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Mis-generalization: an explanation of observed mal-rules

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1. Introduction

Intelligent Tutoring Systems (ITSs) force their implementors to be explicit about domain knowledge, tutoring rules, likely student misunderstandings for a particular domain, etc. Although this explicitness is demanding it does have the advantage that if the system behaves differently than expected, the implementor can determine the reasons for this, modify the suspected rule/knowledge and rerun the system.[1] Further once one has identified misunderstandings which one believes arise pretty consistently in a subject domain, by this or more conventional techniques, then a series of additional investigations are possible. These include:

1. hypothesizing the nature of the processes used by students to solve tasks given the incorrect/buggy/mal-rules.

2. building a remedial subsystem which exploits the inferred student model (this will involve further analysis of teacher-student remedial dialogues).

3. undertaking studies aimed at improving the initial instruction in the domain so as to avoid (some of) the observed difficulties.

In this article I discuss the first of these points in the context of extensive studies undertaken with 14- to 15-year-old algebra students. The Leeds Modelling System, LMS, was implemented and a database of examples, correct- and incorrect, or *mal-* rules had been established which was sufficient to diagnose the majority of difficulties encountered by 15-year-old students, Sleeman [1982]. The same database was then used with 24 14-year-old students and the outcome was very different. A high percentage of the student errors were *not* diagnosed by LMS. The investigator analysed these protocols in some detail and then carried out individual interviews to determine the nature of the students' difficulties, Sleeman [1983a]. The pertinent observations from this latter experiment are:

1. Students appear to regress under cognitive load. That is they are often able to use a particular rule correctly in the context of simple tasks, but make errors with this same rule when the tasks are more complex.[2] See Sleeman [1983a] for examples.

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[1] The approach used within the Expert Systems paradigm.

[2] This analysis *assumes* that domain rules are independent and one rule does not subsume another.

2. There appears to be a number of clearly identifiable *types* of error, (section 2).

3. Students use a number of alternative "methods" to solve tasks of the same type, (section 3).

2. Observed types of student errors

From the protocols and the interviews I concluded that in this domain errors could be classified as: manipulative, parsing, execution/clerical and random. The first two topics will be dealt with in some detail in the rest of this section; see Sleeman [1983b] for details of the others.

2.1 Manipulative Errors

I define a manipulative mal-rule to be a variant on a correct rule which has one substage either omitted or replaced by an inappropriate or incorrect operation, c.f., Young & O'Shea [1981]. For example, MNTORHS[3] is a mal-rule which captures the movement of a number to the other side of the equation, where the student omits to *change* the sign of the number. MXTOLHS is the mal-rule which corresponds to the analogous X-to-lhs rule. (Most of the errors noted with 15-year-old students were of this form.) Note that this schema would ALSO generate many mal-rules, which we have NOT yet observed; in the next paragraph we give an explanation why some of the possible mal-rules are not observed.

<u>a) Analysis of some manipulative mal-rules: A schema for generating manipulative mal-rules</u>
In a recent experiment we noted three (additional) mal-rules which can be explained by this mechanism. Two of them will be analysed in some detail:

1. <u>A variant on SOLVE</u>. The variant on SOLVE transformed:

   $$4 * X = 6 \text{ to } X = 6$$

   whereas SOLVE would change the same expression to X = 6/4. It is suggested that the student realizes he has a task in which the SOLVE rule should be activated and forgets to apply one of the operations, namely dividing by M. SOLVE has three principal actions: noting down N, the divide symbol and M, and so this mal-rule could be said to be omitting some of the principal steps. Furthermore, it appears that students have an idea about the acceptable FORM of answers and so given the above task we have *not* seen X = 6/ or X = /4.

2. <u>A variant on SIMPLIFY</u>. Examples of the two mal-rules noted here, which have occurred reasonably frequently are:

   $$X = 6/4 \Rightarrow X = 3/4$$

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[3] MNTORHS is short for *mal-number-to-rhs* rule.

$$X = 6/4 \Rightarrow X = 6/2$$

(The SIMPLIFY rule transforms the same expression to $X = 3/2$).

Again we argue that the above observations can be explained if we assume that this rule has several principal steps including, calculate the common factor, divide "top" by common factor, divide bottom by common factor, write down the components, and that each of these mal-rules corresponds to one step being omitted.

b) "Grain size" and manipulative mal-rules.

