small compared to those for three and four pions, respectively, since, e.g., a state that can decay into five pions cannot decay into four or six.

Table IV. Relative probabilities of various annihilation modes for the two theories, as functions of their respective parameters.

	P^F		P^{LN}		P^F	P^{LN}
n	$\bar{p}n$	$\bar{p}p$	$\bar{p}n$	$\bar{p}p$	$\bar{p}(p + 1.2n)$	$\bar{p}(p + 1.2n)$
2	$6.34\times10^{-1}\Omega$	$6.34\times10^{-1}\Omega$	$1.16\times10^{-5}\lambda^{-3/2}$	$1.16\times10^{-5}\lambda^{-3/2}$	$6.34\times10^{-1}\Omega$	$1.16\times10^{-5}\lambda^{-3/2}$
3	$1.06\Omega^2$	$7.1\times10^{-1}\Omega^2$	$1.30\times10^{-5}\lambda^{-3}$	$8.68\times10^{-6}\lambda^{-3}$	$9.00\times10^{-1}\Omega^2$	$1.11\times10^{-5}\lambda^{-3}$
4	$2.14\times10^{-1}\Omega^3$	$1.61\times10^{-1}\Omega^3$	$3.73\times10^{-6}\lambda^{-9/2}$	$2.80\times10^{-6}\lambda^{-9/2}$	$1.90\times10^{-1}\Omega^3$	$3.31\times10^{-6}\lambda^{-9/2}$
5	$1.08\times10^{-2}\Omega^4$	$7.55\times10^{-3}\Omega^4$	$5.72\times10^{-7}\lambda^{-6}$	$4.00\times10^{-7}\lambda^{-6}$	$9.32\times10^{-3}\Omega^4$	$4.93\times10^{-7}\lambda^{-6}$
6	$2.89\times10^{-4}\Omega^5$	$2.05\times10^{-4}\Omega^5$	$7.83\times10^{-8}\lambda^{-15/2}$	$5.55\times10^{-8}\lambda^{-15/2}$	$2.50\times10^{-4}\Omega^5$	$6.80\times10^{-8}\lambda^{-15/2}$

Table V. Normalized probabilities of various annihilation modes. The parameters $\Omega = 8.2$ and $\lambda = 0.263$ are those predicted from 2.7-Bev scattering. The annihilation is assumed to have occurred in a nucleus for which the ratio of neutrons to protons is 1.2.

	P^F		P^{LN}	
n	$\Omega = 1$	$\Omega = 8.2$	$\lambda = 1$	$\lambda = .263$
2	36.6	2.2	43.8	1.6
3	51.9	27.2	42.0	11.5
4	10.9	47.2	12.5	25.4
5	0.5	19.0	1.9	28.1
6	~0	4.2	.3	29.1
7				4.3
$\bar{n}$		3.95		4.85

REACTIONS INVOLVING STRANGE PARTICLES

If the assumption is made that all reactions that conserve strangeness are governed by a coupling comparable to the pion-nucleon coupling, one would expect the statistical theory to be applicable to these events, at least as far as it is applicable at all. On this assumption, relative probabilities for production of strange particles can be obtained from Tables I and II for the LN theory. Thus associated K-hyperon production should be about 15% as probable as single pion production at 2.7 Bev, and about 60% as probable at 5.3 Bev. At both energies production of K pairs should be negligible.

The 150 inelastic events at 2.75 Bev contain one definite and two possible examples of associate production. There are 53 single pion events, so that the ratio of associated production to single pion production is $\sim$5%. This is rather low compared with the 15% predicted, but the two are not in definite disagreement, in view of the very small number of events. At 5.3 Bev three strange-particle events are observed in a total of 100, whereas about ten would be expected from the theory--again disagreement by a factor of three.

In annihilation two K particles and two pions are about equally probable, but according to Table V both are very improbable. Thus $2K\pi$ is about one-third as probable as 3π, and $2K2\pi$ about one-fifth of 4π. A very crude estimate based on Fialho's tables indicates that $2K3\pi$ and $4K$ are very improbable, leading to an over-all probability for annihilations involving K particles of about 10%. This estimate is based on Table V with $\lambda = 0.26$, and the previously mentioned effects arising from invariance under charge conjugation have been included. If pion multiplicities of seven and eight are appreciable, which is quite possible in view of the observed mean multiplicity of 5.5, the number of K events should be smaller than that just predicted. Even so the number should still be between 5% and 10% of the total.

In 35 annihilation stars, a total of three involving K particles are observed, giving a ratio of about 8%. This presumably underestimates the actual ratio because of the possibility of neutral K's and K^- absorption. These effects should not increase the ratio to much above 10%, however, so that the agreement between statistical theory and experiment is rather good in annihilation; but again the number of events is too small to provide a stringent check. Thus we conclude that the predictions of the theory for reactions involving strange particles is not in disagreement with experiment, although the theory appears to predict too much associated production.

SUMMARY AND CONCLUSIONS

The Fermi and Lepore-Neuman statistical theories have been compared with the experimental data on p-p collisions at 2.75 and 5.3 Bev, and antiproton annihilation. Charge independence has been assumed throughout. No attempt has been made to include conservation of angular momentum, and energy distributions have not been investigated. No choice of the interaction volume Ω of the Fermi theory can be found that is consistent with all the data. If all the cutoff parameters λ_i of the LN theory are chosen the same and equal to ~ 0.26, theory and experiment are in agreement, so far as both are complete. The choice $\lambda = 0.26$ corresponds to an interaction volume radius of ~ 0.7 pion Compton wave lengths for $n = 5$. The results must be regarded as tentative, since only prong distributions are available at 5.3 Bev, and the calculation has been extended only to pion multiplicities of six for annihilation, while higher multiplicities are probably important. Furthermore, all integrals involving more than four particles have been estimated by means of an approximation of unknown accuracy. Final-state interactions are neglected throughout, but may point to a slightly larger value of λ than 0.26. No comparison has been made here with the pion-production data in np collisions at 2 to 3 Bev, but agreement would not be expected because of the much larger observed ratio of double to single pions. This effect may be due to the final-state interaction, however, as indicated by Kovacs[10] and others. The theory has also been applied to events involving K mesons and hyperons, and is in agreement with experiment for annihilation processes, but appears to predict too large a probability for associated production. However in all cases the number of events is too small for the apparent agreement to be very significant.

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