There is a sense in which detailed analyses of manipulative mal-rules allows one to infer the substep processed by students, and this in turn allows one to predict the set of mal-rules that will be encountered in a domain. (Bearing in mind the idea of acceptable form outlined above). Further, one might argue that the representation of the tasks should be at this "lower" level; the justification for the representation chosen, is that this appears to be more consistent with the collected verbal and written protocols for students solving these tasks. The schema discussed above for generating manipulative mal-rules by omitting, or modifying, one substep is thus consistent with Young and O'Shea's modelling of subtraction.

2.2 Incorrect Representation of the Task or Parse Errors

I assert that many of the students whom we interviewed carried out steps of the computations in ways which would not fall within the definition given earlier for manipulative mal-rules. Below, I give typical protocols for two students working the task 6*X = 3*X + 12:

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I:      6*X = 3*X + 12              II:     6*X = 3*X + 12
        9*X = 12                            X + X = 12 + 3 - 6
        X = 12/9                            2*X = 9
        X = 4/3                             X = 9/2
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When I pressed the "first" student for an explanation of how the original equation was transformed into the second, i.e., 9*X = 12, the student talked about moving the 3*X term across to the left hand side. Thus the interviewer concluded that this was an instance of a student using a variant of the correct rule, namely a manipulative mal-rule. When the "second" student was pressed he simply asserted that the change from the original equation to the second line "was all done in one step". Hence the interviewer concluded it was a very different type of mal-rule involved and not a simple variant on the correct rule. Thus the interviews provided essential additional information as, of course, the second student's protocol could be explained by the use of MXTOLHS and the mal-rule:

M * X => M + X

[4]which some people might wish to argue constitutes a manipulative mal-rule (replacing the * operator by the + operator). Even if we did not have the additional experimental evidence, this investigator would maintain that such a transformation belays a profound misunderstanding of algebraic notation and so should be considered as a parsing mal-rule. See Sleeman [1983b] for additional discussion of this issue.

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[4] Where M stands for an integer, and where in the above example 6*X => 6 + X and 3*X => 3 + X.

3. Bug Migration or Using Alternative Methods

Repair theory gives a neat explanation for the observed phenomena of bug migration in the domain of multi-column arithmetic, Brown & VanLehn [1980], namely that the student will use a related family of mal-rules, and possibly the correct rule, during a single session with one particular task set.

There seems to be an alternative explanation which should also be considered. Although a task-set may have been designed to highlight one particular feature, the student may spot completely different feature(s) and these may dominate his solution.[5] Repair theory accounts for some bugs by hypothesizing that the student had not encountered the appropriate teaching necessary to perform the task. Suppose we make the converse assumption, that the appropriate teaching had been carried out, and further suppose that *some* students[6] do not gain competence in this domain by being told the rules but rather by inferring rules for themselves by noting the transformations which are applied to tasks by the teacher and in texts.[7] It seems reasonable that the student's inference procedure should be guided by his previous knowledge of the domain, in this case the number system, and that the student will normally infer several rules which are consistent with the example, and not just the "correct" rule. Indeed due to some missing knowledge the "correct" rule may not be inferred. (And so the fact that the student never uses the "correct" method along with several "buggy" methods is not evidence that he has NOT encountered the material before). We shall refer to this process as Knowledge Directed Inference of Multiple rules, or mis-generalization for short.

Suppose, the student saw the following stages in an algebraic simplification:

$$3 * X = 6 \quad => \quad X = 6/3$$

Then he might infer

X = RHS number/LHS number OR X = LARGER number/SMALLER number

We will surmise how a student would use such a rule-set. We will suppose that the abler students actively experiment with different "methods", and use their own earlier examples, examples worked by the teacher and in the text to provide discriminatory feedback. From our experiment with 14-year-old students we have direct evidence that some students are aware of having a range of applicable rules and being unsure of when to select a particular method, Sleeman [1983a]. That study did not provided any insights into the rule-selection processes used by these students. We could suggest the common default, i.e., that the process is random. However, studies in cognitive modelling have

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[5] Earlier Sleeman and Brown [1982] have argued: "......Perhaps more immediately, it suggests that a Coach must pay attention to the sequence of worked examples, and encountered task states, from which the student is apt to abstract (invent) functional invariances. This suggests that no matter how carefully an instructional designer plans a sequence of examples, he can never know all the intermediate steps and abstracted structures that a student will generate while solving an exercise. Indeed, the student may well produce illegal steps in his solution and from these invent illegal (algebraic) "principles". Implementing a system with this level of sophistication still presents a major challenge to the ITS/Cognitive Science community... "

[6] Note I am *not* claiming that there is a *single* mechanism.

[7] Independently, VanLehn has come to a similar conclusion, the Sierra system described in his thesis relies heavily on inference, VanLehn [1983].

already discredited this explanation many times, so we will postulate that the process is deterministic but currently "undetermined". It is further suggested that tasks which show a rule is inadequate will weaken belief in the rule, but once a (mal) rule is created it may not be completely eliminated - particularly if the "counter-examples" are not presented to the student for some period. Thus given this view point, the phenomena of bug-migration occurs because the (less able) student has inferred a whole range of rules and selects a rule using a "black-box" process.. Given a further task, he again chooses a method and hence selects the same or an alternative algorithm, influenced partly by the relative strengths of the rules. That is if the relative weights are comparable, it is more likely that the student will select a different method for each task. If one weight "dominates" then it is likely that the corresponding method will be selected frequently. Further, if only one (mal) rule is generated by the induction process then this approach predicts that the student will consistently use that rule.

We suggest that many of the bugs encountered in the subtraction domain can be accounted for by this (inference) mechanism. For instance the Smaller-from-Larger bug, where the smaller number is subtracted from the larger independent of whether the larger number is on top or the bottom row, seems one such example, Brown & Burton [1978] and Young & O'Shea [1981]. Brown & VanLehn [1980] report that because borrowing was introduced, with one group of students, using only tasks with 2 columns, these students inferred that whenever borrowing was involved they should borrow from the left-most column, their "Always-Borrow-Left" bug. So it appears important to ensure that the example set includes some examples to counter previously experienced mal-rules. Indeed it seems as if task-sets can be damaging if they are too preprocessed and contain too little "intellectual ruffage"; Michener [1978] puts a similar argument. Additionally, Ginsburg [1977], quotes several instances of young children inferring the name "three-ty" for 30, given the names for "3", "4", "5", "40", "50", "60". So given the wealth of experimental evidence this alternative explanation should be given serious consideration.

Further, I have two philosophical reservations about repair theory. Firstly, that by some mechanism not articulated all students acquire a common set of impasses, and moreover they consistently observe these. Secondly, repair theory which sets out to explain *major* individual differences at the task level, itself proposes a specific mechanism *common* to all students.[8] On the other hand, mis-generalization predicts that the individual's initial knowledge profoundly influences the knowledge which is subsequently inferred, and captures the sense in which learners are active theory builders trying to find patterns, making sense out of observations, forming hypotheses, and testing them out.

4. Summary

Firstly, there are two hypotheses which explain bug-migration the one given by repair theory and the one put forward here, namely mis-generalization. Of course it is possible that each may be applicable in different situations. Secondly, several "algorithms" have been presented for creating student models. I believe these are suggestive about the processes used when a student solves (these) tasks. Repair theory suggests that it can be explained by making "repairs" to incomplete core-procedures, whereas Young and O'Shea suggest that it is adequate to take a correct procedure and merely delete components. The data for the algebra manipulative mal-rules can be adequately explained by either. However, Young and O'Shea's approach seems inadequate to explain the

[8] Indeed I am concerned that many theories of (child) development do *not* accept the possibility of there being significant individual differences in development, but merely in the individual's *rate* of progress and the level of his final maturation.

parsing mal-rules. Indeed, we have to extend revised repair theory before the results reported here can be accommodated. This paper claims that there are two very different types of malrules at large with algebra students - namely manipulative and parsing mal-rules. And that this second category of algebra errors, and much of the data collected in other areas, appears to be best explained by a further mechanism, namely mis-generalization. However, once *inferred* I believe rules are additionally *applied* incorrectly, and that the mechanism(s) described in Young & O'Shea, repair theory and section 2.1, are appropriate for this stage.

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